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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

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The mechanism of generation of chaotic fluctuations of heating surface temperature in nucleate boiling†

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Abstract

Chaotic fluctuations of the heating surface temperature occur when nucleate boiling with high heat flux takes place. One possible mechanism for these fluctuations has been analysed in this paper. The heat transfer in the heating surface at a single nucleation site has been considered. The generation of bubbles has been simulated by changes of the boundary condition on the heating surface. In this paper the mechanisms of generation of chaotic fluctuations of surface temperature when the conductivity equation is solved using a Control-Volume Finite Element Method has been explained. The model presented in the paper has a qualitative character.

Keywords: Boiling; Chaos

Nomenclature

A	– thermal diffusivity [m^2/s], constant	l	– latent heat of evaporation [J/kgK]
A, B	– constant, nucleation site	m	– constant
b	– constant	N	– number of examined points
C	– constant, specific heat [J/(kg K)]	P	– pressure [Pa]
D	– correlation dimension	Q	– heat flux [W/m^2]
F	– function	R	– vapour bubble radius [m]
F	– area of heating surface [m^2]	T	– temperature [K]
Fo	– Fourier number $\lambda\Delta t/(c\rho(\Delta\delta)^2)$	t	- time [s]
L	– Lyapunov exponent [bit/s]	X, y	– coordinates
δ	– distance [m]	λ	– thermal conductivity [W/(mK)]
$\Delta\delta$	– linear dimension of grid element	ρ	– density [kg/m^3]
ε	– coefficient equal to 4·Fo	τ	– time interval [s]
ΔT	– $T_o - T_{sat}$ [K]		

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Subscripts

1	– time of bubble growth, number of gird element	L	– left
2	– waiting time, number of grid element	l	– largest
A	– referred to point A	max	– maximum
av	– average	o	– constant value
min	– minimum	R	– right
v	– vapour	sat	– saturation
		w	– wall

Superscripts

k	– numbers of time intervals	o	– instant of nucleation

1 Introduction

Despite the fact that many works have been devoted to boiling, there are still numerous fundamental problems, which have yet to be solved. The majority of these problems have dealt with the dynamics of boiling [1]. Originally, nucleate boiling was perceived as a steady state phenomenon. Recently, the dynamic properties of boiling are examined more often [1-2]. Investigation of the dynamics of boiling is important not only from a theoretical but also from a practical point of view, because for example, in the 1980s instabilities of the flow of boiling water in BWR water reactors were discovered. These disturbances increase the coolant temperature, and this can lead to malfunction of the cooling systems or even their breakdown. Analysis of the dynamics of boiling requires an approach different from those used up to now.

The changes of the heating surface temperature occurring in nucleate boiling are irregular. It has been commonly accepted that these irregularities result from stochastic processes. But similar irregularities of system behaviour have also been found in nonlinear dynamic systems. The reasons for such irregularities are by now relatively well understood and the discovered phenomenon has been called the 'deterministic chaos' [3]. 'Deterministic chaos' is understood as irregular (chaotic) changes of characteristic quantities of a non-linear system, for which the laws of dynamics determine its evolution.

The Lyapunov-exponent is one of the quantities determining whether the behaviour of a dynamic system is chaotic or not. It also describes how much information about the system is lost during a given amount of time. Whenever the Lyapunov-exponent is greater than zero the behaviour of the dynamic system is chaotic. In most cases, the evolution equation for the system under investigation is complicated or unknown and only a time series of measurements

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(or numerically determined values of a system variable) can be obtained. Under such conditions, the reliable determination of all Lyapunov-exponents is difficult or even impossible. It is, however, possible to determine a value of the largest Lyapunov exponent – L_l. The condition $L_l > 0$ is sufficient to determine whether chaos exists in the investigated system. In addition, the correlation dimension – one of the essential features of the attractor – allows us to determine the number of degrees of freedom of the system.

The number of papers dealing with the problem of deterministic chaos in boiling is limited. This is due to the complexity of the boiling process. Below, some of the most important achievements identifying and modelling deterministic chaos in boiling are described.

Reconstructions of 3D trajectories from the temperature fluctuations and identification of a correlation dimension for different temperature of heating surface have been presented in [1].

It has been observed that fluctuations of liquid velocity on the heating surface in the area close to vapour bubbles are characterised by a positive value of the largest Lyapunov exponent [1]. The local-dynamic nature of the heating surface temperature has been analysed in [4-6]. In these papers 3D trajectories have been reconstructed from the temperature fluctuations at a single nucleation site and spatial-temporal chaos has been found to exist during nucleate boiling. In [7] the results of experimental investigations of the temperature and pressure fluctuations in a cooling system with natural liquid circulation were described. Theoretical models explaining this phenomenon were described in the papers [8 - 10]. The Coupled Map Lattice method has been used to simulate the heat and mass transfer in boiling [11]. This model was developed by Shoji [1]. Here, too, [1, 11] deterministic chaos can be observed. Simulations of the interaction between two nucleation sites on a thin uniformly heated strip have been presented in [4]. Chaotic fluctuations of the heating surface temperature characterised by a low correlation dimension have been observed. Models simulating the locations of the nucleation sites and the fluctuations of the heating surface temperature have been presented in [12-13]. In [1] a review of experimental investigations and of efforts to model non-linear aspects of boiling have been presented. The paper contains references to many works describing experimental investigations and non-linear modelling of chaos in boiling.

There are many nonlinearities in boiling. In [12] the following nonlinear phenomena of boiling are indicated: on-off behaviour at the nucleation sites, macrolayer formation, meniscus behaviour, macrolayer thinning and growth and departure of a bubble. In other words the changes of boundary conditions at the heating surface can introduce nonlinearities into the process of heat transfer.

The problem of generation of chaotic fluctuations of the heating surface temperature has been analysed in the papers [18-19]. In these papers conductivity equation has been solved using a Control-Volume Finite Element Method. In this paper the mechanisms of generation of chaotic fluctuations of surface temperature when the conductivity equation is solved using a Control-Volume Finite Element Method has been explained. The model presented in the paper has a qualitative character.

2 Modelling of the heating surface temperature fluctuations

Nucleate boiling at an active nucleation site can be subdivided into two phases – bubble growth and departure (time interval τ_1), followed by a waiting time (τ_2), during which no bubble is visible [14]. The heat flux (q) during the phase of bubble growth reaches high values, resulting in a drop of the heater surface temperature T_w. After a bubble departure, the heat flux q drops to values corresponding to single-phase heat transfer, resulting in an increase of the heater surface temperature T_w. Once the surface temperature is high enough, another bubble begins to grow and the cycle starts anew. Typical changes of the heating surface temperature T_w at a single nucleation site are shown schematically in Fig. 1a.

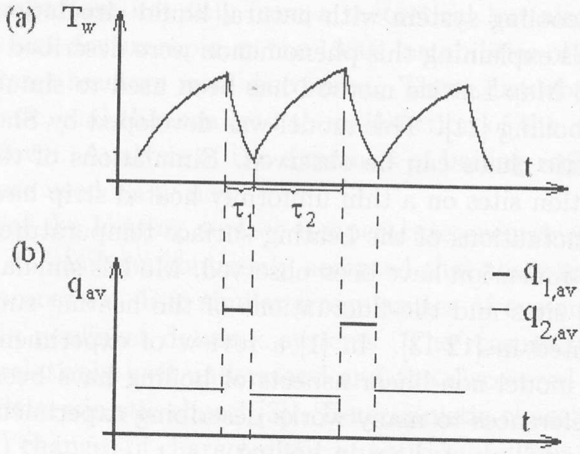

Figure 1. Typical changes of temperature (T_w) and average heat flux at a single nucleation site. a) Temperature changes. b) Changes of average heat flux, $q_{1,av}$ – the heat flux absorbed by the bubble, $q_{2,av}$ – the heat flux absorbed by the liquid without bubbles, τ_l – time interval during which the vapour bubble grows, τ_2 – waiting time to generate the new vapour bubble.

Conditions for the formation of a new bubble are favourable when the surface temperature at the nucleation site, T_w, is greater than temperature of nucleation,

$T_{\max}$. Growing bubble absorbs the heat from the heating surface causing the drop in temperature T_w down to temperature T_{min}. Experimental investigations [14] show that the temperatures $T_{\max}$ and T_{min} are not constant and fluctuate around their average values.

Depending on the behaviour of the boiling liquid and of the bubble, the heat flux absorbed by the boiling liquid during the time intervals τ_l and τ_2 changes accordingly. It is nevertheless possible to calculate the average heat fluxes: $q_{1,av}$ – absorbed by the bubble and $q_{2,av}$ – absorbed by the liquid without bubbles for both intervals. These average heat fluxes, being characteristic for the subsequent time intervals, are shown in the Fig. 1b.

In order to investigate how changes of the boundary condition influence the changes of temperature, T_w, in time, a simple model of heat transfer at a single nucleation site has been analysed. Boiling on a flat and horizontal surface has been investigated. Fig. 2 shows a two-dimensional model of heat transfer under the vapour bubble.

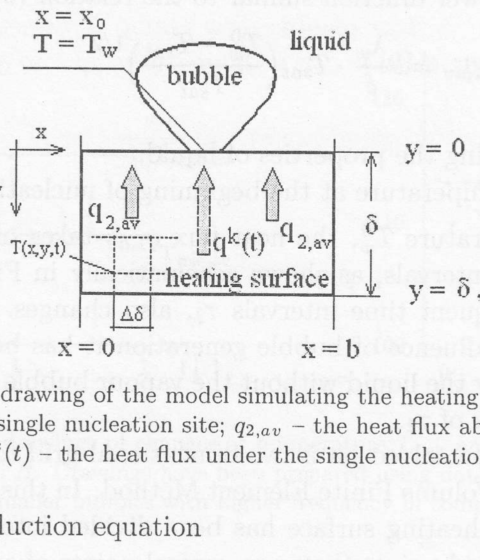

Figure 2. Schematic drawing of the model simulating the heating surface temperature changes under the single nucleation site; $q_{2,av}$ – the heat flux absorbed by liquid without the bubbles, $q^k(t)$ – the heat flux under the single nucleation site.

The heat conduction equation

$$\frac{\partial T}{\partial t} - a\nabla^2 T = 0, \tag{1}$$

with the following boundary conditions

$$\begin{aligned}
&y = \delta, \quad T = T_0 = \text{constant};\\
&y = 0, \quad x \notin (x_0 - \Delta\delta/2, x_0 + \Delta\delta/2), q = q_{2,av}\\
&y = 0, \quad x \in (x_0 - \Delta\delta/2, x_0 + \Delta\delta/2), q^k(t) = \begin{cases} q_{1,av} \text{ in time interval } \tau_1,\\ q_{2,av} \text{ in time interval } \tau_2,\end{cases}\\
&x = 0 \quad \text{and } x = b, T = T(y) \text{ is constant in time,}
\end{aligned} \tag{2}$$

has been solved.

It has been assumed that the boundary conditions change in time at the nucleation site but for other points for $y = 0$ the heat flux absorbed by the liquid is constant and equal to $q_{2,av}$. For $y = \delta$ the temperature of the heating element is constant and equal to T_0. The temperature of vertical 'walls' of the system is a linear function of y and this function is constant in time. The initial temperature distribution is a stationary (linear) function obtained for $q^k(t) = q_{2,av} =$ constant.

The heat flux absorbed by a vapour bubble in the period of time, τ_l, may be expressed by the following function [14]: $q_1 \approx \rho_v l R$. When the vapour bubble, in initially uniformly superheated liquid, is spherical then the Rayleigh's solution of the bubble growth equation shows that in the initial phase of this growth, R is [14]

$$R \approx \left(\frac{2\rho_v l(T_w^0 - T_{sat})}{3\rho_f T_{sat}}\right)^{1/2}. \tag{3}$$

It has been assumed that the average heat flux absorbed by the bubble, $q_{1,av}$, can be expressed by a power function similar to the relation (3)

$$q_{1,av} = b\frac{\lambda_w}{\delta} \cdot T_{sat}\left(\frac{T_w^0 - T_{sat}}{T_{sat}}\right)^{1/2}, \tag{4}$$

where:
b – coefficient representing the properties of liquid,
T_w^0 – heating surface temperature at the beginning of nucleation.

Depending on temperature T_w^0, the heat flux $q_{1,av}$ takes on different values in the subsequent time intervals, as shown schematically in Fig. 1b. The heat flux, $q_{2,av}$, in the subsequent time intervals τ_2, also changes. But in order to focus attention on the influence of bubble generation it has been assumed that the heat flux absorbed by the liquid without the vapour bubble is constant in the subsequent time intervals of τ_2.

Equation (1) has been solved on a two-dimensional grid (x, y) (100x13 grid points) using a Control-Volume Finite Element Method. In this method time and space are discrete. The heating surface has been divided into square elements. The distances between grid points (between central points of squares) have been set to $\Delta x = \Delta y = \Delta\delta$. The temperature at the grid points has been calculated for subsequent time intervals equal to Δt. For elements inside the heating surface the temperature was being calculated according to the following formula [17]

$$T_{i,j}^{k+1} = T_{i,j}^k(1-\varepsilon) + \frac{1}{4}\varepsilon(T_{i-1,j}^k + T_{i+1,j}^k + T_{i,j-1}^k + T_{i,j+1}^k). \tag{5}$$

For elements on heating surface the temperature was being calculated according to the following formula

$$T_{i,j}^{k+1} = T_{i,j}^k\left(1 - \frac{3}{4}\varepsilon\right) + \frac{1}{4}\varepsilon\left(T_{i-1,j}^k + T_{i+1,j}^k + q^k\frac{\Delta\delta}{\lambda_w} + T_{i,j+1}^k\right), \tag{6}$$

where

$$\frac{4}{\varepsilon} = \frac{1}{Fo} = \frac{\delta^2 \cdot \rho_w \cdot c_w}{\lambda_w \cdot \Delta t}. \tag{7}$$

The numerical procedure solving Eq. (1) in the subsequent time intervals τ_1 and τ_2 is stable, when $\mathrm{Fo} = \varepsilon/4 \leq 1/4$. The nucleation site was represented by one square element $\Delta\delta$ in length with the central point in $x = x_o$. The calculations were carried out for $\mathrm{Fo} = 1/8$, $\Delta t = 0.002$ s and $\Delta\delta = 0.0014$ m on the copper surface.

In the paper [6] the results of measurements of temperature fluctuations under the nucleation sites during pool boiling of water on thin, electrically-heated plate have been presented. The underside of the stainless steel heating surface was coated with thermochromic liquid crystal of approximate thickness of 0.01 mm. The temperature fields were recorded by high-speed colour video camera at 200 frames/s. Fig. 3 schematically illustrates changes of temperature $T_{\max}$ and T_{min} in nucleation sites [6].

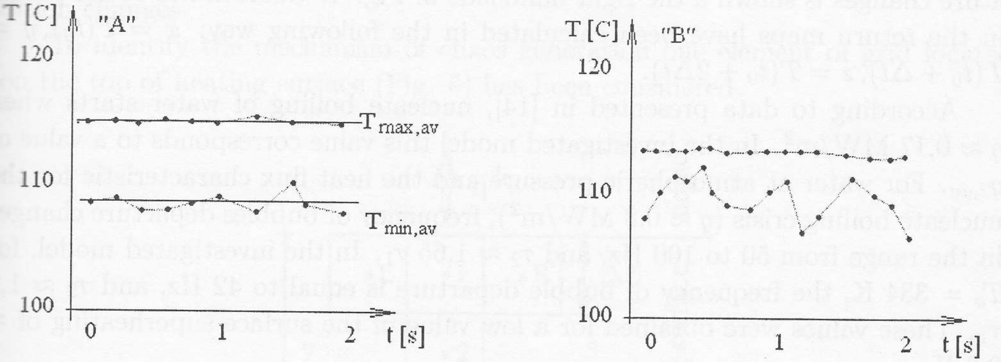

Figure 3. Schematic drawings of changes of temperature $T_{\max}$ and T_{min} under two nucleation sites A and B. Drawings have been prepared using data presented in [6]. The site B produces smaller bubbles with higher frequency in comparison the site A.

The site B produces smaller bubbles with higher frequency as compared with the site A. In the site A temperatures $T_{\max}$ and T_{min} may be treated as constant, but in the site B the temperature T_{min} varies. This is due to the fact that the temperature of bubble departure is determined from the force balance acting on the bubble and such quantities as surface tension and the contact angle. It seems that when the condition of liquid flow around the nucleation site is steady then the temperature fluctuation can appear between two averaged values, $T_{\max,av}$ and $T_{min,av}$ shown in Fig. 3a.

In the presented model values of τ_1 and τ_2 have been determined as the time necessary for the temperature T_w to reach a value of $T_{\max,av}$ and, respectively, as the time required for T_w to drop to a value of $T_{min,av}$ (after the instance of nucleation). Fig. 4 shows an example of the chaotic changes of temperature T_w possible to be obtained.

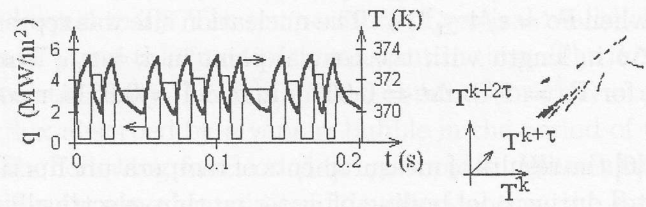

Figure 4. Results of calculations of T_w and q absorbed from the nucleation site for: T_{sat} = 373 K, $T_{max,av}$ = 373.3 K, $T_{min,av}$ = 370 K, b = 0.054, $q_{2,av}$ = 0.12 MW/m^2, Δt = 0.002 s, $\Delta\delta$ = 0.0014 m, copper surface and Fo = 1/8 where T_0 = 386 K. Three dimensional return map of the time series of T_w is shown on the right hand side of graph.

Three-dimensional return map of the time series of the heating surface temperature changes is shown a the right hand side in Fig. 4. Coordinates of the points in the return maps have been calculated in the following way: $x = T(t_0), y = T(t_0 + \Delta t), z = T(t_0 + 2\Delta t)$.

According to data presented in [14], nucleate boiling of water starts when $q \approx 0{,}17$ MW/m^2. In the investigated model this value corresponds to a value of $q_{2,av}$. For water at atmospheric pressure and the heat flux characteristic for the nucleate boiling crisis ($q \approx 0.8$ MW/m^2), frequency of bubbles departure changes in the range from 50 to 100 Hz, and $\tau_2 \approx 1{,}65\ \tau_1$. In the investigated model, for T_o = 384 K, the frequency of bubble departure is equal to 42 Hz, and $\tau_2 \approx 1{,}2\ \tau_1$. These values were obtained for a low value of the surface superheating of $\approx$ 0.3 K.

The result of calculations of the largest Lyapunov exponent of the temperature fluctuations is presented in Fig. 5. The exponent L_l was calculated using the Wolf algorithm [15].

A positive value of the largest Lyapunov exponent indicates that deterministic chaos occurs in the investigated model. The Lyapunov exponent describes the information loss about the system. The calculated value of $1/L_l$[bit/bit/s] $\approx$ 1/70.0 = 0.014 [s] shows how long a certain initial temperature distribution inside the heating surface influences the heating surface temperature. In other words we can say that in the investigated model a single act of nucleation influences the boiling process during a time interval of 0.014 s. This interval corresponds to the average time of the boundary conditions changes (Fig. 4). The obtained results indicate that the changes of boundary conditions destroy long-term memory in the system. The heat transfer during the time intervals τ_1 and τ_2 occurs at constant boundary conditions. The solution of Eq. (1) is stable for $F_0 \leq 1/4$ and

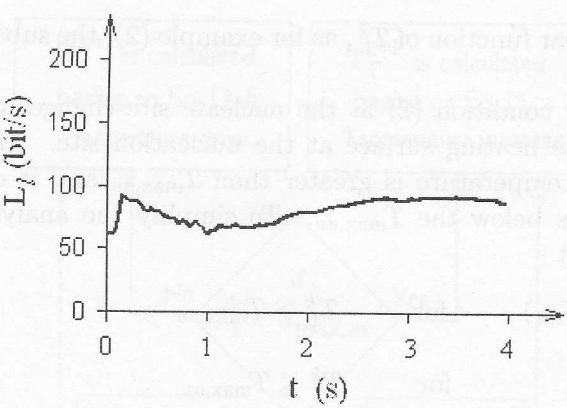

Figure 5. The largest Lyapunov-exponents L_l for time series of T_w as a function of evolution time t for different temperatures T_0. L_l was calculated using the Wolf algorithm [15].

every value of q. The results of the calculation of the largest Lyapunov exponent indicate that destabilisation of the heat transfer is caused by the boundary condition changes.

To identify the mechanism of chaos generation one element of grid located on the top of heating surface (Fig. 6) has been considered.

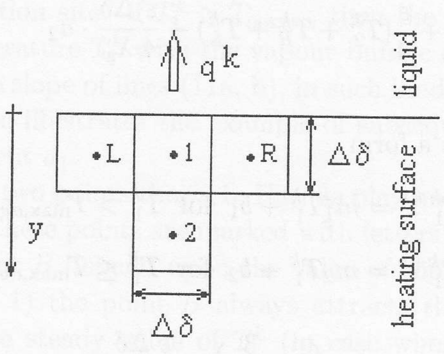

Figure 6. Schematic drawing of two-dimensional grid around the element '1'.

Equation (6) for element '1' shown in Fig. 6 has a form

$$T_1^{k+1} = T_1^k\Big(1 - \frac{3}{4}\varepsilon\Big) + \frac{1}{4}\varepsilon\Big(T_L^k + T_R^k - q^k\frac{\Delta\delta}{\lambda_w} + T_2^k\Big), \tag{8}$$

where: q_k is a heat flux absorbed by liquid from element '1'.

When q_k is constant the function (8), $T_1^{k+1}(T_1^k)$, is a linear function of T_1^k. Feigenbaum has shown that subsequent iterations of all first-order difference equations $T_1^{k+1} = f(T_1^k)$, in the case when $f(T_1^k)$ has (after a proper rescaling of T_1^k) *only a single maximum at the unit interval, can be chaotic [3]. Therefore when* the q_k is a non-linear function of T_l^k, as for example (2), the subsequent iterations can be chaotic.

The boundary condition (2) at the nucleate site changes depending on the temperature of the heating surface at the nucleation site. The bubble is generated when the temperature is greater than $T_{\max,av}$ and it departs when the temperature drops below the $T_{\max,av}$. To simplify the analysis further it has been assumed that

$$q^k = \begin{cases} a_1(T_1^k - T_{sat}) & \text{for} \quad T_1^k > T_{\max,av} \qquad (9a) \\ a_2 & \text{for} \quad T_1^k \le T_{\max,av} \qquad (9b) \end{cases}$$

where a_2 is the heat flux absorbed by boiling liquid, when there is no vapour bubble at the nucleation site and $a_l(T_l^k - T_{sat})$ is the heat flux absorbed by the vapour bubble at the nucleation site. After putting (9) into (8) and making some transformations we can write down Eq. (8) in the following form

$$\begin{aligned} &T_1^{k+1} = \Big[\Big(1 - \frac{3}{4}\varepsilon\Big) - \frac{\varepsilon}{4}\frac{\Delta\delta}{\lambda} \cdot a_1\Big]T_1^k + \frac{\varepsilon}{4}(T_2^k + T_R^k + T_L^k) + \frac{\varepsilon}{4}\frac{\Delta\delta}{\lambda_w} \cdot a_1 T_{sat} \\ &\text{for} \quad T_1^k > T_{\max,av} \\ &T_1^{k+1} = \Big(1 - \frac{3}{4}\varepsilon\Big)T_1^k + \frac{\varepsilon}{4}(T_2^k + T_R^k + T_L^k) - \frac{\varepsilon}{4}\frac{\Delta\delta}{\lambda_w} \cdot a_2 \\ &\text{for} \quad T_1^k \le T_{\max,av} \end{aligned} \qquad (10)$$

The set of Eq. (10) has a form

$$T_1^{k+1} = m_1 T_1^k + b_1 \quad \text{for} \quad T_1^k > T_{\max,av} \qquad (11a)$$

$$T_1^{k+1} = m_2 T_1^k + b_2 \quad \text{for} \quad T_1^k \le T_{\max,av} \qquad (11b)$$

where

$$m_1 = \Big(1 - \frac{3}{4}\varepsilon\Big) - \frac{\varepsilon}{4}\frac{\Delta\delta}{\lambda_w} \cdot a_1, \qquad (12a)$$

$$m_2 = \Big(1 - \frac{3}{4}\varepsilon\Big), \qquad (12b)$$

$$b_1 = \frac{\varepsilon}{4}(T_2^k + T_R^k + T_L^k) + \frac{\varepsilon}{4}\frac{\Delta\delta}{\lambda_w} \cdot a_1 T_{sat}, \qquad (12c)$$

$$b_2 = \frac{\varepsilon}{4}(T_2^k + T_R^k + T_L^k) - \frac{\varepsilon}{4}\frac{\Delta\delta}{\lambda_w} \cdot a_2. \qquad (12d)$$

Figure 7 illustrates the block diagram showing how the temperature T_1^k is calculated in subsequent iterations of the set of Eq. (11). If $T_1^k \le T_{\max,av}$ then the function (11b) describes the changes of temperature T_1^k with no vapour

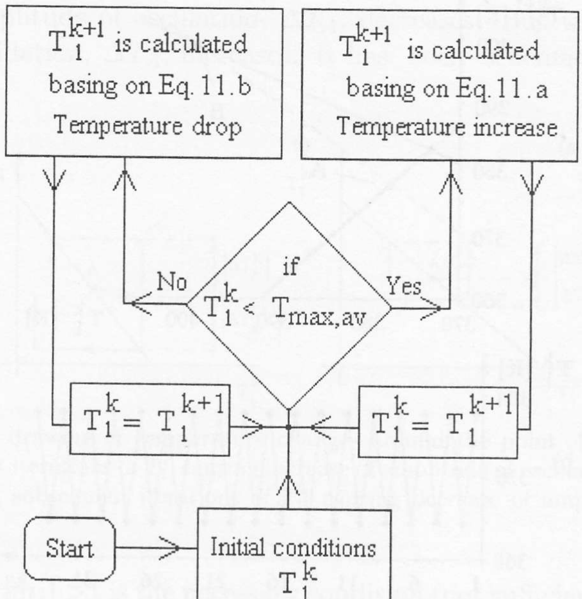

Figure 7. Block diagram to calculate the temperature T_1^k.

bubble at the nucleation site. If $T_1^k > T_{\max,av}$ then the function (11a) describes the changes of temperature T_1^k with the vapour bubble at the nucleation site.

Depending on the slope of lines (11a, b), in such kind of the system chaos can appear [3]. Fig. 8b, c illustrates the example of subsequent iterations of T_1^k for two values of coefficient a_1.

The properties of two points shown in Fig. 8a play important role in evolution of temperature T_1^k. These points are marked with letters A and B (Fig. 8a). The properties of the point B depend upon the value of coefficient m_2. When $\varepsilon \leq 1$ (in this case $|m_2| < 1$) the point B always attracts the subsequent iterations of T_1^k and defines the steady value of T_1^k (in case when there is no bubble at nucleation site). The properties of point A depend on a value of coefficient, m_1. When the point A repels the subsequent iterations of T_1^k, ($|m_1| > 1$), behaviour of the system described by set of Eq. (11) can become chaotic. Fig. 8c shows an example of such kind of behaviour.

For the set of Eq. (11) the Lyapunov exponent can be analytically calculated in an easy way. When iteration formula takes the form $T_1^{ki+1} = f(T_1^k)$ the Lyapunov-exponent equals to [3]

$$L = lim \frac{1}{N} \sum_{i=0}^{N-1} \ln |f'(T_1^k)|. \qquad (13)$$

Putting (11) into (13) and assuming that the same number of iterations has

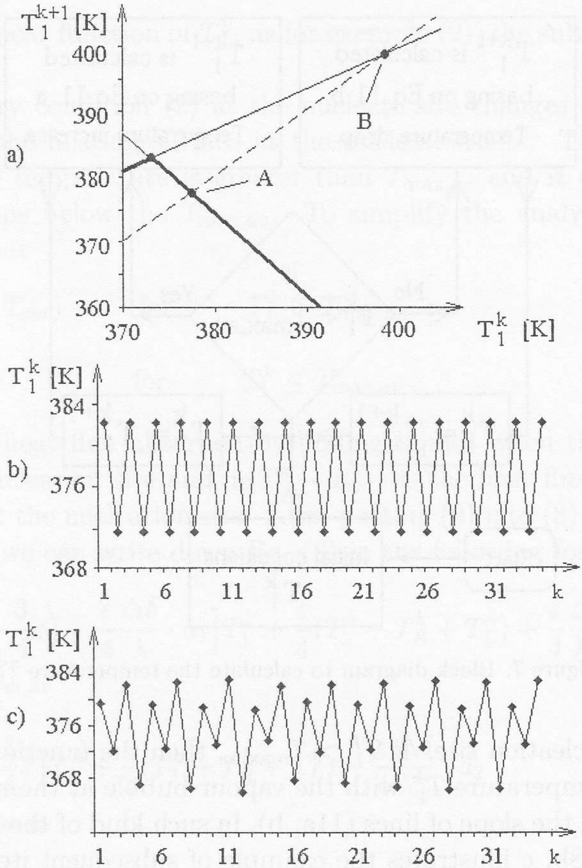

Figure 8. Iterations of set of Eq. (11). Calculations were made for $T_2 = T_R = T_L = 400$ K, $\varepsilon = 0.5$, $T_{sat} = 373$ K, $T_{\max,av} = 373$ K, $\frac{\varepsilon\Delta\delta}{4\lambda_w}a_2 = 1$; a) Two lines described by the set of Eq. (11); b) Two fixed points for $\frac{\varepsilon\Delta\delta}{4\lambda_w}a_1 = 15$; d) Chaotic iterations for $\frac{\varepsilon\Delta\delta}{4\lambda_w}a_1 = 19$.

a place according to Eq. (11a) as well as (11b) we get [3]

$$L = \frac{1}{2}\Big[\ln(|m_1|) + \ln(|m_2|)\Big]. \tag{14}$$

For $\varepsilon \le 1$ the following inequality: $|m_2| < 1$ is true, but a value of m_1 changes depending on a value of coefficient a_1 (Eq. (9)). The changes of coefficient a_1 can lead to a positive value of L. For data presented in the Fig. 8b the Lyapunov exponent L is less than zero, and for data presented in Fig. 8c it is greater than zero.

The temperature T_1^k oscillates around the point A (Fig. 8a) increasing and decreasing its value (Fig. 8b). When $|m_2| \le 1$ then during decreasing temperature T_1^k the amplitude of oscillation, ΔT_A, decreases. But when $|m_2| > 1$ the amplitude of oscillation, ΔT_A, increases. It has been schematically shown in Fig. 9.

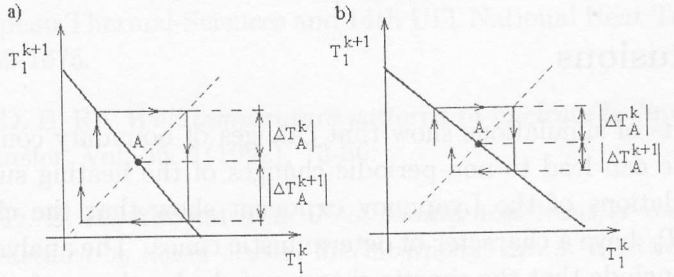

Figure 9. Schematic drawing of temperature changes around the point A; a) Point A repels subsequent iterations of T_1^k causing increase of amplitude of oscillation $|\Delta T_A|$; b) Point A attracts subsequent iterations of T_1^k causing decrease of amplitude of oscillation $|\Delta T_A|$.

An inequality $|m_1| > 1$ is the necessary condition (not sufficient) for the chaotic fluctuations of the heating surface temperature to occur. When $|m_1| > 1$ then the process of the changes of temperature T_1^k described by Eq. (11a) is unstable.

The heat absorption (by boiling liquid) from element 1 decreases its temperature. When heat delivered to the element 1 (from elements L, R and 2 shown in Fig. 6) is large enough the initial difference between calculated temperature T_1^i and steady value of temperature does not increase. But when the heat transfer from elements L, R and 2 (Fig. 6) to element 1 is less than heat absorption by liquid, the initial difference between temperature T_1^i and steady value of temperature, $|\Delta T_A|$, is amplified. The process of amplifying the initial temperature difference, $|\Delta T_A|$, (Fig. 9a) is a property of 'answer' of boiling liquid to changes of heating surface temperature. In the model this 'answer' is represented by boundary condition at the nucleation site.

The coordinates of interception of lines described by Eq. (11) can be obtained from the following equation

$$m_1 T_1^k + b_1 = m_2 T_1^k + b_2. \tag{15}$$

After algebraic transformation the following has been obtained

$$T_1^k = \frac{b_2 - b_1}{m_1 - m_2} = \frac{-\frac{\varepsilon\Delta\delta}{4\lambda}(a_a + a_1 T_{sat})}{-\frac{\varepsilon\Delta\delta}{4\lambda}a_1} = \frac{a_2}{a_1} + T_{sat}. \tag{16}$$

In the case shown in Fig. 8 the temperature T_1^k described by Eq. (16) is equal to 373.1 K. The obtained result explains why chaotic fluctuations of T_w (Fig. 4) appear for $T_{\max,av} \approx T_{sat}$. When $T_{\max,av}$ increases, the bigger number of iterations is realized according to the Eq. (11b). It causes that the Lyapunov exponent (calculated basing on Eq. (14)) becomes less than 0 – then the process becomes periodic.

3 Conclusions

The results of simulations show that changes of boundary conditions at the nucleation site can lead to non-periodic changes of the heating surface temperature. Calculations of the Lyapunov exponent show that the changes of the temperature T_w have a character of deterministic chaos. The analysis carried out allow us to conclude that the chaotic changes of the heating surface temperature occur when the heat transfer to thin layer of heating surface is less than heat absorption by boiling liquid with the vapour bubble from the heating surface.

The temperature of element 1 (Fig. 5) is calculated after each time interval Δt according to the method of heat balance in element 1. The heat flux absorbed from element 1 is calculated after each time interval Δt according to formula (4) or (9). Chaotic fluctuations of temperature of element 1 can appear when function $q(T_1^k)$ causes that the function $T_1^{k+1}(T_1^k)$ has (after rescaling) one maximum in unit interval. This happen not only with function (4) but also for example when function $T_1^{k+1}(T_1^k)$ is a quadratic function of T_1^k. Therefore we can say that function (4), not the method of heat balance in element 1, is responsible for chaotic fluctuations of temperature T_1^k. In the initial phase of calculation the temperature T_w^o always changes near its mean value. When the heat transfer to a thin layer of heating surface is large enough then the initial fluctuations of T_w^o disappear and the temperature of T_w^o stabilises. But when the heat transfer to a thin layer of heating surface is less than heat absorption by boiling liquid then the initial fluctuations of T_w^o are amplified and render deterministic chaos.

To verify the models presented in the paper it is necessary to carry out more fractal analyses of experimental data concerning boiling.

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