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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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ГИДРОФОРУМ

НАУЧНО-ТЕХНИЧЕСКАЯ КОНФЕРЕНЦИЯ

на тему

ПРОБЛЕМЫ ГИДРАВЛИЧЕСКИХ ТУРБОМАШИН

Гданьск-Владыславово, 24-27 сентября 1985 г.

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Some Results of Analysis of the Flow Past Blade Cascades

Some results of numerical analysis of flow past rotating blade cascades used in axial pumps and turbines are presented. Comparisons with other authors' results and experiment are made.

1. Introduction

A computer code for the analysis of the flow past blade cascades used in propeller pumps and turbines was developed and verified in the Department of Fluid Dynamics of the Institute of Fluid-Flow Machinery of the Polish Academy of Sciences in Gdańsk. This work was supported by the Government Project PR-8.

In the present paper a mathematical model, numerical results and a comparison with both numerical and experimental results obtained by other authors are presented. It is scheduled to carry out experimental verification of the code on a propeller pump stand at the laboratory of the Institute of Fluid-Flow Machinery.

2. Flow Model

The numerical results presented in section 4 were obtained using a numerical code which had been developed on the basis of the potential flow model of ideal fluid. It is assumed that the three-dimensional flow through a rotor can be approximated by a set of two-dimensional flows on axisymmetric stream surfaces (in the case of an axial rotor — cylindrical surfaces), which divide the rotor channel into partial channels. Conformal mapping of the chosen stream surface onto complex plane transforms the partial rotor into a straight cascade of profiles. The auxiliary plane, obtained in this way, can be considered as an infinite series of congruent domains D_n, each of them containing an isolated element of the cascade K (Fig. 1). The determination of the flow field within the framework of the potential theory amounts to finding a solution of boundary problems of the elliptic (Laplace) equation.

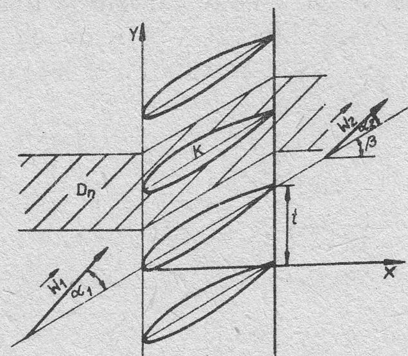

Fig. 1. A straight cascade in the auxiliary plane

Consequently, two mathematically equivalent Fredholm integral equations of th
first and second kind for the velocity component tangential to the contour of a profi
are obtained [1, 2, 3]:

$$\frac{1}{\pi}\int_0^{2\pi} c_t(\varphi')L(\varphi,\varphi')d\varphi' = 2[c_{mx}\dot{y}(\varphi)-c_{my}\dot{x}(\varphi)]+c_n(\varphi)-\frac{1}{\pi}\int_0^{2\pi} c_n(\varphi')K(\varphi,\varphi')d\varphi', \qquad (1)$$

$$c_t(\varphi)-\frac{1}{\pi}\int_0^{2\pi} c_t(\varphi')K(\varphi,\varphi')d\varphi' = 2[c_{mx}\dot{x}(\varphi)+c_{my}\dot{y}(\varphi)]-\frac{1}{\pi}\int_0^{2\pi} c_n(\varphi')L(\varphi,\varphi')d\varphi', \qquad (2)$$

where c_t, c_n — absolute velocity components tangential and normal to the profi
$c_m = (c_1+c_2)/2$ — mean velocity of the flow, φ — angular parameter of the profi
contour; φ varies from 0 to 2π, while $\dot{x} = dx/d\varphi$ and $\dot{y} = dy/d\varphi$.

Integral kernels are given by:

$$K(\varphi,\varphi') = R(\varphi,\varphi')\dot{y}(\varphi)+I(\varphi,\varphi')\dot{x}(\varphi),$$

$$L(\varphi,\varphi') = R(\varphi,\varphi')\dot{x}(\varphi)-I(\varphi,\varphi')\dot{y}(\varphi),$$

$$R(\varphi,\varphi') = \frac{\pi}{t}\frac{\sinh\{2\pi/t[x(\varphi)-x(\varphi')]\}}{\cosh\{2\pi/t[x(\varphi)-x(\varphi')]\}-\cos\{2\pi/t[y(\varphi)-y(\varphi')]\}},$$

$$I(\varphi,\varphi') = -\frac{\pi}{t}\frac{\sin\{2\pi/t[y(\varphi)-y(\varphi')]\}}{\cosh\{2\pi/t[x(\varphi)-x(\varphi')]\}-\cos\{2\pi/t[y(\varphi)-y(\varphi')]\}},$$

where t denotes the cascade pitch.

Equations (1) and (2) describe the most general case of flow past a cascade th
tangential component of velocity is directly equal to vortex intensity distribu
along the profile contour, while the normal component of velocity is directly equ
the intensity of sources modelling the movement effect of the cascade. In case
a stationary cascade $c_n = 0$, while $t \to \infty$ is an assumption made in Eqs (5) and
case of an isolated profile.

If the blade cascade rotates it is more convenient to use a coordinate system related to the rotor with the assumption that the contour of a profile is a streamline of the relative flow. Hence, the normal component of the relative velocity is equalled to 0 at the profile and two following relations hold:

$$c_t = w_t + u_t, \qquad c_n = u_n. \tag{7}$$

Substituting relationship (7) into Eqs (1) and (2) results in the following equations:

$$\frac{1}{\pi}\int_0^{2\pi} w_t(\varphi')L(\varphi,\varphi')d\varphi' = 2[c_{mx}\dot{y}(\varphi) - c_{my}\dot{x}(\varphi)] + u_n(\varphi) - \\ -\frac{1}{\pi}\int_0^{2\pi} u_t(\varphi')L(\varphi,\varphi')d\varphi' - \frac{1}{\pi}\int_0^{2\pi} u_n(\varphi')K(\varphi,\varphi')d\varphi', \tag{8}$$

$$w_t(\varphi) - \frac{1}{\pi}\int_0^{2\pi} w_t(\varphi')K(\varphi,\varphi')d\varphi' = 2[c_{mx}\dot{x}(\varphi) + c_{my}\dot{y}(\varphi)] - u_t(\varphi) - \\ -\frac{1}{\pi}\int_0^{2\pi} u_n(\varphi')L(\varphi,\varphi')d\varphi' + \frac{1}{\pi}\int_0^{2\pi} u_t(\varphi')K(\varphi,\varphi')d\varphi'. \tag{9}$$

In practice equation (9) is used to analyze the flow past profile cascades. Given the assumption of constant peripheral velocity in the chosen cylindrical cross-section of an axial rotor and some properties of integral kernels [3], equation (9) reduces to the equation valid for a stationary cascade:

$$w_t(\varphi) - \frac{1}{\pi}\int_0^{2\pi} w_t(\varphi')K(\varphi,\varphi')d\varphi' = 2[w_{mx}\dot{x}(\varphi) + w_{my}\dot{y}(\varphi)]. \tag{10}$$

The above equation is solved for the relative velocity component tangential to the profile contour. Pressure distribution is obtained from Bernoulli equation.

3. Numerical Method

Integral equation (10) is solved by superposition of solutions of the homogeneous and two non-homogeneous equations with appropriate conditions [4]. The quadrature method is used to reduce the problem to solving a system of three linear equations for discrete unknowns $w_{tm}{}^{(0)}$, $w_{tm}{}^{(1)}$, $w_{tm}{}^{(2)}$:

$$\sum_{m=1}^{M} w_{tm}{}^{(0)} K_{nm} = 0, \tag{11}$$

$$\sum_{m=1}^{M} w_{tm}{}^{(1)} K_{nm} = 2M\dot{x}_n, \tag{12}$$

$$\sum_{m=1}^{M} w_{tm}{}^{(2)} K_{nm} = 2M \dot{y}_n \tag{13}$$

($n = 1, M$; M — the number of points on the profile contour used for calculatio[n]) with the following conditions:

$$\sum_{m=1}^{M} w_{tm}{}^{(0)} = 1, \tag{14}$$

$$\sum_{m=1}^{M} w_{tm}{}^{(1)} = \sum_{m=1}^{M} w_{tm}{}^{(2)} = 0, \tag{15}$$

which are added to each of the equations of an appropriate system (11)-(13) wi[th] the exception of the first equation. To solve the systems of equations Gauss elimina- tion method was applied. The determination of K_{nm} matrix elements was based o[n] the following relation:

$$\sum_{m=0}^{M} K_{mn} = 0. \tag{16}$$

Hence:

$$K_{m,M-m} = -\sum_{\mu=1}^{M} K_{\mu,M-m}. \tag{17}$$

For $m = n$ the diagonal elements of the matrix were determined accordin[g to] Jacob's suggestion using the following formula [5]:

$$K_{mm} = \frac{1}{12}[4(K_{m,m-1} + K_{m,m+1} + K_{m-1,m} + K_{m+1,m}) - \\ - K_{m,m-2} - K_{m,m+2} - K_{m-2,m} - K_{m+2,m}] - M. \tag{18}$$

The profile geometry was represented by two smooth curves which were inter- polated by cubic spline curves [6]. For the profiles analysed in section 4 the an[gular] parametrization with respect to parameter φ_n was used:

$$x_n = \frac{1}{2}(1 + \cos\varphi_n) l, \qquad n = \frac{n-1}{M}\pi, \qquad n = 1, M \tag{19}$$

where x_n — the discrete value of the profile contour x coordinate, l — chord len[gth].

4. Numerical Examples and Conclusions

Blade cascades of pumps and turbines as well as airfoils were analysed un[der] various operating conditions by means of a numerical code which had been deve- loped on the basis of the mathematical model presented herein. Some num[erical] results concerning isolated aerofoils and rotating profile cascades NACA 4[...], NACA 0021 and NACA 2412 are shown in Figures 2a, 2b and 3, respecti[vely]. In each case the pitch/chord ratio is $t/l = 1$. The obtained distributions of the re[la]- tive velocity or the pressure coefficient along the profile coincide well with th[e]

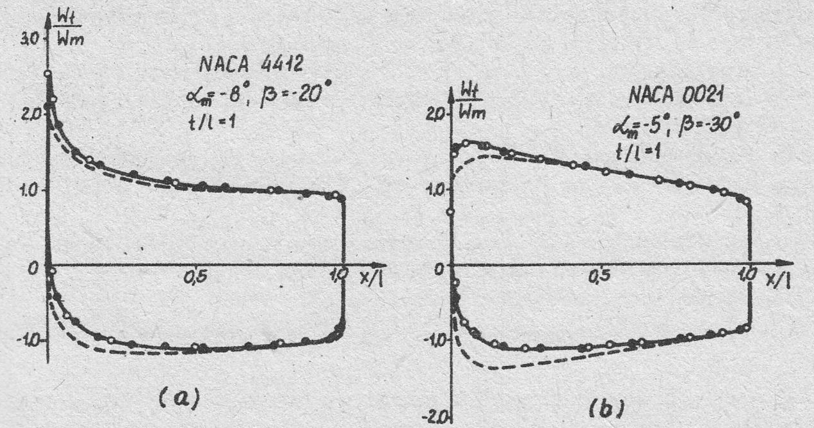

Fig. 2. Relative velocity distribution along the NACA 4412 (a) and NACA 0021 (b) profile contours in the case of a rotating cascade and an isolated aerofoil

——— cascade, ——— aerofoil (● — after [7], ○ — after the author)

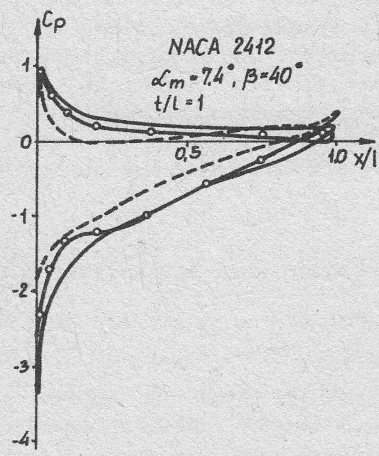

Fig. 3. Pressure coefficient distribution along the NACA 2412 and NACA 0021 profile contours in the case of a rotating cascade and an isolated aerofoil

– – – – cascade, (after the author), ——— aerofoil (calculations by [7] and the author), —○— aerofoil (experimental results after [7]).

reported in [7] (Fig. 2a, 2b, 3). The numerical results concerning the NACA 2412 aerofoil are also in satisfactory agreement with experiment (Fig. 3).

As the next step it is scheduled to analyse the flow through an axial pump by means of the computer code and then to compare theoretical results with experiment. To this end a special model pump was designed and built, and an appropriate test stand is being under construction.

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O pewnych wynikach analizy przepływu przez palisady łopatkowe

Streszczenie

Opracowano i zweryfikowano oparty na metodzie równań całkowych program numeryczny anali przepływu przez palisady łopatkowe pomp i turbin osiowych.

W pracy przedstawiono model matematyczny w zarysie oraz niektóre wyniki numeryczne. Dokon porównania wyników własnych z wynikami teoretycznymi uzyskanymi przez innych autorów ora z wynikami doświadczalnymi (rys. 2 i 3). Porównanie wskazuje na dość dobrą zgodność wyników teore tycznych z doświadczalnymi (rys. 3).

Przewiduje się przeprowadzenie weryfikacji doświadczalnej opracowanego programu na stanowis pompowym w laboratorium Instytutu Maszyn Przepływowych PAN w Gdańsku.

О некоторых результатах анализа течения через лопаточные решётки

Резюме

Разработана и проверена основанная на методе интегральных уравнений численная програм анализа течения через решётки лопаток насосов и осевых турбин.

В работе представлен очерк математической модели и некоторые результаты числен расчётов. Проведены сопоставления собственных результатов с теоретическими результа достигнутыми другими авторами, а также с результатами из эксперимента (рис. 2÷3). Сравнен указывает на довольно хорошую сходимость теоретических и экспериментальных результ исследований (рис. 3).

Предусматривается проведение экспериментальной проверки разработанной программы н стенде для исследования насосов в лаборатории Института проточных машин ПАН в Гданьс