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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

104

WYDAWNICTWO IMP PAN

GDAŃSK 1998

THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

Wydanie publikacji zostało dofinansowane przez PAN ze środków DOT uzyskanych z Komitetu Badań Naukowych

EDITORIAL OFFICE – REDAKCJA

Wydawnictwo Instytutu Maszyn Przepływowych
Polskiej Akademii Nauk
ul. Gen. Józefa Fiszera 14, 80-952 Gdańsk, skr. poczt. 621,
☎ (0-58) 341-12-71 wew. 141, fax: (0-58) 341-61-44,
e-mail: esli@imppan.imp.pg.gda.pl

ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 104, 1998, 105-117

KRZYSZTOF JAN JESIONEK[1]

Prediction of turbulent flow separation in conical diffusers

The prediction of the flow conditions in diffusing passages has long been a major unsolved problem in fluid mechanics. The chief complicating effect is the presence of the large adverse pressure gradient, whose exact influence on the velocity distribution has defied mathematical analysis.

In the paper, the method of flow-separation forecasting for steady, incompressible flows without swirl in axisymmetrical conical diffusers has been described. The separation conditions are shown to depend on the shear stress distribution inside of the turbulent boundary layer. Comparisons between the predictions of boundary-layer theory and the actual data of diffuser passages are given.

1. Introduction

The diffuser channel is one of the substantial elements of fluid-flow turbomachinery and fluid-flow installations. The fluid stream flows in it through subsequent control cross-sections of a larger and larger flow area. Thus, the continuity equation implies a velocity drop of the working medium. Following this, the kinetic energy of the inflowing fluid is, to a certain extent, transformed to potential energy. Subsequently, the medium is displaced towards the increasing values of static pressure. This means that in a diffuser flow there occur positive values of the longitudinal pressure gradient. In elements of fluid-flow machines, depending on the values of longitudinal static pressure gradient dp/dx (see Fig. 1), the flow movement may be of

a confusor – $dp/dx < 0$,
gradientless – $dp/dx = 0$,
or diffuser – $dp/dx > 0$ character.

In a diffuser flow, the boundary-layer thickness grows most intensely of all of the cases mentioned. The velocity profile of the working medium becomes deformed, which in consequence results in a considerable increase of energy losses.

[1]Wrocław University of Technology, Institute of Heat Engineering and Fluid Mechanics, Turbomachinery Department, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław

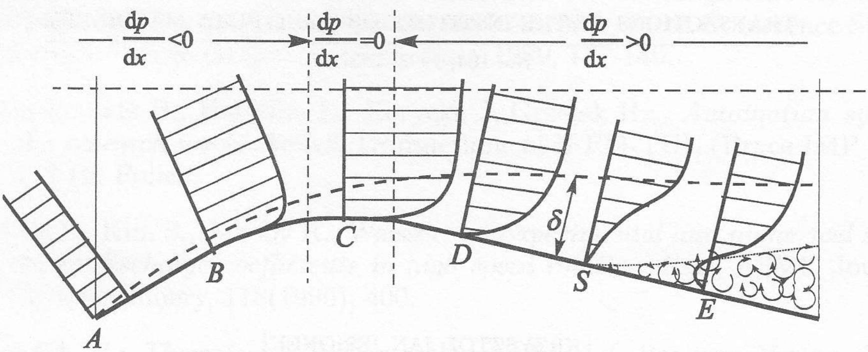

Fig. 1. Flow character in details of fluid-flow machinery: A, B – confusor, C – zero gradient, D, S, E – diffuser.

Flow stability tends to decrease as well, which raises the occurrence probability of stream separation occurrence from the channel wall.

Hence, we can assume that, from the viewpoint of aerodynamics, the scope of issues for improving the effectiveness of fluid-flow installations chiefly resolves itself to increasing flow effectiveness at diffuser sections with simultaneous prevention of separation.

2. Boundary-layer separation

To a large extent, the reliability and effectiveness of fluid-flow installations are determined by working medium flow character and the real velocity field in the cross-sections of the analysed channels. In principle, two kinds of flow are possible:

- unseparated, when all of the cross-sections are totally filled with the fluid flowing along the main movement direction,
- separated, when only part of the channel cross section is used for the movement along the mentioned direction.

In the latter case, the effective flow area, i.e. the area occupied by the active stream is remarkably smaller than the geometrical area.

Following this, in order to assess the work effectiveness of a diffuser channel, it is critically important to find out if, during the fluid movement, there occurs a phenomenon of boundary-layer separation. It presence is highly disadvantageous since the zone where it occurs no longer contributes to the essential task of a diffuser, i.e. kinetic-to-potential energy transfer, e.g. Jesionek & Zarjankin (1992), (1994) and Zarjankin & Jesionek (1990), (1995). The part of the flow channel affected by the separation phenomenon turns out useless.

An additional problem, in the case of the flow-separation occurrence, is also made up by the transition from turbulent quasi-stationary flow to non-stationary flow. Such a condition results in the occurrence of changeable aerodynamic forces in the flow (Zarjankin & Jesionek (1990)), the forces affecting the channel walls or the elements placed in them, which leads to a drop of structural reliability.

In consequence, the problems associated with prevention against stream separation from the channel wall appear to be some of the crucial aerodynamics issues. Their positive resolution will largely influence the further advances in raising the efficiency of fluid machinery and manifold flow devices.

3. Separation criteria

In order to determine whether or not the phenomenon of boundary-layer separation is likely to occur, various criteria are used. In papers of Jesionek (1992), (1998) and Jesionek & Zarjankin (1993) suggested are two such criteria, the first in the form of the inequality

$$K1 = \left[\left(\frac{H}{1.4} - 1\right)\left(\frac{L}{D_1}\right)^{0.2} + 1.0\right]^2 - n \geq 0, \tag{1}$$

where:

H	–	boundary layer shape parameter,
L	–	diffuser length,
D_1	–	diameter of the diffuser inlet
n	–	diffuser area ratio, $n = A_2/A_1$
A_1, A_2	–	inlet and outlet diffuser cross-section area respectively,

and the second criterion of separation, defined as the one below:

$$K2 = \frac{\bar{x}}{\bar{c}_i}\left(\frac{d\bar{c}_i}{d\bar{x}}\right)_{max} + 0.45 \geq 0. \tag{2}$$

In equation (1) the numeric value of the boundary-layer shape parameter H is assumed to be equal to 1.8. It is the lower amount of the value, at which according to A. E. Von Doenhoff and N. Tetervin (1943) the phenomenon of flow separation from the wall channel may occur. On the other hand, expression (2) may also be applied to the specific cases of diffuser channels. Referring to the continuity equation and the A_i/A_1 section ratios determined on the basis of geometric relations, the expression to determine the local dimensionless velocity in conical diffusers is arrived at:

$$\bar{c}_i = \left[1 + \bar{x}(\sqrt{n} - 1)\right]^{-2} \frac{1}{(1 - \bar{\Delta}_2^* \bar{x}^{0.8})}, \tag{3}$$

where $\bar{\Delta}_2^*$ is, according to papers of Jesionek (1992) and (1998), the blocked cross-section area fraction – relative flow area corresponding to the boundary-layer displacement thickness. After determining the derivative value dc/dx, arrived at on the basis of differentiating expression (3) and its substitution in expression (2), the $K2$ criterion resolves itself to the relation

$$K2 = \sum_{i=1}^{k-1} \left(\frac{\bar{x}_{i+1} + \bar{x}_i}{\bar{x}_{i+1} - \bar{x}_i} \right) \left(\frac{\varphi_{i+1} - \varphi_i}{\varphi_{i+1} + \varphi_i} \right) + 0.45, \tag{4}$$

where the dimensionless parameter φ is determined for conical diffusers by means of the following relations:

$$\varphi_{i+1} = \frac{1}{[1 + \bar{x}_{i+1}(\sqrt{n} - 1)]^2 (1 - \bar{\Delta}_2^* \bar{x}_{i+1}^{0.8})}, \tag{5}$$

$$\varphi_i = \frac{1}{[1 + \bar{x}_i(\sqrt{n} - 1)]^2 (1 - \bar{\Delta}_2^* \bar{x}_i^{0.8})}. \tag{6}$$

4. Blocked area fraction

Equations (3), (5) and (6) contain the value $\bar{\Delta}_2^*$, referred to as the blocked cross-section area fraction – the relative flow area corresponding to the boundary-layer displacement thickness. The ‚2' index means that this value corresponds to the ‚2' outlet cross-section of the channel. For the conical diffusers, it is determined (in an implicit form) by the expression:

$$\bar{\Delta}_2^* = Bn^2(1 - \bar{n}_2^*)^3 \int_0^1 \frac{[1 + \bar{x}(\sqrt{n} - 1)]^{-4.6}}{(1 - \bar{x}_2^* \bar{x}^{0.8})^{2.8}} \frac{d\bar{x}}{\bar{x}^{0.2}}. \tag{7}$$

Equation (7) renders it possible to calculate the mentioned relative area of the flow, corresponding to the discharge loss at the outlet section of any diffuser channel. It is important in the case when the boundary-layer separation does not occur in it.

In this equation, dimensionless B – coefficient is determined by the relation:

$$B = \frac{0.1152H}{Re_{D1}^{0.2}} \left(\frac{L}{D_1} \right)^{0.8}, \tag{8}$$

where the Reynolds number is defined by the inlet diameter D_1 of the conical diffuser and velocity c_1 at the inlet:

$$Re_{D_1} = \frac{c_1 D_1}{v}. \tag{9}$$

Boundary layer shape parameter H is determined by the empirical relation

$$H = 1.4 \left[1 + \frac{(\sqrt{n} - 1)}{(\frac{L}{D_1})^{0.2}} \right]. \tag{10}$$

5. Determination of separation place coordinate

In the case of the diffuser flow it is not only important to state that, for instance, the separation phenomenon takes place but also to determine the x_S coordinate of that separation point S, for given flow conditions. The suggested solution includes, at the first stage (for $j = 0$, Fig. 2), the checking out the values of the $K2$ – and, conceivably, $K1$ – criteria.

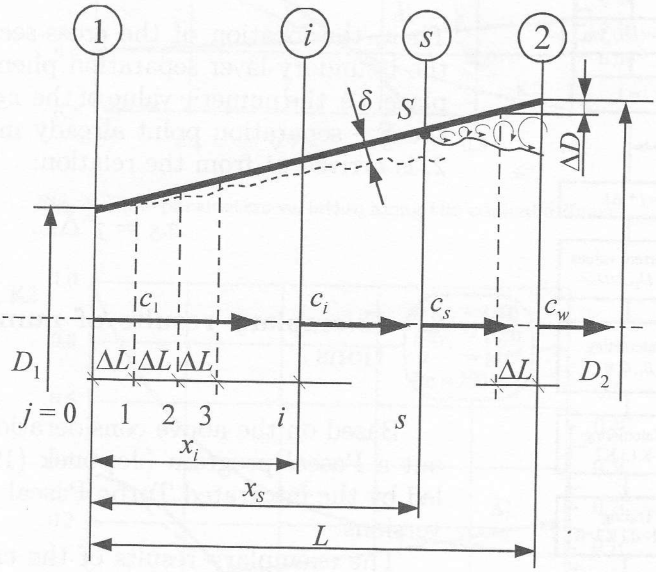

Fig. 2. Geometry of a conical diffuser.

Provided the numeric value of the calculated criterion is not negative, the diffuser is unseparated, which means that the hypothetical separation point S is located at the outlet section or beyond it. On the other hand, if e.g. $K2 < 0$, the cross-section coordinate where the boundary-layer separation can be initiated, is determined according to the following algorithm:

- assume (for index $j = 1$) that the diffuser has got the length equal ΔL,
- for that short diffuser recalculate the current outlet cross-section dimensions,
- determine the new values of the dimensionless geometric parameters,
- calculate the value of the dimensionless flow area – blocked area fraction,
- calculate and verify the separation criterion value $K2$, (and conceivable $K1$).

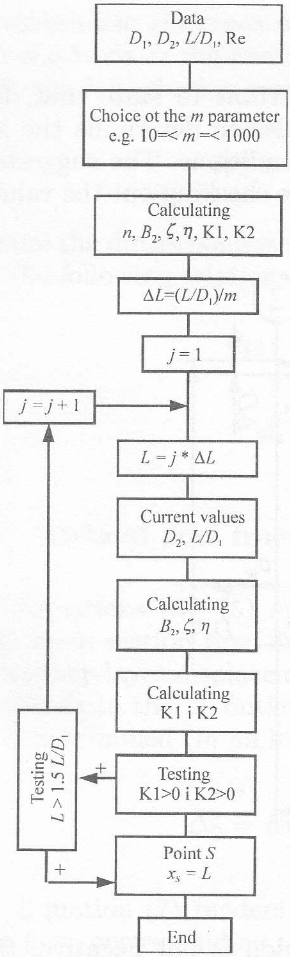

Fig. 3. Scheme of the calculation method for conical diffusers.

If the $K2$ – value is still positive, analyse the subsequent diffuser cases (for j indices $= 2, 3, 4, 5, \ldots$ of longer and longer lengths. The iteration process, Fig. 3, is finished after we reach the point when, for instance, the two criteria meet the condition:

$$K1 \geq 0 \wedge K2 \geq 0. \tag{11}$$

Then, the location of the cross-section at which the boundary-layer separation phenomenon takes place, i.e. the numeric value of the x_S coordinate of the S – separation point already mentioned, Fig. 2, is arrived at from the relation:

$$x_S = j^* \Delta L. \tag{12}$$

6. Exemplary results of numeric calculations

Based on the above considerations, we worked out a Pascal program (Jesionek (1998)), controlled by the integrated Turbo Pascal 4.0 and higher versions.

The exemplary results of the calculations are presented in Figs. 4 and 5, for the cases of conical diffuser displaying the following both:

- geometric ($\alpha = 11^o, n = 2.53$, $L/D_1 = 3.07$ – Fig. 4; and $\alpha = 14^o$, $n = 4.00$, $L/D_1 = 4.00$ – Fig. 5)
- and flow ($Re = 0,206 \cdot 10^6$ – Fig. 4 and $Re = 0,200 \cdot 10^6$ – Fig. 5)

parameters.

Figs. 4 and 5 display the changeability of such values of flow parameters as:

- overall loss (full loss) coefficient – ζ,
- $K1$ criterion,
- $K2$ criterion,
- relative blocked area fraction – $\bar{\Delta}_2^*$.

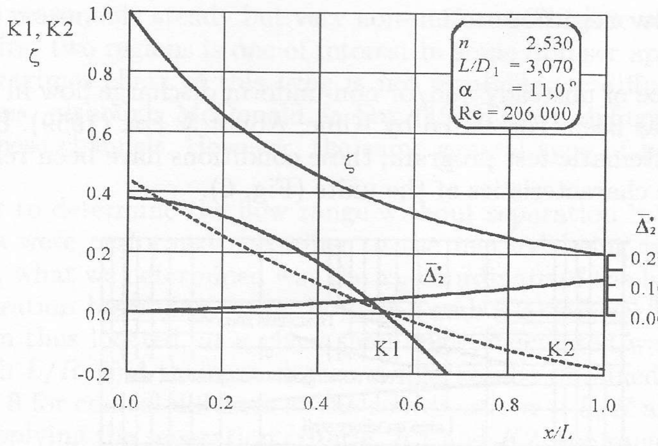

Fig. 4. Flow parameters variation along the conical diffuser.

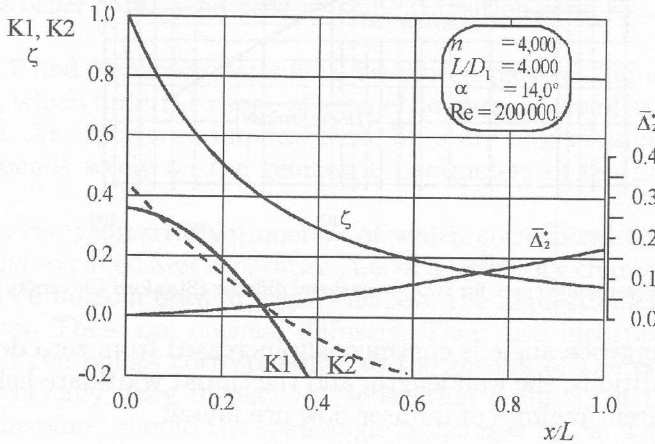

Fig. 5. Flow parameters variation along the conical diffuser.

The figures imply that, in these specific cases, the two separation criteria, both $K1$ and $K2$, almost in the same way determine (tending towards the negative range) the boundary-layer separation cross-section position in the exemplary diffuser channels for the coordinates:

$$\bar{x} = x/L \approx 0.5 \text{ (Fig. 4)}, \tag{13}$$

$$\text{and } \bar{x} = x/L \approx 0.3 \text{ (Fig. 5)}. \tag{14}$$

Because the flow separation phenomenon is possible to occur inside the diffusers, the flow characteristics of the channels can deteriorate to a large extent.

7. Diffuser flow regimes

The occurrence of unsteady and/or non-uniform discharge flow in two-dimensional diffusers has been correlated by Kline, Abbot & Fox (1959). Based on an extensive and systematic test program, these conditions have been related to the overall geometric characteristics of the units (Fig. 6).

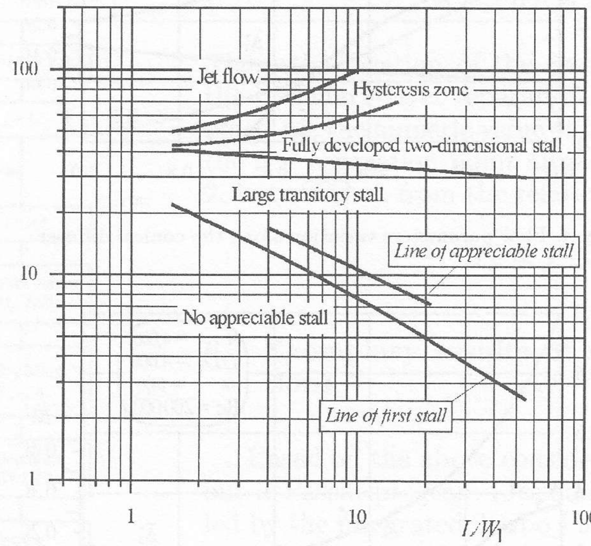

Fig. 6. Flow-regime chart for two-dimensional diffuser (Stanford University).

If diffuser divergence angle is continuously increased from zero degrees while the inlet-flow conditions, the wall length, and the throat width are held constant, the following different regimes of diffuser flow are found:

1. The region of no-appreciable-stall in which the main flow is well behaved and apparently un-separated.

2. The region of large transitory stall in which the separation varies in position, size, and intensity with time.

3. The region of fully developed stall in which the major portion of the diffuser is filled with a large triangular shaped turbulent recirculation region, extending from the diffuser exit toward the diffuser throat.

4. The jet-flow regime in which the main flow is separated from both walls. The separation begins from the diffuser throat. This occurs only at quite high divergence angles.

The region of no-appreciable stall is steady and uniform, the region of transitory stall is unsteady and non-uniform, while the fully-developed and jet-flow regimes are reasonably steady but very non-uniform. The line of demarcation between the first two regions is one of interest in some diffuser applications.

A flow-regimes chart of this type is not available for diffusers with conical cross-sections, although McDonald & Fox (1965) have defined a line of „first stall" for these channels. However, the same general type of behaviour is to be expected.

In order to determine the flow range without separation in conical diffusers, calculations were performed according to the method being proposed. In each design case, what we determined was the x_S-coordinate of the hypothetical point of the separation beginning, using both $K1$ and $K2$ criteria. For the reference cross-section thus located, at a given span angle, calculated was the diffuser relative length L/R_1 and the span degree n. The results obtained are presented in Figs. 7 and 8 for conical diffusers at the coordinates $\alpha - L/R$ and $n - l/R$.

Upon applying the separation criteria, $K1$ and $K2$, the range of diffuser geometric parameters can be divided into two parts. In the first, below the plotted points, the separationless character of the flow is practically preserved. In the second, on the other hand, separated by some transition zone, flows with separation occur.

In Figs. 7 and 8 characteristic lines have been plotted, found out by different researchers, which limit the range of geometric parameters of separationless conical diffusers. As experience implies, these lines are of general significance. Their position depends solely on the geometric parameters of the flow channel being analysed.

Diffusers the geometric parameters of which correspond to the points along the line of the so-called first separation, i.e. of a pulsatory character (these are, for instance, three bottom lines in Fig. 7) achieve the highest efficiency under given circumstances. These are optimal diffusers. They also incorporates the designs marked with the points corresponding to the results of the performed calculations. Theoretically, they display the separation point S, in the outlet section plane and therefore, should the span angle α increase by ca. $2 \div 3^o$ in relation to the values which stem from the position of the analysed point on the boundary line, short diffusers exhibit a slight drop of efficiency only.

Now coming back to the analysis of Figs. 7 and 8, we are justified to state that for conical diffusers the suggested method is applicable like other methods. Separationless channels (or, more precisely, before the boundary layer separation) denoted by triangles and circles, clearly tend to be arranged without scattering, in the area between the first separation lines determined by different researchers.

8. Conclusions

It is concluded that the method of analysis, as represented by Equations (1), (2) and (7) with pictures in Figs. 7 and 8 together, is suitable for prediction of turbulent boundary layer separation in conical diffusers, and that the separation criteria $K1$ and $K2$ are significant factors in predicting the separation.

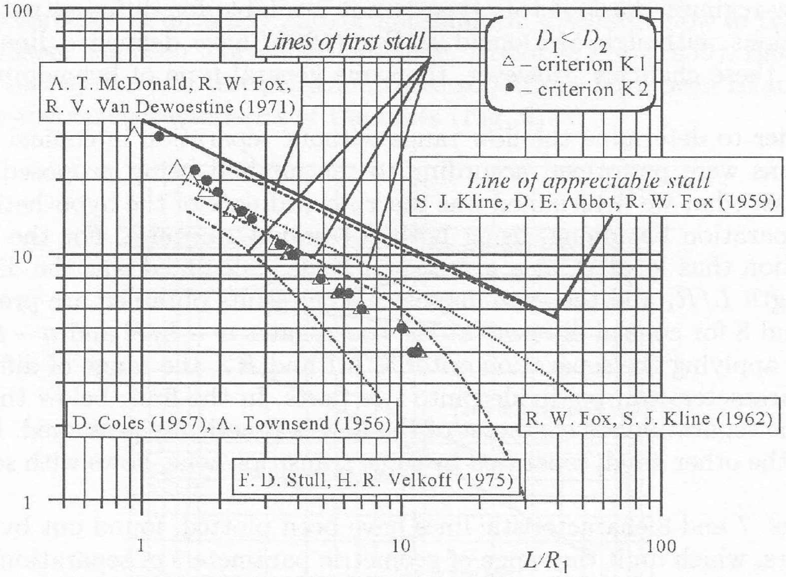

Fig. 7. Conical diffusers flow regimes.

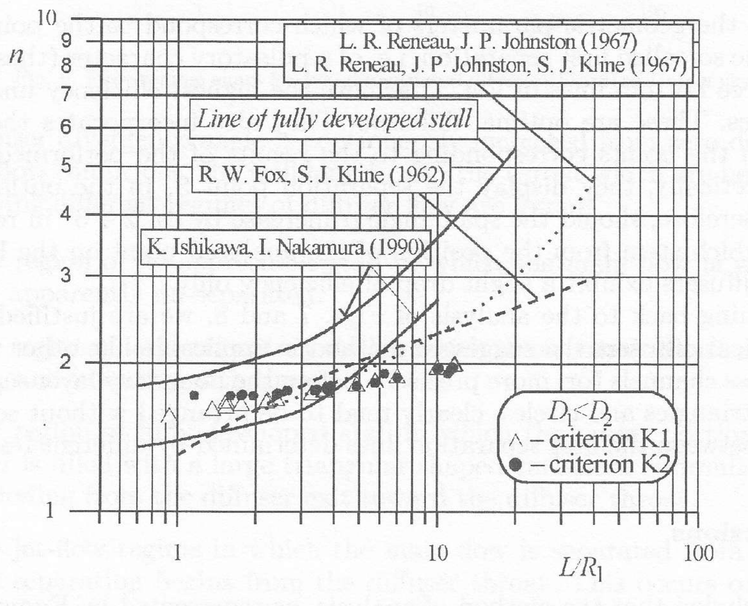

Fig. 8. Chart of the flow regime for conical diffusers.

The predictions of the proposed analysis have been compared with the available experimental results on conical diffusers.

The way to determine the place of the boundary-layer separation submitted in the present paper can be useful in the process of a conical diffuser design. Having the relative blocked area fraction corresponding to the boundary-layer displacement thickness, it is a trivial task to determine such an important parameter as the overall energy loss coefficient. We can use here the known expression

$$\zeta = \frac{1}{n^2(1-\bar{n}_2^*)^2}. \tag{15}$$

The two values, i.e. $\bar{\Delta}_2^*$ and ζ are helpful when optimising (already at the stage of design) the construction of a sought diffuser for a broad range of operating parameters. The method suggested is relatively simple. The loss coefficient calculation results, however, conform well to the experimental results only for short diffusers. Nevertheless, it is possible to improve further the presented method and currently the work aimed at this is in progress.

Finally, it should be stressed that determination of the diffuser's geometric parameters optimal for a given application is extremely difficult since its characteristics largely depend on numerous variables. The present advancement level of fluid mechanics renders it possible to theoretically determine the behaviour of the flow in a diffuser in a rough manner only, even for the simplest cases of its geometry. This stems from the fact that the accurate influence of positive values of the longitudinal pressure gradient fails to be embraced by the methods of mathematical analysis.

Manuscript received in March 1998

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Przewidywanie oderwania przepływu turbulentnego w dyfuzorach stożkowych

Streszczenie

W pracy opisano metodę przewidywania oderwania strumienia od ścianki kanału dla ustalonego, nieściśliwego przepływu bez zawirowań, w przepływie symetrycznym, w dyfuzorze stożkowym. Wykazano, że warunki oderwania przepływu zależą od rozkładu naprężeń stycznych w turbulentnej warstwie przyściennej. Przedstawiono porównanie obliczeń wykonanych na podstawie teorii warstwy przyściennej, z danymi eksperymentalnymi dla kanałow dyfuzorowych.