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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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ZBIGNIEW ZARZYCKI [1]

Friction Factor in a Pulsating Turbulent Pipe Flow[2]

Mathematical models of an unsteady turbulent liquid pipe flow, with a quasi-steady distribution of an eddy viscosity in a pipe cross-section taken into account, were described. These models are based on four-region distributions of the eddy viscosity coefficient. They were adapted for determination of the instantaneous friction factor. For the three-region model - in case of low frequencies - dynamic velocity changes as a function of frequency. It was shown that the instantaneous friction factor value considerably differs from the quasi-steady one.

1. Introduction

The substantial influence on dynamic processes taking place during unsteady flow of liquid exert, among other things, such phenomena as [1]: dissipation of mechanical energy, interaction between pipe wall motion and liquid motion, or possible disruption of a fluid flux as the result of cavitation. This paper is connected with the first phenomenon and refers to the modelling of hydraulic losses during pulsating turbulent flow of a liquid. Such kind of flow, caused most often by an operation of pumps and compressors, is generally met in practice, e. g. in liquid pipelines, in industrial hydraulic systems, in chemical equipment installations, as well as in some hydraulic drive systems.

When computing dynamic processes taking place during pulsating turbulent flow of a liquid in pipes, it is very often assumed that hydraulic resistance is steady [2, 3, 18, 19], or quasi-steady [2, 4, 5], i. e. it has the same form as during steady motion, but for an instantaneous, mean cross-sectional velocity of a liquid. However, such assumption is correct at very low frequencies of liquid vibration [3, 9].

Publications taking into account a change of the hydraulic resistance as the function of frequency are not numerous [6-11]. Considerations contained in them base on Reynolds equation with an application of Boussinesq hypothesis and evidences of eddy viscosity coefficient distribution in a pipe cross-section. This

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[2] Praca wykonana w ramach tematu CPBP 02.18 - 3.2

way of a turbulent flow modelling makes a base for the two-region Popov mo[10], the three-region Brown model [3], the four-region models of Ohmi [8] an Glikman [11]. However, these models contain discontinuities in the eddy viscosi profile. A common feature of the above mentioned models is the assumption of a stationary eddy viscosity distribution in a pipe cross-section. As references [3, 5 show, this assumption is right for higher frequencies range, where inertial effect dominate over viscosity effects.

In this article methematical model of pulsating, turbulent liquid flow in pi-pes, based on a four-region model eddy viscosity coefficient distribution in a pip cross-section has been described. As opposed to the mentioned publications [6-11] continuous and quasi-steady distribution of the eddy viscosity coeficient in a pip cross-section was assumed. This model was adapted to calculate an instanta-ous friction loss factor. The instantaneous friction factor for low frequencies wa determined through the application of the three-region model in which a dyna-mic velocity changes as the function of frequency. Calcutations for this case wer carried out by means of an iterative method. Some results of numerical calcu-lations for the both models were compared with some results of experimenta investigations taken from the literature, and good compatibility was obtained.

2. Mathematical Model of an Unsteady Turbulent Flow of a Liqui for a Circular Cross-Section Pipe

Analytical calculation of the instantaneous friction factor requires an adapta-tion of an appropriate mathematical model for a liquid pipe flow enabling dete-mination of instantaneous distribution of velocity field in a pipe cross-section, a well as instantaneous shear stress on a pipe wall. Unsteady, axisymmetrical tu-bulent flow of a Newtonian liquid in a long pipe with a constant internal radiu and rigid walls is investigated. Moreover, the following assumptions are taken:

- constant distribution of pressure in a pipe cross-section,
- isothermic flow,
- body forces are negligible,
- mean velocity in a pipe cross-section is considerably smaller than soun velocity in a liquid.

Taking into account the above assumptions, the liquid motion is described i cylindrical coordinates:(r, z, θ) by means of Reynolds equation [1]:

$$\rho_0 \frac{\partial \overline{v}_z}{\partial t} = -\frac{\partial \overline{p}}{\partial z} + \rho_o \nu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \overline{v}_z}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(-r\rho_0 \overline{v_z' v_r'}\right), \tag{1}$$

where: $\overline{v_z}$, $\overline{p}$, – averaged in time, respectively: velocity component in the axial direction and pressure, ρ_o – steady density of a liquid, ν – kinematic coefficient of viscosity, t – time.

The prime denotes the turbulent fluctuating component and the overscore denotes the short-time average value (in which $\overline{v'_z} = 0$), which is permitted to include the relatively low frequencies for the sinusoidal travelling waves under cosiderations. Obviously, under the above assumption the frequency of a pulsating flow must be considerably smaller than the frequncy of the turbulent fluctuations. If the length of the periodic waves of liquid $\lambda \gg 2\pi R$ (roughly $\lambda > 20R$, R – radius of a pipe) so, it is shown in the analysis of turbulent energy spectrum [3] that the possibility of correlation of waves of a liquid and turbulent fluctuations is very small. For example, if the velocity of wave propagation equals $c = 1200$ m/s and radius $R = 0.1$ m, so the frequency of pulsating flow $\omega \ll 12000$ $1/s$. The quantity $\overline{v'_z v'_r}$ in the equation (1) is the turbulent shear stress. According to Boussinesq hypothesis, this stress is defined by:

$$\rho_0 \overline{v'_z v'_r} = -\rho_0 \nu_t \frac{\partial \overline{v}_z}{\partial r}, \tag{2}$$

where: ν_t- kinematic coefficient of turbulent viscosity.
After on application of the relationship (2), equation (1) becomes:

$$\frac{\partial \overline{v}_z}{\partial t} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial z} + \nu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{v}_z}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_t\frac{\partial \overline{v}_z}{\partial r}\right). \tag{3}$$

In equation (3) coefficient of turbulent viscosity is unknown. In principle there are two different approaches to the solution of equation (3), closely connected with a way of determination of this coefficient. The first way [12] is related to determination of ν_t on a base of the equation for transport of kinetic turbulent energy and the algebraic equation of a turbulence scale attached to it. It should be emphasized, that number constants in the above mentioned equations are determined on a basis of experimential results for a steady flow. The second way [3, 6, 10] is related to determination of an algebraic function of coefficient ν_t distribution in a pipe cross-section on a basis of experimental results for a steady pipe flow. As the result of this, equation (3) is divided into an equation for viscous sulblayer and equations for the remaining part of the flow area (the number of them depends on the number of layers necessary for determination of ν_t). Both presented above ways of determination of coefficient ν_t appear to be quasi-steady. The second way enables an analytical determination of the velocity field in a vicinity of a pipe wall and, at the same time, allows to define an instantaneous shear stress on the pipe wall. This second way (of determination of ν_t) was used in this paper.

2.1. Four-region Model

Distribution of the turbulent viscosity coefficient for steady flow (which is taken from the references [6, 14, 15]) in overall cross-section of the pipe and in a vicinity of the wall is shown in figures 1 and 2, respectively. Respective symbols on Figs. 1 and 2 mean: y – distance from pipe wall, $y^+ = yv^*/\nu$ – dimensionless distance y, R – internal radius of a pipe, $R^+ = Rv^*/\nu$ – dimensionless radius R, $v^* = \sqrt{\tau_{ws}/\rho_0}$ – dynamic velocity, τ_{ws} – shear stresses on pipe wall for a steady flow.

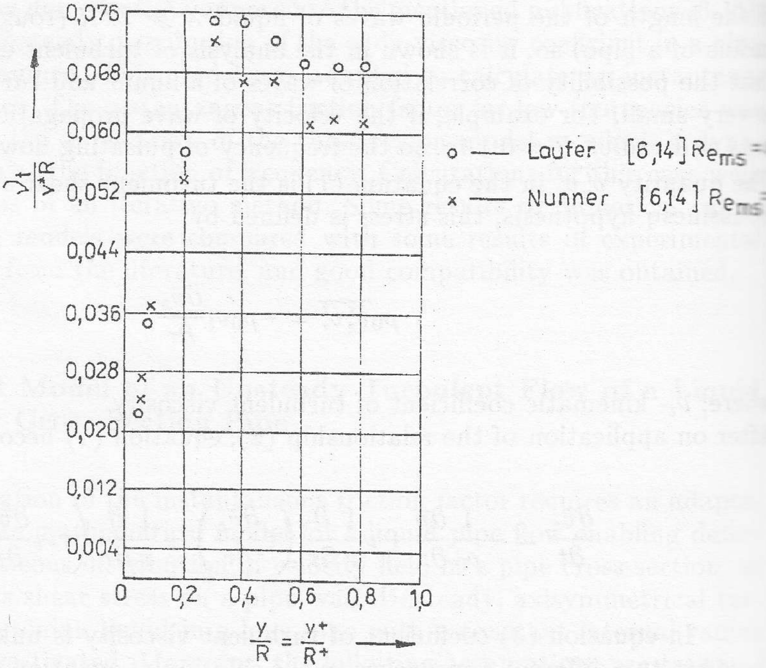

Fig. 1. Distribution of the coefficient ν_t in a pipe cross-section

It will be comfortable for further considerations to present dynamic velocity in the following form [7]:

$$v^* = \sqrt{\frac{\lambda_s}{8}} v_{ms} = \frac{1}{4}\sqrt{\frac{\lambda_s}{2}\frac{\nu}{R}} Re_{ms}, \tag{4}$$

where: λ_s – friction factor for a steady flow, v_{ms} – mean liquid velocity in a pipe cross-section for a steady flow, $Re_{ms} = 2Rv_{ms}/\nu$ – Reynolds number.

To describe the distribution of the coefficient ν_t in a pipe cross-section, the flow region should be divided into several layers, and for each of them $\nu_t = f(r)$ function has to be determined.
By analogy to a steady pipe flow the following regions of the flow are distinquished [16]: the viscous sublayer (VS), the buffer layer (BL), the developed turbulent flow

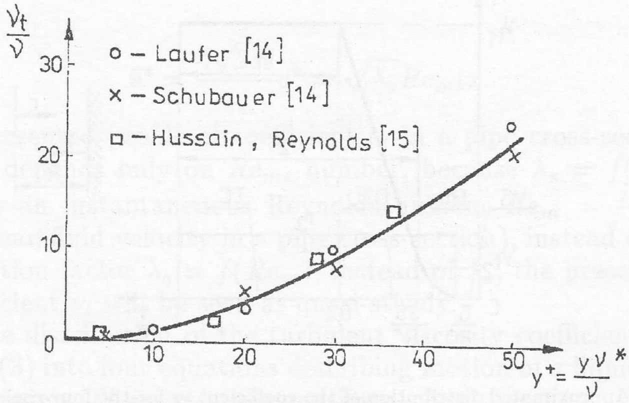

Fig. 2. Distribution of the coefficient ν_t in a pipe wall

layer (DTL), and the turbulent core (TC). Boundaries of the individual layers are presented in the Table 1.

Table 1

Layers of flow area

Flow zone	Zone boundary		Coefficient ν_t
	$y^+ = y\frac{v^*}{\nu}$	$\frac{y}{R}$	
Viscous sublayer	$0 \div 5$	$0 \div \frac{5}{R^+}$	$\nu_t = 0$
Buffer layer	$5 \div 35$	$\frac{5}{R^+} \div \frac{35}{R^+}$	$\nu_t = 0.01(y^+)^2\nu$
Developed flow layer	$35 \div 0.2R^+$	$\frac{35}{R^+} \div 0.2$	$\nu_t = \nu_{t_2} + \frac{\Delta\nu_t}{\Delta r}(r_2 - r)$
Turbulent core	$0.2R^+ \div R^+$	$0.2 \div 1$	$\nu_t = 0.064v^*R$

Basing on an analysis of the results presented in Fig. 1 and 2, it has been ascertained that the coefficient ν_t in the buffer layer can be approximated by means of quadratic dependence, in the developed turbulent flow layer by means of linear dependence, whereas for the turbulent core it can be roughly considered as a constant. Approximated cross-sectional distribution of the coefficient ν_t is presented in Fig. 3. It can be written in the following way:

- for the viscous sublayer ($r_3 \le r < R$):

$$\nu_t = 0, \tag{5}$$

- for the buffer layer ($r_2 \le r < r_3$):

$$\nu_t = 0.01(y^+)^2\nu, \tag{6}$$

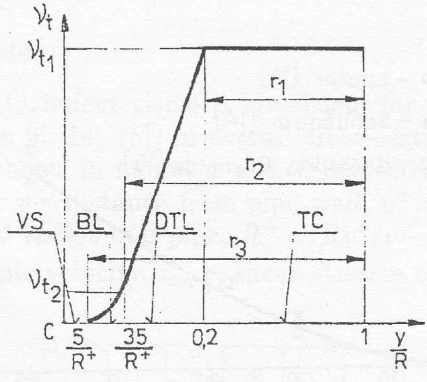

Fig. 3. Approximated distribution of the coefficient ν_t for the four-region model

- for the developed turbulent flow layer ($r_1 \leq r < r_2$):

$$\nu_t = \nu_{t2} + \left(\frac{\nu_{t1} - \nu_{t2}}{r_2 - r_1}\right)(r_2 - r), \tag{7}$$

- for the turbulent core ($0 \leq r < r_1$):

$$\nu_t = \nu_{t1}. \tag{8}$$

Expressions defining quantities: ν_{t1}, ν_{t2}, r_1, r_2 and r_3 have the following forms:

$$\nu_{t1} = 0.016\sqrt{\frac{\lambda_s}{2}} Re_{ms}\nu, \tag{9}$$

$$\nu_{t2} = 12.25\nu, \tag{10}$$

$$r_1 = 0.8R, \tag{11}$$

$$r_2 = \left(1 - \frac{140}{Re_{ms}}\sqrt{\frac{2}{\lambda_s}}\right) R, \tag{12}$$

$$r_3 = \left(1 - \frac{20}{Re_{ms}}\sqrt{\frac{2}{\lambda_s}}\right) R. \tag{13}$$

Many publications [3, 7, 8] leave out the buffer layer when determining th coefficient of turbulent viscosity, which is the cause of discontinuity of a turbu lent viscosity profile. The presented approximation method for a coefficient o turbulent viscosity in a pipe cross-section takes into consideration continuity o its profile. It allows to determine the velocity field of a liquid more precisely, sp cially near a wall, and thus instantaneous shear stress on a pipe wall.

Quantities: ν_{t1}, r_2, r_3 defining distribution of the coefficient ν_t depend on di mensionless dynamic velocity $\tilde{v}^*$, which is determined in the following way:

$$\tilde{v}^* = \frac{4\sqrt{2}R}{\nu}v^* = \sqrt{\lambda_s}Re_{ms}. \tag{14}$$

Therefore the presented profile of coefficient ν_t in a pipe cross-section in a case of *smooth* pipes depends only on Re_{ms} number, because $\lambda_s = f(Re_{ms})$. However, if one takes an instantaneous Reynolds number $Re_m = \frac{2Rv_m}{\nu}$ (v_m – an instantaneuos mean fluid velocity in a pipe cross-section), instead of Re_{ms}, and a quasi-steady friction factor $\lambda_q = f(Re_m)$, instead of λ_s, the presented distrubution of the coefficient ν_t will be seen as quasi-steady.
Knowledge of the distribution of the turbulent viscosity coefficient enables division of equation (3) into four equations describing motion of a liquid in individual layers of a flow region. These are the equations:

- for the viscous sublayer:

$$\frac{\partial \overline{v}_{z1}}{\partial t} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial z} + \nu\left(\frac{\partial^2 \overline{v}_{z1}}{\partial^2} + \frac{1}{r}\frac{\partial \overline{v}_{z1}}{\partial r}\right), \tag{15}$$

- for the remaining layers of flow region:

$$\frac{\partial \overline{v}_{zi}}{\partial t} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial z} + \nu_t\frac{\partial^2 \overline{v}_{zi}}{\partial r^2} + \left(\frac{\nu_t}{r} + \frac{\partial \nu_t}{\partial r}\right)\frac{\partial \overline{v}_{zi}}{\partial r}, \tag{16}$$

where $i = 2, 3, 4$ ($i = 2$ for the buffer layer, $i = 3$ for the developed turbulent flow layer, $i = 4$ for the turbulent core), $\overline{v}_{zi}$ – an axial component of the velocity for the individual layers of a flow region, averaged in time.

Equations (15) and (16) for i = 3,4, with utilization of the relationships (7) and (8), as well as after Laplace transformation for zero initial conditions, resolve themselves into the modified Bessel equations [22], whereas equation (16) for i = 2 when taking into account relationship:

$$\frac{2}{R-r} \gg \frac{1}{r} \quad \text{for} \quad r_2 \leq r < r_3$$

can take a form of a quadratic differential equation with constant factors. Solutions of motion equations for the individual layers become:

- for the viscous sublayer:

$$V_1 = C_1 I_0(\eta_1) + C_2 I_0(\eta_1) - \frac{1}{\rho_0 s}\frac{\partial P}{\partial z}, \tag{17}$$

- for the buffer layer:

$$V_2 = C_3 e^{\eta_2'} + C_4 e^{\eta_2''} - \frac{1}{\rho_0 s}\frac{\partial P}{\partial z}, \tag{18}$$

- for the developed turbulent flow layer:

$$V_3 = C_5 I_0(\eta_3) + C_6 K_0(\eta_3) - \frac{1}{\rho_0 s}\frac{\partial P}{\partial z}, \tag{19}$$

- for the turbulent core:

$$V_4 = C_7 I_0(\eta_4) + C_8 K_0(\eta_4) - \frac{1}{\rho_0 s}\frac{\partial P}{\partial z}, \tag{20}$$

where: I_0, K_0 – modified Bessel functions of the first and second kind of order 0, V_i, P – Laplace transforms for velocity and pressure, i. e. :

$$V_i(r,z,s) = \mathcal{L}(v_{zi}(r,z,t))P(z,s) = \mathcal{L}(p(z,t)),$$

s – is an operator of the Laplace transformation, $\mathcal{L}$ – indicates simple Laplace transformation, e – base of the natural logarithm. Quantities: $\eta_1, \ldots \eta_4$ have the form:

$$\eta_1 = \sqrt{\frac{sr^2}{\nu}}, \tag{17.1}$$

$$\eta_2^{'}, \eta_2^{''} = \frac{1}{2}\left(-1 \pm \sqrt{1 + \frac{400 s\nu}{v^{*2}}}\right)\ln(R-r), \tag{18.1}$$

$$\eta_3 = 2\sqrt{\frac{\Delta r}{\Delta \nu_t} s\left(\tfrac{\nu_{t2}}{\Delta \nu_t}\Delta r + r_2 - r\right)}, \tag{19.1}$$

where: $\Delta r = r_2 - r_1$, $\Delta \nu_t = \nu_{t1} - \nu_{t2}$,

$$\eta_4 = \sqrt{\frac{sr^2}{\nu_{t1}}}. \tag{20.1}$$

Integration constants: $C_1, \ldots C_8$, which are present in the equations (17)–(20), were determined from the bounadry conditions resulting from continuity of velocity and shear stresses at the point of contact for layers. These conditions can be written down in the following way:

$$V_i(r = r_i) = V_{i+1}(r = r_i),$$

$$\frac{\partial V}{\partial r}(r = r_i) = \frac{\partial V_{i+1}}{\partial r}(r = r_i),$$

for $i = 1, 2, 3$
and:

$$V_1(r = R) = 0.$$

Besides, velocity at the pipe axis has a finite value, which causes that $C_8 = 0$, since function $K_0(0)$ takes an infinite value.

2.2. Three-Region Model

Leaving out the buffer layer, and thereby equation (16) for $i = 2$, one can obtain the three-region model of the turbulent viscosity factor distribution in a pipe cross-section, presented in Fig. 4. The point of the viscous sublayer contact with the developed turbulent flow layer is defined by a dimensionless distance $y^+ = 11,5$ at which velocities of the both layers are equal. Equations of liquid motion for the individual layers remain unchanged (these are equations (15) and (16) for $i = 3, 4$), but one should assume:

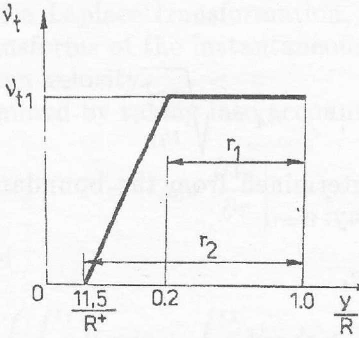

Fig. 4. Distribution of the coefficient ν_t for the three-region model

$$\nu_{t2} = \nu + \frac{\nu_{t1}}{r_2 - r_1}(r_2 - r) \qquad \text{for Eq. (7),}$$
$$r_2 = \left(1 - \frac{46}{Re_{ms}}\sqrt{\frac{2}{\Lambda_s}}\right) R \qquad \text{for Eq. (12)}$$

The analysis of the individual terms for the equations (15) and (16) has shown hat in the case of low frequencies inertia terms in equations of motion for the iscous sublayer and for the developed turbulent flow layer (Eq. (16) for ($i = 3$)) an be omitted. On the other hand, equation of motion for the turbulent core Eq. (16) for ($i = 4$)) remains unchanged. It is right for frequencies:

$$\omega <\approx 2\pi \frac{\nu_{t1}}{r_1^2},$$

king into account the above mentioned, a solution of equations of motion after r Laplace transformation takes a from:

- for the viscous sublayer

$$V_1 = \left(\frac{1}{4\rho_o \nu}\right)\frac{\partial P}{\partial z} r^2 + D_1 \ln\left(\frac{r}{R}\right) + D_2, \tag{24}$$

- for the developed turbulent flow layer

$$V_3 = \left(\frac{r_2 - r_1}{\nu_{t1}\rho_o}\right)\frac{\partial P}{\partial z}(n_3 R) + D_3 \ln(n_3) + D_4, \tag{25}$$

where:

$$n_3 = \left(\frac{\nu + \nu_{t1}}{\nu_{t1}}\right)\left(\frac{r_2 - r}{R}\right), \tag{25.1}$$

- for the turbulent core

$$V_4 = D_5 I_0(n_4) + D_6 K_0(n_4) - \frac{1}{\rho_o s}\frac{\partial P}{\partial z}, \tag{26}$$

where:

$$n_4 = \sqrt{\frac{sr^2}{\nu_{t1}}}. \tag{26.1}$$

Constants $D_1 \ldots D_6$ were determined from the boundary conditions, which are expressed in the following way:

1. $r = 0$; $V_4 =$ finite value,
2. $r = R$; $V_1 = 0$,
3. $r = R$; $-\rho_0 \nu \frac{\partial V_1}{\partial r} = T_w = (\tau_w)$ (shear stress on a pipe wall),
4. $r = r_1$; $V_3 = V_4$,
5. $r = r_2$; $V_1 = V_3$,
6. $r = r_2$; $\frac{\partial V_1}{\partial r} = \frac{\partial V_3}{\partial r}$.

The assumption about a steady distribution of the coefficient ν_t for a pipe cross-section is correct for high frequencies of liquid oscillations [3], whereas for low frequencies a change of the coefficient ν_t due to a non-stationary flow should be taken into account [7]. The only variable quantities defining the profile coefficient ν_t (Fig. 4) are ν_{t1} and r_2. They can be described as functions of the dynamic velocity v^* in the following way:

$$\nu_{t1} = 0.064 v^* R, \tag{27}$$

$$r_2 = R - y(y^+ = 11.5) = R - \frac{11,5\nu}{v^*}, \tag{28}$$

Variability of the coefficient ν_t in a pipe cross-section was obtained on the assumption of a variation of the dynamic velocity v^* as a function of frequency. It will be presented in chapter 4.

3. Determination of Instantaneous Friction Factor

The instantaneous friction factor $\lambda(t)$ was determined from the relationship:

$$\tau_w(t) = \frac{1}{8}\rho_0 \lambda(t) v_m^2, \tag{29}$$

To its determination a transfer function is helpful. The function defines the ratio between shear stress on a pipe wall and a mean velocity, and can be written down in the operator form:

$$W_{TV} = \frac{T(s)}{V(s)}, \tag{30}$$

where: s – is operator of a Laplace transformation, $T = \mathcal{L}(\tau_w)$ and $V = \mathcal{L}(v_m)$ presents the Laplace transforms of the instantaneous shear stress on a pipe wall and of instantaneous mean velocity.
Function (30) was determined by taking into account the following relationships:

$$T = -\rho_0 \nu \left.\frac{\partial V_1}{\partial r}\right|_{r=R}, \tag{31}$$

for the four-region model

$$V = \frac{2}{R^2}\left(\int_0^{r_1} V_4 r dr + \int_{r_1}^{r_2} V_3 r dr + \int_{r_1}^{R} V_1 r dr\right), \tag{32}$$

for the three-region model

$$\rho_0 s V + \frac{\partial P}{\partial z} + \frac{2}{R}T = 0, \tag{33}$$

for the both models.
The last relationship describes the equation of momentum – after Laplace transformation – for an averaged flow. It is necessary to eliminate the pressure gradient when evaluating the transfer function (30).
In the case of a flow which pulsates harmonically with ω frequency i. e.

$$v_m = v_{m0} + v_{mA}\sin\omega t, \tag{34}$$

he substitution of $s = j\omega$ to the relationship (30) one gets the frequency sfer function. It is real part W_1 represents an effective resistance. ng the relationship (29) one gets:

$$\lambda(t) = \frac{16 W_1}{Re_{mo}(1 + A\sin\varphi)}, \tag{35}$$

he evaluation of equation (35) and a mathematical structure of expression W_1 an be found in references [20, 21]. In a general case $\lambda(t)$ has the form:

$$\lambda(t) = \lambda(Re_{mo}, \Omega, A, \varphi), \tag{36}$$

where: $Re_{mo} = \frac{2Rv_{mo}}{\nu}$ – Reynolds number connected with the mean velocity in time v_{mo}, Ω – dimensionless frequency, A – dimensionless amplitude, $\varphi = \omega t$ – phase angle. Quantities Ω and A have the form:

$$\Omega = \frac{\omega R^2}{\nu}, \tag{37}$$

$$A = \frac{v_{mA}}{v_{m0}}, \tag{38}$$

4. Procedure of Calculations

Calculations of the instantaneous friction factor were carried out according to the relationship (35), for the cases of the four-region and the three-region model of turbulent viscosity. In the case of the four-region model, the quasi-steady distribution ν_t in a pipe cross-section was assumed, but instead of Re_{ms} and λ the relationship (14) contains:

- the instantaneous mean Reynolds number Re_m

$$Re_m = Re_{mo}(1 + A \sin\varphi), \tag{39}$$

- the quasi-steady friction factor (acc. to Von Karman-Nikuradse law)λ_q

$$\frac{1}{\sqrt{\lambda_q}} = 0.869 \ln(Re_m \sqrt{\lambda_q}) - 0.8, \tag{40}$$

rescpectively.

In the case of the three-region model, as it was mentioned before, the dynamic velocity changes as a function of frequency. The procedure can be described in the following way. In the case of a harmonic pulsating flow, the transfer function (30) takes a spectral form i. e. :

$$W_{TV}(j\Omega) = W_1(\Omega) + jW_2(\Omega). \tag{41}$$

Thus one can write down:

$$T = A_w V e^{j\psi}, \tag{42}$$

where:

$$A_w = \sqrt{W_1^2 + W_2^2}, \tag{42.1}$$

$$\psi = \arctan\left(\frac{W_2}{W_1}\right), \tag{42.2}$$

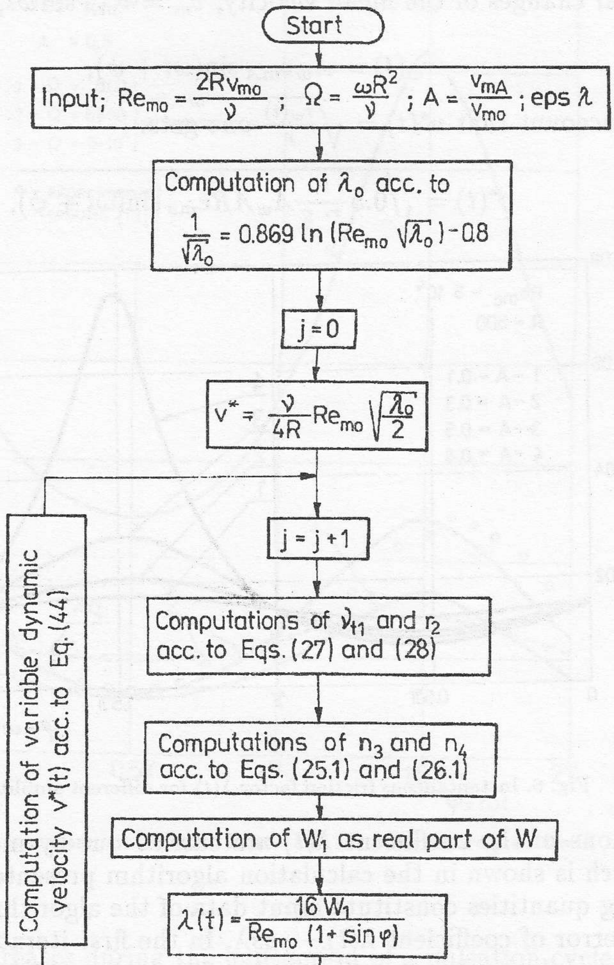

Fig. 5. Algorithm of calculations of $\lambda(t)$

For sinusoidal changes of the mean velocity, $v_m = v_{mA} \sin \omega t$, one gets:

$$\tau_w(t) = A_w v_{mA} \sin(\omega t + \psi), \tag{43}$$

Taking into account that $v^*(t) = \sqrt{\frac{\tau_w(t)}{\rho}}$ one gets:

$$v^*(t) = \sqrt{0.5 \frac{\nu}{D} A_w A Re_{mo} \sin(\omega t + \psi)}. \tag{44}$$

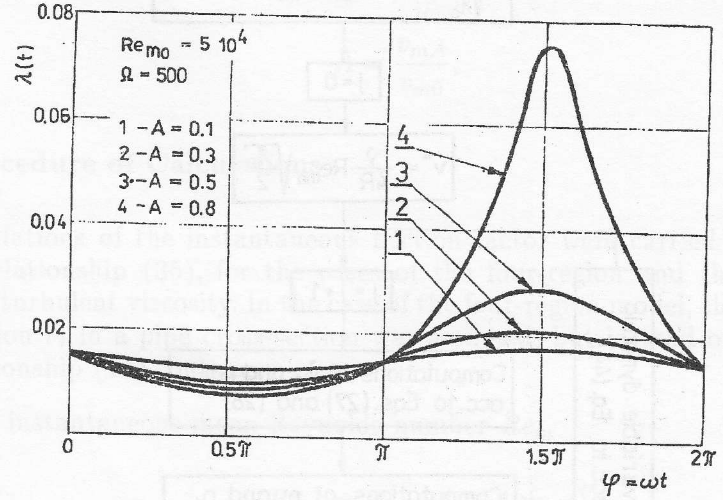

Fig. 6. Instantaneous friction factor $\lambda(t)$ for different amplitudes A.

Calculations of the coefficient $\lambda(t)$ are carried out by means of an iterative method, which is shown in the calculation algorithm presented below (Fig. 5). The following quantities constitute input data of the algorithm: Re_{mo}, Ω, A, and the relative error of coefficient $\lambda(t)$ – $eps\lambda$. In the first iteration calculations are carried out on a base of assumption that $v^* = f(Re_{mo}) = const$. The value of v^*, calculated according to (44), becomes a starting point for the next iteration, and so on. Calculations are carried out till the relative error of the quantity $\lambda(t)$ is smaller than the set value of an error (for the calculations carried out $eps\lambda = 0.001$), i. e. :

$$\left| \frac{\lambda^{n+1}(t) - \lambda^n(t)}{\lambda^n(t)} \right| \leq eps\lambda, \tag{45}$$

where: n – iteration number.

5. Results of Calculations

Some results of the numerical calculations for the instantaneous friction factor are presented in Fig. 6 and Fig. 7. Figure 6 shows that the coefficient $\lambda(t)$ decreases

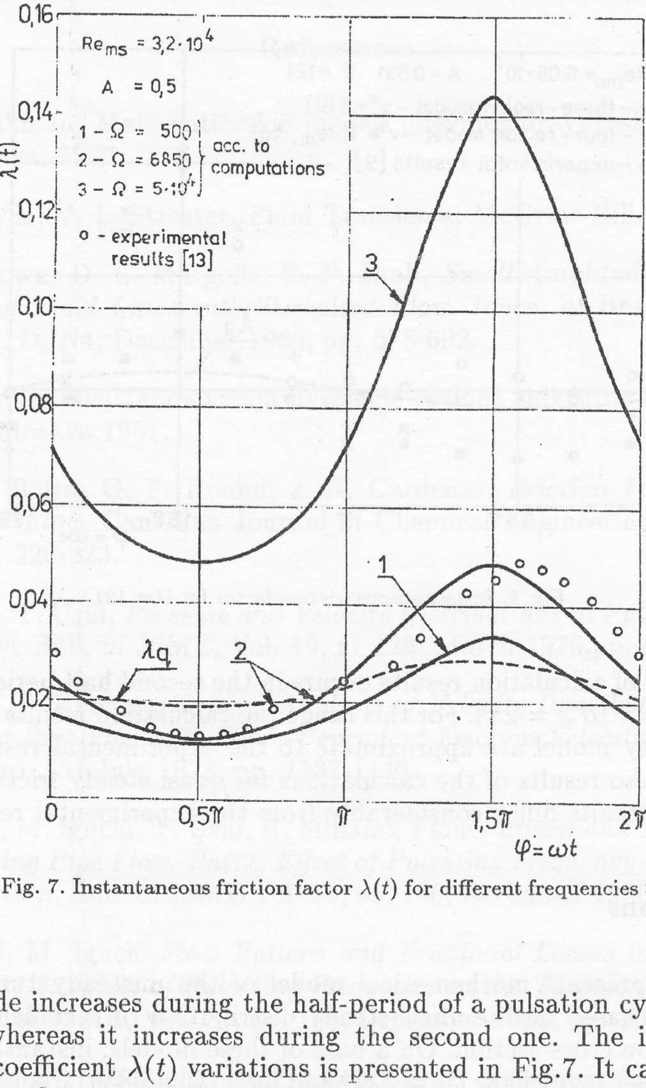

Fig. 7. Instantaneous friction factor $\lambda(t)$ for different frequencies Ω

as an amplitude increases during the half-period of a pulsation cycle (from $\varphi = 0$ to $\varphi = \pi$), whereas it increases during the second one. The influence of the frequency on coefficient $\lambda(t)$ variations is presented in Fig.7. It can be seen, that as the frequency Ω increases this coefficient increases also, for the whole pulsation cycle. Both the increase of velocity amplitude (at the same time Reynolds number) and the frequency increase cause a decrease of the thickness of boundary layer [3, 11]. This fact causes an increase of the velocity gradient in a normal direction and at the same time an increase of the shear stress on the pipe wall. The remarks presended above confirm an increase of the friction factor λ, which is shown in Figs. 6 and 7. Some results of the numerical calculations were compared to results of experiments available from the literature. Fig. 7 compares values of $\lambda(t)$ obtained by means of shear stress measurements on a pipe wall (acc to [13]) with results of the calculations for the four-region model of $\lambda(t)$ (curve 2). A good conformity of the results is evident. Figure 8 compares values of $\lambda(t)$, obtained from an experiment (acc. to [9]) whith the results of the calculations of $\lambda(t)$ for the three-region model in a case of a quasi-steady and a variable dynamic velocity. The

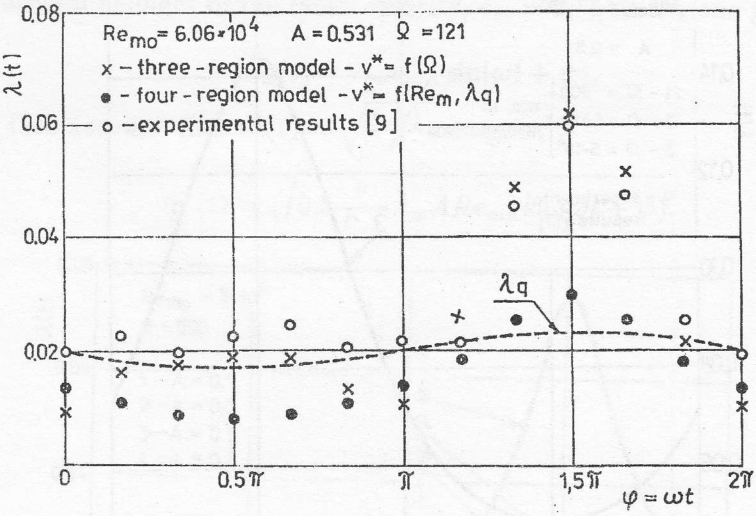

Fig. 8. Instantaneous friction factor for $\Omega = 121$

greatest scatter of calculation results occurs in the second half-period of pulsati[on] cycle (from $\varphi = \pi$ to $\varphi = 2\pi$). For this range the calculation results of the varia[ble] dynamic velocity model are approximate to the experimental results. Figures [7] and 8 present also results of the calculations for quasi-steady friction factor, a[cc.] to (40). These results differ considerably from the experimental results.

6. Conclusions

The paper presents mathematical model of the unsteady turbulent flow [of] liquid in pipes, based on the quasi-steady distribution of turbulent viscosity co-efficient in a pipe cross-section. On a base of these models, instantaneous fricti[on] factor $\lambda(t)$ in the range of low ($\Omega = 121$) and high ($\Omega = 6850$) frequencies confir[ms] usefulness of the worked out models for the determination of hydraulic resista[nce] in pipes. Besides, it has been ascertained that the instantaneous value of t[he] friction factor considerably differes from the quasi-steady one. This fact shou[ld] be taken into account in dynamic calculations of hydraulic systems (e. g. duri[ng] calculations of pressure changes for water hammer by means of characteristic me-thod). In this connection one should tend to work out possibly simple models [of] hydraulic losses functionally dependent on mean velocity changes in time, helpf[ul] for calculations of transients in hydraulic systems. Such models are worked [out] for the case of unsteady laminar pipe flow [17]. Lack of such models results [in] carrying out calculations which are based on *quasi-steady* hydraulic resistance.

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Współczynnik strat tarcia w przewodach zamkniętych podczas pulsującego turbulentnego przepływu cieczy

Streszczenie

W pracy przedstawiono modele matematyczne nieustalonego turbulentnego przepływu cieczy w prze wodach zamkniętych z uwzglednieniem quasi-ustalonego rozkładu współczynnika lepkości turbule w przekroju poprzecznym przewodu. Oparte są one o czwórwarstwowy i trójwarstwowy rozkład wsp czynnika lepkości turbuletnej, przy czym w drugim przypadku dla niskich częstotliwości uzmienn prędkość dynamiczną w funkcji częstotliwości. Modele te adaptowano do wyznaczenia chwilowego wsp czynnika strat tarcia. Wyniki obliczeń numeryczynch chwilowego współczynika strat tarcia porów z odpowiednimi wynikami badań eksperymentalnych, uzuskując dobrą zgodność. Pokazano, że chwi współczynnik strat tarcia znacznie odbiego od wartości quasi-ustalonej.