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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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JAROSŁAW MIKIELEWICZ[1]

Impingement of a liquid jet on an inclined plate[2]

Theoretical analysis of the phenomenon of impingement of a liquid jet on an inclined plate has been carried out. Splitting of the Newtonian liquid on the inclined plate is caused by the inertia and gravity forces. A theoretical model of the phenomenon has been formulated as well a method of its solving has been proposed. The method of solving is based on qualitative analysis of geometrical and topology properties of the integral curves. The existence of turning and singular points in the solution has been pointed out. The method of solving depends on the character of singularity. Turning and singular points of the solution correspond to critical flow condition of the so-called hydraulic jump. Numerical calculations are presented which illustarate the theory. It has been shown that supercritical flow can reach critical parameters approaching the maximal spreading radius. The hydraulic jump may appear due to a variety of disturbances even before the flow reaches the critical conditions. After the hydraulic jump the flow becomes subcritical. The transition of flow from supercritical to subcritical conditions is also possible in the case when the integral curve passes through a singular point.

Nomenclature

C_f	–	friction coefficient,	w	–	z-component of velocity,
g	–	acceleration of gravity,	r, ϕ, z	–	cylindrical coordinates,
Q	–	volumetric flow rate,	α	–	inclination angle,
p	–	pressure,	δ	–	film thickness,
P	–	calculation point,	μ	–	dynamic viscosity,
R_S	–	dimensionless critical radius – Eq. (2.20)	ρ	–	liquid density,
S	–	vector tangent to the flow of the liquid film,	τ	–	wall shear stress,
			$Fr_c = \frac{v_c}{\sqrt{g\delta_c}}$	–	critical Froud number,
u	–	circumferential velocity,	$Fr_{in} = \frac{v_{in}}{\sqrt{g\delta_{in}}}$	–	inlet Froud number,
v	–	radial velocity,	$Re_s = \frac{v_s \delta_s \rho}{\mu}$	–	critical Reynolds number.

Subscripts

φ	–	circumferential,	in		inlet,
r	–	radial,	c		critical,
z	–	vertical,	w	–	at the wall,
o	–	surrounding	s	–	of the singular point circumferential.

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[2] The paper was sponsored by a research project KBN 3 P40403407

Superscripts

+ – dimensionless.

1. Introduction

Liquid flow down the surface of the wall occurs in spray cooled heat exchangers or condensers as well as in a variety of chemical engineering installations. The process of liquid flow in spray cooled heat exchangers is different from that in condensers. Both processes have been investigated theoretically and experimentally. The flow of gravity driven liquid films formed due to vapour condensation is particularly richly represented amongst the literature. This type of flow was investigated in [1,2,3] for different pipe geometry including inclined cylindrical or elliptical pipes and cylindrical finned pipes. In the case of gravity driven flow, the trajectory of the liquid film is determined by the gradient of wall steepness. The inertia forces are neglected then. However, in the case of impingment of a liquid jet on the surface, the inertia forces must be taken into account as they have a great effect on the trajectory of a forming liquid film. The process of flow of a liquid film after the impingment, at a certain speed, of a liquid jet on a flat horizontal surface was investigated theoretically and experimentally in [4,5]. The papers do not offer solutions for inclined surfaces which could be instrumental in investigating flows in complex pipe geometry as in spray cooled heat exchangers.

The aim of the paper is to recognise the phenomenon of liquid jet impingment on surfaces inclined at different angles with respect to the horizontal plane.

Solutions of the model set forth in the paper are confronted with the topological analysis of the system. The analysis enables finding qualitative features of all possible solutions before solving the set of equations describing the investigated phenomenon, [6]. Applying the analysis in advance of executing numerical calculations is of great advantage for the choice of the right method of calculations. The solutions can have regular points belonging to one integral curve only, turning points referring to extreme values, in the appropriate coordinate system, can be amongst them. The solutions can also have singular points belonging to none or more than one integral curves. Both the turning and singular points have clear physical meaning. They could be linked to critical flow conditions of the so-called hydraulic jump. There are three types of singular points - saddle points, nodal points and spiral points. Only saddle points and some nodal points allow for the transition from supercritical to subcritical flow without the hydraulic jump. The analysis of the type of the singular points carried out in the paper is very helpful in better understanding of the flow of liquid films forming at liquid jet impingment. The method of investigations is illustrated by results of numerical calculations.

2. Analysis

A flat surface inclined towards the horizontal plane at an angle α together with a cylindrical coordinate system are sketched in Fig. 1. The coordinate z is assumed vertical to the inclined surface. Let us consider two-dimensional flow described by the coordinates r, ϕ lying in the same plane as the surface vertically impinged by a liquid jet of flow rate Q. Let the following assumptions be taken:

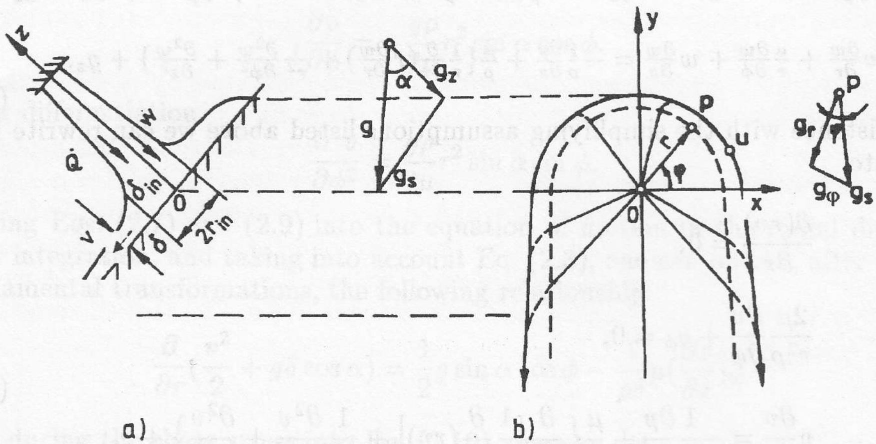

Fig. 1. Impingment of a liquid jet on an flat surface inclined at an angle α: a) liquid jet impingment, b) system of cylindrical coordinates.

1. The flow of the liquid film is steady, laminar and governed solely by the action of the gravity and inertia forces.
2. The surrounding medium is at rest and there is no shear stress at the interfacial surfaces.
3. Liquid-wall and liquid-vapour interfacial surfaces are smooth.
4. Physical properties of the liquid do not change.
5. The liquid film thickness is small which justifies simplifications in the mass and momentum conservation equations.
6. The velocity across the film is constant and equal to the average velocity in the film. The average velocity is a function of the coordinates r, α.
7. The circumferential and vertical components of the velocity vanish ($u = w = 0$), what means that the radial component of the velocity and the film thickness are functions of r, ϕ, except the region very close to the boundary, as shown on Fig. 1.

A complete set of mass and momentum conservation equations can be written as follows [7]

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial(\rho rv)}{\partial r}+\frac{1}{r}\frac{\partial(\rho u)}{\partial\phi}+\frac{\partial(\rho w)}{\partial z}=0,$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+\frac{u}{r}\frac{\partial u}{\partial\phi}+\frac{vu}{r}+w\frac{\partial u}{\partial z}=-\frac{1}{pr}\frac{\partial p}{\partial\phi}+\frac{\mu}{\rho}\{\frac{\partial}{\partial r}[\frac{1}{r}\frac{\partial(ru)}{\partial r}]+\frac{1}{r^2}\frac{\partial^2 u}{\partial\phi^2}+\frac{2}{r^2}\frac{\partial v}{\partial\phi}+\frac{\partial^2 u}{\partial z^2}\}+g_\phi$$

$$\frac{\partial v}{\partial t}+\frac{u\partial v}{r\partial\phi}-\frac{u^2}{r}+v\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\{\frac{\partial}{\partial r}[\frac{1}{r}\frac{\partial}{\partial r}(rv)]+\frac{1}{r^2}\frac{\partial^2 v}{\partial\phi^2}-\frac{2}{r^2}\frac{\partial u}{\partial\phi}+\frac{\partial^2 v}{\partial z^2}\}+g_r$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial r}+\frac{u}{r}\frac{\partial w}{\partial\phi}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\{\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w}{\partial r})+\frac{1}{r^2}\frac{\partial^2 w}{\partial\phi^2}+\frac{\partial^2 w}{\partial z^2}\}+g_z. \tag{2.1}$$

In consistence with the simplifying assumptions listed above we can rewrite Eqs. (2.1) into

$$\begin{aligned}
&\frac{\partial(rv)}{\partial r}=0,\\
&\frac{2\mu}{r^2\rho}\frac{\partial v}{\partial\phi}+g_\phi=0,\\
&v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv)\right]+\frac{1}{r^2}\frac{\partial^2 v}{\partial\phi^2}+\frac{\partial^2 v}{\partial z^2}\right\}+g_r,\\
&-\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z=0.
\end{aligned} \tag{2.2}$$

The components of the gravity forces, see Fig. 1, are as follows:

$$\begin{aligned}
g_r&=g\sin\alpha\cos\phi,\\
g_\phi&=g\sin\alpha\cos\phi,\\
g_z&=g\cos\alpha.
\end{aligned} \tag{2.3}$$

The assumption that the circumferential and vertical components of the velocity vanish leads to a system of four equations with two unknowns: p, v. This means that some equations in (22) are identical to each other. The mass equation for the film can be obtained by integrating the first equation of the system (2.2) within the film thickness:

$$\frac{\partial(rv\delta)}{\partial r}=0. \tag{2.4}$$

Integrating and equating with the flow rate of the impinging liquid jet, we have

$$Q=2\pi rv\delta. \tag{2.5}$$

Then, integrating the equation of motion in z-direction, the distribution of pressure can be found

$$p = \rho g \delta \cos\alpha + p_0. \tag{2.6}$$

Differentiating the above, one can have

$$\frac{\partial p}{\partial r} = \rho g \frac{\partial \delta}{\partial r} \cos\alpha. \tag{2.7}$$

From the equation of motion in the circumferential direction, one can get

$$\frac{\partial v}{\partial \phi} = -\frac{g\rho}{2\mu} r^2 \sin\alpha \cos\phi. \tag{2.8}$$

After differentiation

$$\frac{\partial^2 v}{\partial \phi^2} = \frac{g\rho}{2\mu} r^2 \sin\alpha \sin\phi. \tag{2.9}$$

Placing Eqs. (2.7) and (2.9) into the equation of motion in the radial direction after integration, and taking into account Eq. (2.3), one can obtain, after several fundamental transformations, the following relationship

$$\frac{\partial}{\partial r}\left(\frac{v^2}{2} + g\delta\cos\alpha\right) = \frac{1}{2} g \sin\alpha \cos\phi - \frac{1}{\rho\delta}\mu\left(\frac{\partial v}{\partial z}\right)_w. \tag{2.10}$$

Introducing the shear stress into Eq. (2.10), one can get

$$\frac{\partial}{\partial r}\left(\frac{v^2}{2} + g\delta\cos\alpha\right) = \frac{1}{2} g \sin\alpha \cos\phi - \frac{1}{\rho\delta}\tau. \tag{2.11}$$

The wall shear stress is as follows

$$\tau = C_f \rho \frac{v^2}{2}. \tag{2.12}$$

Then, with the knowledge of the friction coefficient, one can make use of Eq. (2.11) in the case of both laminar and turbulent flow.

Eqs. (2.11) and (2.5) can be considered as a closed set of equations to evaluate the velocity field and film thickness. Depending on whether the relation for the velocity or for the film thickness is used, one can formulate appropriate boundary conditions. Both the velocity and the film thickness at an initial radius r_{in} are assumed arbitrarily and are equal to v_{in} and δ_{in}, respectively. Combining the film thickness from the mass equation (2.5) and Eq. (2.12) into the momentum equation (2.11), one can get, after several transformations, the following differential equation:

$$\frac{dv}{dr} = \frac{\dfrac{gQ\cos\alpha}{2\pi r^2 v} + \dfrac{g\sin\alpha\cos\varphi}{2} - \dfrac{C_f v^3 \pi r}{Q}}{\left(v - \dfrac{gQ\cos\alpha}{2\pi r v^2}\right)} \tag{2.13}$$

where C_f is the friction coefficient, which can be determined from the Blasius formula for the boundary layer:

$$C_f = \frac{0.0664}{\sqrt{\dfrac{v(r - r_{in})}{v}}} \tag{2.14}$$

with r_{in} denoted as the initial radius of the liquid film.

Likewise, a differential equation for the film thickness can be derived based on the above procedure if the velocity is replaced from the mass equation instead of the film thickness.

Integrating Eq. (2.13) gives one a relationship for the velocity as a function of the radius r. Expressing the velocity in terms of the film thickness, Eq. (2.5), a relationship between the film thickness and the film radius can be found. The slope of the integral curve is determined by the value of the derivative of the velocity or film thickness with respect to the film radius. The derivatives go to infinity if the denominator in Eq. (2.13) approaches zero. This situation is characteristic for the so-called turning point and refers to critical flow conditions. Let us find the criticality conditions equating the denominator in Eq. (2.13) with zero. Then we have

$$v_c = \left(\frac{gQ\cos\alpha}{2\pi r_c}\right)^{1/3} \tag{2.15}$$

or in a dimensionless notation

$$\frac{2\pi r v_c^3}{gQ} = \frac{v_c^2}{g\delta_c} = Fr_c^2 = \cos\alpha. \tag{2.16}$$

The relation (2.16) for the flat horizontal surface was obtained in [5]. It follows from Eq. (2.16) that the parameters of the critical flow are local and depend neither on the integration path nor on the friction coefficient. Therefore, they are the same also in the case of inviscid flow without friction.

If both the numerator and denominator in Eq. (2.13) are equal to zero simultaneously, then the derivative, which locally determines the sense of the integration path is indefinite and we have a singular point. The situation also refers to the critical conditions given by Eq. (2.15).

Equating the numerator and denominator of Eq. (2.13) with zero, one can obtain two following equations:

$$r_s = \left(\frac{2Q^2\cos^2\alpha}{\pi^2 g(C_f\cos\alpha - sin\alpha\cos\varphi)^3}\right)^{1/5}, \tag{2.17}$$

$$v_s\left(\frac{gQ\cos\alpha}{2\pi r}\right)^{1/3}. \tag{2.18}$$

Dimensionless coordinates of the singular point are as follows:

$$Fr_s = \sqrt{\cos\alpha}, \tag{2.19}$$

$$R_s^+ = \frac{r}{\frac{6}{C_f}\left(\frac{v^2}{g}\right)^{1/3} Re_s^{2/3}} = \frac{1}{3}\,\frac{\cos^{2/3}\alpha}{\cos\alpha - \sin\alpha\,\frac{\cos\varphi}{C_f}} \tag{2.20}$$

where

$$Re_s = \frac{v_s \delta_s \rho}{\mu}.$$

It comes from Eq. (2.17) that a singular point can appear only in the gravity driven flow, also of an ideal liquid without friction. The Froud number and the dimensionless radius of the film as a function of the inclination angle are given in Figs. 2a and 2b. The relation between the Froud number and the dimensionless film radius is presented in Fig. 2c.

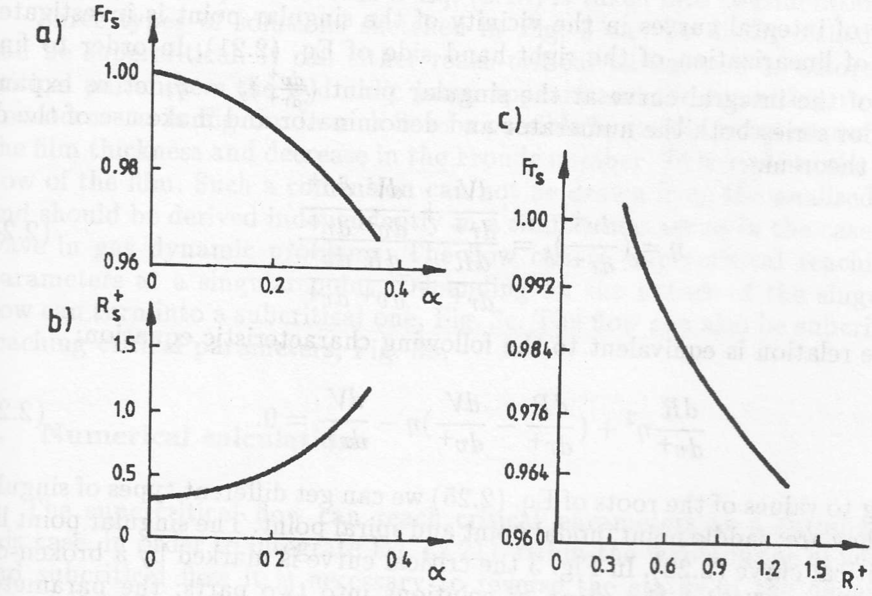

Fig. 2. Critical parameters a) Froud number versus inclination angle α, b) dimensionless film radius versus inclination angle α, c) Froud number versus dimensionless film radius.

For the sake of numerical calculations let us transform Eq. (2.13) into a dimensionless form:

$$\frac{dv^+}{dr^+} = \frac{\frac{\cos\alpha}{r^{+2}v^+} + \frac{1}{2}\sin\alpha\cos\varphi - C_f Fr_{in}^2 \frac{r_{in}}{\delta_{in}} r^+ v^{+3}}{Fr_{in}^2 v^+ - \frac{\cos\alpha}{r^+ v^{+2}}} = \frac{V(r^+, v^+)}{R(r^+, v^+)} \tag{2.21}$$

where

$$r^+ = \frac{r}{r_{in}},\ v^+ = \frac{v}{v_{in}},\ Fr_{in}^2 = \frac{2\pi r_{in} v_{in}^3}{gQ} = \frac{v_{in}^2}{g\delta}.$$

Let us assume the following boundary conditions:

$$r^+ = 1,\ v^+ = 1$$

and transform Eqs. (2.15), (2.17-2.18) into their dimensionless form

$$v_c^+ = \left(\frac{\cos\alpha}{Fr_{in}^2 r_c^+}\right)^{1/3}, \tag{2.22}$$

$$r_s^+ = \left(\frac{8Fr_{in}^2\cos^2\alpha}{(\frac{r_{in}}{\delta_{in}})^3(C_f\cos\alpha - \sin\alpha\cos\varphi)^3}\right)^{1/5},\quad v_s^+ = \left(\frac{\cos\alpha}{Fr_{in}^2 r_s}\right)^{1/3}. \tag{2.23}$$

The shape of integral curves in the vicinity of the singular point is investigated by means of linearization of the right-hand side of Eq. (2.21). In order to find the slope of the integral curve at the singular point $(\frac{dv^+}{dr^+})_s = \eta$, let us expand into a Taylor series both the numerator and denominator and make use of the de l'Hospital theorem.

$$\eta = (\frac{dv^+}{dr^+})_s = \frac{\frac{dV}{dr^+} + \frac{dV}{dv^+}\frac{dv^+}{dr^+}}{\frac{dR}{dr^+} + \frac{dR}{dv^+}\frac{dv^+}{dr^+}}. \tag{2.24}$$

The above relation is equivalent to the following characteristic equation:

$$\frac{dR}{dv^+}\eta^2 + (\frac{dR}{dr^+} - \frac{dV}{dv^+})\eta - \frac{dV}{dr^+} = 0. \tag{2.25}$$

According to values of the roots of Eq. (2.25) we can get different types of singular points. They are: saddle point, nodal point and spiral point. The singular point lies on the critical curve (2.22). In Fig. 3 the critical curve is marked by a broken-dot line. This curve divides the plane of solutions into two parts: the parameters referring to supercritical flow and those of subcritical flow. The solution of Eq. (2.21) refers to supercritical flow when the inlet flow is already supercritical. The condition of flow supercriticality can be expressed as follows:

$$r^+ = 1,\ v_c^+ > 1.$$

Placing the above into Eq. (2.22) one can have

$$Fr_{in}^2 > \cos\alpha. \tag{2.26}$$

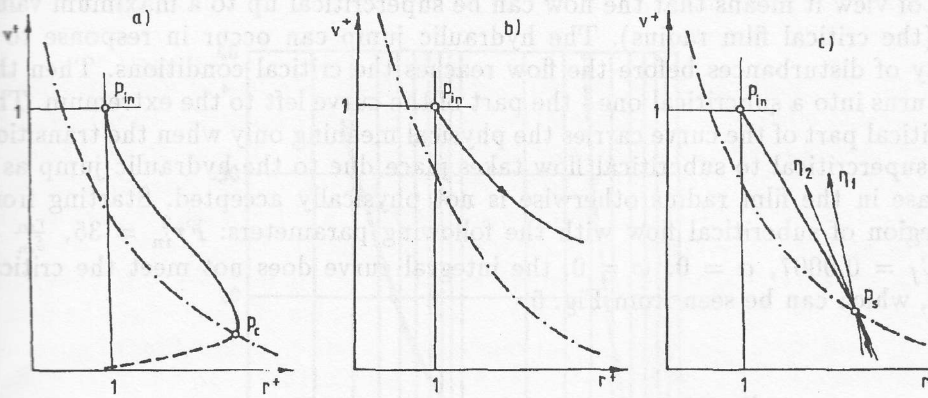

Fig. 3. Posiible solutions a) supercritical flow reaches critical parameters – P_c, b) subcritical flow, c) supercritical flow reaches critical parameters at singular point – P_s.

The result seems quite obvious if Eq. (2.16) is taken into consideration.

Three types of solutions sketched in Fig. 3 can be distinguished. The flow can be supercritical. It can either reach critical parameters or before it reaches critical parameters the hydraulic jump appears and the transition to subcritical flow occurs as in Fig. 3a. In such flows a possible hydraulic jump causes increase in the film thickness and decrease in the Froude number. This leads to the subcritical flow of the film. Such a conclusion can not be drawn from the analised equations and should be derived independently in a similar manner as in the case of a shock wave in gas dynamic problems. The flow can be supercritical reaching critical parameters at a singular point. Depending on the nature of the singularity the flow can turn into a subcritical one, Fig. 3c. The flow can also be subcritical never reaching critical parameters, Fig. 3b.

3. Numerical calculations

The supercritical flow can reach critical parameters at a turning point. In this case, in order to integrate Eq. (2.21) within the whole range of supercritical and subcritical flew it is necessary to reverse the status of the dependent and independent variable. Then the turning point becomes the extremum. Sample calculations were carried out for the inclination angle $\alpha = 0$ and $\varphi = 0$. It was assumed that $Fr_{in}^2 = 400$, $\frac{r_{in}}{\delta_{in}} = 35$, $C_f = 0.0007$. The calculations based on the Runge-Kutta scheme of the forth order. The results of the calculations are presented in Fig. 4. The curve referring to the critical parameters is also indicated in the figure. The curve divides the plane of solutions into regions of supercritical and subcritical flow. Starting from supercritical parameters, it is possible to reach critical parameters. It follows from the figure that the integral curve has a maximum for a certain value of the film radius r. From a physical point of view it means that the flow can be supercritical up to a maximum value of r (the critical film radius). The hydraulic jump can occur in response to a variety of disturbances before the flow reaches the critical conditions. Then the flow turns into a subcritical one - the part of the curve left to the extremum. The subcritical part of the curve carries the physical meaning only when the transition from supercritical to subcritical flow takes place due to the hydraulic jump as a decrease in the film radius otherwise is not physically accepted. Starting from the region of subcritical flow with the following parameters: $Fr_{in}^2 = 35$, $\frac{r_{in}}{\delta_{in}} = 35$, $C_f = 0.0007$, $\alpha = 0$, $\varphi = 0$, the integral curve does not meet the critical curve, which can be seen from Fig. 5.

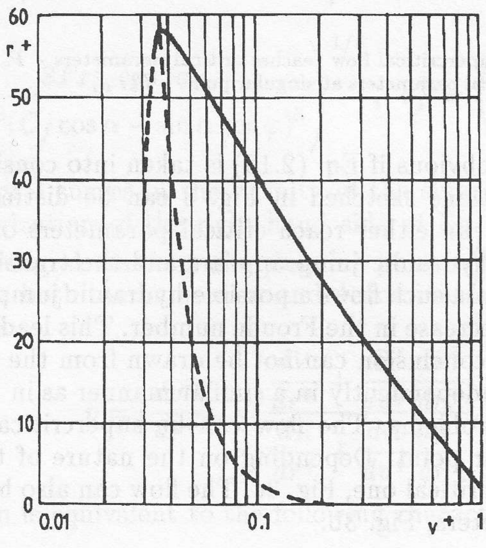

Fig. 4. Integral curve with turning point (——) for $Fr^2 = 400$, $\frac{r_{in}}{\delta_{in}} = 35$, $\alpha = 0$, $\varphi = 0$. Critical curve (- - - -).

Sample calculations of the singular point were performed at: $Fr_{in}^2 = 35$, $\frac{r_{in}}{\delta_{in}} = 150$, $C_f = 0.0007$, $\alpha = 0$, $\varphi = 0$. It was found that $v_c^+ = 0.188$, $r_s^+ = 1.883$. The characteristic directions are: v^+, r^+ : $\eta_1 = -0.157$, $\eta_2 = -0.043$. This suggests the existence of a nodal point. The location of this nodal point and characteristic directions are indicated in Fig. 6. The steepness of the critical curve at the turning point is equal to – 0.0339. If the characteristic direction crosses the critical curve in a manner that passing from the supercritical to subcritical region gives rise to the film radius, then the transition can occur without the hydraulic jump, which is the case of the investigations. Looking into the nature of singular points one can draw a conclusion that the nodal singular point belongs to many integral curves.

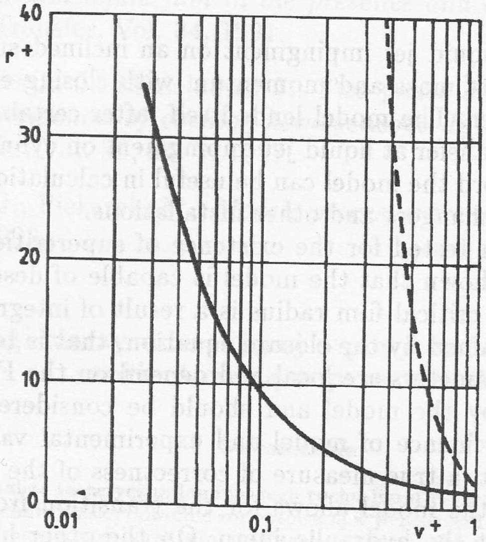

Fig. 5. Integral curve for subcritical flow at $Fr^2 = 0.5$, $\frac{r_{in}}{\delta_{in}} = 35$, $c_f = 0.007$, $\alpha = 0$, $\varphi = 0$. Critical curve (– – – –).

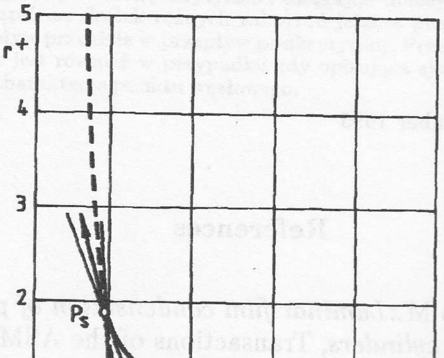

Fig. 6. Integral curve passing through singular point P (———), critical curve (– – – –) at $Fr^2 = 80$, $\frac{r_{in}}{\delta_{in}} = 150$, $\alpha = 0$, $\varphi = 0$, $c_f = 0.02$..

4. Final conclusions

A simple model of liquid jet impingment on an inclined surface consisted of conservation equations of mass and momentum with closing equations has been put forward in the paper. The model lends itself, after certain modifications, to investigations of heat transfer at liquid jet impingment on cylindrical surfaces. As mentioned in the foreword the model can be useful in calculations of performance of spray cooled heat exchangers and other installations.

The model has been tested for the existence of supercritical and subcritical solutions. It has been shown that the model is capable of describing the flow at critical conditions. The critical film radius is a result of integration of governing equations and is determined by the closure equation, that is by the friction coefficient. The critical parameters are local and depend on the Froud number only. They are determined by the model and should be considered part and parcel of the model. The coincidence of model and experimental values of the critical parameters is, no doubt, a true measure of correctness of the model. In the case with a singular point, the model allows for the transition from supercritical to subcritical flow without the hydraulic jump. On the other hand, it seems that the parameters of the singular point assumed for the calculations are not highly realistic. For these parameters, there is a balance of the gravity and viscous forces in the flow and accordingly the friction coefficient is excessively high. Nevertheless, the sample calculations should provide an illustration of the problem under investigations.

Manuscript received in September 1995

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Rozpływ strugi cieczy uderzającej o powierzchnię płaską dowolnie zorientowaną w przestrzeni

Streszczenie

Przeprowadzono analizę teoretyczną laminarnego ruchu filmu cieczowego powstałego wskutek uderzenia strugi o powierzchnię płaską dowolnie zorientowaną w przestrzeni. Rozpływ cieczy newtonowskiej powodowany jest siłą bezwładności oraz siłą grawitacji, w przypadku powierzchni pochylonej. Sformułowano model teoretyczny zjawiska oraz zaproponowano metodę rozwiązania zagadnienia. Opiera się ona o własności geometryczno-topologiczne rozwiązania. Wskazano na istnienie w rozwiązaniu punktów zwrotnych i punktów osobliwych. Analiza jakościowa rozwiązania umożliwia wybór odpowiedniej metody rozwiązania. Punkty zwrotne i singularne rozwiązania odpowiadają warunkom krytycznego rozpływu tzw. uskoku hydraulicznego. Wykonano obliczenia numeryczne ilustrujące teorię. Pokazano, że przepływ nadkrytyczny może osiągnąć parametry krytyczne, osiągając maksymalny promień rozpływu. Uskok hydrauliczny może nastąpić na skutek różnych zaburzeń jeszcze przed osiągnięciem parametrów krytycznych, wówczas przepływ przejdzie w przepływ podkrytyczny. Przejście z przepływu nadkrytycznego w podkrytyczny możliwe jest również w przypadku gdy opisująca zjawisko krzywa całkowa przechodzi przez punkt osobliwy o charakterze punktu węzłowego.