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A nonlinear approach to gas bubbling modelling

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Abstract

In the paper nonlinear features of air bubbling from a submerged glass nozzle are discussed. In order to explain the chaotic characteristics of bubbles departure two simple models, with limited number of freedom degrees, were proposed. In the models a bubble was treated as nonlinear spring where properties depend on the bubble diameter and dumped force was proportional to the bubble growth velocity and bubble radius. It has been shown that increase of velocity of bubble growth leads to appearance of chaotic changes of growing bubble radius. It has been found out that the interaction between subsequent departing bubbles alters dynamics of the bubble growth causing appearance of periodic windows in chaotic area.

Keywords: Bubble dynamics; Nonlinear analysis; Nonlinear model

Nomenclature

A	–	amplitude, mm
d	–	nozzle diameter, mm
F	–	forces, N
f	–	mean frequency, 1/s
K	–	coefficient
M	–	mass, g
P	–	pressure
R	–	radius, mm
t	–	time, s
x	–	coordinates

Greek letters

ω	–	frequency, 1/s
γ	–	dumping coefficient, Ns/m

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δ – distance, mm

Subscripts

d – dumping force
p – position of bubble wall
s – spring force

1 Introduction

According to [1,2] bubbles in free rise in infinite media can be grouped into three categories: spherical, ellipsoidal and spherical or ellipsoidal-cap. Cap shape occurs only at moderately high Reynolds and Eötvös numbers. Most recently, in [3, 4] the model that takes into account instantaneous interactions between successive bubbles as well as incorporates the wake effect of previous bubble has been presented. Therefore new approaches based on fractal and deterministic chaos analyses have been applied to investigate the complex phenomena of multi-phase systems [5-7]. The concept of non-linear dynamic systems has also been applied to study chaotic features of gas bubbling from a single nozzle (orifice) [8-13]. An acoustic technique coupled with high-resolution photographs for bubble sizing has been applied in [14-16].

Investigation of viscous liquid stability is based on the analysis of $e^{-i\omega t}$ terms changing in time. The terms $e^{-i\omega t}$ are elements of motion equation solution. When a certain ω value exists for which the quantity $e^{-i\omega t}$ increases in whole time domain then the system becomes unstable. For a viscous liquid different space scales correspond to particular behavior of the system. For the behaviors in small space scales velocity gradient is very large therefore fluid flow is damped very rapidly. In this case the number of elements $e^{-i\omega t}$ in the solution of motion equation of the viscous liquid is limited. This property allows us to use for investigation of the stability of viscous liquid flow the analysis of stability of mechanical dissipative system with limited number of freedom degrees. The variables in such a system can be treated as amplitudes of subsequent frequencies of liquid velocity (elements of Fourier series) [14].

This kind of simple mechanistic system was used in the present paper to analyse the behaviors of bubbles departure.

In the paper two problems have been addressed:

- interaction of oscillating, growing bubble with surrounding liquid,
- bubble oscillations in case of interaction between subsequent departing bubbles.

2 Chaotic behaviors of bubble wall movement

In the experiment air bubbles were emitted from glass nozzles submerged in distilled water at a depth of 15 cm in a cylindrical tank of 30 cm in diameter and 37 cm height. The volume of the settling chamber was 0.03 m^3. The air volume flow rate was changed from 1 l/h to 45 l/h. The bubbles were produced in pressure-controlled mode. Experiments have been conducted for constant supply overpressure in the settling chamber. Individually calibrated manometer, accurate to $\pm 0.2\%$, has been employed to measure gage pressure in the settling chamber. A hydrophone B&K 8105 was employed to record acoustic signal generated by departing bubbles. A hydrophone was placed 10 mm away from nozzle outlet. Samples of voltage generated by microphone were recorded at the frequency of 11 kHz.

The experimental procedure and results of bubble behaviour investigation have been presented in detail in [10, 11, 13].

As an example typical bubble formation from the nozzle with outlet diameter of d=1.61 mm and for different volume flow rates has been presented in Fig. 1. The same flow structures were observed for all investigated nozzles [13]. For low

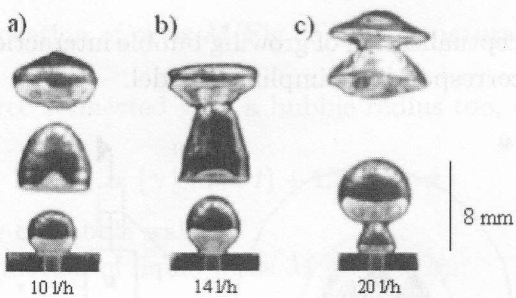

Figure 1. Typical behaviour of bubble train versus volume flow rate for nozzle $d = 1.61$ mm and constant overpressure in the settling chamber $P = 40$ kPa.

volume flow rate no coalescence of subsequent departing bubbles was observed, so the process of bubbles departure was periodic (Fig. 1a). For volume flow rate ca. 14 l/h, bubble coalescence commences as it is shown in Fig. 1b. For high volume flow rate coalescence of bubbles takes place already on nozzle mouth – Fig. 1c. In this case time periods between the departing bubbles become irregular.

The interaction between subsequently departing bubbles seems to be a process responsible for chaotic nature of bubble departure. This vertical coalescence of the bubbles is accompanied by bubble deformation. The phenomenon of bubble deformation caused by preceding bubble is illustrated in Fig. 1b where shape of departing bubbles becomes slender.

Data recorded by acoustic system have been used for identification of bubble dynamics. The correlation dimension and largest Lyapunov exponent have been calculated [13]. Obtained results revealed that the process of waves on bubble surface can be modeled by low dimensional model [13].

3 Modelling

The interaction between the growing bubble and surrounding liquid causes oscillations of bubble wall. These oscillations change the diameter of bubble neck and cause the changes of the time periods between the subsequent bubbles. So, in our investigation it has been assumed that if the bubble oscillations are not chaotic then the disturbance generated by previously departing bubbles does not affect the time between the succeeding bubbles. In this case the additional oscillations of bubble wall are rapidly dumped. But, if the oscillations are chaotic then the amplitude of additional oscillations of bubble wall are increasing. In this case the process of bubble departure is of chaotic nature too.

3.1 Oscillations of single growing bubble

Figure 2a shows conceptualisation of growing bubble interaction with surrounding liquid, and Fig. 2b corresponding simplified model.

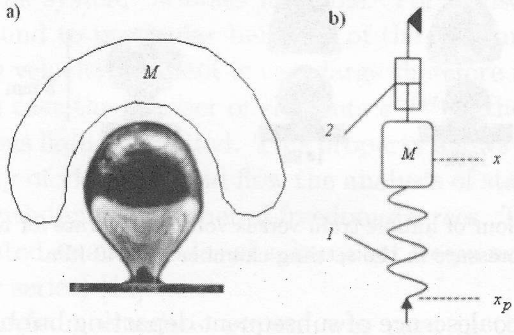

Figure 2. Model of bubble wall deformation; a) interaction between the bubble and liquid bulk, b) model of nonlinear oscillator; 1 – spring simulating elastic reaction of bubble wall, 2 – dumper simulating Stokes force.

Pressure inside a bubble depends on its diameter – the smaller diameter the higher resulting pressure and bigger force necessary for bubble shape deformation. Therefore the bubble was modeled by a nonlinear spring 1 shown in Fig. 2b. The bubble growth is dumped by Stokes force, which in turn is proportional to the bubble wall velocity and bubble radius. This force was modeled by dumper 2 shown in Fig. 2b. In the present model the characteristics of a spring and a dumper are functions of time. In this case the spring and dumper properties depend on the stage of bubble growth.

The motion of the bubble wall was modeled by introducing the concept of "ideal bubble" [12]. In this case the shape of the "ideal bubble" was considered as the theoretical shape of a bubble which doesn't interact with the surrounding liquid. Diameter of the ideal bubble was described by the following function:

$$x_p = A \cdot [\sin(\omega t) + 1.5]/2 \tag{1}$$

where $A/2$ is a diameter of departing bubble, ω represents release frequency and t – time.

The spring force connected with bubble radius described by Eq. (1) can be represented as follows:

$$F_s = \{K[-\sin(\omega t) + 1.5]/2\} \cdot \Delta x^3 \tag{2}$$

where Δx is a bubble wall deformation defined as

$$\Delta x = x - x_p - \delta_o$$

where x represents location of mass M(Fig. 2) and δ_o natural length of a spring, respectively.

The dumping force connected with a bubble radius too, can be expressed in the form:

$$F_d = \{\gamma[\sin(\omega t) + 1.5]/2\} \cdot \dot{x} \tag{3}$$

where $\dot{x}$ is a velocity of bubble wall.

The equation of motion of liquid mass M has a form:

$$\frac{dx^2}{dt^2} = -\frac{F_s}{M} - \frac{F_d}{M}\frac{dx}{dt}\,. \tag{4}$$

Equation (4) was solved with the use of Runge-Kutta method of the fourth order with variable integration step and initial condition $dx/dt = 0$ for minimum value of x_p. In order to obtain high accuracy in numerical calculations, the coefficients in Eq.(4) should take proper values (not too large and not too small). It is possible to solve this problem by changing the units employed. It was assumed that the unit of time is 0.1 s and the unit of length is 1mm.

The equation (4) was solved for $\omega = 2$ and $\delta_o = 2$ mm. A time period of function in Eq. (3) was set to 2π. Thus, the assumption that the time unit is 0.1 s yields the bubble departure frequency $f = 31.4$ Hz. The trial and error calculation yields the most optimized values of parameters and coefficients as follows: $M = 0.04$ g; $K/M = 120$; $\gamma/M = 0.1$ where γ represents damping coefficient; $A = 2.5$ mm; $\omega = 2$. Obtained in simulation the maximum value of spring force is similar to the tension force acting on the bubble interface for the vapor bubble of radius $R = 6$ mm in water. The value of coefficient $\gamma/M = 0.1$ for $M = 0.04$ g corresponds to Stokes force for the sphere of radius $R = 2.13$ mm moving in water. Thus, it can be concluded that forces assumed in the present model take the values not far from real forces acting on an air bubble moving in water.

The oscillations of growing bubbles were investigated as a function of the maximum diameter of an "ideal bubble", which corresponds to investigation of influence of the gas volume flow rate on the bubble oscillations.

It was found that the increase of amplitude of function Eq. (3) leads to decrease of mass motion amplitude and finally yields chaotic changes of location of mass M. Figure 3 presents the Poincare map of system under consideration.

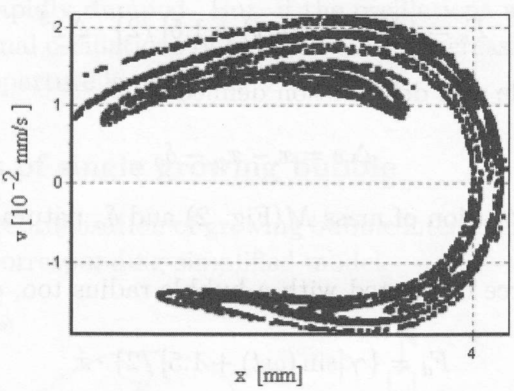

Figure 3. Poincare map for A=1.0 mm K/M=120, γ/M = 0.1, and ω=2.

In Fig. 4 it has been shown the map of chaos appearance in the system. The increase of amplitude A leads to chaos appearance for $A \approx 1.2$.

3.2 Interaction between subsequently departing bubbles

The second investigated problem was the interaction between succeeding bubbles. Figure 5a shows conceptualisation of bubbles interaction, and Fig. 5b corresponding simplified model. Mass (5) – (Fig. 5b), represents the mass of liquid that is moved by movement of elastic bubble wall (1) – (Fig. 5a). The mass (6) represents the mass of liquid that is moved by movement of elastic wall of bubble (2) – (Fig. 5a). The movement of mass (5) – (Fig. 5b), is dumped by stagnant liquid in the vicinity of nozzle outlet. The spring force of spring (3) and dumping force of dumper (1) is described by Eq. (2) and Eq. (3).

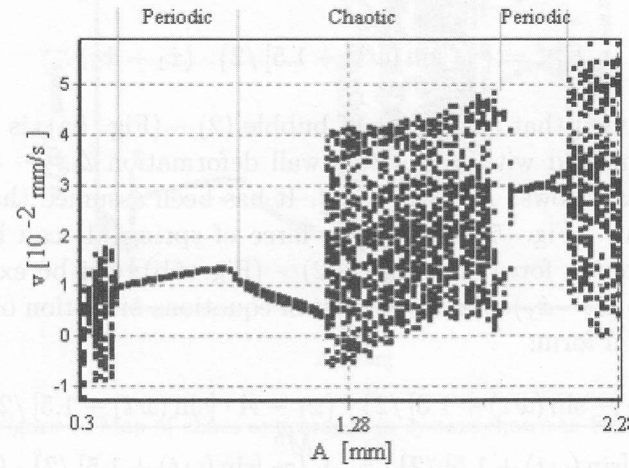

Figure 4. Map of chaos appearance in system shown in Fig. 2.

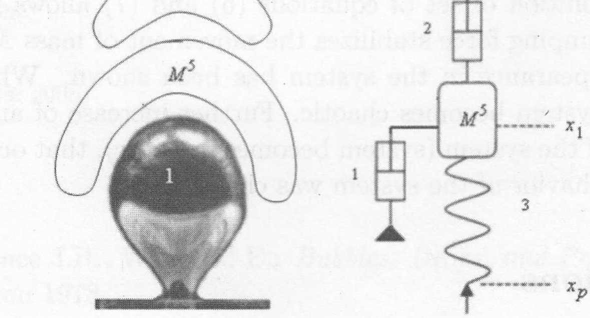

Figure 5. Model of bubbles interaction; a) interaction between the bubbles and liquid bulk, b) model of nonlinear oscillator; 3,4 – springs simulating elastic reaction of bubble wall, 1, 2 – dumpers simulating Stokes force.

The dumping force of dumper (2) – (Fig. 5b), connected with changes of a bubble radius, can be expressed in the form:

$$F_d^{1,2} = \{\gamma_2 \left[\sin(\omega\, t) + 1.5\right]/2\} \cdot (\dot{x}_1 - \dot{x}_2)\,. \tag{5}$$

It has been assumed that the radius of bubble (2) – (Fig. 5a), is constant. The spring force connected with bubble (2) wall deformation Δx_2 – (Fig. 5a), can be represented as follows: $F_s = K_4 \cdot \Delta x_2^3$. It has been assumed that because the bubble (2) is flat – Fig. 5a, the spring force of spring (4) can be assumed as linear. The dumping force of dumper (2) – (Fig. 5b), can be expressed in the form: $F_d^{2,1} = \gamma_2 \cdot (\dot{x}_2 - \dot{x}_1)$. Finally the set of equations of motion of liquid masses M^5 and M^6 has a form:

$$\frac{dx_1^2}{dt^2} = -\frac{\{K_3\left[-\sin(\omega\, t) + 1.5\right]/2\} \cdot \{x_1 - A \cdot \left[\sin(\omega\, t) + 1.5\right]/2 - \delta_o\}^3}{M^5} - \frac{\{\gamma_1 \left[\sin(\omega\, t) + 1.5\right]/2\} \cdot \dot{x}_1 + \{\gamma_2 \left[\sin(\omega\, t) + 1.5\right]/2\} \cdot (\dot{x}_1 - \dot{x}_2)}{M^5}\,, \tag{6}$$

$$\frac{dx_2^2}{dt^2} = -\frac{K_4 \cdot \{x_2 - \delta_1\}}{M^6} - \frac{\gamma_2 \cdot (\dot{x}_2 - \dot{x}_1)}{M^6} \tag{7}$$

where δ_o is the natural location of mass M^5 and δ_1 – the natural location of mass M^6.

The set of equations (6) and (7) was solved with the use of Runge-Kutta method for $\omega = 2$ and $\delta_o = \delta_1 = 2$ mm, $M^5 = M^6 = 0.04$ g, $K_3/M^5 = K_4/M^6 = 120$, $\gamma_1/M^5 = \gamma_2/M^6 = 0.1$, $A = 2.5$ mm.

Behaviors of the interacting bubbles were investigated similar as in the previous presented model by use the concept of "ideal bubble".

Analysis of solution of set of equations (6) and (7) allows to conclude that the additional dumping force stabilizes the movement of mass M^5. In Fig. 6 the map of chaos appearance in the system has been shown. When amplitude A exceeds 1.2 the system becomes chaotic. Further increase of amplitude A leads to stabilization of the system (system becomes periodic), that occurs for $A \approx 1.5$. For $A > 2$ the behavior of the system was chaotic.

4 Conclusions

In the paper it has been shown that increase of the diameter of departing bubble and velocity of bubble growth leads to chaotic oscillations of departing bubble. Bifurcation map illustrates that chaotic oscillations appear when departing bubble diameter exceeds certain value. Obtained bifurcation map is similar to map of chaos appearance obtained for nozzle diameter $d = 0.75$ mm [13].

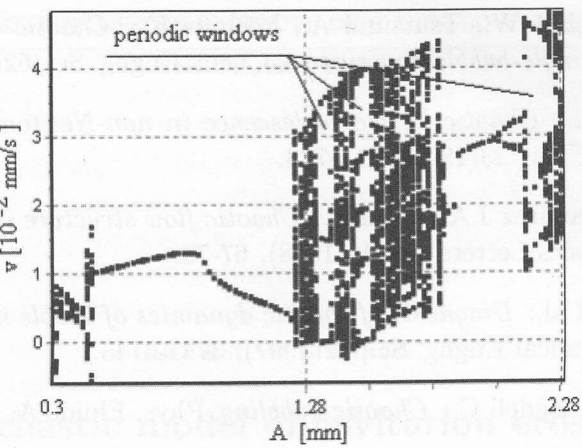

Figure 6. Map of chaos appearance in system shown in Fig. 5.

Analysis of influence of previously departing bubble on the oscillations of growing bubble shows that for small diameter of departing bubbles the liquid flow generated by oscillations of earlier departing bubble stabilizes the chaotic oscillations of the growing bubble. Further increase of departing bubbles diameter leads to appearance of chaotic behaviours of bubble deformations. In the chaotic area (Fig. 6) the periodic windows are created. In this case the chaotic and periodic bubble oscillations appear alternately.

Analyses carried out in the paper reveal that the interaction of springy bubbles with the viscous liquid may lead to chaotic changes of diameter of growing bubble. This type of oscillations leads to chaotic changes of forces causing the bubble motion and consequently leads to chaotic changes of time periods between departing bubbles.

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