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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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Turbulence modelling using various turbulent Prandtl number models[2]

The present work reports the study where the influence of various turbulent Prandtl number models was examined in the light of the models of turbulence response to its variation. Two turbulence models have been examined, namely the mixing length model and the Launder and Sharma two-equation low Reynolds number $k \sim \epsilon$ model. The response of turbulence models have been tested under fully developed constant property forced convection. Simulations have been compared with the Kurganov-Petukhov and Dittus- Boelter correlations. The main outcome of the work is that inclusion of variable turbulent Prandtl number model does not improve the agreement achieved earlier [1].

Nomenclature

C_p	–	specific heat capacity at costant pressure,	Nu	–	Nusselt number, $q_w D/(T_w - T_b)\lambda$,
C_1, C_2	–	constants in modelled dissipation equation,	p	–	pressure,
C_μ	–	constant in constitutive equation of eddy viscosity model,	Pe	–	Peclet number, $Re \cdot Pr$,
			Pr	–	Prandtl number, $C_p\mu/\lambda$,
			q	–	heat flux, W/m^2,
d	–	pipe diameter,	r, z	–	radial and axial coordinates,
D, E	–	terms in low-Reynolds-number k-equation,	R	–	pipe radius,
			Re	–	Reynolds number, $\rho W_b D/\mu$,
f_1, f_2, f_3	–	functions in dissipation equation,	T	–	temperature, °C,
			v', w'	–	fluctuating velocity components in r, z directions,
f_μ	–	function in constitutive equation of $k \sim \epsilon$ model,	V, W	–	mean velocity components in r, z directions,
g	–	acceleration due to gravity,			
h	–	enthalpy,	y	–	distance from the wall,
h'	–	fluctuating component of enthalpy,	y^+	–	non-dimensional distance from the wall.
k	–	turbulence kinetic energy,			

Greek symbols

ϵ	–	rate of dissipation of turbulence kinetic energy,	λ	–	thermal conductivity,
			μ	–	dynamic viscosity,

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$\hat{\epsilon}$	–	modified rate of dissipation of turbulence kinetic energy, $\epsilon = \hat{\epsilon} - D$,	μ_t	–	turbulent dynamic viscosity,
ϵ_H	–	eddy diffusivity of heat,	ν	–	kinematic viscosity,
ϵ_M	–	eddy diffusivity of momentum,	ρ	–	density,
ζ	–	friction coefficient,	Pr_t	–	turbulent Prandtl number,
κ	–	Karman constant,	σ_k, σ_ϵ	–	turbulent Prandtl numbers for diffusion of k, ϵ.

Subscripts

cp	–	const. property forced convection,	in	–	inlet bulk conditions,
w	–	wall,	∞	–	infinity.

Abbreviations

ML	–	mixing length model,	AC	–	Azer and Chao TPN model,
LS	–	Launder and Sharma low-Reynolds-number $k \sim \epsilon$ model,	KC	–	Kays and Crawford TPN model,
			HL	–	Hollingworth et al. TPN model,
TPN	–	turbulent Prandtl number	GF	–	general fit TPN model.

1. Introduction

The present study forms an extension of the work carried out by Mikielewicz at the University of Manchester, where the author was involved in research leading to the degree of Doctor of Philosophy. The author in his research programme there [1] have implemented several different models of turbulence ranging from the early zero-order ones to the advanced $k \sim \epsilon$ models. He then tested these models against extensive range of experimental conditions obtained from literature. All these calculations were, however, done under assumption of uniform turbulent Prandtl number (TPN). This led to further investigations of the influence of the variation of TPN in the boundary layer flows which are one of the merits of the present research granted by the State Scientific Committee for Research.

The point in question was raised in order to find out the benefits of using variable TPN in the boundary layer flows. Another approach would be to model directly the velocity-temperature correlations. However, direct modelling of diffusion of heat in two-dimensional flows is rather computationally expensive and requires solving additional differential equations. Furthermore the TPN distribution in boundary layer flows in smooth vertical pipes have been to a quite good extent covered by experimental investigations. In these cases the diffusion of heat is only in the direction normal to the main flow direction and the use of direct modelling of temperature correlations looses its usefulness. It must also be borne in mind that the concept of eddy viscosity is already a great simplification of the turbulent flow solution. On the other hand, implementation of TPN model into the solution scheme is relatively simple. It is also of interest how the eddy viscosity models would respond to such a refinement. From [1] it is already known that the models examined there agree to within 10% with well established empirical correlations like that due to Kurganov-Petukhov [2] or Dittus-Boelter [3] for constant property fully developed forced convection.

Obviously, there are many other approaches to treat the effects of the variation of TPN. Most of the early approaches to the problem were inspired by the observation that experimentally predicted heat transfer coefficient for the flow of liquid metals compared with that calculated using various empirical correlations for heat transfer coefficient differed significantly. This led to closer examination of the influence of TPN on heat transfer results which resulted with some analytical models, an example of which is the Azer and Chao [8] model. However, all these models were quite mathematically complex and still were not a good representation of the nature. Therefore, the main focus in the present work is on the experimental fits to very recent experimental data on TPN, which shows its correct trends. In recent years there have also been endeavours to calculate TPN directly with the aid of Direct Numerical Simulations. These data are however scarce and do not cover entire range of molecular Prandtl number. It must be also stressed again that the primary objective of the present work was not establishing a new model for TPN but testing the models response to the variation of the TPN across the boundary layer.

The main goal of the present work is therefore to examine presently acknowledged to be best TPN empirical correlations and test them on the selection of eddy viscosity models in two-dimensional flows under conditions of fully developed forced convection with constant properties. Simulated values of Nusselt number are then compared against the Kurganov-Petukhov empirical correlation and Dittus-Boelter [3] empirical correlation.

2. Governing equations

The mean flow equations are written in the 'boundary layer' approximation form, which result from considering flow, where there is a clear principal flow direction and all the important variations occur in the direction normal to the flow direction. However, the problem treated here is that of fully developed forced convection, the equations are presented in their full form for the developing flow and heat transfer as implemented in the code. In order to perform the simulations, the hydrodynamically fully developed profiles of velocity, turbulence quantities must first be obtained and the fully developed Nusselt number can then be calculated. The governing equations are of parabolic type and therefore the finite difference scheme with the marching procedure for solution of the flow field is implemented. The calculations start with approximate, theoretical initial profiles and the code is run with developing flow for 100 diameters in order to ensure that a fully developed fluid flow condition has been reached. The thermal boundary condition is that of uniform heat flux. In the geometry considered in the present study, i.e. axisimmetric, vertical pipe flow, the principal flow direction coincides with the axis of the pipe. Governing equations are therefore written in the following form in cylindrical coordinates:

continuity equation

$$\frac{1}{r}\frac{\partial(\rho r V)}{\partial r}+\frac{\partial(\rho W)}{\partial z}=0, \tag{1}$$

momentum equation

$$\frac{1}{r}\frac{\partial(r\rho VW)}{\partial r}+\frac{\partial(\rho W^2)}{\partial z}=-\frac{dp}{dz}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu\frac{\partial W}{\partial r}-\rho\overline{v'w'}\right)\right]\pm\rho g, \tag{2}$$

energy equation

$$\frac{1}{r}\frac{\partial(r\rho Vh)}{\partial r}+\frac{\partial(\rho Wh)}{\partial z}=\frac{Dp}{Dt}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\lambda}{c_p}\frac{\partial h}{\partial r}-\rho\overline{v'h'}\right)\right]. \tag{3}$$

The form of the energy equation was modified from the form based on temperature. As it was mentioned above the governing equations are for a developing flow with variable properties and the present work is a part of a more extensive programme where the influence of property variation with temperature is also to be examined and therefore it has been decided to present all equations in their developing form rather than simplified one. When equations have been derived it was assumed that the neglected term $\left(T/\rho(\partial p/\partial T)_p\right)$ in the temperature version of the energy equation can introduce some error into the calculations. This problem is alleviated in the enthalpy form of the energy equation, where only the pressure term appears. In the present calculations this term have been neglected.

The eddy diffusivity concept introduces the following definitions:

$$\overline{v'w'}=-\epsilon_M\left(\frac{\partial W}{\partial r}\right), \tag{4}$$

$$\overline{v'h'}=-\epsilon_H\left(\frac{\partial h}{\partial r}\right). \tag{5}$$

Now let us define TPN as:

$$Pr_t=\frac{\epsilon_M}{\epsilon_H}. \tag{6}$$

In order to evaluate TPN at any point in the boundary layer from experimental data it is necessary to measure four quantities: the turbulent shear stress, the turbulent heat flux, the velocity gradient and the temperature gradient. However, it is very difficult to measure all these quantities at a single point in the boundary layer and therefore for a long time these measurements were unreliable and even now are relatively scarce and result in a large experimental scatter.

The closure problem At this stage we possess information which relates eddy diffusivity of momentum and heat. To solve the set of governing equations we still require some means of evaluation of eddy diffusivity of momentum. Some studies have been performed earlier [1] where a comparison have been done of most known turbulence models. Therefore, it was decided that present calculations will be performed using the mixing length model [4] with Nikuradse distribution of mixing length [5] relevant to pipe flows. Another model chosen is due to Launder and Sharma (LS) [7], a two-equation low-Reynolds number turbulence model. The latter have been found [1] to behave best of all other two-equation models under conditions of forced convection in pipes and is quite widely used in a variety of flow problems.

In the LS model the turbulent viscosity μ_t is defined by the equation:

$$\mu_t = C_\mu f_\mu \frac{\rho k^2}{\hat{\epsilon}}, \tag{7}$$

in which $C_\mu = 0.09$ and $f_\mu = exp(-3.4/(1+Re_t/50)^2)$, where $Re_t = k^2/\nu\hat{\epsilon}$, is a damping function which causes the eddy viscosity near the wall to be reduced. The equations, which define transport of k and ϵ equations are as follows:

k-transport

$$\frac{1}{r}\frac{\partial(\rho r V k)}{\partial r} + \frac{\partial(\rho W k)}{\partial z} = \mu_t\left(\frac{dW}{dr}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] - \rho(\hat{\epsilon} + D), \tag{8}$$

$$D = \begin{cases} 2\nu k/y^2 & y^+ \leq 2, \\ 2\nu(\partial k^{1/2}/\partial y)^2 & y^+ \geq 2, \end{cases} \tag{9}$$

ϵ-transport

$$\frac{1}{r}\frac{\partial(\rho r V\hat{\epsilon})}{\partial r} + \frac{\partial(\rho W\hat{\epsilon})}{\partial z} = c_1\frac{\hat{\epsilon}}{k}\mu_t\left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu + \frac{\mu_t}{\sigma_{\hat{\epsilon}}}\right)\frac{\partial\hat{\epsilon}}{\partial r}\right] + $$

$$- C_2 f_2\frac{\rho\hat{\epsilon}^2}{k} + \frac{2\mu\mu_t}{\rho}\left(\frac{\partial^2 W}{\partial r^2}\right)^2, \tag{10}$$

where $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$, $C_1 = 1.44$, $C_2 = 1.92$, $f_2 = 1.0 - 0.3\ exp\ (-Re_t^2)$. $\hat{\epsilon}$ is the modified dissipation variable to which the special boundary condition $\hat{\epsilon} = 0$ applies. At the wall $k = 0$.

Mixing length models use the Prandtl's [4] concept of mixing length hypothesis (MLH) and express the turbulent viscosity in terms of a mixing length, l_m, and the mean velocity gradient by:

$$\mu_t = \rho l_m^2\left|\frac{\partial W}{\partial r}\right|. \tag{11}$$

Various modifications to the original prescription of the mixing length ($l_m = \kappa y$) have been proposed. In the present study, the Nikuradse [5] mixing length distribution, combined with the Van Driest [6] wall damping function, was selected as being representative of this group of models. The Nikuradse distribution of the mixing length has been carefully adjusted to make it applicable for the pipe flows. It reads as follows:

$$\frac{l_m}{R} = 0.14 - 0.08\left(1 - \frac{y}{R}\right)^2 - 0.06\left(1 - \frac{y}{R}\right)^4. \tag{12}$$

It is claimed that this formulation is valid for $Re > 10^5$ for fully developed pipe flow. In order to make this formulation work, without introducing the idea of a separate wall layer, the Van Driest damping term is introduced:

$$l_m = \kappa y[1 - exp(-y^+/A^+)], \qquad \kappa = 0.4;\ A^+ = 26.0\,. \tag{13}$$

3. Empirical correlations used for comparisons in the study

Simple approximate correlation equations of the Dittus-Boelter type (equation 14) are widely used for calculating heat transfer to gases in pipes.

$$Nu_{cp\infty} = 0.023Re^{0,8}Pr^{0.4}. \tag{14}$$

More complicated correlation equations of wider generality such as the Kurganov-Petukhov equations has also been proposed. They have been found to correlate the experimental data to a very satisfactory extent and are therefore now beginning to be favoured by some researchers. The Kurganov-Petukhov correlation has the feature of being able to describe thermal development and includes a correction for evaluation of variable property effects in the case of air. The form of the equation for fully developed constant property forced convection reads:

$$Nu_{cp\infty} = \frac{\zeta/8RePr}{\kappa + 12.7\sqrt{\zeta/8}(Pr^{2/3} - 1)} \tag{15}$$

where

$$\begin{aligned}
\kappa &= 1,07 + \frac{900}{Re} - \frac{0.631}{1 + 10Pr}, \\
\zeta &= \left[1.82\, lg\left(\frac{Re}{8}\right)\right]^{-2}, \qquad Re \geq 10^4, \\
\zeta &= \frac{0.3154}{Re^{0.25}}, \qquad Re < 10^4.
\end{aligned}$$

The thermal development correction, ϵ_l, is calculated using the following relation:

$$Nu_{cp} = \epsilon_l \times Nu_{cp\infty} \tag{16}$$

where

$$\epsilon_l = 1 + \frac{0.48\left(1 + \frac{3600}{Re\sqrt{z/d}}\right)}{\left(\frac{z}{d}\right)^{1/4}} e^{-0.17\frac{z}{d}}.$$

It is claimed that the above equation is applicable for a Reynolds number range from $4 \cdot 10^3$ to $6 \cdot 10^5$ and Prandtl number range from 0.7 to $5 \cdot 10^5$ and that it has an accuracy of better than 4%. It is used here for comparisons with the model simulations of fully developed constant property forced convection.

4. Various approaches to TPN modelling

The Reynolds analogy The simplest of all approaches to treat the turbulent heat transfer leads to an assumption known as the Reynolds analogy where one says that the eddy diffusivity of heat is exactly the same as that of momentum leading to TPN=1.0 and:

$$\epsilon_M = \epsilon_H. \tag{17}$$

The remarkable thing is that the analogy is very useful in theoretical considerations for devising simple analytical solutions to theoretical problems and holds to an appreciable extent for most boundary layer flows. In wide variety of calculations, however, it assumed that the TPN should be corrected and an uniform value of 0.9 is used. In addition, recently it was found that the uniform value of TPN which reflects best the experimental behaviour is TPN=0.85. Results of simulations for TPN=0.85 and TPN=0.9 are presented along with the other correlations in Table 1 and Table 3 for air and water respectively in the constant property mode.

Azer and Chao's (AC) model Azer and Chao [8] used a detailed mixing length model to derive two equations of considerable complexity for TPN, one of these being for fluids of Prandtl number between 0.6 and 15 and the other for liquid metals (where departures from unity of TPN are very significant, molecular Prandtl number is very low and of order 0.01). The authors also presented corresponding simpler approximations to their equations, stating that these gave maximum deviations from the original equations within 14%. The approximate formula for fluids of $0.6 < Pr < 15$ is as follows:

$$Pr_t = \frac{1.0 + \frac{57 f(y/R)}{\left(Re^{0.46} Pr^{0.58}\right)}}{1.0 + \frac{135 f(y/R)}{Re^{0.45}}} \tag{18}$$

where

$$f(y/R) = exp[-(y/R)^{0.25}]. \tag{19}$$

Results of simulations for air and water using this model are presented in Table 1 and Table 3 respectively.

Kays and Crawford (KC) experimental fit for air The correlation which is based on experimental data for air was recommended by Kays and Crawford [10]. It provides a relatively high value of TPN near the wall but approaches 0.85 as ϵ_M/ν and thus y^+ increases. It reads as follows:

$$Pr_t = \frac{1}{0.5882 + 0.228(\epsilon_M/\nu) - 0.0441(\epsilon_M/\nu)^2\left[1 - exp\left(\frac{-5.165}{(\epsilon_M/\nu)}\right)\right]}. \tag{20}$$

The data used for correlation have been very carefully selected from a very wide experience of the above mentioned authors who have been engaged, between the others, in convective heat transfer for a very long time and tried to establish their own models of TPN. Therefore, using this correlation is justified and thought of being the best to the authors ability and can be well trusted. The results of simulations in the case of air are presented in Table 1.

Hollingsworth et al. (HL) experimental fit for water Another comprehensive experiment on establishing the TPN in water was conducted by Hollingsworth [11]. Evaluation of TPN utilized equation (6). The authors proposed the following correlation as a reasonable fit to their data:

$$Pr_t = 1 + 0.855 - tanh[0.2(y^+ - 7.5)]. \tag{21}$$

This equation in the region away from the wall tends to a value of 0.85. The results of simulations utilizing above correlation are presented in Table 3.

A general fit (GF) to experimental data for all fluids Entire molecular Prandtl number range was approximated by Kays [9] to form a reasonably simple formulae for TPN which is as follows:

$$Pr_t = 2.0/Pe_t + 0.85, \qquad Pe_t = (\epsilon_M/\nu)Pr. \tag{22}$$

The simulations were performed for the cases of air and water and presented in Table 1 and Table 3 respectively.

5. Results of simulations for constant property flows of air

Let's first consider the results of simulations of simple constant property case of air flow in a smooth vertical pipe. Computed values of Nusselt number are quoted in Table 1.

As can be noticed there are quite significant differences between computed values of Nusselt number and those calculated using empirical correlations for $Pr = 0.7$. In order to gain a better picture of the differences between various calculations, a table containing percentage differences between calculated values of Nusselt number and Kurganov-Petukhov have been produced. A discussion is

Table 1. Results of simulations of constant property forced convection of air. $d = 0.02\ m, z/d = 100, T_{in} = 20^oC$

Model	TPN	Reynolds number				
		4000	10000	20000	30000	40000
ML	0.85	16.883	31.783	53.012	71.993	89.665
	0.90	16.376	30.701	51.072	69.264	86.139
	AC	18.593	34.585	56.756	76.203	94.089
	KC	16.007	30.201	50.511	68.708	85.674
	GF	13.163	25.238	42.791	58.654	73.516
LS	0.85	14.390	30.008	51.883	71.574	90.020
	0.90	14.136	29.041	50.056	68.949	86.633
	AC	15.580	32.413	55.311	75.532	94.240
	KC	13.297	27.855	48.477	67.138	84.677
	GF	11.527	24.190	42.460	59.106	74.802
Kurganov & Petukhov		14.234	30.289	52.044	71.171	88.862
Dittus & Boelter		15.233	31.705	55.202	76.353	96.112

extened to include the discrepancies with the Dittus-Boelter correlation and the results of calculations using the uniform TPN=0.9 The way in which discrepancies were calculated is shown below:

$$Diff = \frac{Nu_{calc} - Nu_{corr}}{Nu_{corr}}. \tag{23}$$

Comparison with Kurganov-Petukhov correlation At the first glance it is apparent that the best consistency is achieved when a uniform value of TPN=0.85 is used. The simulations agree to within 5% of Kurganov- Petukhov (KP) values. Slightly worse agreement for $Re = 4000$ is because the KP correlations looses its validity in this transition region. The Kays and Crawford (KC) fit also produces very reasonable agreement for this fluid but discrepancy reaches −8%. The Azer and Chao's (AC) model overpredicts the heat transfer coefficient by a similar margin. The general fit (GF) produces values of unacceptable accuracy. The margin of discrepancy is similar for two models considered.

Comparison with Dittus-Boelter correlation Dittus-Boelter correlation is still widely used, however, it is not appropriate for this sort of comparisons. It has a margin of error, compared with experimental data, of about 20%. The results follow the pattern described above. Discrepancies are shifted down compared with

Table 2. Discrepancies between simulated values of Nusselt number and those calculated using Kurganov-Petukhov correlation in the case of air

Model	TPN	Discrepancy [%]				
		4000	10000	20000	30000	40000
ML	0.85	18.6	4.9	1.9	1.2	0.9
	0.90	15.0	1.4	−1.9	−2.7	−3.1
	AC	30.6	14.2	9.1	7.1	5.9
	KC	12.5	−0.3	−2.9	−3.5	−3.6
	GF	−7.5	−16.7	−17.8	−17.6	−17.3
LS	0.85	1.1	0.9	0.3	0.6	1.3
	0.90	−0.7	−4.1	−3.8	−3.1	−2.5
	AC	9.5	7.0	6.3	6.1	6.1
	KC	−6.6	−8.0	−6.9	−5.7	−4.7
	GF	−19.0	−20.1	−18.4	−17.0	−15.8

Table 1 because the values calculated using DB correlation are generally higher than KP values.

Comparison with data simulated at TPN=0.9 This comparison meant to show the reader what are the relative differences in using a particular TPN model compared against commonly used values of TPN=0.9. From Table 2 it is seen that by using TPN=0.85 we obtain approximately 3.5% shift up of heat transfer coefficient. This applies to both turbulence models considered. In the case of AC model, the shift is bigger and reaches 11%. The KC experimental fit results differ in the Nusselt number by about 3%. The worst picture is produced by the GF fit.

6. Results of simulations for constant property case of water

Now let's consider results for a higher molecular Prandtl number i.e. for water ($Pr = 7$). These are presented in Table 3 below.

Similarly as in the case of air table of discrepancies have been produced, where a comparison of calculated values of Nusselt number is compared against KP correlation.

Comparison with Kurganov-Petukhov correlation In the case of water, the discrepancies in heat transfer coefficient are much more pronounced. Only TPN=0.85

Table 3. Results of simulations of constant property forced convection of water $d = 0.02\,m, z/d = 100, T_{in} = 20^{o}C$

Model	TPN	Reynolds number				
		4000	10000	20000	30000	40000
ML	0.85	41.006	85.005	151.07	212.35	270.75
	0.90	40.175	83.279	147.88	207.77	264.82
	AC	52.260	103.70	178.06	245.29	308.36
	HL	34.488	73.456	131.13	179.72	229.46
	GF	34.301	71.625	127.91	180.26	230.24
LS	0.85	28.629	69.257	129.52	185.92	240.01
	0.90	28.207	68.091	127.20	182.49	235.49
	AC	33.940	80.665	147.43	208.57	266.41
	HL	26.040	61.688	114.86	164.72	212.57
	GF	24.930	60.152	112.66	161.99	209.36
Kurganov & Petukhov		38.898	83.961	150.16	211.24	269.33
Dittus & Boelter		38.018	79.130	137.77	190.56	239.88

Table 4. Discrepancies between simulated values of Nusselt number and those calculated using Kurganov-Petukhov correlation in the case of water

Model	TPN	Discrepancy [%]				
		4000	10000	20000	30000	40000
ML	0.85	5.4	1.2	0.6	0.5	0.5
	0.90	3.2	0.8	-1.5	-1.6	-1.7
	AC	34.4	23.5	18.6	16.1	14.5
	HL	-11.3	-12.5	-12.6	-14.9	-14.8
	GF	-11.8	-14.7	-14.8	-14.7	-14.5
LS	0.85	-26.4	-17.5	-13.7	-12.0	-10.9
	0.90	-27.5	-18.9	-15.3	-13.6	-12.6
	AC	-12.7	-3.9	-1.8	-1.3	-1.1
	HL	-33.1	-26.5	-23.5	-22.0	-21.1
	GF	-35.9	-28.4	-25.0	-23.3	-22.3

and TPN=0.9 simulations in the case of ML model agree closely. Again results obtained at TPN=0.85 show better agreement than those calculated using TPN=0.9. For the same TPN models, the LS model underpredicts Nusselt number by about 15%. The striking results are produced when using the AC TPN model. In the case of the ML model, the results are overpredicted by about 20%. However, the situation is different, when the LS model is used. In this case the agreement is very close. The HL and GF correlations of TPN reflect poor agreement as results are underpredicted by at least 20% in both cases.

Comparison with Dittus-Boelter correlation In the case of water, the results of heat transfer coefficient calculated using DB correlation are generally lower than those calculated using the KP correlation. Therefore similar remarks as above apply but discrepancies are slightly higher.

Comparison with data calculated using TPN=0.9 Introduction of non-uniform TPN into calculations results in big discrepancies between simulations utilizing a uniform TPN and a non-uniform TPN. The differences are as large as about 20% in the case of AC model and 10% in the case of HL and GF. This has probably to do with the poor description of the turbulence models in the near wall region. In the case of water, the thermal layer is considerably thinner than the hydrodynamic layer. Therefore, any error in calculation of eddy diffusivity of momentum bears even more error into calculation of thermal layer. This is not such a problem in air flow as molecular Prandtl number is about 0.7 and hence those two layers are of comparable thickness.

7. Conclusions

In the present work a selection of five turbulent Prandtl number concepts have been tested on two models of turbulence namely mixing length model and Launder and Sharma two-equation model. Calculations have been performed for two different molecular Prandtl numbers of 0.7 and 7, which correspond to air and water respectively. The best agreement for all calculations has been achieved when a uniform value of TPN=0.85 was used. This is a slight refinement of widely used value of 0.9. Other TPN models are more suitable for specific range of conditions which is apparent from tables presented earlier. These models are: the Azer and Chao (water) and Kays and Crawford (air). Various TPN concepts describe the distribution of TPN quite well for this rather simple case namely forced convection with constant properties. Using more complex TPN models than uniform value ones (i. e. TPN=0.85) are not suitable for the kind of calculations performed here and are computationally more expensive. However, a great deal of efforts have been recently concentrated on a correct description of the turbulence characteristics in the wall regions in turbulence models, this is still insufficient with regard to calculate correctly the thermal field. Turbulence in forced convection with constant properties is pretty well recognised and investigated. In more complex case, where buoyancy forces are playing an important role and the effects of property appear, it is difficult to judge whether non-uniform TPN models will behave better and which one will be the best. This requires further computational work and the intention of the authors is to find out clarified picture how turbulence models will responds under all such conditions.

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Modelowanie konwekcji wymuszonej przy użyciu różnych modeli turbulentnej liczby Prandtla

Streszczenie

W niniejszej pracy porównano odpowiedzi dwóch różnych modeli turbulencji, a mianowicie modelu drogi mieszania Prandtla oraz modelu $k \sim \epsilon$ Laundera i Sharmy, w warunkach zmienności turbulentnej liczby Prandtla. Przeanalizowano te odpowiedzi w warunkach stałych własnosci termofizycznych płynu w pełni rozwiniętym przepływie turbulentnym. Obliczone liczby Nusselta porównano z korelacjami KurganoVa-Petukhova oraz Dittusa-Boeltera. Z pracy wynika, że zastosowanie takich modeli nie poprawia drastycznie odpowiedzi modeli turbulencji na obliczenia konwekcji wymuszonej przy stałych własnościach.