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Stochastic model of cavitation erosion of low-plasticity metallic materials

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Abstract

Stochastic model of cavitation erosion of low plasticity solids was presented. It is a kinetic type model based on the energy conservation law. Mathematical formulation comprised equations for the energy accumulation and release rates. Micro-cracks were assumed to be the main negative source of energy. In calculations carried out, the random element was omitted and deterministic dependences of energy absorbed and relaxed in the process were found. Conformability of the theoretical and experimental curves was taken in favour of the model. The material fatigue effect was expressed by the delay of micro-cracks appearance with respect to the force action time. The model sets a calculation direction of the temporal development of cavitation erosion. An application to practical cases requires the use of proper loading functions, as well as probabilistic distributions of the absorption and relaxation of energy, being the function of material parameters of the destroyed solid.

Keywords: Cavitation erosion; Random wear process; Stochastic modelling; Non-ductile metal alloys

1 Introduction

Cavitation is an undesirable drawback of operation of hydraulic devices and machines. Loadings originated in cavitation cloud lead to erosion of the solid state surface subjected to its activity. The most important results are noise, decrease of the machine efficiency and its destruction due to fatigue cracking process.

Materials destruction under the cavitation conditions is in fact a stochastic process, depending both on characterisation of the loading and properties of the

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eroded body – properties constituted by its chemical composition, microstructure and physical state. Due to the lack of appropriate model descriptions of the process, thus far such problems were not completely solved as anticipation of the erosion performance (1), setting up a relationship between material parameters and its cavitation resistance in uniform functional formulation (2) or distinguishing the features decisive for constituting the cavitation resistance at given stage and conditions of the process (3).

There are known some phenomenological models of cavitation erosion, all of them based on integral or differential equations with parameters of physical meaning [1-9]. Due to extreme simplifications, their applicability to interpretation of the phenomena is very limited. A large amount of the contemporary works deal with cavitation impingements modeling in a way that the results of their action on the material surface are included (e.g. [10-16]), but, unfortunately not making approach to the effective solution of the listed problems.

Herein, a formulation of cavitation erosion model regarding randomness and multistageness of the process was presented.

2 Characterisation of the process

Surface erosion of the materials placed within the vanishing cavitation cloud zone is caused – according to the standard description [17] – by the forces generated due to single or collective bubbles implosions [18, 19]. Bubbles arising is bound to the decrease in the liquid pressure down to any specific value, approximately equal to the pressure of evaporation. Cyclic or eternal vanishing of the bubbles is a result of violent increase in the pressure [20, 21]. Duration of the phenomena was found to be $10^{-4} \div 10^{-5}$ s [22]. Shock wave pressure generated in the vicinity of the imploding bubble depends on the hydrodynamic and thermodynamic conditions and is probable to reach value of 4 GPa [23]. Variations of the pressure suffered by the solid under the cavitation action are due to differentiation of the distances between the body surface and the impulse source, and due to differentiation of the bubble vanishing topology [24]. E.g. as a result of implosion, a directional liquid micro-jet could be formed [25]. However, according to the authors of the work [26], prevailing role in surface damage process play the shock wave pressure. Spectra of the loading pressures is very broad [27] and can be approximate by exponential-power distribution [28]. Two types of erosive traces could be detected: well-ordered sequence of the indentations caused by a cyclic impingements of the bubble and randomly distributed indentations being a result of collective interactions from over the cavitation cloud [29]. Loadings caused by periodically changing cloud were found to be the most aggressive ones [30]. An appearance of the cavitation phenomena depends on the velocity, pressure and properties of the liquid (as saturated vapor pressure, tensile strength, viscosity or temperature – useful scaling coefficient is so called cavitation number) and, the last but not least, the geometry of the flow imposing appropriate boundary conditions and deciding on such effects as turbulence. Theoretical expectation of cavitation performance is a problem off the unstable, multiphase flow of viscous and compressive liquid category problems [31]. Facing great theoretical difficulties, one can assess the intensiveness of cavitation referring to the experimentally detected loading distributions. Intensity of the material damage due to cavitation impingements is also a function of solid surface state [32]. Local depressions change the stress distribution within the material and could play a role of cavitation inducers. At the beginning of the process, dependence of the hydrodynamic conditions on the wall state is usually negligible and variables could be treated independently. In the face of the randomness of the impulses [33], the erosion of the material, having protected the statistical distribution of the loading, is the stochastic process.

There could be specified some types of cavitation with respect to the conditions of its generation: flowing cavitation in tunnels, vibratory cavitation and the cavitation induced in impinging jets [34]. Those types of cavitation are met both in the operating machines conditions as well as in the laboratory conditions. Some of them were standardized [35, 36].

Density of the easily discernible indentations is being used as a measure of material damage at the inception of the erosion. In metals, the areas of the produced holes and depressions ranges between several square millimeters up to some thousands of square millimeters. Indentations are formed due to local, plastic deformation of microstructure in the surface layer. In case of brittle materials, the process is accompanied by cracking and, subsequently, extracting of the material pieces. Indentations sizes and their multiplication rate are the base of the erosion scalling [37, 38]. A kinetics of the material destruction under the cavitation action is expressed by cumulative erosion curve, i.e. the total volume loss in time. An analysis of the curve let to set the specific phases of the erosion: energy accumulation phase followed by weight loss acceleration and stabilization. Depending on the properties and microstructure of material, a constant or cyclic rate of damage is established [36]. An initial stage of erosion is called the incubation period, the period ranging from the beginning until the time point corresponding to a cut off by the tangent to the curve drawn at the maximum erosion rate. Relative resistance of one material with respect to the other is assessed by comparison of the single-number indicators inferred from the cumulative curves.

Cavitation erosion should be considered as a separate, governed by its own rules type of material damage. It is due to peculiarities of cavitation loadings:

short time of the impulse action (comparable to the time of viscous volume forces of the fluid [39]), very small (infinitesimal) area of the interaction and randomness of the impingements, allowing large amount of different ways of destruction. An observation of the stress fields and ways of the material degradation process reveals the lack of any similarities of that kind of erosion to the other known ways of solid damage [40]. Not taking into account the case of very brittle materials of small impact strength, the cavitation erosion is a fatigue process, as it was proved by Richman [41, 42].

Cavitation action in the incubation period of the erosion is approximately 2-dimensional problem with resultant direction perpendicular to the eroded surface. That time in the scattering of the impulse energy into the fluid medium and (elastically) into the solid body forms a substantial share in the energy loss. In the advanced period of the erosion volume and surface waves generation and energy dissipation in the working liquid are strongly influenced by the complicated morphology of the material surface. An item under loading may preserve a quasi-equilibrium state as a whole [43]. At that stage of damage the local stress fields of 3 axial tensile or compressive nature (depending on the externally forced interrelated translocations of atoms) are generated in a 3-dimensional space. In real materials not complete relaxation of the temporary stresses is observed, which lead to progressive accumulation of the residual stresses [44, 45]. Durable effects of the cavitation forces action in the incubation period involve translocations within crystal net, deformations in microscale and nucleations of the microcracks in the vicinity of any discontinuities, i.e. in places of the maximum Hertz stress.

In iron alloys such deformations of the single grain as twins or slip lines are the most common [46, 47]. After a long time of plastic deformation, the process is accompanied by a brittle damage process, due to advanced disintegration of the structure. Local cleavages appear as a final step of the fatigue process. Generation of the critical cracks is a result of microcracks joining process. Brittle materials can develop the critical cracks yet from the surface. The water pushed-in the open crevice resists its closure and the next impulses forces the crack propagation [46]. It was found that cavitation resistance of homogenous metals depends mainly on their microstructure, bound energy and the ability to deformations and phase transformations [48]. Higher cavitation resistance of the austenitic alloys with respect to the ferritic ones, as commonly met, is explained by the easier destruction of A2 structure than A1 structure at high rate of deformation [49]. Among the A2 alloys of high cavitation resistance, a martensite should be pointed. Its high resistance is achieved due to high hardness and grain refining, which make the incubation period of the erosion prolonged. Further performance is not so distinctly well for its destruction occurs due to brittle cracking [46]. In case of homogenous A1 materials, the surface deformation proceed until the pieces of the solid can be extracted [50]. A high cavitation resistance of the cobalt alloys (at content of over 30% wt.) is to some extent the result of A1 → A3 transformation, induced by mechanical loading [48, 51]. Fundamental mechanisms of energy absorption and dissipation in metastable alloys (as steel 18Cr-8Ni) is cyclic generation and vanishing of the martensite under the cavitation loading, thus contributing to the erosion resistance increase [52, 53]. Likewise, a special power of cyclic transformations of nickel-titanium alloy (TiNi) makes it very resistant to low-frequency fatigue, and soon after it – enormous cavitation resistant [54]. Kinetics of defects evolution as well as the probability of cracks appearance in each separate phase of alloys of multiphase microstructure is not the same [55]. The selective destruction of the heterogeneous materials makes the removal of "weaker" phase at a higher rate. E.g. in carbon steel the α-ferrite is removed before the pearlite [56], similarly as in ferritic-austenitic steels, where the loss of ferrite grains precedes the loss of the austenite ones [57]. The reason for the erosion initiation and development at the grain boundaries are differences in adjoining grains properties [49]. The dependence of cavitation resistance of the material on the presence of hard particles (e.g. carbides) scattered throughout the matrix as contaminants depends on their sizes. At that time, an increase in hardness is accompanied with the decrease in bending strength and microcracks multiplication between the phases. That's why a decrease in cavitation resistance of the material with large-size second phase particles is observed [58] and an increase in cavitation resistance of metals with homogeneously scattered fine particles of hard phases [59].

Yet the simple analysis of the problem allows to conclude that cavitation resistance is not closely related to any single parameter defined in the course of the strength tests, because of the different scales of deformation, rates of deformation and the object of damage, e.i. the surface layer of the material, where the whole history of the deformation is imprinted. E.g. the standardized tests are one-dimensional, whereas the cavitation deformations should be regarded as three-dimensional. Therefore, in the latter case, there arise deformation mechanisms of type absent at the strength tests.

3 Experimental ground for cavitation erosion description

A temporal mass loss of the body is the most important and directly detectable measure of the intensity of cavitation erosion process. Therefore, the correctness of model assumptions is proved by the conformability of the theoretical and experimental cumulative mass loss curves. Obviously, quantitative conformability can be attained only if the proper values of the parameters regarded in the model are chosen, as well as the proper initial and boundary conditions are taken into account. On the other hand, the experimental curves are not each time repeatable, which is the result of the process randomness (there is correlation between the casual place of pit creation and the way of erosion development in successive periods). Thus, one can presume differing fluctuations of the average values of distributions taken for various samples.

Cavitation erosion happens in space confined by the surfaces where the forces are applied. It is the boundary of the model considerations for the free boundary value problem. Due to randomness of the angle between force and surface normal vectors in the advanced stage of the erosion, a space field of stress generated is also random. Nevertheless, an average direction of the force and stress gradient vectors is perpendicular to the surface curvature. It makes possible to use one-dimensional eikonal equations (Kardar, Parisi, Zhang type) for the description of the deformed surface evolution.

According to the results of investigations, elastic materials are featured by longer erosion incubation time than inelastic ones, what is linked to different modes of those materials destruction process. Dumping of the impulse occurs within thinner layer, when the material structure is elastic. Subsequent cycles of loading lead to disintegration of microstructure and an extraction of the material pieces. Elevated brittleness contribute significantly to make the elastic material incubation time shorter. In case of brittle materials, an increase in microcracks density is driven by cavitation loading until their critical level is achieved and an extraction of the body piece occurs. Model conditioning crack generation and fracture on concentration of weak spots in the alloy seems to be the most appropriate for such kind of the materials. Especially, hard precipitations with high level of compressive stress at their boundaries serves as an example of such centers.

Cavitation generates the broad spectrum of the loading allowing for various regimes of fatigue fracturing, regimes linked to various rates of the crack length increase da/dN, the latter depends on the conducting force diversity range. Exact dependences could be found in [60]. Hence, in fact, two rates of two imposed erosion processes could be distinguished during cavitation erosion: (1) continuous loss of small pieces of the material at their specific rate and (2) extraction of big pieces at another rate. In former case, the crack development depends mainly on average level of stress and the time of its relaxation, being the function of loading frequency. An application of the continuum approximation is then allowed. In case (2), cracking begins at the material surface and its development is strongly influenced by both microstructure and stress level.

Material properties of the surface layer subjected to cavitation loading change, thus detailed investigations of stress-strain relationship could be regarded as one of the convenient ways of cavitation erosion modelling.

Intensity of cracking under the mechanical loading is linked to the material heterogeneity. An increase in microcracks density could be regarded as the process of new phase creation, which leads to destabilization of the whole body. Following a Sornette idea ([61, 62]) it could be assumed that fracture is a climax of microcracks interaction and joining process, crowned with fractal nets generation.

4 Analysis of the process and physical model foundations

Crack appearance under the external loading is proceeded by generation the critical conditions for the erosion, a process of probabilistic nature. Statistical base for thermodynamic criteria of stability is an Einstein's formula for probability of fluctuations around the steady state: $p = p_0 \exp(\delta^2 S/2k)$, where $\delta^2 S$ is a variance of entropy and k – Boltzmann's constant. Structural destruction is a result of overloading or fatigue state of the body. Determination of the material durability is a practical aim, which can be realized by derivation either a distribution of the critical loading or appropriate curve of damage accumulation rate. In case of cavitation erosion, a Griffith's idea can be used. According to his concept, cracking occurs rather after energetic conditions are complied and not due to high level of stress is reached [63]. This reasoning supports the suggestion that cracking should be regarded as a process of the external sources energy transfer into surface energy of crevices [64].

General stages of energy transfer up till the result of permanent change of the material state is achieved involve random excitation and generation of the places of condensed energy, it subsequent spreading and dissipatively annihilating or trapping within heterogeneous structures. From thermodynamic point of view a persistent change of the material state due to the loading forces depends on the probability if the potential barrier is overcome.

Having assumed that in the course of fatigue process the potential barrier lowers down to the cracking level, one can regard the process as probabilistically determined. Due to the nature of the phenomena up to the critical state is achieved, the material degradation is modeled by chosen stochastic process in the continuous time domain [65, 66]. One can also presume an idea on the probability of overcoming an unchangeable potential barrier, but in that case the probability should be depended on time, e.g. the fatigue effect should be expressed implicitly by introducing the interaction time. The efficiency of the energy trapping is influenced by density of the energy acceptors ("weak spots"). The process of energy accumulation is conducted until culmination (e.g. cracking) and qualitative change within the material. Weak spots density is functionally linked to the dynamic interaction of impingements and the material.

It should be kept in mind that heterogeneous body (e.g. with microcracks included) is hardly probable to be described properly by differential equations. The range of the forces action in elastic materials is non-local. The effect is an answer to resultant stress field, built up within the confined volume of the body. Its size is presumably high enough that specific statistics of non-homogeneities could be set up. It is concluded that an increase in micro-cracks density depends on deformation and is the way of energy release from any element of the material. Volume of the element is a function of microcracks sizes and mutual distances [67]. Specification of the space of considerations makes the simulations of the statistical distributions possible. Such attitude is a base for non-local probabilistic models of the erosion. An integral formulation eliminate many paradoxes and difficulties fixed to differential formulation. It gives a chance for connecting the energy delivered to the body in the succeeding cycles of the loadings and the critical level of the material tensile strength, remembering about the free energy potential variations in each cycle and resulting state equations.

Some ways of defining the thermodynamic potential for a volume erosion involved were proposed [68-71]. However, defining of a suitable type of the potential is still an unresolved problem.

For the modelling purpose it is important to determine if the material performance is brittle or ductile. When plastic deformations are dominant, the kinetics of deformation in heterogeneous environment is described by distribution functions for deformation velocity or by their statistical moments. Defining a non-equilibrium process and finding a dependence of the particles velocities for that process are of paramount importance in case of plastic deformation. The base phases at a macro-scale are then non-equilibrium initial phase and quasi-equilibrium phase of advanced erosion [72].

In case of both brittle and elasto-plastic materials, parallel to the process of the surface layer erosion, the process of generation and association of micro-cracks within deeper layer occurs. It leads to extraction of the big material pieces in longer periods of time, presumably in accordance with statistics of strong cavitation loadings. Spallation, a process dominant at the surface is induced by interaction of falling and reflected waves [73].

A process of nucleation and joining of micro-cracks inside the deeper layer of the eroded material is a stochastic Markov process [74]. At the small density of the cracks or defects, a description of their evolution in a phase space could be done with the aid of the Poisson's process [75]. Micro-cracks nucleation could be treated as singularities in the phase space. An assumption on leading role of the micro-cracks in the cavitation erosion process is linked to very high rate of the loading force pressing (according to [76], a higher rate of force action, the higher probability of the increase in micro-crack density) (1), and fatigue nature of the process (2).

Completing the reasoning, one should mention that decisive for cavitation erosion are not only loadings and internal structure of the body (and hence resultant material properties as a strength), but also state of the surface: surface energy and geometry of the solid boundary. However, an influence of the surface state on the erosion process is minor. In the cumulative phenomenological models the space variables are not present, therefore the boundary problem does not exists explicitly. Results of the model calculations are valuable if there is assumed a priori a proper distribution of damage probability. At approach involving the physical processes, a dimensional or non-dimensional space description is required and the boundary conditions are to be included. That time, the free boundary problem is met, in which the edge characterisation influences the course of the process.

5 Assumptions and simplifications

A model of the cavitation erosion of heterogeneous solids is based on the empirical observation that material fracture is proceeded by generation of a set of micro-cracks, indispensable for creation an appropriate number of the critical cracks. Therefore, the process is assumed to consist of such main stages of the erosion as energy accumulation, its relaxation due to micro-cracks appearance and next the generation some macro-cracks with certain probability. Such assumption can refer to non-ductile materials. A developed model is of kinetic nature.

Detailed assumptions are following:

1. The mass loss at the steady process is proportional to that fraction of the absorbed energy, which was not released in the course of micro-cracks generation; i.e. to the amount of energy accumulated in the extracted blocks and used for critical cracks generation in unit time. Mass loss is not dependent on the spatial distribution of the cracks.

2. The initial distribution of micro-cracks density in the layer is known.
3. Constant coefficients of the temporal density distribution of the cracks generation probability are attributed to given material parameters of the body. The shape of the distribution is assumed a'priori.
4. An average value of the loading is introduced as a parameter of the probability distribution and it is assumed constant.
5. The greater the number of micro-cracks inside the layer, the smaller the rate of their increase.
6. Generation of micro-cracks is a result of energy accumulation.
7. Generation of the critical cracks depends on the number of micro-cracks inside the layer. The probability distribution of the critical cracks generation is set for the given level of the micro-cracks density with accuracy governed by the random element.
8. The fluctuations of the parameters in the probabilistic field of random numbers are admissible.
9. Statistically, an increase in micro-cracks generation rate at the point is connected to the increase in micro-cracks generation in the surrounding space having displayed the same temporal function as delivered energy.

An important simplification made to facility the solving procedure was neglecting the dependence of the density distribution of the micro-cracks generation probability on the amount of absorbed energy. Generally, that function depends on energy accumulated in the surface layer. In order to carry out the calculations by using statistical distributions given a'priori and with no random numbers, the following additional simplifications have been introduced: (1) time distribution of the micro-cracks generation rate is invariable and (2) the rate of the energy absorption does not depend on the micro-cracks concentration within the loaded layer.

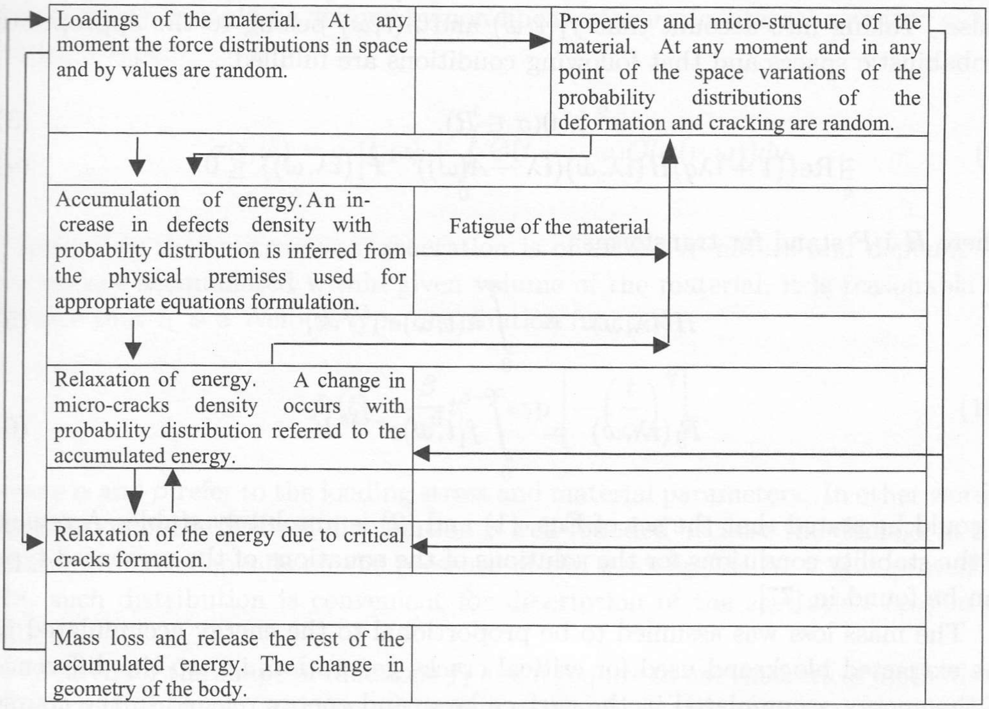
A scheme of causality of the cavitation erosion process is presented in the Tab. 1.

6 Equations for the erosion

Under the assumptions presented above, the equations for energy accumulation rate and for production of micro-cracks in the unit volume of the surface layer in the unit of time take the form:

$$\frac{dx(t,\omega)}{dt} = A(\omega)x(t,\omega) + \int_0^t \lambda_1 f_1\Big([\alpha_i], t-\tau, \omega\Big)\, g\Big([\alpha_i], \tau, \omega\Big)\, d\tau\,, \qquad (1)$$

Table 1. Scheme of causality of the cavitation erosion proces.



$$\sigma(t,\omega) \;=\; g\Big([\alpha_i],t,\omega\Big) + \int_0^t \lambda_2\, h(t-\tau,\omega),\; x(\tau,\omega)d\tau \qquad (2)$$

where:

- x – amount of energy relaxed at micro-cracks manufacturing in the unit volume of the surface layer;
- σ – amount of energy absorbed in the unit volume of the surface layer in the unit time;
- f_1 – density of the temporal probability distribution of micro-cracks generation in the unit of the loaded layer;
- α_i – material parameters and the loadings, given as parameters of the distribution functions g, h, f_1;
- g – a function for the rate of energy accumulation in the material due to plastic deformation (it depends on the material properties and the average loading);
- h – a function for the loss of the energy store in the layer due to critical cracks generation (micro-cracks joining);
- A, λ_1, λ_2 – coefficients of proportionality, $A < 0$;
- ω – element of randomness.

The function g could also be made dependent on the element of randomness, linked to the microstructure, but additional complexity of the equations it would cause. Taking into account that $f_1(t,\omega)$ and $h(t,\omega)$ belong to the appropriate probabilistic spaces and that following conditions are fulfilled:

$$\sigma^2 > 0 (\sigma \in R), \tag{3}$$

$$\underset{q}{\exists} \operatorname{Re}\{(1 + i\lambda q) H(i\lambda,\omega)(i\lambda - A(\omega))^{-1} F_1(i\lambda,\omega)\} \leq 0 \tag{4}$$

where H i F stand for transforms:

$$H(i\lambda,\omega) = \int_0^\infty h(t,\omega) e^{-i\lambda t} dt, \tag{5}$$

$$F_1(i\lambda,\omega) = \int_0^\infty f_1(t,\omega) e^{-i\lambda t} dt \tag{6}$$

it could be stated that the set of Eqs. (1) and (2) is absolutely stable. A review of the stability conditions for the solutions of the equations of the presented type can be found in [77].

The mass loss was assumed to be proportional to the energy accumulated in the extracted blocksand used for critical cracks generation, i.e. to the difference of the energy accumulated in the surface layer and energy relaxed in the course of the micro-cracks and cracks generation. It could be calculated according to the following formula:

$$\sigma_1(t) = \sigma(t) - \frac{dx(t)}{dt}. \tag{7}$$

The meaning of the constituents of Eqs. (1) and (2) is following:

(a) the first constituent on the right side of Eq. (1) refers to the dumping of the micro-cracks generation due to development of existing cracks;

(b) the second constituent means the micro-cracks generation rate at the given level of energy accumulation rate g;

(c) the second component on the right side of Eq. (2) stands for the rate of energy relaxation due to critical cracks inception and/or energy relaxation rate in the process of micro-cracks disappearance due to their joining and absorption by the critical cracks.

The solution to the set of Eqs. (1, 2) may be given in the following form:

$$x(t,\omega) = e^{A(\omega)t} x_0(\omega) + \int_0^t e^{A(\omega)(t-s)} \int_0^s f_1\Big([\alpha_i], s-\tau, \omega\Big) g\Big([\alpha_1], \tau, \omega\Big) d\tau ds. \tag{8}$$

After (8) is substituted into (2) and after some transformations are done (a change of the integrate sequence and use of the commutative properties of the convolution), σ could be defined by non-linear, stochastic integral equation of the Volterra type:

$$\sigma(t,\omega) = g_1(t,\omega) + \int_0^t \Theta(t-\nu,\omega)G(\sigma(\nu,\omega))d\nu. \tag{9}$$

Considering that micro-crack generation is of extensive nature and depends on the energy accumulated within given volume of the material, it is reasonable to assume that f_1 is a Weibull type distribution function:

$$f_1(t) = \frac{\beta}{\alpha^\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right] \tag{10}$$

where α and β refer to the loading stress and material parameters. In other words, application of the Weibull distribution is well-founded because the damage in the fatigue process depends on the presence of the weak spots. As it was proved in [78] such distribution is convenient for description of the cavitation erosion of solids.

Search for the shape of functions f_1 and h require to use theoretical methods of fracture mechanics. On basis of the free assumptions that temporal distribution of the loading is Gaussian and the time variations of the strength parameters are exponential, the function $g([\alpha_i], t, \omega)$ could be presented as an integral of the distribution K[1]:

$$g(t) = \int_0^t \frac{2}{b\Gamma(a)} \left(\frac{\tau}{2b}\right)^a K_{a-1}\left(\frac{\tau}{b}\right) d\tau\,. \tag{11}$$

The cumulative curve of energy used in material extraction process is conformable to the curve obtained by integration of the difference of energy rates delivering and relaxing – for micro-cracks generation (an integral of σ_1). An illustration of the concept is schematically presented in Figs. 1, 2. The results of calculations made under severe simplifications are presented in Fig. 3.

Optional values of the model parameters were assumed. Curves (1) in Fig. 3 present the difference in the rates of the energy absorption and relaxation due to micro-cracks generation in a given volume of the material surface layer. In

[1] K is a distribution of the complex stochastic process $K(x,z) = p(x,y) \cdot q(y,z)$, where distribution of the process X is $p(x,y)$ and the distribution of the process Y is $q(y,z)$ [79]. In our case p is an exponential function and the distribution of q is Γ.

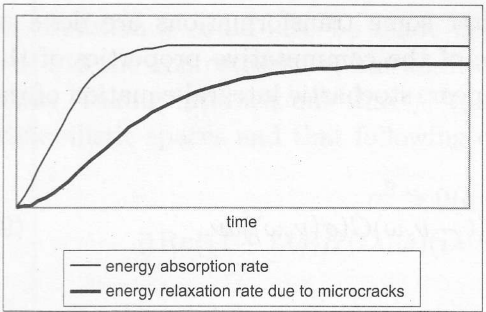


Figure 1. Conceptual curves of energy apsorption rate and energy relaxation rate.

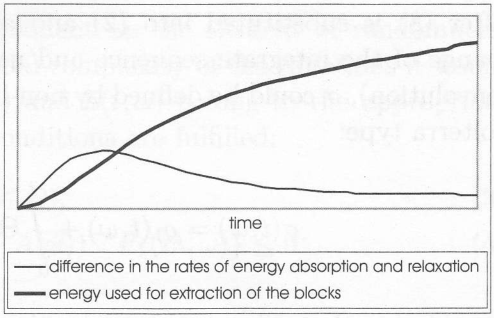


Figure 2. Conceptual curves of the difference in the rates of energy apsorption and relaxation and energy used for extraction of the blocks.

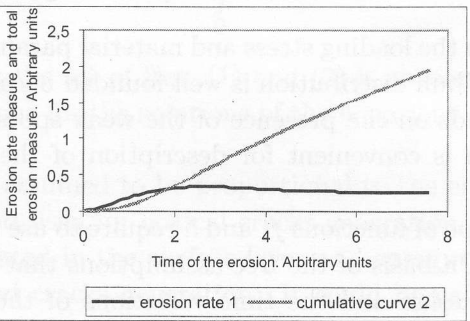


Figure 3. Exemplary curves: cumulative (2) and erosion rate (1) of the material for optimal values of the model parameters.

the case of Fig. 3 calculations were carried out for simplified case, where energy delivering rate is an integral of the distribution function: $(z/2)K_{0.001}(z)$. Curve (2) is an integral of the curve (1).

7 Discussion and verification of the model

The model presented is of kinetic type. Space coordinates are not present in its formulation. From the physical point of view it is an energetic model. The ways of the energy dissipation during the process of the erosion are expressed in the mathematical formulation of the model. It should be noted that energy is the only parameter preserving its meaning at various scales. The measure of the erosion is the sum of the extracted blocks energy and energy used for critical cracaks generation.

There is an unvarying statistical relationship between material loss and the energy released due to generation of the critical cracks. The latter could be defined explicitly from the model.

An extension of the erosion description by the dynamic equations with free boundary conditions would enable to find the performance of the process not only in time but also in space. However, such a set of equations may be not solvable. In order to regard the spatial coordinate (e.g. depth), the fundamental for stochastic processes Fokker-Planck equation could be used. That is an equation for the description of particle movement in non-local conditions. In general, Eqs. (1, 2) give presentation of the processes, which depend on the element of randomness and constitute the statistically deterministic system. Within the system, the probability distribution for obtaining $\sigma(t)$ and $x(t)$ values could be derived unambiguously.

At exemplary calculations of cumulative curves (Fig. 3), the element of randomness has been omitted, thus deterministic distributions of $\sigma(t)$ and $x(t)$ were obtained. In presented model, the forms of probability distributions were proposed, especially, the Weibull distribution for f_1. Application of other distributions is also possible. Correctness of the assumptions should be verified by deriving the cumulative curves for real materials and conditions and by their comparison to the experimental results.

Some typical experimental curves obtained in support of International Cavitation Erosion Test program are shown (Figs. 5, 6). Their similarity to the calculated curves should be stressed.

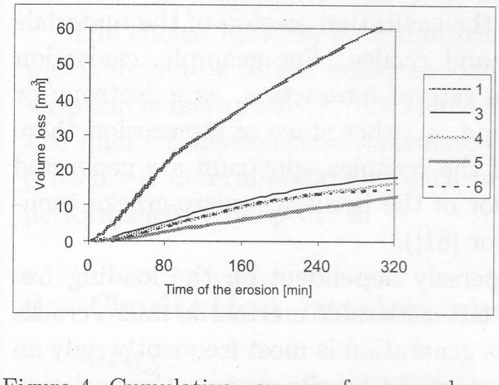


Figure 4. Cumulative curves for samples made of the brass M63. Testing was completed at various vibrating rigs: (1) CISE, Milan; (3) University of Hiroshima; (4) IFFM Gdańsk; (5) University of Cape Town; (6) University of Hull [80].

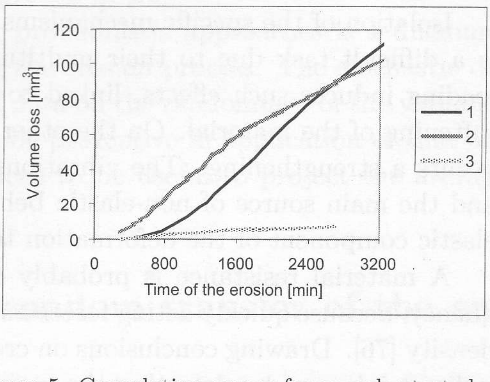


Figure 5. Cumulative curves for samples tested under the International Cavitation Erosion Test: (1) steel 45, cavitational tunnel in VK-AG Peitz; (2) steel 0H18N9T, cavitational tunnel in VK-AG Peitz; (3) Armco iron, cavitating jet, Univ. of Hannower [80].

The sentence that shape of theoretical curve preserves all specific features of typical experimental curve is justified. Among others, achieved conformability is indicative of the correctness of the description of critical spots generation within the fractured material, which is linked to the process of energy migration. According to that reasoning, an increase in weak spots in time ensure their critical level is achieved and fracture occurs. That critical level is not expressed in the model. Instead, a probability of fracture is attributed to each level of micro-cracks density, being the certain simplification.

Conceivable inconformity of the theoretical and experimental results is probably due to deviation from the brittle mode of cracking and impossibility of proper including other various causes of the critical cracks generation.

8 Cognitive limitations and scope of theoretical models applicability

Tracing in situ the erosion process under the cavitation loading is extremely difficult due to space and temporal scale as well as the limited measurability. Any usefulness could be appreciated in using the acoustic emission method. Thus, the conclusions on the process mechanisms could be only drawn out after investigation of the cavitation loading effects. Differentiating of the material state in the non-equilibrium stage are allowed only by the observations of the microstructure. Unfortunately, these observations refer to different samples, being the source of inaccuracy when comparison the results and assessing of the erosion performance are done.

Isolation of the specific mechanisms of the cavitation erosion of the materials is a difficult task due to their multitude and rivalry. For example, cavitation loading induces such effects, linked to the rate of interaction, as anisotropy or softening of the material. On the other hand, in other stage of the erosion there occurs a strengthening. The vibrations of the complex spectrum are generated and the main source of non-elastic behavior of the body are micro-cracks (non-elastic component of the deformation tensor [81]).

A material resistance is probably dispersely dependent on the loading frequency, because quickly acting forces make a considerable increase in micro-cracks density [76]. Drawing conclusions on cracks generation is most frequently only an indirect job, e.g. by detecting the variations of the tensile stress – the variation which depend on the distances between cracks and on the size of the plastic zone around the cut line [82].

Another source of inaccuracy is randomness of the boundary and initial conditions, which decide on the place of damage initiation and subsequent development of the erosion. Inaccuracy refers also to the casual nature of the generation of the critical damage. It depends on the dimensions of the eroded body. The larger its surface, the more probabilistic character of the phenomena it is.

The damage steps at the grain level undoubtedly undergo statistical description. It is decided by randomness of both the loadings and structural formation of the material (among others, the directions of the grains arrangement). An achieving the willful effect at given impulse and microstructure would be a deterministic task. Assuming that material is homogeneous, the continuity approximation could be used for the description. However, even in that case, a slight disturbances of the conditions annihilate the uniqueness of the solutions [83]. Besides, it is impossible to simulate the cracking distributions basing on the local continuum models, i.e. the set of equilibrium equations for mass, momentum, energy and entropy transfer, because an increase in micro-cracks density depends on deformation of the substantial volume of the material element and the energy from the whole volume of "deformation element" is released by generation of the micro-crack. Thus there is a suggestion to use the integral formulation of the erosion, even if the spatial image of the process would be lost.

One possible description of the surface layer destruction is a fractal description (e.g. [84]). The fractality of the process results from the following physical premises: casuality of interactions and micro-cracks joining, multi-way of the process performance as well as the crack growth conditions and granulation of the material microstructure [85-88]. Correctness of the fractal description also arise from the validity of the Gutenberg and Richter law (presented in [89]), which was many times confirmed for damage occurred due to earthquakes. A relationship between properties of the micro-crack fractal nets and a fracture energy of the multiphase materials was proved in [90].

The choice between deterministic or probabilistic approaches is a dilemma, which should be solved before modelling the erosion process. The stochastic description is more realistic, but the lack of data on the randomness of the variables and high calculation requirements could be preventive in application of that approach. A deterministic approach happens to be useful to project the average performance of the process.

9 Cavitation erosion and random transfer of the energy

External forces action causes the persistent change in its physical state of the material, on condition that relaxation processes are not complete and a part of the impulse energy do the work increasing the internal energy of the body. External forces induce the temporal stress field, approximately thoroughly all over the material. The approximation is justified by a great velocity of phonons propagation.

At that time, the energy is simultaneously trapped in many places of the body, analogically to the case of overheated liquid. In case of very short interactions, in nanosecond time scale, the limited velocity of stress impulse propagation could be regarded. At that time, the source of the excitation energy in deep layers of material are particles of the surface layer, deflected from the equilibrium state. An energy transformed is an energy of impulse – a wave packet of the frequencies equal to the frequencies of various modes of control volume cell in the layer. The impulses of different forces take part in the erosion process. They could be divided into some classes, i.e. the impulses capable to initiate: the elastic vibration with hysteresis, the fatigue micro-deformations, the plastic deformations, the micro-cracks and the macro-cracks.

Evolution of the defects in the course of deformation is a step-like process, performed by overcoming subsequent energetic barriers. Propagation of the mechanical wave in environment with defects is getting more nonlinear. The source of non-linearity are the stress concentrators (e.g. the source of the second harmonic generation [91]). In non-linear process an even function could be transformed into function with singularities ([92]) and in dynamical conditions the shock waves could even arise. The most frequent way of energy dissipation therein is due to micro-cracks formation. Inward evolution of the shape results in shrinking and formation of the space non-linearity. An element of the non-linearity class is a micro-crack. Thus, we deal with 2-stage erosion process: an increasing in "deformation inhibitors" and increasing in critical cracks number due to forced joining of micro-cracks and relaxation of deformation energy. Generalizing, the processes consist in energy transfer to acceptors and their multiplication, and can be recognized as migration controlled process. Under the external loading a non-equilibrium state of material is forced. Energy needed to generate an initial amount of micro-cracks is greater than energy needed to generate the next portion of micro-cracks in the deformed micro-structure or to their further development [93]. Thermally activated processes of micro-cracks joining are intensified after the incubation period due to developed disorder [94].

Cavitation erosion is essentially a stochastic process [95]. The double randomness, referred to correlated processes of loading and micro-crack generation as well as considerable variances of their probability distributions lead to unambiguous erosive results. The higher the rate of the processes, the more stochastic they are, mainly because of their rivalry and hierarchy. High rate promotes the role of geometric factor in probability of the cracking. At that time, an importance of the statistical arrangement of grains with respect to the loading direction is increased.

The casualty of the process influence the effectiveness of the energy transfer within the defined volume of the material. The effectiveness, being the result of appropriate stochastic process, depends on statistical distributions of the loadings and material structure in time and in space. Another contributors to the effectiveness of the erosion are obviously the initial conditions, decisive for the duration of the incubation period and bifurcations, and the interaction scale. At the beginning the erosion yield is a time function. It is also a function of the space scale of interaction and a scale of energy loading. There is a difference in effectiveness of the energy transfer at various ranges of the mentioned variables values.

10 Summing-up

Cavitation erosion is a stochastic process, governed by the probabilities of the applying an impact loading of a given value and an appearance of deformations or cracks in the body. It is therefore a function of probability distributions:

1. distribution of the loading force values,
2. distribution of the energy absorbed in the material,
3. distribution of the local damage of the body at given level of the accumulated energy.

Simplifications involving an assumption on heterogeneity of the loadings and omission of the interaction directions let the dynamic equations be disregarded, without considerable loss of the calculated results precision. However, the probability distribution of the local defect should be chosen properly. An assumption on leading role of the micro-cracks in the cavitation erosion refers to the fatigue nature of the process. The latter effect can be made present in the description of the erosion by introducing a delay of micro-cracks generation with respect to the time of the loading forces action. An appropriate model was formulated. Both randomness and multistageness of the cavitation erosion process of the metallic materials was regarded in the model.

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