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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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JAN KICIŃSKI[1]

Nonlinear model of vibrations in rotor-bearings system. Part I. Calculation algorithm

The paper presents a theoretical model affording possibilities for a non-linear description of the oil vibrations occurring in the rotor-bearings-supports system after exceeding the stability threshold. A new approach to the determination of the elasto-damping properties of the oil film, not based on the commonly used principle of superposition of pressure or the hydrodynamic forces in the bearing, is put forward. The dynamic features of the oil film are determined by the coefficients of stiffness and damping, non-linearly variable in time, calculated by using a fast and modified perturbation method. The method applied to describe the oil film properties makes it possible to take advantage of the equations of motion, relating to the entire system, derived on the basis of the finite elements method.

The theoretical model takes also into account the effect of heat exchange within the bearing. With regard to kinetostatics problems the temperature distribution in the film is calculated on the basis of a complicated, so-called diathermic model of the bearing. On this basis it is possible to determine the mean "effective" dynamic viscosity of the oil, futher applied to the problems of the system dynamics.

There has been presented a general calculation algorithm, as well as a computer program consisting of two parts clearly separated from each other, the kinetostatic one and the dynamic one.

The paper also gives examples of experimental verification of the model at a special large-size test ring. The experimental tests and the verification were carried out in the sphere of small oil vibrations.

Part II of the paper presents results of the simulation investigation related to the oil vibrations (the whirls and the whipping) in the form of trajectories of selected points in the system, characteristics of the amplitudes and the phase angles, and also in the form of vibration spectra. The investigations were carried out both on a two-support testing rotor and on a model of a real 200 MW turbine.

Nomenclature

c – oil specific heat $[\mathrm{J \cdot kg^{-1} \cdot K^{-1}}]$,
$c_{i,k}$ – stiffness coefficients of the oil film $[\mathrm{N \cdot m^{-1}}]$,
$d_{i,k}$ – damping coefficients of the oil film $[\mathrm{N \cdot s \cdot m^{-1}}]$,
$\mathbf{D}$ – global matrix of the system damping $[\mathrm{N \cdot s \cdot m^{-1}}]$,
e – eccentricity of the bearing [m],
h – oil film thickness [m],
k_{ol} – oil thermal conductivity $[\mathrm{W \cdot m^{-1} \cdot K^{-1}}]$,
$\mathbf{K}$ – global matrix of the system stiffness $[\mathrm{N \cdot m^{-1}}]$,
$\mathbf{M}$ – global matrix of the system inertia $[\mathrm{N \cdot s^2 \cdot m^{-1}}]$,
L – width of the bearings [m],
n – rotational speed of the rotor [rpm],
$(O_p)_{st}(O_c)_{st}$ – position of static equlibrium of the bearing shell and journal,
O_c – temporary position of the journal centre during dynamical loadings,

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p – hydrodynamic pressure in the oil film $[\mathrm{N \cdot m^{-2}}]$,
$\mathbf{P}_0, \mathbf{P}$ – vector of static and dynamic external forces of the system [N],
$\mathbf{q}$ – vector of displacements [m],
r – radial coordinate (Fig. 2) [m],
R – radius of the bearing journal [m],
ΔR – absolute radial clearance [m],
t – time [s],
T_w – temperature of the oil flowing out of the bearing,
T – temperature in the lubricating film,
T_p – temperature in the bearing shell,
W_x, W_y – components of the hydrodynamical load capacity of the oil film [N],
x, y – rectangular set of coordinates connected with the static equlibrium position of the shell (Figs. 1 and 2),
z – coordinate along the bearings axis; rotor geodesic coordinate (Fig. 5),
γ – angle of line of centres [rad],
μ – oil dynamic viscosity $[\mathrm{N \cdot s \cdot m^{-2}}]$,
$\overline{\mu}$ – oil mean "effective" viscosity $[\mathrm{N \cdot s \cdot m^{-2}}]$,
ξ, η – set of coordinates circumferential and along the bearing axis (Fig. 2),
ξ_1^*, ξ_2^* – coordinate defining the beginning and the end of a continuous noncavitated oil film,
ψ – angular coordinate $\psi = \xi/R$ [rad],
$\lambda_{11}, \lambda_{12}$ – stiffness coefficients of the external fixings of the bearing shells, horizontally and vertically [N/m],
ρ – oil density $[\mathrm{kg \cdot m^{-3}}]$,
τ – angle determining the position of the unbalance vector of the disk in relation to axis x [in degrees]; $\tau = \omega t$,
ω – angular velocity of the rotor [rad/s].

1. Preliminary remarks – the objective of the paper

Strongly nonlinear properties of the oil film in slide bearings cause that under certain conditions the entire rotor-bearings system generates some characteristic vibrations known as oil vibrations. In general, small oil vibrations also referred to as oil whirls or half-frequency vibrations and big oil vibrations usually defined as oil whip can be distinguished. The whirling oil film participates thus actively in the dynamics of the whole system causing the loss of its stability and further, already dangerous development of the vibrations.

The whirls and the whipping of oil have already found much space in the literature [1-9]. First systematic investigations on the nature of these phenomena were published in the paper by Hori, about 34 years ago [1]. The next papers [2-9], also including the work by Muszyńska [2] contribute much to this case by a number of new interesting aspects connected with the effect of oil vibrations upon the dynamics of the rotor-bearing system.

It must be admitted that the question of oil vibrations has been elaborated relatively well mainly owing to experimental investigations. Attempts made to prepare the theoretical description are based on extensive simplifications relating to the operation system concerning mainly the slide bearings. The operation of the bearing working under conditions of great vibrations of the journal, that is in the nonlinear range, is not easily described. Some methods that have been known so far, such as the ones by Butenschön [10], Booker [11], Blok [12] or others, are based on the principle of superposition of the pressure distribution, or resultant forces originating from the "associated" velocities of the rotating film and the "squeezing" of the oil film. This type of superposition results in serious errors due to the necessity of using different boundary conditions for the film being "squeezed" and for the rotating film.

In the theoretical investigations related to oil whirls and oil whip, the effect of thermal phenomena occurring in the lubrication film is, as a rule, omitted. The objective of Part I of the paper is to present a new theoretical model deprived of the above mentioned simplifications, and also to provide a calculation algorithm and a computer program that will enable one to carry out a complete computer simulation of the oil vibrations taking place in the rotor-bearing system. By the application of such an advanced tool to the simulation tests an attempt will be made in Part II to find the moment of generating the oil whirls and to follow their further development. Using the amplitudes and phase angles analyses, and an analysis of the vibration spectra we will obtain some information and get more familiarised with the nature of these fascinating phenomena.

The reader will find some useful information on taking advantage of the model and the computer program in simulation diagnostics of rotors with slide bearings (on the example of a real turbine model of 200 MW).

2. The theoretical model

Oil vibrations are self-excited vibrations resulting from evidently non-linear characteristics of the slide bearing. Their occurrence indicates the loss of stability of the system and signals that some greater vibrations may take place. The development of these vibrations can therefore be simulated exlusively on the basis of a non-linear description, that is a non-linear equation of motion of the slide bearings and the whole system. The non-linear analysis creates new qualitative possibilities for the simulation investigation of rotors. It makes provision for generating the vibration spectra (the base of the vibration diagnostics), which can not be obtained even by using the most complex linear description.

Let us now present the adopted non-linear model of the rotor-bearing system beginning with the major, and at the same time, the most difficult sub-system to be theoretically modelled, namely the slide bearings.

Using the system of coordinates and the denotations as in Figs. 1 and 2 the hydrodynamic pressure distribution in the lubricating film can be described by the Reynolds equation in the form of:

$$\frac{\partial}{\partial\xi}\left[h^3\frac{\partial p}{\partial\xi}\right]+\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right]=6R\omega\overline{\mu}\frac{\partial h}{\partial\xi}-12\overline{\mu}\frac{\partial e}{\partial t}\cos(\psi-\gamma)-12\overline{\mu}e\frac{\partial\gamma}{\partial t}\sin(\psi-\gamma). \quad (1)$$

The equation (1) is integrated within boundaries determined by the zero pressure gradient condition along the continuity zone and the zero value of pressure on the bearing edges, that is:

$$\frac{\partial p}{\partial\xi}\Big|_{\xi=\xi^*_{1,2}}=0, \qquad p=0\big|_{z=\pm L/2}\,. \quad (2)$$

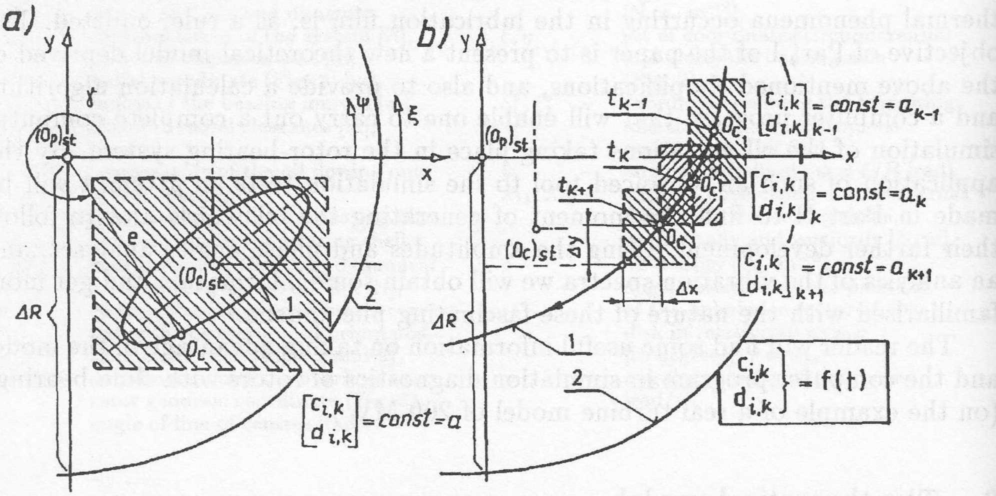

Fig. 1. Significant difference between the linear a) and nonlinear b) analysis of slide bearings using the stiffness coefficients $c_{i,k}$ and the damping coefficients $d_{i,k}$ of the lubricating film; 1 – trajectory of the journal centre during dynamic loading, 2 – circle of clearance, 3 – semi-circle of static equlibrium.

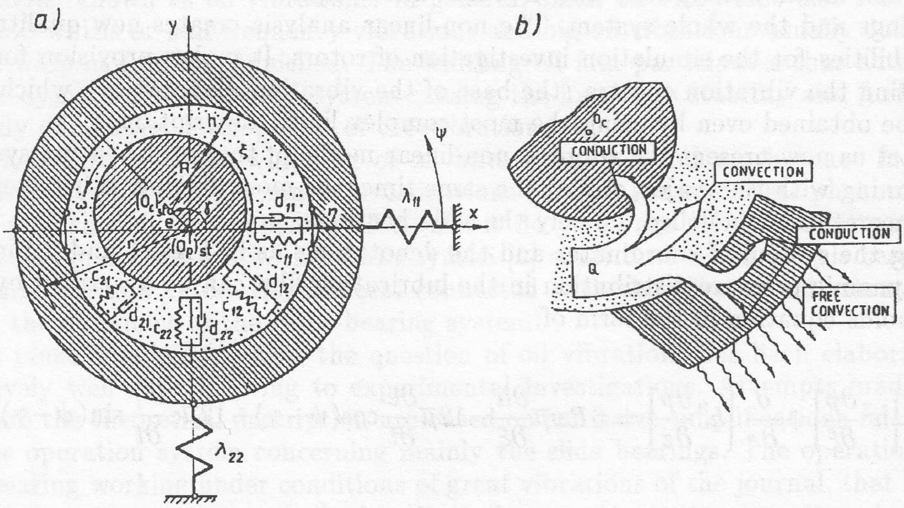

Fig. 2. Diagram of elasto-damping characteristics of the oil film a) and the model of heat exchange in the bearing b).

where ξ_1^*, ξ_2^* are coordinates of the position of the continuous lubricating film boundaries. These boundaries are variable in time, i.e. $\xi_1^*(t)$ and $\xi_2^*(t)$. A physical explanation of the condition (2) is given in Fig. 3. The determination of the varia-

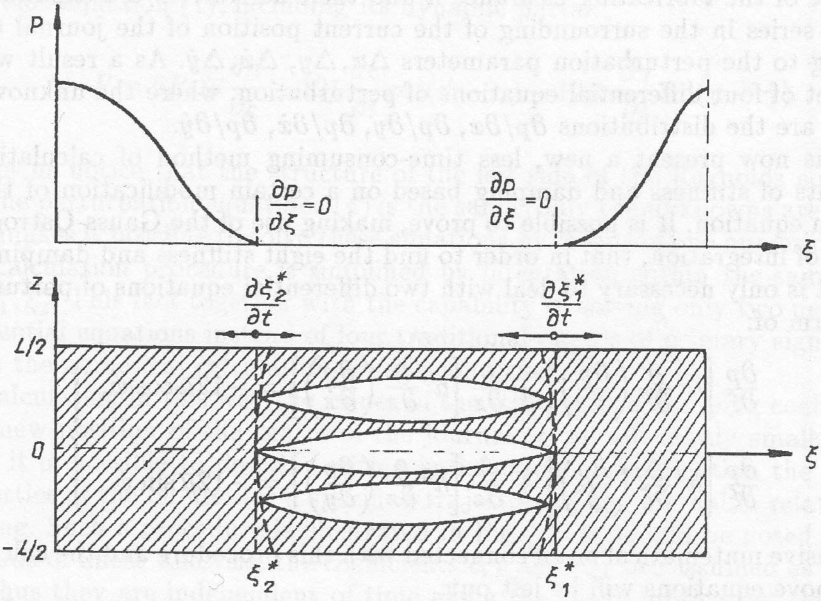

Fig. 3. Physical interpretation of the boundaries ξ_1^*, ξ_2^* of the continuous and cavitated oil film obtained from condition (2) related to dynamic loadings of the bearing; 1 – continuous oil film, 2 – cavitation bubbles.

tion of functions ξ_1^* and ξ_2^* is a different, very complicated problem. In a general case these functions should be determined on the basis of the equation of flow continuity in the cavitation zone, treating time t as a normal independent variable (the effect of the so-called prehistory). It is possible to show ([13-14]) that in the case of abundant bearing lubrication (which often occurs) the determination of functions ξ_1^*, ξ_2^* by means of the condition (2) is sufficient.

The Reynolds equation (1) together with the conditions (2) defines the hydrodynamic pressure distributions, and thus the time-variable functions of the hydrodynamic load capacity components W_x, W_y of the bearing. Further considerations will also require data concerning variations in time of elastic and damping properties of the lubricating film. Such properties can be determined by applying four stiffness and four damping coefficients of the oil film defined in the following way (Fig. 2):

$$\left.\begin{aligned} c_{11} &= \frac{\partial W_x}{\partial x}; \quad c_{12} = \frac{\partial W_x}{\partial y}; \quad c_{21} = \frac{\partial W_y}{\partial x}; \quad c_{22} = \frac{\partial W_y}{\partial y}; \\ d_{11} &= \frac{\partial W_x}{\partial \dot{x}}; \quad d_{12} = \frac{\partial W_x}{\partial \dot{y}}; \quad d_{21} = \frac{\partial W_y}{\partial \dot{x}}; \quad d_{22} = \frac{\partial W_y}{\partial \dot{y}}; \end{aligned}\right\} \tag{3}$$

where $\dot{} = \partial/\partial t$.

Coefficients c_{ik}, d_{ik} similarly to the pressure distribution in Eq. (1) vary in time in a significantly non-linear way, that is $c_{ik}(t)$ and $d_{ik}(t)$. To calculate them advantage is taken of the perturbation calculus. The distribution of pressure p, the shape of the lubricating clearance h and their derivatives are expanded into a Taylor series in the surrounding of the current position of the journal O_c and according to the perturbation parameters $\Delta x, \Delta y, \Delta\dot{x}, \Delta\dot{y}$. As a result we shall have a set of four differential equations of perturbation, where the unknown magnitudes are the distributions $\partial p/\partial x, \partial p/\partial y, \partial p/\partial\dot{x}, \partial p/\partial\dot{y}$.

Let us now present a new, less time-consuming method of calculating the coefficients of stiffness and damping based on a certain modification of the perturbation equation. It is possible to prove, making use of the Gauss-Ostrogradski theorem of integration, that in order to find the eight stiffness and damping coefficients it is only necessary to deal with two differential equations of perturbation in the form of:

$$\left.\begin{aligned}
\frac{\partial p}{\partial\xi}\left[h^3\frac{\partial}{\partial\xi}\left(\frac{\partial p}{\dot{x}}\right)\right]+\frac{\partial}{\partial z}\left[h^3\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial\dot{x}}\right)\right]&=-12\overline{\mu}\cos\psi,\\
\frac{\partial p}{\partial\xi}\left[h^3\frac{\partial}{\partial\xi}\left(\frac{\partial p}{\dot{y}}\right)\right]+\frac{\partial}{\partial z}\left[h^3\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial\dot{y}}\right)\right]&=-12\overline{\mu}\sin\psi.
\end{aligned}\right\}\tag{4}$$

An extensive matematical proof connected with this procedure and the calculation of the above equations will be left out.

Eqs. (4) are integrated exactly within the same area as the Reynolds equation (1), i.e. within the boundaries ξ_1^* and ξ_2^*. Appriopriate integrals of distribution $\partial p/\partial\dot{x}$, $\partial p/\partial\dot{y}$ define the searched coefficients of stiffness and damping of the lubricating film:

$$\left.\begin{aligned}
c_{11}&=-\omega\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{x}}R_x^*\,dz\,d\xi, & c_{21}&=-\omega\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{y}}R_x^*\,dz\,d\xi,\\
c_{12}&=-\omega\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{x}}R_y^*\,dz\,d\xi, & c_{22}&=-\omega\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{y}}R_y^*\,dz\,d\xi,\\
d_{11}&=-\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{x}}\cos\psi\,dz\,d\xi, & d_{11}&=-\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{x}}\sin\psi\,dz\,d\xi,\\
d_{12}&=d_{21}, & d_{22}&=-\int_{-L/2}^{+L/2}\int_{\xi_1^*}^{\xi_2^*}\frac{\partial p}{\partial\dot{y}}\sin\psi\,dz\,d\xi.
\end{aligned}\right\}\tag{5}$$

However,

$$R_x^*=-\frac{1}{2}\sin\psi+\frac{1}{4R\overline{\mu}\omega}\left\{h\frac{\partial p}{\partial\xi}\left[h\sin\psi+\frac{\partial h}{\partial\xi}R\cos\psi\right]-\frac{P}{h}R\cos\psi\right\},$$

$$R_y^* = -\frac{1}{2}\cos\psi + \frac{1}{4R\overline{\mu}\omega}\left\{h\frac{\partial p}{\partial \xi}\left[h\cos\psi + \frac{\partial h}{\partial \xi}R\sin\psi\right] - \frac{P}{h}R\sin\psi\right\}.$$

Expression P denotes in the above relationships the whole right-hand side of the Reynolds equation (1), including its dynamic parts:

$$P = 6R\omega\overline{\mu}\frac{\partial h}{\partial \xi} - 12\overline{\mu}\frac{\partial e}{\partial t}\cos(\psi - \gamma) - 12\overline{\mu}e\frac{\partial \gamma}{\partial t}\sin(\psi - \gamma).$$

Let us notice that the structure of the left side of the Reynolds equation (1) and the perturbation equations (4) is the same (only the unknowns are different). This makes it possible to solve these equations by means of one numerical scheme. The calculation procedure is simplified by integration within the same boundaries ξ_1^*, ξ_2^*. This fact together with the capability of solving only two perturbation differential equations instead of four traditional ones is of primary significance as far as the numerical calculation time is concerned.

Calculating in this way the values of the stiffness and damping coefficients for each new position of the centre of the journal O_c at sufficiently small time intervals, it is possible to describe with a satisfactory approximation the non-linear properties of the oil film for a specified trajectory of the journal in relation to the bushing. Such a procedure is illustrated in Fig. 1b. It should be noted that in all methods of linear analysis, the coefficients $c_{i,k}$ and $d_{i,k}$ are assumed as constants, and thus they are independent of time along the entire lubricating clearance, or its great part. Their values are calculated with respect to a specified kinetostatic equilibrium position of the journal $(O_c)_{st}$ - Fig. 1a.

Let us now proceed to the problems connected with the assessment of the heat exchange effect in bearings. The form of the formulae (1) and (4) indicates that in the case of dynamic loading a constant dynamic viscosity $\overline{\mu}$ of the oil appropriate for the thermal conditions corresponding to the kinetostatic equilibrium position of the rotor has been assumed. The determination of the mean value of the "effective" viscosity $\overline{\mu}$ presents a different problem. For a given geometry and static loading of the system, the rotational speed of the rotor n, and for a specific type of oil and known supply temperature T_0, the viscosity $\overline{\mu}$ will depend on complex conditions of the bearings heat exchange and the equilibrium position of the journals $(O_c)_{st}$. As an assumption, the viscosity $\overline{\mu}$ will be determined by the mean temperature T_w of the outlet oil understood as:

$$T_w = \frac{1}{2\pi hR}\int_0^h\int_0^{2\pi R} T(\xi,\eta)\,d\eta\,d\xi. \tag{6}$$

Now it is possible to determine the viscosity by making use of the known oil characteristics (viscosity-temperature) or according to various theoretical relations, for example

$$\overline{\mu} = \overline{\mu}_0\exp(-b(T_w - T_0)).$$

Temperature distribution $T(\xi,\eta)$ in the lubrication film will be determined by solving three mutually coupled equations:
– the "three-dimensional" Reynolds equation

$$\frac{\partial}{\partial\xi}\left[\frac{\partial p}{\partial\xi}A\right]+\frac{\partial}{\partial z}\left[\frac{\partial p}{\partial z}A\right]=-\omega R\left[\frac{\partial h}{\partial\xi}-\frac{\partial B}{\partial\xi}\right], \tag{7}$$

where:

$$A=\int\limits_0^h\left[\int\limits_0^P\eta\frac{\eta}{\mu}\,d\eta-\frac{\int\limits_0^h\frac{\eta}{\mu}\,d\eta}{\int\limits_0^h\frac{1}{\mu}\,d\eta}\int\limits_0^\eta\frac{1}{\mu}\,d\eta\right]d\eta \qquad B=\int_0^h\left[\frac{\int\limits_0^\eta\frac{1}{\mu}\,d\eta}{\int\limits_0^h\frac{1}{\mu}\,d\eta}\right]d\eta,$$

– the energy equation defining the searched temperature distribution $T(\xi,\eta)$

$$\rho c u_m\frac{\partial T}{\partial\xi}-k_{ol}\frac{\partial^2 T}{\partial\eta^2}=\mu\left[\left(\frac{\partial u}{\partial\eta}\right)_m^2+\left(\frac{\partial w}{\partial\eta}\right)_m^2\right], \tag{8}$$

where u_m denotes the mean velocity of the oil stream in the circumferential direction, while $(\partial u\ \partial\eta)_m,(\partial w/\partial\eta)_m$ are the mean derivatives of the velocity components with respect to η
– the conductivity equation (for the bearing bush)

$$\frac{\partial^2 T_p}{\partial r^2}+\frac{1}{r}\frac{\partial T_p}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T_p}{\partial\psi^2}=0. \tag{9}$$

The boundary conditions for the energy equation (8) include the temperature distribution on the internal surface of the bearing shell (calculated from the conductivity equation (9)) and the given mean temperature of the journal T_c assumed as the mean value of the temperature distribution on the internal surface of the bearing shell. It is assumed that the heat from the external surface of the shell is carried away by free convection. A thermal model of the bearing is given in Fig. 2b.

3. Algorithm of calculation

Eqs. (7), (8) and (9) enable us to determine the effective hydrodynamic load capacity for each bearing node for the assumed position of the journals in the bearings $(\xi_0)_j,(\gamma_0)_j$. The opposite procedure, where attempt is made to find positions $(\xi_0)_j,(\gamma_0)_j$ for some given external loads (reaction of the system supports), requires the application of the iterative procedure, in which Eqs. (7), (8), (9) are additionally coupled with the formula for kinetostatic deformation of the rotor and the supports in a general form:

$$\mathbf{Kq}=\mathbf{P}_0. \tag{10}$$

In consequence it is possible to find the equilibrium positions $(\xi_0)_j, (\gamma_0)_j$ of the journals, the reaction of the supports and the rotor kinetostatic deformation line. The static characteristics of the bearings, including also the mean viscosity $\overline{\mu}$ are calculated in the equilibrium position.

Now it is time to return to the dynamics problems. The presented way of determining the elasto-damping properties of the lubricating film at large displacements of the journal (time-variable coefficients $c_{i,k}.d_{i,k}$) enables us to easily analyze the dynamic properties of the entire rotor-bearing system on the basis of the finite elements method. The equations of motion of the system will take the non-linear form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}}, t)\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q}, \dot{\mathbf{q}}, t)\mathbf{q} = \mathbf{P}(t). \tag{11}$$

At each time step t_k, the stiffness and damping coefficients $(c_{i,k})_j, (d_{i,k})_j$ derived from Eqn.(4) and the relationship (5) modify the system global matrices of stiffness $\mathbf{K}$ and damping $\mathbf{D}$. Matrices $\mathbf{D}$ and $\mathbf{K}$ in Eq. (11) are, therefore not constant, but variable in time in a definitely non-linear manner. The equation (11) is integrated numerically using the Newmark method.

The above calculation algorithm is presented in Fig. 4, where the kinetostatic part with the main iterative loop I_k is clearly distinguished from the dynamic part with the loop illustrating the stepwise proceeding I_d. With arbitrarily variable but periodical loadings $\mathbf{P}(t)$, the above algorithm provides, after numerous repetitions, convergent results. On the basis of this algorithm a computer code referred to as NLDW-01 was constructed. The results presented in Part II of the paper we obtained with the help of this code.

The matrices used in equations (10) and (11) contain elements related both to the rotor itself and to the bearings and more extensively understood supports (external fixings of bearing bush, foundation) of substitute stiffness and damping $\lambda_{i,k}, \Lambda_{i,k}$ – as seen in Fig. 5a. Making use of the finite elements method it is possible to model the rotor by the beam finite elements discretization, while the reaction of the bearings (coefficients $c_{i,k}, d_{i,k}$) and the supports (coefficients) $\lambda_{i,k}, \Lambda_{i,k}$ can be modelled using the point finite elements. The reaction of the disks is modelled by means of stiff finite elements of known moments of inertia M_B and gyro moments M_g – Fig. 5b.

The presented method of analyzing the non-linear problems in the form of the mutually connected differential equations (7), (8), (9) and (10) applied to kinetostatics of the system, and Eqs. (1), (4), (5) and (11) for its dynamics, gives an idea of how time-consuming the numerical calculations are. To obtain an appropriate accuracy and convergence of calculations, Eqs. (1), (4), (5) and (11) must be calculated hundreds of times to find only one variation period of the exciting force. Taking into consideration that the calculations should be carried on with respect to at least some scores of periods, and that the system may have several supports, the set of equations is calculated thousands of times to obtain a single datum relating to, for instance, one speed of the rotor n. The most time-consuming calculations are the ones that refer to stiffness and damping coefficients of the oil film (in spite of applying a modified perturbation method).

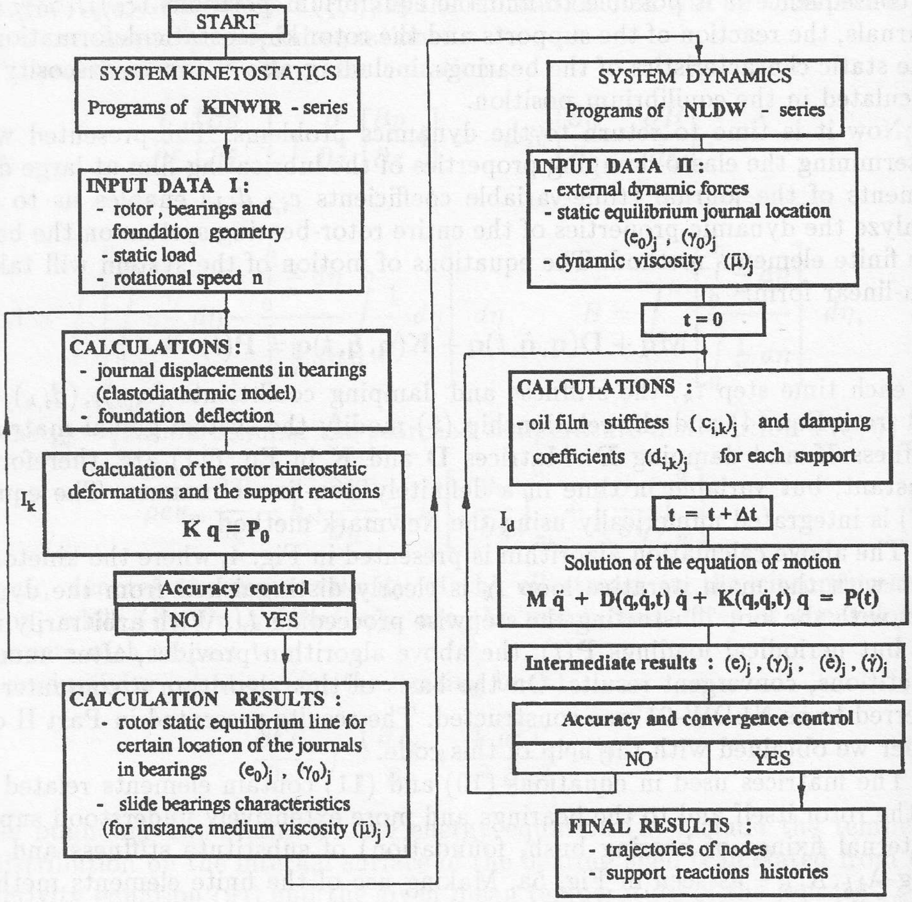

Fig. 4. Calculation algorithm of program NLDW-01. I_k – the main "kinetostatic" iterative loop, I_d - the main "dynamic" loop responsible for the stepwise proceeding.

That might be the reason why in the literature it is hard to find solutions based on the "method of coefficients" entirely consistent with the physics of the phenomenon. The Reynolds equation is usually solved by supperposition of pressure or hydrodynamic forces.

4. Experimental verification

A broad experimental verification of the model, with respect to the kinetostatic characteristics of the sliding bearings is presented in the paper [15], while with reference to the rotor itself with the disks, in the paper [16]. To verify the complex nonlinear model, that is the combined model of bearings, supports (foundation)

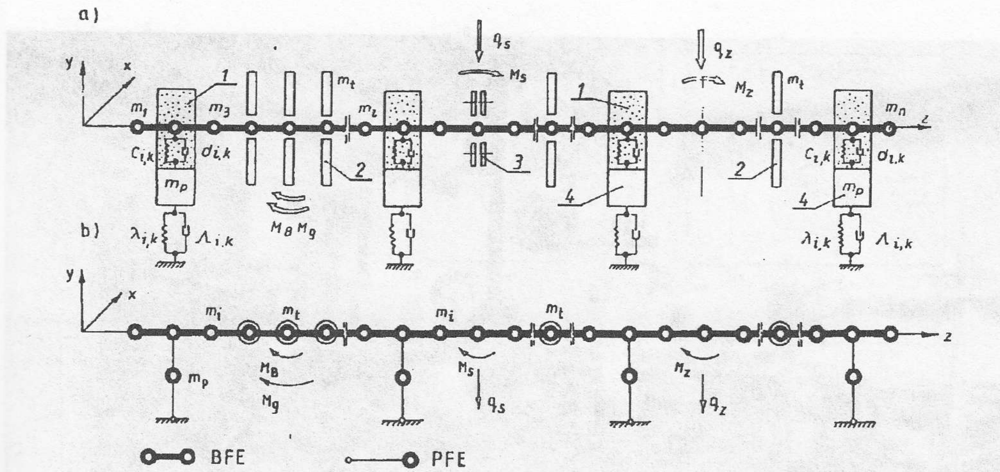

Fig. 5. Model of the rotor-bearings-supports system a) and its discretization by means of finite elements b); BFE – finite elements of the beam, PFE – point finite elements
1 – oil film, 2 – disks(plate), 3 – clutch plate, 4 – bushes of the slide bearings, m_i – mass of the i-th element of the rotor, m_t – mass of the disk(plate, m_p) – mass of the bearing bush, M_B, M_g – moments of inertia and disk gyro, M_s, q_s – moments and forces due to the interaction of the coupling system, M_z, q_z – arbitrary external moments and forces under the system loading $\lambda_{i,k}, \Lambda_{i,k}$ – coefficients of stiffness and damping of the bush external fixing.

and the rotor, analyzed in this paper, a special large-sized test rig as shown in Fig. 6 has been constructed . The test rig is also designed for the verification of the model in the zone of oil vibration occurrence, i.e. after exceeding the stability threshold of the system.

Some general parameters of the test rig are as follows: diameter of the shaft $D = 0.1$m, diameter of the disk $D_t = 0.4$m, thickness of the disk $g = 0.2$m, spacing of the bearing pedestals $L_s = 1.4$m, two slide bearings of cylindrical clearance $\Delta R = 90 \times 10^{-6}$m and breadth $L = 0.05$m, range of rotational speed $n = 0 \div 6000$rpm. The parameters assumed for the tests: stiffness of fixing the bearing bush $\lambda_{11} = \lambda_{22} = 1.0 \times 10^7$ N/m, viscosity of the machine oil $\mu_0 = 0.08$ Pa-s at feeding temp. $T_0 = 30^o$C, radius of disk unbalance $\hat{r} = 50 \times 10^{-6}$m. For the measurement of the displacements of the bearing journals and elements of the rotor, vortex sensors of Bently-Nevada type were used. Advantage was also taken of entirely numerical technique in processing the measuring signals, including the computer method of filtering the disturbances originating from material heterogeneity (known as "runout").

Extensive experimental tests of forced vibrations caused by unbalanced rotating disk have been carried out for symmetric configuration of the rotor (a simple disk in the centre, two identical slide bearings) and various magnitudes of clearance ΔR and the stiffness of fixing the external bearing shells $\lambda_{11}, \lambda_{22}$. On account of the dimensions of the test rig and safety measures it was impossible to carry on the experimental research in the case of large oil vibrations (oil "whip").

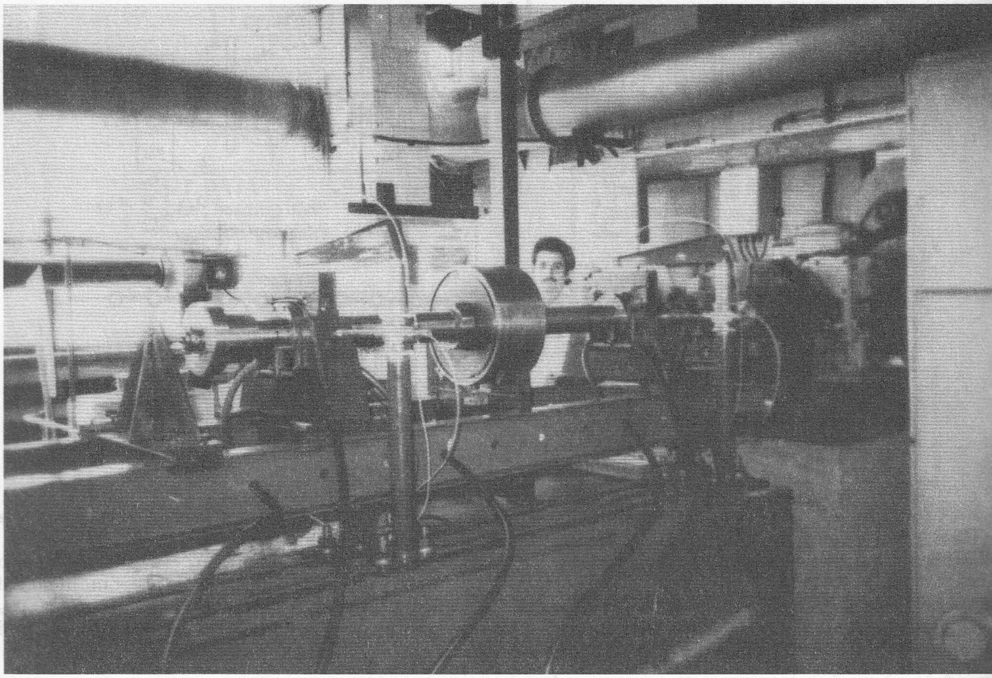

Fig. 6. Photograph of the experimental test rig.

The vibration amplitude of the entire system exceeded too soon the admissible level. Thus, the investigation was confined to the upper range of rotational velocities, that is to the sphere of small oil vibrations, or in other words, to low hydrodynamic instability.

Since the conclusions of the investigations carried out for various values of clearance ΔR and fixing stiffness $\lambda_{11}, \lambda_{22}$ are similar, we will restrict ourselves to the discussion of a few examples only of some trajectories picked out. The results presented in Fig. 7 concern relative vibrations of the journal and the bearing bush (vibrations of the oil film). The results in Fig. 8 are with respect to vibrations of the disk centre. It is evident that the simulated development of the oil whirls (characteristic "fission" of trajectory and formation of "half-loop") finds experimental confirmation. Also a sudden change of the phase angles (location of angles $\tau = 0^o, 720^o$, and $\tau = 360^o$ – Figs. 7b,c, and Figs. 8b,c) predicted by the theory is experimentally proved (Figs.7e,f, and Figs. 8e,f).

The basic difficulty lay in the theoretical modelling of the real shape of the lubrication gap of the slide bearings as technological errors of shape of the order of 0.01 mm are usually unavoidable. It was also hard to foresee the thermal deformations of the gap, in particular at high rotational velocities. For this reason some gap shape modifications of the order of $10 \div 30\%$ with respect to the con-

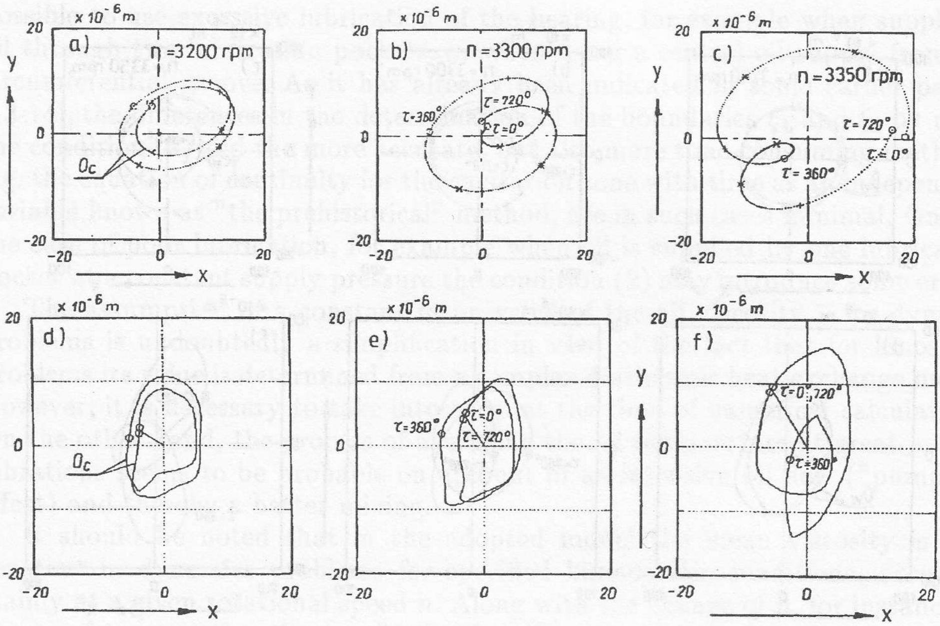

Fig. 7. Trajectory of the journal in relation to the bearing shell within the area of the occurence of small oil vibrations (oil whirls) at assumed radius of disk unbalance $\hat{r} = 50 \times 10^{-6}$ m; a, b, c – computer simulation (program NLDW-01), d, e, f – experimental measurements (after computer "elimination" of the effect of material heterogeneity).

structional shape were possible, which could change in a significant the value of the stiffness and damping coefficients. This fact can account for the noticeable differences in shape of the very trajectories, though the conformity of the calculated amplitude value and the measured one can be regarded as satisfactory.

On the basis of the verification carried out it seems reasonable to state that the elaborated theoretical model and the computer program are a reliable tool for further simulation research.

5. Analysis of the model

The significance of the presented method of the non-linear description of the rotor-bearing system lies in the way of defining the elasto-damping properties of the oil film. The properties have been defined on the basis of time-variable coefficients of stiffness and damping of the oil film obtained by integration of the Reynolds equation (1) and perturbation equation (4) with boundaries ξ_1^*, ξ_2^* with a simultaneous consideration of all the elements of the right-side of the Reynolds equation. These boundaries determine, for each instantaneous position of the

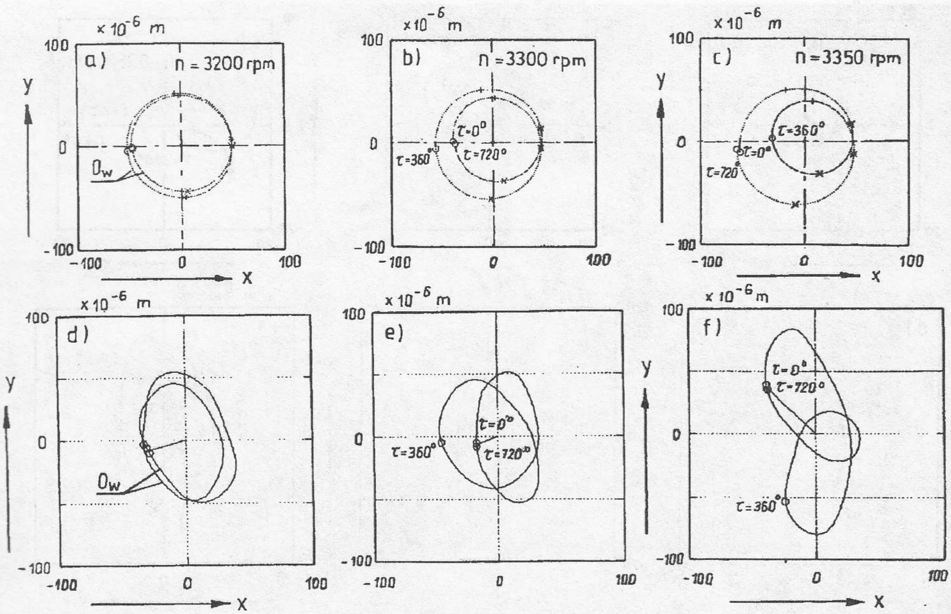

Fig. 8. Trajectory of the disk (centre of the rotor) in the area of the occurence of small oil vibrations (oil whirls) at assumed radius of unbalance $\hat{r} = 50 \times 10^{-6}$ m;
a, b, c – computer simulation (program NLDW-01), d, e, f – experimental measurements (after computer "elimination" of the effect of material heterogeneity).

journal in the bearing, the area where the lubricating gap may be totally filled up with oil. Only in such an area the hydrodynamic pressure, responsible for the properties of the lubricating film, will be generated. This proceeding is therefore quite consistent with the physics of the phenomena occurring in the lubricating gap. Integration within the same boundaries determining a continuous lubricating film is of particular importance in the case of great vibrations in the system, that is in the situation of the oil vibration simulation. The principle of superposition, so commonly accepted in the slide bearings dynamics should therefore not be applied to this type of cases.

The theoretical model presented in the paper is based, among others, on two important assumptions which can be a matter of discussion, namely:

- zone boundaries of a continuous and cavitated oil film are determined by the condition of zero pressure gradient (2),
- the oil dynamic viscosity, in the case of great vibrations of the system, can be averaged to the value $\overline{\mu}$ corresponding to the outlet oil temperature T_w under the condition of kinetostatic equilibrium of the system.

The zero-pressure gradients condition (2) can be applied to determine the boundaries of a continuous lubricating film ξ_1 and ξ_2 in all cases where it is possible to use excessive lubrication of the bearing, for example when supplying oil through two lubrication pockets, or when using a central oil supply from the circumferential groove. As it has already been indicated in some earlier papers [13-14], the differences in the determination of the boundaries ξ_1 and ξ_2 by using the condition (2) and the more accurate, but also more time-consuming methods, e.g. the equation of continuity for the cavitation zone with time as an independent variable known as "the prehistorical" method, are in such cases minimal. Only in the case of poor lubrication, for example when oil is supplied by one lubricating pocket at a constant supply pressure the condition (2) may introduce some errors.

The assumption of a constant mean value of the oil viscosity $\overline{\mu}$ for dynamic problems is undoubtedly a simplification in view of the fact that for kinostatic problems its value is determined from a complex diathermic heat exchange model. However, it is necessary to take into account the time of numerical calculations. On the other hand, the process of averaging the oil temperature at great journal vibrations seems to be probable on account of an intensive oil flow ("pumping" effect) and thereby a better mixing.

It should be noted that in the adopted model the mean viscosity is only constant in dynamics problems for specified kinetostatic conditions, occurring mainly at a given rotational speed n. Along with the change of n, for instance, in course of constructing the amplitude-phase characteristics of the system, or the simulation of whirls developing with the rising rotational speed of the rotor, the value $\overline{\mu}$ is certainly variable. Values of $\overline{\mu}$ at rotor speeds in the range of hundreds of revolutions per minute in comparison with speeds of rotor reaching thousands of revolutions per minute, differ by an order of magnitude. Such high differences of oil viscosity have a significant effect upon the elasto-damping properties of the oil film and thereby upon the operation of the whole system. The results of the experimental verification given in the previous chapter seem to confirm the correctness of the assumptions.

To sum up the discussion it may be concluded that the model presented in the paper involves three basic novel elements related principally to the description of the operation of the slide bearings, namely:

I – the elasto-damping properties of the oil film are determined by a system of mutually interrelated differential perturbation equations (4) and the Reynolds equation (1) integrated within physically justified boundaries ξ_1^*, ξ_2^* with a simultaneous consideration of all terms, including the dynamic ones of the right-hand side of the Reynolds equation (1),

II – in question of dynamics the model takes into consideration the oil viscosity variations $\overline{\mu}$ caused by the change of the mean temperature T_w of the outlet oil depending on the kinetostatic conditions of the system operation,

III – the applied modified perturbation method (relationships (5)) enables us to reduce the number the perturbation equations from four down to two indispensable to be solved.

All the other sub-systems relating to the rotor itself, the foundation, external fixings of the bushing, etc. do not include any elements of novelty. Their analysis of kinetostatic (10) or dynamic properties (11) is based on complex, but known equations derived from the finite elements method. For this reason they have not received a wider presentation.

On account of a new, more approaching the reality, description of the slide bearings operation the presented model can, of course, not only be utilized in computer simulation of oil vibrations, but also in simulating various dynamic reactions occurring within the rotor-bearing-supports system. Applying at any point the non-linear variable forces and moments resulting from the interaction of clutches, various external forces, or unbalanced rotating disks (Fig. 4), it is possible to obtain some valuable data relevant to the behaviour of the entire system, and thereby to construct catalogues of diagnostic relations, for example on the basis of the vibration spectra.

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Nieliniowy model drgań w układzie wirnik-łożyska
Cz. I Algorytm obliczeń

Streszczenie

W pracy przedstawiono model teoretyczny umożliwiający nieliniowy opis drgań olejowych, występujących w układzie wirnik-łożyska-podpory po przekroczeniu granicy stabilności.

Zaoprezentowana została nowa metoda określania spężysto-tłumiących własności filmu olejowego, algorytm obliczeń i program komputerowy z wyraźnym rozdzieleniem części kinestatycznej i dynamicznej oraz przykłady weryfikacji eksperymentalnej modelu na specjalnym wielkogabarytowym stoisku doświadczalnym.