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POLISH ACADEMY OF SCIENCES

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

115

WYDAWNICTWO IMP PAN

GDAŃSK 2004

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ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 115, 2004, 37-68

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Coupled forms of non-linear vibrations as a new tool of crack detection in rotating shaft

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Abstract

It is widely acknowledged from literature that material or structural imperfections of the shaft crack type are capable, in some cases, of generating coupled forms of lateral-axial-torsional vibrations in the rotating machine. Such information, however, pertains to small or model objects of that type. In the paper presented are investigations into the influence of the shaft crack on the dynamic state of a large, multi-supported rotating machine. Determined have been the cases where the crack is capable of inducing pronounced forms of vibrations, which can serve as basis for more reliable and effective diagnostics, or the cases, where the mechanism of couplings deteriorates, despite the existence of cracks with significant depths. Determined has been the degree of sensitivity of non-linear and mutually coupled system vibrations on crack propagation and conducted had been the assessment of their applicability as a diagnostic determinant of a the dynamic state in a large rotating machine. Indicated have also been the cases, where traditional diagnostics, based on the analysis of constituent vibration spectra of 2X type in the case of lateral vibrations and 1X and 2X for axial and torsional vibrations can be misleading. Conducted have also been investigations into the influence of crack location and propagation on operation of rotating machine after exceeding the stability threshold and hence system operation under conditions of existence and formation of oil whirls. It turned out that the crack can sustain and reinforce existing sub-harmonic components in the vibration spectrum whereas oil whirls efficiently impair the effect of vibrations coupling. Investigations have been conducted based on the advanced models and suites of computer codes for the analysis of complex systems of the type: line of rotors-slide bearings-foundation, enabling generation of diagnostic relations defect-symptom also in cases of large displacements and strongly non-linear external interactions occurring the system. Presented have been the examples of experimental verification of applied research tools.

Keywords: Rotor dynamics; Crack detection; Vibrations

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1 Introduction

Issues of early detection of material and construction imperfections in rotating machines have been the merit of numerous publications for several years now [1-17, 20, 24, 25]. Despite such abundant investigations conducted in that area all over the world there are still some issues unresolved. This regards particularly such problems as coupled forms of non-linear vibrations of multi-supported rotors induced by for example the shaft crack or the issue of appropriate determination of diagnostic determinant of such state.

Tracing the results of investigations on the assessment on the dynamic state of cracked rotors and early detection of that kind of defects [1-10] there can be several more general conclusions drawn, namely:

- directions of investigations tend to focus on the 1X and 2X harmonic responses in vibration spectra ,
- the 2X harmonic component is the most practical crack indicator to be implemented in a monitoring system [2,21],
- the 1X component of lateral vibrations is not sensitive to t he crack depth, but the 1X component of axial vibrations and 2X component of lateral vibrations are very sensitive.

Obviously above conclusions are correct in principal, especially in specific cases analysed by authors of those publications. There can however be indicated situations, where the above conclusions are valid in a limited range and their generalisation may lead to significant errors in the analysis of the state of the object. This regards particularly large and multi-supported rotating machinery founded in slide bearings. One of the objectives of the present work is to indicate such cases.

Another issue requiring a closer recognition is the answer to a question on the influence of rotor crack on the dynamic state of the entire machine after exceeding the stability threshold, hence in the non-linear range of operation, which is often linked to the appearance of oil whirls. There is a lack of literature contributions to that topic. In the present paper an attempt has been made to answer that question.

The principal objective of the paper, however, is to search for a more efficient diagnostic determinant of the dynamic state of a large object with material or construction imperfections of rotating elements. It turns out that a good tool in the analysis of the state of such cases are coupled, non-linear forms of system vibrations induced by a specific defect. That fact has already received attention in [12,13]. Simultaneous analysis of bending-torsional-axial vibrations, induced for example by the rotor crack, gives much better effects than classical analysis of lateral vibrations only. Such conclusion could be perceived at the first glance apparent, but its practical application, due to extremely difficult theoretical description and difficult experimental measurements on real objects, requires completely different approach to the entire problem.

Analysis of coupled, non-linear forms of vibrations of complex systems such as rotor-bearings-foundation requires development of qualitatively new research tools in the form of adequate models and computer codes capable of generating defected vibration spectra in a linear and moreover non-linear regime of operation of considered object. Question arises, why is the non-linear description in such case so important? The answer is, that only the non-linear description (even in the stable range of system operation) generates non-elliptical trajectories and vibration spectra, in the shape of which the considered effects can be coded in. Such tools together with investigations on that topic will be presented in the present paper. At the same time an attempt will be made to justify the thesis that in the case of large and complex rotating systems the coupled forms of vibrations can serve in many cases as a better diagnostic determinant of the dynamic state of the object.

The shaft crack has been considered in the present paper as one of the possible origins of the mechanism of vibrations coupling and, at the same time, the sources of structural non-linearities in the system. The same effect could be induced for example by a significant misalignment of the line of rotors, anisotrophy of shaft cross-section or large non-linear external excitations acting on the system. The general conclusions, however, particularly the principal thesis of the paper, will remain similar.

Let's pass over first to presentation of research tools and their verification, and then to investigations focused on the search for a better diagnostic determinant of the dynamic state in a large multi-supported rotating machine with a cracked shaft and operating both in the stable range of rotational velocities and beyond the threshold of stability of the entire system.

2 Research tools

In the classical rotating machine there can be discerned three principal subsystems:

- rotor line with discs, clutches and imperfections like cracks or misalignments,
- hydrodynamic journal bearings and labyrinth seals,
- foundations with supports and bearings external fixings,

Particularly difficult in theoretical modelling are slide bearings and labirynth sealings. At IFFM PAS in Gdańsk there has been developed a so called diathermal model of heat transfer in bearings (the code DIATER), which consists of coupled Reynolds, energy and conduction equations and the model of hybrid lubrication in the case of supplying from the siphon pockets and possible bush skewness (the code IZOSLEW) – Fig. 1.

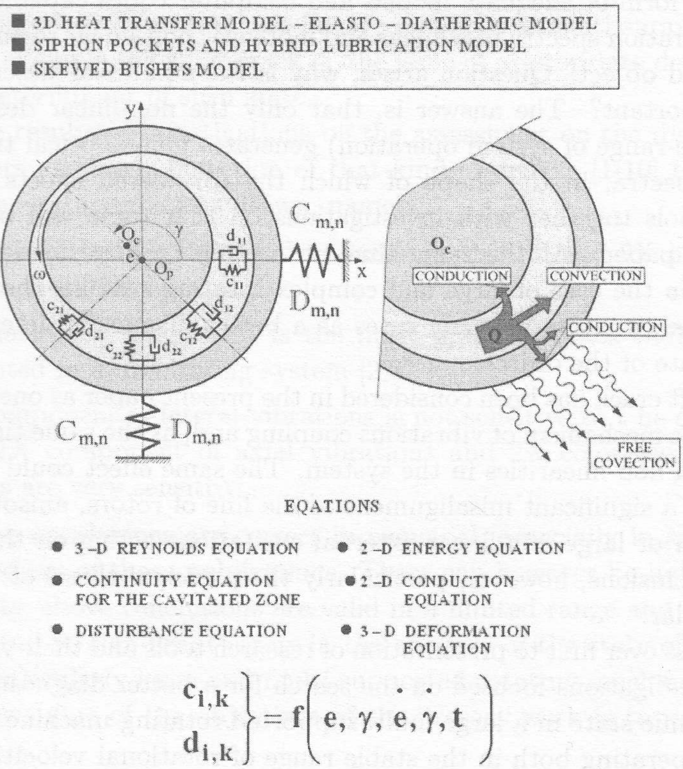

Figure 1. The thermal model of bearing assumed in analysis (DIATER) and corresponding hybrid lubrication model (IZOSLEW).

The above bearings models have been described in detail in [23] or partially in [22, 24, 25] and hence will not be presented in the present paper. It is not even the intention of the latter. It is however worth reminding that at larger shaft displacements the coefficients of stiffness and damping of oil film $\mathbf{c}_{ik}$ and $\mathbf{d}_{ik}$ are time dependent in the strongly non-linear fashion. That remark will prove crucial in further considerations.

The line of rotors with discs has been modeled by means of the FEM method featuring typical beam elements with 6 degrees of freedom in each node [16, 17, 22]. In order to account for a transverse shaft crack there has been applied, known from literature, model of the element with transverse crack due to Knott et al.

[16, 17] of the fully open – fully closed type. A dynamic influence coefficient matrix of supporting structure and foundation has been determined by means of known commercial codes such as ABAQUS and ADINA, based on the FEM method.

A key issue is now to develop a general algorithm of calculations, both kinetostatic and dynamic, combining all mentioned above sub-systems of the entire system, and also development of the algorithm of dynamic calculations incorporating possible non-linear external excitations of the system and large displacements of shafts in bearings. The idea of such type of mutual interactions has been presented in Fig. 2.

It can be noticed that, on one hand, coupling of the line of rotors with labirynth sealings can base on varying non-linearly in time coefficients of stiffness and damping of oil film, and on the other hand, on linear matrices of dynamic influence of supporting structure and foundation. In effect we obtain a general equation of motion of the line of rotors, also in the non-linear form (Fig. 2), as the global matrices of damping **D** and stiffness **K**, which are further functions of generalized parameters of displacements **q**, velocities $\dot{q}$ and time **t** in the non-linear way. Equation of motion for the entire system are therefore a set of several (tens or hundreds with respect to assumed FEM discretisation of the line of rotors) mutually coupled non-linear ordinary differential equations.

Obviously, one possible way of solving of such system of equations is the iterative process. In Fig. 3 presented is a general algorithm of calculations, where at each time step calculated are new coefficients of shaft stiffness with imperfections corresponding to their actual values of displacement. Iterative process is conducted up to the moment of obtaining a full convergence of results of calculation and satisfactory accuracy. In effect we will obtain non-elliptical trajectories of displacement and spectra in selected nodes for all coupled forms of lateral-axial-torsional vibrations.

It is worth stressing at that point the most important issue, namely the way of combination of the shaft "breathing" process with its kinematic and dynamic displacements. Suggested algorithm of non-linear calculations for the entire system (Fig. 2 and Fig. 3) enables combination of a linear model of the element itself with the crack in the way enabling non-linear analysis of final results. It is possible as analysed dynamic and kinetostatic displacements of the entire line of rotors have a non-linear character. Therefore the process of opening and closing of a crack depends on its location on the line of kinetostatic deflections φ_k as well as instantaneous dynamic displacements φ_d as schematically presented in Fig. 4. In the assumed model it is possible to locate the crack on the circumference with respect to the plane of action of external excitation forces denoted by the angle α_p. In effect the process of crack "breathing" (opening and closing) is a very complex function of following parameters: φ_k, φ_d and α_p.

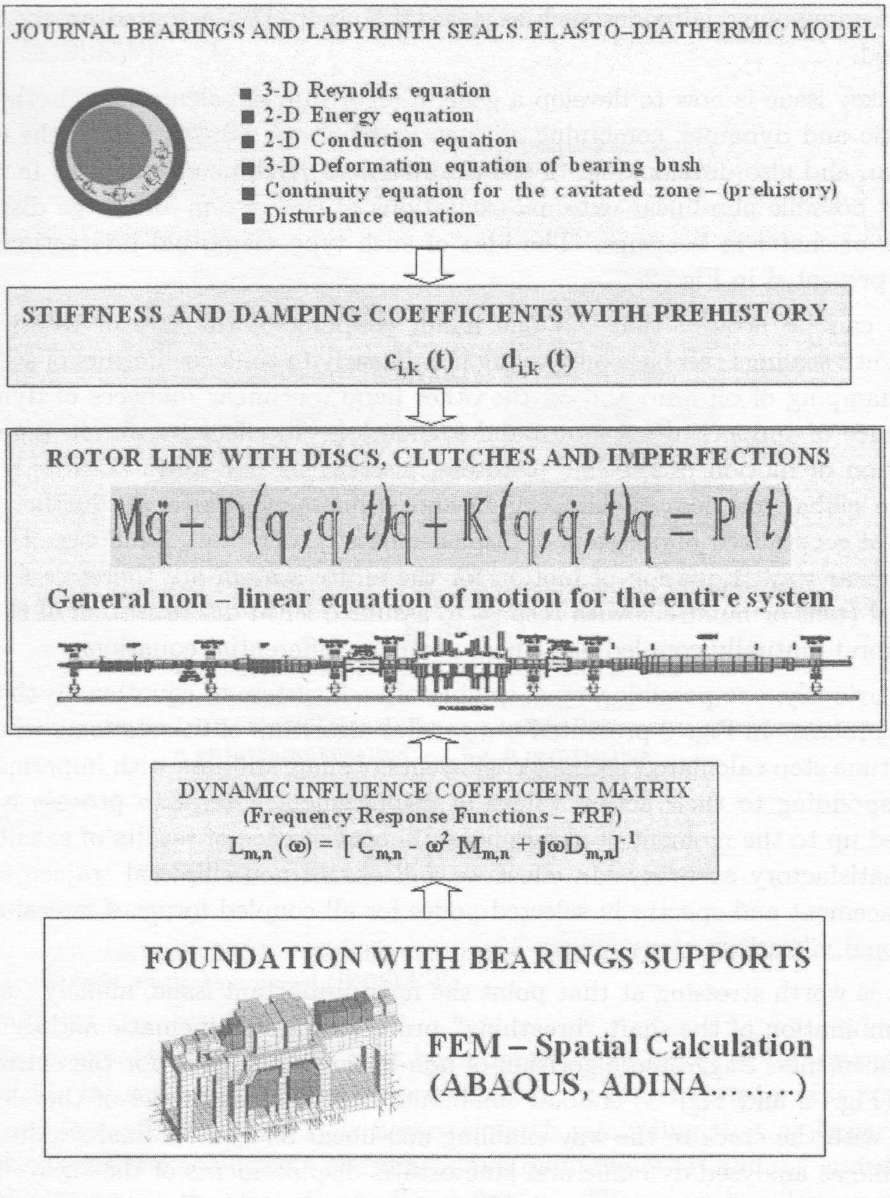

Figure 2. Relations between major components of the system; general non-linear equations of motion for the entire system.

The influence of kinetostatic deflections of the line of rotors on the form of crack interaction can be of very significant importance in some cases. This is illustrated schematically in Fig. 5. At relatively high values of kinetostatic deflections compared to instantaneous dynamic deflections the "breathing" process may cease and the crack may remain open or closed at all times – see Fig. 5.

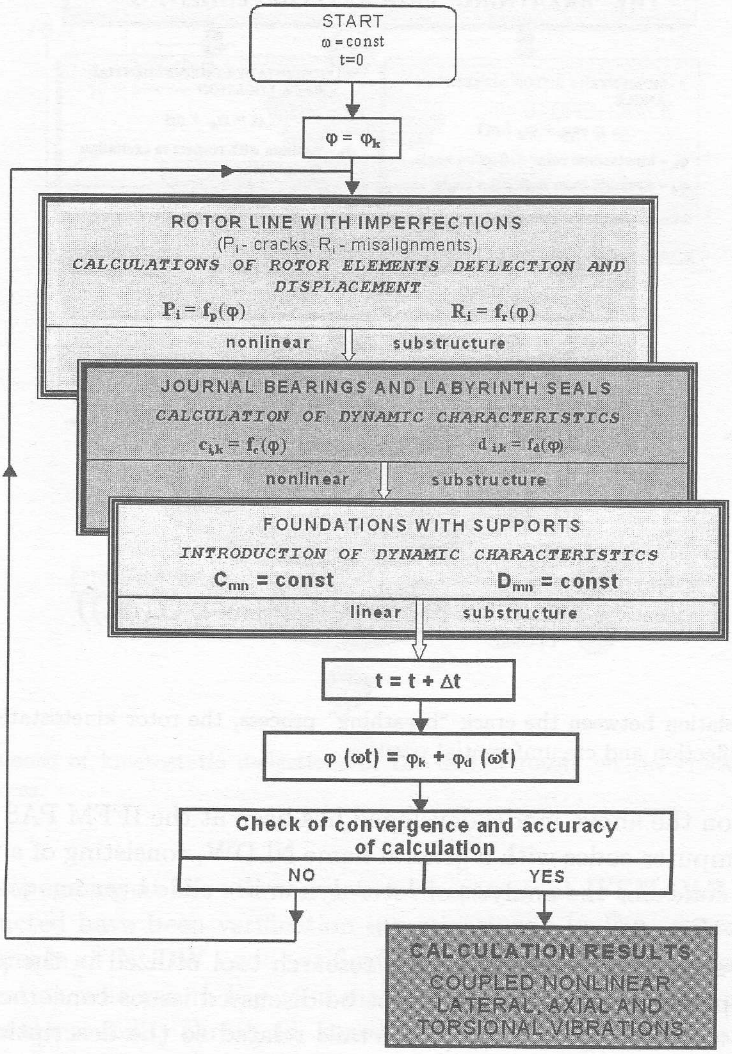

Figure 3. A general algorithm of iterative procedure enabling non-linear analysis of coupled forms of vibrations with imperfections.

Suggested algorithm of calculations (Figs. 2 and 3) and particularly the way of incorporation into the procedure of the model of element with the crack (Figs. 4 and 5) with account of kinetostatic and dynamic rotor shaft deflections in the linear and non-linear range of displacements opens qualitatively new possibilities of assessment of the state of rotating machine with shaft imperfections and forms, in author's opinion, an important element of novelty in the present work.

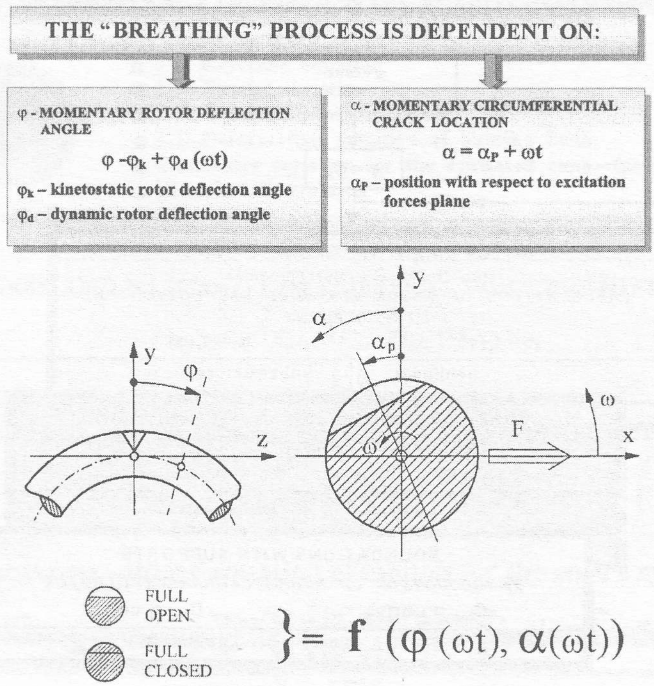

Figure 4. Relation between the crack "breathing" process, the rotor kinetostatic and dynamic deflection and circumferential position.

Based on the above model developed has been at the IFFM PAS in Gdańsk a suite of computer codes with a general name NLDW, consisting of a whole family of specific codes for the analysis of rotor dynamics, slide bearings and supporting structure – Fig. 6.

The system NLDW forms a basic research tool utilized in the present work. Due to apparent reasons there will not be discussed issues concerned with capabilities of that vast system nor the details related to the description of utilized equations and simplifying assumptions. Such information can be found in [18-20, 22, 24, 25].

A separate issue is experimental verification of developed research tools. The NLDW system has been exposed to a detailed verification procedure both in the laboratory scale and real objects, such as power engineering turbosets of large power. Presented next will be two examples of code verification on the research stand whilst neglected will be abundant reports on the code verification on real objects. In Fig. 7 presented is a photograph of a multi-scale and multi-supported

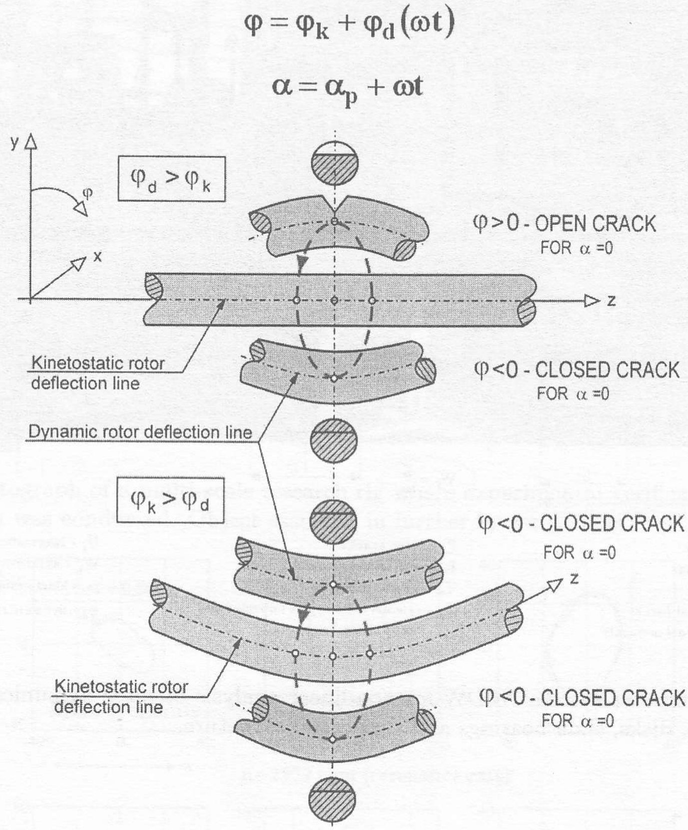

Figure 5. Influence of kinetostatic deflections of the line of rotors on the crack „breathing" process.

research rig operating at the vibroacoustics laboratory at IFFM PAS in Gdańsk, where conducted have been verification investigations. In Fig. 8 presented has been an example of NLDW code verification under typical stable conditions of the rig operation whereas in Fig. 9 a more interesting case of verification after surpassing of the stability threshold, hence in the region of existence of oil whirls, is shown. It is worth noticing that a sudden change of phase angles during propagation of oil whirls, predicted by computer simulations (development of characteristic half-loops), has found experimental confirmation. It is of significant importance as modeling of oil whirls under conditions of stability loss of a large rotating machine is very difficult. Therefore the results presented in Fig. 8 and Fig. 9 can be regarded as satisfactory.

It ought to be noticed, however, that the results presented in Figs. 8 and 9 have been obtained at the assumption of a lack of cracks in the rotor shafts. In

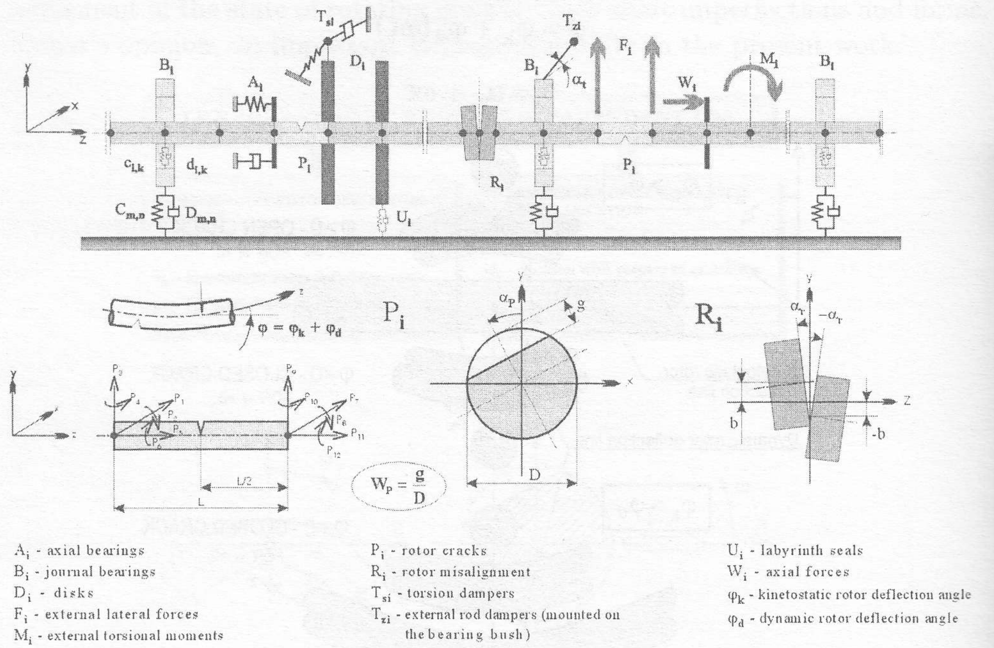

Figure 6. A computer system NLDW for non-linear analysis of rotor dynamics with imperfections, disks, slide bearings and supporting structure.

the case of a large research stand (leaving out the real objects such as power engineering turbosets) there are practically no way, due to operational safety and costs, of conducting investigations into the influence of propagation of real cracks in shafts on dynamic properties of the entire machine. Available in literature experimental data [1-15] regard, in principle, very small and simple (modeling) objects which are difficult to be transferred, due to the effect of scale and other structure, onto the bigger and more complex objects. Heaving that in mind assumed has been a concept, according to which a verified model accompanied by the computer code of a large, multi-supported rotating machine (without shaft cracks), in combination with a widely used

FEM model of the crack itself, can be a source of reliable information regarding simulations of crack propagation in the entire machine. Such idea is schematically presented in Fig. 10. Obviously the author of the present paper is aware of the fact that combination of two fully reliable and separately verified models does not necessarily means that the resulting model, being the sum of the two, will also be reliable.

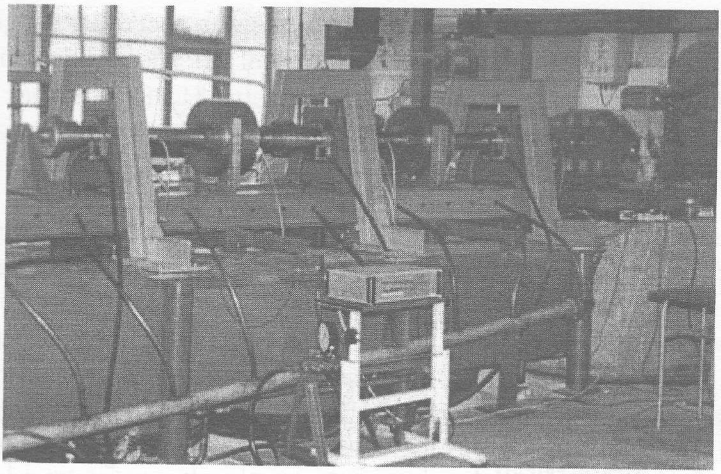

Figure 7. A photograph of a multi-scale research rig where experimental verification of NLDW system was conducted. Object assumed in further investigations.

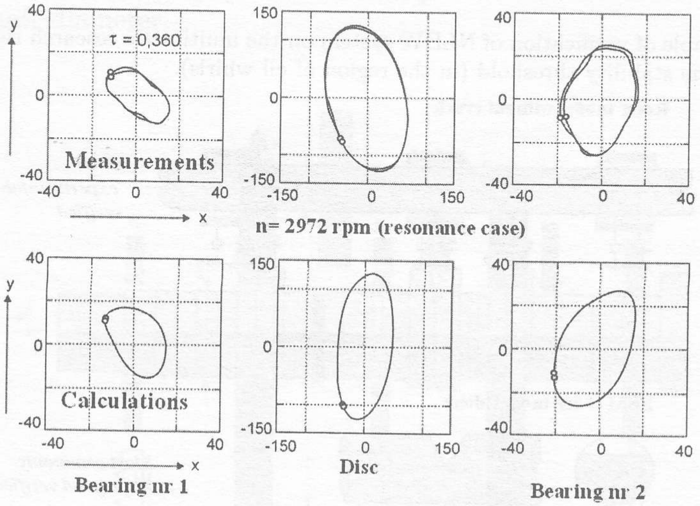

Figure 8. Example of verification of NLDW system on the multi-scale research rig in the range of its stable operation (in resonance).

3 Object of investigations

Let's assume in further investigations the object presented in Fig. 7. It is a three-supported rotating machine with a shaft diameter $d = 0.1$ m, its total length $L = 3.2$ m with two discs of diameter $D = 0.4$ m, supported in three slide bearings with cylindrical clearance. Matrices of dynamic influence of supporting structure have been experimentally measured. The system is influenced by two,

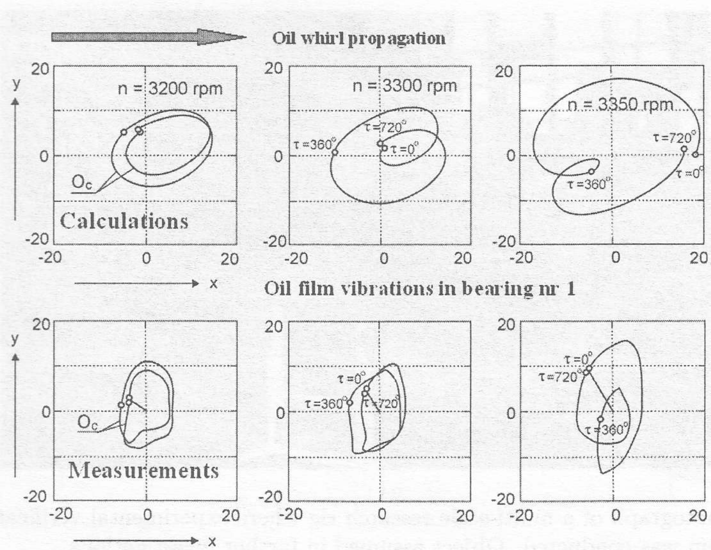

Figure 9. Example of verification of NLDW system on the multi-scale research rig after surpassing the stability threshold (in the region of oil whirls).

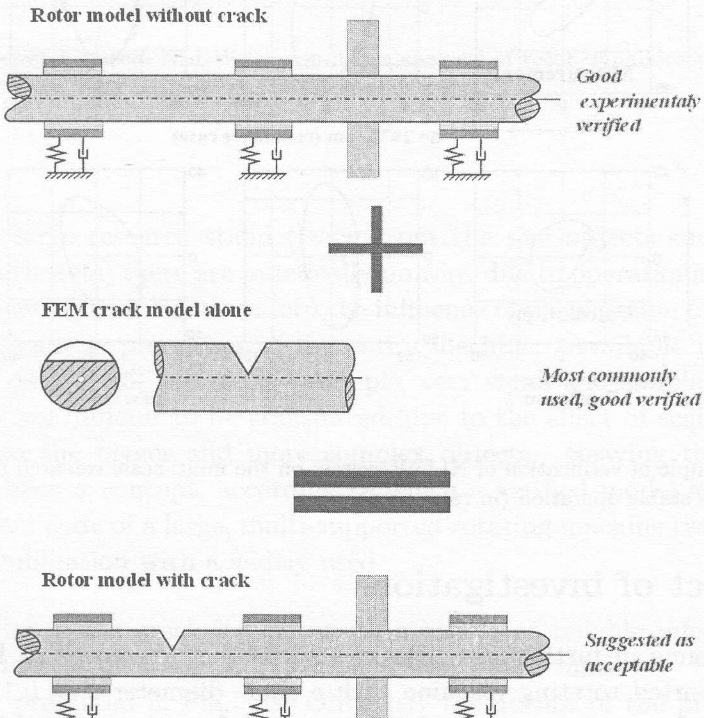

Figure 10. Illustration of a general concept assumed in the paper: a verified model of the rotating machine itself (without shaft crack) in combination with a generally assumed model of the cracked element itself can in effect form a reliable research tool for simulation of the influence of crack propagation.

counteracting in the phase domain, external excitation forces F1 and F2, which are the effect of action of mass imbalance $M1 = M2 = 0.0154$ kg applied to both discs of the radius $r = 0.18$ m. In Fig. 11 presented is FEM discretisation of the line of rotors in assumed object together with location of external excitation forces.

Let's assume now that in the system, defined in such a fashion, there can exist interchangeably two cracks, namely **crack** situated in the rigid clutch, hence in the vicinity of a bearing No. 2 and **crack 2** situated in the middle part of the line of rotors between bearings Nos. 2 and 3, hence in the part where large shaft displacements are expected. Obviously, the cracks **crack 1** and **crack 2**will assume a different depth and will be situated in various locations on the circumference with respect to the plane of action of external excitation forces. Investigations will be conducted for four selected angles α_p of circumferential location of a crack, as illustrated in Fig. 11. In the present work assumed has been a definition of a relative crack depth W_p assumed as a ratio of the crack depth to a shaft diameter.

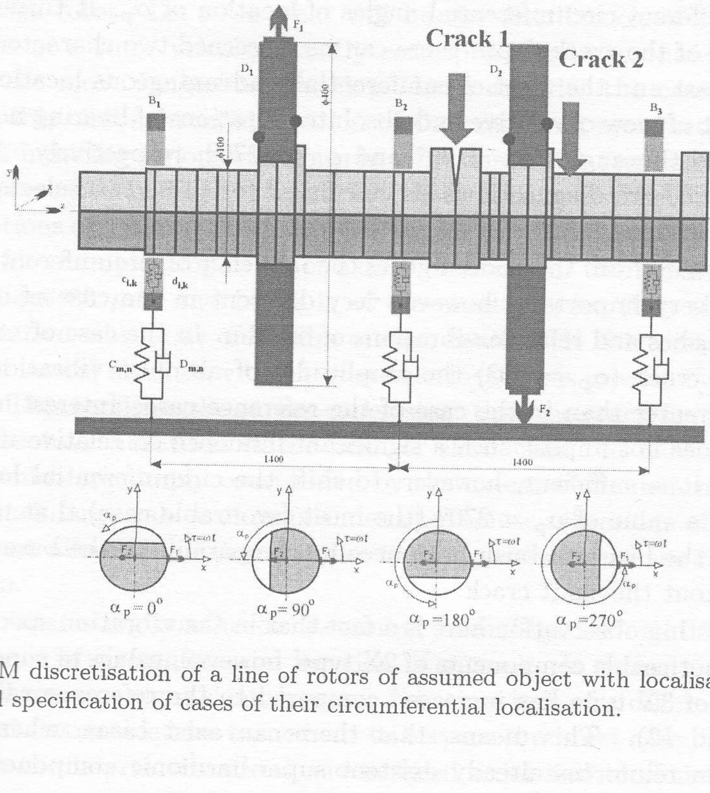

Figure 11. FEM discretisation of a line of rotors of assumed object with localisation of cracks and specification of cases of their circumferential localisation.

4 Investigations into the influence of crack location and propagation

4.1 Influence of circumferential location of crack on transverse vibrations

Let's start our considerations from depicting the pattern of vibrations in the assumed object without cracks in the rotor shaft. Such case is regarded as a reference, very helpful in comparative analysis of cases with the crack. The results of calculations for the reference (base) case, in the form of so called diagnostic card, have been presented in Fig. 12. Calculations have been conducted for the system resonance velocity in the stable range of rotor operation (below the stability threshold limit). It is worth noticing here that assumed multi-supported rotor configuration generates dominant synchronous components of 1X type, but also generates notable super-harmonic components of 3X type (which is perceptible particularly in the case of absolute vibrations of the bearings bushes No. 1). Let's assume now, that in the assumed object we are dealing with a crack situated in the clutch of the line of rotors, i.e. with the case of a **crack1** , as denoted in Fig. 11. Conducted has been a series of calculations for different crack depths and different circumferential angles of location of α_p. It turned out that for each value of the crack depth there can be discerned two characteristic cases, namely the least and the most circumferentially advantageous location of crack, from the point of view of relative and absolute vibrations of bearing nodes, which corresponds to the angles $\alpha_p = 90°$ and $\alpha_p = 270°$, respectively. In Figs. 13 and 14 presented are diagnostic cards calculated for these two selected cases for the lateral vibrations.

As can be seen from the above figures the influence of circumferential location of a crack is very important, however very different in the case of absolute vibrations of bushes and relative vibrations of oil film. In the case of unfavourable location of a crack ($\alpha_p = 90°$) the amplitudes of absolute vibrations are over three times greater than in the case of the reference case. Interesting, that the same crack does not impose such a significant influence on relative vibrations of the oil film. It is sufficient, however, to shift the circumferential location of a crack toward a value of $\alpha_p = 270°$ (the most favourable case) that the absolute vibrations of the bearings bush have become comparable to the base case, hence the case without the shaft crack.

An interesting observation here is a fact that in the vibration spectrum we do not observe noticeable components of 2X type, however, values of super-harmonic components of 3X type has increased compared to the reference case (compare Figs. 12 and 13). This means, that there can exist cases, where the crack in the system reinforces already existent super-harmonic components and not necessarily generates a typical and expected spectrum component of 2X type. The above conclusion can be very helpful in the case of monitoring systems of the state of the object, which bases generally on the analysis of spectrum components of 2X type (in the case of lateral vibrations). In Fig. 15 schematically is presented the explanation why circumferential location of a crack with respect to the plane of action of external forces exerts such an influence on system operation.

4.2 Coupled forms of vibrations induced by a crack

Let's proceed now to considerations related to coupled forms of vibrations and assessment of their applicability as a diagnostic determinant of a dynamic state of a large rotating machine. The first question to be answered is whether the rotor shaft crack is capable of generating, apart from apparent changes in distributions of lateral vibrations, significant axial and torsional vibration, despite the fact that in these directions there are no action of any external forces? In order to provide answer to that question conducted have been investigations, the results of which have been presented in Figs. 16-18.

From Fig. 16 it results that a crack with a moderate depth $W_p = 0.2$ induced a second, very strong and dangerous resonance in the system **R2** at lower rotational velocity of the rotor. Let's see what will be the distribution of coupled forms of vibrations at a rotor velocity exactly corresponding to resonance **R2**, hence potentially the most dangerous situation. Let's consider the case of least and most favourable circumferential location of the crack and see whether the crack is capable of inducing serious couplings in vibrations. The results of calculations have been presented in Figs. 17 and 18. It results from them that the crack under conditions of a strong resonance can render very strong couplings of system vibrations, for example amplitude of axial vibrations in the vicinity of bearing No. 2 is about 80 μm, and hence is of the similar order than the amplitude of lateral relative vibrations of oil film of that bearing! It is interesting that the amplitude of lateral absolute vibrations at that location is greater by one order of magnitude. This confirms the earlier conclusion, that the crack more formidably influences absolute vibrations of the bush or shaft than the relative vibrations of oil film, which are simply constrained by the extent of the bearing clearance. Obviously, such large values of amplitudes of absolute vibrations of the shaft or bushes are not allowable in practice due to the possibility of the failure of the entire system.

Another interesting conclusion stemming from Figs. 17 and 18 is related to the influence of circumferential location of a crack. It turned out that such influence still remains significant, however this time it is of a different tendency. Hitherto most favourable case turned out to become the worst case. That remark regards both the lateral vibrations and coupled axial and torsional vibrations.

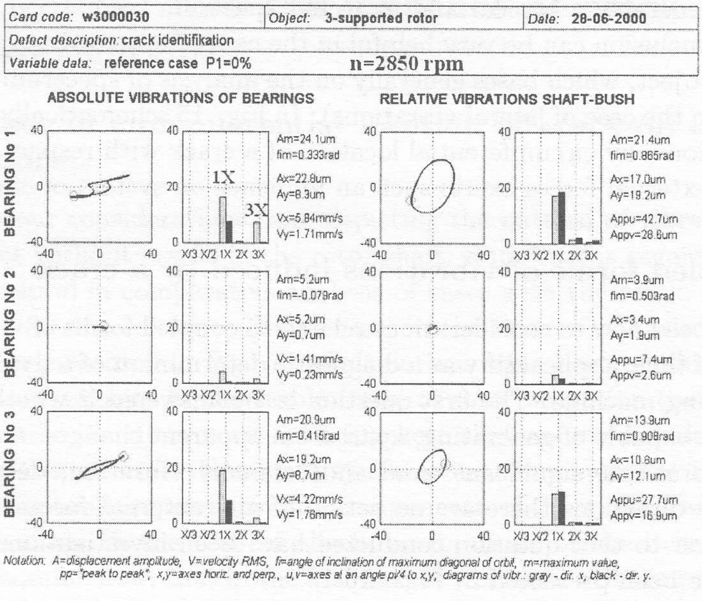

Figure 12. The results of calculations in the form of diagnostic card for the reference case without the crack. Trajectories and spectra of absolute bush vibrations and relative oil film vibrations for three bearings of multi-supported rotor from Figs. 7 and 11.

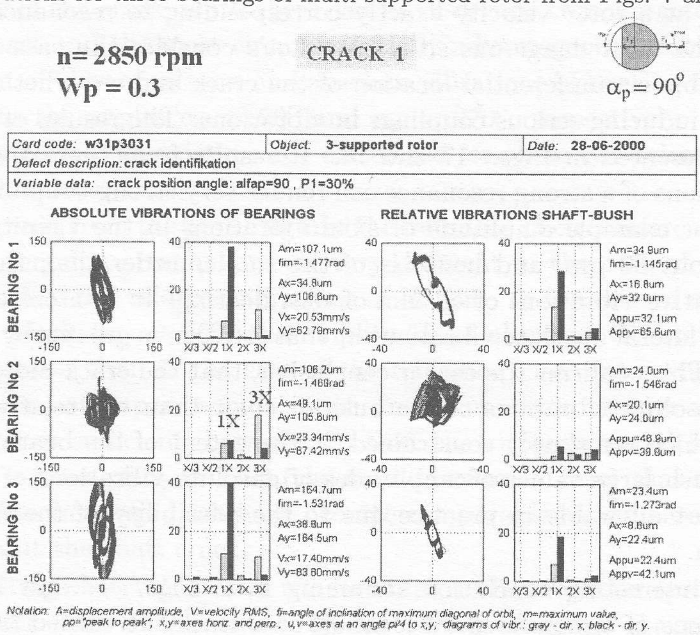

Figure 13. Diagnostic card calculated for a crack with the depth of $W_p = 0.3$ and the least favourable circumferential location $\alpha_p = 90°$.

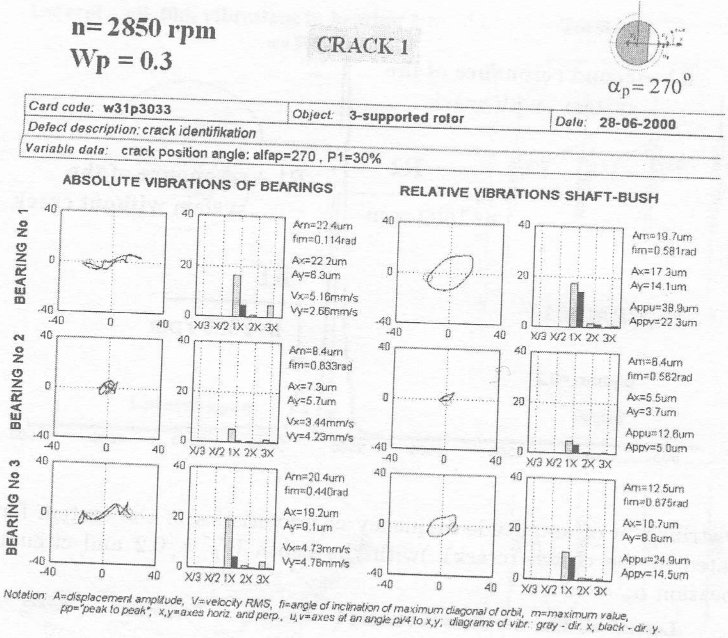

Figure 14. Diagnostic card calculated for a crack with the depth of $W_p = 0.3$ and the most favourable circumferential location $\alpha_p = 270°$.

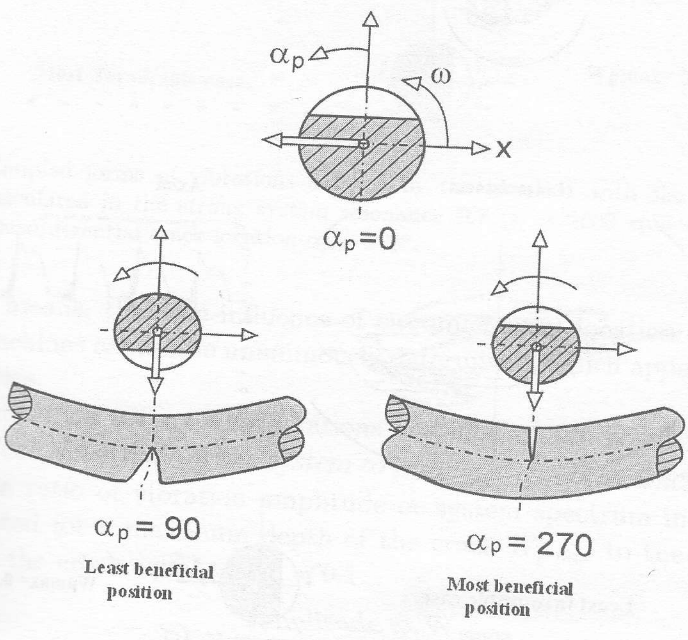

Figure 15. Explanation of the reasons of a strong circumferential interaction of crack location. For $\alpha_p = 90°$ the external forces and kinetostatic deflection aid the crack to remain in the open state, for $\alpha_p = 270°$ situation is opposite – the crack has a tendency to close and at the same time to impair its influence on the system.

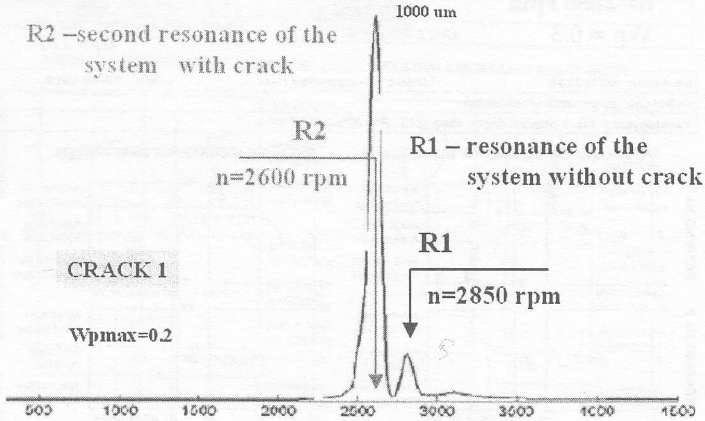

Figure 16. Distribution of amplitude-frequency characteristics of the system for the crack situated in the clutch (crack1)with the depth $W_p = 0.2$ and circumferential crack location $\alpha_p = 90°$.

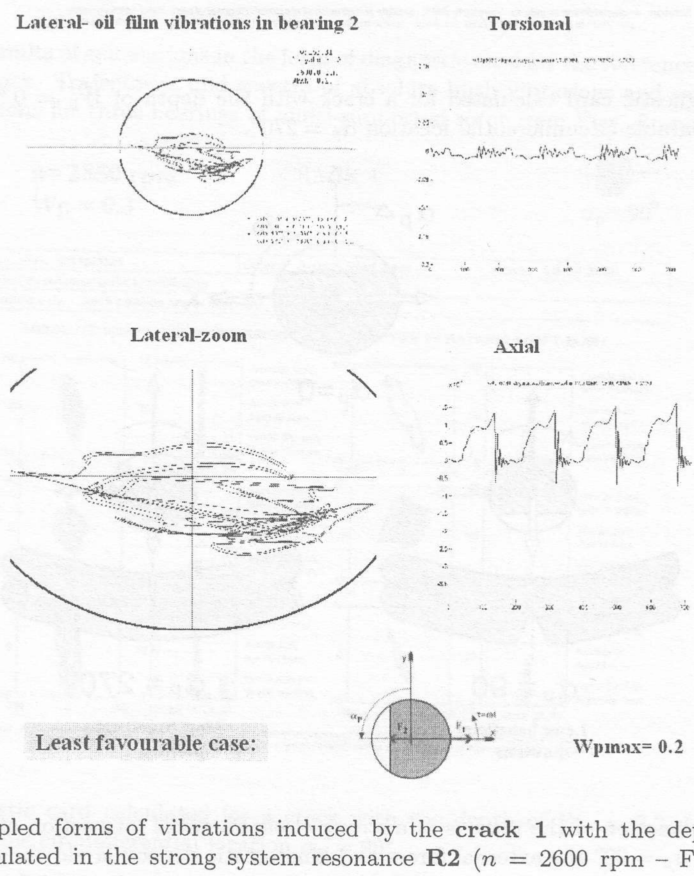

Figure 17. Coupled forms of vibrations induced by the **crack 1** with the depth $W_p = 0.2$ calculated in the strong system resonance **R2** (n = 2600 rpm – Fig. 16) for the circumferential crack location $\alpha_p = 90°$.

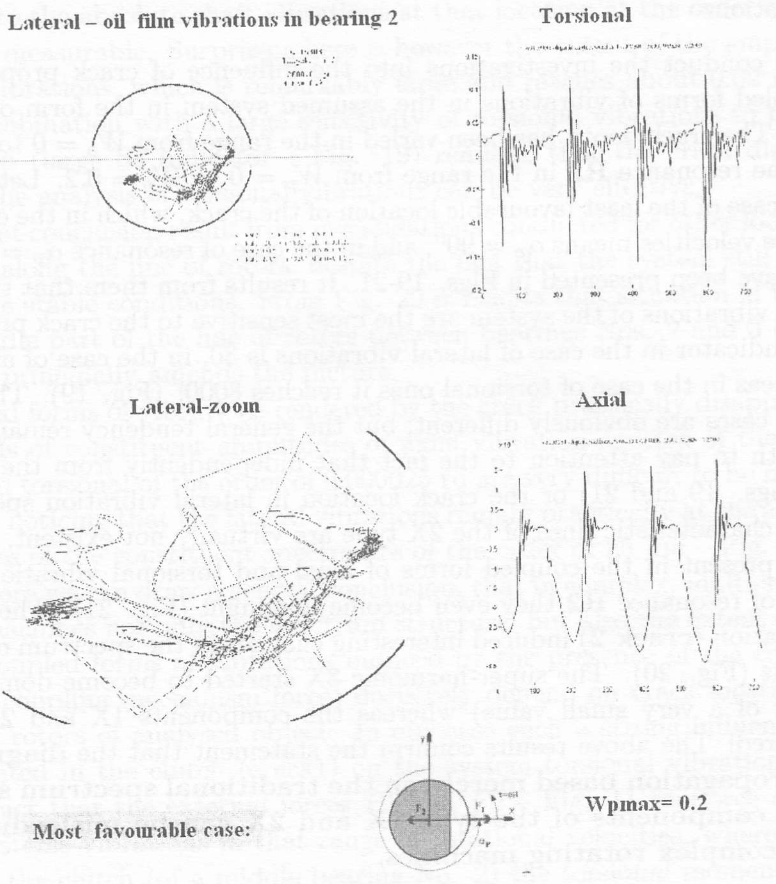

Figure 18. Coupled forms of vibrations induced by the **crack 1** with the depth $W_p = 0.2$ calculated in the strong system resonance **R2** (*n = 2600 rpm – Fig. 16*) for the circumferential crack location $\alpha_p = 270°$.

The above means, that the influence of circumferential location of the crack in complex machines cannot be unanimously determined, which apparently impedes its diagnostics.

Let's introduce to our considerations the idea of the coefficient **SI**, which determines the sensitivity of the system to propagation of the crack depth. Let's define it as a ratio of vibration amplitude or system spectrum in the specified mode calculated for a maximum depth of the crack W_{pmax} to the amplitude at the depth of the crack equal to $W_p = 0.1$.

$$\mathbf{SI} = \frac{\text{Amplitude at } W_{pmax}}{\text{Amplitude at } W_p = 0.1}.$$

SI is obviously a precisely conventional indicator, which will be useful in further considerations.

Let's conduct the investigations into the influence of crack propagation on the coupled forms of vibrations in the assumed system in the form of vibration spectra. The crack depth has been varied in the range from $W_p = 0$ to $W_p = 0.4$, and in the resonance **R2** in the range from $W_p = 0$ to $W_p = 0.2$. Let's consider now the case of the least favourable location of the crack, which in the case of sub-resonance velocities means $\alpha_p = 90°$, and in the case of resonance $\alpha_p = 270°$. The results have been presented in Figs. 19-21. It results from them that the coupled torsional vibrations of the system are the most sensitive to the crack propagation. The **SI** indicator in the case of lateral vibrations is 30, in the case of axial ones – 100 whereas in the case of torsional ones it reaches 8000! (Fig. 19). These values for other cases are obviously different, but the general tendency remains similar. It is worth to pay attention to the fact that independently from the operation range (Figs. 19 and 21) or the crack location in lateral vibration spectra (Fig. 20) such characteristic lines of the 2X type are virtually not existent. These are however present in the coupled forms of axial and torsional vibrations, and in the case of resonance **R2** they even become dominant (Fig. 21). The change of crack location (**crack 2**) induced interesting changes in the spectrum of torsional vibrations (Fig. 20). The super-harmonic 3X started to become dominant here (however of a very small value) whereas the components 1X and 2X literally disappeared! The above results confirm the statement that the **diagnostics of crack propagation based merely on the traditional spectrum structure, i.e. the components of the type 1X and 2X can be misleading in the case of complex rotating machines.**

Let's come back now to the issue of the extent of amplitudes of coupled forms of vibrations, hence the key question, whether the shaft crack is capable of generating **significant** vibrations also in directions, where external excitation forces are not acting. Important is not only the spectrum structure itself, which was discussed earlier, but also the extent of a potential signal. As has been concluded, in the case of resonance rotor revolutions (for example in resonance **R2** – Fig. 16) such coupling were remarkably strong (Figs. 17, 18 and 21). A strong and dangerous system resonance induced by the crack is a very interesting case, but from the point of view of machine operation rather non-typical, as in practice all efforts are made to surpass the resonance reasonably quickly. Let's see then how that issue is reflected in the case of typical, sub-resonance, hence stable, operational conditions of the system (which in the considered case corresponds to the rotor velocity of for example $n = 2000$ rpm). As stems from Fig. 10 in the case of the crack with a large depth of $W_p = 0.4$ situated in the rotor clutch (**crack 1**) generated have been axial vibrations in the vicinity of a middle bearing No. 2 with the amplitude of about 1.6 μm, which is relatively little compared to the absolute shaft vibrations at that location at the order of 15 μm, however is measurable. Surprising here is however the extent of the amplitude of torsional vibrations, which is remarkably large and reaches about 0.02 rd. That fact, in combination with a large sensitivity of torsional vibrations to the crack propagation (large **SI** indicator – Fig. 19) renders, that the crack diagnostics based on the analysis of torsional vibrations can be very effective.

Different conclusion result from investigations conducted for other localization of a crack along the line of rotors, despite the fact that the system still operates in the same stable conditions. From Fig. 20 it results that situation of the crack in the middle part of the line of rotors between bearings Nos. 2 and 3 (**crack 2** – Fig. 11) principally amends the picture.

Coupled forms of vibrations rendered by the crack practically disappeared, as the extends of constituent amplitudes of axial vibration spectra of the order of 0.1 μm and torsional of the order of 0.000025 rd are very difficult to be measured! It is worth noticing that the lateral vibrations remain practically at the same level (amplitudes of the constituent spectra are of the order of 25 μm – Fig. 20).

Therefore we can draw a general conclusion, that in complex, multi-supported rotating machines not only the spectrum structure, but also the extent of amplitudes of coupled forms of vibrations induced by the presence of the crack (and hence the coupling mechanism force) decisively depend on crack location along the line of rotors of analysed object. In our case such a strong influence of the crack situated in the clutch (crack1) on the system torsional vibrations results from the fact that the external forces $F1$ and $F2$ – Fig. 11 reinforce a natural form of systems vibrations in that range of rotational velocities, whereas in the vicinity of the clutch (of a middle bearing No. 2) the torsional moments on the shaft cumulate.

5 Investigations of crack propagation after surpassing the stability limit

Hitherto conducted investigations have been performed in the sub-resonance range of rotational velocities of the assumed object, and also in the strong resonance **R2** induced by the presence of the crack. Investigations referred, generally speaking, to the stable regime of operation of the entire system.

Let's consider now investigations into the influence of crack propagation after surpassing the threshold of system stability, hence under conditions of existence of oil whirls. In the case of assumed objects such conditions are already present at rotor velocities of about $n = 4700$ rpm. The results of calculations are presented in Figs. 22-24. In Fig. 22 presented is the distribution of trajectory deformation of the oil whirl rendered by the crack propagation and calculated for two

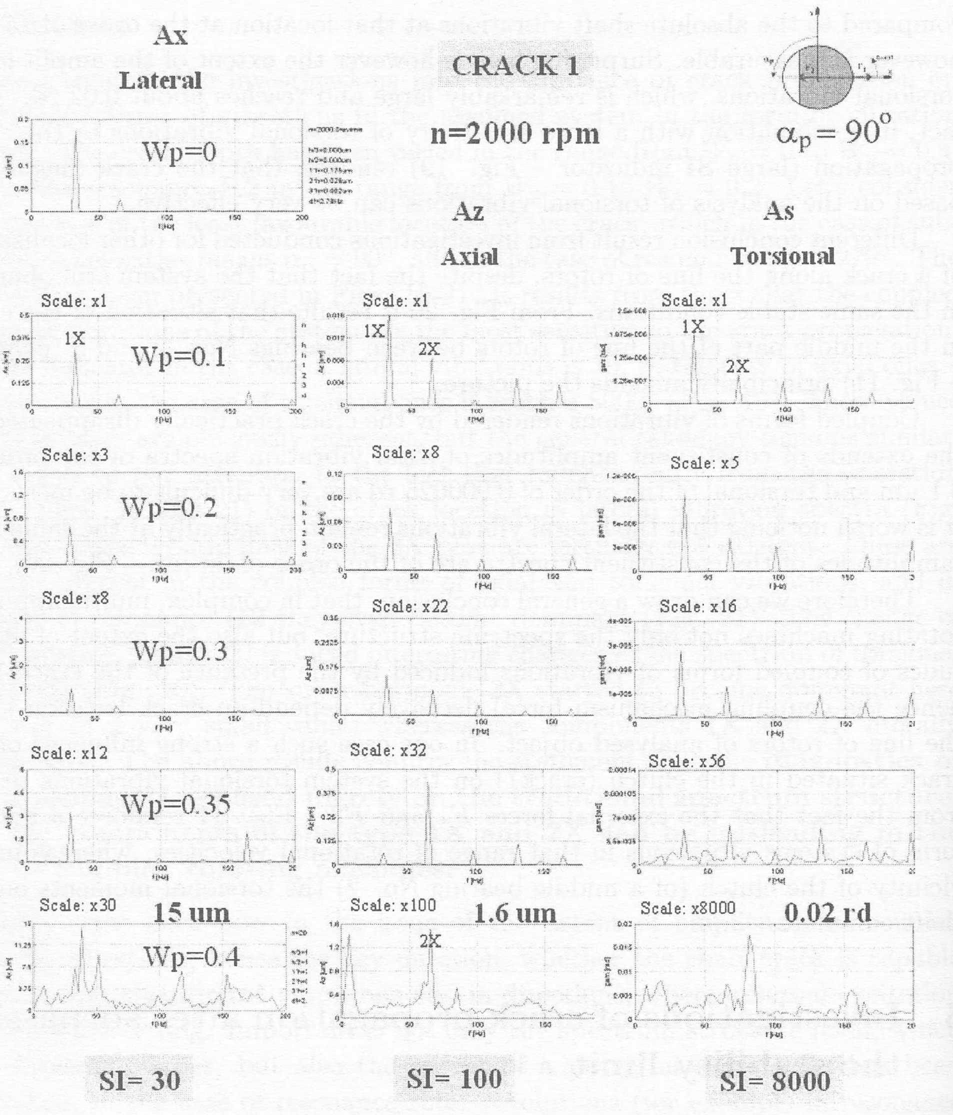

Figure 19. Coupled spectra of lateral-axial-torsional vibrations induced by the **crack 1** with the depth ranging from $W_p = 0$ to $W_p = 0.4$. Stable sub-resonance range of rotor operation. A case of strong coupled torsional vibrations.

characteristic circumferential crack locations ($\alpha_p = 90^o$ and $\alpha_p = 270^o$). Worth mentioning here is a small influence of circumferential location of the crack, which is a completely different conclusion as compared to the earlier considered cases (in the stable range of machine operation).

More interesting information is supplied from the analysis of coupled spectra

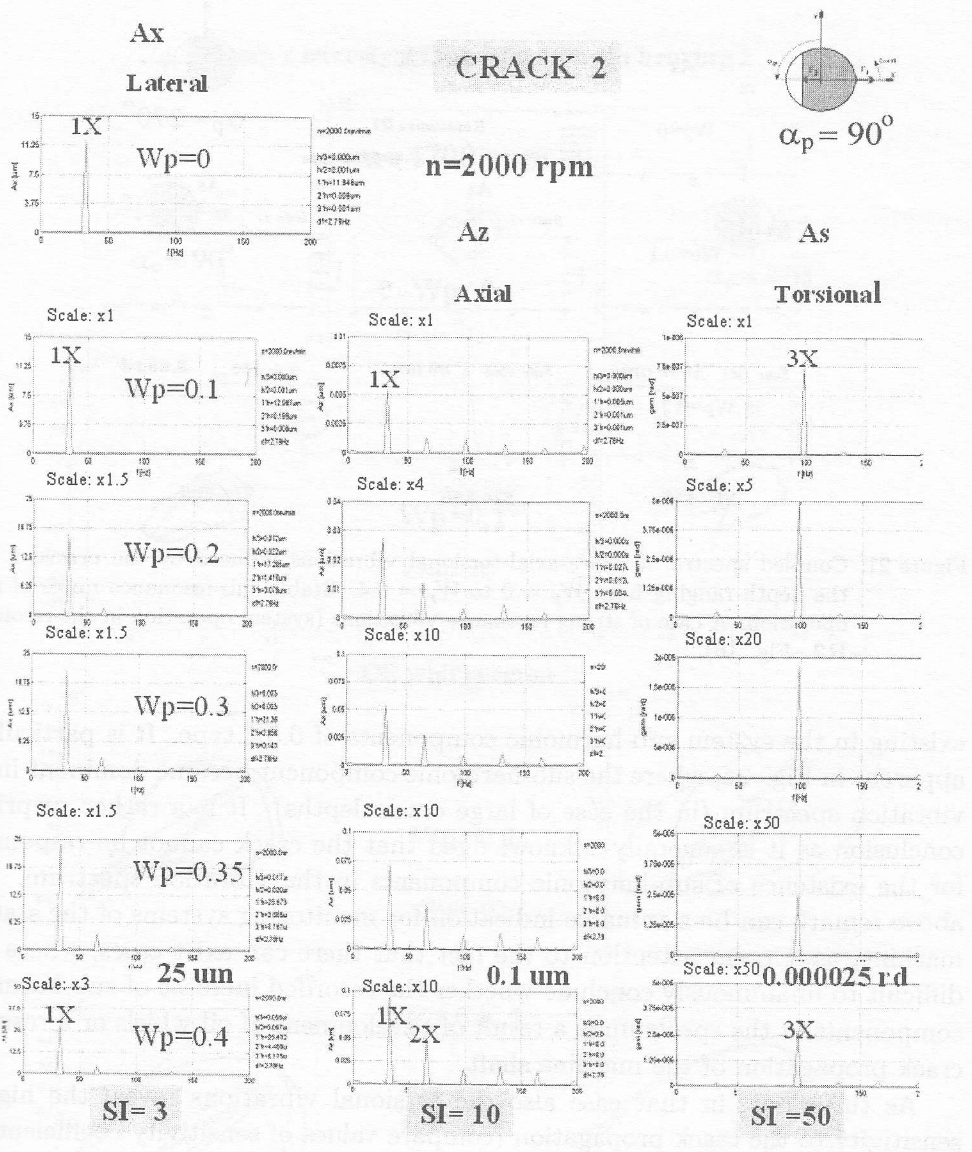

Figure 20. Coupled spectra of lateral-axial-torsional vibrations induced by the crack **crack 2** with the depth ranging from $W_p = 0$ to $W_p = 0.4$. Stable sub-resonance range of rotor operation. A case of small values of axial and torsional vibrations (diminishing of the coupling mechanism).

of vibrations rendered by the crack presence – Figs. 23 and 24. In the case of oil whirls characteristic are sub-harmonic spectral components close to 0.5X. It turns out, that the crack with the increase of its depth sustains and reinforces

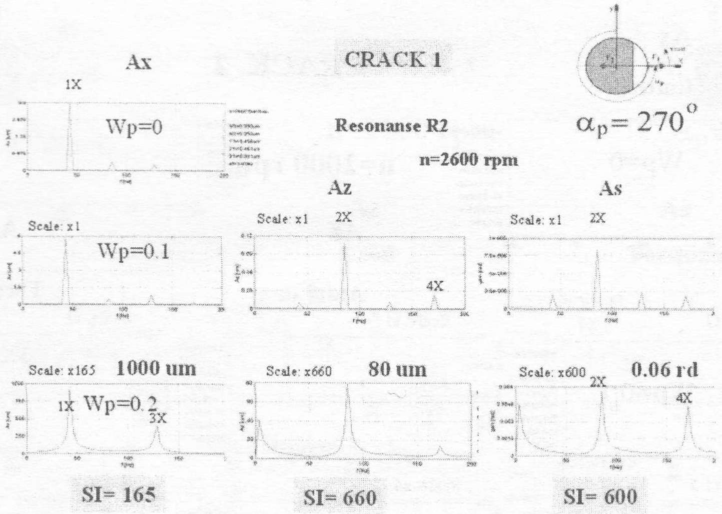

Figure 21. Coupled spectra of latera-axial-torsional vibrations induced by the **crack 1** with the depth ranging from $W_p = 0$ to $W_p = 0.4$. Stable sub-resonance range of rotor operation. A case of strong resonance vibrations (system operation in the resonance **R2** –Fig. 16).

existing in the system sub-harmonic components of 0.5X type. It is particularly apparent in Fig. 24, where the sub-harmonic component became dominant in the vibration spectrum (in the case of large crack depths). It is a rather surprising conclusion as it is generally acknowledged that the crack cannot be responsible for the existence of sub-harmonic components in the vibration spectrum. The above remark can be a valuable indication for monitoring systems of the state of machine, as it point attention to the fact that there can exist cases, where it is difficult to unanimously conclude whether the recorded increase of sub-harmonic components in the spectrum is a result of development of oil whirls or a result of crack propagation of the machine shaft.

As turns out, in that case also the torsional vibrations reveal the highest sensitivity to the crack propagation (compare values of sensitivity coefficients **SI** – Figs. 23 and 24).

The most surprising is however the fact that after surpassing the system stability threshold even large values of the shaft crack, independently from its location on the line of rotors, are not capable of generating significant coupled forms of axial and torsional vibrations (amplitudes of axial vibrations are of the order of 0.2 to 0.5 μm, whereas the torsional vibrations are of the order of 0.00005 rd - Figs. 23 and 24). That conclusion is interesting because under same conditions amplitudes of constituent spectra of lateral vibrations depend strongly on the crack location (10 μm for the crack **crack 1** and 150 μm for **crack 2** –

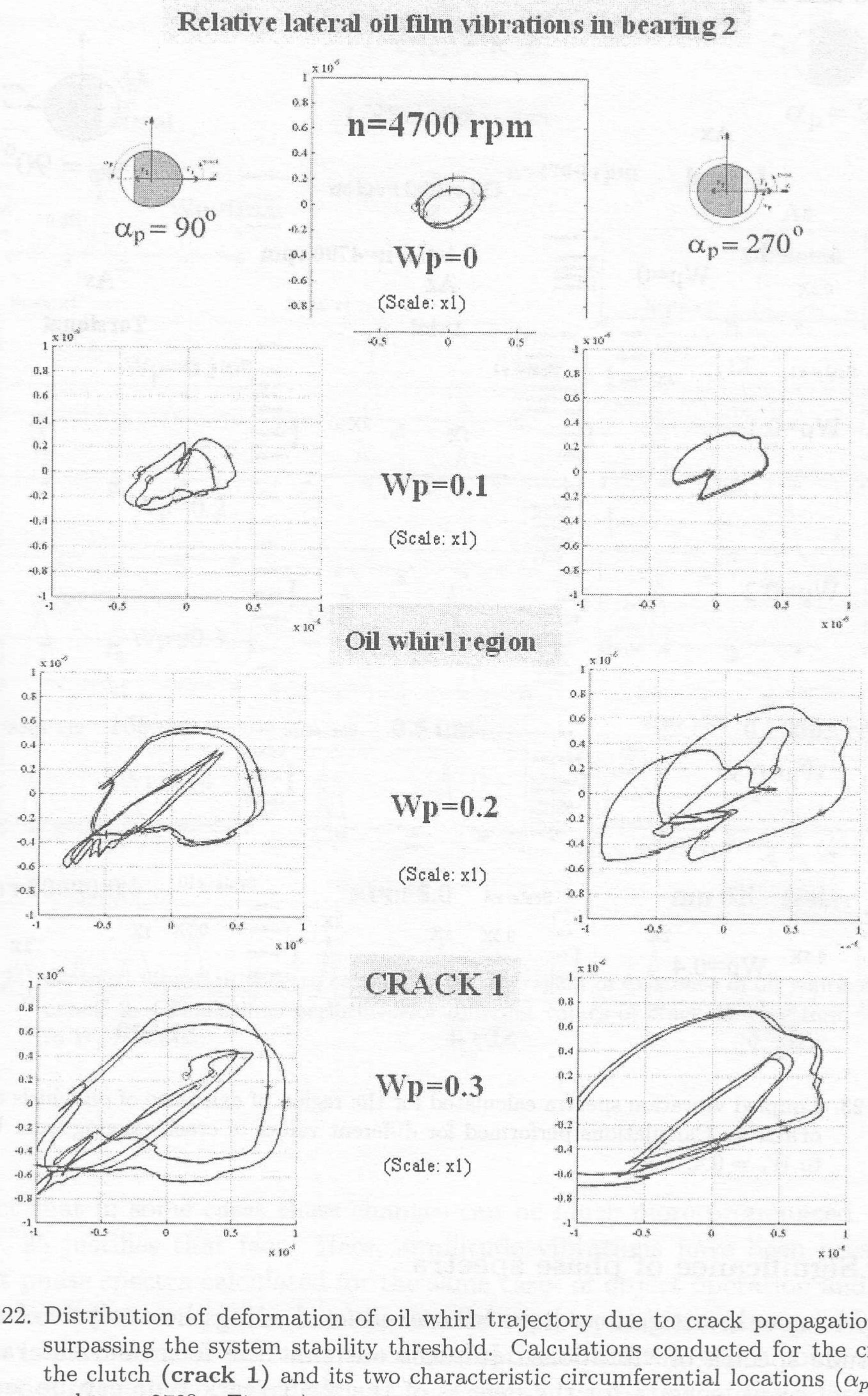

Figure 22. Distribution of deformation of oil whirl trajectory due to crack propagation after surpassing the system stability threshold. Calculations conducted for the crack in the clutch **(crack 1)** and its two characteristic circumferential locations ($\alpha_p = 90°$ and $\alpha_p = 270°$ – *Fig. 11*).

Figs. 23 and 24).

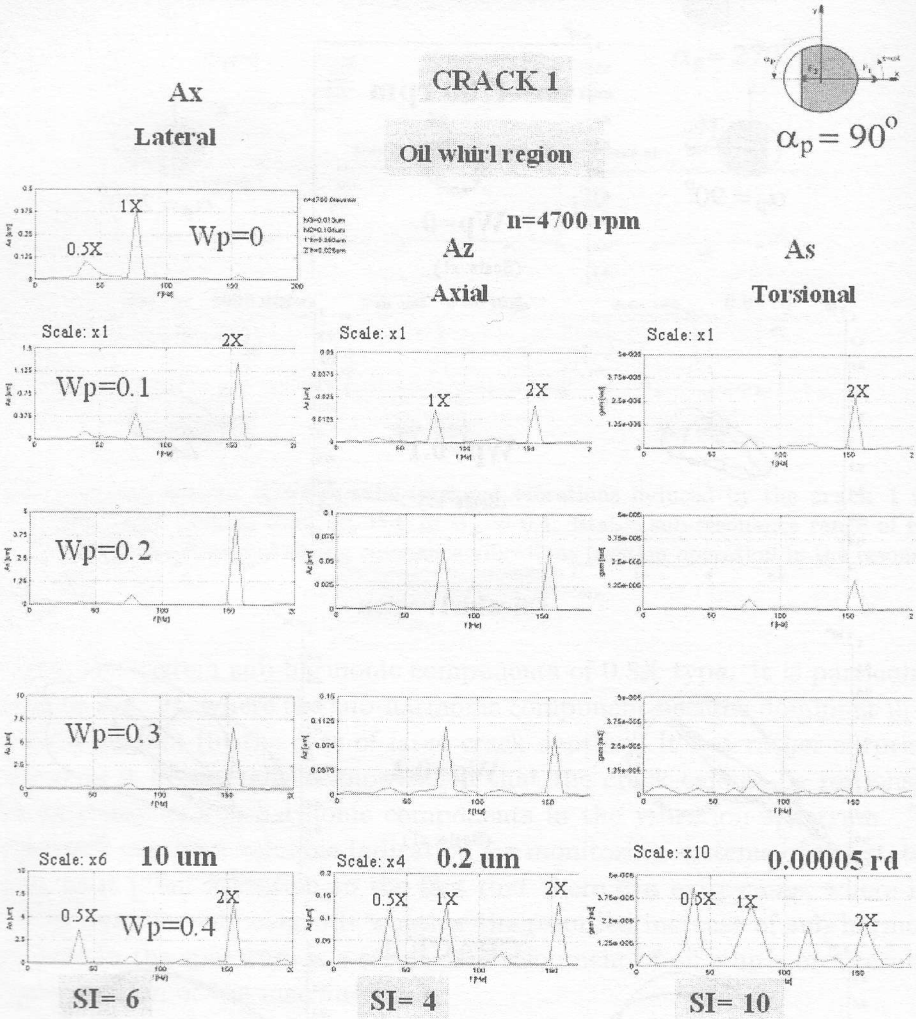

Figure 23. Coupled vibration spectra calculated for the region of existence of oil whirls and the **crack 1**. Calculations performed for different values of crack ranging from $W_p = 0$ to $W_p = 0.4$.

5.1 Significance of phase spectra

Up to now investigations have been conducted using the analysis of merely amplitude spectra of vibrations. It turns out that the form and the range of change of phase spectra for the defects of the shaft crack type can be an additional and valuable carrier of diagnostic information, especially in the light of

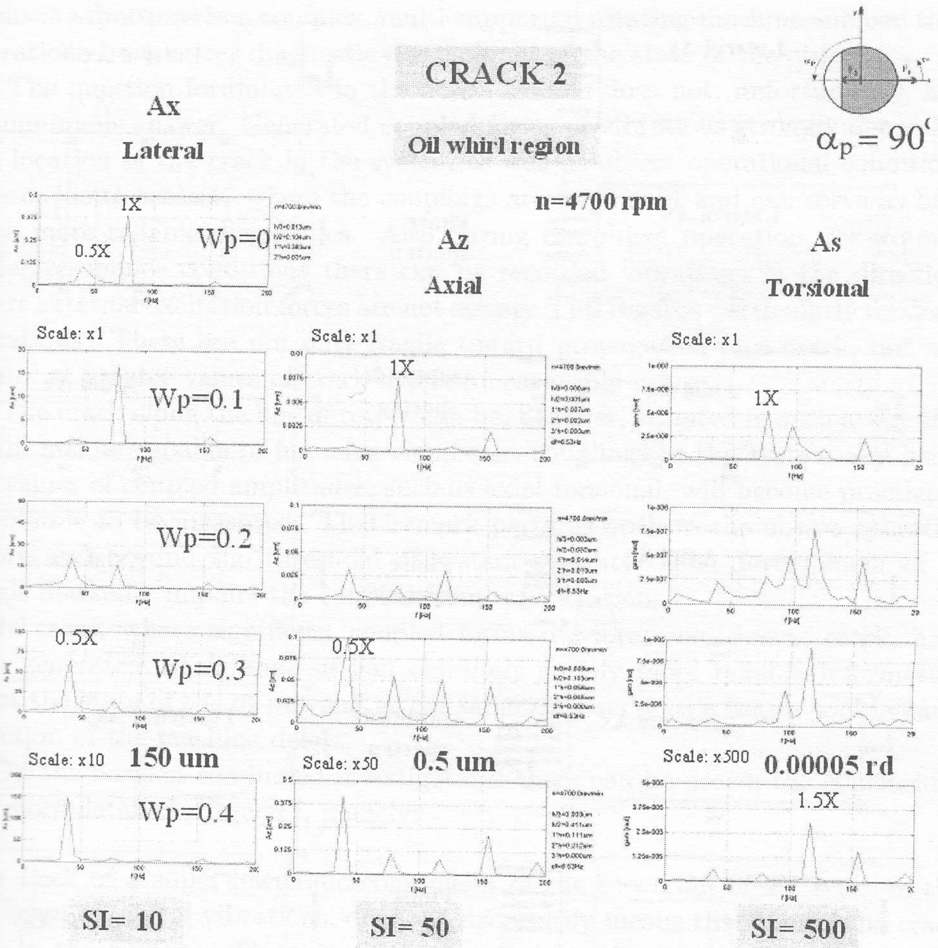

Figure 24. Coupled vibration spectra calculated for the region of existence of oil whirls and the **crack 2**. Calculations performed for different values of crack ranging from $W_p = 0$ to $W_p = 0.4$

the fact that in some cases these changes can be much more pronounced. Data in Fig. 25 justifies that fact. Here, amplitude vibrations have been presented against phase spectra calculated for the same cases of object operation and identical defect (shaft crack). As can be seen, the representation of changes of phase spectra is much greater and assumes a qualitative character, which could enable diagnostics of analysed defect such as for example assessment of the influence of circumferential location of crack.

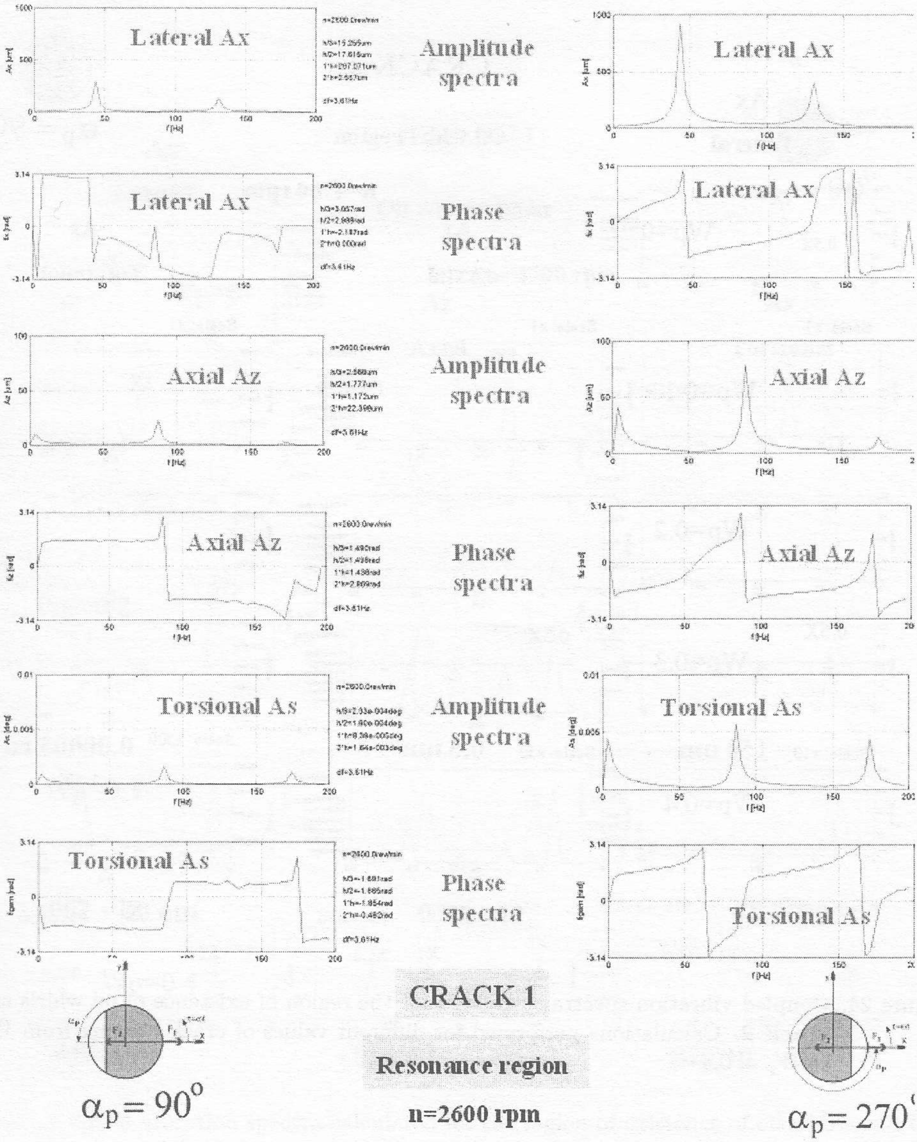

Figure 25. Significance of phase spectra. Phase spectra tabulated against amplitude spectra for all coupled forms of vibrations induced by the crack **crack 1** with the depth $W_p = 0.2$. Calculations performed in the resonance **R2** of the object from Fig. 11.

6 Discussion and concluding remarks

Let's return now to the principal thesis in the paper and the question whether the defect of the shaft crack type is capable of generating significant coupled forms of vibrations in a complex, multi-supported rotating machine and can these vibrations be a better diagnostic determinant of the state of the object.

The question formulated in the above fashion does not, unfortunately, have a unanimous answer. Generated coupled forms of vibrations strongly depend on the location of the crack in the system as well as object operational conditions. There exist locations, where the couplings are important and can serve as basis for a more reliable diagnostics. Also during the object operation, for example under resonance conditions there can be recorded vibrations in the directions where external excitation forces are not acting. This regards particularly torsional vibrations. These are not only fragile toward propagation of a crack, but also attain, at greater values of crack depths, measurable values.

The crack along the line of rotors can be, however, situated in such a way that it will not be capable of inducing significant couplings in the system and hence the values of coupled amplitudes, such as axial-torsional, will become practically impossible to be measured. That remark pertains both to the object operation before and beyond the threshold of system stability. The development of oil whirls markedly impairs the coupled forms of vibrations.

In cases, where significant coupled forms of vibrations, due to crack, have been generated, their analysis can definitely supply more reliable information about the state of the object and at the same time serve as a better tool in early detection of the machine defect.

On the basis of conducted investigations there can be drawn the conclusions of a more detailed character, namely:

- Lack of a super-harmonic component of the spectrum of 2X type, in the case of lateral vibrations, does not necessarily means that there is no crack in the system. There are situations, where the crack reinforces already existing components of the higher orders at the presence of traces only of the second harmonical.
- After surpassing of the system stability threshold, under conditions of existence and development of oil whirls, the crack shaft reinforces characteristic sub-harmonic components in the vibration spectrum. Values of amplitudes of coupled forms of axial-torsional vibrations are very small.
- In the case, when significant coupled forms of vibrations have been generated, the components of spectrum of 1X and 2X type, in the case of axial and torsional in particular, are particularly prone to the crack propagation compared to classical 2X components of lateral vibrations.
- Circumferential location of a crack has a significant, however differentiated influence on the system operation under stable operation conditions. After surpassing of the stability threshold that effect disappears.

- Phase spectra of vibrations can serve as a valuable supplement to amplitude spectra.

It is worth nothing that despite the fact that the analysis of torsional vibrations and phase spectra was much more effective in several cases, but its practical implementation is impaired by measurement difficulties on real objects and the cost related to these activities.

Summarising the considerations conducted in the present work we can conclude that in the case of large rotating machines a unanimous and early detection of defects of the shaft crack type is not always possible. There are situations, where the lack of characteristic components of the vibration spectrum (such as 2X type) or the lack of coupled forms of vibrations in the systems does not automatically implies that there are no cracks. This implies the necessity of individual treating and analysis of each object and also places high requirements to the contemporary diagnostic systems. Such requirements are even higher in the light of the fact that usually we do not know *a priori* about the crack location nor about amended resonance velocities of the object, amended by its influence, which indicates a potential possibility of ambiguous relations of the defect-symptom type and difficulties in identification of adequate diagnostic determinant of the state of the machine. On the other hand, however, modern methods of computer analysis, based on new models and computer codes, as shown in the results in the present paper, lay grounds for a better and safer diagnostics of emergency states of rotating machines in future.

Received 5 October 2003

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