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Unsteady friction modelling in transient pipe flow simulation

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Abstract

The paper presents comparative analysis of unsteady pressure course calculation in closed conduits using selected hydraulic resistance models incorporating quasi-steady and unsteady friction models. Calculation was carried out for laminar and turbulent initial flows as tested by E. L. Holmboe and W. T. Rouleau. The results enabled qualitative and quantitative assessment of methods used to calculate friction losses. The assessment has confirmed that the commonly applied method of hydraulic resistance calculation (compatible with the quasi-steady flow hypothesis) does not guarantee satisfactory agreement between calculated and recorded hydraulic transients. It has been proven that the unsteady friction models based on weighting function of past flow velocity changes show superiority over models based on assumption that the portion of energy losses related to flow unsteadiness is proportional to the first derivative of liquid velocity.

Keywords: Friction modelling; Hydraulic transients in pipes; Pressure wave propagation

Nomenclature

a	–	celerity of pressure wave propagation, m/s
f_q	–	steady flow friction coefficient
g	–	acceleration of gravity, m/s^2
k_3	–	empirical coefficient in Eq. (15)
m_i, n	–	coefficients in Eq. (10)
t	–	time, s
u	–	variable of integration, s
x	–	distance along pipe axis, m
y_i	–	variables in Eq. (9), m/s
A_i, B_i	–	coefficients in Eq. (8)

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A^*, C^*	–	coefficients in Eq. (11)
C_1, C_2, m	–	coefficients in Eq. (12)
C, n	–	coefficients in Eq. (13)
D	–	pipe internal diameter, m
H	–	pressure head, m
ΔH	–	pressure head rise, m
I	–	rate of convergence in areas
J	–	head loss on pipe length unit, impedance
K	–	equivalent sand roughness height, m
L	–	penstock length, m
Re	–	Reynolds number VD/ν
V	–	average flow velocity in the pipe, m/s
W	–	weighting function
ν	–	kinematic viscosity, m^2/s
τ	–	dimensionless time

Subscripts and Superscripts

app	–	approximate
c	–	calculated value
m	–	measured value
$c-s/c-u$	–	critical value under steady/unsteady conditions
l/t	–	laminar/turbulent flow
o/e	–	initial conditions/end of simulation
q	–	according to the quasi-steady flow hypothesis
u	–	according to an unsteady flow model

1 Introduction

The proper prediction of transient flow in closed conduits is an engineering task of particular significance. This significance results from destructive character of phenomena accompanying fast flow rate variations in hydraulic systems. Pressure waves of high amplitude, propagating along the pressure conduit, pose considerable danger not only to the pipe coating shells but also to other components of hydraulic systems (valves, fluid-flow machines, etc.) [2, 4, 6]. Reliable simulation of hydraulic transients is therefore a problem of great significance both at the stage of design and operation of hydraulic systems.

In practice, engineering calculations are conducted assuming that the velocity profile of liquid in pipes under unsteady flow conditions does not differ from that corresponding to steady flow with the same mean velocity. In that way the energy losses resulting from unsteady friction forces are disregarded. This simplification seems to be one of the basic causes of discrepancy observed between calculation and experimental results.

In recent years, calculation methods of friction losses during unsteady liquid flow in closed conduits are subject of research in numerous scientific centres all over the world. Thanks to multiyear effort of numerous researchers, a variety of hydraulic resistance models are available when solving transient flow problems. The most satisfactory results have been achieved in case of laminar flows. The credit for this goes to W. Zielke [20] who derived the relationship between the wall shear stress, the integral flow acceleration and some time-dependent weighting function. A. K. Trikha [13] simplified this weighting function to the form much easier to use for the numerical computation purposes by applying appropriate approximation of this function. Trikha also suggested to use his slightly modified model both to transient turbulent and laminar flows.

Due to the random nature of turbulence, a universal model of the transient turbulent flow is still an open question. There are two trends to be distinguished in modelling unsteady turbulent flows in pipes. The first one is represented by models based on the 2D (axisymmetric) Reynolds equation and the Boussinesq hypothesis on empirical eddy viscosity distribution in a pipe cross-section. Depending on approximation of the observed radial distribution of eddy viscosity in a cross-section, the multi-region models have been developed. In the two-region eddy viscosity model (A. E. Vardy et al. [14]) the friction effects are assumed to be concentrated in the steady laminar boundary layer (sub-layer) whereas inertia and compressibility effects play a leading role in the turbulent core region of the flow [16]. Such an approach allowed to specify the weighting function form in the similar way as in Zielke model. This function enables to calculate the unsteady component of the wall shear stress in case of a transient turbulent flow with correct results in the range of high frequencies and Reynolds numbers up to $\mathrm{Re} = 10^5$. Still, there is a need to work out models for predicting unsteady flows at higher Reynolds numbers.

More complicated, four-region eddy viscosity model, was developed by Z. Zarzycki [18]. After decomposing eddy viscosity coefficient onto four layers within the conduit cross-section and adopting adequate boundary conditions, Zarzycki derived the function describing hydraulic resistance. Afterwards, due to its complicated form the function had to be approximated with simpler expressions. Correctness of Zarzycki model was checked in case of a transient laminar flow and its good coincidence with Zielke models was showed. The comparison between the Zarzycki and Vardy-Brown model reveals that both models are consistent with each other in the range of small Reynolds numbers ($\mathrm{Re} < 10^4$) [19]. As Re rises, the differences get more significant. Although all these comparisons may confirm correctness of the Zarzycki model, an experimental verification is still lacking.

The multi-region models are featured by discontinuity in the eddy viscosity distributions, which are calculated assuming steady-state conditions in each region. Numerical methods based on these models demand high processor capacity and substantial CPU time which is their additional disadvantage. It is unquestionable virtue of these models that no empirical factors have to be evaluated that results from the generally accepted turbulent flow models used for determining weighting functions.

The second trend in modelling hydraulic resistance in case of transient pipe flows results from the assumption that the wall shear stress due to flow unsteadiness is proportional to the variable flow acceleration. This approach was introduced by a group of researchers working under guidance of W. L. Daily [9]. The factor of proportionality was established basing on experimental measurements carried out by M. R. Cartens and J. E. Roller [8]. Modifications of this model were subject of many subsequent studies (Safwat and Polder [12], Brunone et al. [7], Vitkovsky [15], Bergant and Simpson [5]). Easy applicability to numerical computations is a significant advantage of this approach. However, the need to determine the empirical factor mentioned above is an obvious demerit. Recently, there appeared suggestions that the friction factor in this model depends also on the velocity derivative of the second or even higher order.

It follows from this unsteady friction models review that the approaches mentioned require assessments based on comparisons between each other and with results of experiments. Comparative analysis of calculations achieved from numerical methods based on selected friction models is presented in this paper. Authors also verify these calculations with experimental measurements taken from accessible literature. Results of these analyses form a basis for qualitative and quantitative assessment of selected models.

2 Governing equations

The most common mathematical model of unsteady liquid flow in closed conduit is based on the following system of partial differential equations (continuity and momentum equations) [3,17]:

$$\frac{\partial H}{\partial t} + \frac{a^2}{g} \cdot \frac{\partial V}{\partial x} = 0, \tag{1}$$

$$\frac{\partial V}{\partial t} + g \cdot \frac{\partial H}{\partial x} + g \cdot J = 0. \tag{2}$$

These equations result from the following basic assumptions:

- flow of liquid is one-dimensional with a uniform cross-sectional velocity profile,
- liquid and pipe walls deform elastically, but their dynamic interaction is negligible (it is sufficient to account only for quasi-steady pipe wall reaction on pressure variations),

- the mean velocity of liquid is relatively small in comparison with pressure wave celerity[1].

Elasticity of pipe walls and liquid is represented by a. Quantity J stands for head loss per length unit of pipe so it represents hydraulic resistance caused by internal friction in liquid and friction at pipe wall. As already mentioned, it is general practice to model this term following the quasi-steady state hypothesis which does not guarantee sufficient conformity between calculation and measurement.

Equations (1) and (2) form a closed system of hyperbolic type partial differential equations with unknown functions of pressure head, $H = H(x,t)$, and flow velocity, $V = V(x,t)$.

There are many different numerical methods used to solve Eqs. (1) and (2)[2]. We may mention the Method of Characteristic (MOC), Finite Difference Method (FDM), Finite Element Method (FEM) and Operatory Methods. MOC is currently the method most widely used for predicting unsteady flows in closed conduits. Its popularity follows from the virtue of excellent reflecting the physical sense of the flow disturbances propagation and relatively easy incorporation of complex initial-boundary conditions into the computational process.

MOC allows replacing Eqs. (1) and (2) with a system of ordinary differential equations as follows:

$$\text{for } \mathbf{C}^{+}: \frac{\mathrm{d}x}{\mathrm{d}t} = a\ ; \mathrm{d}V + \frac{g}{a}\mathrm{d}H + gJ \cdot \mathrm{d}t = 0, \tag{3}$$

$$\text{for } \mathbf{C}^{-}: \frac{\mathrm{d}x}{\mathrm{d}t} = -a\ ; \mathrm{d}V - \frac{g}{a}\mathrm{d}H + gJ \cdot \mathrm{d}t = 0. \tag{4}$$

After integrating Eqs. (3) and (4) along corresponding characteristic lines $\mathbf{C}^+$ and $\mathbf{C}^-$, finite difference equations, applicable for computational lines on the grid of characteristics, are derived. The grid of characteristics is homogeneous with constant time step Δt and pipe reach length $\Delta x = a\Delta t$ (Fig. 1) for pipes with constant diameter and wall thickness. Eqs. (3) and (4), which are fulfilled on characteristic lines $\mathbf{C}^+$ and $\mathbf{C}^-$ and cross at point P, enable to calculate unknown values of H_P and V_P (H and V values at points A and B required for this calculation are known from previous time steps):

$$\mathbf{C}^{+} : (V_P - V_A) + \frac{g}{a}(H_P - H_A) + (gJ)^{+}\,\Delta t = 0, \tag{5}$$

[1]In practice, flows in closed conduits are characterized by maximum velocity not exceeding few m/s whereas local changes proceed with speed equal to the celerity of flow disturbance propagation, usually higher than 1000-1200 m/s. Hence, disregarding terms consisting of the convective derivatives of velocity and pressure head is not expected to cause relative inaccuracies greater than 1 %.

[2]Equations (1) and (2) can be solved analytically in some simple cases – linearization of terms in these equations is required.

$$\mathrm{C}^{-}: (V_P - V_B) - \frac{g}{a}(H_P - H_B) + (gJ)^{-}\,\Delta t = 0. \quad (6)$$

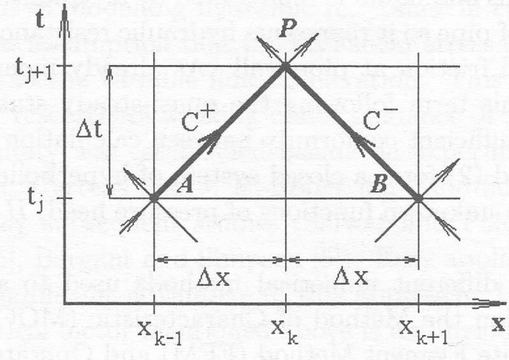

Figure 1. Grid of characteristics

Solving system of Eqs. (5) and (6) requires proper modelling of term gJ along C^+ and C^-characteristic lines. The form of this term depends on the adopted method of friction modelling. In the next sections of this paper authors present models of hydraulic losses selected for further analyses. It is necessary to emphasize that presented considerations are limited to single-phase flows without the liquid column separation phenomenon appearing when pressure falls beneath the critical pressure level – close to vapour pressure of the liquid.

3 Friction models

Quantity J, the so-called impedance, is the sum of the quasi-steady flow pipeline resistance J_q (effective resistance resulting from viscous friction at the pipe wall) and pipeline inertance J_u (reactance including effect of inertia forces in flowing liquid):

$$J = J_q + J_u. \quad (7)$$

3.1 Quasi-steady flow friction

Friction modelling according the quasi-steady flow hypothesis assumes that pipeline inertance J_u equals zero. The pipeline resistance J_q is usually described basing on the Darcy-Weisbach equation:

$$J_q = \frac{f_q}{gD} \cdot \frac{V|V|}{2}. \quad (8)$$

Calculation of the quasi-steady friction factor (Darcy friction factor) depends on:

- flow characteristic (Re) in the Hagen-Poiseuille law used for laminar flows ($\mathrm{Re} \le \mathrm{Re}_{c-s}$ where $\mathrm{Re}_{c-s} = 2320$)

$$f_{q-l} = \frac{64}{\mathrm{Re}} \tag{9}$$

- and additionally on absolute pipe-wall roughness (K/D) in the Colebrook-White formula used for turbulent flows ($\mathrm{Re} > \mathrm{Re}_{c-s}$)

$$\frac{1}{\sqrt{f_{q-t}}} = -2\lg\left(\frac{2.51}{\mathrm{Re}\sqrt{f_{q-t}}} + \frac{K/D}{3.71}\right). \tag{10}$$

3.2 Unsteady flow friction

According to Zielke model, the pipeline inertance can be related to the local acceleration of flow by means of the weighting function $W(t)$ [20]:

$$J_u = \frac{16\nu}{gD^2}\int_0^t \frac{\partial V}{\partial t}(u)\cdot W(t-u)\,du, \tag{11}$$

where:

$$\begin{aligned}
&W(t) \approx W_1(\tau) = \sum_{i=1}^{5} e^{-A_i\tau} \text{ for } \tau = \tfrac{4t\nu}{D^2} \ge 0.02,\\
&W(t) \approx W_2(\tau) = \sum_{i=1}^{6} B_i\cdot\tau^{\frac{i-2}{2}} \text{ for } \tau < 0.02,\\
&A_i = \{26.3744;\ 70.8493;\ 135.0198;\ 218.9216;\ 322.5544\},\\
&B_i = \{0.282095;\ -1.25;\ 1.057855;\ 0.9375;\ 0.396696;\ -0.351563\}.
\end{aligned} \tag{12}$$

Equation (11) is grounded on solid theoretical basis and remains valid in case of any law of mean velocity vs. time variation. It is worthwhile to pay special attention to the weighting function W, Eq. (12), that should be interpreted as a function, which relates inertance at pipe cross-section to past flow velocity changes in this cross-section.

The simplified inertance expression proposed in Trikha model has the following form [13]:

$$J_u = \frac{16\nu}{gD^2}(y_1 + y_2 + y_3), \tag{13}$$

where

$$\begin{aligned}
&y_i(t+\Delta t) = y_i(t)\cdot e^{-n_i\frac{4\nu\cdot\Delta t}{D^2}} + m_i\left[V(t+\Delta t) - V(t)\right],\\
&W_{app}(\tau) = \sum_{i=1}^{3} m_i e^{-n_i\tau},
\end{aligned} \tag{14}$$

where $\quad m_i = \{40; 8.1; 1\}; \quad n_i = \{8000; 200; 26.4\}; \quad \tau > 0.00005.$

According to Vardy-Brown model, the pipeline inertance expression proposed by Zielke is correct for $\mathrm{Re} < 10^5$ with weighting function as follows [14]:

$$W(t) \approx W_{app} = \left(\frac{A^*}{\sqrt{\tau}}\right) \exp\left(\frac{-\tau}{C^*}\right),$$
with:
$$A^* = \frac{1}{2\sqrt{\pi}}; \quad C^* = \frac{7.41}{\log_{10}\left(14.3/\mathrm{Re}^{0.05}\right)} \tag{15}$$

This approach relates W not only to time but also to the Reynolds number.

The analysis of axisymmetric flow using a four-region eddy viscosity distribution enabled to define the weighting function in Zarzycki model [19] by means of the following equations:

a. for laminar flows ($\mathrm{Re} \leq \mathrm{Re}_{c-u}$)

$$W(t) \approx W_{app} = C_1 \tau^{-0.5} + C_2 \cdot e^{-m\tau},$$
with:
$$\tau = \frac{4\nu t}{D^2};\ C_1 = 0.2812;\ C_2 = -1.5821;\ m = 8.8553. \tag{16}$$

b. for turbulent flows ($\mathrm{Re} > \mathrm{Re}_{c-u}$)

$$W(t) \approx W_{app} = \left(\frac{C}{\sqrt{\tau}} \cdot \mathrm{Re}^n\right),$$
with:
$$\tau = \frac{4\nu t}{D^2};\ C = 0.299635;\ n = -0.005535. \tag{17}$$

In case of unsteady flows there exist different criteria of flow area zoning. The Re_{c-u} critical value of Reynolds number, used to distinguish between the laminar and turbulent flow, is given as follows:

$$\mathrm{Re}_{c-u} = 800\sqrt{\frac{\pi D^2 a}{8L\nu}}. \tag{18}$$

The Brunone et al. model is related to the second trend in hydraulic resistance modelling. It assumes the pipeline inertance can be defined as:

$$J_u = \frac{k_3}{g}\left(\frac{\partial V}{\partial t} - a\frac{\partial V}{\partial x}\right). \tag{19}$$

Originally [1], the k_3 coefficient was fixed so as to match computational and experimental results on a sufficient level of conformity. Vardy and Brown proposed the following empirical relationship to derive this coefficient analytically [14]

$$k_3 = \frac{\sqrt{C^*}}{2} \tag{20}$$

4 Comparison between calculation and available experimental results

Basing on the Eqs. (1) and (2) and friction models presented above, a computer code for simulating hydraulic transients in simple hydraulic system has been developed. The hydraulic system consists of an upper reservoir with head water level sustaining constant, pressure pipeline of constant diameter and wall thickness and fast-closing valve providing sudden flow cut-off. Calculations were conducted for flow cases tested by E. L. Holmboe and W. T. Rouleau on an experimental apparatus described in their paper [10] (copper pipe embedded in concrete with $D = 0.0254$ m and $L = 36.088$ m). These experimental results are the most often cited in the literature on transient flows in closed conduits.

Two Holmboe-Rouleau experiments were carried out for two flow regimes at initial steady-state conditions – laminar and turbulent, respectively. In the first case, the hydraulic system was filled with high viscosity oil ($\nu = 3.967 \cdot 10^{-5}$ m^2/s) and the flow under initial conditions was laminar (Re $\cong$ 82). In the second case, tests were run with water ($\nu = 0.86 \cdot 10^{-6}$ m^2/s) under turbulent flow (Re $\cong$ 7200) initial conditions. Calculations for the first and second case were carried out using the following friction models:

- frictionless analysis ($J = 0$);
- quasi-steady friction model ($J = J_q$ in Eq. (1));
- unsteady friction models:
  - Zielke (correct for laminar flows – according to assumptions),
  - Trikha,
  - Vardy-Brown,
  - Zarzycki[3],
  - Brunone et al. with k_3 coefficient in Eq. (19) obtained empirically by means of the trial-and-error method,
  - Brunone et al. with k_3 coefficient in Eq. (19) obtained analytically according to Eq. (20).

All calculations were accomplished using the same grid of characteristics with density defined by fixing 16 computational intervals along the pipe length. It is worthwhile to stress that fixing more grid nodes did not change significantly the results.

[3] Re_{c-u} value in turbulent case is about 84000. It means that for this case (with initial conditions of Re $\cong$ 7200) calculation have to be carried out using the weighting function of Eq. (16).

Wave celerity was calculated basing on the experimental data and was $a = 1324.4$ m/s in the first case (for oil) and $a = 1350.0$ m/s in the second case (for water). Sudden flow cut-off at the valve (flow velocity given by a stepwise function of time) and constant pressure head in the reservoir were included in the boundary conditions. Initial conditions concerned pressure head and flow velocity distribution along the pipeline length.

Results of calculations (time courses of pressure head variations at the valve – cross-section 1 – and in the middle of the pipe – cross-section 2) are presented in Figs. 2 to 9. Calculated courses have been put together with experimental results enabling thus a qualitative assessment of applied models. The following parameter has been introduced in order to enable a quantitative analysis:

$$I = \frac{\int_{t_o}^{t_e} |\Delta H_c| dt}{\int_{t_o}^{t_e} |\Delta H_m| dt}. \tag{21}$$

As Eq. (21) shows the I parameter (so-called rate of convergence in areas) is given by the ratio of areas limited by pressure head line and axis x for calculated and measured ΔH values. The I parameter may be an additional criterion used in verification process of friction models. Calculated I values, corresponding to the results shown in Figs. 2 to 9 are presented in Tab. 1.

Table 1. Rates of convergence in areas

Model	Re = 82		Re = 82	
	valve I_1 [%]	midpoint I_2 [%]	valve I_1 [%]	midpoint I_2 [%]
Frictionless	148.4	133.8	112.0	106.9
Quasi-steady	123.3	114.5	109.8	104.9
Zielke	88.8	96.4	101.5	100.0
Trikha	86.9	98.7	101.4	99.7
Vardy-Brown	82.1	89.7	101.4	100.0
Zarzycki	90.8	99.0	101.3	100.0
Brunone et al. ($k_3$experimental)	90.0	91.5	-	-
Brunone et al. (k_3 analytical)	84.2	86.6	100.5	97.6

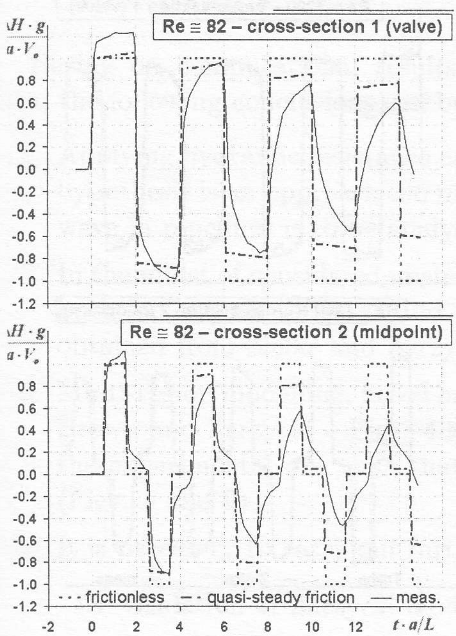

Figure 2. Comparison between measured and calculated pressure courses.

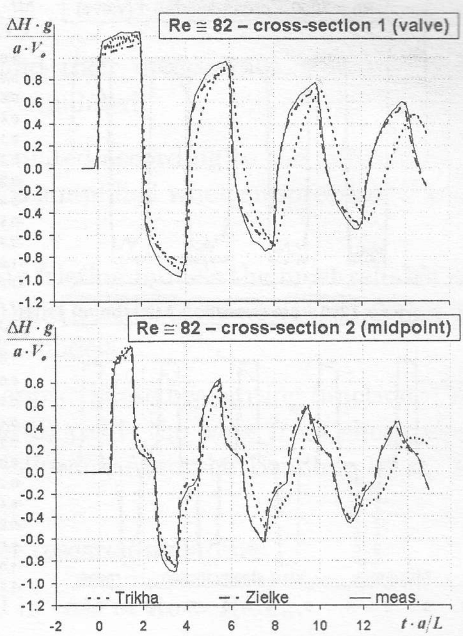

Figure 3. Comparison between measured and calculated pressure courses.

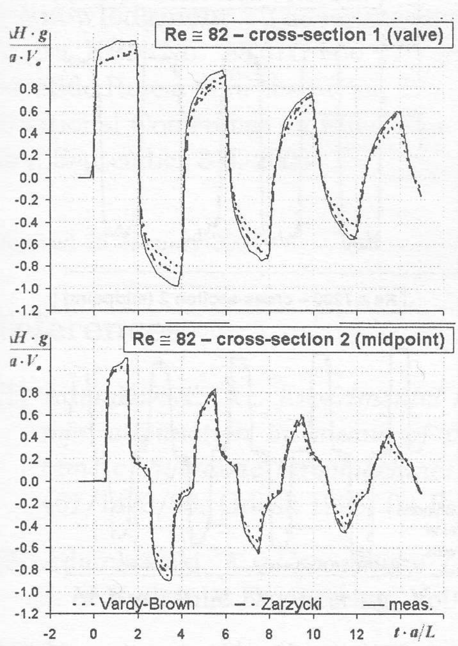

Figure 4. Comparison between measured and calculated pressure courses.

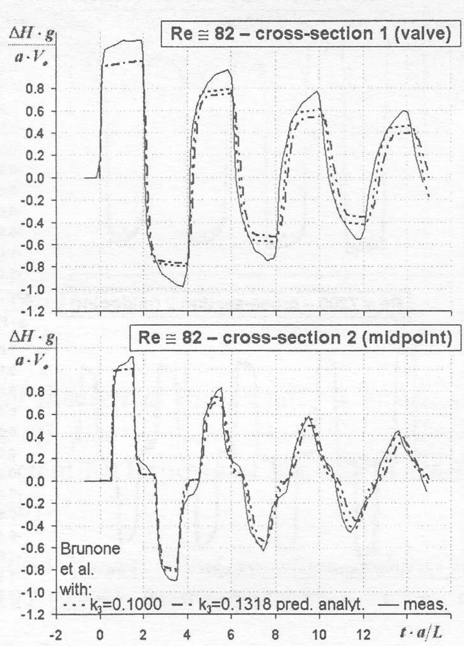

Figure 5. Comparison between measured and calculated pressure courses.

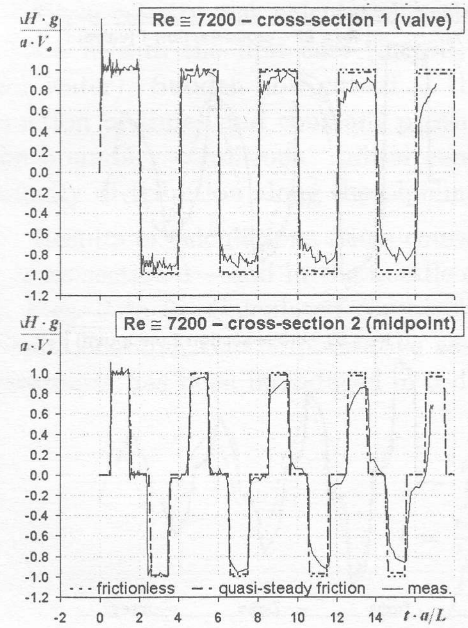

Figure 6. Comparison between measured and calculated pressure courses.

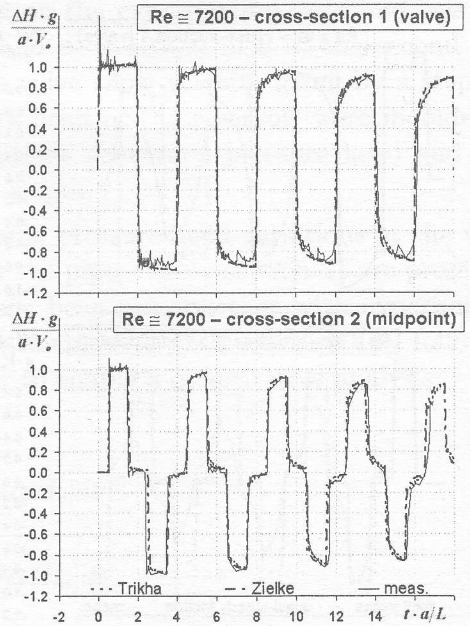

Figure 7. Comparison between measured and calculated pressure courses.

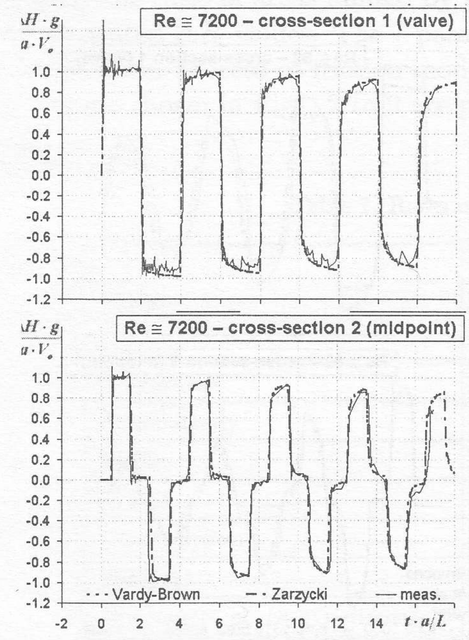

Figure 8. Comparison between measured and calculated pressure courses.

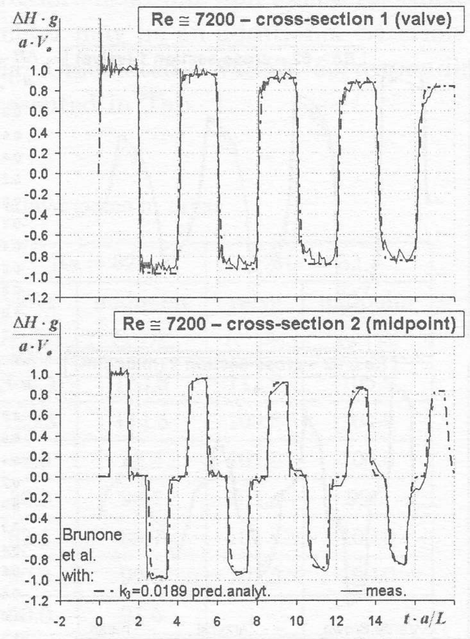

Figure 9. Comparison between measured and calculated pressure courses.

5 Conclusions

Basing on computational results and their comparison with experimental data, the following conclusions can be formulated:

1. Applying hydraulic resistance calculated according to the quasi-steady flow hypothesis is an approach too much simplified when suppression of pressure wave in pipelines is to be analysed.
2. In the midst of considered unsteady friction models the most reliable results both for laminar (Figs. 3 and 4) and turbulent flows (Figs. 7 and 8) are obtained from Zielke and Zarzycki models.
3. Multi-region modelling, based on approximated weighting functions (Vardy-Brown and Zarzycki – Figs. 4 and 8) yields the wave front shape closer to the experimental one than that following from the Brunone et al. model (Figs. 5 and 9).
4. It is necessary to carry out further research including:
   - validation of Zarzycki model in case of $Re > Re_{c-u}$,
   - comparison of computational and experimental results within a wide range of Re, including the cases of water column separation.

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