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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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Stefan-Convection Effects on Evaporation and Condensation of Droplets in Surrounding Vapour*

The influence of phase-change induced convection ("Stefan flux") on droplet-to-vapour mass and heat transfer has been examined. Methods of determining transfer fluxes at various transfer conditions have been developed and new, more general expressions for correction factors have been derived. Calculations of evaporation and condensation processes have been carried out and compared with results of previous methods.

Notation

a – gas phase thermal diffusivity,
B – auxiliary function in (3.14),
b – averaged collision distance, (2.56),
c – specific heat,
e – specific internal energy,
F – free path distribution function,
g – share in incident molecule flux,
H – auxiliary function, (2.32),
h – phase change enthalpy ("latent heat"),
i – specific enthalpy,
Kn – Knudsen number, (2.60),
l – free path of a molecule,
m – mass of a molecule,
p – static pressure,
$R=pv''_s/T_s$ – pseudo-gas constant,
r – radial coordinate,
T – absolute temperature,
t – time (or: auxiliary variable),
$U=\bar{u}/u_x$ – averaged velocity ratio, (2.58),
u – bulk velocity,
v – specific volume,
W – molecular velocity,
z – Cartesian coordinate,
α – bulk flow attack angle,
β – mass transfer coefficient,
Γ – mass flux density,
γ – heat transfer coefficient,
$\delta=T-T_s$ – temperature deviation,
η – dimensionless velocity, (2.12),
$\varkappa$ – isentropic exponent,
Λ – heat flux density,
λ – thermal conductivity,
μ – coefficient combination, (2.48),
ν – geometrical factor, (2.27),
ξ – condensation coefficient,
σ – surface tension,
Φ – Maxwellian velocity distribution function,
φ – elevation angle,
χ – accomodation coefficient of molecule reflection,
Ψ – energy flux density,
ψ – meridional angle,
Ω – auxiliary expression in (2.15), (2.36).

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Indices

b – distance of averaged collision effects,
ev – evaporated molecule stream,
m – mean value, – intermediate boundary,
s – saturation,
T – transport energy,
0 – reference values (without convection influence),
∞ – plane phase boundary, – far field conditions,
$( )'$ – liquid phase, emitted fluxes,
$( )''$ – gas phase, incident fluxes,
$(\tilde{})$ – denotes averaging over ensamble of striking molecules (contrary to the ensamble of resident molecules).

1. Introduction and Main Assumptions

Investigation of droplet-to-vapour transfer processes in wet steam containing small liquid droplets is important as the knowledge of interphase transfer fluxes is essential in determining the course of wet steam processes occurring in many technical applications, such as e.g. in power plant machines and equipment [3, 5, 6]. In particular, the coupled interphase heat and mass transfer processes determine the evaporation or condensation rate of droplets.

These phase change processes on the droplet surface cause a displacement of the phase boundary, occurring with a finite velocity in accordance to the relevant mass transfer rate. If the densities of the phases differ from each other, bulk substance movements will arise, which are called "Stefan-flow". The phenomenon becomes pronounced in liquid-vapour systems where the density differences are substantial. It causes phase-change induced convection flows in the less dense (i.e. gaseous) phase, which result in distortion of temperature resp. concentration profiles around the droplet and thus exert an additional effect on the heat transfer and evaporation rates.

These effects are in essence non-linear. With the assumption of a quasi-steady situation and some additional simplifications, their description may be fitted into linear calculation models.

Analytical treatments of these phenomena, encountered in literature, are dealing mainly with melting/solidifaction processes, with attention focussed on the phase boundary movement rather than on the effects of bulk flow velocity.

Stefan-flow effects in gaseous media have been considered mostly in connection with diffusion problems [2,17]. A comprehensive equation system containing also Stefan-convection terms (for the case of continuous-medium transfer regime) has been presented in [12] without an attempt to analyze their influence on transfer rates. A specific case of condensation on a flat liquid film with account taken of Stefan-flow influence has been treated by the present author in [7].

Stefan-flow effects in the region of free-molecular transfer regime have been treated analytically by Schrage [16] who provided a simplified solution based on consideration of a flat film condensation or evaporation. It will be shown in this paper, that Schrage's solution is not applicable to spherical droplets and other than fully free-molecular transfer mechanisms.

In [2] it is reported that the Stefan-flow around droplets could in some instances be observed visually as it causes small dust particles to be repulsed by evaporation or attracted by condensation on the droplet surface. The observations have been made on meteorogical rain droplets, several orders of magnitude bigger than those contained in technical wet steam.

The present paper is an attempt to examine the influence of Stefan-flow effects on droplet-to-vapour heat and mass transfer and thus on the rate of droplet evaporation or condensation processes. The aim is to assess the influence of non-uniform radial convection velocity field, caused by phase change on the droplet surface, upon the intensity of interphase processes and to develop suitable calculation methods. Examination of analytical and calculation results, as compared with respective reference results not containing Stefan terms, should help to determine the relevant influence ratios and their dependence upon thermal and geometrical parameters of the system at hand.

The following main assumptions will be made:

a. The considered system consists of a single spherical droplet surrounded by vapour phase of the same substance (one-component medium). No slip velocity exists between droplet and gas phase. There are no neighbourhood ("proximity") effects originating from other droplets or from vessel resp. flow duct boundaries. Hence the system at hand is radially symmetric.

b. The phase change processes take place on the droplet surface only; no spontaneous phase change occurs in either phase.

c. All occurring processes may be treated as quasi-static i.e. all thermal parameters, as well as the droplet size, are supposed to be maintained at their momentary values during the considered elementary time-step of the real process. This implicates the assumption that all characteristic times of the transfer processes at hand are sufficiently small in comparison to the time scale of the process the whole medium is undergoing.

Obviously, this assumption does not exclude non-equilibrium effects; interphase non-equilibrium processes are considered as caused by parameter differences between droplet and gas phase, which act as driving forces of heat and mass transfer fluxes.

d. The considered temperature range is well below the critical temperature of the substance at hand; thus the specific volume of the liquid may be regarded as small compared with that of the gas phase

$$\frac{v'}{v''} \ll 1. \tag{1.1}$$

On the strength of that the velocity of phase boundary displacement (advancing or receding of the droplet surface as the drop condenses or evaporates) is regarded as negligible, compared to the radial gas flow velocity caused by phase change processes. Hence the physical model reduces itself to a droplet of a temporarily constant size, on the surface of which a source (positive or negative) of gas phase is acting. Due to this source, there is a non-uniform radial velocity field around the droplet.

e. Within the range of involved temperature differences, the gas phase may be approximately treated as a perfect gas a pseudo-gas-constant taken formally from relevant saturation values.

In the following, all directions and resultant fluxes will be regarded as positive if directed from the liquid towards the gas phase. All flux densities will be related to a unit of droplet surface.

Considerations on evaporation and condensation processes have to be based on a notion of the physical mechanism providing interphase mass and energy transfer. The nature of this transfer mechanism depends on prevailing physical and topological conditions, expressed by the thermal parameters of the system and the droplet size. These two kinds of features are combined in the Knudsen-number, which is usually employed as a characteristic measure of transfer conditions.

Reviewing the application scope of various droplet-vapour systems, two main border cases may be selected.

The first case, marked by extremely high *Kn*-values, corresponds to a highly rariefied gas phase and/or very small droplet sizes; in this case the molecular-kinetic transfer mechanism prevails. The treatment of this case has to be based on application of kinetic gas theory [11, 15].

Conversely, in the case of extremely low Knudsen numbers, the continuous-medium transfer mechanism is acting.

More complicated, from the analytical viewpoint, are the complex cases of intermediate *Kn*-number values, where both mechanisms are interacting; these cases make up a substantial part of encountered two-phase processes.

In the following, chapter 2 is devoted to the consideration of Stefan-convection in molecular-kinetic transfer processes, while chapter 3 deals with the cases where continuous-medium transfer may be assumed. Chapter 4 presents a composite model, designed for the treatment of cases, determined by intermediate Knudsen number values.

2. Molecular-Kinetic Transfer Processes

2.1. Analytical Treatment

The molecular transfer processes between the surface of the liquid droplet and the surrounding vapour will be considered. The state of the gas phase is determined by its thermal parameters: temperature and static pressure. Both temperatures – that of the droplet surface and that of the gas phase – differ from each other and neither of them has to be equal to the saturation temperature corresponding to the pressure (with the capillarity correction due to the droplet size taken into account). Thus the non-equilibrium heat and mass transfer may be presented as dependent on relevant temperature differences (e.g. [4, 5, 6]).

A uniform distribution of thermal parameters in the whole gas phase space is assumed; this uniformity is maintained constant, unaffected by the real transfer

processes with the droplet surface. The only quantity which is not uniformly distributed throughout the gas space is the bulk vapour velocity, induced by phase change on the droplet surface. In each gas element this velocity vector is directed radially with regard to the droplet centre and its value is a function of the element's radial coordinate. This bulk gas flow is assumed to be stationary and its influence on the pressure and temperature distribution to be negligible.

The molecular velocity distribution in the gas phase is the Maxwellian equilibrium distribution (as seen by an observer moving with the steady bulk gas velocity) related to the thermal parameters prevailing in the vapour. As a result of the foregoing assumptions it is assumed that the transfer of each quantity conveyed by molecules leaving the droplet surface occurs at their first collisions in the gas space. Conversely, each molecule which strikes the droplet surface carries the pertinent amount of the transferred quantity received at the last collision in the gas phase.

As the Maxwellian distribution function depends on the bulk gas velocity, it has to be assumed that each space element has its own Maxwellian distribution; thus there is a non-uniform field of Maxwellian distributions in the gas space. This does not affect the fact that the molecular velocity distribution at each location is considered as an equilibrium one.

In order to determine transfer fluxes, a droplet surface element interacting with gas phase molecules will be considered.

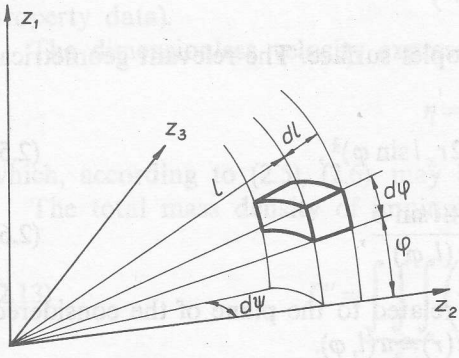

Fig. 1. Geometry of the gas space element

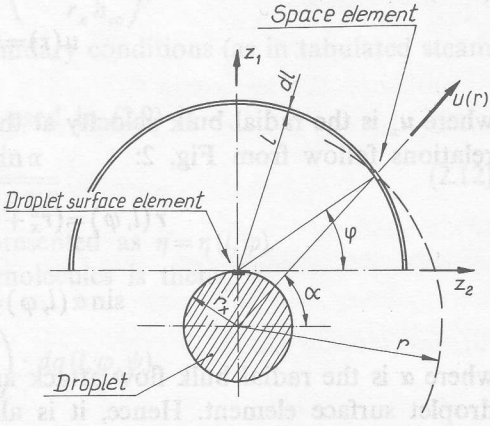

Fig. 2. Position of gas space and droplet surface elements (cross section in the z_1, z_2 plane)

From the incident molecule stream a class of molecules will be selected, which after their last gas space collision travelled the distance of l to $(l+dl)$ before reaching the droplet surface element. The share of these molecules in the total incident molecule stream is (e.g. [11])

$$F(l)\,dl=\frac{1}{l_m}\cdot\exp\left(\frac{-l}{l_m}\right)\cdot dl, \tag{2.1}$$

where l_m is the mean free path of a molecule in the gas phase.

The locus of the last gas space collisions of these molecules is a half-spherical layer with the inner radius l and radial thickness dl and with its centre at the considered droplet surface element.

Consider a space element carved out of that layer, with its location defined by the elevation angle φ and meridional angle ψ, as shown in Fig. 1; its volume being:

$$dV = l^2 \cos\varphi \, d\varphi \, d\psi \, dl. \tag{2.2}$$

On the droplet surface element, the impingement density produced by molecules of the said class, which arrive from various directions, is proportional to the cosine of the complementary elevation angle $(\pi/2 - \varphi)$. By averaging over the range of all possible striking directions one obtains a weighing factor to be taken into account as equal to $(2 \sin\varphi)$.Hence, with the use of geometrical relations from Fig. 1 and 2, the share of impingement flux density caused by molecules whose last collisions occurred within the described space element is

$$dg = F(l) \cdot \frac{\cos 2\varphi}{2\pi} \cdot dl \, d\varphi \, d\psi. \tag{2.3}$$

The gas phase contained in the considered space element has radial bulk velocity, the value of which follows immediately from the continuity condition

$$u(r) = u_x \left(\frac{r_x}{r}\right)^2, \tag{2.4}$$

where u_x is the radial bulk velocity at the droplet surface. The relevant geometrical relations follow from Fig. 2:

$$r(l, \varphi) = (r_x^2 + l^2 + 2 r_x l \sin\varphi)^{\frac{1}{2}}, \tag{2.5}$$

$$\sin\alpha(l, \varphi) = \frac{r_x + l \sin\varphi}{r(l, \varphi)}, \tag{2.6}$$

where α is the radial bulk flow attack angle related to the plane of the considered droplet surface element. Hence, it is also $u(r) = u(l, \varphi)$.

The mass flux of molecules arriving at the droplet surface may be determined by applying the Hertz-Knudsen approach (e.g. [4, 15]) to the above described model of a space element containing last collision sites. For the sake of convenience, a Cartesian coordinate system, with origin at the centre of the droplet surface element, will be chosen in such a way that the considered space element appears in the (z_1, z_2) plane. Then the Maxwellian molecular velocity distribution, assumed valid for this particular space element, has the form

$$\Phi = \frac{1}{mv''} \cdot \frac{1}{(2\pi RT'')^{3/2}} \cdot \exp\left[-\frac{(W_1 - u\sin\alpha)^2 + (W_2 - u\cos\alpha)^2 + W_3^2}{2RT''}\right] \tag{2.7}$$

and the partial mass flux density ascribed to molecules which have their last collisions in that space element is

$$\frac{d\Gamma''}{dg}=m\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\Phi(l,\varphi)\cdot W_1\,dW_1\,dW_2\,dW_3. \tag{2.8}$$

The above quantity may be written down in the known form:

$$\frac{d\Gamma''}{dg}=\Gamma_s\cdot\sqrt{\frac{T_s}{T''}}\cdot[\exp(-\eta^2)+\sqrt{\pi}\,\eta\cdot(1+\operatorname{erf}\eta)], \tag{2.9}$$

where

$$\Gamma_s=\frac{p}{\sqrt{2\pi RT_s}} \tag{2.10}$$

is the mass flux density between liquid droplet surface and vapour under complete equilibrium conditions i.e. with both liquid and vapour temperatures equal to the saturation temperature and with no relative bulk movement of the phases.

The appropriate expression of the saturation temperature, used here, has to take into account curvature effects which are essential for the case of small droplets. With minor simplifications [4] there is

$$T_s(p)=T_{s\infty}(p)\cdot\left(1-\frac{2v'\sigma}{r_x h_\infty}\right), \tag{2.11}$$

where the index ∞ indicates flat phase boundary conditions (as in tabulated steam property data).

The dimensionless velocity expression used in (2.9) is

$$\eta=\frac{u\sin\alpha}{\sqrt{2RT''}} \tag{2.12}$$

which, according to (2.5), (2.6), may be presented as $\eta=\eta(l,\varphi)$.

The total mass density of impinging molecules is then

$$\Gamma''=\underbrace{\int\int\int_{g}}\left(\frac{d\Gamma''}{dg}\right)\cdot dg(l,\varphi,\psi). \tag{2.13}$$

After introducing (2.9) and (2.3) with (2.1) and carrying out the obvious integration over ψ from $\psi=0$ to 2π, one arrives at

$$\Gamma''=\Gamma_s\cdot\sqrt{\frac{T_s}{T''}}\cdot\Omega_I, \tag{2.14}$$

where

$$\Omega_I=\frac{1}{l_m}\int_0^{\pi/2}\int_0^{\infty}[\exp(-\eta^2)+\sqrt{\pi}\,\eta(1+\operatorname{erf}\eta)]\cdot\exp\left(\frac{-l}{l_m}\right)\cdot\sin 2\varphi\cdot dl\,d\varphi. \tag{2.15}$$

It is to be noted that in a stagnant medium i.e. with no bulk flow taking place

$$(\Omega_I)_{u=0}=1. \tag{2.16}$$

A part of the impinging molecule stream condenses at the droplet surface i.e. becomes incorporated into the liquid molecular surface layer; the share of these molecules will be defined in the usual way by introduction of the condesation coefficient ξ (also called "mass accommodation coefficient"). The rest of impinging molecules is reflected back from the liquid surface.

Respectively, the outward bound mass flux of molecules emitted from the droplet surface consists of the evaporated molecule flux and of the reflected molecule flux

$$\Gamma'=\Gamma'_{ev}+(1-\xi)\Gamma''. \tag{2.17}$$

The evaporated mass flux value, which appears in the above equation, may be determined by considering a fictitious vapour, being in equilibrium with the droplet at its actual thermal state. In such a vapour temperature and pressure would have to be equal to the droplet surface temperature and the respective saturation pressure. The sought value of the evaporating mass flux would then be equal to the mass flux of condensing molecules arriving from the stagnating fictitious vapour i.e.

$$\Gamma'_{ev}=\xi\frac{p'(T')}{\sqrt{2\pi RT'}} \tag{2.18}$$

or, with (2.10):

$$\Gamma'_{ev}=\xi\,\Gamma_s\cdot\frac{p'_s(T')}{p}\sqrt{\frac{T_s}{T'}}, \tag{2.19}$$

where, taking account of curvature effects on small droplets,

$$p_s(T')=p_s^{\infty}(T')\cdot\exp\frac{2v'\sigma}{r_x RT'}. \tag{2.20}$$

Finally, the resultant net mass flux density is

$$\Gamma=\Gamma'-\Gamma'' \tag{2.21}$$

or, introducing (2.17)

$$\Gamma=\Gamma'_{ev}-\xi\cdot\Gamma''. \tag{2.22}$$

According to the adopted convention, resultant mass flux $\Gamma>0$ denotes net evaporation of the droplet and $\Gamma<0$ net condensation on the droplet surface. The rate of the droplet size change is

$$\frac{dr_x}{dt}=-\Gamma v'. \tag{2.23}$$

It can be seen that in the equilibrium case

$$\Gamma'(T_s)=\Gamma''(T_s)=\Gamma_s$$

and

$$\Gamma=0. \tag{2.24}$$

The radial vapour velocity at the droplet surface follows directly from the continuity condition, with the influence of droplet size change already defined as negligible because of (1.1):

$$u_x = \Gamma\, v''. \tag{2.25}$$

The use of v'' in (2.25) results from already adopted assumptions which require a uniform, equilibrium-like temperature field throughout the entire gas phase space, irrespectively of the non-equilibrium nature of transfer processes actually taking place between the phases.

With the help of foregoing relations (2.4) to (2.6) and (2.15), the dimensionless radial velocity (2.12) may be presented as

$$\eta = \frac{\Gamma}{\Gamma_s}\sqrt{\frac{T''}{T_s}}\, v(l,\varphi), \tag{2.26}$$

where the geometrical factor

$$v(l,\varphi) = \frac{1}{2\sqrt{\pi}} \cdot \frac{1+\frac{l}{r_x}\sin\varphi}{\left(1+\frac{l^2}{r_x^2}+2\frac{l}{r_x}\sin\varphi\right)^{3/2}}. \tag{2.27}$$

Thus in (2.15) the expression

$$\Omega_I = \Omega_I(\Gamma). \tag{2.28}$$

Finally, introducing (2.14) and (2.19) into (2.22), one arrives at the equation

$$\Gamma = \xi \cdot \Gamma_s \left[\frac{P_s'(T')}{P}\sqrt{\frac{T_s}{T'}} - \sqrt{\frac{T_s}{T''}} \cdot \Omega_I(\Gamma)\right] \tag{2.29}$$

from which the value of the resultant net mass flux Γ is to be determined; the expression $\Omega_I(\Gamma)$ represents here the influence of phase change induced radial flow on the mass transfer i.e. on the droplet evaporation resp. condensation rate. Because of the form of expression (2.28), equation (2.29) cannot be solved in an elementary way.

It is to be noted that, having obtained Γ from (2.29), the way of calculating other quantities, such as Γ'', is opened with the help of (2.14), (2.15) and (2.26).

By a similar way of reasoning, the energy and heat fluxes between the phases will be determined with account taken of Stefan-flow influence. The general way of approach will follow that used in the author's previous paper [4].

The flux density of translational energy carried by the stream of incident molecules, having their last collision in the space element defined by (l, φ, ψ), is

$$\frac{d\Psi_{tr}''}{dg} = \frac{1}{2}m \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \Phi(l,\varphi)\cdot W^2 W_1\, dW_1\, dW_2\, dW_3 \tag{2.30}$$

or, after integration

$$\frac{d\Psi''_{tr}}{dg} = \Gamma_s \cdot RT_s \sqrt{\frac{T''}{T_s}} \cdot H, \tag{2.31}$$

where

$$H = \left[\left(\frac{\eta}{\sin\alpha}\right)^2 + 2\right] \cdot \exp(-\eta^2) + \sqrt{\pi}\left[\left(\frac{\eta}{\sin\alpha}\right)^2 + \tfrac{5}{2}\right] \cdot \eta(1 + \operatorname{erf}\eta). \tag{2.32}$$

Using (2.26), (2.27) and (2.6) one may also obtain the expression

$$\left(\frac{\eta}{\sin\alpha}\right)^2 = \frac{1}{4\pi} \cdot \frac{1}{1 + \frac{l^2}{r_x^2} + 2\frac{l}{r_x}\sin\varphi} \cdot \frac{T''}{T_s} \cdot \left(\frac{\Gamma}{\Gamma_s}\right)^2. \tag{2.33}$$

Hence, the translational energy flux of the whole stream of incident molecules is

$$\Psi''_{tr} = \underbrace{\int\int\int}_{g} \left(\frac{d\Psi''_{tr}}{dg}\right) \cdot dg(l, \varphi, \psi), \tag{2.34}$$

which after simple integration over ψ as in (2.15), yields

$$\Psi''_{tr} = 2\Gamma_s RT_s \sqrt{\frac{T'}{T_s}} \cdot \Omega_{II}, \tag{2.35}$$

where

$$\Omega_{II} = \frac{1}{2l_m} \int_0^{\pi/2} \int_0^{\infty} H(l, \varphi) \exp\left(\frac{-l}{l_m}\right) \cdot \sin 2\varphi \, dl \, d\varphi. \tag{2.36}$$

The average translational energy per unit mass of impinging molecules is

$$\tilde{e}''_{tr} \equiv \frac{\Psi''_{tr}}{\Gamma''_{tr}} = 2RT'' \frac{\Omega_{II}}{\Omega_I}. \tag{2.37}$$

In a stagnant gas phase i.e. with the radial bulk flow disregarded

$$(\Omega_{II})_{u=0} = 1 \tag{2.38}$$

in which case (2.37) reduces itself to the known value

$$\tilde{e}''_{tr} = 2RT'' \tag{2.39}$$

while the average amount of translational energy per unit mass of molecules resident in a given volume is, in accordance to the principle of equipartition,

$$e''_{tr} = \tfrac{3}{2} RT''. \tag{2.39a}$$

By the same principle the amounts of other energy components of impinging molecules may be defined. The number of degrees of freedom of a gas molecule is

$$j=\frac{2}{\varkappa-1} \tag{2.40}$$

with the value of $\varkappa$ related to the existing gas phase*).

Then the average rotational energy of striking molecules per mass unit is

$$e''_{\mathrm{rot}}=\frac{j-3}{2}RT'' \tag{2.41}$$

and the whole energy flux, carried by the incident molecule stream, may be written down as

$$\Psi''=\Gamma''(\tilde{e}''_{tr}+e''_{\mathrm{rot}}+e''_{\mathrm{pot}}). \tag{2.42}$$

The energy component due to molecule oscillation has been neglected, which is appropriate in the range of moderate temperatures covering most technical two-phase processes. The term e''_{pot} refers to potential energy in the field of mutual interaction forces of molecules. In accordance with the assumptions made:

$$e_{\mathrm{pot}}(T)=i''(T)-\frac{\varkappa}{\varkappa-1}RT. \tag{2.43}$$

The distribution of gas molecule energy (per mass unit) is shown on Fig. 3.

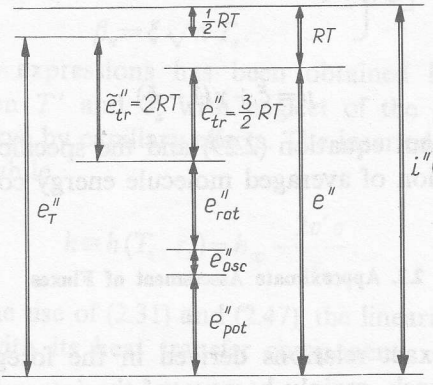

Fig. 3. Averaged molecular energy components in a stagnant gas phase

The stream of molecules emitted from the droplet surface is made up of evaporated and reflected molecules. The partial accommodation of both translational and rotational energy of the reflected molecules will be accounted for by

*) One should note that with the value of $\varkappa$ computed from tabulated data for real vapour, the formal application of (2.40) yields a value of j which may not be a whole number; novertheless it reflects properly – though approximately – the distribution of molecule energy.

a common average accommodation coefficient χ. Then the energy flux emitted from a unit of the droplet surface is

$$\Psi' = (1-\xi)\Gamma''[\tilde{e}''_{tr} + e''_{rot} + e''_{pot} + \chi(\tilde{e}'_{tr} + e'_{rot} - \tilde{e}''_{tr} - e''_{rot})] + \Gamma'_{ev}(\tilde{e}'_{tr} + e'_{rot} + e'_{pot}), \quad (2.44)$$

where the first term refers to the energy of reflected molecules and the second one to the evaporated molecule energy. Note, that in the stream of evaporated molecules

$$\tilde{e}'_{tr} = 2RT'.$$

The resulting net energy flux density is

$$\Psi = \Psi' - \Psi'' \quad (2.45)$$

which may be split up into two terms: one, representing "pure" heat transfer between droplet surface and vapour, due to the existing temperature difference, and the other, associated with the emergence of a resultant net mass flux (i.e. the unbalanced part of the mutual mass transfer), represents the energy transported with the bulk mass flow

$$\Psi = \Lambda + \Gamma e_T. \quad (2.46)$$

where the heat flux, after rearrangement and introduction of (2.14), (2.22), (2.37) and (2.41), becomes

$$\Lambda = \mu\Gamma_s \sqrt{\frac{T_s}{T''}} \left[\frac{1}{2} \frac{\varkappa+1}{\varkappa-1} R(T''-T')\,\Omega_I - 2RT''(\Omega_{II}-\Omega_I) \right], \quad (2.47)$$

where

$$\mu = \xi + \chi(1-\xi) \quad (2.48)$$

while Γ is determined from equation (2.29) and the specific transported energy e_T results from the summation of averaged molecule energy components (see Fig. 3).

2.2. Approximate Assessment of Fluxes

The application of exact relations derived in the foregoing chapter leads to tedious computational work, mainly because of the integrals Ω which have to be solved separately for each given case. Hence it is necessary to introduce a simplified method to assess the influence of the radial, phase change induced, convection flow on the transfer fluxes.

For this purpose the following two further simplifications will be introduced:

a) linearization of driving forces appearing in the considered model,

b) additional assumptions on the locus of the first (respectively last) collisions of molecules in the gas space.

Linearization of driving forces is in accordance with the already adopted Maxwellian distribution, which is an assumption permissible for thermal states not

too far from equilibrium. Assuming that both temperature deviations and the radial velocity are small as compared to their reference quantities i.e.

$$\left.\begin{aligned} \frac{|\delta'|}{T_s}&=\frac{|T'-T_s|}{T_s}\ll 1,\\ \frac{|\delta''|}{T_s}&=\frac{|T''-T_s|}{T_s}\ll 1,\\ |\eta|&\ll 1, \end{aligned}\right\} \tag{2.49}$$

linearization of (2.9) and (2.18) with the help of (2.22) leads to the following expression for the resultant mass flux density

$$\Gamma=\beta'_0\frac{\delta'}{T_s}+\beta''_0\frac{\delta''}{T_s}+\beta_u\bar{\eta}, \tag{2.50}$$

where an averaged dimensionless velocity $\bar{\eta}$ has been introduced, the value of which is to be defined. The coefficients are

$$\left.\begin{aligned} \beta'_0&=\xi\,\Gamma_s\left(\frac{h}{RT_s}-\frac{1}{2}\right),\\ \beta''_0&=\frac{1}{2}\,\xi\,\Gamma_s,\\ \beta_u&=\xi\sqrt{\pi}\,\Gamma_s. \end{aligned}\right\} \tag{2.51}$$

The first of the above expressions has been obtained by linearization of the saturation curve between T' and T_s with neglect of the slight distortion of the Clausius-Clappeyron curve by capillary effects. The inserted saturation temperature is derived from (2.11) while

$$h\equiv h(T_s, r_x)=h_\infty-\frac{2v'\sigma}{r_x}. \tag{2.52}$$

In a similar way, with the use of (2.31) and (2.47), the linearized energy flux may be presented as in (2.46) with its heat transfer component:

$$\Lambda=\gamma_0\frac{\delta'-\delta''}{T_s}+\gamma_u\bar{\eta}, \tag{2.53}$$

where the coefficients are:

$$\left.\begin{aligned} \gamma_0&=\frac{1}{2}\frac{\varkappa+1}{\varkappa-1}\,\mu\cdot RT_s\cdot\Gamma_s,\\ \gamma_u&=\frac{\sqrt{\pi}}{2}\,\mu\cdot RT_s\cdot\Gamma_s \end{aligned}\right\} \tag{2.54}$$

*) In [4] a slightly different notation has been used.

and the specific transport energy

$$e_T = i''(T_s) - \tfrac{1}{2} R T_s . \tag{2.55}$$

The coefficients, introduced here, are related to relevant quantities at saturation conditions. Their values, in function of saturation temperature, have been computed and presented in diagrams in a previous paper of this author [4] *).

The locus of the first (respectively last) gas space collisions will be presented in a simplified manner. The simplification consists in lumping together all such collisions associated with a particular value of l and regarding their results as manifesting themselves at one point; this point represents a "reduced gravity centre" of collisions taking place within the half-spherical layer with the inner radius l and thickness dl.

Obviously, the "reduced gravity centre" is located vertically above the considered droplet surface element. Its radial coordinate has to be determined with account taken of the influence of the elevation angle on the molecule flux density, as in the derivation of (2.3). Hence

$$b = \frac{1}{2\pi l^2} \int_0^{\pi/2} 2\pi l^3 \sin^2\varphi \cos\varphi \, d\varphi = \tfrac{2}{3} l. \tag{2.56}$$

Thus, with the use of (2.4), the mean radial velocity value associated with the free path value l will be assumed as

$$\frac{u(r_b)}{u_x} = \frac{r_x^2}{(r_x + b)^2}, \tag{2.57}$$

while the angle $\alpha = \pi/2$ (see Fig. 4).

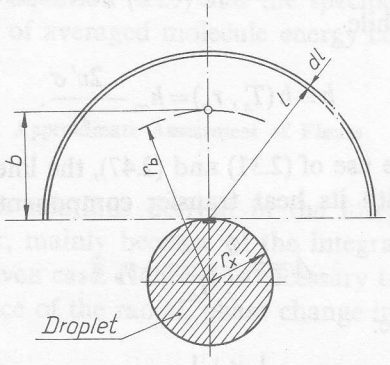

Fig. 4. Sketch to the approximate assessment of collision effects

The averaged value of bulk radial velocity $\bar{u}$, to be used in linearized expressions (2.50) and (2.53), will be determined by averaging (2.57) over the whole range of free path values l. With (2.56) and the distribution function (2.1) the averaged radial

velocity value will be obtained from

$$U \equiv \frac{\bar{u}}{u_x} = \int_0^\infty \frac{r_x^2}{(r_x + \frac{2}{3} l)^2} F(l)\, dl. \tag{2.58}$$

The solution of the above integral may be presented in the form:

$$U \equiv U(Kn) = \frac{3}{4Kn}\left[1 + \frac{3}{4Kn} \exp\left(\frac{3}{4Kn}\right) Ei\left(\frac{-3}{4Kn}\right)\right], \tag{2.59}$$

where

$$Kn = \frac{l_m}{2 r_x} \tag{2.60}$$

is the Knudsen-number of the droplet-vapour system under consideration and Ei denotes the integral-exponential function*). The limit values of $U(Kn)$ are

$$U(0) = 1, \quad U(\infty) = 0.$$

Then the averaged dimensionless velocity $\bar{\eta}$, as defined in (2.12), becomes in accordance with (2.25) and (2.58)

$$\bar{\eta} = \frac{\sqrt{RT''}}{p\sqrt{2}} \Gamma\, U(Kn), \tag{2.61}$$

the above value is to be inserted into (2.50) and (2.53).

The quantity $\bar{\eta}$ may be also assessed in a more crude way in order to avoid the use of Ei – tables for computation of (2.59). Instead of solving the integral (2.58), an averaged "reduced gravity centre" distance may be assumed directly using (2.56)

$$b_m = \tfrac{2}{3}\, l_m. \tag{2.62}$$

Then, analogically to (2.57):

$$U \equiv \frac{\bar{u}}{u_x} \approx \frac{1}{(1 + \frac{4}{3} Kn)^2} \tag{2.63}$$

and, with (2.61)

$$\bar{\eta} \approx \frac{\sqrt{RT''}}{p\sqrt{2}} \cdot \frac{\Gamma}{(1 + \frac{4}{3} Kn)^2}. \tag{2.64}$$

The more simplified expression (2.63) based on the adoption of b_m is, in fact, the result of disregarding the non-uniformity of the radial bulk velocity field. In effect, (2.62) overestimates the averaged distance $\bar{b}$ and thus underestimates the value of U in comparison to its more exact value qiven by (2.59); hence the effect of the radial convection becomes underestimated.

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*) $Ei(x) = \int_{-\infty}^{x} \frac{\exp t}{t}\, dt$, see e.g. [14,18].

The computed values of $U(Kn)$ are presented on Fig. 5.

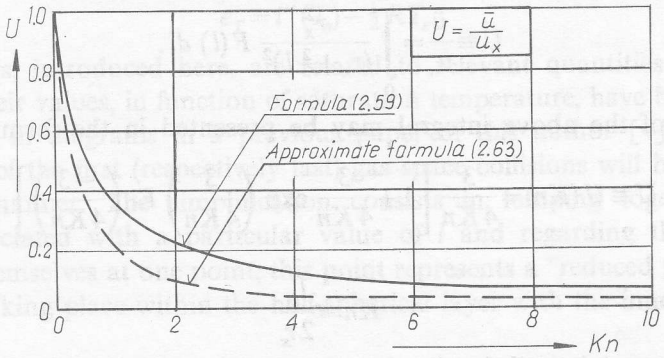

Fig. 5. Averaged radial velocity ratio in function of the Knudsen number

Inserting the averaged dimensionless velocity $\bar{\eta}$ – which may be taken from (2.61) or (2.64) – into (2.50) one arrives at the equation

$$\Gamma=\beta'_0\frac{\delta'}{T_s}+\beta''_0\frac{\delta''}{T_s}+\beta_u\frac{\sqrt{RT''}}{p\sqrt{2}}\,\Gamma\,U(Kn) \tag{2.65}$$

which, after solving with regard to Γ, yields the final expression for the mass flux density

$$\Gamma=\beta'\frac{\delta'}{T_s}+\beta''\frac{\delta''}{T_s}, \tag{2.66}$$

where the coefficients β', β'', as well as the mass flux rate, are determined by the correction:

$$\frac{\Gamma}{\Gamma_0}=\frac{\beta'}{\beta'_0}=\frac{\beta''}{\beta''_0}=\frac{2}{2-U(Kn)\cdot\xi}. \tag{2.67}$$

The above correction factor accounts for the influence of radial convection (Stefan) flow on the resultant mass transfer flux i.e. on the evaporation or condensation rate of the droplet. As $U\leqslant 1$ this influence will be essentially weaker on droplet phase change than on respective plane film processes.

By inserting (2.50) and (2.61) into (2.53) one obtains, after rearrangements, the heat flux as

$$\Lambda=\gamma'\frac{\delta'}{T_s}+\gamma''\frac{\delta''}{T_s} \tag{2.68}$$

with the coefficients determined by

$$\left.\begin{aligned}\frac{\gamma'}{\gamma_0}&=1+\frac{\varkappa-1}{\varkappa+1}\cdot\left(\frac{h}{RT_s}-\frac{1}{2}\right)\cdot\frac{U(Kn)\,\xi}{2-U(Kn)\,\xi},\\ \frac{\gamma''}{\gamma_0}&=-1+\frac{1}{2}\cdot\frac{\varkappa-1}{\varkappa+1}\cdot\frac{U(Kn)\,\xi}{2-U(Kn)\,\xi}.\end{aligned}\right\} \tag{2.69}$$

2.3. Evaluation of Results

Numerical calculations of the influence quantities related to mass and heat transfer have been carried out; their results are presented on Figs. 6, 7 and 8.

From (2.64) and Fig. 6 follows that the effect of phase change induced radial convection is always enhancing the actual evaporation or condensation flux by a constant factor which depends on the condensation coefficient and on the Knudsen-number value. The proportion at which each of the temperature deviations δ' and δ'' influences the resultant mass flux remains unchanged by the introduced correction. As the data in Fig. 6 indicate, the mass transfer flux correction is substantial in the range of smaller Kn – numbers only.

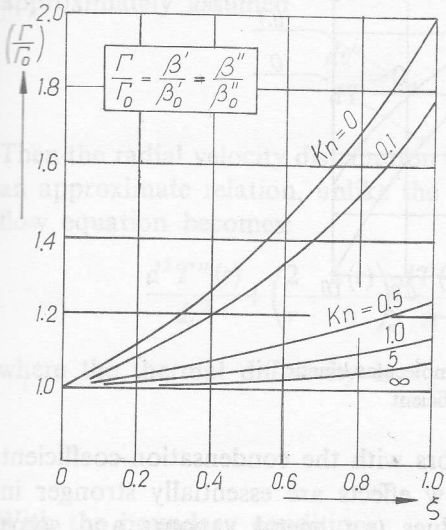

Fig. 6. Mass transfer correction factor in function of the condensation cofficient for various Knudsen numbers

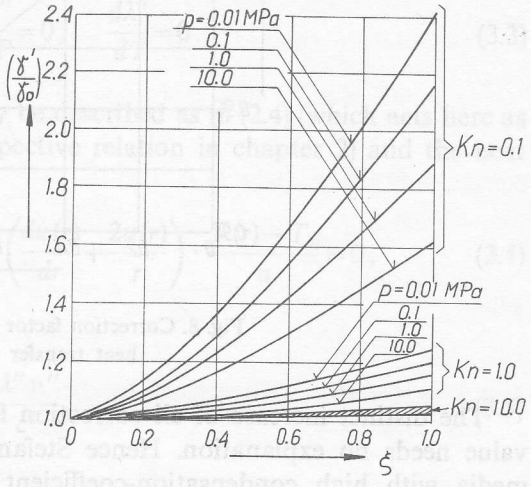

Fig. 7. Correction factor for molecular-kinetic heat transfer coefficient

In the limit case $Kn=0$ the expression (2.67) assumes its highest value, equal to the correction factor of Schrage [16]. This particular limit case is also equivalent to the case of flat film phase change.

In the opposite limit case $Kn\to\infty$, which corresponds to the case of extremely rarefied gas phase, the correction factor is unity; hence there is no influence of Stefan-convection.

In the general case there is a distinct effect of geometry and thermal conditions on the correction factor. Thus the use of the Schrage-correction for droplet-to-gas transfer calculations (occasionally occurring in literature) is incorrect.

As the calculated data indicate, application of the correction factor is essential in the case of bigger droplets and in transfer calculation methods designed for the transition transfer regime (as in composite transfer models described in chapter 4).

Regarding the heat flux, one notices that the effect, each of the temperature deviations (δ', δ'') exerts on the resultant heat flux, is altered by the Stefan-flow correction in a different way; this differentiation manifests itself by different correction factors to each of the respective heat transfer coefficients in (2.68). As can be seen from Figs. 7, 8, the coefficient γ' is particularly sensitive to these effects. This is a reflection of the general property of mass transfer (which is decisive in Stefan-flow effects) being more strongly influenced by liquid phase than by gas phase temperature deviations.

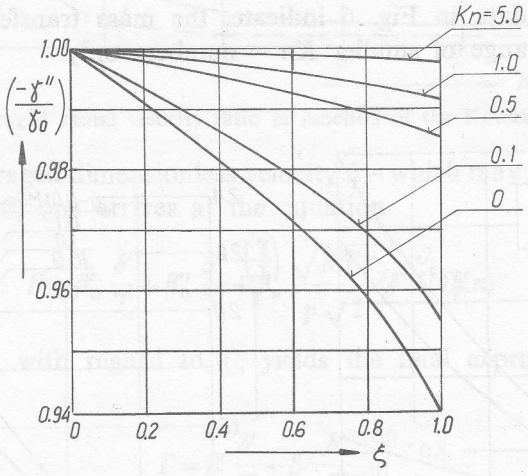

Fig. 8. Correction factor for molecular-kinetic heat transfer coefficient

The distinct increase of all correction factors with the condensation-coefficient value needs no explanation. Hence Stefan-flow effects are essentially stronger in media with high condensation-coefficient values (e.g. metal vapours and some organic compounds where ξ is near unity). In transfer calculations concerning H_2O vapour, ξ often appears as an uncertainty factor. The difficulty in assessing its pertinent value is due partly to the peculiar, semi-crystalline structure of water surface and partly to the strong dependence on thermal conditions and surface contamination. Problems of condensation-coefficient assessment and compilation of relevant data are to be found in [8, 10]. Generally, for H_2O in the low temperature region values $\xi < 0.1$ are recommended, while at boiling (saturation) temperature ξ is believed to have values near unity.

3. Continous Medium Transfer Processess

The transfer model, considered here, is a limit model opposite to the limit model of molecular-kinetic transfer treated in the foregoing chapter. The gas phase processes are reduced to spherically symmetric, quasi-static heat conduction associated with a steady radial convection flow, its velocity depending on the radial

coordinate of the space element at hand. The inner spherical boundary will be assumed as identical with the droplet surface and a value of the far-field temperature T''_∞ will be considered as given.

The equations governing the so described process are:

$$\operatorname{div}\left[\lambda'' \operatorname{grad} T'' - \frac{c''}{v''} u(T''-T''_\infty)\right]=0, \tag{3.1}$$

$$\frac{d}{dr}\left(\frac{ur^2}{v''}\right)=0. \tag{3.2}$$

The data describing vapour properties are temperature-dependent. With the additional assumption that temperature differences are relatively small, it may be approximately assumed

$$\frac{dv''}{dT}=0, \quad \frac{dc''}{dT}=0, \quad \frac{d\lambda''}{dT}=0. \tag{3.3}$$

Then the radial velocity distribution may be described as in (2.4) (which acts here as an approximate relation, unlike the respective relation in chapter 2) and the heat flow equation becomes:

$$\frac{d^2T''(r)}{dr^2}+\left(\frac{2}{r}-\frac{u(r)}{a}\right)\frac{dT(r)}{dr}-\left(\frac{du(r)}{dr}+\frac{2u(r)}{r}\right)\cdot\frac{T(r)-T_\infty}{a}=0, \tag{3.4}$$

where the thermal diffusivity is

$$a=\frac{\lambda'' v''}{c''}. \tag{3.5}$$

With the boundary conditions

$$T''(r_x)=T''_x, \qquad u(r_x)=u_x, \qquad T''(\infty)=T_\infty, \tag{3.6}$$

the temperature distribution may be obtained as

$$T''(r)=T''_\infty+(T''_x-T''_\infty)\frac{1-\exp\left(\frac{-u_x r_x^2}{ar}\right)}{1-\exp\left(\frac{-u_x r_x}{a}\right)} \tag{3.7}$$

and the gas phase temperature gradient at the inner boundary

$$\left(\frac{dT''}{dr}\right)_x=(T''_\infty-T''_x)\cdot\frac{u_x}{a}\cdot\frac{\exp\left(\frac{-u_x r_x}{a}\right)}{1-\exp\left(\frac{-u_x r_x}{a}\right)}. \tag{3.8}$$

The heat flux density (measured at the droplet's surface) is

$$\Lambda = -\lambda'' \left(\frac{dT''}{dr}\right)_x . \tag{3.9}$$

Analogically to (2.25), the resultant mass flux on the droplet surface is defined by

$$\Gamma = \frac{u_x}{v''_x} . \tag{3.10}$$

The balance of energy fluxes at the phase boundary yields:

$$-\lambda' \left(\frac{dT'}{dr}\right)_x + \Gamma i'_x = -\lambda'' \left(\frac{dT''}{dr}\right)_x + \Gamma i''_x . \tag{3.11}$$

Considering that

$$\begin{aligned} i'_x &= i'_s + c' \delta'_x , \\ i''_x &= i''_s + c'' \delta''_x \end{aligned} \tag{3.12}$$

and postulating that (due to the quasi-static assumption)

$$\left(\frac{dT'}{dr}\right)_x = 0$$

equation (3.11) renders

$$\Gamma = \frac{\lambda''}{h - B} \left(\frac{dT''}{dr}\right)_x , \tag{3.13}$$

where

$$B = c' \delta'_x - c'' \delta''_x \tag{3.14}$$

and h is defined, with regard to the curvature effects, in (2.52).

Insertion of (3.8) into (3.13) renders the mass flux

$$\Gamma = \frac{a}{v''} \frac{1}{r_x} \ln \left[1 - \frac{c''(T''_x - T''_\infty)}{h - B}\right] . \tag{3.15}$$

The above expression may be used to write down $\left(\frac{dT''}{dr}\right)_x$ in function of the temperature difference; insertion into (3.8) with the help of (3.10) yields

$$\left(\frac{dT''}{dr}\right)_x = \frac{h - B}{c'' r_x} \ln \left[1 - \frac{c''(T''_x - T''_\infty)}{h - B}\right] . \tag{3.16}$$

For the reference case of neglected radial convection influence (i.e. by inserting $u = 0$ in eq. (3.4)) one obtains

$$\Lambda_0 = -\lambda'' \frac{T''_\infty - T''_x}{r_x} , \tag{3.17}$$

$$\Gamma_0 = \frac{\lambda''}{h - b} \frac{T''_\infty - T''_x}{r_x} . \tag{3.18}$$

Thus the correction factor which accounts for the radial convection influence is

$$\frac{\Gamma}{\Gamma_0}=\frac{\Lambda}{\Lambda_0}=\frac{h-B}{c''(T''_\infty-T''_x)}\ln\left[1-\frac{c''(T''_x-T''_\infty)}{h-B}\right]. \tag{3.19}$$

Interpretation of the above derived relations becomes more distinct with the assumption that the droplet surface is at saturation conditions and that there is no temperature jump at the phase boundary i.e.

$$T'_x=T''_x=T_s;$$

then the considered system reduces itself to the "classical" model of a saturated liquid droplet which evaporates or grows due to the existence of a far-field temperature deviation (being the only non-equilibrium parameter). The correction factor for both mass and heat flux takes then the form:

$$\frac{\Gamma}{\Gamma_0}=\frac{\Lambda}{\Lambda_0}=\frac{h}{c''\delta''_\infty}\ln\left(1+\frac{c''\delta''_\infty}{h}\right). \tag{3.20}$$

The typical behaviour of mass flux values is shown on the sketch Fig. 9.

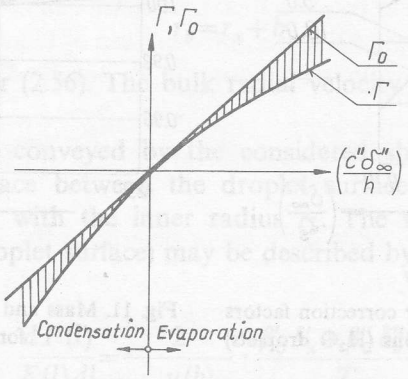

Fig. 9. Sketch of Stefan-convection influence on continuous mass transfer

It can be seen that the influence of the radial Stefan-convection in vapour, treated as continous medium, is basically different from that exerted on the molecular transfer mechanism. The most prominent feature of this influence is its asymmetry; as can be seen from (3.20) and Fig. 9 it enhances the condensation rate but causes a decrease of the evaporation rate of the droplet.

Such behaviour can be explained by the non-uniformity of the temperature field and by the fact that the mass flow moves always in the direction opposite to the heat flux. The evaporated mass flux, moving outwards into the region of higher temperature, absorbes heat; hence the temperature distribution becomes flattened as compared to the temperature curve in a stagnant medium, thus diminishing the intensity of the evaporation process. The same effect causes steeper gradients i.e. process enhancement in the case of condensation.

Unlike in the molecular-kinetic transfer, the correction factor depends on the temperature deviation and vapour properties but is independent of the droplet radius.

Calculation results of the correction factor for H_2O vapour are presented in Fig. 10. As can be seen, the Stefan-influence on continuous transfer is rather slight at low and moderate pressures, becoming substantial in the high pressure region (as e.g. in nuclear turbine flow processes).

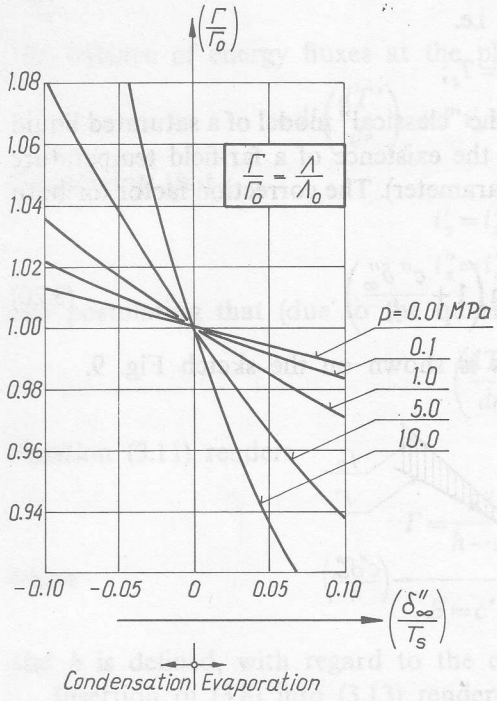

Fig. 10. Mass and heat transfer correction factors in continuous transfer conditions (H_2O droplets)

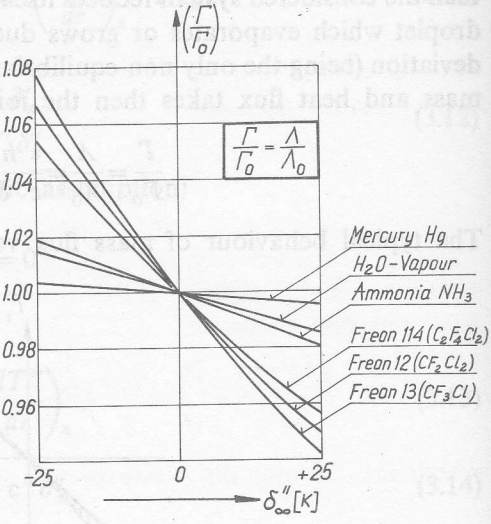

Fig. 11. Mass and heat transfer correction factors for various media

However, this influence becomes more pronounced in some other substances. A comparison of calculated correction factors for various media is shown in Fig. 11. As a common base for comparison the pressure $p=0.1$ MPa and the related saturation temperature for each medium has been adopted. The calculation results indicate that Stefan-flow effects are particularly strong in some organic refrigerants.

4. Composite Models

4.1. Derivation of Calculation Models

Both transfer mechanisms treated in the foregoing chapters (molecular-kinetic and continuous) are extreme limit models of the real transfer phenomena occurring in two phase processes. Their strict validity would require conditions described by external values $Kn=0$, respectively $Kn\to\infty$. In most technical applications neither of

those assumptions holds; droplets, which grow by condensation or evaporate, pass through a wide range of *Kn*-values during their lifetime (e.g. wet steam processes in turbines). Hence for the intermediate *Kn*-number regions the use of composite ("multirange") calculation models is recommended.

The most obvious way to build up such a model consists in subdividing the gas space into two concentric spherical regions: the inner region where molecular-kinetic transfer is supposed to take place and the surrounding infinite continuous transfer region. The border between those two regions is an imagined spherical surface with a radius r_m, to which a temperature T_m is ascribed. Models based on such concepts have been presented in previous papers of this author [5, 6].

In the following a similar model, which takes account of the free-path distribution will be employed.

First, attention will be focussed on those molecules, interacting with the droplet surface, which travel the free path between l and $(l+dl)$ before or after striking the considered surface element. Their share in the whole molecule stream is given by (2.1) and the summarized effects of their first resp. last collisions may be ascribed to a point (see also Fig. 4) having a radial coordinate

$$r_b = r_x + b \tag{4.1}$$

with b determined after (2.56). The bulk radial velocity existing at that location is described by (2.57).

The mass transfer, conveyed by the considered share of molecules, may be regarded as taking place between the droplet surface and a spherical layer of infinitesimal thickness, with the inner radius r_b. The resultant mass flux of this transfer (per unit of droplet surface) may be described by a derivation analogous to (2.66) – (2.68)

$$\frac{d\Gamma(l)}{F(l)\,dl} = \frac{2}{2-\dfrac{u(b)}{u_x}\xi} \cdot \frac{\beta'_0\,\delta'_x + \beta''_0\,\delta''_b(b)}{T_s} \tag{4.2}$$

the velocity ratio which appears here has to be taken from (2.57).

In the region outside the radius r_b, continuous transfer, as described in chapter 3, is acting. The relevant expression for mass transfer can be taken from (3.15) with the alterations

$$(\,)_x \to (\,)_b, \qquad \Gamma \to \Gamma \cdot \left(\frac{r_b}{r_x}\right)^2 \tag{4.3}$$

bearing in mind that all flux densities have to be related to the radius of the droplet surface. Thus the mass flux density in the continuous region is:

$$\frac{d\Gamma(l)}{F(l)\,dl} = \left(\frac{r_b}{r_x}\right)\frac{1}{r_x}\,\frac{\lambda''}{c''}\ln\left[1-\frac{c''(\delta''_b-\delta''_\infty)}{h-c'\,\delta'_x-c''\,\delta''_b}\right]. \tag{4.4}$$

Equating the relevant mass transfer expressions leads, after rearrangements, to the equation

$$\left(1+\frac{b}{r_x}\right)\frac{1}{r_x}\frac{\lambda''}{c''}\ln\left[1-\frac{c''(\delta_b''-\delta_\infty'')}{h-c'\delta_x'-c''\delta_\infty''}\right]=\frac{2}{2-\frac{\xi r_x^2}{(r_x+b)^2}}\cdot\frac{\beta_0'\delta_x'+\beta_0''\delta_b''}{T_s}. \tag{4.5}$$

At given conditions, described by $(r_x, T_s, \delta_x', \delta_\infty'')$, the above equation has to be solved with regard to δ_b'' for the whole ensamble of b-values. Each of these solutions applies to the share of mass transfer ascribed to molecules travelling the free path between l and $(l+dl)$. Having so determined $\delta_b''(b)$, the resultant mass flux density follows, with the use of (2.57) and (4.2), from

$$\Gamma=\int_0^\infty\left(\frac{d\Gamma(l)}{F(l)\,dl}\right)\cdot F(l)\,dl. \tag{4.6}$$

The above indicated procedure, though leading to exact results, is impractical because it requires tedious computations in order to solve (4.5) and (4.6).

A more approximate mass transfer flux expression can be derived by adopting an "averaged distance" b_m as in (2.62). After introduction of U as in (2.63) and with some further simplifications, the following equation is obtained

$$\frac{2}{2-U(Kn)\,\xi}\cdot\frac{\beta_0'\delta_x'+\beta_0''\delta_m''}{T_s}=(1+\tfrac{4}{3}Kn)\cdot\frac{1}{r_x}\cdot\frac{\lambda''}{c''}\ln\left[1-\frac{c''(\delta_m''-\delta_\infty'')}{h}\right] \tag{4.7}$$

which has to be solved only once in order to determine the value δ_m''. The mass flux density is then

$$\Gamma=\frac{2}{2-U(Kn)\,\xi}\cdot\frac{\beta_0'\delta_x'+\beta_0''\delta_m''}{T_s}. \tag{4.8}$$

The so obtained mass flux value can be compared to its reference value i.e. to the respective mass flux determined without taking Stefan-convection into account. The way of solving the reference case is basically the same as above. The determining equation is then

$$\frac{\beta_0'\cdot\delta_x'+\beta_0''\cdot\delta_{0m}''}{T_s}=(1+\tfrac{4}{3}Kn)\cdot\frac{\lambda''}{r_x}\cdot\frac{\delta_\infty''-\delta_{0m}''}{h} \tag{4.9}$$

from which δ_{0m}'' may be written down explicitly

$$\delta_{0m}''=\frac{(1+\frac{4}{3}Kn)\frac{\lambda''}{r_x}\frac{\delta_\infty''}{h}-\frac{\beta^0\cdot\delta_x'}{T_s}}{\frac{\beta_0''}{T_s}+\frac{\lambda''}{r_x h}(1+\frac{4}{3}Kn)} \tag{4.10}$$

and the resultant mass flux density becomes

$$\Gamma_0 = \frac{1+\frac{4}{3}Kn}{(1+\frac{4}{3}Kn)\,T_s + \frac{\beta_0'' h r_x}{\lambda''}} \cdot (\beta_0' \delta_x' + \beta_0'' \delta_\infty''). \tag{4.11}$$

4.2. Calculation Results

In order to assess the Stefan-convection influence in the intermediate region between molecular-kinetic and continuous transfer regimes, series of calculation examples have been carried out. As independent variables static pressure and Knudsen numbers have been chosen, from which the saturation temperature and respective radii of H_2O – droplets have been determined with account taken of surface curvature effects. To facilitate comparisons, it has been assumed that the droplet surface is at saturation temperature, while the only non-equilibrium

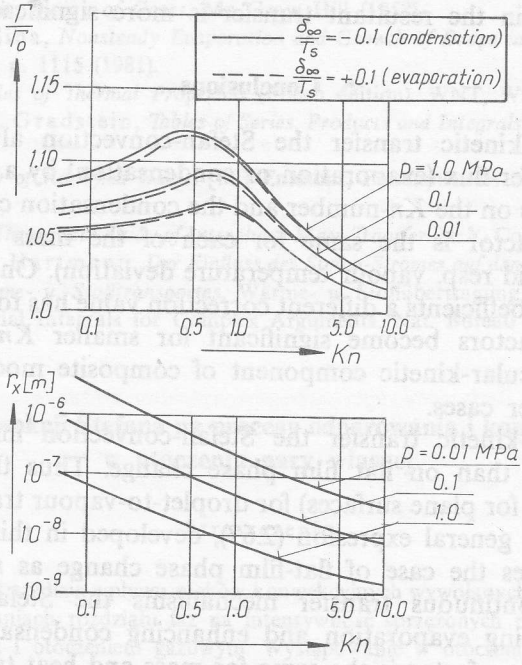

Fig. 12. Calculation results of Stefan-convection influence on a composite transfer model. Upper diagram: mass transfer rates, lower diagram: respective droplet radii

parameter in the system is the far-field temperature deviation δ_∞''. Ranges of Kn and p values have been chosen so as to cover the intermediate transfer region. At $r \to \infty$ two values of relative temperature deviations have been assumed as ± 0.1, corresponding to cases of rather strong condensation resp. evaporation of the droplet.

The calculation procedure indicated in the foregoing chapter 4.1 has been employed. Having determined the intermediate value of δ''_m from eq. (4.7) (with the condensation coefficient assumed as $\xi = 1$), the mass flux values Γ and Γ_0 have been obtained from (4.8) and (4.11).

The results are shown on Fig. 12. It can be seen that the Stefan-flow influence becomes stronger when the static pressure increases, an effect for which the continuous transfer component is responsible. The non-monotonous run of the curves in the upper diagram is due to the fact, that the *Kn*-range covers regions where either molecular-kinetic or continuous transfer prevails; in each of these regions the transfer flux is governed by different relations reflecting different transfer mechanisms. Hence the appearance of maximums of correction factor values. It is obvious that for $Kn \to \infty$, $\Gamma/\Gamma_0 \to 1$, while for $Kn \to 0$ the correction factor assumes a certain limit value different from unity.

It can also be seen that the Stefan-convection influence on droplet evaporation (dashed lines in Fig. 12) is slightly weaker than on condensation processes. This effect is due to the asymmetric convection influence in the continuous transfer region (see chapter 3) and becomes perceptible at smaller *Kn*-values where the continuous transfer component in the resultant transfer is more significant.

5. Conclusions

a. In molecular-kinetic transfer the Stefan-convection always enhances the resultant mass transfer flux (evaporation or condensation) by a constant correction factor which depends on the *Kn*-number and the condensation coefficient. The value of this correction factor is the same for each of the mass transfer coefficients (attached to the liquid resp. vapour temperature deviation). On the other hand, for each of the transfer coefficients a different correction value has to be used. The values of the correction factors become significant for smaller *Kn*-values, thus being important for molecular-kinetic component of composite models rather than for free-molecule transfer cases.

b. In molecular-kinetic transfer the Stefan-convection influence is generally weaker on droplets than on flat film phase change. Thus the use of Schrage's formulae (developed for plane surfaces) for droplet-to-vapour transfer calculations is incorrect. The more general expresion (2.67), developed in this paper, is valid for droplets and includes the case of flat-film phase change as a limit case.

c. In purely continuous transfer mechanisms the Stefan-flow influence is asymmetric, decreasing evaporation and enhancing condensation on the droplet surface. The correction factor is the same for mass and heat transfer and does not depend on the droplet size. The numerical quantity of the this influence on H_2O – droplets is rather slight, except for high-pressure processes (e.g. nuclear steam turbine processes); it also becomes significant for some organic substances (e.g. refrigerants).

d. In the intermediate transfer region there is a distinct Stefan-convection influence, depending on the geometry and the thermal conditions of the system at hand.

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Wpływ zjawiska konwekcji Stefana na procesy odparowania i kondensacji na kroplach w otoczeniu pary własnej

Streszczenie

Praca poświęcona jest analizie wpływu ruchów konwekcyjnych wywołanych przez przemiany fazowe zachodzące na powierzchniach rozdziału faz na intensywność sprzężonych procesów wymiany masy i ciepła pomiędzy kroplą i otoczeniem gazowym. Występowanie w otoczeniu kropli promieniowych ruchów konwekcyjnych wpływa na zmianę rozkładu temperatur wokół kropli co z kolei powoduje zmianę wartości międzyfazowych strumieni wymiany masy i energii w stosunku do ich wartości, wyznaczonych bez uwzględnienia wpływu konwekcji Stefana. Podobnie w zakresie wymiany molekularno-kinetycznej uwzględnienie występowania wtórnej konwekcji promieniowej prowadzi do zmiany wyznaczonych wartości wymiany masy i energii, przenoszonych przez strumienie molekuł fazy gazowej, oddziaływających z powierzchnią cieczy.

Analizę wpływu konwekcji Stefana w molekularno-kinetycznej wymianie masy i energii przeprowadzono w rozdziale 2. Przy użyciu uproszczeń uzyskano m.in. wartość poprawki, uwzględniającej wpływ konwekcji wtórnej w postaci (2.67); jest to wyrażenie ogólniejsze niż stosowane czasami – w stosunku do wymiany z kroplami w sposób niepoprawny – poprawki Schrage'go. Stwierdzono, że

wpływ konwekcji wtórnej powoduje zawsze zwiększenie strumienia odparowania względnie kondensacji. Wyznaczono również odpowiednie poprawki dla molekularnej wymiany ciepła.

W rozdziale 3 przedstawiono analizę wpływu konwekcji wtórnej na procesy wymiany między kroplą i gazem, traktowanym jako ośrodek ciągły. Najistotniejszą cechą tego zjawiska jest występująca asymetria oddziaływań: powodowanie zmniejszenia strumienia odparowania, natomiast zwiększenie intensywności kondensacji na kroplach. Stwierdzono też, że wpływ konwekcji wtórnej staje się istotny dla pary H_2O w zakresie wysokich ciśnień, a także dla niektórych innych substancji organicznych.

W rozdziale 4 omówiono metody obliczania wymiany międzyfazowej oparte na „składanych modelach; modele takie przydatne są w obliczeniach procesów wymiany w tzw. przejściowym zakresie liczb Knudsena. Przeprowadzono analizę wpływu parametrów geometrycznych i termicznych na procesy wymiany i rolę konwekcji wtórnej w takich układach oraz wykonano szereg przykładów liczbowych, których wyniki ilustrują wyprowadzone zależności.

Влияние явления конвекции Стефана на процессы испарения и конденсации на каплях в окружности собственного пара

Резюме

Работа посвящена анализу влияния конвекционных движений, вызванных фазовыми превращениями происходящими на поверхностях раздела фаз, на интенсивность сопряжённых процессов тепло – и массовмена между каплей и газовой окружностью. Выступание в окружности капли радиальных конвекционных движений влияет на изменение распределения температур вокруг капли, что по очереди вызывает изменение значений межфазных потоков массо- и энергообмена по отношению к их значениям, определённым без учёта влияния конвекции Стефана. Подобным образом в пределах молекулярно-кинетического обмена учёт выступания вторичной радиальной конвекции ведёт к изменениям определённых значений массо- и энергообмена, осуществляемого потоками молекул газовой фазы, воздействующих с поверхостью жидкости.

Анализ влияния конвекции Стефана в молекулярно-кинетическом массо- и энергообмене произведён а разделе 2. Пользуясь упрощениями получено м.пр. значение поправки, учитывающей влияние вторичной конфекции в форме (2.67); это является выражением более общими чем иногда применяемые – по отношению к обмену с каплями неправильно – поправки Шраге. Констатируется, что влияние вторучной конвекции способствует всегда увеличению потока испарения или конденсации. Определены также соответственные поправки для молекулярного теплообмена.

В разделе 3 представлен анализ влияния вторичной конвекции на процессы обмена между каплей и газом, трактованным как сполшная среда. Наиболее истинной чертой этого явления оказывается асимметрия воздействий: способствование уменьшению потока испарения и увеличению интенсивности конденсации на каплях. Констатируется также, что влияние вторичной конвекции становится истинным для водяного пара в переделах высоких давлений а также для некоторых других органических веществ.

В разделе 4 обсуждаются методы расчёта межфазного обмена основанные на "составных" моделях; такие модели очень пригодны в расчётах процессов обмена в т. наз. переходном пределе чисел Кнудсена. Произведён анализ влияния геометрических и термических параметров на процессы обмена и роль вторичной конвекции в таких системах, а также выполнен ряд численных примеров, которых результаты иллюстрируют выведенные зависимости.