INSTITUTE OF FLUID-FLOW MACHINERY
POLISH ACADEMY OF SCIENCES

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

109

Selected papers from the National Seminar
devoted to 60th birthday anniversary of
Professor Jarosław Mikielewicz,
Gdańsk, Poland, April 10th, 2001

WYDAWNICTWO IMP PAN

GDAŃSK 2001

EDITORIAL AND PUBLISHING OFFICE

IFFM Publishers (Wydawnictwo IMP), Institute of Fluid Flow Machinery, Fiszera 14, 80-952 Gdańsk, Poland, Tel.: +48(58)3411271 ext. 141, Fax: +48(58)3416144, E-mail: esli@imp.gda.pl

Financial support of publication of this journal is provided by the State Committee for Scientific Research, Warsaw, Poland

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ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY Appears since 1960

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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY
No. 109, 2001, 87-100

JAN TALER[1]

Inverse determination of local heat transfer coefficients

Two methods for the solution of the nonlinear inverse heat conduction problem encountered in the experimental determination of the local heat transfer coefficient distributions are presented in the paper. The methods are formulated as linear and nonlinear least-squares problems. The uncertainties in the estimated heat transfer coefficients are determined for the temperature measurements with known and unknown standard deviations. The main advantage of the presented methods is that they do not require any knowledge, or solution to, the complex fluid flow field.

1. Introduction

There are many different methods for measuring local heat transfer. One common category is mass transfer method [1,2]. The naphtalene sublimation method is one of the most convenient mass transfer methods, that is particularly useful in complex flows and geometries. Another widely used technique is the transient method [3-5]. Liquid crystals have been used extensively for the surface temperature measurements with this method. Both techniques have been widely used with considerable success but they have certain difficulties. The limitations to the naphtalene sublimation method are high temperatures. Liquid crystal thermography has a limited temperature range from -25^0C to 250^0C, so it is not feasible for examination of the objects at high temperature. An alternative method to obtain the local convective heat transfer coefficient, that does not have any disadvantages noted above, is the inverse procedure. Determination of the space-variable heat transfer coefficient on a complex shape surface requires the solution of the nonlinear inverse heat conduction problem (IHCP) [6-8].

The unknown parameters associated with the solution are selected to achieve the closest agreement in a least squares sense between the computed and measured temperatures using the Gauss-Newton method in conjunction with the singular value decomposition or modified Gram-Schmidt methods. Hensel and Hills [9]

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approached the two-dimensional steady-state inverse heat conduction problem using the linear least-squares method. Linearization of the least squares problem is accomplished by assuming unknown temperatures [9] or temperatures and heat fluxes [8,10] on the boundary.

The boundary is divided into a large number of elements whereas the temperatures or heat fluxes are assumed to be constant over each element. Having determined the boundary values of temperature and the heat flux from the solution of the IHCP, the convective heat transfer coefficients are determined from the Newton‘s Law of Cooling. Numerical [9-10] and experimental tests [19] demonstrated that the spatial distribution of the heat transfer coefficient can be estimated with satisfactory accuracy if the division of the boundary into elements is very fine. If the number of segments on the boundary is too small, than the constant value of temperature or the heat flux over an element cannot be assumed. In order to solve the over-determined IHCP the number of interior temperature measurement points should be greater than the number of boundary temperature or heat flux components. However, mounting numerous thermocouples inside the solid disturbs the temperature distribution and is impractical. In this paper two different problems are considered. In both cases the distribution of heat transfer coefficient is deduced from the internal temperature measurements. In the first case, the number of temperature measurement points is equal to the number of heat transfer components to be estimated. In the second case of the over- specified IHCP, the number of internal temperature measurements is greater than the number of unknown heat transfer coefficients on the boundary. Thermal conductivity of the solid $k(T)$ may be temperature-dependent. Both linear and nonlinear least-squares formulations are studied. The least-squares problems are parameterized by assuming the stepped changes of heat transfer coefficient on the boundary or expressing the space variations of the heat transfer coefficient in the functional form. The confidence intervals of IHCP solutions are calculated using the error propagation rule of Gauss [11].

2. Numerical formulation of the inverse heat conduction problem

The temperature distribution in the body is governed by the nonlinear partial differential equation

$$\nabla \cdot [k(T)\nabla T] = 0 . \tag{1}$$

The unknown boundary conditions may be expressed as:

- boundary conditions of the first kind

$$T = f(\mathbf{r}_s) \quad \text{on} \quad S , \tag{2}$$

- boundary conditions of the second kind

$$\lambda(T)\frac{\partial T}{\partial n} = \dot{q}(\mathbf{r}_s) \quad \text{on} \quad S , \tag{3}$$

- boundary conditions of the third kind

$$k(T)\frac{\partial T}{\partial n} = h(\mathbf{r}_s)(T_\infty - T_s)\ , \tag{4}$$

where $\mathbf{r}_s$ represents points on the boundary, S.

In addition to the unknown boundary conditions the internal temperature measurements f_i are included in the analysis:

$$T(\mathbf{r}_i) = f_i, \quad i = 1, \ldots, m\ , \quad m \geq n\ . \tag{5}$$

The objective of the present approach is to determine the spatial distribution of the heat transfer coefficient from measured temperatures at m interior locations. The measured temperatures $\mathbf{f} = (f_1, \ldots, f_m)^T$ depend on the transfer coefficients and can, therefore, be used for their determination. The problem may be formulated as linear or nonlinear IHCP.

3. Linear IHCP

First, the part of the boundary with unknown conditions is discretized into n intervals. The unknown boundary temperature or heat flux is then approximated by a piecewise constant function (stepped function) (Fig.1).

The parameters $\mathbf{x}$ are unknown temperatures or heat fluxes.

The Kirchhoff transformation can be introduced to transform Eqs(1-3) to the linear problem. The new variable is the integral over temperature of the thermal conductivity. For known boundary condition of the third kind, however, the transformed boundary condition remains nonlinear. In this paper, the method of successive substitution will be used to linearize the problem (1-3). The spatial discretization of Eq.(1) subjected to boundary conditions (2-3) using finite difference, or finite element methods yields the nonlinear system of N_E algebraic equations

$$\mathbf{A}(\mathbf{T})\mathbf{T} = \mathbf{B}(\mathbf{T})\mathbf{x} \tag{6}$$

where vector $\mathbf{x}$ contains n unknown surface temperatures and heat fluxes, $\mathbf{A}$ is $N_E \times N_E$ matrix, $\mathbf{T}$ is the N_E dimensional column vector of node temperatures and $\mathbf{B}$ is $N_E \times n$ matrix. The right side of Eq. (6) $\mathbf{Bx}$ contains all of the applied boundary and volumetric source functions.

Using the method of successive substitution Eqs (6) can be written as ,

$$\mathbf{A}(\mathbf{T}^{(k)})\mathbf{T}^{(k+1)} = \mathbf{B}(\mathbf{T}^{(k)})\mathbf{x}^{(k+1)}\ , \tag{7}$$

where the superscript indicates the iteration number. The algorithm has a reasonably large radius of convergence. After selecting the starting vector $\mathbf{T}^{(0)}$ the convergent solution is obtained in about six iterations. If the thermal conductivity is constant then iteration is not needed. The linear system (7) has the unique solution

$$\mathbf{T}^{(k+1)} = [\mathbf{A}(\mathbf{T}^{(k)})]^{-1}\mathbf{B}(\mathbf{T}^{(k)})\mathbf{x}^{(k+1)}, \quad k = 0, 1, \ldots \tag{8}$$

Temperature of the body is measured at m interior locations which are assumed to be coinciding with selected nodes in the mesh.

The temperature at the j^{th} node, where the temperature sensor is placed, can be calculated in the following way:

$$T_j^{(k+1)} = \mathbf{e}_j^T[\mathbf{A}(\mathbf{T}^{(k)})]^{-1}\mathbf{B}(\mathbf{T}^{(k)})\mathbf{x}^{(k+1)}, \tag{9}$$

where $\mathbf{e}_j$ denotes j^{th} column vector of the identity matrix $\mathbf{I}_{N_E}$. The vector of calculated temperatures $\mathbf{T}_m^{(k+1)}$ at nodes coincident with sensor locations, is then formed as follows

$$\mathbf{T}_m^{(k+1)} = \mathbf{C}_m^{(k)}\mathbf{x}^{(k+1)} \tag{10}$$

where $\mathbf{T}_m^{(k+1)}$ is the m-dimensional vector, and $\mathbf{C}_m^{(k)}$ is the $m \times n$ matrix. The object is to choose $\mathbf{x}$ such that the computed temperatures agree within the certain limits with the experimentally measured temperatures. This may be expressed as

$$\mathbf{T}_m^{(k+1)} - \mathbf{f} \cong 0, \tag{11}$$

where $\mathbf{T}_m^{(k+1)}$ is given by Eq. (10).

If the number m of measured temperatures is equal to the number of unknown boundary temperatures and heat fluxes n, then the $n \times n$ system of linear equations (11) has the unique solution

$$\mathbf{x}^{(k+1)} = (\mathbf{C}_m^{(k)})^{-1}\mathbf{f}, \quad k = 0, 1, \ldots \tag{12}$$

provided the inverse matrix $(\mathbf{C}_m^{(k)})^{-1}$ exists.
If thermal conductivity k does not depend on temperature then iteration is not needed.
The uncertainty of the calculated parameters $\mathbf{x}$ can be determined using the error propagation rule [11].

A standard procedure is to take more temperature measurements than the number of unknown parameters x_i. The least-squares method is used to determine $x_1 \ldots x_n$ when $m > n$. To measure how well the calculated temperature agree with data, the chi-square merit function is used

$$S = (\mathbf{f} - \mathbf{T}^{(k+1)})^T\mathbf{G}_f(\mathbf{f} - \mathbf{T}_m^{(k+1)}), \tag{13}$$

where $\mathbf{G}_f$ is know positive definite matrix. If the measurement errors are known then the matrix $\mathbf{G}_f$ is

$$\mathbf{G}_f = \begin{bmatrix} 1/\sigma_1^2 & 0 & \ldots & 0 \\ 0 & 1/\sigma_2^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & 0 \\ 0 & 0 & \ldots & 1/\sigma_m^2 \end{bmatrix} \tag{14}$$

where σ_i is the standard deviation of the ith measured temperature f_i. The least-squares solution that minimizes the sum (13) is [12]

$$\mathbf{x}^{(k+1)} = [(\mathbf{C}_m^T)^{(k)}\mathbf{G}_f\mathbf{C}_m^{(k)}]^{-1}(\mathbf{C}_m^T)^{(k)}\mathbf{G}_f\mathbf{f} . \tag{15}$$

Once the boundary temperatures and heat fluxes are determined, the heat transfer coefficient is determined from

$$h = \frac{-k(T)}{(T|_S - T_\infty)}\frac{\partial T}{\partial n}\Big|_S , \tag{16}$$

where the fluid temperature T_∞ is known from measurements.

The measure of the accuracy of the estimated parameters $\hat{\mathbf{x}}$ is 95% confidence interval. Having finished iteration the 95% confidence intervals are calculated according to

$$\hat{x}_i - 2\sigma(x_i) \le x_i \le \hat{x}_i + 2\sigma(x_i) , \tag{17}$$

where $\sigma(x_i) = \sqrt{C_{ii}}$.

The notation C_{ii} represents the diagonal element of the covariance matrix

$$\mathbf{C}_{\hat{x}} = (\mathbf{C}_m^T\mathbf{G}_f\mathbf{C}_m)^{-1} . \tag{18}$$

The variance associated with the estimate T_j can be found from

$$\hat{T}_j - 2\sigma(T_j) \le T_j \le T_j + 2\sigma(T_j) , \tag{19}$$

where $\sigma(T_j)$ are square roots of the diagonal elements of the matrix

$$\mathbf{C}_T = \mathbf{C}_m\mathbf{C}_{\hat{x}}\mathbf{C}_m^T . \tag{20}$$

In some cases the standard deviations associated with a set of temperature measurements are not known in advance and the sum of squares are used to derive a value σ.

If we assume that $\mathbf{G}_f = \mathbf{I}_m$ the $100(1-\alpha)$ % confidence intervals for x_i are calculated using the formulas in the inequality

$$\hat{x}_i - t_{m-n}^{\alpha/2}\sqrt{\frac{S}{m-n}}\cdot\sqrt{C_{ii}} \le x_i \le \hat{x}_i + t_{m-n}^{\alpha/2}\sqrt{\frac{S}{m-n}}\cdot\sqrt{C_{ii}} , \tag{21}$$

where the value $t_{m-n}^{\alpha/2}$ is the $(1-\alpha/2)th$ quantile of the Student's t-distribution with $(m-n)$ degrees of freedom. The least-squares sum S is computed from Eq. (13) using the fitted parameters $\hat{\mathbf{x}}$ (boundary temperatures or heat fluxes).

The covariance matrix (18) simplifies to

$$\mathbf{C}_{\hat{x}} = (\mathbf{C}_m^T\mathbf{C}_m)^{-1} . \tag{22}$$

4. Nonlinear IHCP

If the number of the unknown heat transfer components is equal to the number of the temperature measurement points, the method presented in [6] can be used.

The heat transfer coefficients are estimated so that the computed temperatures agree with the experimentally measured temperatures, e.g. Eqs (5) should be satisfied. The predicted temperatures depend implicitly on the heat transfer coefficients $\mathbf{x}$. In order to solve the system on nonlinear equations (5) the SOR-secant method or SOR-Newton methods can be used [6]. The SOR (= successive over-relaxation) algorithm works well in practice and is very efficient in terms of the programming and computing time.

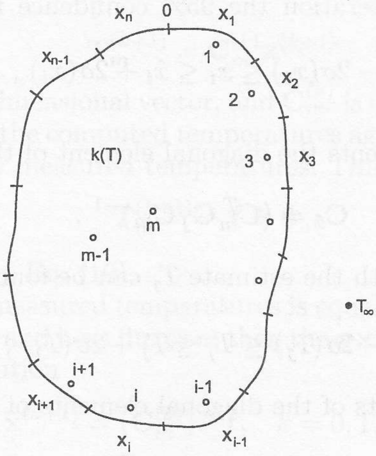

Fig. 1. Location of temperature sensors and boundary discretization; Method I – boundary conditions of the first or second kinds – $x_1, ..., x_n$ represent unknown temperatures or heat fluxes; Method II – boundary condition of the third kind – $x_1, ..., x_n$ represent unknown heat transfer coefficients.

When unknown parameters are heat transfer components and $m > n$ then the least-squares problem becomes nonlinear (Fig. 1). The numerical solutions of the nonlinear overdetermined IHCP encountered in determining heat transfer coefficients, are presented in [7]. The Gauss-Newton method in conjunction with the singular value decomposition was used to determine the space components of the heat transfer coefficient. The methods are very efficient provided the starting value $\mathbf{x}^{(0)}$ is sufficiently close to the global minimum $\hat{\mathbf{x}}$. This weakness of the Gauss-Newton method is not inherent in the Levenberg-Marquardt algorithm [12], that is fast and robust. The Levenberg-Marquardt method is one of the best general-purpose Gauss-Newton-based algorithms for solving nonlinear least squares problems.

The problem of determining space-variable heat transfer coefficient can be formulated as a parameter estimation problem by selecting the functional form for the heat transfer coefficient or by approximating the spatial changes of the heat transfer coefficient by the stepped function. In both cases, there are n parameters: $\mathbf{x} = (x_1, \ldots, x_n)^T$ to be determined such that the computed temperatures T_i agree in the least-squares sense with the experimentally acquired temperatures f_i. The Levenberg-Marquardt method performs the k^{th} iteration as

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta^{(k)} , \tag{23}$$

where

$$\delta^{(k)} = [(\mathbf{J}_m^{(k)})^T \mathbf{G}_f \mathbf{J}_m^{(k)} + \mu^{(k)} \mathbf{I}_n]^{-1} (\mathbf{J}^{(k)})^T \mathbf{G}_f [\mathbf{f} - \mathbf{T}_m(\mathbf{x})] , \tag{24}$$

and

$$\mathbf{J}_m = \frac{\partial \mathbf{T}_m(\mathbf{x})}{\partial \mathbf{x}^T} = \left[\left(\frac{\partial T_i(\mathbf{x})}{\partial \mathbf{x}_j} \right) \right]_{m \times n} . \tag{25}$$

Suppose we have converged to the solution of the nonlinear IHCP.

To obtain approximate $100(1-\alpha)\%$ confidence intervals for $\mathbf{x}$, Eqs. (17) or (21) may be used as in the case of the method I. However, the matrix $\mathbf{J}_m$ should be used in Eqs (17) and (21) instead of $\mathbf{C}_m$.

5. Test cases

First numerical experiment with simulated data is presented to demonstrate the method I and II. The local heat transfer coefficient from a hot surface to boiling water is often measured using a heat flux transducer. The transducer body is made of stainless steel that incorporates three thermocouples placed in different positions (Fig. 2). Unidirectional steady-state heat transfer is assumed to occur within the transducer between the resistance heater and boiling water. Similar constructions have heat flux meters inserted in inspection ports [13]. Based on temperature measurements at three locations, the heat flux $x_1 = \dot{q}_s$ on the surface $x = 0$ and the surface temperature $x_2 = T|_{x=L}$ at $x = L$ (method I) or the heat transfer coefficient $x_2 = h$ (method II) are determined. Since the heat flux x_1 is a constant independent of y, then the heat transfer coefficient in the method I can be calculated from the estimated values of the heat flux x_1 sensor surface temperature x_2 and from measured fluid temperature T_∞:

$$h = \frac{x_1}{x_2 - T_\infty} . \tag{26}$$

The governing heat conduction equation with temperature-dependent thermal conductivity is

$$\frac{d}{dy}\left[k(T) \frac{dT}{dy} \right] = 0 . \tag{27}$$

The objective is to determine the boundary conditions

$$-\left[k(T) \frac{\partial T}{\partial y} \right]\Big|_{y=0} = x_1 , \tag{28}$$

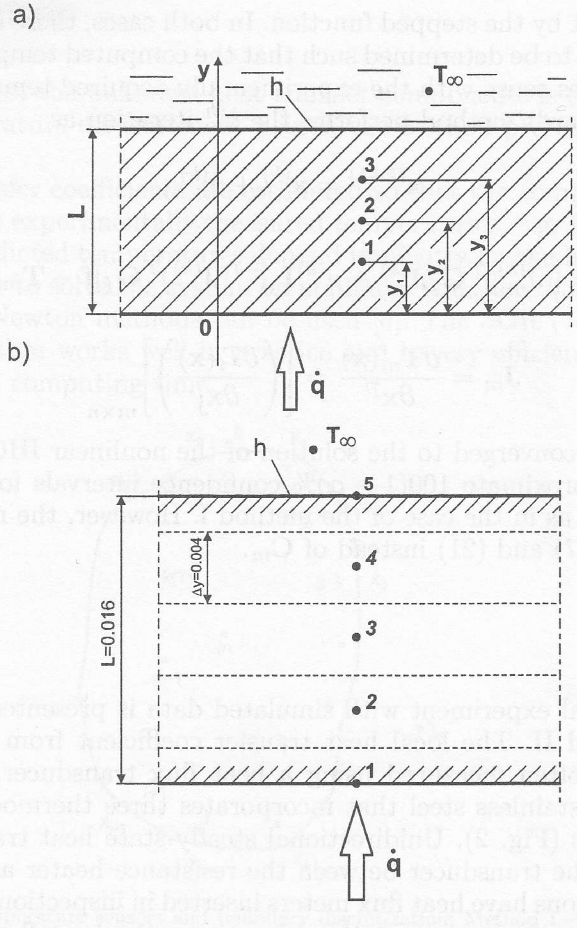

Fig. 2. Location of thermocouples in the heat flux sensor (a) and division of the sensor into control volumes (b).

$$T\Big|_{y=L} = x_2 \ , \quad \text{(method I)} \ , \tag{29}$$

or

$$-\left[k(T)\frac{\partial T}{\partial y}\right]\Bigg|_{y=L} = x_2(T|_{x=L} - T_\infty) \ , \quad \text{(methodII)} \ . \tag{30}$$

Using, the interior conditions known from temperature measurements (Fig. 2a)

$$T|_{y=y_1} \cong f_1 \ , \tag{31}$$

$$T|_{y=y_2} \cong f_2 \ , \tag{32}$$

$$T|_{y=y_3} \cong f_3 \ , \tag{33}$$

the parameters x_1 and x_2 will be determined using the methods I and II. The thermal conductivity is temperature dependent and represented as follows

$$k(T) = 14.65 + 0.0144 \cdot T\,, \tag{34}$$

where k is expressed in W/mK and T in ^oC. To simulate 'exact' measurement data the direct heat conduction problem was solved using: $\dot{q}_s = 274800$ W/m^2, $h = 24000$ W/m^2, $T_\infty = 15^oC$, $L = 0.016$ m, $y_1 = 0.004$ m, $y_2 = 0.008$ m and $y_3 = 0.012$ m. In order to study the effect of errors in the measured temperatures, small numbers ε_i, $i = 1, 2, 3$ were added to 'exact' data

$$\begin{aligned} f_1 &= T|_{y=y_1} + \varepsilon_1 = 314.29 + 1.01 = 315.3^oC \\ f_2 &= T|_{y=y_2} + \varepsilon_2 = 255.68 + (-2.98) = 252.7^oC \\ f_3 &= T|_{y=y_3} + \varepsilon_3 = 194.23 + 1.47 = 195.7^oC \end{aligned} \tag{35}$$

Standard deviations of the measured temperatures are: $\sigma_1 = 1^oC$, $\sigma_2 = 2^oC$ and $\sigma_3 = 1.5^oC$. The differential equation (27) with boundary conditions (28)-(29) is solved numerically using the control volume method.
The resulting system of nonlinear, algebraic equations is solved using the matrix inversion method in conjunction with successive iteration (method I) or by using the Gauss-Seidel iteration (method II). The results given in Tab. 1 are in excellent agreement with the input data: $x_1 = \dot{q}_s = 274800$ W/m^2 and $\dot{x}_2 = h = 2400$ W/m^{2}K.

Table 1. Results obtained by the inverse methods

	Analytical method (exact solution)	Method I	Method II
Estimated parameters	$\dot{q}_s = 274800$ W/m^2, $h = 2400$ W/m^{2}K, $T\|_{x=L} = 129.5^oC$	$\hat{x}_1 = \dot{q}_s = 275045$ W/m^2, $\hat{x}_2 = T\|_{x=L} = 129.97^oC$, $h = 2392.3$ W/m^{2}K	$\hat{x}_1 = \dot{q}_s = 274964$ W/m^2, $\hat{x}_2 = h = 2390.6$ W/m^2, $T\|_{x=L} = 130.01^oC$
Standard Deviations		$\sigma(\hat{x}_1) = 4082.3$ W/m^2 $\sigma(\hat{x}_2) = 2.28^oC$	$\sigma(\hat{x}_1) = 4027.4$ W/m^2 $\sigma(\hat{x}_2) = 82.7$ W/m^{2}K
Sum of squares $S(\hat{\mathbf{x}})$		$3.695(^oC)^2$	$3.702(^oC)^2$
	Temperature distribution $T(y)$, ^oC		
y [m]	Analytical method	Method I	Method II
0.000	370.43	371.02	370.99
0.004	314.29	314.85	314.84
0.008	255.68	256.21	256.22
0.012	194.23	194.74	194.76
0.016	129.50	129.97	130.01

In the second example the actual experimental data are used. Experiments were performed with an array of vertical finned tubes with two longitudinal fins arranged in staggered pattern. The experimental results reported herein are among the first that show the variation of the local heat transfer coefficients over the circumference of the finned tube. Most data reported previously were acquired

for smooth tubes at low temperatures [3, 14-20].
With uniform heat flux at the inner surface of the tube placed in a cross flow, heat flows by conduction in the peripheral direction due to the asymmetric nature of the air flow around the perimeter of the tube. The peripheral heat flow affects the wall temperature distribution to such an extent that in some cases significantly different results may be obtained for geometrically identical surfaces [15] . In this study IHCP is solved to account for the peripheral wall heat conduction.

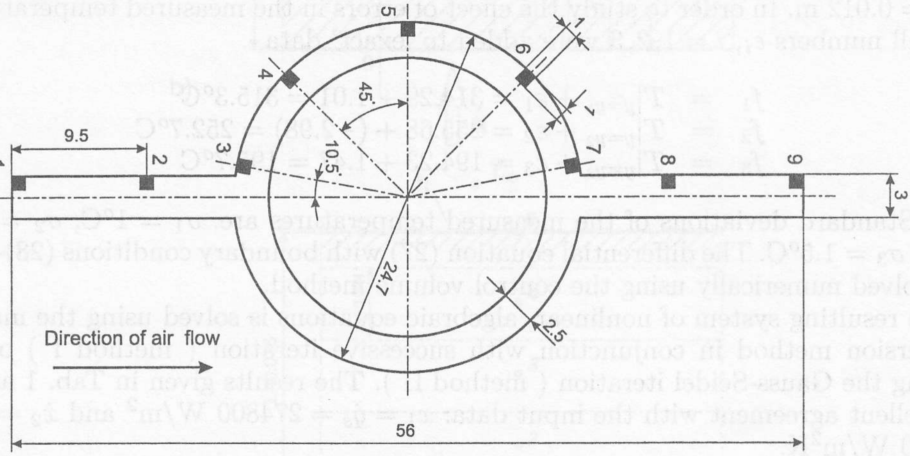

Fig. 3. Cross section of the finned tube showing locations of nine thermocouples.

Fig. 3 shows a cross through the finned tube and dimensions needed in calculation of the local heat transfer values around the tube periphery. The temperature of the tube was measured at nine locations shown in Fig. 3. The midplane of the finned tube surface was instrumented with K-type sheeted thermocouples of 1 mm in diameter.

The tube with an outside diameter of 24.7 mm was constructed from the K18 low alloy steel and was centrally located at the fourth row in the array. An array of 36 electrically heated tubes was placed in the wind tunnel in the 0.27×0.30 m rectangular test section. The heated length of the tube was 0.30 m. The tubes were mounted in a staggered array with nine successive rows each consisting of five tubes. The pitch of tubes in direction of flow S_L and the pitch of tubes in plane perpendicular to flow S_T were: $S_L = 0.031$ m and $S_T = 0.05325$ m, respectively. The air temperature at the inlet and outlet of the bundle was measured with K-type thermocouples of the diameter of 1 mm in diameter placed in the two cross sections of the air channel. The Reynolds number was calculated based on the outer diameter and the maximum air velocity in the narrowest cross section of the bundle. The heat flux on the inner surface of the tube $\dot{q}_{in}$, is given by $\dot{q}_{in} = \dot{Q}/(\pi d_{in} H)$, where $\dot{Q}$ denotes the electric power of the resistance heater placed inside the tube, d_{in} is the inner diameter and H is the heated length of the tube.

The local heat transfer coefficient $h(L)$ is approximated by the following function

$$h(L) = \sum_{i=1}^{5} x_i \cdot \cos\left[(i-1)\frac{L}{L_c}\pi\right], \tag{36}$$

where x_i are the coefficients to be estimated, L is the distance from the point 1 at the tip of the fin in the direction of flow (path coordinate along tube) and L_c is the extended distance between points 1 and 9 in Fig. 3. Figs 4 and 5 show spatial distributions of the temperature and heat transfer coefficient as a function of path coordinate along the finned tube for two different values of the Reynolds number. The experiments and calculations were carried out at the following conditions:
$\dot{Q} = 103.9$ W, $T_\infty = 23.4^o$C for Re = 47962,
and
$\dot{Q} = 123.5$ W, $T_\infty = 29.2^o$C for Re = 14514,
where T_∞ denotes the air temperature. The thermal conductivity of the tube material was $k = 53$ W/mK. The unknown coefficients x_i, $i = 1, \ldots, 5$ in Eq. (35) were estimated using the method II.

The temperature distribution in the finned tube cross section was calculated using the control volume method. Because of the symmetry, only a half of the tube was considered. It was divided into 75 control volumes. The following 95% confidence intervals for the parameters x_i are obtained for Re = 47962: $x_1 = (154.1 \pm 8.3)$ W/m^{2}K, $x_2 = (56.4 \pm 4.1)$ W/m^{2}K, $x_4 = (1.0 \pm 8.5)$ W/m^{2}K and $x_5 = (0.97 \pm 6.7)$ W/m^{2}K.

The heat transfer conditions within the bundle are dominated by the boundary layer separation effects and by the wake interactions. The local heat transfer coefficient decreases from the fin tip up to the angular position, $\theta \cong 105^o$ on the tube circumference (Fig. 5), where the boundary layer separation occurs, then increases due to the turbulence in the wake. Similar results are obtained when spatial variation of the heat transfer coefficient was approximated by the stepped function.

6. Conclusions

Two iterative inverse heat conduction methods are developed to determine unknown distributions of the local heat transfer coefficient on surfaces of arbitrarily shaped solids. The distribution of the heat transfer coefficient is estimated by utilizing temperature measurements inside the solid. The fluid temperature is also measured. The boundary of the solid is divided into elements, on which temperatures or heat fluxes are assumed to be constant (method I) or the inverse heat conduction problem (IHCP) is parametrized by assuming the stepped changes of heat transfer coefficient or expressing it as a function of space (method II). The overdetermined IHCP is solved using the linear (method I) and nonlinear (method II) least-squares methods. The thermal conductivity of the solid may be dependent on temperature.

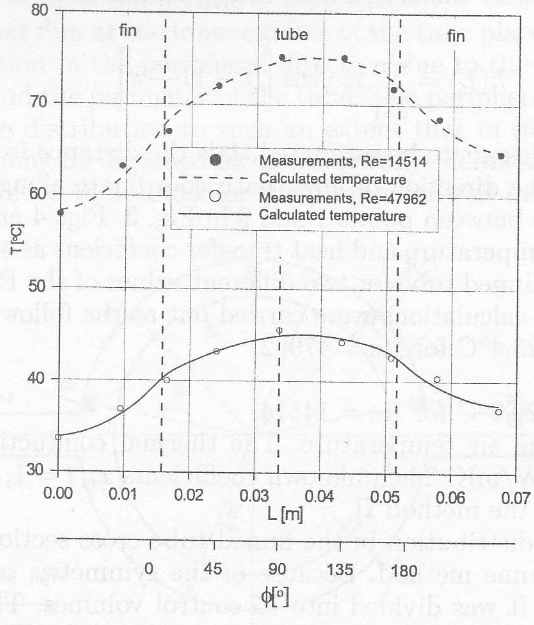

Fig. 4. Measured temperature at locations 1-9 and determined temperature distribution versus the path coordinate along tube or the angular position at two different Reynolds numbers.

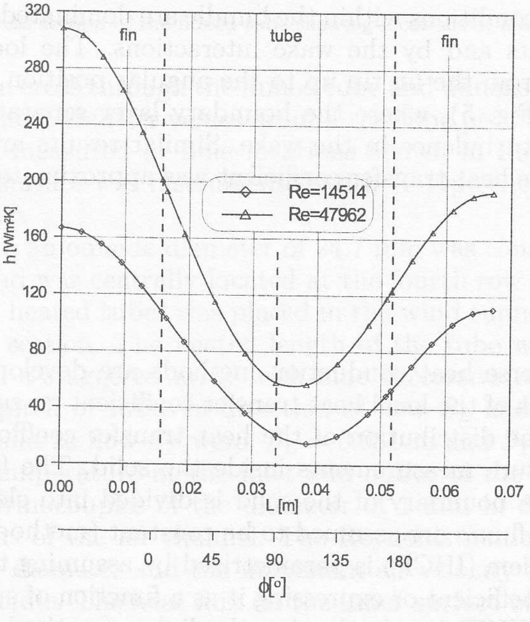

Fig. 5. Spatial distribution of the heat transfer coefficient on the outer surface of the finned tube at two different Reynolds numbers.

The uncertainties in the estimated components of the heat transfer coefficient or in the estimated parameters are determined for the temperature measurements with known and unknown standard deviations. The determination of the circumferential heat transfer coefficient distribution on the heated tube with two longitudinal fins in cross flow demonstrates the accuracy and the potential use of the second method.

The main advantage of presented methods is that they do not require any knowledge, or solution to, the complex fluid flow field.

It should be noted that determining unknown steady distribution of heat transfer coefficients by using the developed methods is inexpensive, since they require only one fluid temperature probe and a few thermocouples for temperature measurements inside the solid.

Received 15 July 2001

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