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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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PROBLEMS OF HYDRAULIC TURBOMACHINES

Gdańsk-Władysławowo, September 24-27, 1985

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ГИДРОФОРУМ

НАУЧНО-ТЕХНИЧЕСКАЯ КОНФЕРЕНЦИЯ

на тему

ПРОБЛЕМЫ ГИДРАВЛИЧЕСКИХ ТУРБОМАШИН

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Numerical Water Hammer Simulation

A block diagram of a model of the water hammer in a pipe is presented. Results of water hammer calculations obtained with the use of the simulation language IBM-CSMP are discussed.

1. Introduction

When designing hydraulic systems and analyzing existing ones, it is necessary to determine the maximum and minimum pressures which can occur. The largest pressure changes occur during the so called water hammer. Excessive pressure changes are dangerous because of the limited mechanical strength of pipes and fittings.

Several methods of calculating the water hammer are presented in available literature [1]. At present, the method of characteristics is most commonly used for solving the equations describing the water hammer. However, this method requires a computer program in one of the standard program languages such as e.g. ALGOL, FORTRAN. Many designers and engineers concerned with hydraulic systems have no experience in programming in these languages.

Development of numerical methods resulting mainly in introducing the higher order languages open up the possibility of making water hammer calculations easier. The paper presents a method of water hammer calculations based on the use of the Laplace transformation. The solution of the system of equations describing the water hammer in the form of Laplace transforms [2] is presented in block diagram form, which makes writing a computer program in the IBM-CSMP language easier. An example of calculations of pressure changes in a discharge pipe is presented in the final part of the paper.

2. Solving of Equations Describing the Water Hammer

Depending on required accuracy of results and on assumed method of solving, the system of hiperbolic partial differential equations [1.4] — the most general description of the water hammer — is simplified to various degrees. After introducing small

simplifications the basic equations are presented in linearized form in reference [2]:
— the equation of fluid motion

$$\frac{1}{\rho}\frac{\partial(\Delta p)}{\partial x} = -\frac{\partial(\Delta v)}{\partial t} - K_h(\Delta v) \tag{1}$$

— the flow continuity equations

$$\frac{\partial(\Delta v)}{\partial x} = -\frac{1}{\rho w^2}\frac{\partial(\Delta p)}{\partial t} \tag{2}$$

where: K_h — $\lambda(v_0/3D)$, — coefficient of linear hydraulic resistance, v_0 — initial flow velocity [m/s], D — pipe diameter [m], w — velocity of propagation of the changed pressure wave [m/s], ρ — density of liquid [kg/m^3].

In these equations Δp and Δv designate deviation of pressure and flow velocity in the pipe, from the steady state, respectively. The Laplace integral transformation was used for solving the above equations. The independent variable t was taken as the transformation variable. The independent variable x was treated as a parameter. Initial conditions are equal to zero since deviation from steady state is considered [2].

After the Laplace transformation, the system of partial differential equations (1) and (2) is reduced to a system of ordinary differential equations:

$$\frac{1}{\rho}\frac{d\,\Delta p(x,s)}{dx} + s\,\Delta v(x,s) + K_h\,\Delta v(x,s) = 0 \tag{3}$$

$$\frac{d\,\Delta v(x,s)}{dx} + \frac{1}{\rho w^2}\,s\,\Delta p(x,s) = 0 \tag{4}$$

where: s — variable of the Laplace transformation.

By determining $\Delta v(x,s)$ from equation (3) and putting it into (4), the II-nd order linear differential equation is obtained:

$$\frac{d^2\Delta p(x,s)}{dx^2} - \frac{(K_h+s)s}{w^2}\,\Delta p(x,s) = 0 \tag{5}$$

The general solution of equation (5) is:

$$\Delta p(x,s) = c_1 e^{rx} + c_2 e^{-rx}. \tag{6}$$

Putting (6) into (3), the solution for the velocity change Δv is obtained in the form

$$\Delta v(x,s) = -\frac{1}{Z_c}(c_1 e^{rx} - c_2 e^{-rx}) \tag{7}$$

where:

$$r = \frac{1}{w}\sqrt{s(K_h+s)}, \qquad Z_c = \frac{\rho}{r}(K_h+s).$$

The value of constants c_1 and c_2 is determined from boundary conditions of the considered system. These constants have been determined with the assumption that the pressure changes at the end of the pipe and velocities at its beginning are known (which most often is true for the discharge pipes). After taking into account the expressions for c_1 and c_2, the equations (6) and (7) have been used to find the relation between the pressure and the flow velocity changes at the beginning and end of the pipe:

$$\Delta v(0,s)=\frac{1}{Z_c}\frac{e^{rL}-e^{-rL}}{2}\Delta p(L,s)+\frac{e^{rL}+e^{-rL}}{2}\Delta v(L,s) \tag{8}$$

$$\Delta p(0,s)=\frac{e^{rL}+e^{-rL}}{2}\Delta p(L,s)+Z_c\frac{e^{rL}-e^{-rL}}{2}\Delta v(L,s) \tag{9}$$

These relations constitute a mathematical model of a pipe of constant diameter situated at a constant angle of inclination. In order to formulate a model of a pipe of by step varying diameter and variable inclination, of each section, the above equations should be used, and additionally, at the points of diameter or inclination change, the continuity and the force balance equations should be taken into account.

3. Block Diagram of the Mathematical Model of the Water Hammer

Equations (8) and (9) represent pressure and flow velocity changes in the form of Laplace transforms. In order to determine the change in time of pressure and flow velocity, the inverse transformation must carried out. The values of pressure and velocity may be also obtained by modelling equations (8) and (9) on a computer, using the IBM-CSMP language. To make the programming easier, the equatio: s are presented in a block diagram form (Fig. 1).

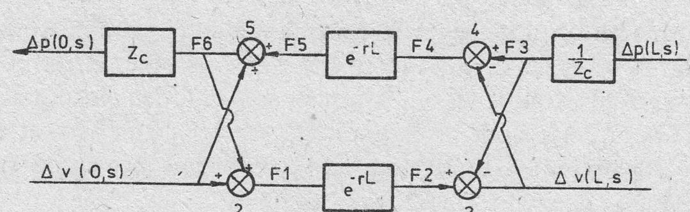

Fig. 1. Block diagram of water hammer in a pipe

Elements with transmittance Z_c and e^{-rL}, appearing in the block diagram, are not standard functions of the IBM-CSMP language. Therefore they must be expanded into a power series.

By expanding into a power series the expression for r has been obtained in the following form:

$$r=\frac{1}{w}\sqrt{s(K_h+s)}\cong\frac{s}{w}\left(1+\frac{K_h}{2}\frac{1}{s}-\frac{K_h^2}{8}\frac{1}{s^2}+\frac{K_h^3}{16}\frac{1}{s^3}\cdots\right) \tag{10}$$

The third and the further terms of the series, are very small in comparison with the first two terms. This results from the value of K_h, which for water pipes is much smaller than 1.

Taking into account relationship (10), the element of transmittance e^{-rL} appearing in the block diagram may be replaced by a series connection of blocks with transmittance

$$e^{-\frac{K_h L}{2w}} \quad \text{and} \quad e^{-\frac{L}{w}s}.$$

Now IBM-CSMP standard functions can be used for writing the program.

The block containing transmittance Z_c can also be replaced. Expanding the root present in this expression into a power series the following formula has been obtained:

$$Z_c = w\sqrt{1+\frac{K_h}{s}} \cong \rho w\left(1+\frac{K_h}{2}\frac{1}{s}-\frac{K_h^2}{8}\frac{1}{s^2}+\frac{K_h^3}{16}\frac{1}{s^3}\cdots\right) \tag{11}$$

As in expression (10), only the two first terms of the expansion have been used in further calculations. Therefore, the block with transmittance Z_c can be replaced by a parallel connection of blocks with transmission function ρw and $\rho w\frac{K_h}{2}\frac{1}{s}$, which are standard IBM-CSMP functions. Often, when a real hydraulic system is considered, the block diagram becomes simpler. In the further part of the paper a diagram and calculations for a straight discharge pipe are presented.

4. Calculations of Water Hammer in a Discharge Pipe

Discharge pipes are often a part of hydraulic systems. For the calculations a discharge pipe with a pump at its beginning was taken into account. At the other end of the pipe is a large open tank. Most often the water hammer effect in such a pipe is due to sudden switching on or off of the pump, or to sudden closing of the gate valve dowstream the pump. Methods of determining the velocities downstream the pump or gate valve are presented in reference [4]. For thus presented example a rectilinear flow velocity decay in time T_A was assumed.

During the water hammer of short duration, the pressure at the connection of pipe and tank does not change. Therefore, the block diagram model presented in Fig. 1 can be simplified. Since $\Delta p(L, s)$ designates the transform of pressure deviation from the initial state, then during water hammer occurrence $\Delta p(L, s) = 0$ at the tank. It results that the discharge pipe model may be presented in the form shown in Fig. 2.

The common factor ρw is separated from the blocks defining the transmittance Z_c. It is also assumed that velocity changes at the end of the pipe (at the tank) will not be determined during computations. Therefore only one signal is supplied to and carried away from summing knots 3 and 4. These knots constitute one line, and the negative sign of the signal coming from knot 4 is replaced to knot 2.

Transmittances appearing in the presented diagram are standard IBM-CSMP functions, and the system can be programmed on a digital computer in order to calculate the pressure changes.

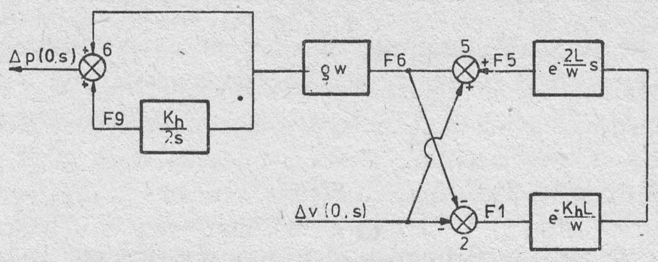

Fig. 2. Block diagram of water hammer in a discharge pipe

To give an example, a discharge pipe of length $L = 800.0$ m, diameter $D = 0.5$ m and hydraulic resistance coefficient $\lambda = 0.026$ was considered. Initial flow velocity was $v_0 = 1.5$ m/s. The calculated velocity of the changed pressure wave is $w = 1073.0$ m/s.

The developed program is presented below:

```
TITLE      PRESSURE VARIATION COURSE IN THE DELIVERY
           SYSTEM BEHIND THE GATE
INITIAL
CONSTANT   L = 800.0, D = 0.5, V0 = 1.5, TA = 1.0, W = 1073.0
CONSTANT   LAM = 0.026
           KH = LAM*V0//3*D/
           HP0 = KH*L*V0*1000
           C3 = KH*W*500
           C11 = L/W
           C12 = 2*C11
           C13 = KH*C11
           C14 = EXP/-C13/
DYNAMIC    VD = -V0/TA*LIMIT/0.0; TA; TIME/
           F1 = -VD - F6
           F5 = DELAY/100, C12, C14*F1/
           F6 = F5 + VD
           F9 = INTGRL/HP0, F6*C3/
           PD = F9 + F6*W*1000
TERMINAL
TIMER      FINTIM = 22.5, DELMIN = 1.0E-9, OUTDEL = 0.15
OUTPUT     PD
PAGE       NTAB = 0
LABEL      PRESSURE VARIATION COURSE BEHIND THE GATE
END
STOP
ENDJOB
```

Structural instructions describing the considered system appear in the DYNAMIC part. The remaining part of the program deals with preliminary calculations and organization instructions.

Pressure changes in the pipe due to the fact that the flow at its beginning stops after the time $T_A = 5$ s are shown in Fig. 3.

In order to evaluate the credibility of obtained results, they were compared with the results obtained by solving the basic system of equations for the considered pipe using the method of characteristics. These last results show good agreement [5] with experimental results. The obtained results were put in together with the previously made curves (Fig. 3). It can be stated, that there is a good agreement of results obtained by both methods, especially for the first few time-stages of the water hammer curve.

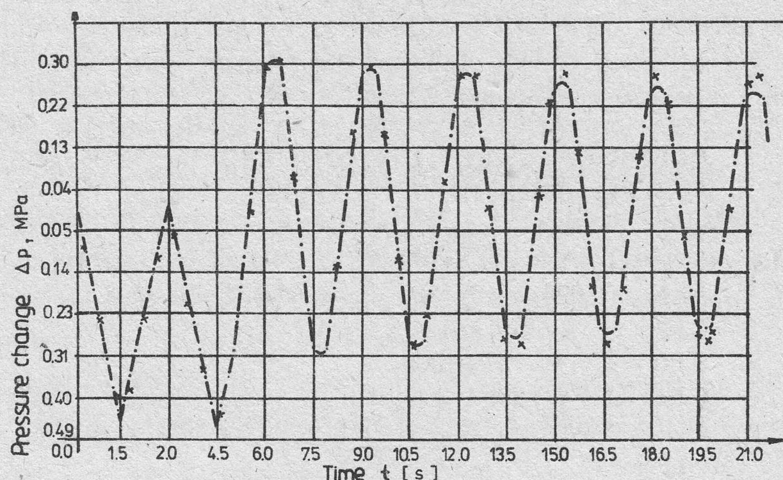

Fig. 3. Comparison of pressure change curves calculated with presented method and with the method of characteristics

——•—— operator method, —— × —— method of characteristics

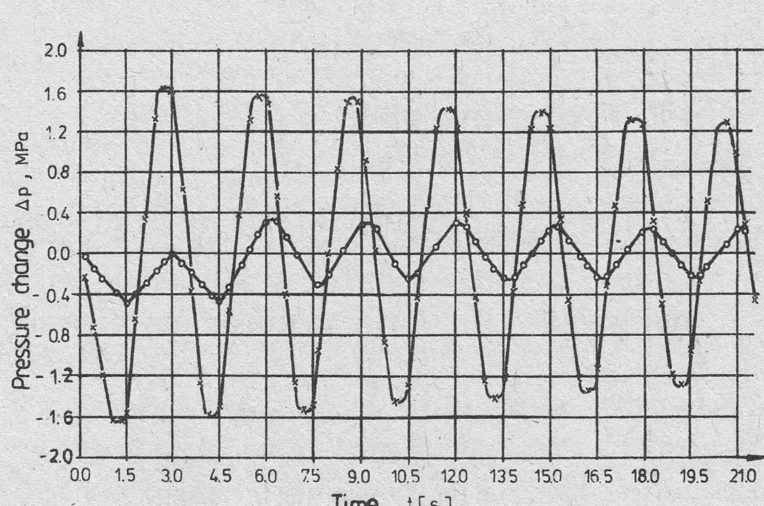

Fig. 4. Influence of flow stopping time on pressure change

——○—— time $T_A = 5$ s, —— × —— time $T_A = 1$ s

The next curve (Fig. 4) illustrates the influence of flow stopping time on the pressure change curve. The pressure change curves for flow stopping times $T_A = 1$ s and $T_A = 5$ s are shown. The shorter time is selected in such a way that the so called simple water hammer occurs in the system. For $T_A = 5$ s, a non-simple water hammer takes place in the pipe.

The curve indicates that the maximum deviation of pressure from the static pressure is 1.61 MPa. The pressure change calculated from the Allievi formula [4] is for this system:

$$\Delta p = \rho w \Delta v = 1.61 \text{ MPa}.$$

These values also confirm the credibility of the presented method of pressure change calculation.

It may be also observed from the curve that increase in time during which the flow is stopped (leading to the non-simple water hammer) results in a decisive decrease of extreme pressure changes during the water hammer. Therefore, slow changing of flow velocity is one of the methods of preventing excessive pressure changes. The curve shows that decrease in the initial velocity of flow results in significant drop of pressure changes occurring during the water hammer. The influence of other parameters characterizing the system on pressure changes occurring during the water hammer can be examined in a similar way. Printout in the form of curves allows quick evaluation of results obtained.

5. Summary

The results obtained with the use of the presented method are in good agreement with results obtained by solving the basic system of equations using the method of characteristics as well as with the results obtained from the Allievi formula. This warrants the credibility of curves obtained.

The model for water hammer calculations is presented in a block diagram form. This makes utilization of the IBM-CSMP simulation language for computer calculations easier. The language enables performing calculations on a computer with only very little knowledge of programming. Results are obtained directly in the form of curves. This allows to check the results more quickly than in case when they are given in the form of tables. The above feature of the presented method of water hammer analysis suggests that it can be recommended for use in design calculations.

The method can be also used for calculating water hammer in more complex hydraulic systems and for selecting water hammer damping devices [5].

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Symulacja cyfrowa przebiegu uderzenia hydraulicznego

Streszczenie

W pracy przedstawiono metodę modelowania i analizy uderzenia hydraulicznego w przewodzie hydraulicznym. Układ równań opisujących przebieg uderzenia, który jest szczegółowo omawiany w pracach 1 i 4, po zlinearyzowaniu przedstawiono w postaci równań (1) i (2). Rozwiązanie tych równań, zależności (8) i (9), przedstawia związek pomiędzy zmianami ciśnienia i prędkości przepływu na początku przewodu $p(0, s)$, $v(0, s)$ oraz na końcu przewodu $p(L, s)$, $v(L, s)$. Do wyznaczenia wartości liczbowych ciśnienia i prędkości w przewodzie na komputerze korzystano z języka programowania IBM-CSMP. W celu ułatwienia programowania równania (8) i (9) przedstawiono w postaci schematu blokowego (rys. 1).

Dla przykładowych obliczeń przyjęto przewód tłoczny, na początku którego znajduje się pompa. Na drugim końcu przewód zakończony jest dużym zbiornikiem otwartym. Schemat blokowy przebiegu uderzenia w tym przewodzie przedstawiono na rysunku 2. Wyniki obliczeń na komputerze przedstawia rysunek 3. Celem oceny wiarygodności otrzymanych wyników porównano je z wynikami uzyskanymi z rozwiązania podstawowego układu równań dla rozpatrywanego układu za pomocą metody charakterystyk i uzyskano dużą zgodność.

Przedstawiony model przebiegu uderzenia może być wykorzystany do obliczeń w układach bardziej złożonych (przewody łączone równolegle i szeregowo). Schemat blokowy pozwala na uwzględnienie zmian struktury badanego układu wykorzystując metody przekształcania schematów blokowych. Możliwość zastosowania języka programowania IBM-CSMP pozwala na korzystanie z przedstawionej metody już przy niewielkiej znajomości programowania. Wyniki otrzymuje się bezpośrednio w postaci wykresów, co ułatwia analizę układu podczas obliczeń inżynierskich.

Цифровая симуляция хода гидравлического удара

Резюме

В работе представлен метод моделирования и анализа гидравлического удара в гидравлическом трубопроводе. Система уравнений описывающих ход удара, подробно описанная в трудах [1] и [4], после линеаризации представлена в форме уравнений (1) и (2). Решение этих уравнений, т.е. зависимостей (8) и (9), представляет связь между изменениями давления и скорости течения в начале трубопровода $\Delta p(0, s)$, $\Delta v(0, s)$ и в конце трубопровода $\Delta p(L, s)$, $\Delta v(L, s)$.

Для определения численных значений давления и скорости в трубопроводе на ЭВМ использован язык программирования IBM-CSMP. С целью облегуения программирования уравнений (8) и (9) представлено в виде блочной схемы (рис. 1).

Для примерных расчётов принят напорный трубопровод, в начале которого находится насос. На другом конце трубопровод входит в большой открытый резервуар. Блочная схема хода удара в этом трубопроводе представлена на рис. 2. Результаты расчётов на ЭВМ представлены на рис. 3.

С целью оценки достоверности полученных результатов произведено их сравнение с результатами решения основной системы уравнений для рассматриваемой системы с помощью метода характеристик. Достигнуто хорошее соответствие.

Представленная модель хода удара может быть использована для расчётов в более сложных системах (трубопроводы соединяемые параллельно и последовательно). Блочная схема позволяет учитывать изменения структуры исследуемой системы, используя методы превращений блочных схем. Возможность применения языка программирования IBM-CSMP позволяет использовать представленный метод уже при небольшом ознакомлении с программированием. Результаты получаются непосредственно в форме графиков. Это облегчает анализ системы во время инженерных расчётов.