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Computation of transient turbulent flow of liquid in pipe using unsteady friction formula

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Abstract

The paper deals with computation and simulation of transient turbulent flow of liquid in pipes. The unsteady shear stress at pipe wall is expressed as integral convolution of weighting function and mean acceleration of liquid. Finally, the unsteady flow of liquid in the pipe is described by two equations, i.e. integro-differential motion equation and continuity equation. The above equations have been solved by means of the method of characteristics using finite difference method. Additionally, calculations have been carried out using relation for quasi-steady wall shear stress. For water hammer, the computed results have been compared to the experimental ones. The comparison of computational and experimental data confirms a high accuracy of the solutions obtained from the calculations based on the presented formulae.

Keywords: Transient; Turbulent flow; Unsteady friction

Nomenclature

c_o	–	acoustic wave speed, m s^{-1}
p	–	pressure, kg m^{-1}s^{-2}
t	–	time, s
v	–	instantaneous mean flow velocity, m s^{-1}
$\hat{t} = \nu t/R^2$	–	dimensionless time
v_z	–	axial velocity, m s^{-1}
w	–	weighting function
z	–	distance along pipe axis, m
H	–	piezometric head, m
L	–	pipe length, m
R	–	radius of pipe, m

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$Re = 2Rv/\nu$	–	Reynolds number
μ	–	dynamic viscosity, kg m^{-1}s^{-1}
ν	–	kinematic viscosity, m^{2}s^{-1}
ν_t	–	eddy viscosity, m^{2}s^{-1}
ν_Σ	–	modified eddy viscosity, m^{2}s^{-1}
λ	–	Darcy-Weisbach friction factor
ρ_0	–	fluid density (constant), kg m^{-3}
τ_w	–	wall shear stress, kg m^{-1}s^{-2}
τ_{wq}	–	wall shear stress for quasi-steady flow, kg m^{-1}s^{-2}
τ_{wu}	–	unsteady wall shear stress, kg m^{-1}s^{-2}
$\Omega = \omega R^2/\nu$	–	dimensionless frequency

1 Introduction

Most papers dealing with numeric calculations and simulations of unsteady liquid pipe flow are based on an assumption, that the formulae for quasi-steady friction can be applied. Moreover, there are formulae for unsteady friction for laminar flow [7,14]. However, there are only very few papers presenting formulas for unsteady, turbulent flow.

One of the earliest models was developed by Daily et al. [3], in which the unsteady friction factor depends on the instantaneous mean flow velocity and instantaneous acceleration. Trikha [7] adapted the Zielke [14] model (which presents a weighting model of transient laminar pipe friction) for unsteady friction in laminar flow to turbulent flow. Brunone et al. [2] deduced the unsteady friction model, which is the function of an instantaneous mean velocity, instantaneous local acceleration and the instantaneous convective acceleration. Vardy et al. [8] and Vardy and Brown [9, 10] worked out a model friction factor, which depends on the instantaneous mean velocity and weighted past velocity changes.

The expressions for determining hydraulic losses during a transient turbulent pipe flow are, in this paper, based on a two-dimensional Reynolds equation and the algebraic equations for the four regions of turbulent viscosity are incorporated into the model. The unsteady shear stress at the pipe wall is expressed as the integral convolution of weighting function and the mean acceleration of the liquid. However, the weighting function does not depend only on dimensionless time, as is it the case in Zielke's model for laminar flow [14], but it depends additionally on instantaneous Reynolds number connected with the mean flow velocity.

Finally, the unsteady flow of slightly compressible fluid in the pipe is described by two equations, i.e. an integro-differential motion equation and the continuity equation (together with the equation of state attached to it). Taking into account the unknowns (pressure and mean velocity) one can treat the equation system as closed. The task of solving the above equation system is not an easy one, because the mean velocity is included in the weighting function which may lead to some difficulties during calculations. Nevertheless, the governing equations have been solved using the finite difference method. Additionally, the calculations have been carried out using a relationship for quasi-steady wall shear stress. For water hammer, the computed results have been compared to the experimental ones, i.e. those given by Holmboe and Rouleau [6] as well as Bergant and Simpson [1]. The comparison of computational and experimental data confirms a high accuracy of solutions obtained from the calculations based on the presented formulae of unsteady friction factor.

2 Fundamental equations

Unsteady flow of liquid in pipes is often represented by two one-dimensional hyperbolic partial differential equations. The momentum and continuity linearized equations are [5, 11÷13]:

$$\rho_o \frac{\partial v}{\partial t} + \frac{\partial p}{\partial z} + \frac{2}{R}\tau_w = 0\,, \tag{1}$$

$$\frac{\partial p}{\partial t} + \rho_o c_o^2 \frac{\partial v}{\partial z} = 0\,, \tag{2}$$

where:

t	–	time,
z	–	distance along the pipe axis,
$v = v(z,t)$	–	average value of velocity in cross-section of pipe,
$p = p(z,t)$	–	average value of pressure in cross-section of pipe,
τ_w	–	shear stress at the pipe wall,
ρ_o	–	density of liquid (constant),
R	–	inside radius of pipe,
c_o	–	acoustic wave speed.

The equations system (1)-(2) is not closed, because it contains 3 unknows: v, p, τ_w. In order to eliminate τ_w from Eq. (1), a relation between τ_w and v, was created. This function depends on dimensionless time and Reynolds number. As a result of inverse Laplace transform, the wall shear stress is expressed as a convolution of liquid acceleration of fluid $\frac{\partial v}{\partial t}$ and step response. It is shown in detail in papers [12, 13].

Finally, the instantaneous stress τ_w may be regarded as the sum of two components: quasi-steady state shear component τ_{wq} and an unsteady state shear component τ_{wu}. Similarly to the laminar unsteady flow it may be expressed as follows [4, 7, 8, 12]

$$\tau_w = \frac{1}{8}\rho_o \lambda \cdot v \cdot |v| + \frac{2\mu}{R}\int_0^t w\,(t-u)\,\frac{\partial v}{\partial t}\,(u)\,du\,, \tag{3}$$

where:

λ – Darcy-Weisbach friction coefficient,

w – the weighting function,

μ – dynamic viscosity,

u – time, used in convolution integral.

The first component in Eq. (3) presents the quasi-steady state of wall shear stress, the second is the additional contribution due to unsteadiness. Eq. (3) relates the wall shear stress to the instantaneous average velocity and to the weighted past velocity changes.

3 The weighting function model

In order to determine the weighting function for wall shear stress the 2D Reynolds equation is used [12, 13]. This equation has the following form

$$\frac{\partial \bar{v}_z}{\partial t} = -\frac{1}{\rho_o}\frac{\partial p}{\partial z} + \nu_\Sigma \frac{\partial^2 \bar{v}_z}{\partial r^2} + \left(\frac{d\nu_\Sigma}{dr} + \frac{\nu_\Sigma}{r}\right)\frac{\partial \bar{v}_z}{\partial r}, \tag{4}$$

where:

$\nu_\Sigma = \nu + \nu_t$ – modified turbulent viscosity,

ν – kinematic viscosity,

ν_t – turbulent kinematic viscosity,

$\bar{v}_z$ – averaged in time (a short time) axial velocity,

r – radial distance from pipe axis.

For presenting the profile of ν_Σ a four – region model [12, 13] (viscous sublayer, buffer layer, developed turbulent layer and turbulent core) has been adopted. Solving the equations for the individual layers (after Laplace transform) we obtain the transfer function relating the wall shear stress to the mean velocity. Making the inverse Laplace transform we obtain the instantaneous wall shear stress, which contains the weighting function (Eq. (3)). This approach is similar to that proposed by Zielke [14] for a laminar flow.

The weighting function depends on the dimensionless time and on the Reynolds number of the averaged flow. Since the function has a complex mathematical structure, its simplified form is proposed. It is more useful for the practice of unsteady flow simulation. The weighting function has been approximated to the following form [15]

$$w(\hat{t}) = C\mathrm{Re}^n \frac{1}{\sqrt{\hat{t}}}, \tag{5}$$

where: $C, n = const$, $C = 0.299635$, $n = -0.005535$, Re – Reynolds number, $\hat{t}$ – dimensionless time

$$\hat{t} = \frac{\nu t}{R^2}. \tag{6}$$

4 Finite – difference equations and calculations procedure

Equations (1), (2), (3), (5) may be presented as follows

$$a\frac{\partial v}{\partial t}+b\frac{\partial p}{\partial z}+c\lambda v\left|v\right|+d\int\limits_0^t\left|v\left(t-u,z\right)\right|^n\frac{1}{\sqrt{t-u}}\frac{\partial v(u)}{\partial u}du=0\,, \tag{7}$$

$$e\frac{\partial p}{\partial t}+f\frac{\partial v}{\partial z}=0\,, \tag{8}$$

where:

$$a=\rho_o, \tag{9.1}$$

$$b=e=1, \tag{9.2}$$

$$c=\frac{1}{4}\frac{\rho_o}{R}, \tag{9.3}$$

$$d=\frac{4\mu}{R^2}C\left(\frac{2R}{\nu}\right)^n, C=0.299635,\ \ n=-0.005535, \tag{9.4}$$

$$f=\rho_o c_o^2, \tag{9.5}$$

$$g=\frac{2c_o}{R}. \tag{9.6}$$

Equations (7) and (8) form a closed system of nonlinear hyperbolic integropartial differential equations with the kernel of Voltera type. The nonlinearity is caused also by the Darcy-Weisbach friction factor λ, which depends on the unknown instantaneous velocity. The friction factor λ is determined by the Prandtl formulae:

$$\frac{1}{\sqrt{\lambda}}=0.869\ln\left(\mathrm{Re}\sqrt{\lambda}\right)-0.8. \tag{10}$$

Equations (7), (8) can be transformed into a pair of differential equations by the method of characteristics (MOC). We have:

$$ae\frac{dp}{dt}\pm\frac{f}{c_o}\frac{dv}{dt}\pm g\tau_w=0, \tag{11}$$

where:

$$\tau_w=c\lambda v\left|v\right|+d\int\limits_0^t\left|v\left(t-u,z\right)\right|^n\frac{1}{\sqrt{t-u}}\frac{\partial v(u)}{\partial u}du=0. \tag{12}$$

Using the Eqs. (9.1)-(9.6) we have

$$\pm dp \pm \rho_o c_o dv + \frac{2\tau_w c_o}{R} dt = 0, \tag{13}$$

$$\text{if} \quad dz = \pm c_o \cdot dt. \tag{14}$$

The fixed grid for the method of characteristics is shown in Fig. 1.

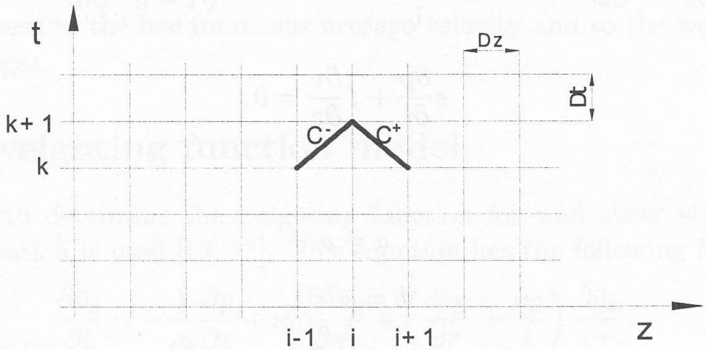

Figure 1. Fixed grid for the method of characteristics (used in the study).

Equations (13) and (14) are rewritten in a finite difference form. It allows us to compute average instantaneous pressure and fluid velocity during water hammer effect. The results are given as two arrays (array of pressure and array of velocity) in Fig. 2. The columns represent adequate points on the hydraulic line and the rows are the counterparts of the successive time steps. These arrays allow reading variation of pressure and flow velocity dependent on time in points of the hydraulic line determined by the number of divisions.

Referring to the grid given in Fig. 1, the solution of system (13), (14) is as follows. The pressure and velocity flow is computed by means of the following equations

$$p_{k+1,i} = \\ = 0.5\left[(p_{k,i-1}+p_{k,i+1})+\rho_o a\left(V_{k,i-1}-V_{k,i+1}\right)+\tfrac{2\Delta z}{R}\left(\tau_{w(k,i+1)}-\tau_{w(k,i-1)}\right)\right], \tag{15}$$

$$v_{k+1,i} = \\ = 0.5\Big[(v_{k,i-1}+v_{k,i+1})+\tfrac{1}{\rho_o c_o}[(p_{k,i-1}-p_{k,i+1})+\tfrac{2\Delta z}{R}\left(\tau_{w(k,i+1)}+\tau_{w(k,i-1)}\right)]\Big]. \tag{16}$$

In order to compute the first column of pressure array, shown in Fig. 3, we must write additional equations:

$$p_{k+1,1} = p_{k,2} + \rho_o c_o \left(v_{k+1,1} - v_{k,2}\right) + \frac{2 \cdot \Delta z}{R} \tau_{w(k,2)}, \tag{17}$$

$$v_{k+1,i} = v_{k,i-1} - \frac{1}{\rho_o c_o}\left[(p_{k+1,i} - p_{k,i-1}) + \frac{2 \cdot \Delta z}{R}\tau_{w(k,i-1)}\right], \tag{18}$$

where:

i = 2, 3, ...h,

k = 1, 2, ...m,

m – number of time steps,

h – number of divisions along pipe length,

$$\tau_{w(k,i)} = \tau_{wq(k,i)} + \tau_{wn(k,i)}, \tag{19}$$

$$\tau_{wq(k,i)} = \frac{1}{8}\rho_o \lambda(\mathrm{Re}_{k,i}) \cdot v_{k,i} \cdot |v_{k,i}|, \tag{20}$$

$$\tau_{wn(k,i)} = \frac{2\mu}{R}\Big[(v_{k,i} - v_{k-1,i})\,W_{1,i} + \\ + (v_{k-1,i} - v_{k-2,i})\,W_{2,i} + ... + (v_{2,i} - v_{1,i})\,W_{k-1,i}\Big], \tag{21}$$

$$W_{i,k} = C\frac{1}{\sqrt{k\Delta t}} \cdot \mathrm{Re}^n_{k,i}, \tag{22}$$

$$\mathrm{Re}_{k,i} = \frac{2R}{\nu}\left(|v_{k,i}| + a\right). \tag{23}$$

Where a is a very small number added to the Reynolds number expression in order to avoid dividing by zero and obtain positive results during calculations of Eq. (22) (Reynolds number expression has a negative exponent). Reynolds number amounts to zero when velocity of the flow amounts to zero. During the water hammer phenomenon caused by a sudden closure of valve, flow velocity oscillates around zero and finally approaches it in steady state. So it is obvious that in calculations zero – value does occurs. In calculations authors assume that $a = 10^{-10}$.

Calculation procedure concerns the system of Eqs. (15)-(23). The flow chart of the calculations procedure for a turbulent flow of a liquid in pipe is shown in Fig. 3. Quantity Re_c is critical Reynolds number [5]:

$$\mathrm{Re}_c = 800\sqrt{\Omega}, \tag{24}$$

where: $\Omega = \frac{\omega \cdot R^2}{\nu}$, $\omega = \frac{2\pi}{T}$, $T = \frac{4L}{c_o}$.

In Fig. 3 quantities d and L represent respectively: the diameter and length of a pipe.

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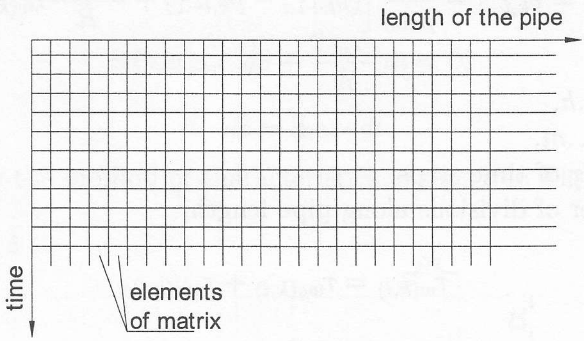

Figure 2. Scheme of results array.

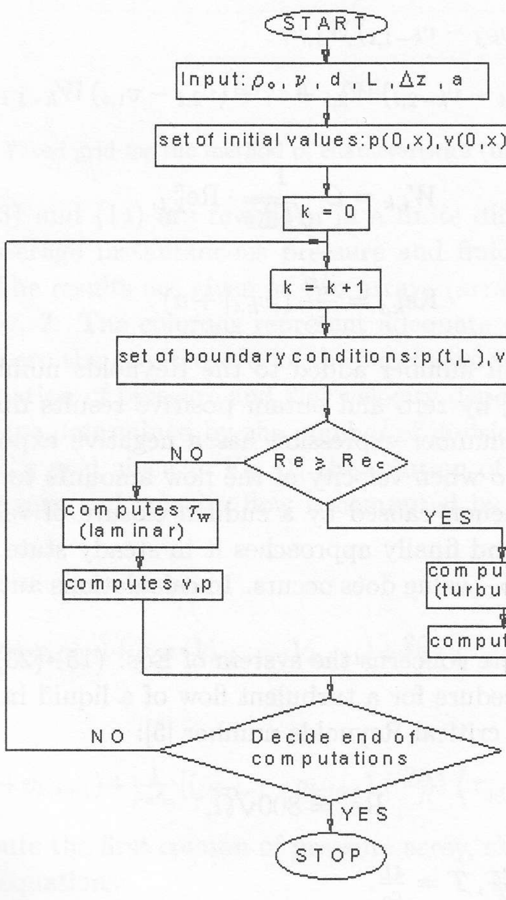

Figure 3. Flow chart.

5 Numerical results

Test 1

The water hammer generated by a rapid closure of the valve for the system is shown in Fig. 4. The liquid having an initial velocity v_o flowed through a pipe of diameter d and a length L. One of the ends of the pipe has been connected to a reservoir where the pressure is always constant. The other end of the pipe finishes in the valve. On the basis of the experiment carried out by Bergant

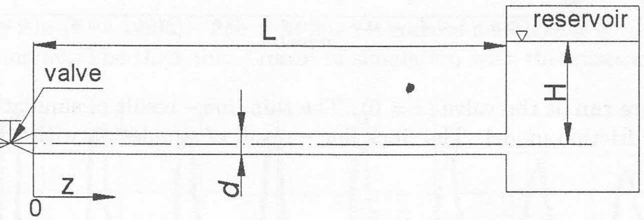

Figure 4. Hydraulic system scheme.

and Simpson [1] numerical simulations were performed. The simulations using the quasi-steady friction model are compared with those using unsteady friction model. Finally, the experimental run is compared with numerical simulations with both models of friction.

The following data were used in the simulations [1]:

- length of the pipe $L = 37.2$ m ,
- pipe diameter $d = 0.022$ m ,
- acoustic wave speed of the fluid $c_o = 1319$ m/s ,
- initial velocity $v = 0.29$ m/s ,
- fluid density 1000 kg/m^3 ,
- kinematic viscosity $1.139 \cdot 10^{-6}$ m^2/s .

In this simulation an assumption has been made that the valve closes for $\Delta t < T/2 = 2L/c_o$
The results of the computation are given as follows:

- for $z = 0$ (pressure at the valve)

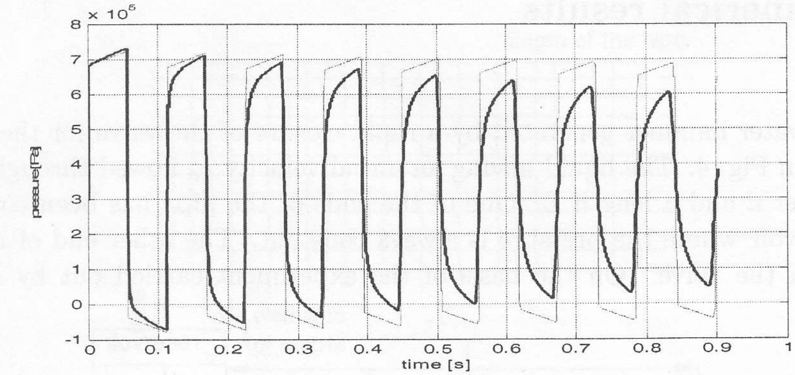

Figure 5. Pressure run at the valve ($z = 0$). The thin line – result of simulation with the quasi-steady friction model. The thick line – result of simulation with the unsteady friction model.

- for $z = 37.2$ (velocity flow at the reservoir)

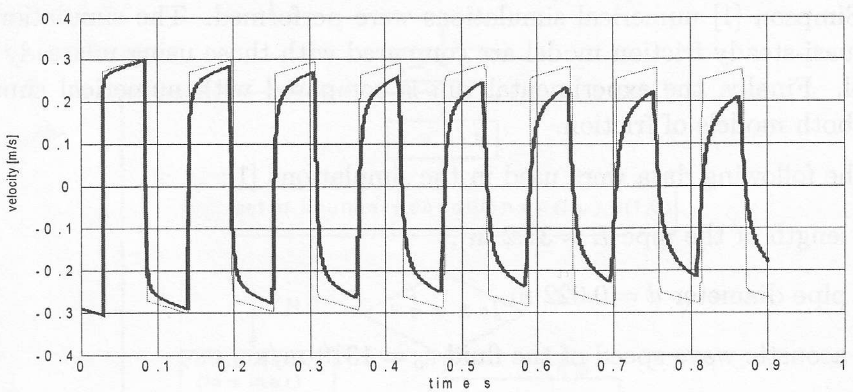

Figure 6. Velocity flow run at the reservoir ($z = 37.2$). The thin line represents the model with quasi-steady state friction model. The thick line-result of simulation with the unsteady friction model.

- for $z = 14.88$ m (pressure)
- for $z = 14.88$ m (velocity)
- for $z = 22.32$ m (pressure)
- for $z = 22.32$ m (velocity)

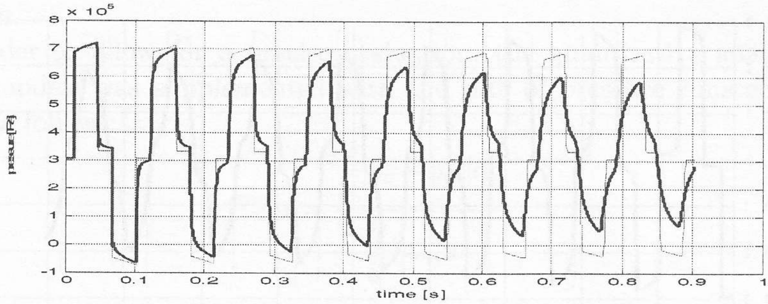

Figure 7. Pressure run ($z = 14.88$). The thin line – result of simulation with the quasi-steady friction model. The thick line – result of simulation with the unsteady friction model.

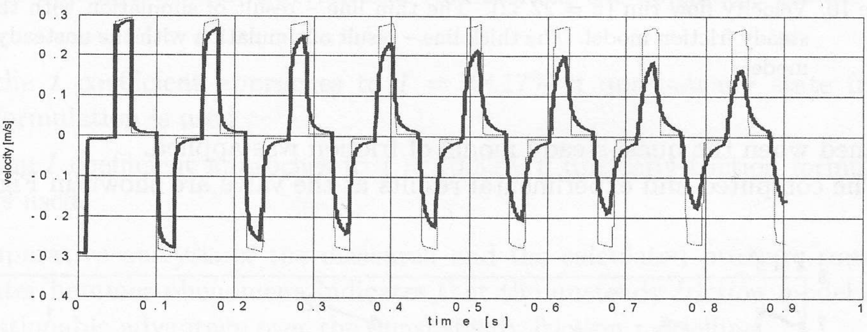

Figure 8. Velocity flow run ($z = 14.88$). The thin line – result of simulation with the quasi-steady friction model. The thick line – result of simulation with the unsteady friction model.

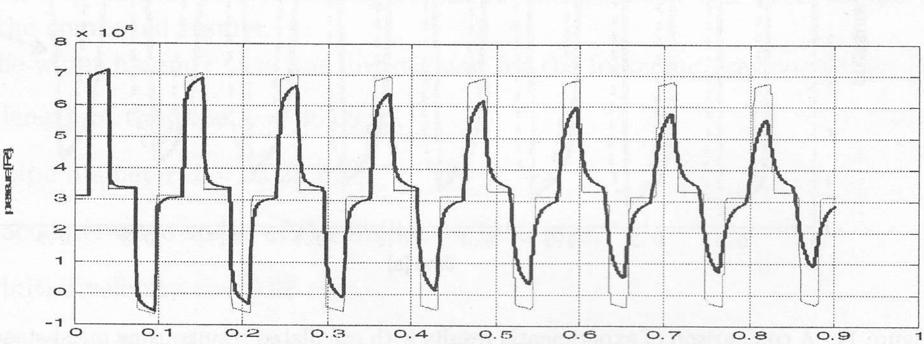

Figure 9. Pressure run ($z = 22.32$). The thin line – result of simulation with the quasi-steady friction model. The thick line – result of simulation with the unsteady friction model.

In the all above analysed cases the computational results obtained using unsteady model of friction reveal much more pressure wave attenuation than those

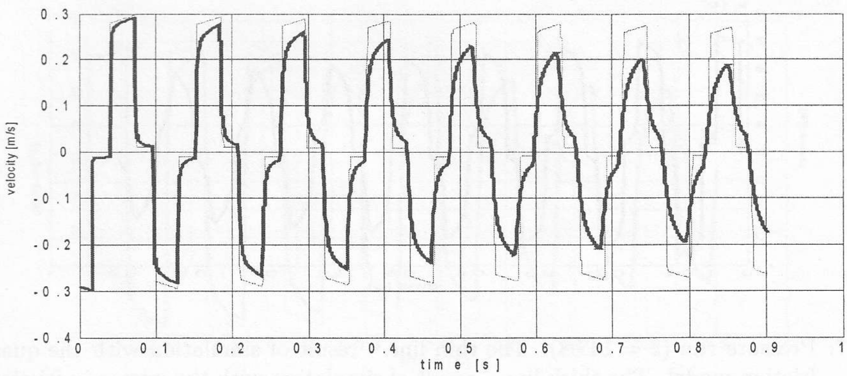

Figure 10. Velocity flow run ($z = 22.32$). The thin line – result of simulation with the quasi-steady friction model. The thick line – result of simulation with the unsteady friction model.

obtained when the quasi-steady model of friction was applied.

The computed and experimental results at the valve are shown in Fig. 11.

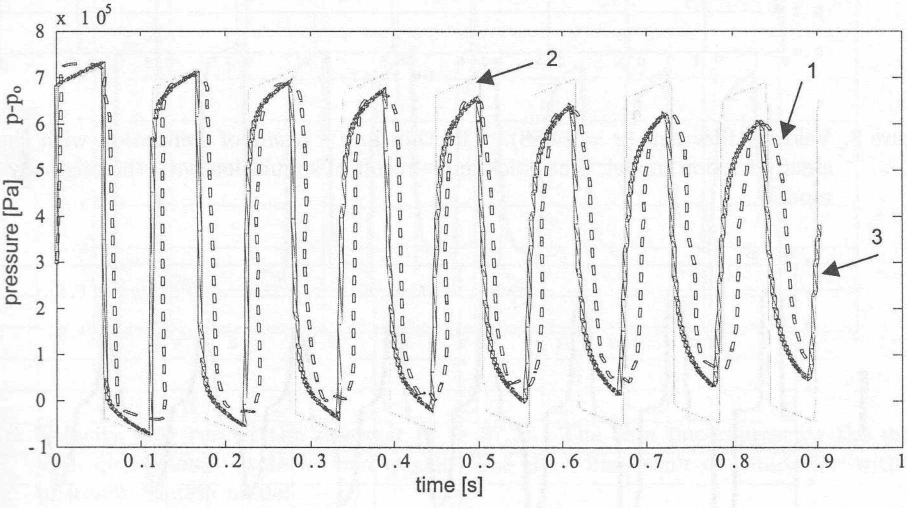

Figure 11. A comparison of experimental results with calculated results using quasi-steady state and unsteady friction: 1 – measurements, 2 – quasi-steady state friction, 3 – unsteady friction.

Fig. 11 presents a comparison between the numerical simulations using both models of friction (quasi-steady and unsteady) and the experimental result. It is clear from Fig. 11 that the run predicted by the unsteady friction model fit experimental data better than the run obtained by using the quasi-steady model of friction.

In order to allow for a precise evaluation, the quantitative analysis of the friction models was supplemented with the rate of pressure runs convergence, defined as follows:

$$I = \frac{\int\limits_0^{t_e} |p_c - p_m| \mathrm{d}t}{\int\limits_0^{t_e} |p_m| \mathrm{d}t}, \tag{25}$$

where: p_m – measured pressure, p_c – calculated pressure.

If the value of I coefficient is closer to zero the simulations fit the experimental data better.

A comparison of the experimental results with the calculated results show that:

- the I coefficient approches to $I = 48.17\%$ if quasi-steady state friction formulation is used,
- the I coefficient approches to $I = 10.32\%$ if unsteady friction formulation is used.

A comparative analysis of the measured and the calculated pressure runs during water hammer phenomena indicates that the unsteady friction model shows unquestionable advantage over the quasi-steady friction modelling.

Test 2

The experiment carried out by Holmboe and Rouleau has been compared with the computed results.

The water hammer case was investigated for the following flow conditions [6]:

- length of the pipe $L = 36.09$ m,
- pipe diameter $d = 0.025$ m,
- acoustic wave speed of the fluid $c_o = 1350$ m/s,
- initial velocity $v = 0.28$ m/s,
- fluid density 1000 kg/m^3,
- kinematic viscosity $1.141 \cdot 10^{-6}$ m^2/s.

The computed and the experimental results at the pipe midpoint are shown in Fig. 12. The computed results for unsteady friction factor are close to the experimental results.

Test of the rate of pressure runs convergence I in this test amounts to:

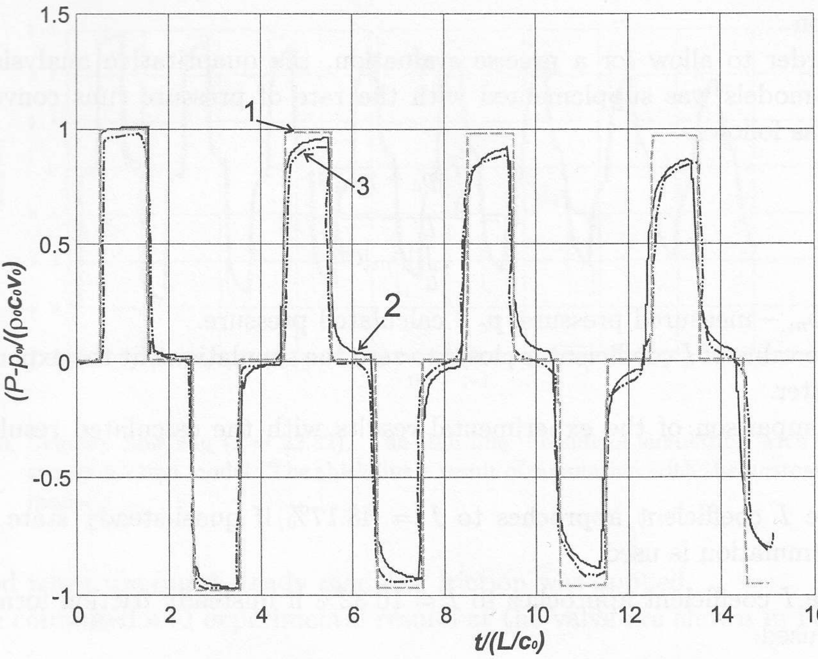

Figure 12. A comparison of the experimental results investigated by Holmboe and Rouleau with the calculated results using the steady state and unsteady friction: 1 – steady state friction, 2 – unsteady friction, 3 – measurements.

- $I = 30.81\%$ if qusi-steady state friction formulation is used,
- $I = 2.09\%$ if unsteady friction formulation is used.

In this test, as we can see on the basis of the rate of pressure runs convergence, the values predicted by the simulation which uses unsteady friction model give a better fit than in the previous test even though conditions of experiment in both cases were similar. This is due to other special conditions of the experiment carried out by Holmboe and Rouleau. The copper tube was embedded in concrete in order to reduce vibrations to an acceptably low level as well as in order to avoid deformation of conduit.

6 Summary

The unsteady wall shear model for turbulent flow has a similar form as in laminar flow. The unsteady friction term depends on the instantaneous mean velocity and weighted past velocity changes. The weighting function depends on dimensionless time and Reynolds number. The approximated formula for weighting function has a simple shape and can be used in calculations and simulations of transients flow in liquid pipe using the method of characteristics (MOC).

Numerical calculations of a transient turbulent flow, after instantaneous closure of a downstream valve in a pipe with an upstream reservoir, have been made using MOC (the case of water hammer). The calculations have been carried out using relations for unsteady and quasi-steady wall shear stress models. The computed results have been compared to the experimental results (Bergant and Simpson as well as Holmboe and Roleau).

The comparison between computational and the experimental data confirms high accuracy of the solutions obtained from the calculations based on the presented formulae of an unsteady friction factor. In the numerical calculations the value of Courant number amounts to 1. This is due to an assumption that convective derivative was neglected and acoustic wave speed was constant.

For a simulation where the convective derivative cannot be neglected and what is more important acoustic wave speed varies considerably, as for highly deformable tubes or very compressible fluids, it is necessary to use the method of characteristics with interpolations. It will be the subject of further research by the authors.

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