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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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MARIAN TRELA[1], IRENEUSZ KORNECKI[1]

The flow of gravity-driven rivulets in the entrance region[2]

The formation and development of flow of gravity-driven liquid rivulets down a vertical wall is analysed in the paper. The shape of the rivulets changes due to the interaction of gravitational, viscous and surface tension forces. A simple model based on the momentum balance is put forward showing reasonable agreement with available experimental data.

Nomenclature

A	–	function given by Eq. (10),	x, y, z	–	rectangular coordinates,
B	–	half width of the rivulet $B = \pi \cdot r$,	x^+	–	dimensionless coordinate, $x^+ = x/r$,
C_0	–	Coriolis constant, $C_0 = 1.2$,	y^+	–	dimensionless coordinate, $y^+ = x/r$,
CS	–	control surface,	α	–	angle defined in Fig. 2,
CV	–	control volume,	δ	–	film thickness,
F	–	force,	Γ	–	unit mass flow rate, $\Gamma = \rho \cdot q$,
g	–	acceleration of gravity,	μ	–	dynamic viscosity,
q	–	unit volumetric flow rate, $g = \overline{v} \cdot \delta$,	ν	–	kinematic viscosity, $\nu = \mu/\rho$,
r	–	tube radius,	Θ	–	contact angle,
s	–	length of the rivulet – arc length,	ρ	–	liquid density,
t	–	temperature,	σ	–	surface tension.
v	–	velocity,			

Subscripts

0	–	initial value (for $x = 0$),	$+$	–	dimensionless,
x	–	component in x direction,	n	–	normal,
—	–	average,	s	–	tangential.

1. Introduction

The flow of liquid in the form of rivulets along the walls of solid bodies is a common phenomenon in a variety of technical devices like evaporators, condensers, spray-cooled heat exchangers, cooling towers and many others. However,

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knowledge of mechanisms governing this phenomenon is far from complete. Design engineers usually consider rivulets undesirable in terms of heat and mass exchange and make efforts to prevent their occurrence. The most favourable conditions are when the whole surface of a covered by a continuous liquid sheet called "liquid film". The rivulet flows usually form during failures or break-downs. Boiling crisis of the second degree (burnout) can occur in annular flow when the liquid film on the inside surface of the tube breaks down into rivulets leaving the surface partly dry. As a result the process of heat exchange becomes less efficient leading to burnout and break-down of the device, [1].

Among other interesting cases where natural formation of rivulets takes place are: flow of a cooling two-phase mixture of air and water droplets through a bundle of horizontal tubes [2] or vertical flow in tube bundle condensers. In general the intensity of heat exchange depends mainly on the area over which the rivulets remain in contact with the body, thus on the shape of the rivulets.

The present paper is concerned with the problem of formation of a gravity-driven rivulet flow on a vertical wall. This problem occurs in devices where very thin liquid films flow down the vertical walls of the tubes. When the flow rate decreases below a certain critical value, the liquid film loses its stability. In the aftermath of little disturbances the film breaks down into rivulets at the stagnation point, see Fig. 1. Newly formed rivulets quickly change their width (shape). The problem of evaluation of the shape of rivulets flowing on a horizontal cylindrical surface was analysed by first author of the present paper in [3, 4] whereas relevant experimental data can be found in [5]. Rivulet flow down the vertical walls and its formation due to break-down of a thin liquid film is a particularly interesting phenomenon and will be analysed here.

2. Theoretical analysis

A very thin layer of liquid flows freely down the wall (of a tube) driven by the gravity forces. Due to accidental small disturbances the stability of the flow is lost at a point S and the film breaks down into rivulets. The conditions referring to the film break-down were presented in detail in [3]. The rivulets are separated by dry patches in the form of parabola [6] and a collar-shaped thickening of the liquid layer, growing with the distance covered by the flow, occurs at the edges of the rivulets. In this chapter we are going to theoretically trace the change in shape of the rivulet due to the change of its width in the region of its formation beginning from the point S. For the sake of our analysis the following assumptions are made:

1. the surface of the liquid film and the rivulets is flat,
2. the existence of a collar-like fold formed by the liquid at the edge of the rivulet is neglected; apart from the boundary region the profile of the rivulet in the plane perpendicular to the flow is rectangular as in Fig. 2,

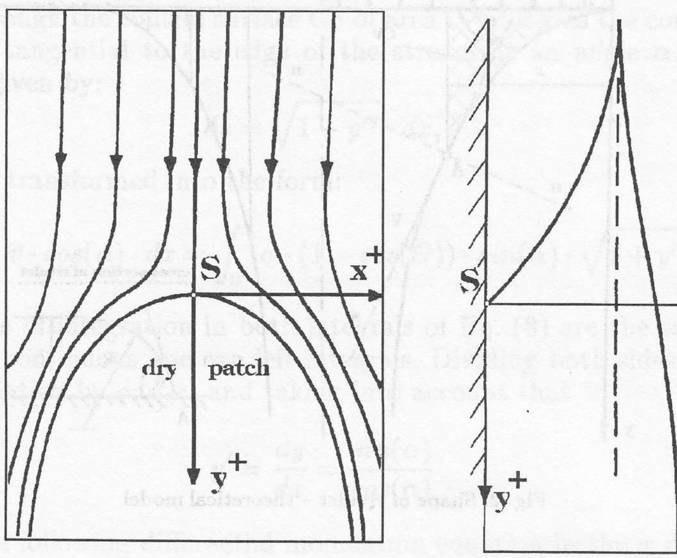

Fig. 1. Flow of liquid in the vicinity of the stagnation point S

3. the width of the boundary region is negligibly small compared with the width of the rivulet,
4. apart from the boundary region the streamlines of the rivulet are vertical, the effect of surface tension is essential in the boundary region, surface tension is responsible for streamline curvature in the boundary region and decreasing width of the rivulet,
5. the flow is laminar, the inertia forces due to change in rivulet thickness are ignored,
6. the liquid properties do not change.

Assumption 4 is based on experimental observations made in [7] of free falling liquid streams driven by the gravity forces. As the inertia forces are neglected, the equation of motion outside the boundary region takes the following form:

$$\frac{\mu}{\rho} \cdot \frac{d^2 v}{dz^2} + g = 0, \tag{1}$$

The boundary conditions are as follows:

$$\begin{aligned} z = 0, \quad & v = 0, \\ z = \delta, \quad & \tfrac{dv}{dz} = 0, \end{aligned} \tag{2}$$

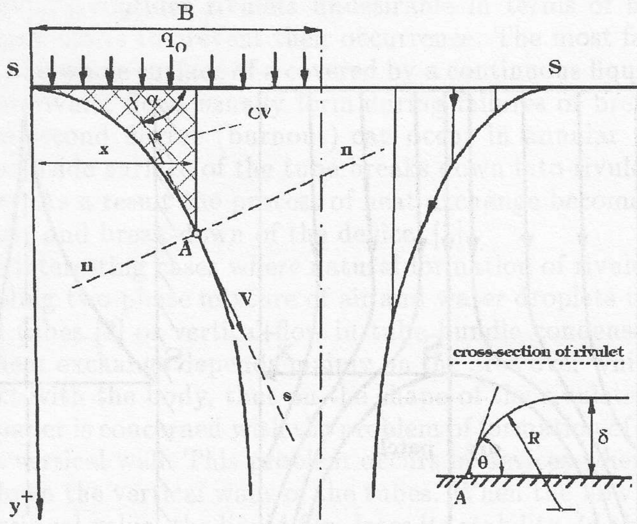

Fig. 2. Shape of rivulet – theoretical model

From Eq. (1) and boundary conditions (2) one can derive familiar relations describing the parabolic-type velocity profile in the liquid layer, the average velocity and the thickness of the layer:

$$v = \frac{\rho \cdot g}{2 \cdot \mu} \cdot (2 \cdot z \cdot \delta - z^2), \tag{3}$$

$$\overline{v} = \frac{\rho \cdot g}{3 \cdot \mu} \cdot \delta^2 = \left(\frac{\rho \cdot g}{3 \cdot \mu}\right)^{\frac{1}{3}} \cdot q^{\frac{2}{3}}, \tag{4}$$

$$\delta = \left(\frac{3 \cdot \mu q}{\rho \cdot g}\right)^{\frac{1}{3}} . \tag{5}$$

In order to determine the evolution of the rivulet width let us consider a control volume CV as in Fig. 2. In steady conditions the equation of motion in the x direction:

$$\int_{cs} v \cdot \rho \cdot v \cdot dA = \sum F_x \tag{6}$$

can be rewritten in the form:

$$\int_0^x C_0 \cdot \Gamma \cdot \overline{v} \cdot cos(\alpha) \cdot dx = \int_0^s \sigma \cdot (1 - cos(\Theta)) \cdot sin(\alpha) \cdot ds \tag{7}$$

due to the fact that the only unbalanced force acting on the stream is the surface tension force as the effect of gravity is neutralized by the friction forces. Constant

C_0 appears in the left side of Eq. (7), because the momentum flux through the control surface is expressed by the average velocity $\overline{v}$ instead of the local velocity distribution given by Eq. (3). It may be proved that for this case $C_0 = 1.2$ In accordance with the assumptions of the model, the mass flux into the control volume CV through the control surface CS of area $x \cdot \delta_0$, leaves the control volume in a direction tangential to the edge of the stream at an angle α. As the arc differential is given by:

$$ds = \sqrt{1 + y'^2} \cdot dx,$$

Eq. (7) can be transformed into the form:

$$\int_0^x C_0 \cdot \Gamma \cdot \overline{v} \cdot cos(\alpha) \cdot dx = \int_0^x \sigma \cdot (1 - cos(\Theta)) \cdot sin(\alpha) \cdot \sqrt{1 + y'^2} \cdot dx \qquad (8)$$

Since the limits of integration in both integrals of Eq. (8) are the same and the integrands are continuous one can left integrals. Dividing both sides of resulting differential equation by $cos(\alpha)$ and taking into account that

$$y' = \frac{dy}{dx} = \frac{sin(\alpha)}{cos(\alpha)}$$

one can get the following differential momentum equation in the x direction:

$$C_0 \cdot \rho \cdot q \cdot \overline{v} = \sigma \cdot (1 - cos(\Theta)) \cdot y' \sqrt{1 + y'^2}, \qquad (9a)$$

or

$$C_0 \cdot \rho \cdot \left(\frac{\rho \cdot g}{3 \cdot \mu}\right)^{\frac{1}{3}} \cdot q^{\frac{5}{3}} = \sigma \cdot (1 - cos(\Theta)) \cdot y' \cdot \sqrt{1 + y'^2}. \qquad (9)$$

For further analysis it is convenient to rewrite Eq. (9) in the form:

$$y' \cdot \sqrt{1 + y'^2} = A \qquad (10)$$

where

$$A = \frac{C_0 \cdot \rho \cdot \left(\frac{\rho \cdot g}{3 \cdot \mu}\right)^{\frac{1}{3}} \cdot q^{\frac{5}{3}}}{\sigma \cdot \left(1 - cos(\Theta)\right)}, \qquad (11)$$

since unit volume flow rate q is a function of x, i. e. $q = q(x)$. Taking squares of both sides of Eq. (10) one can obtain the equation:

$$y'^4 + y'^2 - A^2 = 0 \qquad (12)$$

whose physical solution is:

$$y' = \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A^2} - 1}. \qquad (13)$$

The symmetry of the flow in the region of the stagnation point S requires an initial condition:

$$x = 0, \qquad y = 0, \qquad y' = y_o' = 0. \tag{14}$$

It follows from Eq. (13) that the function y' does not vanish at $x = 0$ and equals:

$$y_o' = \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A_0^2} - 1}. \tag{15}$$

Bearing in mind Eq. (11), we have that for $x = 0$,

$$A_0 = \frac{C_0 \cdot \rho \cdot \left(\frac{\rho \cdot g}{3 \cdot \mu}\right)^{\frac{1}{3}} \cdot q_0^{\frac{5}{3}}}{\sigma \cdot (1 - cos(\Theta))}. \tag{16}$$

Thus, the differential equation of the rivulet shape satisfying the boundary condition imposed is as follows:

$$y' = \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A^2} - 1} - y_o' \tag{17a}$$

or

$$y' = \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A^2} - 1} - \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A_0^2} - 1}. \tag{17}$$

The shape of the rivulet is described by the equation:

$$y(x) = \frac{1}{\sqrt{2}} \cdot \int_0^x \sqrt{\sqrt{1 + 4 \cdot A^2} - 1} \cdot dx - \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A_0^2} - 1} \cdot x. \tag{18}$$

This equation assures that the function y as well as its derivative vanish at $x = 0$ which is in consistence with the mechanism of distribution of the liquid into rivulets in the vicinity of the stagnation point. A closing equation is the continuity equation which can be written as follows:

$$q = q_0 \cdot \frac{B}{B - x}. \tag{19}$$

Combining Eqs. (18) and (19) one can obtain the final equation to describe the shape of the rivulet in the region of its formation :

$$y(x) = \frac{1}{\sqrt{2}} \cdot \int_0^x \sqrt{\sqrt{1 + 4 \cdot A_0^2 \cdot \left(\frac{B}{B - x}\right)^{\frac{10}{3}}} - 1} \cdot dx - \frac{1}{\sqrt{2}} \cdot \sqrt{\sqrt{1 + 4 \cdot A_0^2} - 1} \cdot x. \tag{20}$$

The contact angle Θ appears here as a parameter. In fact its value varies with properties of the three-phase medium of gas, solid state and liquid as well as with the flow conditions, see [3, 8]. It should also be expected that in real flows the contact angle can vary along the edge of the rivulet, that is $\Theta = \Theta(x)$.

A weak point of the model presented in this paper is lack of physical reality of the flow at the edge of the stream due to Eq. (7). This equation says that the mass of the liquid at the interval $0 \div x$ is concentrated at the edge of the stream whereas in real flows it is distributed in the collar of non-zero thickness. Therefore, it should be expected that the model suits better the case of relatively wide rivulets. Then, the area of the boundary region forms a minor part of the whole area taken by the rivulet. Let us note that this is the core of assumption 3.

3. Experimental verification

Well-documented experimental investigations of the shape of gravity-driven liquid streams over a horizontal cylindrical surface can be found in [5]. Unfortunately, validation of the present model on the basis of experimental data from this paper seems to be hardly possible due to the fact that the liquid supply was different there, namely the flow took place under conditions of varying tangential component of gravitational acceleration as well as the initial condition was different ($y' \neq 0$).

However, the paper of Ponter et al. [9] lends itself to the comparative analysis. The results of investigations of the minimum flow rate of a cooling liquid film (and also its minimum thickness) at which the film does not break up into rivulets during the mass exchange between the liquid film of a solution of ethanol and pure water and the flow of air and ethanol vapour for four different tube materials are presented in [9]. The flow of the gas phase was countercurrent to the freely flowing liquid film. The authors also recorded, by means of a photographic technique, the shape of the liquid film in several cases of interest. Applying these records to validate our model, one can come across certain difficulties due to lack of precise means of determination of the contact angle and surface tension. In [9], the contact angle was measured under steady conditions for a droplet remaining at rest.

As already mentioned, the contact angle depends on physical properties of the three-phase liquid-gas-solid system and the initial conditions of the flow. A priori determination of the contact angle is still obscure for the scientists. The surface tension varied in the course of investigations due to lack of thermodynamic equilibrium between the air-ethanal vapour mixture and the liquid film. The gas phase alone may be considered to have been in the state of equilibrium. However, absorption of the ethanol vapour took place and consequently the surface tension varied. The results presented in [9] concerning the minimum flow rate of the liquid film obtained at the beginning of the series and at the end of the series exhibit considerable discrepancies as high as $50 \div 100\%$, (depending on the surface material). These observations were explained in [9] as a consequence of changes in surface tension due to absorption and adsorption. Some photographs of the liquid streams enclosed in [9] were not provided with signatures showing exact times of their taking which makes the proper interpretation difficult. The properties of the liquid film and the gas mixture were taken from the tables. However, the surface tension depends not only on the contents of the mixture but also on the presence of pollutant particles which may lead to additional discrepancies in the results.

Calculated and experimental shapes of the liquid streams are compared in Figs. 3, 4, 5.

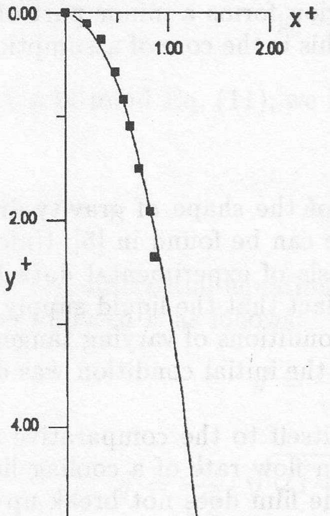

Fig. 3. Shape of liquid stream – calculations and experiment. Experimental conditions: $t = 20°C$, liquid film – pure water, gas – air, tube material – perspex, $r = 12.7\ 10^{-3}$ m, $\Theta = 6.0$ deg, $\Gamma = 0.18$ kg/m s, $\nu = 1.006\ 10^{-6}$ m^2/s, $\sigma = 72.7\ 10^{-3}$ N/m, $\rho = 998.2$ kg/m^3

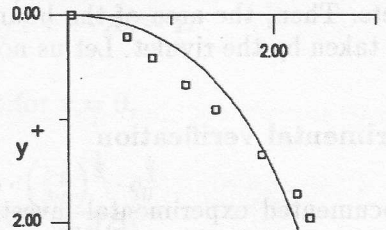

Fig. 4. Shape of liquid stream – calculations and experiment. Experimental conditions: $t = 20°C$, liquid film – 4.6% solution of ethanol in water, gas – equilibrium mixture of air and ethanol, tube material – perspex, $r = 12.7\ 10^{-3}$ m, $\Theta = 17.0$ deg, $\Gamma = 0.032$ kg/m s, $\nu = 1.24\ 10^{-6}$ m^2/s, $\sigma = 58.0\ 10^{-3}$ N/m, $\rho = 992.5$ kg/m^3

However, it should be stipulated that the measuring data were collected from low-quality photographs giving rise to a sort of uncertainty over the presented comparison. The figures are provided with notes about experimental conditions. The contact angle Θ in our model is assumed constant for the whole stream, its value calculated so as to reproduce best the experimental results. It seems that the calculated and experimental curves agree well.

4. Final conclusions

The problem of flow of liquid films down the vertical walls under the action of gravity forces has been considered in the paper. Main attention has been focused on the formation of rivulets after film breakdown. A simple model to describe the shape of the liquid stream has been put forward. The model takes into conside-

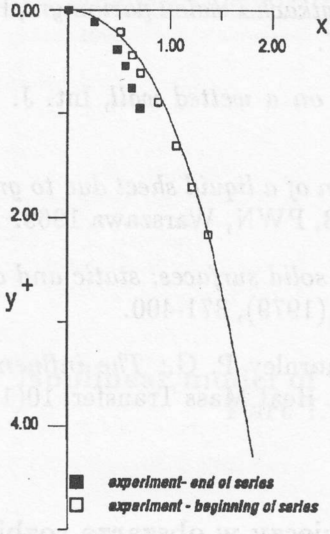

Fig. 5. Shape of liquid stream – calculations and experiment. Experimental conditions: $t = 20^oC$, liquid film – 4.6% solution of ethanol in water, gas – equilibrium mixture of air and ethanol, tube material – perspex, $r = 12.7\ 10^{-3}$ m, $\Theta = 5.5$ deg, $\Gamma = 0.043$ kg/m s, $\nu = 1.24\ 10^{-6}$ m^2/s, $\sigma = 58.0\ 10^{-3}$ N/m, $\rho = 992.5$ kg/m^3

ration the effects of surface tension on the shape of the streams but does not account for the existence in real flows of a collar-like boundary region. Therefore, the model suits better the case of relatively wide streams where the boundary region forms a minor part of the rivulet. However, further attempts to develop theoretical description of rivulet flows should allow for a more realistic representation of the boundary region with a view to modelling also relatively narrow rivulets. Another very important thing is access to reliable experimental data concerning change of shape of the rivulet in the region of its formation to enable the comparison with the theoretical models the simplified model without a collar and a more advanced one with a collar.

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Modelowanie ruchu grawitacyjnych strug cieczy w obszarze rozbiegu hydraulicznego

Streszczenie

W pracy rozpatrzono zagadnienie spływu grawitacyjnych strug cieczy po pionowej ścianie w obszarze rozbiegu hydraulicznego. Szczególną uwagę zwrócono na problem określenia zmiany szerokości strugi wzdłuż przepływu, czyli jej kształtu. Przedstawiono model teoretyczny tego zagadnienia a uzyskane wyniki numeryczne porównano z dostępnymi badaniami eksperymentalnymi.