INSTITUTE OF FLUID-FLOW MACHINERY
POLISH ACADEMY OF SCIENCES

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

110

Papers from the National Seminar
on *Fluid Mechanics in Machine Construction and Exploitation*
Sopot, May 21st, 2002

GDAŃSK 2002

EDITORIAL AND PUBLISHING OFFICE

IFFM Publishers (Wydawnictwo IMP), Institute of Fluid Flow Machinery, Fiszera 14, 80-952 Gdańsk, Poland, Tel.: +48(58)3411271 ext. 141, Fax: +48(58)3416144, E-mail: esli@imp.gda.pl

Financial support of publication of this journal is provided by the State Committee for Scientific Research, Warsaw, Poland

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ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

Appears since 1960

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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 110, 2002, 93-101

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Water hammer pressure in thick-walled conduits

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Abstract

The paper presents a method allowing to determine water hammer pressure in thick-walled closed conduits. This method may be employed in hydrodynamic calculations and strength analyses in the case of hydraulic installations which are part of high-pressure hydraulic systems. The results of calculations concerning simple water hammer pressure are compared with the results obtained on the basis of formulas derived earlier by other authors, and the results of measurements.

Keywords: Water hammer pressure; Hydrodynamic calculations

Nomenclature

- c – mean instantaneous liquid velocity, m/s
- c_o – mean liquid velocity in a steady flow, m/s
- d – internal diameter of a hydraulic conduit, mm
- E – modulus of wall elasticity, MPa
- K – modulus of liquid elasticity, MPa
- m – relative thickness of the conduit wall
- p_o – liquid pressure in a steady state, MPa
- t – time, s
- t_c – time of transition of a pressure shock wave, s
- t_z – time of hydraulic system adjustment, s
- u – propagation velocity of a pressure shock wave, ms
- ν – Poisson ratio of the conduit wall
- δ – wall thickness, mm
- Δp – water hammer pressure, MPa
- ρ – fluid density, kg/m^3
- $\varepsilon_t, \varepsilon_r, \varepsilon_l$ – relative strains: circumferential, radial and longitudinal
- $\sigma_t, \sigma_r, \sigma_l$ – stresses: circumferential, radial and longitudinal, MPa
- ω – coefficient depending on the state of stress, state of strain and conduit relative thickness

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1 Introduction

Water hammer is one of the phenomena commonly observed in hydraulic systems of machines and devices. Such phenomena cause an increase in the dynamic load of installations and particular units of hydraulic systems. Rapid pressure changes, accompanying the water hammer phenomenon, result from quick changes in flow velocity. Changes in velocity, e.g. through adjustment (of a valve, regulator, etc.), cause an increase or a decrease in pressure and produce positive or negative water hammer.

Pressure changes in conduits accompanying the water hammer phenomenon may be presented in the following form [1]:

$$p(t) = p_o + \Delta p = p_o + \rho u[c_o - c(t)] . \qquad (1)$$

The function $c(t)$ fulfils the following conditions: at the moment $t = 0$, $c(t) \doteq c_o$, whereas for $t = t_z$, $c(t) = 0$.
In hydraulic systems the time of adjustment t_z is usually shorter than the time of transition of a shock wave t_c, so

$$t_z \leq t_c , \qquad (2)$$

where

$$t_c = \frac{2l}{u} ,$$

l denotes the length of the conduit segment where the water hammer phenomenon occurs. In this case

$$\Delta c = [c_o - c(t)] = c_o ,$$

and water hammer pressure reaches its maximum value. This phenomenon is referred to as a simple water hammer phenomenon.

The pressure Δp, observed during a simple water hammer phenomenon, is usually determined from the Żukowski formula:

$$\Delta p = \rho u c_o , \qquad (3)$$

where the propagation velocity of a pressure shock wave, reported for the first time by Kortewega, is

$$u = \sqrt{\frac{\frac{K}{\rho}}{1 + \frac{K}{E}\frac{d}{\delta}}} . \qquad (4)$$

After introducing dependence (4) to (3), the water hammer pressure is equal to

$$\Delta p = \rho c_o \sqrt{\frac{\frac{K}{\rho}}{1 + \frac{K}{E}\frac{d}{\delta}}} . \qquad (5)$$

The dependence (5), known as the Żukowski-Allievi [2, 3] formula, was derived assuming that during the water hammer phenomenon only simple stresses (circumferential stresses σ_t) arise in the conduit walls. The results of previous investigations show that the relation (5) may be applied to determine water hammer pressure in thin-walled conduits [4], e.g. derivation ones [5]. Hydraulic conduits are considered thin-walled it they fulfill the conditions of a membranous shell. The state of stress and strain of a membranous shell may be described by means of the Laplace equations, and its relative thickness as $m = \frac{\delta}{d} \leq 0.025$, [6].

In the case of hydraulic installations employed in control and high-pressure hydraulic systems, relative thickness m usually exceeds considerably the limit value of 0.025. Because of that the application of the formula (5) to calculations concerning thick-walled hydraulic conduits may lead to erroneous results.

2 Effect of the state of stress, state of strain and conduit relative thickness on water hammer pressure

In order to determine precisely the water hammer pressure in thick-walled conduits, an attempt was made to devise new formulas, taking into consideration not only the relative thickness m, but also the relationships between the state of stress and state of strain in thick-walled shells. The Lame equations were used, which constitute a particular solution of the Airy stress function [7, 8]. A general state of stress in a wall of a thick-walled hydraulic conduit is presented in Fig. 1, and the distribution of stresses – in Fig. 2. It was found that the way of the conduit mounting has a significant effect on the general state of stress and strain in its walls. There are three main variants of conduit mounting in hydraulic systems:

- on both sides, without dilatation,
- on both sides, with dilatation,
- on one side.

The definition of propagation of small disturbances and the Lame equations were used to devise formulas describing the water hammer pressure in thick-walled conduits [9]. Analogous results were obtained employing the energy method [10]. These formulas may be presented as:

$$\Delta p = \rho c_o \sqrt{\frac{\frac{K}{\rho}}{1 + \frac{K}{E}\omega}} \; . \qquad (6)$$

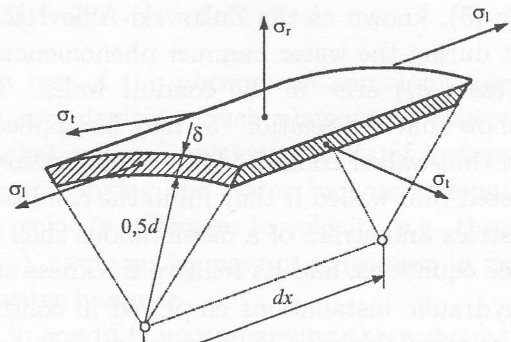


Figure 1. General state od stress in the wall of a thick-walled conduit, mounted on both sides, without dilation.

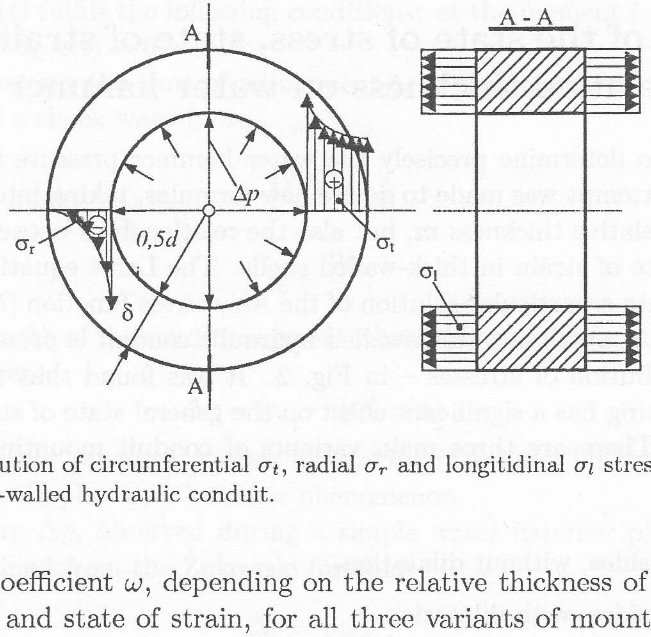


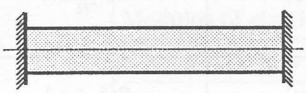
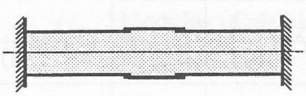
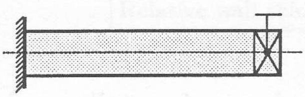
Figure 2. Distribution of circumferential σ_t, radial σ_r and longitidinal σ_l stresses in the wall of a thick-walled hydraulic conduit.

The value of coefficient ω, depending on the relative thickness of conduit walls, state of stress and state of strain, for all three variants of mounting is given in Tab. 1.

3 Analysis and comparison of the results of calculations and studies

The above formulas were verified comparing the results of calculations with the results obtained on the basis of formulas derived earlier by other authors. All formulas describing the simple water hammer pressure may be presented in the form of (6). The differences between particular formulas, for all variants of

Table 1. Dependence of the coefficient ω on the way of hydraulic conduit mounting, taking into consideration general state of strain and stress

Way of hydraulic conduit mounting	State of stress and strain	Coefficient ω
Conduit mounted on both sides, without dilataion 	$\varepsilon_t \neq 0 \quad \sigma_t \neq 0$ $\varepsilon_r \neq 0 \quad \sigma_r \neq 0$ $\varepsilon_l = 0 \quad \sigma_l \neq 0$	$\frac{1-\nu^2}{m+m^2} + 2(1+\nu)$
Conduit mounted on both sides, with dilatation 	$\varepsilon_t \neq 0 \quad \sigma_t \neq 0$ $\varepsilon_r \neq 0 \quad \sigma_r \neq 0$ $\varepsilon_l = 0 \quad \sigma_l = 0$	$\frac{1}{m+m^2} + 2(1+\nu)$
Conduit mounted on one side 	$\varepsilon_t \neq 0 \quad \sigma_t \neq 0$ $\varepsilon_r \neq 0 \quad \sigma_r \neq 0$ $\varepsilon_l \neq 0 \quad \sigma_l \neq 0$	$\frac{1.25-\nu}{m+m^2} + 2(1+\nu)$

hydraulic conduit mounting, concern the coefficient ω, as presented in Tab. 2. The data included in this table show that the relative thickness of conduits, two-way state of stress and strains – circumferential and longitudinal – are taken into consideration in the formulas devised by S. Bednarczyk [11] and J. Parmakian [2]. The formula derived by Y. Kono [12] disregards the relationships between stress and strain, and the relative thickness of conduits walls m. It follows that according to Y. Kono the coefficient ω is the same for all three variants of conduit mounting. Also according to G. Preissler [13] the value of this coefficient remains the same irrespective of the mounting variant used. He, however, takes into account the relative thickness of conduit walls, ignoring the relationships between stress and strain operating in various directions – this is confirmed by lack of the Poisson ratio ν in the formula. The formulas describing the coefficient ω for conduits mounted on both sides and dilated, published by E. B. Wylie and V. L. Streeter [14], are analogous with those proposed by author. However, the equations concerning conduits mounted on one side are different in form.

Table 2. Formulas defining the coefficient ω

No.	Author of the formula	Coefficient ω		
		Conduit mounted on both sides, without dilatation	Conduit mounted on both sides, with dilatation	Conduit mounted on one side
1	S. Bednarczyk J. Parmakian	$\frac{1-\nu^2}{m}$	$\frac{1-0.5\nu}{m}$	$\frac{1.25-\nu}{m}$
2	Y. Kono	$2(1+\nu)$	$2(1+\nu)$	$2(1+\nu)$
3	T. J. Nałęcz	$\frac{1-\nu^2}{m+m^2}+2(1+\nu)$	$\frac{1}{m+m^2}+2(1+\nu)$	$\frac{1.25-\nu}{m+m^2}+2(1+\nu)$
4	G. Pressler	$2\left[1+\frac{1}{2(m^2+m)}\right]$	$2\left[1+\frac{1}{2(m^2+m)}\right]$	$2\left[1+\frac{1}{2(m^2+m)}\right]$
5	E. B. Wylie V. L. Streeter	$\frac{1-\nu^2}{m+m^2}+2(1+\nu)$	$\frac{1}{m+m^2}+2(1+\nu)$	$\frac{1-0.5\nu}{m+m^2}+2(1+\nu)$

The formulas derived were verified analytically and experimentally, and compared with those proposed by the above-mentioned authors. The calculations were done for a thick-walled steel hydraulic conduit, mounted on both sides, without dilatation. Laboratory tests were conducted on universal test stand, designed for testing hydraulic elements by the throttling method. A computerized system of pressure measurement was applied in the studies. The system consisted of a micro-computer, a control-measuring module (measuring card), a signal concentrator and piezoelectric pressure transducers. The relative error of pressure transducer did not exceed 0.3% within the whole measuring range. The courses of pressure were measured during the occurrence of the water-hammer phenomenon, in a conduit mounted on both sides, between the distributor and flow regulator. The characteristic values used in the experiment and calculations are presented in Tab. 3. The results of calculations concerning water hammer pressure, and relative percentage deviations Δ between the results of calculations and measurements are included in Tab. 4.

4 Conclusions

The analysis made in the studies indicates that the formulas devised may be applied to determine water hammer pressure in thick-walled monolithic con-

Table 3. Characteristics of working liquid and a hydraulic conduit

Quantity	Symbol	Value	Unit
Working liquid-hydraulic oil L-HL-46			
Density	ρ	869.61	kg/m^3
Modulus of elasticity	K	$1.72{\cdot}10^3$	MPa
Volumentric flow rate	Q	$6.67{\cdot}10^{-4}$	m^3/s
Pressure in a steady state	p_o	1.96	MPa
Mean velocity in a steady state	c_o	6.13	m/s
Steel hydraulic conduit			
Modulus of elasticity	E	$2.06{\cdot}10^5$	MPa
Poisson ratio	ν	0.30	
Internal diameter	d	11.80	mm
Wall thickness	δ	2.00	mm
Relative wall thickness	m	0.1695	

Table 4. Results of calculations concerning water hammer pressure, and relative percentage deviations Δ

No.	Basis for determining water hammer pressure	Values of water hammmer pressure Δp MPa	Relative deviations Δ %
1	S. Bednarczyk J. Parmakian	7.33	1.66
2.	Y. Kono	7.42	2.19
3.	T. J. Nałęcz	7.28	0.97
4.	G. Preissler	7.29	1.11
5.	E. B. Wylie V. L. Streeter	7.28	0.97
6.	measurement	7.21	

duits, where the simple water hammer phenomenon takes place. The differences between analytical and experimental results may be caused by the Poisson coupling mechanism. This phenomenon considerably complicates computational processes, as it is difficult to determine the difference between the propagation velocity of a pressure shock wave and the velocity of a stress wave, whose value is always slightly higher.

Received 28 February 2002

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