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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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Kielce

Energetic Possibilities of One-Membrane Osmotic Motor (Practical Description)

The main idea of a practical method of solving problems connected with osmotic or osmotic-and-diffusive conversion of free energy of solutions with different concentrations is presented in this work. We propose an application of the method to studies of the energetic efficiency of the one-membrane osmotic motor, which is described by the Kedem-Katchalsky formalism. Proper equations for useful power, dissipated power, total power and the energetic efficiency have been derived for that osmotic motor. These equations make possible detailed analysis of energetic problems referring to the work of the motor. They also enable us to perform its energetic optimization just at the stage of designing.

List of symbols

J_v – volume flux,
$\Delta\Pi$ – osmotic pressure difference,
$\Delta P = P_m - P_o$ – mechanical pressure difference,
M, M^l – membranes (see Fig. 1),
L_p – filtration coefficient of the membrane M,
σ – reflection coefficient of the membrane M,
ω – permeability,
c_1, c_2, c_m – concentrations,
L – filtration coefficient of the membrane M^l,
$\sigma = 0$ – reflection coefficient of the membrane M^l,
j_s – solute flux,
$\bar{c} \approx (c_1 + c_2)/2$ – average concentration,
j_D – diffusion flux,
j_u – solute flux carried away by osmotic flow J_v,
S – active membrane surface,
M' – power,
M_{SD} – difference of powers,
M_c – total power,
η_{oD} – energetic efficiency,
J_{vo} – volume flux through membrane of the osmotic motor,
J_{vp} – volume flux pumped by pump P,
J_{vt} – volume flux flowing through turbine T,
j_{up}, j_{ut} – solute fluxes (carried away by fluxes J_{vp} and J_{vt}),
M_{ut} – power absorbed by the turbine T,
M_{up} – power given by pump in the shape of mechanical work,
M_{ip} – power connected with transport of molecules of solute by flux J_{vp} to compartment A (see Fig. 2),
M_{it} – power emitted outside the system as free energy caused by flux j_{ut},
M_p – power of pump feeder,
$M_i = M_{ip} - M_{it}$ – net power,
η_p – energetic efficiency of pump P,
η_t – energetic efficiency of turbine T,

F – force,
v – velocity,
M_{ro} – power dissipated in membrane M,
M_{uo} – effective power,
M'_{co} – sum of power,
M_{uD} – power (result of existence of flux J_v),
M_{co} – total osmotic power,
j_{vs}, j_{vD}, j_{vu} – fluxes (in volume units),
M_{rD} – power dissipated as a result of flux j_D.
M_{um} – power emitted by the turbine as mechanical work,
M_o – osmotic component of effective power emitted by the turbine,
M_{rp} – power dissipated in the pump,
M_{rt} – power dissipated in the turbine,
T – temperature,
R – gas constant,
$\Delta c = c_m - c_o$ – concentration difference (see Fig.2),
S_p – cross-section of flux J_{vp},
S_t – cross-section of flux J_{vt},

1. Introduction

Osmosis, which is a process of passive permeation of solvent across semipermeable or selective membranes, is also one of the ways to convert free energy of solutions to effective work. It is the result of the fact that osmotic fluxes can be directed against external forces. It should be added here that the existence of solution concentration difference ΔC is the necessary condition for such energy conversion, analogously to the heat device, for which the temperature difference ΔT is necessary.

Osmosis regarded as an energy conversion device was a subject of numerous works [1–13]. These studies were undertaken mainly for technological and biophysical points. An interest in osmosis from technological point of view follows from the existence of great volumes of salted (e.g. sea) and saltless water (e.g. river water) in nature. Free energy contained in the water can be osmotically converted to effective work. In general we can say that technological solution of problems of osmotic energy conversion resolves itself into designing and construction of such membrane (osmotic) systems which could work in a permanent way and which could be treated as osmotic motors.

Up to now three types of such machines (motors) have been developed.
1. In 1974 Levenspiel and Nevers [3] put forward an idea of a two-membrane osmotic motor. Their motor based on semipermeable membranes is not likely to work in practice as they would have to be affected by very high hydrostatic pressures $\Delta P = \rho g h$ (amounting to $2.3 \cdot 10^6[\frac{N}{m^2}]$).
2. A different type of osmotic machine, satisfying the criteria of the motor, was proposed by us. Its physico-technical principles of operation are given in [4,11,12]. This particular motor was based on two selective membranes differing in parameters of permeation.
3. Another example of machine capable of converting permanently free energy into effective work is the one-membrane osmotic motor described, e.g. in the work of Lee, Baker and Lonsdale [1]. This motor works under laboratory-technical conditions at the mouth of the Jordan into the Dead Sea.

Generally speaking, if the process of balancing concentrations, within motor operation, proceeded in a reversible manner, the obtained work would be maximum and its values would be easy to calculate from the principles of ther-

modynamics. However, in the actual motors selective membranes are used and they let pass not only water but also solute to a certain extent. Thus, there is a question of dissipation of energy due to water flow with a certain velocity against the forces of viscosity resistance and to diffusion of solute particles across a membrane. These and other energetic problems as well as the high costs of producing membranes [1] still make all the types of osmotic motors in use economically unprofitable. Further investigations are necessary to achieve, among others, optimization of their work. It also refers to the one-membrane motor which has been incompletely discussed in this respect so far. Now we can deal with it thoroughly owing to our analytic method of the quantitative testing of osmotic-and-diffusive conversion of free energy, which was once described in Kargol [13].
In the present paper we first introduce the idea of this method, and, then, show how it was used to test this one-membrane osmotic motor from the energetic point of view. The transport equations, the power equation and the energetic efficiency expressions concerning this motor are derived. These equations make it possible to consider the energetic possibilities of the motor in detail, which is accounted for by some exemplary numeric calculations. Let us add that the suggested method is competitive with the idea of Kedem and Caplan [10] or Peusner [6,7] because of its simplicity and because it gives us the possibility of complex energetic analysis of osmotic and osmotic-and-diffusive energy conversion.

2. Idea of the practical method of investigation of osmotic-and-diffusive energy conversion

In this section we will briefly describe our practical method [13] to investigate osmotic or osmotic-and-diffusive conversion of free energy of solutions with different concentrations. For that purpose we will consider a suitable membrane system. Our method consists of several steps. First we have to write phenomenological transport equations for given osmotic-and-diffusive system and next we derive so-called global power equations. From them one can directly obtain equations for: effective, dissipated and total power. The last step is to derive the expression for energetic efficiency of osmotic-and-diffusive energy conversion (i.e. effective to total power ratio).

2.1. Membrane system and transport equations

A two-membrane system as shown in Fig. 1 will be considered. In this system, membrane M with surface S, filtration coefficent L_p, reflection coefficent σ and permeability ω separates two compartments A and B, each filled with a wellmixed solution differing in concentration ($C_1 < C_2$). As a result, osmotic volume flow J_v is generated on the membrane. Membrane M^l with filtration coefficent L, permeability ω_l and reflection coefficent $\sigma_l = 0$ separates compartments B and C both filled with solutions with concentration C_2. This membrane will be considered as energy acceptor. Let us assume, for the sake of simplification, that mechanical pressure in the outer compartments are identical ($P_A = P_C = P_0$). We will also assume that the volumes of all the three compartments are sufficiently large so that permeation processes can be regarded as quasi-stationary (with fixed active surfaces S of the membranes).

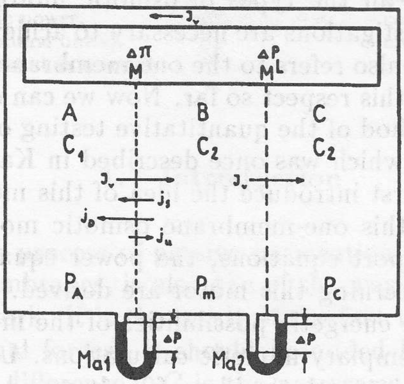

Fig. 1. Schematic representation of the osmotic system (A, B, C – compartments, M, M^l – membranes, c_1, c_2 – concentrations of solutions, M_{a1}, M_{a2} – manometers, $\Delta\Pi$ – osmotic pressure difference, ΔP – mechanic pressure difference, J_v – volume flow).

According to the Kedem-Katchalsky formalism (Katchalsky and Curran [15]) we can write, for the considered membrane system:

$$J_v = L_p \sigma \Delta\pi - L_p \Delta P, \tag{1}$$

where: $\Delta P = P_m - P_o$, and P_m is the osmotically generated mechanical pressure in compartment B. In agreement with the principles of hydrodynamics, a pressure ΔP will occure across both membranes and will be indicated by manometers M_{a1} and M_{a2} (Fig. 1). Pressure difference ΔP across membrane M^l will be considered as external pressure. Flux J_v flows across membrane M^l. Therefore we can write:

$$J_v = L\Delta P \tag{2}$$

Using Eqs. (1) and (2) we get:

$$j_v = \frac{L L_p \sigma \Delta\pi}{L + L_p} \tag{3}$$

With a selective membrane (i.e. $0 < \sigma < 1$) the solute flux j_s across the membrane will also occur. This flux will be described by:

$$j_s = \omega\Delta\pi - (1-\sigma)\bar{c}J_v\,, \tag{4}$$

where: $\bar{c}$ is the average concentration ($\bar{c} = (C_1 - C_2)/(\ln(C_1/C_2))$. The particular terms in equation (4):

$$j_D = \omega\Delta\pi\,, \tag{5}$$

$$j_u = (1-\sigma)\bar{c}J_v \tag{6}$$

represent the diffusion flux (j_D) and the solvent flux (j_u) carried by osmotic flow J_v in membrane pores.

2.2. Power equations

If membrane M (see Fig. 1) is selective ($0 < \sigma < 1$), then, due to the existence of fluxes J_v and j_s, both osmotic and diffusive energy conversion are possible. We will consider free energy conversion to effective work, free energy dissipation and conversion efficiency. Eqs. (3) and (4) will be used for the analysis of the osmotic and diffusive conversion respectively.

Osmotic energy conversion.

Let us multiple equation (3) by factor the $S(J_v/L_p)$ and rewrite it in the form

$$S\sigma\Delta\pi J_v = S\frac{J_v^2}{L_p} + S\frac{J_v^2}{L}\,, \tag{7}$$

where S is the membrane surface.

We will term the equation obtained the global power equation [13]. In accordance with $M = Fv = S\Delta Pv$ (where: M – power, F – force, v – velocity) the particular terms

$$M'_{co} = S\sigma\Delta\pi J_v\,, \tag{8}$$

$$M_{ro} = S\frac{J_v^2}{L_p}, \tag{9}$$

$$M_{uo} = S\frac{J_v^2}{L} = S\Delta PJ_v, \quad \text{(because } J_v = L\Delta P) \tag{10}$$

denote sum of powers (M'_{co}); power (M_{ro}) dissipated in membrane M, i.e. the amount of energy dissipated in a unit of time because of solution viscosity; and effective power (M_{uo}) absorbed and emitted by membrane M^l considered as energy acceptor. According to Eq. (7),

$$M'_{co} = M_{ro} + M_{uo} \,. \tag{11}$$

Diffusive energy conversion.

Let us now consider the diffusive conversion of energy [13]. First of all it should be noted that, because $0 < \sigma < 1$, fluxes J_v and j_s will flow across membrane M simultaneously. This means that both the osmotic and the diffusive conversion of energy will occur in the system. To solve the problem of diffusive energy conversion fluxes j_s, j_D and j_u are related to volume units:

$$j_{vs} = \frac{j_s}{\bar{c}} = \frac{\omega}{\bar{c}}\Delta\pi - (1-\sigma)J_v \,, \tag{12}$$

$$j_{vD} = \frac{\omega}{\bar{c}}\Delta\pi \,, \tag{13}$$

$$j_{vu} = (1-\sigma)J_v \,. \tag{14}$$

Multiplying Eq. (12) by $S\Delta\pi$ the following power is obtained:

$$Sj_{vs}\Delta\pi = S\frac{\omega}{\bar{c}}\Delta\pi^2 - S(1-\sigma)J_v\Delta\pi \,. \tag{15}$$

In this equation, according to the basic formula of mechanics $M = Fv$, the particular terms

$$M_{rD} = S\frac{\omega}{\bar{c}}\Delta\Pi^2 \tag{16}$$

$$M_{uD} = S(1-\sigma)J_{vp}\Delta\Pi \,, \tag{17}$$

$$M_{sD} = Sj_{vs}\Delta\Pi \tag{18}$$

express: M_{rD} the power dissipated due to the presence of flux j_D; M_{uD} the power recovered in terms of free energy as a result of removing solute molecules by flow J_v (in poress of membrane M). M_{sD} is the difference between M_{rD} and M_{uD}. According to Eq. (15)

$$M_{sD} + M_{uD} = M_{rD} \,. \tag{19}$$

2.3. The energetic efficiency of system

According to our previous considerations and using the energy conservation law we can write (see (11) and (19)):

$$M_{co} + M_{sD} = M_{uo} + M_{ro} + M_{rD} = M_c\,, \tag{20}$$

where: $M_{co} = M'_{co} + M_{uD}$.
Next, applying the general definition of efficiency:

$$\eta = \frac{M_u}{M_c}\,,$$

(where: M_u – effective power, M_c – total power) and Eqs. (1), (2), (9), (10), (16), (20) we obtain the expression for total energetic efficiency of the osmotic-and-diffusive conversion of energy [13]:

$$\eta_{oD} = \frac{M_{uo}}{M_{uo} + M_{ro} + M_{rD}} = \frac{\frac{J_v^2}{L}}{\frac{J_v^2}{L} + \frac{J_v^2}{L_p} + \frac{\omega}{c}\Delta\pi^2}\,. \tag{21}$$

3. One-membrane osmotic motor and its energetic analysis

3.1. One-membrane osmotic motor and transport equations

In this section we will apply the above-mentioned method to energetic analysis of the one-membrane osmotic motor described in the paper by Lee, Baker and Lonsdale [1]. This motor, partly treated so far as to its energetic possibilities, is shown schematically in Fig. 2. It consists of pump P, turbine T and chamber A with membrane M. The permeability of the membrane is described by parameters L_p, σ, and ω. We will consider only the case when concentrations fulfill the condition $C_2 > C_o$.

According to the laws of hydrodynamics we can write, for that motor (see Fig. 2):

$$J_{vt} = J_{vp} + J_{vo}\,, \tag{22}$$

where: J_{vp} – volume flux pumped by pump P; J_{vo} – osmotic volume flux which permeates across membrane M; J_{vt} – volume flux flowing through turbine T_r. If the reflection coefficient σ of the membrane fulfills the condition $0 < \sigma < 1$ then, apart from flux J_{vo}, solute flux j_s permeates across the membrane.

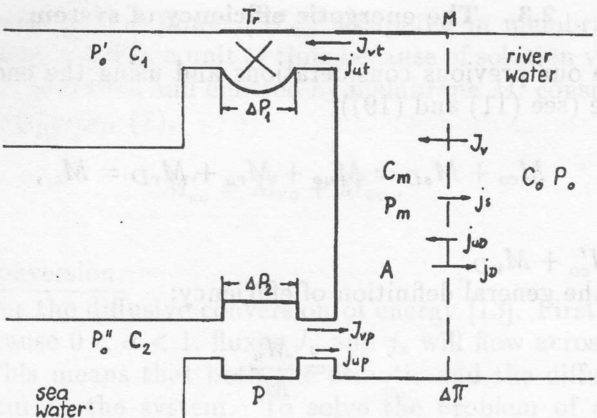

Fig. 2. One-membrane osmotic motor (M – membrane, P – pump, T_r – turbine).

These fluxes, according to the Kedem-Katchalsky formalism, (see eq. (1) and (4)) are described by the equations:

$$J_v = L_p \sigma RT(C_m - C_o) - L_p \Delta P \, , \tag{23}$$

$$j_s = \omega RT(C_m - C_o) - (1-\sigma)\frac{C_m + C_o}{2} J_{vo} \, , \tag{24}$$

where: C_m – concentration of solution in chamber A, $\Delta P = P_m - P_o$ – mechanical pressure difference on membrane, $\frac{C_m + C_o}{2} = \bar{c}$.

It should be mentioned that fluxes J_{vp} and J_{vt} are always accompanied by appropriate solute fluxes j_{up} and j_{ut} (carried away by fluxes J_{vp} and J_{vt}). These fluxes can be described in the following way:

$$j_{up} = C_2 J_{vp} \, , \tag{25}$$

$$j_{ut} = C_m J_{vt} \, , \tag{26}$$

where: C_2 – concentration of the solution pumped by pump P; C_m – concentration of the solution which flows through turbine T_r.

In stationary state the following equation holds:

$$j_{up} = j_{ut} + j_s \, . \tag{27}$$

On the basis of equations (22), (24), (25), (26) and (27) it's easy to get the following expression of concentration C_m in the chamber A of the motor

$$C_m = \frac{C_2 J_{vp} + \omega RTC_o + \frac{1}{2}(1-\sigma)C_o J_{vo}}{J_{vp} + J_{vo} + \omega RT - \frac{1}{2}(1-\sigma)J_{vo}}. \tag{28}$$

Let us discuss this equation. According to our expectations we got from the discussion the following results.
1. If the membrane M is semipermaeble (it means that $\omega = 0$ and $\sigma = 1$) then:

$$C_m = \frac{C_2 J_{vp}}{J_{vp} + J_{vo}}.$$

Hence it follows that with $J_{vp} \gg J_{vo}$ we have $C_m \simeq C_2$. If however $J_{vp} = J_{vo}$, we get $C_m = 0,5C_2$.
2. Assuming that membrane M is permeable ($\sigma = 0$) and the pressure difference ΔP on it is zero (this implying that $J_{vo} = 0$) and $C_o = 0$, from equation (28) it follows that:

$$C_m = \frac{C_2 J_{vp}}{J_{vp} + \omega RT}.$$

Assuming, moreover, that $\omega = 0$ we get $C_m = C_2$.
Further investigation of the motor we conduct assuming that $C_o = 0$. This assumption facilitates the calculations markedly, while practically it does not diminish the interpretation possibilities concerning the engine. So assuming that $C_o = 0$, equations (23) and (28) can be written as

$$J_{vo} = L_p \sigma RTC_m - L_p \Delta P, \tag{29}$$

$$C_m = \frac{C_2 J_{vp}}{J_{vp} + \omega RT + \frac{1}{2}(1+\sigma)J_{vo}}. \tag{30}$$

In the above set of equations the unknowns are C_m and J_{vo}. Solving them we get the following square trinomial:

$$AC_m^2 + BC_m + C = 0, \tag{31}$$

where: $A = 0.5(1+\sigma)L_p\sigma RT$, $B = J_{vp} + \omega RT - 0.5(1+\sigma)L_p\Delta P$, $C = -C_2 J_{vp}$.
Discriminant of the trinomial $\Delta = B^2 - 4AC > 0$.
Thus the trinomial has two solutions: $C_{m1} < 0$ and $C_{m2} > 0$. Only the second solution has a physical meaning. So we can write:

$$C_m = C_{m2} = \frac{-B + \sqrt{\Delta}}{2A}, \tag{32}$$

where: $\Delta = [J_{vp} + \omega RT - \frac{1}{2}(1+\sigma)L_p\Delta P]^2 + 2(1+\sigma)L_p\sigma RTC_2J_{vp}$. Using the last expression and equation (29) we find the sought expression fo osmotic volume flow J_{vo} permeating the membrane

$$J_{vo} = \frac{-[J_{vp} + \omega RT + \frac{1}{2}(1+\sigma)L_p\Delta P] + \sqrt{\Delta}}{(1+\sigma)}. \tag{33}$$

It should be added here that for equal P_o' and P_o (see Fig. 2) the difference $\Delta P = P_m - P_o$ on the membrane is a measure of loading the turbine with external forces.
From equations (32) and (33) it follows that concentration C_m and flux J_{vo} are functions of many variables, pressure difference ΔP among them.

3.2. Power equations of one-membrane osmotic motor

Now we can proceed with analysis of the motor from energetic point of view. We will apply the above-mentioned practical and analytic method of investigation. According to the method, on the basis of equation (29), we obtain the global equation of power for the process of osmosis:

$$S\sigma RTC_mJ_{vo} = S\frac{J_{vo}^2}{L_p} + S\Delta PJ_{vo}\,, \tag{34}$$

or, taking into account the expression $J_{vo} = J_{vt} - J_{vp}$:

$$S\sigma RTC_mJ_{vo} = S\frac{J_{vo}^2}{L_p} + S\Delta PJ_{vt} - S\Delta PJ_{vp}\,. \tag{35}$$

The fluxes J_{vp}, J_{vt} and J_{vo} are expressed in the units of velocity $[\frac{m}{s}]$. Each part of this equation satisfies the expression of mechanics: $M = Fv$.
If we introduce notations:

$$M'_{co} = S\sigma RTC_mJ_{vo}\,, \tag{36}$$

$$M_{ro} = S\frac{J_{vo}^2}{L_p}\,, \tag{37}$$

$$M_{uo} = S\Delta PJ_{vo}\,, \tag{38}$$

$$M_{ut} = S\Delta P J_{vt}\,, \tag{39}$$

$$M_{up} = S\Delta P J_{vp} \tag{40}$$

then equation (35) can be written in form:

$$M'_{co} = M_{ro} + M_{uo} \tag{41}$$

or

$$M'_{co} = M_{ro} + M_{ut} - M_{up}\,,$$

where: M'_{co} – power emitted in the process of osmosis; M_{ro} – power dissipated on membrane M; M_{ut} – power absorbed by the turbine as mechanical work; M_{up} – power given by pump in the shape of mechanical work; $M_{uo} = M_{ut} - M_{up}$ – osmotic component of useful of power absorbed by the turbine.
Let us now consider the question of dissipated power in the process of solute flux j_s permeation across membrane M. The flux, given by equation (24), can be expressed, putting $C_o = 0$, in units of velocity

$$j_{vs} = \frac{j_s}{\bar{c}} = \frac{\omega}{\bar{c}} RTC_m - (1-\sigma)J_{vo}\,.$$

Each part of this equation is expressed as velocity $[\frac{m}{s}]$.

Proceeding according to the proposed method (on multiplication by $SRTC_m$) we obtain the following global power equation for diffusion:

$$Sj_{vs}RTC_m = S\frac{\omega}{\bar{c}}(RT)^2C_m^2 - S(1-\sigma)J_{vo}RTC_m\,. \tag{42}$$

Each part of this equation satisfies the expression of mechanics $M = Fv$.

Introducing the notation:

$$M_{sD} = Sj_{vs}RTC_m\,, \tag{43}$$

$$M_{rD} = S\frac{\omega}{\bar{c}}(RT)^2C_m^2 \tag{44}$$

$$M_{uD} = S(1-\sigma)J_{vo}RTC_m\,, \tag{45}$$

equation (42) can be written as:

$$M_{sD} = M_{rD} - M_{uD}\,. \tag{46}$$

where, let us remind: M_{rD} – power dissipated in the process of diffusion, M_{uD} – power which is reclaimed by the system because the solute is carried away by flux J_{vo}, M_{sD} – net power.
Using (41) and (46), satisfying the energy conservation law, we get:

$$M_c = M_{co} + M_{sD} = M_{uo} + M_{ro} + M_{rD}\,, \tag{47}$$

where: $M_{co} = M'_{co} + M_{uD}$.
It should be added that pumping of solution C_2 by pump P means that besides supplying the motor with external power described by equation (40), there also takes place supplying it with power M_{ip} in the form of internal energy. This power is connected with transport of molecules of solute compartment A with speed $v = J_{vp}$, because the substance is translocated against osmotic pressure $\Delta\pi_2 = RT(C_2 - C_m)$. We can express this power as:

$$M_{ip} = S_p \Delta\pi_2 J_{vp} = S_p RT(C_2 - C_m)J_{vp}\,, \tag{48}$$

where: S_p – cross-section of flux J_{vp}.
The existence of flux J_{vt} results in driving the turbine and therefore in emission of power M_{ut} in the form of mechanical work. This also causes transport of solute from chamber A (with solution concentration C_m) to solution with concentration C_o smaller that C_m (see Fig. 2). This means that the system loses power in the form of free energy. This power can be evaluated from the formula:

$$M_{it} = S_t \Delta\pi_2 J_{vt} = S_t RT(C_m - C_o)J_{vt}\,, \tag{49}$$

where: S_t – cross-section of flux J_{vt}.

The above considerations result in the following conclusions:
1. The system (the motor) is supplied with power M_{up} and M_{ip}. Besides, the motor is provided with the M_{uD} power as a result of carrying the solute across the flux J_{vo}.
2. The system emits: power M_{ut} and power M_{it} as free energy, moreover the system dissipates power M_{ro} and M_{rD} (in membrane). These conclusions can be written in the form of the energy conservation law:

$$M_{up} + M_{ip} + M_{uD} = M_{it} + M_{ut} + M_{ro} + M_{rD}\,, \tag{50}$$

or:

$$M_i + M_{uD} = M_{uo} + M_{ro} + M_{rD} = M_c\,, \tag{51}$$

where: $M_{uo} = M_{ut} - M_{up}$ – power emitted in an osmotic way, $M_i = M_{ip} - M_{it}$ – net power, the system is supplied with (in the form of free energy), M_c – total power.

3.3. The energetic efficiency of the one-membrane osmotic motor

Following the method presented in section 2 one can obtain an expression for the efficiency of osmotic-and-diffusive conversion of free energy in the osmotic motor:

$$\eta_{oD} = \frac{M_{uo}}{M_c} = \frac{M_{uo}}{M_{uo} + M_{ro} + M_{rD}}, \tag{52}$$

where: $M_{uo} = S\Delta P J_{vo}$, $M_{ro} = S\frac{J_{vo}^2}{L_p}$, $M_{rD} = S\frac{\omega}{\bar{c}}\Delta\pi^2 = S\frac{\omega}{\bar{c}}(RT)^2 C_m^2 = 2S\omega(RT)^2 C_m$; because if $C_o = 0$ then $\bar{c} = \frac{1}{2}C_m$ (see Fig. 2).

It should be mentioned that in a real osmotic motor the pump and the turbine work with certain energetic efficiencies. Assuming that the pump efficiency is equal to η_p we obtain that the pump supplies the system with the power (in the form of mechanical work):

$$M_{up} = \eta_p M_p , \tag{53}$$

where: M_p – power of the pump feeder.
The energy receiver (the turbine) is supplied with the total power:

$$M_{ut} = \eta_p M_p + M_{uo} = \eta_p M_p + \eta_{oD} M_c , \tag{54}$$

where: $M_{uo} = \eta_{oD} M_c$ – the component of osmotic power (because $\eta_{oD} = \frac{M_{uo}}{M_c}$). The turbine works with the efficiency η_t therefore it emits power outside (as mechanical work):

$$M_{um} = \eta_t M_{ut} . \tag{55}$$

Using the expression (54) we get:

$$M_{um} = \eta_t(\eta_p M_p + \eta_{oD} M_c) = \eta_t(M_u p + M_{uo}) . \tag{56}$$

If we subtract power M_p from that power M_{um} then we get only osmotic component of useful power M_o emitted in the form of mechanical work:

$$M_o = M_{um} - M_p = M_p(\eta_t\eta_t - 1) + \eta_t\eta_{oD} M_c . \tag{57}$$

From this equation we can observe that η_t and η_p are given the motor can be profitable from the economical point of view if $M_o > 0$. In particular case if $\eta_t = 1$ and $\eta_p = 1$ then

$$M_o = \eta_{oD} M_c = M_{uo}\,. \tag{58}$$

In order to sum up the above considerations, we have shown in Fig. 3 a block scheme of the discussed engine with the respective powers specified i.e. power input, power dissipated and power output in the form of mechanical work.

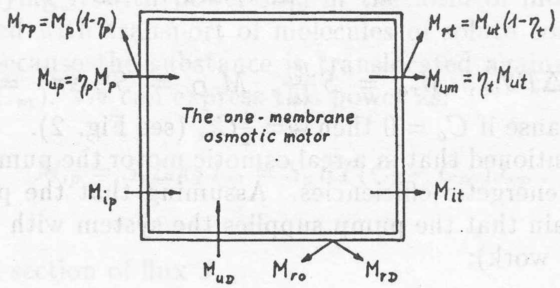

Fig. 3. Block diagram of 1-membrane osmotic motor (M_{up} – power given by the pump in the shape of mechanical work, M_{ip} – power given by the pump in the shape of free energy, M_{uD} – power recovered in the process of osmosis, M_{rp} – power dissipated in the pump, M_{ro} – power dissipated in the membrane, M_{rD} – power dissipated by diffusion, M_{rt} – power dissipated in the turbine, M_{um} – power emitted as mechanical work, M_{it} – power emitted as free energy).

In conclusion of the sented considerations one can state that the obtained equations enable us to perform a detailed analysis of osmotic-hydro-mechanical and energetic possibilities of the one-membrane osmotic motor. These equations are of great importance for optimization of the work of the considered osmotic motor.

It is verified by effects the calculations presented in the next part of this work.

4. Results of calculations and conclusions drawn from them

Having in view the usefulness of the above considerations, we now present some selected results of the calculations. They refer to energetic problems in the functioning of 1-membrane osmotic engine, including optimization problems. The calculations were performed with some simplifying assumptions. It was assumed, for instance, that energy efficiency of the pump and turbine are: $\eta_p = 1$ and $\eta_t = 1$. It was also assumed that flux J_{vp} through the pump has constant

value ($J_{vp} = const.$), independent of the load on the turbine (i.e. of the pressure ΔP). Further, it was assumed that $P'_o \cong P_o$ (see Fig. 3) which implies that $\Delta P = P_m - P_o$ is the pressure difference on the membrane M. The following data were taken for the calculations:
$c_o = 0[M]$, $c_m = 0,4[M]$, $L_p = 5 \cdot 10^{-12}[m^3N^{-1}s^{-1}]$, $\sigma = 0.044$, $\omega = 6.8 \cdot 10^{-11}[molN^{-1}s^{-1}]$, $S = 1[m^2]$, $T = 300[K], R = 8.31[Jmol^{-1}K^{-1}]$, $J_{vp} = 9 \cdot 10^{-8}[ms^{-1}]$, $J_{vp1} = 9 \cdot 10^{-7}[ms^{-1}]$, $J_{vp2} = 9 \cdot 10^{-6}[ms^{-1}]$.
A change of load on the turbine (i.e. of pressure ΔP) induces a change in concentration c_m (according to eq. 32), a change in flux J_{vo} (according to eq. 33) and in flux $J_{vt} = J_{vp} + J_{vo}$. For each pressure ΔP the above quantities have fixed values in stationary state (corresponding to each ΔP). We can thus say that changing ΔP and reaching a stationary state of the engine we change its working conditions. Changes in working conditions of the engine can also be considered by changing other quantities, e.g. fluxes J_{vp}, or parameters L_p, σ and ω using different membranes. In the present work we are interested mainly in working of 1-membrane engine as dependent on loading ΔP (at certain fixed values of flux J_{vp}).
1. Relationships: $c_m = f(\Delta P)$ and $J_{vo} = f(\Delta P)$.
Using the above numerical data and formulas (32) and (33) the relationships $c_m = f(\Delta P)$ and $J_{vo} = f(\Delta P)$ were calculated and plotted for the three different values of flux J_{vp} : $9 \cdot 10^{-8}[ms^{-1}]$, $9 \cdot 10^{-7}[ms^{-1}]$ and $9 \cdot 10^{-6}[ms^{-1}]$. Figures 4 and 5 show the relationships. From the plots in Fig. 4 it can be seen that concentration c_m weakly depends on load ΔP of the turbine. That concentrations depends, however, markedly on fluxes J_{vo}, this being evidenced by the separation of the curves 1, 2 and 3.

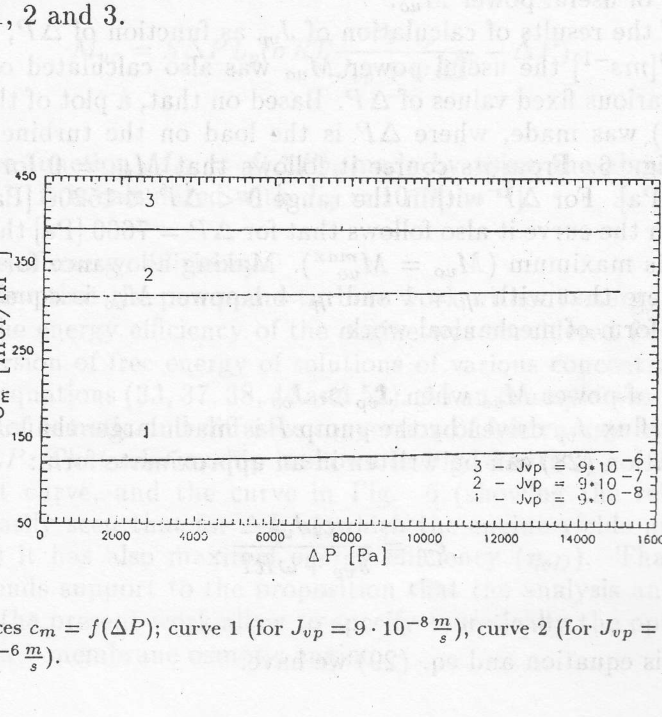

Fig. 4. Dependences $c_m = f(\Delta P)$; curve 1 (for $J_{vp} = 9 \cdot 10^{-8}\,\frac{m}{s}$), curve 2 (for $J_{vp} = 9 \cdot 10^{-7}\,\frac{m}{s}$), curve 3 (for $J_{vp} = 9 \cdot 10^{-6}\,\frac{m}{s}$).

From plots 1, 2, 3 in Fig. 5 it follows that flux J_{vo} depends on the load ΔP markedly; decreases with increasing ΔP (at fixed J_{vp}). The separation of the curves 1, 2 and 3 means that flux J_{vo} markedly depends on flux J_{vp}: the greater is the flux J_{vp} (at constant ΔP) the greater the value of flux J_{vo}.

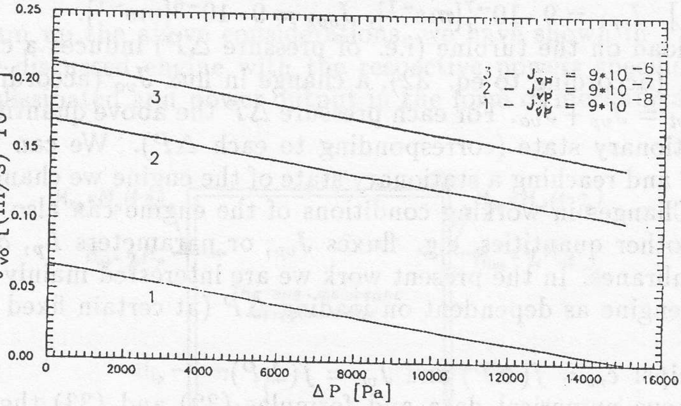

Fig. 5. Dependences $J_v = f(\Delta P)$; curve 1 (for $J_{vp} = 9 \cdot 10^{-8} \frac{m}{s}$), curve 2 (for $J_{vp} = 9 \cdot 10^{-7} \frac{m}{s}$), curve 3 (for $J_{vp} = 9 \cdot 10^{-6} \frac{m}{s}$).

2. Calculation of useful power M_{uo}.
Making use of the results of calculation of J_{vp} as function of ΔP, obtained with $J_{vp} = 9 \cdot 10^{-8} [ms^{-1}]$ the useful power M_{uo} was also calculated on the basis of Eq. (38), for various fixed values of ΔP. Based on that, a plot of the relationship $M_{uo} = f(\Delta P)$ was made, where ΔP is the load on the turbine T. The plot is shown in Fig. 6. From its course it follows that $M_{uo} = 0$ for $\Delta P = 0$ and $\Delta P \simeq 15200$ [Pa]. For ΔP within the range $0 < \Delta P < 15200$ [Pa] the value of $M_{uo} > 0$. From the curve it also follows that for $\Delta P = 7600$ [Pa] the useful power M_{uo} reaches its maximum ($M_{uo} = M_{uo}^{max}$). Making allowance for equation (58) one can add here that with $\eta_t = 1$ and $\eta_p = 1$ power M_{uo} is equal to power M_e yielded in the form of mechanical work.

3. Calculation of power M_{uo} when $J_{vp} \gg J_{vo}$.
Assuming that flux J_{vp} driven by the pump P is much larger than J_{vo} (even when $\Delta P = 0$), equation (28) can be written in an approximate form:

$$c_m = \frac{c_2 J_{vp}}{J_{vp} + \omega RT}.$$

Then using this equation and eq. (29) we have:

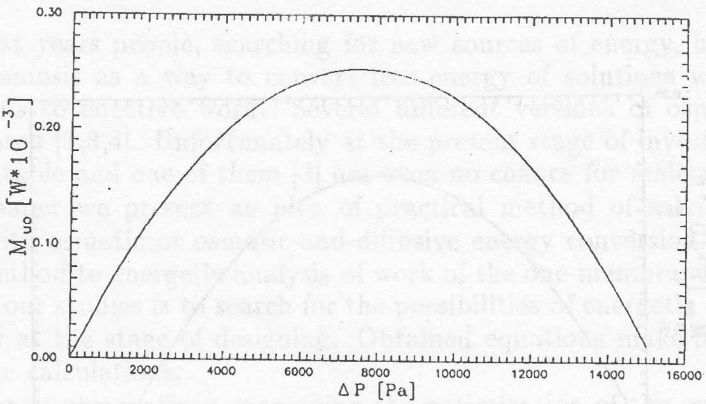

Fig. 6. Dependences $M_o = f(\Delta P)_{J_{vp}=9\cdot 10^{-8}\frac{m}{s}}$.

$$J_{vo} = L_p \sigma RT \frac{c_2 J_{vp}}{J_{vp} + \omega RT} - L_p \Delta P \, .$$

Finally, taking into account (38), we get:

$$M_{uo} = S \Delta P L_p (\sigma RT \frac{c_2 J_{vp}}{J_{vp} + \omega RT} - \Delta P) \, .$$

The plot of the function $M_{uo} = f(\Delta P)$, made by using the above equation, is shown in Fig. 7. It is calculated with $J_{vp} = 10^{-5}[ms^{-2}]$.

4. Calculations of energy efficiency.

Assuming further that the pump and turbine work without energy losses ($\eta_t = 1$ and $\eta_p = 1$), the energy efficiency of the engine was considered for osmotic-and-diffusive conversion of free energy of solutions of various concentration. To this end were used equations (33, 37, 38, 44 and 52). As an example was calculated the coefficient η_{oD} of osmotic-and-diffusive conversion of free energy as dependent on turbine load ΔP. That relationship is shown in Fig. 8 with $J_{vp} = 9 \cdot 10^{-8}[ms^{-1}]$. Analysing that curve, and the curve in Fig. 6 (showing the relation $M_{uo} = f(\Delta P)$) it is easily seen that for ΔP at which the engine yields maximal useful power (M_{uo}^{max}) it has also maximal energy efficiency (η_{oD}). That observation in particular lends support to the proposition that the analysis and calculations carried out in the present work allow to specify teoretically the optimal working conditions for a 1-membrane osmotic engine.

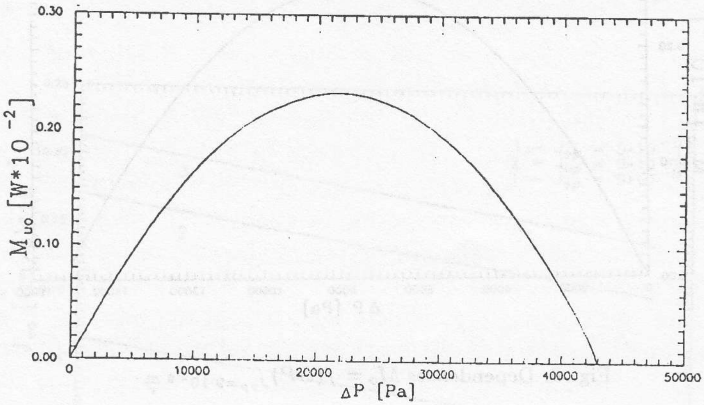

Fig. 7. Dependences $M_{uo} = f(\Delta P)_{J_{vp}=10^{-5}\frac{m}{s} \gg J_{vo}}$.

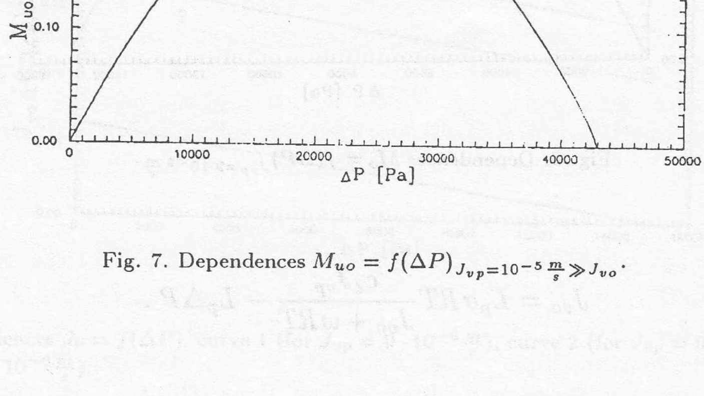

Fig. 8. Dependences $\eta_{oD} = f(\Delta P)_{J_{vp}=9\cdot 10^{-8}\frac{m}{s}}$.

5. Conclusion

In the last years people, searching for new sources of energy, have paid attention to osmosis as a way to convert free energy of solutions with different concentrations to effective work. Several different versions of osmotic motors were elaborated [1,3,4]. Unfortunately at the present stage of investigation they are not profitable and one of them [3] has even no chance for realization.

In this paper we present an idea of practical method of solving problems connected with osmotic or osmotic-and-diffusive energy conversion devices. We apply the method to energetic analysis of work of the one-membrane motor. The main aim of our studies is to search for the possibilities of energetic optimization of the motor at the stage of designing. Obtained equations make it possible by way of simple calculations.

A number of observations concerning the optimization of the one-membrane motor have been made in the present paper. In particular it has been shown that this motor works as its maximum energy efficiency for certain values of external pressures. At this pressures the motor also yields the maximum effective power.

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Praktyczny opis możliwości energetycznych 1-membranowego silnika osmotycznego

Streszczenie

W pracy przedstawiono ideę praktycznej metody rozpatrywania w sposób kompleksowy zagadnień osmotyczno-dyfuzyjnego przetwarzania energii swobodnej roztworów o różnych stężeniach. Następnie zastosowano tę metodę do badania efektywności energetycznej 1-membranowego silnika osmotycznego opisanego formalizmem Kedem-Katchalskyego. Wyprowadzono dla tego silnika odpowiednie równania na moc użyteczną, moc rozpraszaną, moc całkowitą oraz równania na sprawność energetyczną. Równania te pozwalają na szczegółowe rozpatrywanie zagadnień energetycznych dotyczących działania owego silnika. Umożliwiają one także, co po części wykazano w niniejszej pracy, na osiąganie efektów optymalizacji energetycznej rozważanego silnika już w fazie jego projektowania.

Практическая опись энергетических возможностей 1-но мембранного осмотического двигателя

Резюме

В статье представлена идея практического метода рассмотрения комплексным способом проблем осмотично-диффузионного преобразования свободной энергии растворов с разными концентрациями.
Потом был применен этот метод для испытания энергетической эффективности 1-но мембранного осмотического двигателя описанного формализмом Кедем-Качальского. Выведено дла этого двигателя соответствующее уравнение на эффективную, рассеянную, полную мощность, а также уравнения на энергетический коэффициент полезного действия.

Эти уравнения разрешают подробно рассмотреть энергетические проблеммы касающиеся работы этого двигателя. Делают они также возможным, что частично указано в настоыащей статье, на достижение эффектов энергетической оптимализации рассматриваемого двигателя уже в фазе его проектирования.