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Multigroup interfacial area transport equations for bubbly and cap/slug flows

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Abstract

In this paper we obtain two groups of coupled transport equations for the interfacial area concentration (IAC), the void fraction and the density of bubbles starting from the Liouville equation. Also we study the source and sink terms of these transport equations. These terms are produced by interactions between bubbles, by the impact of turbulent eddies, or by generation of new bubbles by nucleation. Special attention has been paid to the term that considers the rate of volume change of existing bubbles along their trajectories; this term takes into account the change in IAC produced by pressure changes.

Keywords: Interfacial area; Multi-phase models; Bubble collisions

Nomenclature

$a_i(\vec{r},t)$ – interfacial area concentration at point $\vec{r}$, and time t, m^{-1}

$a_{i,k}(\vec{r},t)$ – interfacial area concentration of the bubbles belonging to the k-th group, m^{-1}

$A_i(V)$ – interfacial area of the bubbles with volume V, m^2

$\langle A_i \rangle_k$ – average interfacial area of the bubbles belonging to group k, m^2

$\langle \Delta A_i \rangle_j^{(k)}$ – interfacial area change in group k produced by one collision of type j, m^2

D_c – critical diameter of the bubbles, m

$f(\vec{r},V,t)$ – bubble size (volume) distribution function

g – gravity acceleration, m/s^2

$m_j^{(k)}$ – average number of bubbles being destroyed at group k per collision of type j

$n_{b,k}(\vec{r},t)$ – number density of bubbles belonging to the k-th group, at point $\vec{r}$, and at time t, $1/m^3$

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$n_j^{(k)}$	–	average number of bubbles being generated at group k per collision of type j
$R_j(\vec{r},t)$	–	rate of collisions of the j-th type per unit of volume, at point $\vec{r}$, and time t, $1/(\mathrm{m}^3\mathrm{s})$
$S_{O,j}^{(k)}(\vec{r},t)$	–	rate of bubbles being generated (output) in the group k per volume unit by the j-th collision mechanism,
$S_{I,j}^{(k)}(\vec{r},t)$	–	rate of bubbles being destroyed in the group k (input) per volume unit by the j-th collision mechanism,
$\vec{v}$	–	velocity vector, m/s
$\vec{v}_i(\vec{r},t)$	–	average value of the interfacial velocity of the bubbles belonging to the k-th group, m/s
$\vec{v}_{pm,k}(\vec{r},t)$	–	average local particle velocity weighted with the particle number, m/s
$\vec{v}_{g,k}(\vec{r},t)$	–	velocity of the bubbles belonging to group k, m/s
V	–	volume, m^3
$V_{min,k}$	–	minimum volume of the bubbles belonging to group k, m^3
$V_{max,k}$	–	maximum volume of the bubbles belonging to group k, m^3

Greek letters

α	–	void fraction
α_k	–	partial void fraction of the bubbles belonging to group k
$\phi_j^{(k)}$	–	source/sink term of interfacial area concentration in group k due to collisions of type j, $\mathrm{m}^2/(\mathrm{m}^3\mathrm{s})$
Γ_g	–	steam generation rate per unit volume, $\mathrm{kg/m^3s}$
σ	–	surface tension (N/m)
$\Delta\rho$	=	$\rho_l - \rho_g$, $\mathrm{kg/m^3}$

Subscripts

c	–	critical volume
i	–	interfacial
j	–	collision type
k	–	bubble's group ($k = 1, 2$)
RC	–	random collisions
TI	–	turbulent impact with eddies
SI	–	surface instability
SO	–	shearing off
WE	–	wake entrainment

1 Introduction

The accurate prediction of two-phase flow systems is a a crucial and challenging task for nuclear reactor safety analysis, chemical industry, oil industry, and other industrial applications. Among the different models that are used to describe two-phase flows, the most general one is perhaps the two-fluid model. In this model each phase is described separately, and the conservation equations of mass, energy and momentum are established for each separate phase. Then these equations are averaged in time and space, to yield time averaged macroscopic conservation equations. These equations are closed using postulated constitutive relationships that are deduced from considerations of heat, mass and momentum transfer [1]. The exchange of mass, energy and momentum between the phases in a two-phase fluid depends strongly on the interfacial area concentration $a_i(\vec{r},t)$ (IAC), which is a local property. However, there exist several mechanisms of interaction of the bubbles with eddies and with other bubbles, like break-up of bubbles by turbulent impact, coalescence of bubbles due to wake entrainment, random collisions and so on, that do not modify the void fraction but change the IAC. As a consequence, the classical closure relations to obtain the IAC based on the void fraction and the flow regime can lead to important errors in its evaluation.

Kocamustafaogullari and Ishii [2] emphasized the importance of IAC transport, and generalized the population balance approach of Reyes [3]. Then, they developed an IAC transport equation. Guido-Lavalle and Clausse [4] followed a statistical formulation based on the Liouville-type equation for the probability density function $f(\vec{r},\vec{\xi},t)$, of the dispersed flow of particles (bubbles, droplets...), where $\vec{r}$ defines the position of the particles, and the components of the vector $\vec{\xi} \in R^m$ are a set of properties of the particles such as, volume, shape factor, and so on. This particle density function is defined in such a way that $f(\vec{r},\vec{\xi},t)\, d^3r\, d^m\xi$ gives the expected number of particles inside a volume element d^3r, at position $\vec{r}$, and having a vector of characteristic properties with components belonging to the intervals $(\xi_1, \xi_1 + d\xi_1)$,.. $(\xi_m, \xi_m + d\xi_m)$, and obeys the Liouville-type equation:

$$\frac{\partial f}{\partial t} + \vec{\nabla}\cdot(\vec{v}\, f) + \sum_{j=1}^{m} \frac{\partial}{\partial \xi_j}(f\dot{\xi}_j) = S(\vec{r},\vec{\xi},t) \qquad (1)$$

where $\dot{\xi}_j$ is the rate of change of the j characteristic property with time, $S(\vec{r},\vec{\xi},t)$ is a source or sink term due to the different processes and interactions that can create or destroy particles, and $\vec{v}(\vec{r},\vec{\xi},t)$ is the particle velocity.

Later, in order to model the IAC transport phenomena in all the two-phase flow regimes, Ishii et al. [5, 6] took into account the differences in interfacial area among the bubbles due to their size and shape. These authors considered two IAC transport equations for two different groups. The first group, up to a critical volume V_c with diameter $D_c = 4\sqrt{\sigma/(g\Delta\rho)})$, is formed by the small dispersed and distorted bubbles, while group two is formed by the cap/slug bubbles. The modeling of the different collision terms, which produce interfacial area changes, was performed by Wu et al. [7], and Hibiki and Ishii [8].

In this paper in Sec. 2, we start from the Liouville-type equation for the bubble's density function, and we compute the different moments of this equation obtaining a set of six coupled transport equations for the density of bubbles n_{b1} and n_{b2} in both groups, the partial void fractions α_1 and α_2 of the bubbly and the cap/slug group respectively, and the interfacial area concentrations a_{i1} and a_{i2}.

Also in Sec. 2, we discuss the different source and sink terms arising from the different types of bubble interactions, specially the inter-group collisions. Finally, we discuss the simplifications of this system of partial differential equations, and the closure relations. In Sec. 3 we present the main conclusions of the paper.

2 Governing transport equations for two groups of bubbles

2.1 Bubble distribution function transport equation

We started with defining the bubble's density function $f(\vec{r}, V, t)d^3r dV$ in an arbitrary two phase flow mixture. This function gives us the probable number of bubbles at time t, located inside the volume element d^3r, at position $\vec{r}$, and with volumes belonging to the interval $V, V{+}dV$. The bubble's density function f must obey a conservation Liouville-type equation [9], of the form:

$$\frac{\partial f}{\partial t} + \vec{\nabla} \cdot (\vec{v} f) + \frac{\partial}{\partial V}\left(\frac{dV}{dt} f\right) = \sum_j S_j + S_{ph} \tag{2}$$

where dV/dt is the volume change rate of the bubbles of size V, and velocity $\vec{v}(\vec{r}, V, t)$. $S_{ph}\,(\vec{r}, V, t\,)$ is the source strength per unit of phase space due to phase change. $S_j\,(\vec{r}, V, t)\,d^3r\; dV$ is a number of bubbles being generated or destroyed per unit time in the j-th interaction type of the bubbles within a d^3r at $\vec{r}$, and with volumes belonging to the interval $(V, V{+}dV)$. The different types of interactions of the bubbles with other bubbles or with eddies are summarized in Tab. 1.

2.2 Transport equations for the interfacial area

We consider two groups of bubbles, the first one up to a critical volume V_c is formed by the small bubbly flow bubbles, while the second group is formed by the cap/slug bubbles (see Fig. 1). The density of bubbles $n_{b,k}(\vec{r}, t)$ for group k is:

$$n_{b,k}(\vec{r}, t) = \int_{V_{\min,k}}^{V_{\max,k}} dV f(\vec{r}, V, t). \tag{3}$$

Multiplying Eq. (2) by the interfacial area $A_i(V)$ and integrating over all the bubble volumes of group k, one get the transport equation for the IAC for group

k. In this case we define the IAC for the bubbles of group k as:

$$a_{ik}(\vec{r},t) = \int_{V_{\min,k}}^{V_{\max,k}} dV f(\vec{r},V,t)\, A_i(V) = n_{bk}(\vec{r},t)\, \langle A_i \rangle_k \tag{4}$$

where $\langle A_i \rangle_k$ is the average interfacial area per single bubble of the group k.

The main steps to obtain the IAC transport equations are:

i) Multiplying the terms of Eq. (2) that contain the time derivative and the $\vec{\nabla}$ operator, by the interfacial area $A_i(V)$, and integrating over the bubble volumes of group k, gives on account of the Leibnitz rule the following result:

$$\begin{aligned}\int_{V_{\min,k}}^{V_{\max,k}} dV A_i(V)\left(\frac{\partial f}{\partial t} + \vec{\nabla}\cdot(\vec{v}f)\right) = \frac{\partial}{\partial t} a_{ik}(\vec{r},t) + \vec{\nabla}\cdot(\vec{v}_{ik} a_{ik}(\vec{r},t)) + \\ - \left(\left[A_i f \frac{dV}{dt}\right]_{\max} - \left[A_i f \frac{dV}{dt}\right]_{\min}\right)_k\end{aligned} \tag{5}$$

where the average value of the interfacial velocity of the bubbles belonging to group k has been defined by the following equation:

$$\vec{v}_{ik}(\vec{r},t) = \frac{1}{a_{ik}(\vec{r},t)} \int_{V_{\min k}}^{V_{\max k}} dV \vec{v}(\vec{r},V,t) A_i(V)\, f(\vec{r},V,t) \tag{6}$$

where we have used the same definition as introduced by Hibiki and Ishii for the interfacial velocity of the bubbles belonging to the k-th group [8]. In this definition the bubbles that have more interfacial area and are more abundant have more weight in the average value of the interfacial velocity.

ii) Multiplying the term of Eq. (2) that contains the partial derivative with respect to the bubble volume by the interfacial area $A_i(V)$, and integrating over the bubble volumes of group k, gives by performing the integration by parts the following result:

$$\begin{aligned}\int_{V_{\min,k}}^{V_{\max,k}} dV A_i(V)\, \frac{\partial}{\partial V}\left(f \frac{dV}{dt}\right) = \\ = \left(\left[A_i f \frac{dV}{dt}\right]_{\max} - \left[A_i f \frac{dV}{dt}\right]_{\min}\right)_k - \left\langle \frac{\partial A_i(V)}{\partial V} \right\rangle_k \int_{V_{\min k}}^{V_{\max k}} f \frac{dV}{dt} dV\end{aligned} \tag{7}$$

where $\langle \partial A_i(V)/\partial V \rangle_k$ is the average value of the derivative of the interfacial area with respect to the bubble volume, for the bubbles of the group k. We can choose the lower limit V_{min} of group 1, in such a way that the density function $f(\vec{r}, V_{\min,1}, t)$ has a negligible value; in the same way we can choose the upper limit of group two V_{max2}, so that $f(r, V_{\max,2}, t) \approx 0$.

Now for spherical bubbles we can deduce an expression for the average value of the derivative of interfacial area with respect to the volume, i.e. $\langle \partial A_i/\partial V \rangle_1$, taking into account that the partial derivative of the interfacial area of a spherical bubble with respect to the bubble volume is equal to $2A_i(V)/(3V)$, and therefore:

$$\int_{V_{\min}}^{V_c} V f \frac{\partial A_i}{\partial V} dV = \left\langle \frac{\partial A_i}{\partial V} \right\rangle_1 \alpha_1 = \frac{2}{3} \int_{V_{\min}}^{V_c} A_i(V) f \, dV = \frac{2\, a_{i1}}{3} \tag{8}$$

where α_1 is the void fraction of the bubbles belonging to group 1, and a_{i1} is the interfacial area of the bubbles belonging to group 1. A similar proof can be given for group 2. Therefore one arrives at:

$$\left\langle \frac{\partial A_i}{\partial V} \right\rangle_k = \frac{2}{3} \frac{a_{ik}}{\alpha_k}. \tag{9}$$

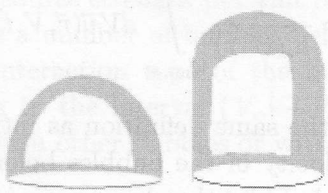

Figure 1. The left side figure displays a cap bubble, while the right side figure displays a slug bubble.

Next we define the average variation rate $\langle dV/dt \rangle_k$, in bubble volume at point $\vec{r}$ and time t, for the bubbles of group k, by means of expression:

$$\int_{V_{\min k}}^{V_{\max k}} dV\ f\,(\vec{r}\,,\,V\,,\,t)\,\frac{dV}{dt} = n_{bk}\,(\vec{r}\,,\,t)\ \left\langle \frac{dV}{dt} \right\rangle_k. \tag{10}$$

Therefore $n_{bk}\,(\vec{r}\,,\,t)\left\langle \frac{dV}{dt} \right\rangle_k$ gives the total variation rate in the volume of the existing bubbles of group k, at point $\vec{r}$, and time t, along the bubble's trajectory per unit of biphasic mixture volume.

Now on account of expressions (9) and (10) we can write Eqs. (7) for group k as follows:

$$\int_{V_{\min,k}}^{V_{\max,k}} dV A_i(V)\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right) = -\frac{2}{3}\frac{a_{ik}}{\alpha_k}\left(n_{bk}(\vec{r},t)\left\langle\frac{dV}{dt}\right\rangle_k\right) \pm A_{i,c} f_c \frac{dV_c}{dt} \quad (11)$$

where the plus sign is for group 1, and the minus for group 2.

iii) Source and sink terms.

According to Ishii et al. [5, 6], and Wu et al. [7], the main source and sink terms for the IAC are:

(a) the bubble breakup due to the impact of turbulent eddies which produces an increase in the IAC;

(b) the bubble's coalescence due to random collisions which produces a decrease in the interfacial area after each collision;

(c) the wake entrainment of small bubbles in the wake of large bubbles, which diminishes the IAC;

(d) the surface instability of large bubbles that increase the IAC because large bubbles are split in smaller ones and finally;

(e) the shear-off of small bubbles from cap/slug bubbles.

Some of these processes are displayed in Fig. 2.

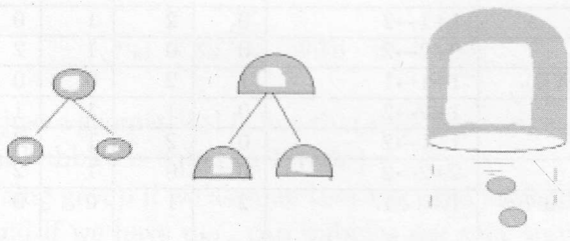

Figure 2. This figure displays: i) the coalescence of two bubbles of group 1 into one bubble of group 1, ii) the coalescence of two caps into a single cap, and 3) the wake entrainment of bubbles of the group 1 by a slug bubble.

Each collision between bubbles, or the break-up of one bubble, produces a change in the interfacial area, that for the j-th interaction process will be denoted by ΔA_j. Each mechanism, for instance the bubble coalescence due to random collisions, has a source (output) contribution $S_{oj}(\vec{r}V,t)$, due to the new bubbles created, and a sink term or input $-S_{Ij}(\vec{r},V,t)$, due to the old bubbles destroyed. So we write:

$$S_j(\vec{r},V,t) = S_{oj}(\vec{r},V,t) - S_{Ij}(\vec{r},V,t)\,. \quad (12)$$

Integrating the source and sink contributions, over the bubble volume of the k-th group, we can write:

$$\int_{V_{\min,k}}^{V_{\max,k}} dV(S_{oj}(\vec{r},V,t) - S_{Ij}(r,V,t)) = \\ = S_{oj}^{(k)}(\vec{r},t) - S_{Ij}^{(k)}(\vec{r},t) = (n_j^{(k)} - m_j^{(k)})R_j(\vec{r},t) \tag{13}$$

where $S_{oj}^{(k)}(\vec{r},\, t)$ and $S_{Ij}^{(k)}(\vec{r},t)$ are the rate of bubbles belonging to the group k, being generated and destroyed respectively per unit volume by the j-th mechanism; $R_j(\vec{r},t)$ is the rate of collisions of the j-th type per unit volume; and $n_j^{(k)}$ and $m_j^{(k)}$ are the average number of bubbles of the group k produced and destroyed respectively per collision of type j. Tab. 1 displays the main interaction types, and the number of bubbles being created and destroyed in groups (1) and (2), for each interaction type. The table notation $1 + 2 \rightarrow 2$ means that one bubble of group 1 collides with one bubble of group 2, and yields one bubble of group 2, the simplified notation is (12, 2).

Therefore the contribution of the j-th interaction source/sink term to the interfacial area concentration, in the k-th group, will be:

Table 1. Collision types and source and sink terms for groups 1 and 2.

Collision Mechanism	Interaction Type	$n_j^{(1)}$	$m_j^{(1)}$	$n_j^{(2)}$	$m_j^{(2)}$	$R_j(\vec{r},t)$
Random Collisions (RC)	1+1→1	1	2	0	0	$R_{RC(1)}$
(RC)	1+2→2	0	1	1	1	$R_{RC(12,2)}$
(RC)	1+1→2	0	2	1	0	$R_{RC(11,2)}$
(RC)	2+2→2	0	0	1	2	$R_{RC(2)}$
Wake Entrainment (WE)	1+1→1	1	2	0	0	$R_{WE(1)}$
(WE)	1+2→2	0	1	1	1	$R_{WE(12,2)}$
(WE)	1+1→2	0	2	1	0	$R_{WE(11,2)}$
(WE)	2+2→2	0	0	1	2	$R_{WE(2)}$
Turbulent Impact with Eddies (TI)	1→1+1	2	1	0	0	$R_{TI(1)}$
(TI)	2→1+2	1	0	1	1	$R_{TI(2,12)}$
(TI)	2→2+2	0	0	2	1	$R_{TI(2)}$
Surface Instability (SI)	2→1+2	1	0	1	1	$R_{SI(2,12)}$
(SI)	2→2+2	0	0	2	1	$R_{SI(2)}$
Shearing Off (SO)	2→1+2	1	0	1	1	R_{SO}

$$\phi_j^{(k)} = \int_{V_{\min,k}}^{V_{\max,k}} dV\,(\,A_i\,(V)\,S_{Oj}\,(\vec{r},V,t) - A_i\,(V)\,S_{Ij}\,(\vec{r},V,t)) \tag{14}$$

which, on account of expressions (12) and (13), can be recast in the form [10]:

$$\phi_j^{(k)} = \left(n_j^{(k)} \langle A_i \rangle_{Oj} - m_j^{(k)} \langle A_i \rangle_{Ij} \right) R_j(\vec{r}, t) = \langle \Delta A_i \rangle_j^{(k)} R_j(\vec{r}, t) \tag{15}$$

where we have defined:

$$\langle A_i \rangle_{o,j}^{(k)} = \frac{\int_{V_{\min,k}}^{V_{\max,k}} dV A_i(V) S_{o,j}^{(k)}(\vec{r}, V, t)}{n_j^{(k)} R_j(\vec{r}, t)}$$

and (16)

$$\langle A_i \rangle_{I,j}^{(k)} = \frac{\int_{V_{\min,k}}^{V_{\max,k}} dV A_i(V) S_{I,j}^{(k)}(\vec{r}, V, t)}{m_j^{(k)} R_j(\vec{r}, t)}$$

as the average interfacial areas of the bubbles, which are produced and destroyed by the j-th interaction mechanism, in the k-th group. Finally, $\langle \Delta A_i \rangle_j^{(k)}$ is the interfacial area variation in the k-th group per single collision of the j-th type.

There are two different ways to compute the variation in the interfacial area per single collision. The first method is to start from the expression that relates the average number of bubbles with the void fraction α, and the IAC a_i [11], and to extend this expression to the bubbly group:

$$n_{b1} = \psi_1 \frac{a_{i1}^3}{\alpha_1^2} \quad \text{with} \quad \psi_1 = \frac{1}{36\pi}. \tag{17}$$

The factor ψ_1 is just a geometrical factor that depends on the shape of the bubbles and for spherical bubbles is equal to $1/(36\pi)$.

For the cap/slug group if we assume that the caps are hemispheres of average diameter D_{2c}, and if we have $n_{b2,c}$ cap bubbles per unit volume, then the interfacial area concentration and the void fraction due to the cap bubbles are given by the following expressions:

$$a_{i2,c} = n_{b2,c} \left(\frac{3}{4} \pi D_{2c}^2 \right) \quad \text{and} \quad \alpha_{2,c} = n_{b2,c} \left(\frac{1}{12} \pi D_{2c}^3 \right). \tag{18}$$

Now computing the third power of the IAC for the cap bubbles and then dividing by the square of the partial void fraction of the cap bubbles one obtains:

$$n_{b2,c} = \psi_{2,c} \frac{a_{i2,c}^3}{\alpha_{2,c}^2} \quad \text{with} \quad \psi_{2,c} = \frac{1}{60.75\pi} \tag{19}$$

assuming that the slugs displayed in Fig. 1, are composed of a cylinder of diameter D and height $h_s = 2D$, plus a hemisphere of diameter D, one arrives to:

$$n_{b2,s} = \psi_{2,s} \frac{a_{i2,s}^3}{\alpha_{2,s}^2} \quad \text{with} \quad \psi_{2,s} = \frac{1}{61.14\pi} \tag{20}$$

where $n_{b2,s}$ is a number of slugs per volume unit. Therefore the expression:

$$n_{b2} = \psi_2 \frac{a_{i2}^3}{\alpha_2^2} \quad \text{with} \quad \psi_2 = \frac{1}{61\,\pi} \tag{21}$$

with $n_{b2} = n_{b2,c} + n_{b2,s}$, $\alpha_2 = \alpha_{2,c} + \alpha_{2,s}$ and $a_{i2} = a_{i2,c} + a_{i2,s}$, is only true when caps or slugs are predominant, but not when we have both types of particles. Differentiating Eq. (17), we get

$$dn_{b1} = 3\,\psi_1 \left(\frac{a_{i,1}}{\alpha_1}\right)^2 da_{i,1} \; - \; 2\,\psi_1 \left(\frac{a_{i,1}}{\alpha_1}\right)^3 d\alpha_1 \,. \tag{22}$$

Next we observe that in the bubble break-up 1→1+1, and coalescence 1+1→1 processes respectively, the steam volume of the group 1 is preserved but the interfacial area of this group changes, so we must apply to Eq. (22), for these previous processes, the condition of α_1 constant, which yields:

$$\left(\frac{da_{i1}}{dn_{b1}}\right)_{\alpha_1 = cte} = \frac{1}{3\,\psi_1} \left(\frac{\alpha_1}{a_{i1}}\right)^2 = \langle \Delta A_{i1} \rangle \,. \tag{23}$$

However, this expression is not true for coalescence by random collisions 1+2→2, because the steam volume in group 1 is not preserved. The preservation is true at the global level of both groups simultaneously.

The same limitations are applicable to group 2, in this case, for break up processes 2→2+2, or coalescence processes 2+2→2, it is obtained

$$\left(\frac{da_{i2}}{dn_{b2}}\right)_{\alpha_2 = cte} = \frac{1}{3\,\psi_2} \left(\frac{\alpha_2}{a_{i2}}\right)^2 = \langle \Delta A_{i2} \rangle \,. \tag{24}$$

The other method departs from the average change in interfacial area obtained from Eq. (15), and given for group 1 by:

$$\langle \Delta A_i \rangle_j^{(1)} = \left(n_j^{(1)} \langle A_i \rangle_{oj}^{(1)} - m_j^{(1)} \langle A_i \rangle_{Ij}^{(1)} \right) . \tag{25}$$

Expression (25) can be estimated for the different types of collisions; however an exact calculation requires computing expression (16). To estimate expression (25), for coalescence due to random collisions in group 1, we observe that for j=*RC(1)*, $n_{RC(1)}^{(1)} = 1$ and $m_{RC(1)}^{(1)} = 2$, therefore for binary collisions between two identical spherical bubbles of diameters D_1, with volume preservation i.e. $2V_1 = V_2$, one gets:

$$\langle \Delta A_i \rangle^{(1)}_{RC(1)} = \left(\langle A_i \rangle^{(1)}_{o,RC(1)} - 2 \langle A_i \rangle^{(1)}_{I,RC(1)} \right) = -0.413 \langle A_i \rangle^{(1)} \tag{26}$$

where $A_i = \pi D_1^2$. To estimate $\langle A_i \rangle^{(1)}$, we take into account the equations:

$$\begin{aligned} \alpha_1 &= \int_{V_{\min}}^{V_c} dV\, V\, f(\vec{r}, V, t) = n_{b1}(\vec{r}, t)\, \langle V \rangle^{(1)}, \\ a_{i1} &= \int_{V_{\min}}^{V_c} dV\, A_i(V) f(\vec{r}, V, t) = n_{b1}(r, t)\, \langle A_i \rangle^{(1)} \end{aligned} \tag{27}$$

where $\langle V \rangle^{(1)}$, and $\langle A_i \rangle^{(1)}$ are the average volume and the average interfacial area of the bubbles of group 1, respectively. Now, dividing the second equation of (27) by the first one, and on account of Eq. (17), one obtains:

$$\langle A_i \rangle^{(1)} = \frac{a_{i1}}{\alpha_1} \langle V \rangle^{(1)} = \frac{1}{\psi_1} \left(\frac{\alpha_1}{a_{i1}} \right)^2 . \tag{28}$$

Therefore, on account of Eq. (26) and (28), the random collision IAC source term $\phi^{(1)}_{RC(1)}$, in group 1 due to collisions of the form 1+1→1, is given by:

$$\phi^{(1)}_{RC(1)} = \frac{1}{3\psi_1} \left(\frac{\alpha_1}{a_{i1}} \right)^2 \left(-1.24\, R_{RC(1)}\, (\vec{r}, t) \right) \tag{29}$$

because in general the coalescence is asymmetric, i.e. the bubbles have different sizes; expression (29) is only an approximation. Proceeding in the same way we can estimate the source term for the coalescence of two cap bubbles.

For random collision of the form 1+2→2, we have a sink term of interfacial area in group (1) and a change of interfacial area in group (2). These two terms can be estimated as follows, the average change of interfacial area in group 1 per collision, can be obtained from equations (25) and (28), on account that $n^{(1)}_{RC(12,2)} = 0$, and $m^{(1)}_{RC(12,2)} = 1$, this calculation yields:

$$\langle \Delta A_i \rangle^{(1)}_{RC(12,2)} = - \langle A_i \rangle^{(1)} = -\frac{1}{\psi_1} \left(\frac{\alpha_1}{a_{i1}} \right)^2 . \tag{30}$$

The change in interfacial area in group 2 is given, on account of Eq. (15), by:

$$\langle \Delta A_i \rangle^{(2)}_{RC(12,2)} = \left(n^{(2)}_{RC(12,2)} \langle A_i \rangle^{(2)}_{o,RC(12,2)} - m^{(2)}_{RC(12,2)} \langle A_i \rangle^{(2)}_{I,RC(12,2)} \right) . \tag{31}$$

The output in group 2 is a bubble belonging to group 2 and with average volume:

$$\langle V \rangle^{(1+2)} = \left(\langle V \rangle^{(1)} + \langle V \rangle^{(2)} \right) = \left(\frac{\alpha_1}{n_{b1}} + \frac{\alpha_2}{n_{b2}} \right) . \tag{32}$$

If the cap bubbles are the most abundant in group 2, and we approximate the cap bubble shape by a hemisphere, then we can obtain the interfacial area from the bubble volume, as follows:

$$\langle A\rangle^{(1+2)} = \frac{3}{4}\pi\left(D^{(1+2)}\right)^2 = \frac{3}{4}\pi\left(\frac{12\ \langle V\rangle^{(1+2)}}{\pi}\right)^{2/3} = 5.64\left(\langle V\rangle^{(1+2)}\right)^{2/3}. \quad (33)$$

Therefore the change in the interfacial area in group 2, due to random collisions 1+2→2, will be given on account of equations (32), and (33) by:

$$\langle \Delta A_i\rangle^{(2)}_{rc(12,2)} = 5.64\left(\frac{1}{\psi_1}\left\{\frac{\alpha_1}{a_{i1}}\right\}^3 + \frac{1}{\psi_2}\left\{\frac{\alpha_2}{a_{i2}}\right\}^3\right)^{2/3} - \frac{1}{\psi_2}\left\{\frac{\alpha_2}{a_{i2}}\right\}^2. \quad (34)$$

Finally the source term $\phi^{(2)}_{RC(12,2)}$ in the group 2 is:

$$\phi^{(2)}_{RC(12,2)} = \langle \Delta A_i\rangle^{(2)}_{RC(12,2)}\ R_{RC(12,2)}(\vec{r},t) \quad (35)$$

where $R_{RC(12,2)}(\vec{r},t)$ is the number of collisions of type *RC(12,2)* per unit of volume and time, and the rest of symbols have been defined previously.

The source terms due to break up of bubbles produced by turbulent impact with eddies are of the types defined in Table 1. The simplest cases are the break up processes, 1→1+1, and 2→2+2. Assuming volume conservation, and symmetric break-up, the change in interfacial area in group 1, produced in collisions of type *TI(1)*, is:

$$\langle \Delta A_i\rangle^{(1)}_{TI(1)} = \left(2\,\langle A_i\rangle^{(1)}_{o,TI(1)} - \langle A_i\rangle^{(1)}_{I,TI(1)}\right) = \frac{1}{3\psi_1}\left(\frac{\alpha_1}{a_{i1}}\right)^2 0.78\,. \quad (36)$$

Therefore the source terms $\phi^{(1)}_{TI(1)}(\vec{r},t)$ and $\phi^{(2)}_{TI(2)}(\vec{r},t)$, of interfacial area in groups 1 and 2, due to breakup processes 1→1+1 and 2→2+2, produced by turbulent impact with eddies, assuming that each bubble breaks into two identical bubbles, are:

$$\begin{aligned}\phi^{(1)}_{TI(1)} &= \frac{1}{3\psi_1}\left(\frac{\alpha_1}{a_{i1}}\right)^2 0.78R_{TI(1)}\left(\vec{r},t\right),\\ \phi^{(2)}_{TI(2)} &= \frac{1}{3\psi_2}\left(\frac{\alpha_2}{a_{i2}}\right)^2 0.78R_{TI(2)}\left(\vec{r},t\right)\end{aligned} \quad (37)$$

where $R_{TI(1)}(\vec{r},t)$ and $R_{TI(2)}(\vec{r},t)$ are the rate of collisions of types *TI(1)*, and *TI(2)*, respectively per unit of volume.

In the case of coalescence induced by wake entrainment of two identical bubbles of group 1, the source term $\phi^{(1)}_{WE(1)}$, of interfacial area, is given by an equation similar in form to Eq. (37). There are inter-group terms in the wake entrainment that will be discussed in forthcoming papers, also the shear-off source term will be discussed later. Finally, on account of equations (5), (11), and (15), the following transport equation is obtained for the interfacial area concentration of group 1:

$$\frac{\partial}{\partial t}a_{i1}(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{i1}(\vec{r},t)\,\vec{a}_{i1}(\vec{r},t)\right) = \\ = R_{ph1}(\vec{r},t)\pi D_{dp}^2 + \frac{2}{3}\frac{a_{i1}}{\alpha_1}n_{b1}(\vec{r},t)\left\langle\frac{dV}{dt}\right\rangle_1 + \sum_j R_j\,(\vec{r},t)\,\langle\Delta A_i\rangle_j^{(1)} \tag{38}$$

where $R_{ph1}(\vec{r},t)$ is the rate of bubbles generated by wall nucleation per unit volume and D_{dp} is the departure diameter of the bubbles that we assume that belong to group 1, and the last term gives the change in the interfacial area concentration for group 1, produced by the different types of collisions.

The transport equation for the interfacial area concentration of group 2 is given by:

$$\frac{\partial}{\partial t}a_{i2}(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{i2}(\vec{r},t)\,\vec{a}_{i2}(\vec{r},t)\right) = \\ = \frac{2}{3}\frac{a_{i2}}{\alpha_2}n_{b2}(\vec{r},t)\left\langle\frac{dV}{dt}\right\rangle_2 + \sum_j R_j(\vec{r},t)\,\langle\Delta A_i\rangle_j^{(2)}. \tag{39}$$

2.3 Transport equations for the partial void fractions α_1 and α_2

The void fraction α_k, for group k, can be obtained from f as follows:

$$\alpha_k = \int_{V_{\min k}}^{V_{\max k}} dV\,V f(\vec{r},V,t). \tag{40}$$

In the IAC transport equations appear the unknowns $n_{bk}(\vec{r},t)\left\langle\frac{dV}{dt}\right\rangle_k$. These unknowns are obtained by multiplying the bubble density function transport equation for f, by the volume V, and then integrating the resulting equation over the bubble's volume belonging to group k. This set of calculations yields a transport equation for the void fraction of the bubbles belonging to group k. For group 1, the term of the Liouville equation that contains the time derivative of the volume yields after multiplication by V followed by integration by parts, and neglecting f_{min}:

$$\int_{V_{\min}}^{V_c} dV\,V\,\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right) = V_c f_c\frac{dV_c}{dt} - n_{b1}(r,t)\left\langle\frac{dV}{dt}\right\rangle_1. \tag{41}$$

Therefore, the application of the operator $\int_{V_{\min,k}}^{V_{\max,kc}} dV V$, to the Liouville equation yields the following transport equation for the void fraction of group k:

$$\frac{\partial}{\partial t}\alpha_k(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{gk}(\vec{r},t)\,\alpha_k(\vec{r},t)\right) = \tag{42}$$

$$= n_{bk}(\vec{r},t)\left\langle \frac{dV}{dt}\right\rangle_k + \eta_{ph,k} + \int_{V_{\min k}}^{V_{\max k}} V \sum_j S_j(\vec{r},V,t)$$

where the source term $\eta_{ph,k}$ is due to the new bubbles produced in group k, by nucleation and at the walls, and is given by:

$$\eta_{ph,k}(\vec{r},t) = \int_{V_{\min k}}^{V_{\max k}} dV\, V\, S_{ph}(\vec{r},V,t). \tag{43}$$

This change of phase term gives the rate of steam volume being generated per unit of mixture volume at group k, due to the new bubbles that are being created by nucleation and at the walls.

In expression (42) $\vec{v}_{gk}$, $k = 1, 2$ is the average velocity of the bubbles belonging to the k-th group, which have been defined by:

$$\vec{v}_{gk}(\vec{r},t) = \frac{1}{\alpha_k}\int_{V_{\min,k}}^{V_{\max,k}} dV \vec{v} V\, f(\vec{r},V,t). \tag{44}$$

The last term in Eq. (42), is due to the gain or loss in the group void fractions produced by the bubble's interactions. This term can be written as follows:

$$\sum_j \int_{V_{\min,k}}^{V_{\max,k}} dV\, V\, S_j(\vec{r},V,t) = \sum_j \int_{V_{\min,k}}^{V_{\max,k}} dV\, V\left(S_{o,j}(\vec{r},V,t) - S_{I,j}(\vec{r},V,t)\right) \tag{45}$$

where $S_{o,j}(\vec{r},V,t)\,dV$ and $S_{I,j}(\vec{r},V,t)dV$ are the average number of bubbles with volume belonging to the interval $(V,\ V{+}dV)$ being generated and destroyed respectively per unit volume and unit time at point $\vec{r}$, by collisions of type j. Next we define the average volumes $\langle V\rangle_{o,j}^{(k)}$ and $\langle V\rangle_{I,j}^{(k)}$ of the bubbles of the group k, being created and destroyed in a collision of the type j, as follows:

$$\langle V\rangle_{oj}^{(k)} = \frac{\int_{V_{\min,k}}^{V_{\max,k}} dV V S_{o,j}(\vec{r},V,t)}{n_j^{(k)}\, R_j(\vec{r},t)} \quad\text{and}\quad \langle V\rangle_{I,j}^{(k)} = \frac{\int_{V_{\min,k}}^{V_{\max,k}} dV\, V\, S_{I,j}(\vec{r},V,t)}{m_j^{(k)}\, R_j(\vec{r},t)} \tag{46}$$

where $n_j^{(k)}$ and $m_j^{(k)}$ are the number of bubbles created and destroyed respectively in the group k in a collision of the type j-th, this quantity depends on the group, and of the type of collision and can be obtained from table 1. Therefore on account of definitions (46), Eq. (45) can be written as:

$$\begin{aligned}\sum_j \int_{V_{\min,k}}^{V_{\max,k}} dV\, V\, S_j(\vec{r}, V, t) = \\ = \sum_j \left(n_j^{(k)} \langle V\rangle_{o,j}^{(k)} - m_j^{(k)} \langle V\rangle_{I,j}^{(k)}\right) R_j(\vec{r}, t) = \sum_j \langle \Delta V\rangle_j^{(k)} R_j(\vec{r}, t).\end{aligned} \tag{47}$$

Now we observe that the term between brackets is the average volume change in the bubbles of group k, as result of a single collision of the j-th class.

Therefore on account of Eq. (47), the transport equations of the partial void fractions α_k , can be written in the form:

$$\begin{aligned}\frac{\partial}{\partial t}\alpha_k(\vec{r}, t) + \vec{\nabla}\cdot\left(\vec{v}_{gk}(\vec{r}, t)\,\alpha_k(\vec{r}, t)\right) \\ = n_{bk}(\vec{r}, t)\left\langle\frac{dV}{dt}\right\rangle_k + \eta_{ph,k} + \sum_j \langle\Delta V\rangle_j^{(k)} R_j(\vec{r}, t)\,.\end{aligned} \tag{48}$$

Now adding up the equations for α_1 and α_2, and on account of the following obvious identities:

$$\alpha = \alpha_1 + \alpha_2; \quad \eta_{ph} = \eta_{ph1} + \eta_{ph2}; \quad \langle\Delta V\rangle_j^{(1)} + \langle\Delta V\rangle_j^{(2)} = 0 \tag{49}$$

we obtain the transport equation for the average void fraction:

$$\frac{\partial}{\partial t}\alpha(\vec{r}, t) + \vec{\nabla}\cdot\left(\vec{v}_g(\vec{r}, t)\,\alpha(\vec{r}, t)\right) = n_b(\vec{r}, t)\left\langle\frac{dV}{dt}\right\rangle + \eta_{ph}. \tag{50}$$

Now from Eq. (50), and on account of the mass conservation equation for the steam, the following result is obtained:

$$n_b(\vec{r}, t)\left\langle\frac{dV}{dt}\right\rangle = \frac{\partial\alpha}{\partial t} + \vec{\nabla}\cdot\alpha\vec{v}_g - \eta_{ph} = \frac{1}{\rho_g}\left(\Gamma_g - \alpha\frac{D\rho_g}{Dt}\right) - \eta_{ph} \tag{51}$$

where Γ_g is the steam generation rate per unit volume (kg/m^{3}s), and D/Dt is the material derivative.

2.4 The transport equations for the number densities n_{b1} and n_{b2}

Integrating Eq. (2), over the bubble's volume belonging to the group k, yields a transport equation for the number density n_{bk}:

$$\frac{\partial}{\partial t} n_{bk}(\vec{r}, t) + \vec{\nabla} \cdot (\vec{v}_{pmk}\, n_{bk}(\vec{r}, t)) = \sum_j S_j^{(k)}(\vec{r}, t) + S_{ph}^{(k)} \tag{52}$$

where in the previous equations we have defined the following source terms: $S_j^{(k)}(\vec{r}, t)$ as the net number of bubbles belonging to the k-th group that are produced or destroyed per unit of volume and unit of time in the collisions of type j, at point $\vec{r}$; $S_{ph}^{(k)}(\vec{r}, t)$ is the number of bubbles that are being generated in the k-th group per unit of volume and unit of time due to change of phase processes (nucleation and detachment from the walls).

The velocity that appears in equations (52), denoted by $\vec{v}_{pmk}$, is the average local particle velocity weighted by the particle number, which is defined by:

$$\vec{v}_{pmk}(\vec{r}, t) = \frac{1}{n_{bk}(\vec{r}, t)} \int_{V_{\min,k}}^{V_{\max,k}} dV \vec{v} f(\vec{r}, V, t)\,. \tag{53}$$

Adding up the equations for both groups we recover the one group transport equation for the total bubble's density $n_b(\vec{r}, t)$, which obeys the equation:

$$\frac{\partial}{\partial t} n_b(\vec{r}, t) + \vec{\nabla} \cdot (\vec{v}_{pm} n_b(\vec{r}, t)) = \sum_j S_j(\vec{r}, t) + S_{ph}(\vec{r}, t) \tag{54}$$

where $\vec{v}_{pm}$ is the average local particle velocity and we have the following closure relations:

$$n_b = n_{b1} + n_{b2}; \qquad S_j = S_j^{(1)} + S_j^{(2)}; \qquad S_{ph} = S_{ph}^{(1)} + S_{ph}^{(2)}\,. \tag{55}$$

2.5 The coupled system of transport equations for the group interfacial areas, the group void fractions, and the bubble's density

In the previous sections we have taken the moments of the Liouville equation for the density function $f(\vec{r}, V, t)$. When integrating this equation over the bubble's volume of each group, we have obtained two transport equations for the bubble's densities $n_{b1}(\vec{r}, t)$, and $n_{b2}(\vec{r}, t)$. Then we have multiplied the Liouville equation by V and we have integrated over the bubble's volume of each group, in this way we have obtained the transport equations for the partial void fractions, α_1 and α_2 of each group. Finally we have multiplied the Liouville equation by $A_i(V)$, and we have integrated over the bubble's volume of each group, obtaining two transport equations for the interfacial area concentrations $a_{i1}(\vec{r},t)$, and $a_{i2}(\vec{r},t)$.

These six previous equations are coupled and should be solved in conjunction with the conservation equations of mass, energy and momentum of the two-fluid model (steam and liquid), and the closure relations.

We can eliminate the unknowns $n_{b2}(\vec{r},t)\left\langle \frac{dV}{dt} \right\rangle_2$, in Eqs. (38), and (39), using Eq. (48). These substitutions yield the following transport equations for the interfacial area concentrations of both groups, neglecting $\eta_{ph,2}$:

$$\frac{\partial}{\partial t} a_{i1}(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{i1}(\vec{r},t)\,\vec{a}_{i1}(\vec{r},t)\right) = R_{ph1}(\vec{r},t)\pi D_{dp}^2 + \sum_j R_j(\vec{r},t)\left\langle \Delta A_i\right\rangle_j^{(1)} + \frac{2}{3}\frac{a_{i1}}{\alpha_1}\left(\frac{\partial}{\partial t}\alpha_1(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{g1}(\vec{r},t)\,\alpha_1(\vec{r},t)\right)\eta_{ph,1} - \sum_j \left\langle \Delta V\right\rangle_j^{(1)} R_j(\vec{r},t)\right) \tag{56}$$

$$\frac{\partial}{\partial t} a_{i2}(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{i2}(\vec{r},t)\,\vec{a}_{i2}(\vec{r},t)\right) = \sum_j R_j(\vec{r},t)\left\langle \Delta A_i\right\rangle_j^{(2)} + \frac{2}{3}\frac{a_{i2}}{\alpha_2}\left(\frac{\partial}{\partial t}\alpha_2(\vec{r},t) + \vec{\nabla}\cdot\left(\vec{v}_{g1}(\vec{r},t)\,\alpha_2(\vec{r},t)\right) - \sum_j \left\langle \Delta V\right\rangle_j^{(2)} R_j(\vec{r},t)\right). \tag{57}$$

3 Conclusions

In this paper we start with the statistical formulation of the probability density function $f(\vec{r},V,t)$ based on the Liouville type equation, for a dispersed flow of particles (bubbles, droplets...). Then we have split according to Ishii criterion the bubbles in two groups: small spherical and distorted bubbles form the first group, while cap/slug bubbles (displayed at Fig. 1) form the second group.

Then we have taken the following moments of the Liouville-type equation:

i) The zero order moment is simply the integral of the Liouville equation over the range of volumes of each group, which is equivalent to applying the operators $\int_{V_{\min}}^{V_c} dV$ and $\int_{V_c}^{V_{\max}} dV$ to the Liouville Eq. (2). These operations yield two transport equations for the density of bubbles in groups 1 and 2, i.e. n_{b1}, and n_{b2}.

ii) The order one moment for the volume is obtained applying the operators $\int_{V_{\min}}^{V_c} V dV$ and $\int_{V_c}^{V_{\max}} V\, dV$ to the Liouville-like transport equation. This operation yields two transport equations for the partial void fractions; from these equations one can obtain the unknowns $n_{bk}(\vec{r},t)\left\langle \frac{dV}{dt} \right\rangle_k$ that give the rate of change of the volume of existing bubbles per unit of volume mixture. These two unknowns can be obtained from Eq. (48). We observe in this equation that to obtain such variation rate for group k, we must subtract from the rate of change of the void fraction α_k, the volume generated or destroyed by change of phase, and by collisions in group k, or by inter-group collisions. Also we observe that when summing up the two transport equations for α_1 and α_2, the collision terms cancel in the transport equation for $\alpha = \alpha_1 + \alpha_2$, because the volume gained in one collision by one group is equal to the volume lost by the other group, as expressed by equation (49).

iii) The application of the operators $\int_{V_{\min}}^{V_c} A_i(V)dV$, and $\int_{V_c}^{V_{\max}} A_i(V)dV$, to the Liouville-like equation for the particle density function, gives two transport equations for the IAC of group 1 and 2. Also it is important that we have obtained general expressions for the source and sink terms of the IAC and the partial void fraction transport equations, especially the inter-group collision terms discussed in expressions (34), and (35). A special interest has the coalescence produced by random collision of the type 1+2→2. We have obtained general expressions for the average change of interfacial area produced in groups 1 and 2 when these collisions take place.

Finally we must mention that we have presented a general discussion about the interfacial area change produced in groups 1 and 2, during the collisions and the method to compute these average changes.

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