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ZBIGNIEW BILICKI, ROMAN KWIDZIŃSKI¹Viscous term in the model of two-phase bubble flow²

The equations of the homogeneous two-phase flow model with a viscous term have been presented. The coefficient of substitutional viscosity has been introduced. Its value has been calculated from a comparison of the entropy production in a real bubble flow and in the continuous model of two-phase medium. In the case of bubble flow it follows that this coefficient should be several orders of magnitude higher than the molecular viscosity of the liquid.

Nomenclature

- deformation tensor,	x	- dryness fraction,
- unit mass force [N/kg],	z	- length coordinate [m],
- specific enthalpy [J/kg],	δ_h	- thickness of hydraulic boundary layer [m],
- mass [kg],	Φ	- dissipation function [W/kg],
- ratio of viscosity coefficients,	φ	- void fraction,
- pressure [Pa],	λ	- coefficient of thermal conductivity [W/mK],
- heat flux vector [W/m],	μ	- coefficient of dynamic viscosity [kg/ms],
- stress tensor,	$\tilde{\mu}$	- coefficient of substitutional viscosity in homogeneous two-phase model [kg/ms],
- temperature [K],	ρ	- density [kg/m ³],
- viscous stress tensor,	σ	- entropy production [W/kgK],
- time [s],	ζ	- coefficient of bulk viscosity [m ² /s],
- specific internal energy [J/kg],	$\tilde{\zeta}$	- coefficient of substitutional bulk viscosity [m ² /s].
- volume [m ³],		
- vector of barycentric velocity [m/s],		

Subscripts

- liquid,	ν	- vapour.
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Introduction

Dispersed flow is an important type of two-phase flow. In this flow the dispersed phase occupies separate regions, namely gas or vapour bubbles or liquid

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droplets, which are distributed in the continuous phase.

The two-phase flow is described by the basic laws of mass, momentum and energy conservation. The laws are in the form of differential or integral equations. Differential equations can be used to describe continuous media only. Therefore, modelling of two-phase media requires appropriate assumptions about the continuity of both phases [1]. It is possible to formulate the model of the two-phase medium that is continuous in the microscale. On the ground of this model each particle of the dispersed phase should be treated individually. The solution of the model should provide complete information about the flow. It should be possible, for example, to follow the trajectory of each droplet or bubble. However, such detailed description is not needed in practice because the microscale properties of the medium like the radius of droplets or bubbles have random values different in every run of the same experiment. Only values of parameters averaged in the scale larger than characteristic scale of the dispersed phase, i. e. in the mesoscopic scale, are reproducible in successive runs of the experiment. Taking this into account only averaged parameters are useful. This observation underlies the model of continuous two-phase media. In this model both phases occupy the entire flow region. The question arises if the continuous two-phase medium reflects the properties of the real medium. In this paper the influence of viscosity on the properties of the continuous two-phase medium has been investigated.

2. Model of the homogeneous two-phase flow

The continuous two-phase medium has the feature that the same point of the flow domain is occupied simultaneously by both phases. Properties of the phases are usually described by the ratio of mass or volume of the dispersed phase to the mass/volume of the mixture in the control volume.

Let us define the void fraction φ in the liquid-vapour flow as a ratio of the volume of the vapour phase V_ν to the volume of the mixture $V = V_l + V_\nu$, namely

$$\varphi = \frac{V_\nu}{V_l + V_\nu} = \frac{V_\nu}{V}. \quad (1)$$

The control volume V should be selected in the way that, being much smaller than the characteristic macroscopic size of the flow, it contains sufficient number of bubbles or droplets. Similarly, in this volume the quality of the two-phase mixture x can be defined as a ratio of the mass of the vapour phase m_ν to the total mass of the mixture m :

$$x = \frac{m_\nu}{m_l + m_\nu} = \frac{m_\nu}{m}. \quad (2)$$

One can find the relations between the void fraction and the quality in the form

$$x = \frac{\varphi \rho_\nu}{\varphi \rho_\nu + (1 - \varphi) \rho_l}, \quad \varphi = \left[1 + \frac{(1 - x) \rho_\nu}{x \rho_l} \right]^{-1}. \quad (3)$$

Now the average density ρ of the continuous two-phase medium can be calculated from the densities of the components

$$\rho = \varphi \rho_\nu + (1 - \varphi) \rho_l. \quad (4)$$

However, the average specific values of extensive parameters should be calculated with the use of the quality, for example the average specific enthalpy should be calculated from

$$h = x h_\nu + (1 - x) h_l. \quad (5)$$

The conservation laws written for the flow of a continuous two-phase medium are the foundations of the two-fluid model and the homogeneous model. In terms of the two-fluid model, it is assumed that the phases move at different velocities. The balance equations are written separately for each phase. The mass, momentum and energy exchange between phases is described by an additional relation. This model is relatively complex because even in the simplest case of one-dimensional flow it consists of six differential equations. Moreover, formulation of the relations for the interfacial exchange comes across certain difficulties. On the other hand, in the frame of the homogeneous model it is assumed that the phases travel at the same velocity or that the difference between the velocities of the phases is negligible. The advantage of the homogeneous model is its simplicity and for this reason this model is used in the following study.

The homogeneous model describes the flow of the homogeneous two-phase mixture whose parameters have been averaged in space according to the proportions of the phases. The set of conservation equations of the model has the same form as in the case of a single-phase flow. In a general case it consists of the equations of mass conservation

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{w} = 0, \quad (6)$$

momentum conservation

$$\rho \frac{d\mathbf{w}}{dt} = \rho \mathbf{g} + \nabla \cdot \mathbf{S} \quad (7)$$

and energy conservation

$$\rho \frac{d}{dt} \left(h + \frac{\mathbf{w}^2}{2} \right) = \frac{\partial P}{\partial t} + \nabla \cdot (\mathbf{w} \cdot \mathbf{T}) + \rho (\mathbf{w} \cdot \mathbf{g}) - \nabla \cdot \mathbf{q}. \quad (8)$$

In the above equations P is an average pressure, the same in both phases, ρ is the average density calculated from (4), h – the average specific enthalpy (5) and $\mathbf{w}$ is a velocity of the mass centre of the mixture (i. e. the barycentric velocity). The stress tensor $\mathbf{S}$ defines the relation between the stress in the fluid and the pressure, density, temperature and velocity at each point of the continuous medium. The tensor $\mathbf{S}$ is symmetric and depends on physical properties of the medium. If the normal stress (pressure P) and the shear stress are distinguished, the stress tensor can be written as a sum

$$\mathbf{S} = \mathbf{T} - P\mathbf{I}, \quad (9)$$

Here, $\mathbf{T}$ is the viscous stress tensor, which describes the influence of viscosity in the medium. It is also a symmetric tensor. Noting that the medium is isotropic and assuming that the linear relation holds between the shear stress and velocity gradient (Newton's hypothesis), viscous stress tensor can be written in the form [2]

$$\mathbf{T} = 2\mu\mathbf{D} + \left[\left(\zeta - \frac{2}{3}\mu \right) \nabla \cdot \mathbf{w} \right] \mathbf{I}, \quad (10)$$

where $\mathbf{D}$ is the deformation tensor. In the above relation the coefficient of dynamic viscosity μ and the coefficient of bulk viscosity ζ also appear. Values of the coefficients should be found experimentally. The measurement of the dynamic viscosity is relatively simple in the single-phase medium. It appears that in general, μ depends on the thermodynamic state of the medium $\mu = \mu(P, T)$, but the dependence on the pressure is weak. The measurement of the bulk viscosity ζ , in turn, comes across certain difficulties. In the case of incompressible liquid this value is unimportant because $\nabla \cdot \mathbf{w} = 0$. The kinetic theory of gases gives $\zeta = 2/3$ for monatomic gases, which agrees well with measurements for noble gases. For polyatomic gases, on the other hand, we observe from the damping of ultrasonic waves that $\zeta \neq 0$. It can be proved that it is the result of the relaxation processes in the gas [3]. The measurements of the viscosity in a two-phase medium are difficult because the results depend on the composition of the medium as well as on the phase distribution. Many definitions of the viscosity coefficient in the homogeneous the two-phase mixture can be found in the literature. Some of them are as follows [4]:

$$\begin{aligned} \mu &= \mu_l & (\text{Owen}) \quad \mu &= x\mu_\nu + (1-x)\mu_l, & (\text{Cicchitti et al.}) \\ \frac{1}{\mu} &= \frac{x}{\mu_\nu} + \frac{1-x}{\mu_l} & (\text{Isbin}) \quad \mu &= x \frac{\rho}{\rho_\nu} \mu_\nu + (1-x) \frac{\rho}{\rho_l} \mu_l, & (\text{Dukler}) \end{aligned} \quad (11)$$

They have a common feature that the value of the viscosity in the two-phase mixture lies between the viscosity of the vapour μ_ν and the viscosity of the liquid μ_l . There is no data considering the bulk viscosity ζ in two-phase medium but presumably its influence on properties of the medium is negligible.

The heat flux vector $\mathbf{q}$, which appears in the equation of energy conservation (8), depends on the temperature gradient and can be evaluated with the aid of Fourier's law

$$\mathbf{q} = -\lambda \nabla T, \quad (12)$$

where λ denotes the coefficient of thermal conductivity. This coefficient like the viscosity depends on the temperature and slightly on the pressure. The value of λ should also be found experimentally.

Bearing in mind the above assumptions concerning the tensors $\mathbf{S}$, $\mathbf{T}$ and the vector $\mathbf{q}$, the set of the conservation equations for the case of one-dimensional flow in the direction z can be written in the form

$$\frac{d\rho}{dt} + \rho \frac{\partial w}{\partial z} = 0, \quad (13)$$

$$\rho \frac{dw}{dt} = \rho g \cos \alpha - \frac{\partial P}{\partial z} + \frac{\partial}{\partial z} \left[\left(\zeta + \frac{1}{3} \mu \right) \frac{\partial w}{\partial z} \right], \quad (14)$$

$$\rho \frac{d}{dt} \left(h + \frac{w^2}{2} \right) = \frac{\partial P}{\partial t} + \frac{\partial}{\partial z} \left[\left(\zeta + \frac{4}{3} \mu \right) w \frac{\partial w}{\partial z} \right] + \lambda \frac{\partial^2 T}{\partial z^2} + \rho w g \cos \alpha. \quad (15)$$

The angle α is measured between the flow direction and the direction of the mass force $\mathbf{g}$. The unknown quantities in the above set of equations are pressure P , density ρ , temperature T , specific enthalpy h and flow velocity w . The equations of state

$$\rho = \rho(P, h), \quad T = T(P, h) \quad (16)$$

should be added to close the set (13-15). The form of the equations of state (16) is the simplest when the thermodynamic equilibrium between the phases is assumed. However, it is possible to take into account the thermal nonequilibrium between the phases provided that the equation describing the nonequilibrium heat and mass exchange on the interfacial surface will be added to the proceeding set.

3. Energy dissipation in two-phase flow

The observations of the propagation of compression waves in two-phase media showed their significant dispersion, i. e. broadening of their width. This phenomenon can not be described in terms of previously developed homogeneous models. It turns out that some of the wave properties of the medium are lost in averaging procedures. It also seems that the energy dissipation in the real flow is much greater than that predicted by the homogeneous model. Up to now, most of the models of two-phase flow do not take account of the fluid viscosity and thermal conductivity. Some interesting results have been obtained when the dissipation in the homogeneous flow had been increased allowing for the thermal conductivity [5]. An additional dissipation term can be included in the set of conservation equations when the viscosity of the medium is considered. However, the problem arises how to determine the value of the viscosity coefficient in the two-phase inhomogeneous flow.

In viscous flow the energy dissipation, i. e. irreversible exchange of mechanical energy into heat, is present. Let us denote by dE_k the infinitesimal change of the kinetic energy in the control volume dV and time dt . It follows from the equation of energy conservation that the difference between dE_k and the work of the mass forces δA^V as well as the surface forces δ^S equals

$$(\delta A^V + \delta A^S) - dt dE_k = dt \int_V \Phi dV, \quad (17)$$

where

$$\Phi = \mathbf{S} : \mathbf{D} = \nabla(\mathbf{w} \cdot \mathbf{T}) - (\mathbf{w} \cdot \nabla \mathbf{T})$$

is called the dissipation function. It can be interpreted as the irreversible dissipation of mechanical energy into heat caused by the viscosity per unit time and per unit volume. In one-dimensional flow of Newtonian fluid the dissipation function has a form

$$\Phi = \left(\zeta + \frac{4}{3}\mu\right) \left(\frac{dw}{dz}\right)^2. \quad (19)$$

Making use of the above equation, the entropy production due to shear stress in viscous flow is defined as

$$\sigma_f = \frac{\Phi}{T} = \frac{1}{T} \left(\zeta + \frac{4}{3}\mu\right) \left(\frac{dw}{dz}\right)^2. \quad (20)$$

The second law of thermodynamics requires that $\sigma_f \geq 0$. Therefore $\zeta \geq 0$ as well as $\mu \geq 0$ for any velocity field in the fluid.

4. Viscosity in the homogeneous two-phase flow

The entropy production caused by viscous dissipation in the two-phase medium should be the same in the homogeneous model and in the real flow. The following discussion will be restricted to the case of bubble flow. It follows from the equation (20) that the entropy production is proportional to the velocity gradient. The velocity field in the real bubble flow is very complicated. First, it should be noted that the vapour bubbles can move with velocities different from the velocity of the surrounding liquid. This phenomenon is called a slip. As a result, there exists a boundary layer around the bubbles. In the case of flow through a channel the boundary layer also exists at the channel wall. The velocity gradient there is much higher than the gradient of the average velocity calculated in terms of the continuous medium concept. Therefore, it is expected that the energy dissipation due to the viscosity takes place mainly in the boundary layers. The entropy production in the boundary layer at the channel wall may be considered a result of the influence of external forces on the two-phase medium, while the entropy production in the boundary layers around the bubbles can be thought as an internal property of the medium. For that reason energy dissipation arising from the slip of the bubbles should be included in the model of the homogeneous two-phase medium. It can be done when the replacement viscosity coefficient is introduced.

Let us estimate the entropy production σ_{b_i} in the boundary layer around a single bubble. If the average thickness of the hydraulic boundary layer equals δ_h then the velocity gradient in the layer can be found from the relation

$$\frac{dw}{dz} \approx \frac{w_l - w_v}{\delta_h}, \quad (21)$$

where w_l and w_v denote the average velocity of the liquid and the vapour in the control volume, respectively. It can be seen that the velocity gradient around

the bubble is nonzero only when $w_l \neq w_\nu$ that is when the slip between phases $\Delta w = w_l - w_\nu \neq 0$. Thus, from eq.(20) the entropy production can be calculated as

$$\sigma_{b_i} = \frac{1}{T_l} V_h (\zeta_l + \frac{4}{3} \mu_l) \left(\frac{\Delta w}{\delta_h} \right)^2, \quad (22)$$

where $V_h = 4\pi R^2 \delta_h$ is the volume of the boundary layer. Then, the specific entropy production in the control volume V containing N bubbles of equal radius R is

$$\sigma_{fb} = \frac{N \sigma_{b_i}}{V} = \frac{3\varphi}{R} \frac{1}{T_l} (\zeta_l + \frac{4}{3} \mu_l) \frac{(\Delta w)^2}{\delta_h}. \quad (23)$$

In the above equation the relation between the void fraction φ and the number of bubbles N in the volume V

$$\varphi = \frac{V_l}{V} = \frac{\frac{4}{3} \pi R^3 N}{V} \quad (24)$$

has been introduced. The entropy production σ_f in the homogeneous medium should be calculated with the help of the macroscopic values. Particularly the velocity gradient present in the equation (20) should be evaluated from the distribution of average velocity in the homogeneous two-phase flow. It should be remembered that this gradient is evaluated from the average velocity at which the mass centre of the phases in the control volume moves. In the flow with non-zero slip this gradient is considerably different from the velocity gradient in the boundary layer (21). Thus, if the equality of the entropy production calculated from the macroscopic average values (20) as well as calculated with the aid of the local values at the mesoscopic level of description (23) is required

$$\sigma_f = \sigma_{fb}, \quad (25)$$

then the coefficients of viscosity should be redefined. Let us rewrite the equation (20)

$$\sigma_f = \frac{1}{T_l} (\tilde{\zeta} + \frac{4}{3} \tilde{\mu}) \left(\frac{dw}{dz} \right)^2, \quad (26)$$

denoting by $\tilde{\mu}$ and $\tilde{\zeta}$ the coefficients of the substitutional dynamic viscosity and substitutional bulk viscosity, respectively. Their sum can be found from the equation (25)

$$\tilde{\zeta} + \frac{4}{3} \tilde{\mu} = \frac{3\varphi}{R} (\zeta_l + \frac{4}{3} \mu_l) \frac{(\Delta w)^2}{\delta_h} \left(\frac{dw}{dz} \right)^{-2}. \quad (27)$$

In the flow with no intense relaxation processes the bulk viscosity can be neglected because it has insignificant influence on the properties of the medium. Therefore, the equation (27) can be simplified

$$\tilde{\mu} = \mu_l \frac{3\varphi}{R} \frac{(\Delta w)^2}{\delta_h} \left(\frac{dw}{dz} \right)^{-2}. \quad (28)$$

The coefficients of the substitutional viscosity calculated from (27) or (28) should be used in the homogeneous two-phase model formulated by the equations (13-15).

Let us estimate the difference between the coefficients of the substitutional viscosity of the continuous two-phase medium and the viscosity of the liquid

$$M = \frac{\tilde{\mu}}{\mu_l} = 3\varphi \frac{\delta_h}{R} \left(\frac{\Delta w}{\delta_h} \right)^2 \left(\frac{dw}{dz} \right)^{-2}. \quad (29)$$

In order to complete the calculation of M , the slip between phases Δw is required. This information is easily obtained on the grounds of the two-fluid model. However, the ratio (29) can be estimated in terms of the homogeneous model if it is assumed that the slip has no significant influence on the flow properties except for the viscous dissipation. The slip, of course, can not be directly calculated from equations of the homogeneous model. To do this the equation of momentum conservation for an isolated spherical bubble travelling in the liquid can be used. Taking into account only the force resulting from the pressure gradient in the liquid and the drag of the bubble, it is obtained

$$\rho_\nu w_\nu \frac{dw_\nu}{dz} = -\frac{dP}{dz} - \frac{3}{8} \frac{C_D}{R} \rho_l (w_\nu - w_l) |w_\nu - w_l|, \quad (30)$$

where C_D is the bubble drag coefficient. For spherical bubbles empirical correlation holds [6]

$$C_D = 6.3 Re^{-0.385}, \quad 10 < Re < 1200. \quad (31)$$

Reynolds number Re is based here on the slip $|w_l - w_\nu|$ and the bubble radius R

$$Re = \frac{2\rho_l |w_l - w_\nu| R}{\mu_l}. \quad (32)$$

Solution of the equation (30) is obtained numerically, provided that the pressure and velocity distributions in the liquid are known. In the case of the two-phase homogeneous flow in a straight channel, when $\varphi = O(10^{-1})$, $R = O(10^{-4})$, $\delta_h = O(10^{-5})$, $dP/dz = O(10^4)$, $dw/dz = O(10^1)$, the slip evaluated from the equation (30) equals $\Delta w = O(10^{-1})$. Thus, it follows that $M = O(10^6)$. It is seen that it is necessary to introduce the substitutional viscosity in the continuous two-phase medium of order six times higher than the liquid viscosity if the condition of equality of the entropy production due to viscous losses (25) is required. The difference between the coefficients of viscosity is lower in the case of flow inside a compression wave. When the pressure gradient in the wave is of order of magnitude $O(dP/dz) = 10^6$, the velocity gradient $O(dw/dz) = 10^2$ and other values are kept the same as before, then $O(\Delta w) = 1$ is obtained and consequently $O(M) = 10^4$. In an analogous way it is estimated that the entropy production due to the slip of the bubbles is at least one order of magnitude higher than those in the boundary layer at the channel wall.

5. Conclusions

It is concluded that the viscous dissipation can be modelled in the continuous two-phase medium if the coefficient of substitutional viscosity is introduced to the homogeneous model of the two-phase flow. The substitutional viscosity is a parameter describing the continuous two-phase medium at the macroscopic level. Its value can be evaluated from the analysis of the local properties of the flow at the mesoscopic level, namely from the velocity distribution around an individual vapour bubble. A considerable fraction of the total mechanical energy losses takes place inside the hydraulic boundary layer around the bubble, even if the difference between the bubble velocity and the velocity of the surrounding liquid is relatively small. It comes from the fact that the velocity gradient in the boundary layer is much higher than the gradient of averaged velocity in the two-phase homogeneous flow. The condition of equal entropy production in the model of the continuous two-phase medium and in the real medium is satisfied provided that the value of the substitutional viscosity is several orders higher than the molecular viscosity in the liquid or vapour.

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Człon lepkościowy w modelu pęcherzykowego przepływu dwufazowego

Streszczenie

Przedstawiono równania jednorodnego modelu przepływu dwufazowego zawierające człony lepkościowe. Na podstawie porównania produkcji entropii w rzeczywistym przepływie pęcherzykowym i w ciągłym ośrodku dwufazowym wprowadzono zastępczy współczynnik lepkości dla ciągłego i jednorodnego ośrodku dwufazowego. W przypadku przepływu pęcherzykowego oszacowano, że jego wartość może być o kilka rzędów wielkości wyższa od lepkości dynamicznej cieczy.