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Transactions of the Institute of Fluid-Flow Machinery have primarily been established to publish papers from four disciplines represented at the Institute of Fluid-Flow Machinery of Polish Academy of Sciences, such as:

- Liquid flows in hydraulic machinery including exploitation problems,
- Gas and liquid flows with heat transport, particularly two-phase flows,
- Various aspects of development of plasma and laser engineering,
- Solid mechanics, machine mechanics including exploitation problems.

The periodical, where originally were published papers describing the research conducted at the Institute, has now appeared to be the place for publication of works by authors both from Poland and abroad. A traditional scope of topics has been preserved.

Only original and written in English works are published, which represent both theoretical and applied sciences. All papers are reviewed by two independent referees.

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Editorial

These Special Issues of the *Transactions of Fluid-Flow Machinery*, Nos. 113 and 114 contain selected papers from the International Conference on *Turbines of Large Output* devoted to commemorate 100$^{\text{th}}$ Birthday Anniversary of Prof. Robert Szewalski. The conference was held in Gdańsk, Poland on September 22-24, 2003.

The Conference is a continuation of previous conferences held at the Institute of Fluid-Flow Machinery PAS in former years dedicated to technology of steam turbines. Series of conferences bearing the same name took place in the years 1962, 1965, 1968, 1993. In 1997 organised has been a conference on steam turbines and related topics, but with a slightly amended title – *Problems of Fluid-Flow Machinery*. At present at the Institute of Fluid Flow Machinery there are conduced research of fundamental character encompassing both the issues related to steam turbines and fundamentals of power engineering.

Organisers of the present conference have returned to the traditional name, i.e. Conference on *Turbines of Large Output* to mark the respect to the memory of Professor Robert Szewalski (1903-1993), the founder and a director of the Institute of Fluid-Flow Machinery for several years, and initiator of the conference series devoted to the problems of steam turbines.

The Editors are very grateful to the referees of the papers presented in this issue of the *Transactions of Fluid-Flow Machinery*: J. Badur, W. Batko, E. S. Burka, J.T. Cieśliński, P. Doerffer, A. Gardzilewicz, B. Grochal, J. Kiciński, G. Kosman, T. Król, J. Krzyżanowski, J. Mikielewicz, A. Neyman, W. Ostachowicz, R. Puzyrewski, R Rządkowski, J. Świrydczuk, M. Trela, T. Uhl and Z. Walczyk.

We appreciate cordially the manifested authors interest in our conference. Special thanks are conveyed to Ms 'Maya' Bagińska, for her fruitful editorial help related to directing the papers to reviewers and correspondence with the authors of contributions.

Prof. Jarosław Mikielewicz
Editor-in-Chief

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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

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JACEK SOKOŁOWSKI[a], ROMUALD RZĄDKOWSKI[a,b*] and LESZEK KWAPISZ[a]

Influence of the blade length on natural frequencies and modes shapes of two bladed rigid discs on the shaft

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Abstract

The dynamic behaviour of a rotor consisting of two bladed discs on a solid shaft is considered. The effect of shaft flexibility on the dynamic characteristics of the bladed discs and the coupling effects between the shaft and bladed disc modes are investigated. Results presented for various cases with differing blades flexibility show clearly the coupling effects in a bladed disc-shaft system. Calculated natural frequencies obtained from blade, shaft, bladed disc and shaft with two discs are checked to discover resonance conditions and the coupling effects. The calculations show the influence of the shaft on the natural frequencies of the bladed discs up to one nodal diameter frequencies. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameters modes of the single bladed discs. The bending modes of the shaft are coupled with one nodal diameter modes of the bladed discs. It is shown that including the shaft in the bladed discs model is important from the designer's point of view and can change the spectrum of frequencies considerably.

Keywords: Blades; Bladed discs; Shaft

Nomenclature

f	–	frequency, Hz
k	–	number of nodal diameters
N	–	number of rotor blades

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1 Introduction

In turbomachinery the study of vibration characteristics of individual components such as blades, discs and shafts has been established as an important part of the design. However, vibration characteristics of an individual component can change considerably when these components are assembled together to form one system, due to the coupling effects among these constituent components. The vibration characteristics of a blade in a gas or a steam turbine can change due to the flexibility of the discs and that of the shaft carrying the discs. Furthermore there may be interactions between blades on different stages of the turbine due to the flexibility of the discs and the shaft.

Two independent approaches are commonly used to analyze the dynamic behavior of turbomachinery rotating assemblies. In the first, the rotordynamics approach is concerned with disc-shaft systems. The shaft is mostly modeled using beam finite elements without account of disc flexibility then (Berger at el.[1]), or with consideration of the latter (Chivens at el.[3]). In the second approach, the bladed disc approach deals with flexible discs (Rządkowski [4]). There are few papers dealing with free vibrations of the bladed disc on the shaft (Dubiegeon at el.[5], Huang at el. [6], Jacquest-Richardet at el. 1996 [7], Kanki et al. [8], Khader at el. [9], Okabe at el. [10]). In these papers it is said that the inertial effects generated on the shaft by the vibration of the bladed disc are important for the modes associated with zero and one nodal diameter modes.

In this paper natural frequencies of the single blade, the bladed disc, the discs on the shaft, the shaft and two bladed discs on the shaft are checked to discover resonance conditions and the coupling effects. Calculations show the influence of the shaft on the natural frequencies of the bladed discs up to one nodal diameter frequencies. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameters modes of the single bladed discs. From the calculations it is shown that including the shaft in the bladed discs model is important from the designer's point of view and can change the spectrum of frequencies considerably. When flexible bladed discs are mounted on a flexible shaft the resultant system has vibration characteristics which depend on the coupling between the modes of vibration of the individual components. In studies of such vibration characteristics the system can not be treated as two independent systems, one being flexible bladed discs on a rigid shaft and the other being discs with rigid blades on a flexible shaft.

2 Numerical model

The structure is composed of 24 blades, mounted rigidly on a simply supported –clamped shaft with two discs (see Fig. 1). The main dimensions are as follows: the disc outer diameter is equal to 0.3808 m, the inner diameter is equal to 0.12 m, the height of the blade is 0.484 m and the length of the shaft is 3.52 m. The isoparametric brick elements with 8 nodes and 3 degrees of freedom per node are used. Natural frequencies obtained from the cantilever blade, the shaft,

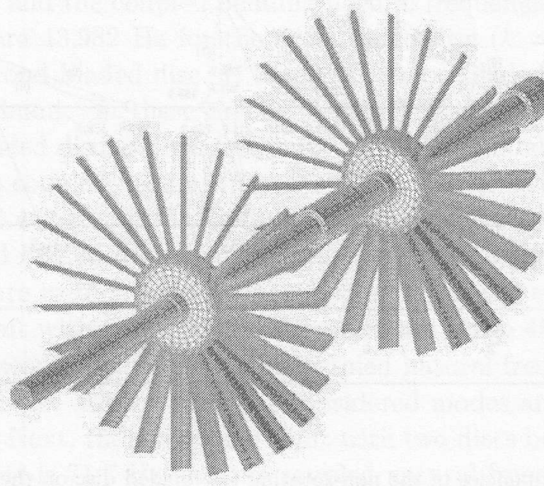

Figure 1. Two bladed discs on the shaft.

the bladed disc and the shaft with two discs are checked to discover resonance conditions and the coupling effects.

In this paper geometrical cross-section blade parameters were taken from the 4^{th} Standard Configuration [2]. The length of the blade was assumed to be $L = 0.484$ m in order to get the first natural frequency close to the first natural frequency of the shaft with two discs. The natural frequencies of the non-rotating disc of thickness $h = 0.105$ m, the inner diameters $d_i = 0.12$ m and the outer diameters $d_o = 0.3808$ m are presented in Fig. 2. The natural frequencies of the shaft and the shaft with two discs also are presented in Fig. 2. The boundary conditions in the cylindrical coordinates (r, φ, z) from the left are $z = 0$, $\varphi = 0$ and in the position of bearings assumed $r = 0$. The natural frequencies of the shaft with discs consist of a single torsion, the double bending and a single axial. Thus, the natural frequencies of the non-rotating bladed disc with 24 blades of $L = 0.484$ m were calculated.

The modes of the bladed disc are classified by using an analogy with axisymmetric modes which are mainly characterized by nodal lines lying along the diameters of the structure and having constant angular spacing. They are either zero ($k = 0$), one ($k = l$), two ($k = 2$), or more ($k > 2$) nodal diameter bending

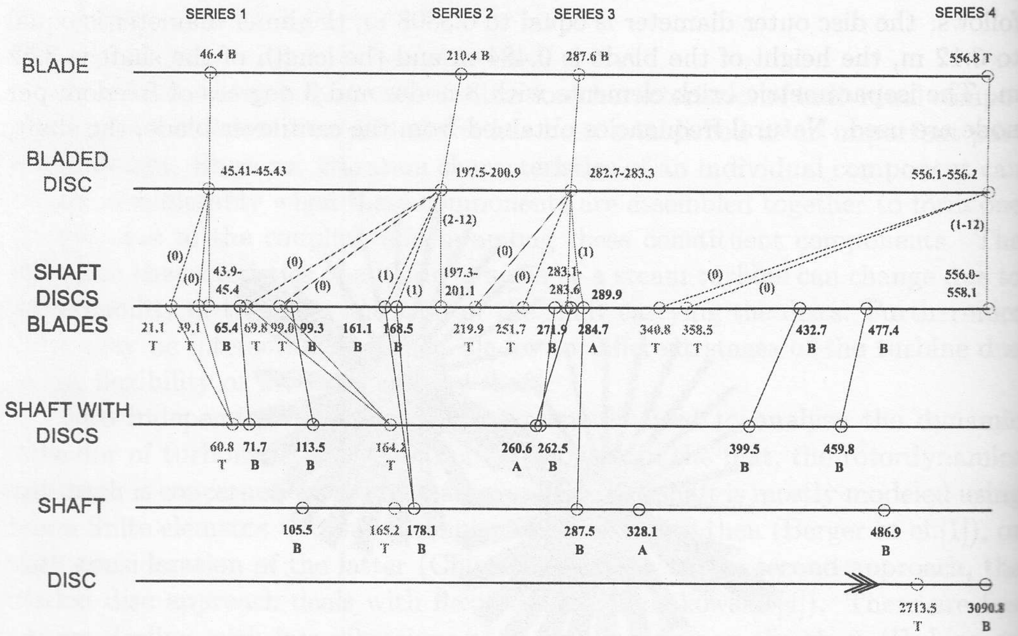

Figure 2. Natural frequencies of the non-rotating two bladed disc on the shaft ($L = 0.484$ m).

or torsion modes. Series 1 is associated with the first natural frequency of the single cantilever blade. Series 2 is associated with the second natural frequency of the single cantilever blade; and so on k is the number of nodal diameters.

In Fig. 2, the upper axis indicates the uncoupled natural frequencies of the cantilever blade. Thus, the next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs and the shaft were seen. The lower axis indicates the uncoupled natural frequencies of the disc. The numbers in brackets show the number of nodal diameters of the bladed disc.

The spectrum of natural frequencies of two bladed discs on the shaft consist of the natural frequencies connected with the natural frequencies of the first bladed disc (24 frequencies), second bladed disc (24 frequencies) and the bending and axial frequencies of the shaft with two discs. The torsion natural frequencies of the shaft with two discs are coupled with the zero nodal diameters frequencies of the bladed discs.

The uncoupled bladed disc natural frequency of the zero nodal diameter mode is 45.413 Hz and the uncoupled torsion natural frequency of the shaft with two discs is 60.8 Hz. Both are so close that the coupled natural frequency of the bladed discs on the shaft with zero nodal diameter split to values 21.1 Hz for the first bladed disc and 39.1 Hz for the second bladed disc. As shown by this example, the frequency split due to coupling effects cannot be ignored in certain situations.

The uncoupled bladed disc natural frequency of the one nodal diameters mode is 45.413 Hz and the coupled bending natural frequencies of the shaft with two bladed discs are 43.982 Hz for the first bladed disc ($k = 1$) and 44.886 Hz ($k = 1$) for the second bladed disc. It is seen that two bladed discs vibrate with one being predominant. In these modes the shaft bending vibration is visible. The uncoupled bladed disc natural frequency of the two nodal diameters mode is 45.414 Hz and the coupled natural frequency of the shaft with two bladed discs are in the range 45.442 Hz for the first and 45.442 Hz for the second bladed disc.

The uncoupled bladed disc natural frequencies of the three to twelve nodal diameters modes are in the range 45.414-45.434 Hz and the coupled natural frequencies of the shaft with two bladed discs are in the range 45.423-45.442 Hz for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes. Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 71.7 Hz and the coupled natural frequencies of the shaft with two bladed discs are 65.4 Hz for the first and second bladed disc ($k = 1$). It is seen that two bladed discs vibrate, where one is predominant. In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

The coupled modes of the second series of the bladed discs are similar to that in the coupled first series. The torsion uncoupled mode of shaft with two discs equal to 164.2 Hz have an influence on the coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter. These frequencies split to value 69.829 Hz for the first blades disc and 99.086 Hz for the second bladed disc. The uncoupled bladed disc natural frequency of the zero nodal diameters mode is 197.575 Hz. Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 113.5 Hz and the coupled natural frequencies of the shaft with two bladed discs are 99.3 Hz for the first and second bladed discs ($k = 1$). (Fig. 2)

It is seen that two bladed discs vibrate, where one is predominant. In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

The uncoupled bladed disc natural frequency of the one nodal diameters mode is 197.661 Hz and the coupled natural frequencies of the shaft with two bladed discs are 161.1 Hz for the first bladed disc ($k = 1$) and 168.5 Hz ($k = 1$) for the second bladed disc. In this case the coupling between the shaft bending mode of

178.1 Hz and the one nodal diameter modes of the bladed discs is observed. It is seen that two bladed discs vibrate, where one is predominant. In these modes the shaft bending vibration is visible. The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 197.662-200.991 Hz and the coupled natural frequencies of the shaft with two bladed discs are in the range 197.31-201.01 Hz for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are less than uncoupled modes. The coupled modes of the third series of the bladed discs are similar to that in the coupled modes of the second series. The uncoupled bladed disc natural frequency of the zero nodal diameters mode is 282.729 Hz and the coupled natural frequencies of the shaft with two bladed discs are 219.97 Hz for the first bladed discs ($k = 0$) and 251.75 Hz ($k = 0$) for the second bladed disc. The influence of the coupling between the blades-discs and shaft is visible here, although the uncoupled particular structure frequencies are not close in the considered region. Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 262.5 Hz and the coupled natural frequencies of the shaft with two bladed discs are 271.9 Hz for the first and second bladed discs ($k = 1$). In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs. The uncoupled bladed disc natural frequency of the one nodal diameters mode is 283.086 Hz and the coupled natural frequencies of the shaft with two bladed discs are 283.6 Hz for the first bladed discs ($k = 1$) and 289.9 Hz ($k = 1$) for the second bladed disc. It is seen that two bladed discs vibrate, where one is predominant. As shown by this example, the frequency split due to coupling effects with the uncoupled shaft mode of 287.5 Hz. In these modes the shaft bending vibration is visible. The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 283.198-283.344 Hz and the coupled natural frequencies of the shaft with two bladed discs are in the range 283.18-283.39 Hz for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes. Next, the uncoupled shaft with two discs axial mode of the zero nodal diameters is 260.6 Hz and the coupled axial natural frequencies of the shaft with two bladed discs are 284.7 Hz.

In order to consider the influence of the blade length on the dynamic behavior of the shaft-discs-blades system the length of the blade was decreased from $L = 0.484$ m to 0.330 m. The geometrical cross-section blade parameters were taken from the 4^{th} Standard Configuration [2]. In Fig. 3, the left axis indicates the uncoupled natural frequencies of the cantilever blade. Thus, the next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs are seen. The right axis indicates the uncoupled natural frequencies of the shaft. The numbers in brackets show the number of nodal diameters of the bladed discs modes. The uncoupled bladed disc natural frequency of the zero nodal diameters mode is 96.969 Hz and the uncoupled torsion natural frequency of the shaft with two discs is 60.8 Hz. The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter split to values 31.998 Hz for the first blades disc and 70.519 Hz for the second bladed disc. Next, the uncoupled shaft with two discs bending mode of the one nodal diameter has frequency equal to 71.7 Hz and the coupled natural frequencies of the shaft with two bladed discs are 63.664 Hz for the first and second bladed disc ($k = 1$). The uncoupled shaft with two discs bending mode of the one nodal diameters has frequency 113.5 Hz and the coupled natural frequencies of the shaft with two bladed discs are 111.35 Hz for the first and second bladed disc ($k = 1$).

In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode. The uncoupled bladed disc natural frequencies of one to twelve nodal diameters modes are in the range 97.005-97.065 Hz and the coupled natural frequencies of the shaft with two bladed discs are in the range 97.005-99.739 Hz for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes. In this case the splitting of the coupled frequencies corresponding to one nodal diameter is not observed.

In the next series the uncoupled bladed disc natural frequency of the zero nodal diameters mode is 405.339 Hz and the uncoupled torsion natural frequency of the shaft with two discs is 164.2 Hz. The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter split to values 129.48 Hz for the first blades disc and 154.66 Hz for the second bladed disc. The uncoupled shaft with two discs axial mode of the zero nodal diameters has frequency 260.6 Hz and the coupled axial natural frequencies of the shaft with two bladed discs are 246.8 Hz. The uncoupled shaft with two discs bending mode of the one nodal diameters has frequency 262.5 Hz and the coupled natural frequencies of the shaft with two bladed discs are 233.6 Hz for the first and second bladed disc ($k = 1$). The uncoupled bladed disc natural frequency of the one nodal diameters mode has frequency 405.855 Hz and the uncoupled bending natural frequency of the shaft with two discs has frequency 399.5 Hz. The coupled natural frequencies of the bladed discs on the shaft with one nodal diameter split to values 273.1 Hz for the first blades disc and 325.6 Hz for the second bladed disc. The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 407.291-417.69 Hz and the coupled natural frequencies of the shaft with two bladed discs are in the range 404.95-417.69 Hz for the first and the second

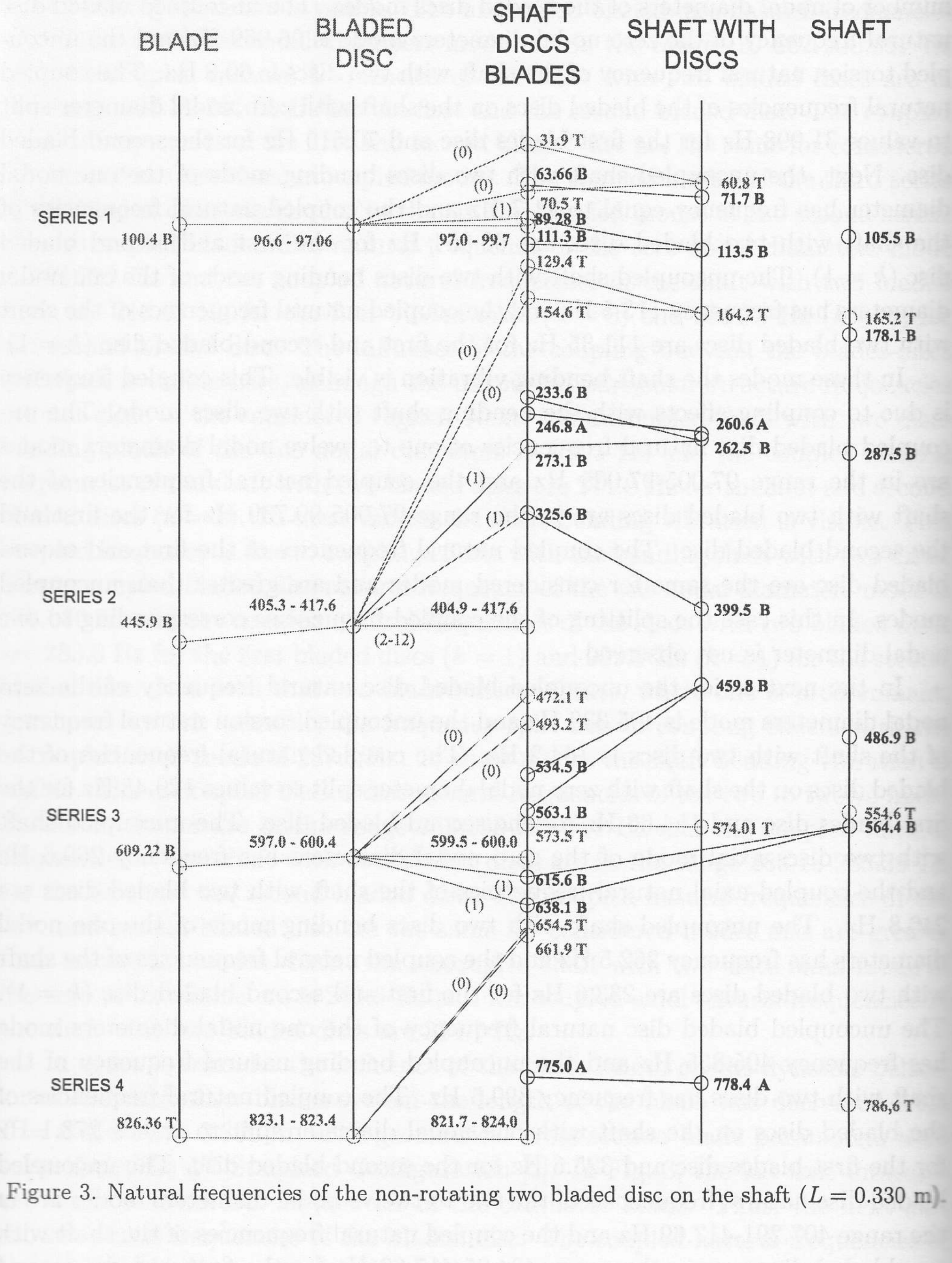

Figure 3. Natural frequencies of the non-rotating two bladed disc on the shaft ($L = 0.330$ m).

bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes.

In the third series the uncoupled bladed disc natural frequency of the zero nodal diameters mode is 597.016 Hz. The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameters split to values 472.1 Hz for the first blades disc and 493.2 Hz for the second bladed disc. The uncoupled shaft with two discs bending mode of the one nodal diameters has frequency equal to 459.8 Hz and the coupled bending natural frequencies of the shaft with two bladed discs are 534.5 Hz and 563.1 Hz. The uncoupled shaft frequency of 486.9 Hz influence these modes. The uncoupled shaft with two discs torsion mode has frequency 574.01 Hz and the coupled torsion natural frequency of the shaft with two bladed discs is 573.5 Hz. This shaft torsion mode does not influence the zero nodal diameters modes of bladed discs. The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 599.515-600.487 Hz and the coupled natural frequencies of the shaft with two bladed discs are in the range 599.2-600.49 Hz for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes. The uncoupled shaft bending mode is 564.0 Hz and the coupled natural frequencies of the shaft with two bladed discs are 615.6 Hz for the first and 638.1 Hz for the second bladed disc ($k = 1$).

In order to consider the influence of the blade length on the dynamic behavior of the shaft-discs-blades system the length of the blade was decreased from $L = 0.330$ m to $L = 0.165$ m and subsequently to $L = 0.094$ m. The geometrical cross-section blade parameters were taken from the 4$^{\text{th}}$ Standard Configuration [2]. The coupled modes of the all series of the bladed discs are similar to that in the coupled modes of the earlier cases.

In Fig. 4 and Fig. 5, the left axis indicates the uncoupled natural frequencies of the cantilever blade. Thus, the next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. The right indicates the uncoupled natural frequencies of the shaft with two discs. The numbers in brackets show the number of nodal diameters of the bladed discs modes. Decrease of the length of blade from 0.330 m to 0.094 m causes the increase of natural frequencies of the blades – discs – shaft system. The torsion frequencies of the discs on the shaft influence the zero nodal diameter bladed disc modes considerably.

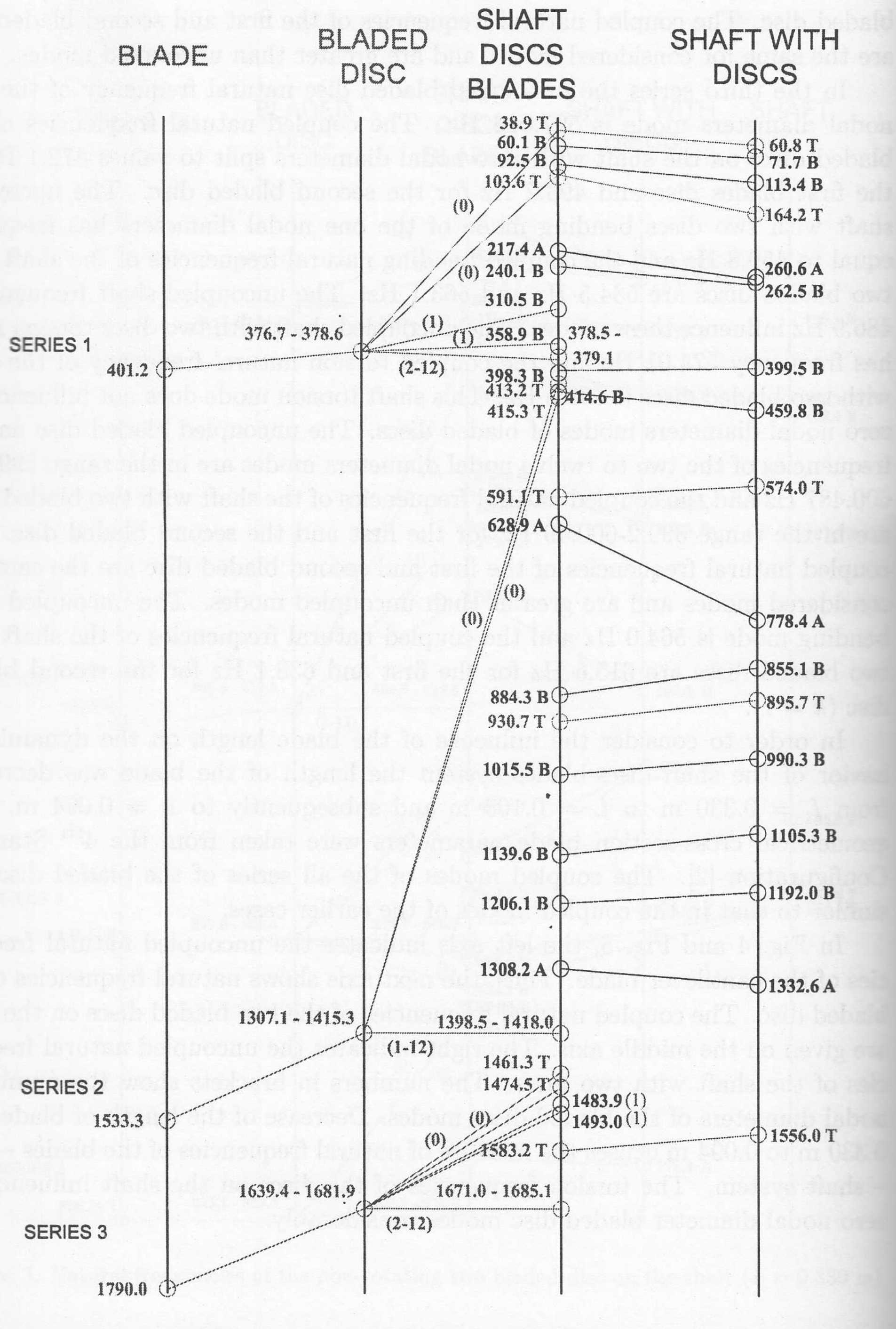

Figure 4. Natural frequencies of the non-rotating two bladed disc on the shaft ($L = 0.165$ m)

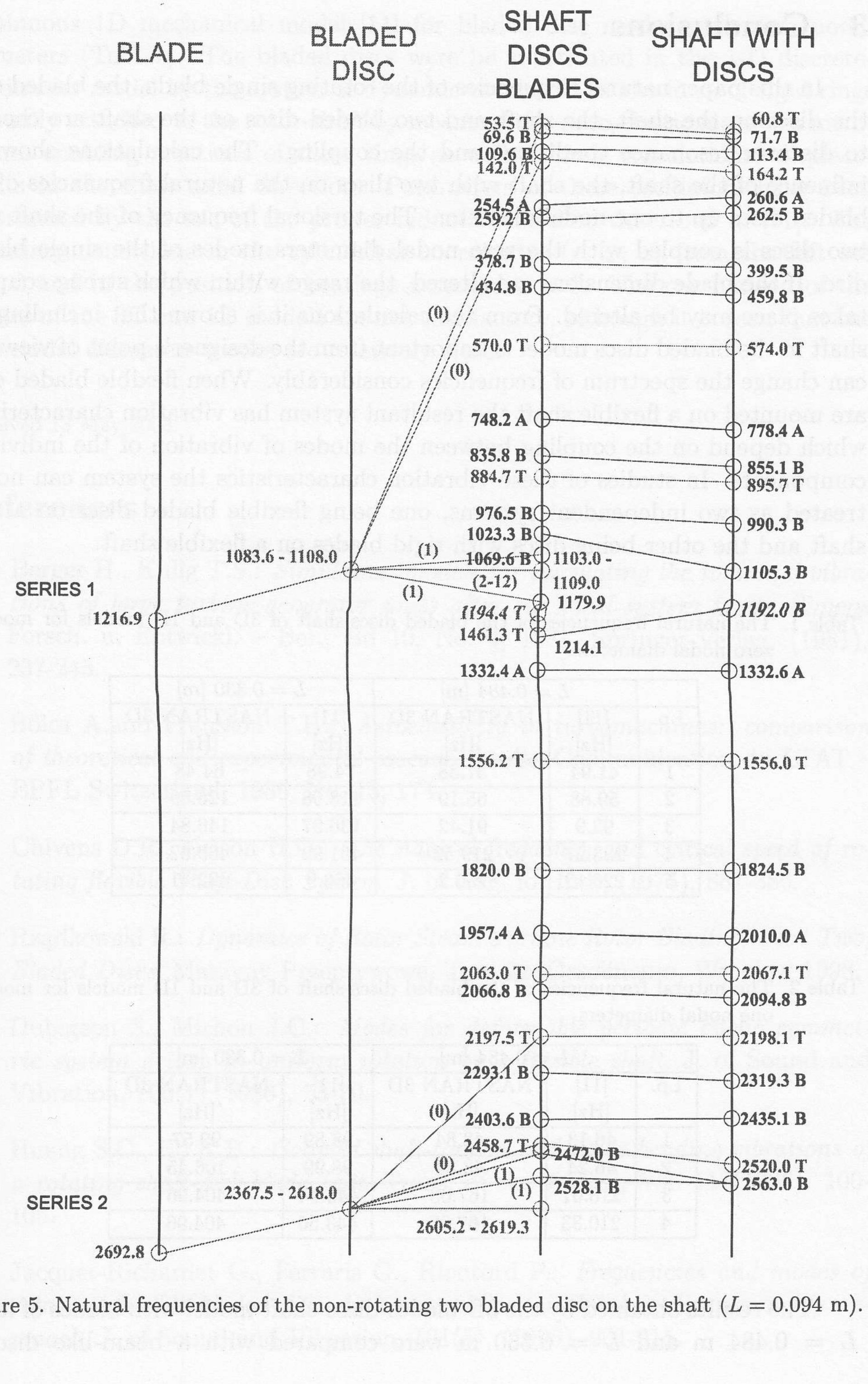

Figure 5. Natural frequencies of the non-rotating two bladed disc on the shaft ($L = 0.094$ m).

3 Conclusions

In this paper natural frequencies of the rotating single blade, the bladed disc, the discs on the shaft, the shaft and two bladed discs on the shaft are checked to discover resonance conditions and the coupling. The calculations show the influence of the shaft, the shaft with two discs on the natural frequencies of the bladed discs up to one nodal diameter. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameters modes of the single bladed disc. If the blade dimensions are altered, the range within which strong coupling takes place may be altered. From the calculations it is shown that including the shaft in the bladed discs model is important from the designer's point of view and can change the spectrum of frequencies considerably. When flexible bladed discs are mounted on a flexible shaft the resultant system has vibration characteristics which depend on the coupling between the modes of vibration of the individual components. In studies of these vibration characteristics the system can not be treated as two independent systems, one being flexible bladed discs on a rigid shaft and the other being discs with rigid blades on a flexible shaft.

Table 1. The natural frequencies of the bladed discs-shaft of 3D and 1D models for modes of zero nodal diameters.

	$L = 0.484$ [m]		$L = 0.330$ [m]	
Lp.	[11] [Hz]	NASTRAN 3D [Hz]	[11] [Hz]	NASTRAN 3D [Hz]
1	41.94	37.38	74.98	64.48
2	59.88	65.19	118.98	126.59
3	92.9	91.42	136.97	146.84
4	223.27	219.52	461.92	469.62
5	226.12	250.2	466.9	492.41

Table 2. The natural frequencies of the bladed discs-shaft of 3D and 1D models for modes of one nodal diameters.

	$L = 0.484$ [m]		$L = 0.330$ [m]	
Lp.	[11] [Hz]	NASTRAN 3D [Hz]	[11] [Hz]	NASTRAN 3D [Hz]
1	46.13	43.84	98.89	99.57
2	46.24	43.84	98.99	106.45
3	210.01	167.08	443.14	404.96
4	210.33	167.08	443.56	404.96

The results obtained by the 3D blades-discs-shaft model with blades of length $L = 0.484$ m and $L = 0.330$ m were compared with a beam-like discrete-continuous 1D mechanical model [11] for bladed disc modes with zero nodal diameters (Tab. 1). The bladed discs were be substituted in the 1-D discrete-continuous model by the system of dynamic oscillators in the form of rigid rings mutually attached to the rotor-shaft by means of the visco-elastic mass-less membranes enabling rotations of these rings as well as their translational displacements in the shaft axial direction. Parameters of these oscillators have been determined by the use of the proper reduction method described in [11]. The boundary conditions in the cylindrical coordinates (r, φ, z) from the left are $z = 0$ and in the position of bearings assumed $r = 0$. The comparison of the results of the 3D and 1D models are not satisfactory, although it is seen that the zero nodal diameters modes are relatively close.

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