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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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JAROSŁAW MIKIELEWICZ[1], DARIUSZ MIKIELEWICZ[2]

Analysis of the influence of the design parameters of the probe for the dew-point temperature measurements on its static and dynamic characteristics[3]

In the work presented are the considerations of the influence of the design parameters of the probe for the dew point temperature measurements on its static and dynamic characteristics. The analysis is based on the solution of the differential equation describing the probe dynamics. From the presented analysis it results that the biggest influence on the probe dynamics comes from the mass of the probe radiator and the heat transfer to surroundings. The mass of the probe membrane and the gap between the membrane and the heater have smaller influence. The static response is governed by the steady-state part of the solution. The influence of other probe parameters on temperature measurements is also presented.

Nomenclature

A_p	– heat transfer surface,	λ	– thermal conductivity,
A, B, C, D	– coefficients in Eq. (11)	M	– mass,
c_p	– heat capacity,	p	– initial condition for temperature,
δ	– plate thickness,	q	– initial condition for cooling heat flux
F	– heat transfer surface,	Q_g	– heat flux,
k	– heat transmission coefficient,	t	– time,
K	– general heat transmission coefficient,	T	– temperature.

Subscripts

m	– radiator,	pp	– heater,
p	– membrane	0	– ambient conditions.

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1. Introduction

The common availability of digital readout thermocouples with built-in cold junctions has led to the increased use of thermocouple-based psychrometers. Since relatively small thermocouple junctions can be used, their response to changes in temperature and hence humidity is fast, and they are more appropriate than thermometer-based psychrometers in situations where the air state changes rapidly with time. Trela [1] developed a concept of dew point temperature measurements around the dew point based on the change of dynamical temperature of the specially designed probe.

Using the mathematical model of the plate, representing the membrane of the probe for the dew point temperature measurements, presented in [2], the analysis of the dynamic response of the probe has been conducted. The influence of various elements of the probe on the time response of the probe has been analysed.

In the present work we analyse the influence of various probe parameters, such as the masses of the membrane (plate) and the heater, the influence of the material of the membrane and the heater, the influence of the gap between the membrane and the heater, and finally the heat transfer to the surroundings, on the response of the probe.

2. The fundamentals of modelling

Presented here concept of dynamical investigations of the probe for the measurements of the dew point temperature is the extension of the theoretical analysis of the influence of thermal parameters on the non-stationary temperature measurements presented in [2]. The model presented there gave consistent results both from qualitative and quantitative point of view. The method adopted here to analyse the influence of various construction parameters is based on the decomposition of heat fluxes into subsystems. We have assumed that the masses of subsystems are homogeneous and we treat the subsystems as a material points.

The heat balance for each subsystem can be generally cast in the following form:

$$\sum_{i=1}^{N} K_i \Delta T_{i,j} + M_i \frac{dT_i}{d\tau} = S_i \tag{1}$$

where:

i – denotes the number of the considered subsystem,

j – denotes the number of the subsystem exchanging heat with subsystem i

N – denotes the number of subsystems in the analysed system.

The terms in equation (1) denote:

$K_i \Delta T_{i,j}$	–	the heat transmission between subsystems i and j,
$M_i \frac{dT_i}{d\tau}$	–	heat storage in the system i,
S_i	–	heat delivered to the subsystem i from the heat source.

The heat flow between particular subsystems can take the following forms:

a) without the mass transfer between the subsystems and then

$$K_i = k_{i,j} A_i \tag{2}$$

where:

$k_{i,j}$	–	heat transmission coefficient,
$A_{i,j}$	–	heat transfer area,

b) with the mass transfer between the subsystems and then

$$K_i = m_{i,j} C_p \tag{3}$$

where:

$m_{i,j}$	–	mass flux between the subsystems exchanging heat,
C_p	–	specific heat of the mass flux.

In order to determine the temperatures of particular subsystems we need to solve the set of equations similar to equation (1). These are the ordinary differential equations of the first order.

3. Theoretical model of the phenomenon

A schematic of the probe is presented in Fig. 1. In the considered problem the model of the probe has been divided into three subsystems, i.e. the plate, the heater and the radiator. It has been assumed in the considerations that

- material properties are constant,
- heat flux due to cooling is constant,
- the plate thickness is small so that the linear temperature distribution across the plate can be assumed,
- the influence of the surroundings on the probe is reflected in the constant heat transfer coefficient and the constant ambient temperature.

Using the method of decomposition of the heat flux the following equations have been derived for the particular subsystems:

- plate

$$M_p C_{pp} \frac{dT_p}{dt} + (T_p - T_0) A_p k_{p0} + (T_p - T_{pp}) A_p k_{pp} = 0, \tag{4}$$

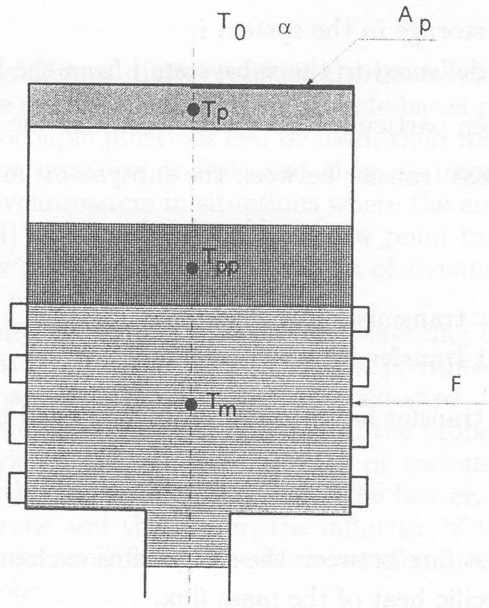

Fig. 1. Schematic of the probe for the dew point temperature measurements.

- heater and the air gap

$$(T_{pp} - T_p)A_p k_{pp} + (T_{pp} - T_m)A_p k_{pm} = Q_g, \tag{5}$$

- radiator

$$M_m C_{pm} \frac{dT_m}{dt} + (T_m - T_{pp})A_p k_{pm} + (T_m - T_0)F k_{m0} = 0. \tag{6}$$

Equations (4) and (6) are the ordinary differential equations of the first order and equation (5) is an algebraic equation. The method of solution of the above system is based on the simple algorithm presented next. We are primarily interested here in determination of the influence of various probe parameters on the temperature of the membrane hence determination of T_p is our primary objective.
First we determine from equation (5) the temperature T_{pp},

$$T_{pp} = \frac{k_{pp}}{k_{pp} + k_{pm}} T_p + \frac{k_{pp}}{k_{pp} + k_{pm}} T_m + \frac{Q_g}{A_p(k_{pp} + k_{pm})}. \tag{7}$$

This temperature T_{pp} is then substituted into equations (4) and (6) and in the end we obtain the following form of equation (4)

$$\frac{M_pC_{pp}}{A_p}\frac{dT_p}{dt}=$$
$$=\frac{k_{pp}k_{pm}}{k_{pp}+k_{pm}}T_m-\left[k_{p0}+\frac{k_{pp}k_{pm}}{k_{pp}+k_{pm}}\right]T_p+T_0k_{p0}+\frac{Q_gk_{pp}}{A_p(k_{pp}+k_{pm})} \tag{8}$$

as well as equation (6) in the form

$$\frac{M_mC_{pm}}{A_p}\frac{dT_m}{dt}=\frac{k_{pp}k_{pm}}{k_{pp}+k_{pm}}T_p-\left[\frac{k_{pp}k_{pm}}{k_{pp}+k_{pm}}+\frac{F}{A_p}k_{m0}\right]T_m+$$
$$+\frac{F}{A_p}k_{m0}T_0+\frac{Q_gk_{pm}}{A_p(k_{pp}+k_{pm})}. \tag{9}$$

As a result of these rearrangements we obtain two linear differential equations with two unknowns T_p and T_m. As it was mentioned earlier, we are primarily interested in the solution of the plate temperature T_p, hence we will conduct our transformations in this direction. From equation (9) we determine T_m and from the differentiated form of equation (8) (derivative with respect to time) let's determine dT_m/dt. Substituting to equation (9) we obtain the linear differential equation of the second order with re spect to T_p of the following form

$$\frac{M_mC_{pm}}{A_p}\frac{M_pC_{pp}}{RA_p}\frac{d^2T_p}{dt^2}+\left[\left(\frac{k_{p0}}{R}+1\right)\frac{M_mC_{pm}}{A_p}+\right.$$
$$\left.+\left(1+\frac{F}{A_p}\frac{k_{m0}}{R}\right)\frac{M_pC_{pp}}{A_p}\right]\frac{dT_p}{dt}+\left[k_{p0}+\frac{Fk_{m0}}{A_p}\left(1+\frac{k_{p0}}{R}\right)\right]T_p=$$
$$=\left[k_{p0}+\frac{Fk_{m0}}{A_p}\left(1+\frac{k_{p0}}{R}\right)\right]T_0+\left(1+\frac{F}{A_p}\frac{k_{m0}}{k_{pm}}\right)\frac{Q_g}{A_p} \tag{10}$$

where $R=\dfrac{k_{pp}k_{pm}}{k_{pp}+k_{pm}}$.

The above form of the differential equation can be cast the simplified form:

$$A\frac{d^2T_p}{dt^2}+B\frac{dT_p}{dt}+CT_p=D \tag{11}$$

where the terms A, B, C, D have the following values:

$$\begin{aligned}
A &= \frac{M_mC_{pm}}{A_p}\frac{M_pC_{pp}}{RA_p}\\
B &= \left[\left(\frac{k_{p0}}{R}+1\right)\frac{M_mC_{pm}}{A_p}+\left(1+\frac{F}{A_p}\frac{k_{m0}}{R}\right)\frac{M_pC_{pp}}{A_p}\right],\\
C &= \left[k_{p0}+\frac{Fk_{m0}}{A_p}\left(1+\frac{k_{p0}}{R}\right)\right],\\
D &= \left[k_{p0}+\frac{Fk_{m0}}{A_p}\left(\frac{k_{p0}}{R}-1\right)\right]T_0+\left[1+\frac{F}{A_p}\frac{k_{m0}}{k_{pm}}\right]\frac{Q_g}{A_p}.
\end{aligned} \tag{12}$$

Solution to this equation using the classical methods means finding the general integral of the homogeneous equation and the particular integral of the non-homogeneous equation. In our case the shape of the solution to equation (11) with respect to the plate temperature T_p has the following form

$$T_p = \frac{D}{C} + C_1 \exp\left(-\frac{1}{2}\frac{B-\sqrt{b^2-4AC}}{A}t_i\right) + C_2 \exp\left(-\frac{1}{2}\frac{B-\sqrt{b^2-4AC}}{A}t_i\right). \quad (13)$$

In order to determine the constants C_1 and C_2 we assume the following initial conditions

$$\begin{aligned} &\text{for} \quad t=0 \quad T_p = p, \\ &\text{for} \quad t=0 \quad \frac{dT_p}{dt} = q. \end{aligned} \quad (14)$$

Substituting the above conditions to equation (13) we obtain

$$\begin{aligned} p &= \frac{D}{C} + C_1 + C_2, \\ q &= -\tfrac{1}{2}C_1\frac{B-\sqrt{B^2-4AC}}{A} - \tfrac{1}{2}C_2\frac{B+\sqrt{B^2-4AC}}{A}. \end{aligned} \quad (15)$$

Solving the above equation with respect to C_1 and C_2 gives us

$$\begin{aligned} C_1 &= \tfrac{1}{4}(2pC + qB - \sqrt{B^2-4AC} - 2D)\frac{(-B^2+4AC-B\sqrt{B^2-4AC})}{[C(-B^2+4AC)]}, \\ C_2 &= \tfrac{1}{4}(2pC + qB + \sqrt{B^2-4AC} - 2D)\frac{(-B^2+4AC+B\sqrt{B^2-4AC})}{[C(-B^2+4AC)]}. \end{aligned} \quad (16)$$

The solution of equation (11) consists of the free response of the plate and the particular solution (term D/C). The particular solution is responsible for the performance of the probe under steady-state conditions. The steady-state temperature must be lower than the expected dew point temperature, otherwise the vapour condensation will not take place. This information should be taken into account during the design stage of the probe. The free response of the probe provides information about the dynamics of the system and enables better selection of the probe dimensions and the construction materials.

4. Results

The model derived above has been tested on the example of the probe MELCOR FC0.6-32-05TL [3]. Calculations have been performed for the following dimensions given in Table 1 and Table 2.

The values of thermal resistance between the particular elements are as follows:

Table 1.

Given quantities	Value
Mass of the plate	0.000018 kg
Mass of the radiator	0.046 kg
Plate heat capacity	400 J/kg K
Radiator heat capacity	400 J/kg K
Width of the air gap	0.0000 75 m
Thermoelement height	0.0025 m
Plate thermal conductivity	370 W/mK
Radiator thermal conductivity	370 W/mK

- heat transmission between the surroundings and a membrane

$$k_{p0} = \frac{1}{\frac{\delta}{2\lambda_{air}} + \frac{1}{\alpha}} \tag{17}$$

where δ is the membrane thickness,

- heat transmission between the membrane and a heater (including the air gap)

$$k_{pp} = \frac{1}{\frac{\delta}{2\lambda_p} + \frac{\delta_1}{\lambda_{air}} + \frac{\delta_2}{2\lambda_{npn}}} \tag{18}$$

where δ_1 is the width of the air gap, δ_2 is the thermoelement thickness, λ_{air} is the thermal conductivity of air and λ_{npn} is a mean thermal conductivity of the thermoelement (heater),

- heat transmission between the heater and a radiator

$$k_{pm} = \frac{1}{\frac{\delta_2}{2\lambda_{npn}} + \frac{h}{2\lambda_m}} \tag{19}$$

where h is the radiator height and λ_m is its thermal conductivity,

- heat transmission between the radiator and the surroundings

$$k_{m0} = \frac{1}{\frac{h}{2\lambda_m} + \frac{A_p}{F\alpha}}. \tag{20}$$

Above relations have been derived under assumption of one-dimensional axial heat flow and that only the radiator exchanges heat with ambience.

Table 2.

Quantities assumed in calculations	Value
Heat transfer coefficient between the surroundings and a plate	30 W/m^2K
Heat transfer coefficient between the plate and thermoelement	266.7 W/m^2K
Heat transfer coefficient between the thermoelement and a radiator	$5.137 \times 10^4 W/m^2K$
Heat transfer coefficient between the radiator and the surroundings	503.8 W/m^2K
Heat transfer coefficient to surroundings	30 W/m^2K
Ambient temperature	25°C
Mean thermal conductivity of the thermoelement	210 W/mK
Thermal conductivity for air	0.0267 W/mK
Initial temperature	100°C
Initial heat flux	– 0.1 W
Cooling heat flux	– 0.1 W

Calculations of the temperature distribution of the membrane have been presented in Fig. 2. The fully developed temperature of the membrane is $T_{p\ fullydev.} =$ 24.96°C.

As can be seen from the figure, the temperature stabilises after about 30 seconds, which is similar when compared against the experimental data from [4]. This results must be deemed satisfactory.

Let's consider now the influence of major components of the model on the time distributions of the membrane.

4.1. Influence of the membrane mass on the dynamic characteristics of the probe

First, we consider the influence of the membrane mass on the dynamic characteristics of the probe. During the experiments [4], this was found to be the major reason for the changes of the probe construction. Results of calculations are presented in Fig. 3.

In Fig. 3 presented are three curves. These correspond to three different masses of the membrane, namely:

- $m_{p1} = 0.00009$ kg,
- $m_{p2} = 0.00018$ kg,
- $m_{p3} = 0.00144$ kg.

Selected values are twice reduced when compared to original case, or eight times larger compared to the original case. The other parameters remain unchanged and have the values from the Tables 1 and 2.

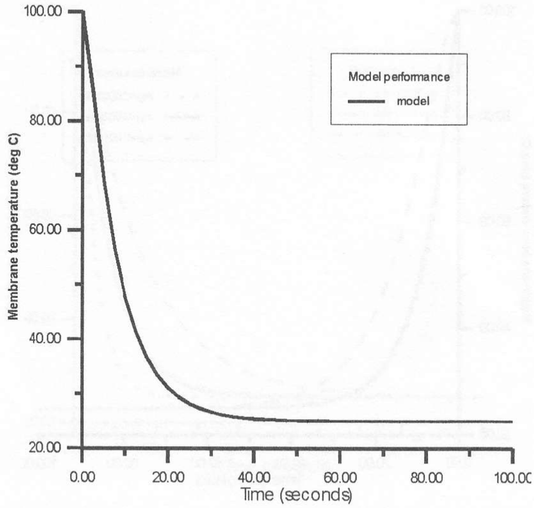

Fig. 2. Temperature distribution of the membrane.

The influence of the mass of the membrane shows a significant effect on the distribution of the plate temperature. Increasing the mass of the membrane renders that the time required to obtain the fully developed temperature is longer. Reduction of the membrane mass shortens the time of reaching fully developed conditions. There is however, a construction limit for the dimensions of the membrane. The membrane thickness cannot be reduced infinitely and masses m_{p1} and m_{p2} correspond to the technological limits attainable in practice.

4.2. Influence of the radiator mass on the probe dynamic characteristics

The next parameter considered was the influence of the radiator mass on the probe dynamic characteristics. This is the largest element of the probe which is responsible for the heat transfer with the surroundings. Therefore it contains the fins. The results of calculations are presented in Fig. 4.

In the Fig. 4 presented are three curves. These correspond to three different masses of the radiator, namely:

- $m_{m1} = 0.023$ kg,
- $m_{m2} = 0.046$ kg,
- $m_{m3} = 0.092$ kg.

Selected values are twice reduced when compared to original case, or twice larger compared to the original case. The other parameters remain unchanged and have the values from the Tables 1 and 2.

As can be seen from the figure, the influence of the mass of the radiator on the membrane temperature distribution is also significant. The larger mass of the

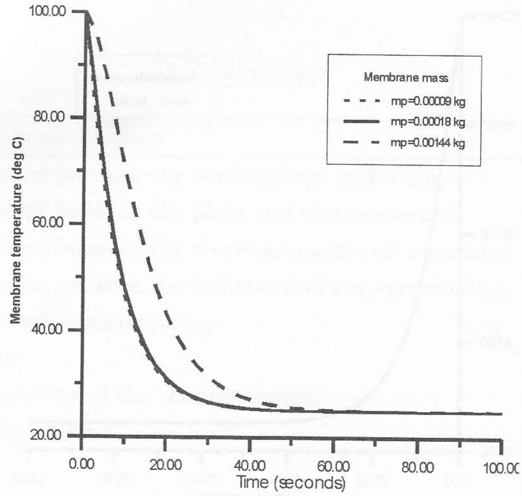

Fig. 3. Influence of the membrane mass on the membrane temperature distribution.

radiator renders greater thermal inertia of the probe, and hence longer times for achievement of the fully developed temperature by the probe.

4.3. Influence of the membrane and radiator material on the probe dynamic response

The next parameter investigated was the material of the membrane and the radiator. Analysed have been the influence of two other materials with significantly different thermal properties. The results of calculations have been presented in Fig. 6.

In Fig. 5 presented are three distributions corresponding to the following materials:

- $\lambda_1 = 58$ W/mK and $C_{m1} = C_{p1} = 377$ J/kgK (brass),
- $\lambda_2 = 370$ W/mK and $C_{m2} = C_{p2} = 400$ J/kgK (copper),
- $\lambda_3 = 418$ W/mK and $C_{m3} = C_{p3} = 234$ J/kgK (silver).

As can be seen from the presented calculations, the values of the thermal conductivity are twice lower for the case number 2 or slightly higher for the case three compared to the original case. These values correspond to the possible different materials for the probe. Thermal capacity of copper and brass is similar but for silver it is different. Other parameters remain unchanged from the Tables 1 and 2.

The influence of the membrane and radiator material is not significant or even not perceptible. This provides guidance that other material could be considered a substitute as for copper. It seems that the fundamental influence comes from the thermal capacity of the considered material and the thermal conductivity plays less significant role.

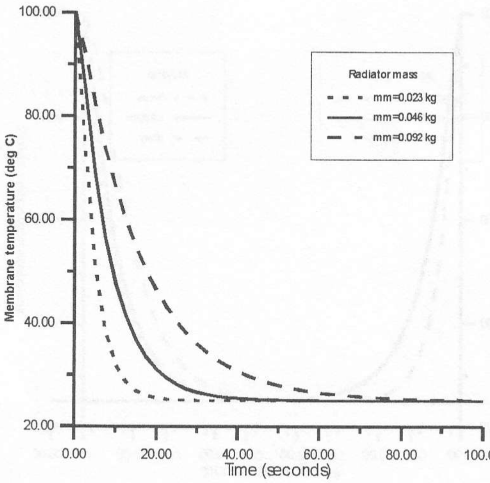

Fig. 4. Influence of the radiator mass on the membrane temperature distributions.

4.4. Influence of the air gap on the dynamic characteristic of the probe

During experiments presented in [4] it was thought that the height of the air gap between the membrane and the thermoelement could play a significant role. To test the model for this influence some sample calculations have been conducted and the results are presented in Fig. 6.

In Fig. 6 presented are three distributions corresponding to the following conditions:

- $\delta_1 = 0.00001$ m,
- $\delta_2 = 0.0001$ m,
- $\delta_3 = 0.001$ m.

These are the values, which are ten times increased for the second case or ten times decreased for the third case compared to the original height of the gap (case 1). Other parameters remain unchanged with respect to the Table 1 and 2.

From presented results it is clear that the height of the air gap is not a negligible influence but it does not play significant role in the response of the membrane.

4.5. Influence of heat transfer to surroundings on the dynamic response of the probe

Another parameter studied here was the influence of heat transfer to surroundings on the dynamic characteristics of the probe. The results are presented in Fig. 7.

In Fig. 7 presented are three distributions corresponding to the following conditions:

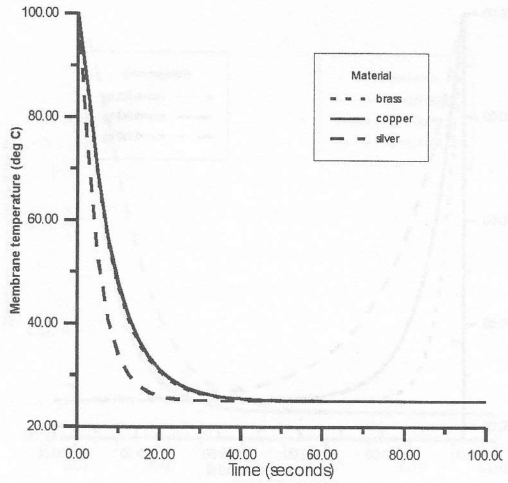

Fig. 5. Influence of the probe and radiator material selection on the membrane temperature distribution.

- $\alpha_1 = 15$ W/m^{2}K,
- $\alpha_2 = 30$ W/m^{2}K,
- $\alpha_3 = 60$ W/m^{2}K.

These are the values, which are twice increased, the second case, or twice reduced, the third case. Other parameters remain unchanged compared to the Tables 1 and 2.

It must be said that the influence of heat transfer to surroundings has a very significant contribution to the membrane temperature distribution. Increasing the convective heat transfer coefficient (finning) decreases the time required to achieve fully developed conditions.

4.6. Influence of cooling on the dynamic characteristics of the probe

The last parameter studied was the influence of cooling of the probe on the dynamic characteristics of the probe. The results are presented in Fig. 8. There are presented three distributions corresponding to the following conditions:

- $Q_{g1} = 0.0$ W (no cooling),
- $Q_{g2} = -0.1$ W,
- $Q_{g3} = -1.0$ W.

As can be seen these are the values, which for the first case correspond to no cooling of the probe and in the third case cooling is increased ten times. The other parameters remain unchanged compared to the Tables 1 and 2.

It should be said that the influence of cooling has a small influence on the temperature distribution of the membrane.

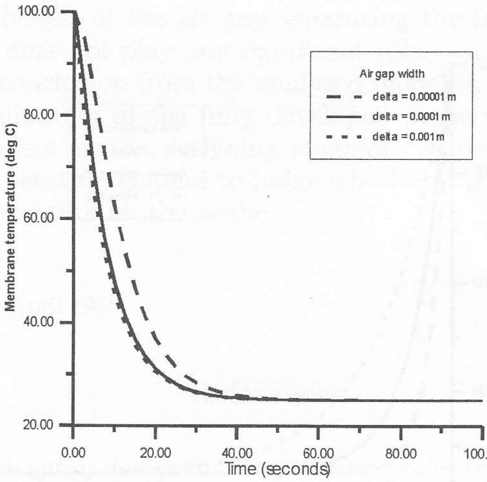

Fig. 6. Influence of the air gap on the temperature distribution of the membrane.

5. Conclusions

In the work presented here analysed is the influence of the construction parameters of the probe for the measurements of the dew point temperature on its static and dynamic characteristics. Using the model, developed in [2], the analysis of the dynamics of the probe has been conducted. As a result, the model has been developed by means of which the influence of particular parameters on the static and dynamic characteristics can be studied. The model is in the form of the ordinary linear differential equation of the second order with constant coefficients. The solution to the model is given in analytical form which enables fast and accurate analysis of the influence of various parameters.

In the work analysed have been the influence of the following parameters on the dynamic characteristics of the probe:

- mass of the membrane,
- mass of the radiator,
- membrane and radiator material,
- height of the air gap,
- heat transfer to the surroundings,
- cooling of the membrane.

From presented results it is apparent that the most significant influence on the dynamic characteristics of the probe comes from the mass of the radiator, as an element exchanging heat with surroundings. The second most significant contribution comes from the mass of the membrane and the heat transfer to

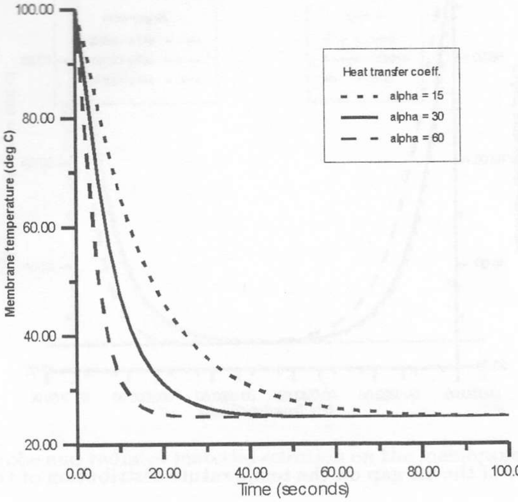

Fig. 7. Influence of the heat transfer to surroundings on the membrane temperature distribution.

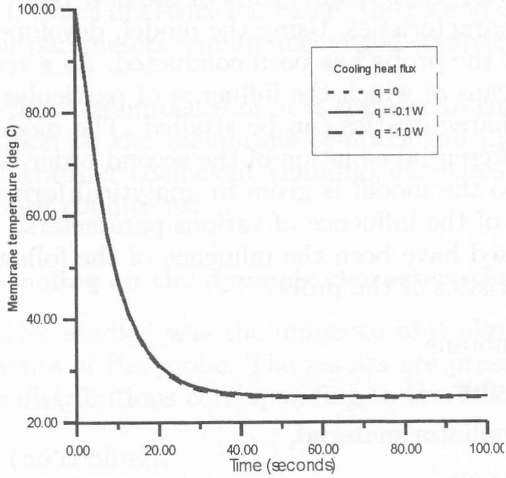

Fig. 8. Influence of cooling on the membrane temperature distribution.

surroundings. The height of the air gap separating the heater (thermoelement) and the membrane does not play any significant role.

Another useful conclusion from the analysed model is the possibility of quick and efficient determination of the fully developed probe temperature. This is a very important element in the designing stage of the probe. Knowledge of the fully developed temperature enables to judge whether it is at all possible to reach the dew point temperature by the probe.

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Analiza wpływu parametrów konstrukcyjnych czujnika do pomiaru wilgotności na jego charakterystyki statyczne i dynamiczne

Streszczenie

W pracy przedstawiono rozważania dotyczące pomiarów temperatury przy pomocy czujnika płytkowego służącego do pomiaru temperatury rosy. Analiza oparta jest na rozwiązaniu równań różniczkowych opisujących dynamikę czujnika. Analizowano wpływ poszczególnych elementów czujnika na jego odpowiedzi dynamiczne. Z przedstawionej analizy wynika, że największy wpływ na dynamikę czujnika ma masa radiatora oraz warunki wymiany ciepła z otoczeniem. Drugorzędny wpływ ma masa membrany oraz odległość pomiędzy elementem grzejnym a membraną. Na statykę czujnika ma największy wpływ rozwiązanie szczególne równania różniczkowego, które przedstawia temperaturę ustaloną układu. Przedstawiono dyskusję wpływu różnych parametrów płytki na czas odpowiedzi na zmiany temperatury.