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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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на тему

ПРОБЛЕМЫ РАЗВИТИЯ ГИДРАВЛИЧЕСКИХ РОТОРНЫХ МАШИН С ОСОБЫМ УЧЕТОМ НУЖД ЭНЕРГЕТИКИ

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Budapest*

Determination of Invariants in Connection with Examination of Long Pipelines with Air Chambers

Nomenclature

- s – spot coordinate along the pipeline,
- c – velocity of the liquid in the pipe,
- t – time; $t=0$ at the beginning of examination,
- ρ – density of liquid,
- p – pressure,
- g – acceleration of gravity,
- z – coordinate of height,
- D, d, d_c – diameters of pipes,
- w – wave velocity,
- E_r – reduced elastic modulus of the pipe and the liquid in it,
- L – length of the elastic pipe,
- λ – friction coefficient in the elastic pipe,
- W_g – gas volume in airchamber,
- n – polytropic exponent,

Indices

- 0 – starting condition,
- * – dimensionless quantity.

For reduction of pressure oscillations during transient processes in pipelines, air chambers built in downstream of the pump are often used. The movement of the liquid level in the air chamber can only be traced by solving the mathematical model made for the particular elastic system. Since in this case a model of distributed parameter system may be used, this leads to a quasi-linear hyperbolic boundary value problem. To solve this only numerical methods can be suggested for which, however, special computer programs are needed. On the basis of the mathematical model it is expedient to look for such invariants which are characteristic for the dimensionless system. With the adequate choice of the set of values of the invariants the solutions of cases expected in the design practice can be produced. If the number of invariants were large their handling would again make the procedure complicated. Therefore it is advisable to strive after production of possibly fewest, i.e. optimally chosen invariants. One method of achieving this is simplification of the model to such an extent which is still allowable.

For an air chamber protecting the system against the pressure oscillation starting in consequence of the current storage of the pump the simplified model as shown in Fig. 1 can be used. Before starting of the examination the system was supplied by a pump at the point A in steady-state conditions. At the instant of starting the examination the supply

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ceases and only the air chamber is connected to the point A through F throttling. L is a long pipeline at the B point of which constant pressure is set as a boundary condition.
The point-coordinate along the pipeline is s.

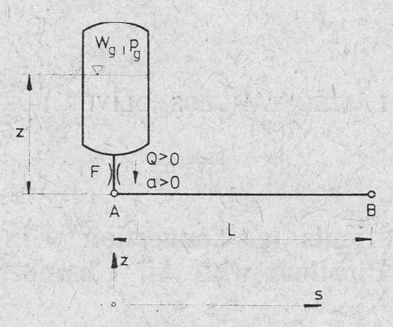

Fig. 1

It will also lead to the reduction of the number of invariants if we state in the same form the invariants equally characteristic for the two essential elements:

1) for the long elastic pipeline and,

2) for the boundary conditions describing the air chamber.

Let us start from the long elastic pipeline with the following constraints:

a) the cross section and wall thickness of the pipe is constant,

b) the density of the liquid filling the pipe is the function of the pressure only, $\rho=\rho(p)$,

c) we disregard the transversal changes.

Equations characteristic for the pipe:

$$\frac{\partial c}{\partial t}+c\frac{\partial c}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial s}+g\frac{dz}{ds}+\frac{\lambda}{2D}|c|c=0, \tag{1}$$

continuity

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial s}(\rho c)=0 \tag{2}$$

and the velocity of wave propagation

$$\sqrt{\frac{dp}{d\rho}}=w=\sqrt{\frac{E_r}{\rho}}. \tag{3}$$

The simplifying conditions b) and c) lead to the following form of the continuity equation

$$\frac{d\rho}{dp}\frac{\partial p}{\partial t}+c\frac{d\rho}{dp}\frac{\partial p}{\partial s}+\rho\frac{\partial c}{\partial s}=0 \tag{4}$$

from which upon substitution of (3) we get the equation

$$\frac{1}{w^2}\left(\frac{\partial p}{\partial t}+c\frac{\partial p}{\partial s}\right)+\rho\frac{\partial c}{\partial s}=0. \tag{5}$$

Let us make the variables of equations (1) and (5) dimensionless in the following way: Let us denote the dimensionless quantities with asterisk, subscript 0 denoting the quantity making them dimensionless

$$s^*=\frac{s}{L_0}, \quad p^*=\frac{p}{p_0}, \tag{6), (7}$$

$$\rho^*=\frac{\rho}{\rho_0}, \quad c^*=\frac{c}{c_0}. \tag{8), (9}$$

The dimensionless time is $t^*=t/t_0$, where

$$t_0=L_0\int_0^1 \frac{ds^*}{w(s^*)}. \tag{10}$$

Hence the dimensionless wave velocity

$$w^*=\frac{w}{w_0}, \tag{11}$$

where

$$w_0=\frac{L_0}{t_0}.$$

Substituting the above into equation (1) results in the dimensionless equation of motion

$$\frac{c_0}{t_0}\frac{\partial c^*}{\partial t^*}+\frac{c_0^2}{L_0}c^*\frac{\partial c^*}{\partial s^*}+\frac{p_0}{\rho_0 L_0}\frac{1}{\rho^*}\frac{\partial p^*}{\partial s^*}+g\frac{p_0}{L_0 \rho_0}\frac{\partial z^*}{\partial s^*}+\frac{c_0^2}{L_0}\frac{\lambda}{2D^*}c^*|c^*|=0. \tag{12}$$

After converting identically and arranging the terms

$$\frac{c_0 w_0 \rho_0}{p_0}\left[\frac{\partial c^*}{\partial t^*}+\frac{c_0}{w_0}\left(c^*\frac{\partial c^*}{\partial s^*}+\frac{\lambda}{2D^*}c^*|c^*|\right)\right]+\frac{1}{\rho^*}\frac{\partial p^*}{\partial \rho^*}+g\frac{\partial z^*}{\partial s^*}=0. \tag{13}$$

In equation (13)

$$\varepsilon=\frac{c_0 w_0 \rho_0}{p_0}, \quad \alpha=\frac{c_0}{w_0} \tag{14), (15}$$

and λ as well, are invariants.

It can be seen that equation (5) brings no more invariants. Making equation (5) dimensionless yields

$$\frac{1}{w_0^2 w^{*2}}\left(\frac{p_0}{t_0}\frac{\partial p^*}{\partial t^*}+\frac{c_0 p_0}{L_0}c^*\frac{\partial p^*}{\partial s^*}\right)+\rho_0\frac{c_0}{L_0}\rho^*\frac{\partial c^*}{\partial s^*}=0. \tag{16}$$

After converting the terms we obtain

$$\frac{1}{\varepsilon w^{*2}}\left(\frac{\partial p^*}{\partial t^*}+\alpha c^*\frac{\partial p^*}{\partial s^*}\right)+\rho^*\frac{\partial c^*}{\partial s^*}=0. \tag{17}$$

To solve numerically equations (13) and (17) we determine the division of independent variables in such a way that

$$\Delta s^* = w^* \Delta t^* . \tag{18}$$

From this and from equations (13) and (17) at the point A the following condition can be obtained for the pressure and the velocity

$$p^* + \varepsilon \rho^* w^* c^* = P_0^* , \tag{19}$$

(see [2]), where P_0^* is a known function which can be calculated from data originated for time earlier by Δt^*. For equation (19) at the point A the air chamber gives the further boundary condition prescriptions.

Supposing a polytropic gas change of state for the W_g gas volume of the air chamber we have

$$p_g W_g^n = K_0 . \tag{20}$$

The liquid volume flowing through the F throttle is

$$Q = \frac{dW_g}{dt} \tag{21}$$

The pressure at the bottom-point of the air chamber in case of positive direction given in Fig. 1 can be expressed as:

$$p_A = \frac{K_0}{W_g^n} + \rho g z - \zeta \frac{8}{\pi^2} \rho \left| \frac{dW_g}{dt} \right| \frac{dW_g}{dt} \frac{1}{d^4} - \rho z \frac{d^2 W_g}{dt^2} \frac{4}{\pi} \frac{1}{d^2} . \tag{22}$$

In the equation (7) let p_0 be the pressure in the air chamber in case of steady operation. Let us introduce further

$$W_g^* = W_g / W_{0g} , \qquad z^* = z / z_0 , \tag{23}$$

$$p_0 W_{0g}^n = K_0 . \tag{24}$$

Making equation (22) dimensionless according to the above, making use of (24) and substituting all the invariants introduced so far we have

$$p_A^* = \frac{1}{W_g^{*n}} + \frac{\rho_0 z_0 g}{p_0} \rho^* z^* - \zeta \frac{\varepsilon}{\alpha} \left(\frac{W_{0g}}{L_0^3} \right)^2 \frac{8}{\pi^2 d^{*4}} \rho^* \left| \frac{dW_g^*}{dt^*} \right| \frac{dW_g^*}{dt^*} -$$

$$- \frac{W_{0g}}{L_0^3} \frac{\varepsilon}{\alpha} \frac{z_0}{L_0} \frac{4}{d^{*2} \pi} \frac{d^2 W_g}{dt^{*2}} . \tag{25}$$

The velocity in the pipe is equal

$$c = \frac{4}{d_c^2 \pi} \frac{dW_g}{dt} . \tag{26}$$

Making it dimensionless and substituting the invariants introduced yields

$$c^* = \frac{W_{0g}}{L_0^3} \alpha \frac{4}{\pi} \frac{1}{d_c^{*2}} \frac{dW_g^*}{dt^*} . \tag{27}$$

Substituting equations (25) and (27) into (19)

$$\frac{1}{W_g^{*n}}+\frac{\rho_0 g z_0}{p_0}\rho^* z^* - \zeta\frac{\varepsilon}{\alpha}\left(\frac{W_{0g}}{L_0^3}\right)^2\frac{8}{\pi^2 d^{*4}}\rho^*\left|\frac{dW_g^*}{dt^*}\right|\frac{dW_g^*}{dt^*} -$$

$$-\frac{W_{0g}}{L_0^3}\frac{\varepsilon}{\alpha}\frac{z_0}{L_0}\frac{4}{\pi d^{*2}}\frac{d^2W_g^*}{dt^{*2}}+\varepsilon\alpha\frac{W_{0g}}{L_0^3}\rho^* w^*\frac{4}{d_c^{*2}\pi}\frac{dW_g^*}{dt^*}=P_0^*. \tag{28}$$

If p_0 is the pressure for the steady-state operation and z_0 is the level height in the air chamber then the term of the equation (28)

$$\frac{\rho_0 g z_0}{p_0}=\frac{p_{A\,\mathrm{stac}}-p_0}{p_0}=\frac{p_{A\,\mathrm{stac}}}{p_0}-1=p_{A\,\mathrm{stac}}^*-1 \tag{29}$$

needs not be regarded in consequance of (29) as a new invariant.

From equation (28) we have now the invariants needed to characterize the system

- ε see equation (14),
- α see equation (15),
- λ friction factor in the elastic pipe,
- ζ drag coefficient of F throttle,
- $K_1=W_{0g}/L_0^3$,
- $K_2=z_0/L_0$,
- n polytropic exponent.

At the Department of Hydraulic Machinery of the T. U. Budapest investigation is made currently in order to reduce further the numbers of the above invariants for particular series of calculations and to achieve better understanding of their influence.

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[1] O. Füzy, *Analysis of hydraulic systems by means of digital simulation.* Proceedings of the Sixth Conference of Fluid Machinery, Budapest 1979.

[2] O. Füzy, L. Kullmann, *Rapid method for calculating the liquid flow in a pipe network.* Proceedings of the Sixth Conference of Fluid Machinery, Budapest 1979.

Wyznaczanie niezmienników dlą badania długich rurociągów z komorami powietrznymi

Streszczenie

Artykuł poświęcony jest badaniu zachowania komory powietrznej w czasie trwania nieustalonych procesów hydraulicznych. Opierając się na równaniach różniczkowych, charakteryzujących zjawisko, szuka się niezmienników charakteryzujących procesy przejściowe.

Traktując przyjęte niezmienniki jako parametry prowadzi się badania porównawcze uproszczonego modelu układu (w postaci nieskończenie długiej rury połączonej z komorą powietrzną) i symulację numeryczną dokonywaną na podstawie nieuproszczonego układu równań różniczkowych. Celem tego porównania jest określenie czynników pozwalających na wyznaczenie, przy danych wartościach niezmienników, wymiarów komory powietrznej na podstawie uproszczonego modelu układu.

Определение нонвариант для исследования длинных трубопроводов с воздушными камерами

Резюме

Статья посвящена исследованию поведения воздушной камеры во время неустановившихся гидравлических процессов. На основе дифференциальных уравнений, характеризующих это явление, ведутся поиски нонвариант определяющих переходные процессы.

Принимая нонварианты в виде параметров ведутся сравнительные исследования упрощенной модели системы (в форме бесконечно длинной трубы, соединенной с воздушной камерой) и проводится численная симуляция основанная на неупрощенной системе дифференциальных уравнений. Целью этого сравнения является определение факторов позволяющих определить, при данных значениях нонвариант, размеры воздушной камеры, на основе упрощенной модели системы.