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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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MARIAN TRELA
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Prediction of Heat Transfer to a Coal Slurry Flow*

The paper presents an analysis of the heat transfer to a three-phase coal slurry flow. This highly complex problem is simplified by adopting the homogeneous and separated flow models as used mainly in two-phase flow. The analysis indicates that the homogeneous flow model is more adequate in this case. This is valid both for Newtonian and non-Newtonian behaviour of the coal slurry.

List of symbols

A – area,
α – hold-up,
C_p – specific heat,
D – pipe diameter,
f – friction factor,
G – mass velocity,
h – heat transfer coefficient,
j – superficial velocity,
k – conductivity,
K – property index,
K' – consistency index,
Nu – Nusselt number,
n' – flow behaviour index,
Pr – Prandtl number,
p – pressure,
Re – Reynolds number,
St – Stanton number,
T – temperature,
X – mass quality fraction,
$\dot{Q}$ – volumetric flow rate,
$\dot{W}$ – mass flow rate,
Z – volumetric factor,
V – velocity,
ρ – density,
μ – viscosity,
Θ – experimental parameter.

Subscripts

1 – low pressure and temperature,
2 – high pressure and temperature,
l – liquid,
lo – liquid only,
g – gas,
go – gas only,
s – solid,
so – solid only,
lsg – three-phase flow (liquid, solid, gas).

1. Introduction

Hydrodynamics and heat transfer in a stream consisting of three phases, namely liquid, solid and gas were not investigated in the past. In a few years, the interest has grown remarkably, in particular in the chemical engineering.

*This study was carried out at the Institute for Mining and Minerals Research, University of Kentucky, USA.

In this paper the three-phase flow and heat transfer will be investigated for the coal liquefaction processes. The three phases are: oil, coal and hydrogen.

Heat transfer to the three-phase flow is too complex for a strictly theoretical treatment. It may be solved only by replacing the real flow by a simpler one. Such approach to the problem is characteristic for multiphase flows (mainly two-phase) where one considers „homogeneous" or „separated" flow instead of the real one.

In this paper the same approach is used to develop the methods for the prediction of heat transfer to the coal slurry flow. Assuming that slurry is Newtonian in behaviour and considering the flow as „homogeneous" or „separated" two methods are obtained. The same may be done for a non-Newtonian slurry but first some characteristic parameters must be determined experimentally.

2. Theoretical Prediction of the Heat Transfer to a Newtonian Coal Slurry

Assuming that the coal slurry is Newtonian in behaviour we may adopt the approach to the problem from the two-phase flows, in which the flows are considered to be either „homogeneous" or „separated". In the both cases the flow is assumed to be turbulent.

2.1. Some Basic Terminology

Before proceeding with the analysis of the three-phase coal slurry flow it is necessary to define some of the relevant terminology.

Hold-up α. The hold-up of a phase in a multiphase flow is defined as the fraction of the cross-section area of the pipe occupied by that phase. The following relations are obvious:

$$\alpha_g=\frac{A_g}{A}, \quad \alpha_l=\frac{A_l}{A}, \quad \alpha_s=\frac{A_s}{A} \tag{1}$$

and

$$\alpha_g+\alpha_l+\alpha_s=1. \tag{2}$$

Mass flow rate $\dot{W}$. Mass flow rate $\dot{W}$ will be the sum of the individual mass flow rates

$$\dot{W}=\dot{W}_g+\dot{W}_l+\dot{W}_s. \tag{3}$$

Phase velocities. Phase velocities are expressed by

$$V_g=\frac{\dot{W}_g}{\rho_g A_g}, \quad V_l=\frac{\dot{W}_l}{\rho_l A_l}, \quad V_s=\frac{\dot{W}_s}{\rho_s A_s}. \tag{4}$$

Volumetric flow rate $\dot{Q}$. It will be the sum of the individual flow rates

$$\dot{Q}=\dot{Q}_g+\dot{Q}_l+\dot{Q}_s. \tag{5}$$

Mass quality fraction. The mass quality fraction is expressed as

$$X_g=\frac{\dot{W}_g}{\dot{W}}, \quad X_l=\frac{\dot{W}_l}{\dot{W}}, \quad X_s=\frac{\dot{W}_s}{\dot{W}} \tag{6}$$

and

$$X_g+X_l+X_s=1. \tag{7}$$

Mass velocity

$$G=\frac{\dot{W}}{A}. \tag{8}$$

Superficial velocities

$$\left.\begin{aligned} j&=\frac{\dot{Q}}{A}=\frac{\dot{W}}{\rho A}=\frac{G}{\rho}=V, \quad j_g=\frac{\dot{Q}_g}{A}=V_{go}, \\ j_l&=\frac{\dot{Q}_l}{A}=V_{lo}, \quad j_s=\frac{\dot{Q}_s}{A}=V_{so} \end{aligned}\right\} \tag{9}$$

and

$$j_g+j_l+j_s=j. \tag{10}$$

Volumetric factor Z. It is defined as

$$Z=\frac{Q_g}{\dot{Q}_l+\dot{Q}_s}. \tag{11}$$

2.2. The Physical Properties of the Slurry Components

Liquid phase. It was intended to use for heat transfer calculations the data for tetralin because tetralin is to be used in the future experimental research. As its thermal properties were not available, the naphthalene data will be used instead for heat transfer calculations. The thermal properties of naphthalene are shown in Figs 1 - 4. The viscosity as a function of temperature was taken from a nomogram in [1]. The density at the boiling point was calculated from Koppa and Le Bos formulae given in [2]. For other temperatures the density was assumed to have the same gradient as other similar compounds. The thermal conductivity and specific heat were taken from [1]. All these values are valid for atmospheric pressure.

Gas phase. Nitrogen instead of hydrogen is to be used during the future experimental research (to reduce the cost of the experiments), so nitrogen properties will be used in the calculation. They are taken from [3].

Solid phase. It is assumed that coal used for slurry preparation has specific gravity 1.1. The thermal conductivity and specific heat are available from [4].

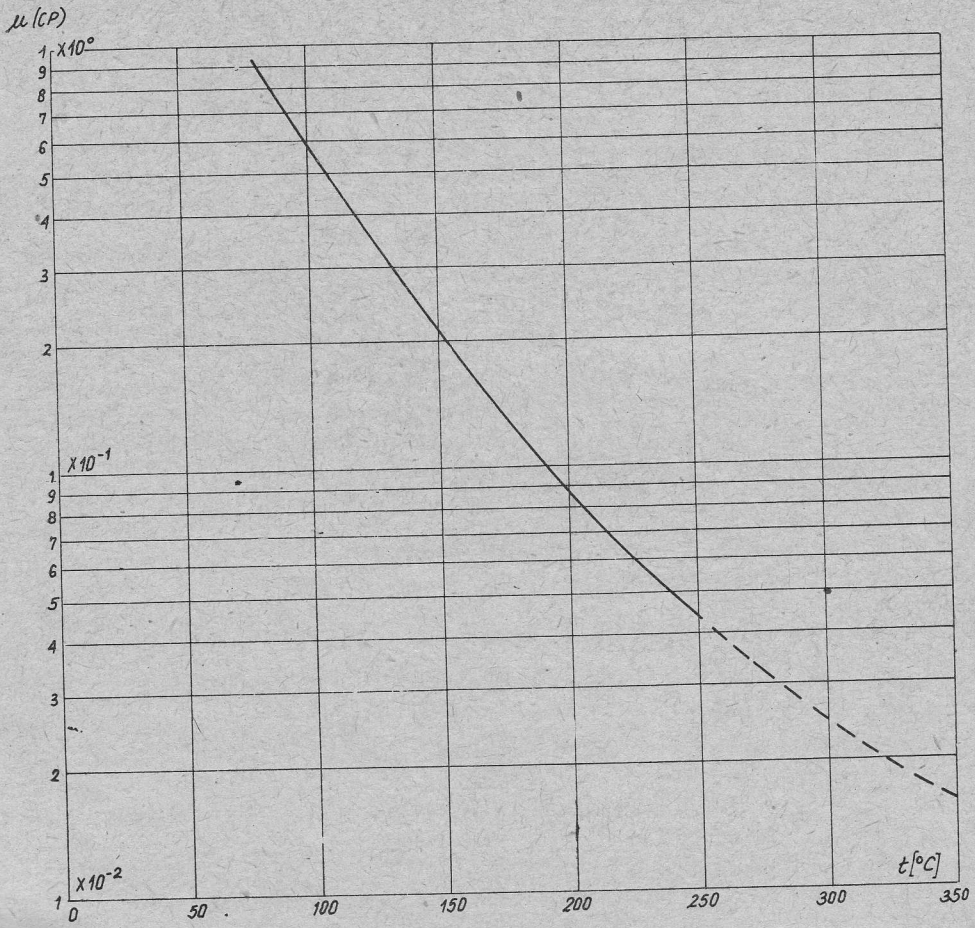

Fig. 1. Naphthalene viscosity

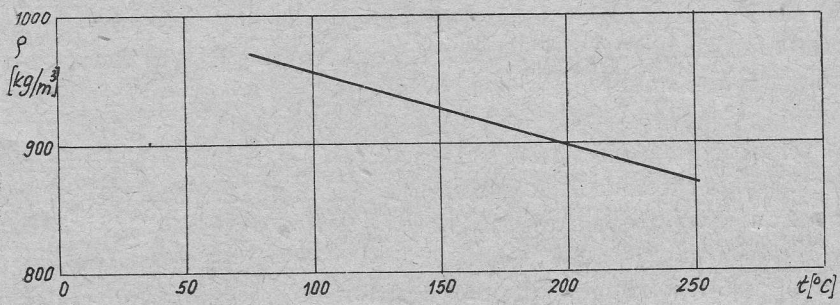

Fig. 2. Naphthalene density

2.3. Heat Transfer to the Coal Slurry Flow – Homogeneous Flow Model

The homogeneous flow theory provides the simplest technique for analyzing a multiphase flow. This model considers the phases to flow as a single phase possessing mean fluid properties [5, 6].

The basic assumptions of the model are:

a) equal velocities of each phase,

b) the attainment of the thermodynamic equilibrium between the phases,

c) the use of a suitably defined simple-phase friction factor for a multiphase flow.

The pseudo properties of the fluid (velocity, temperature, density, viscosity and conductivity) are weighted averages. Sometimes there are some deviations from the above

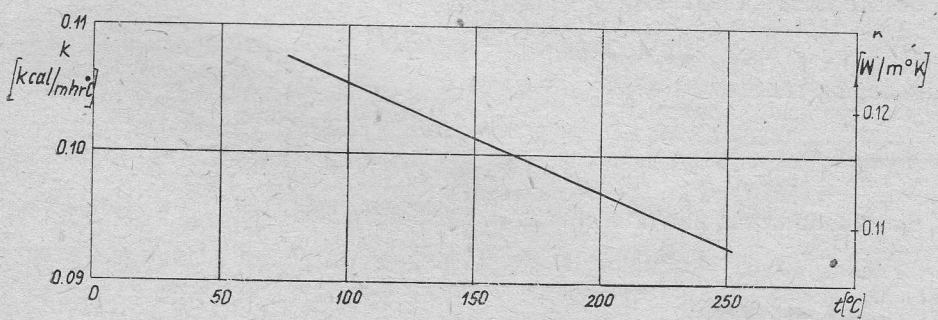

Fig. 3. Thermal conductivity

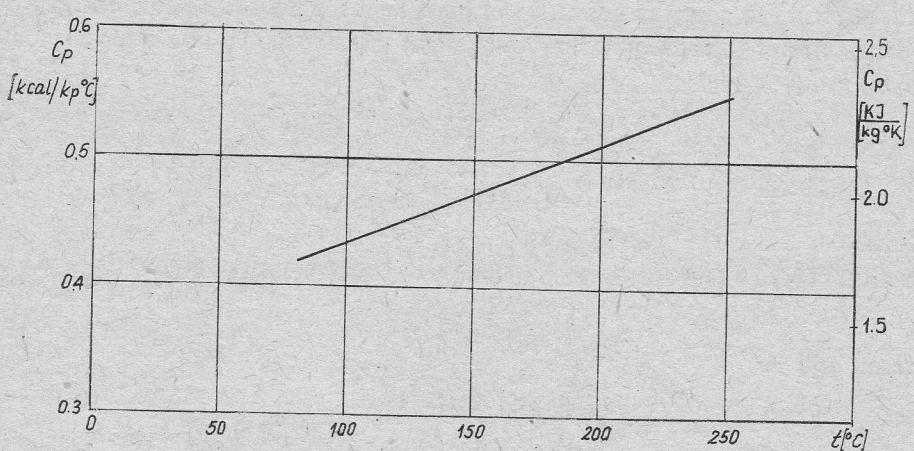

Fig. 4. Specific heat

assumptions and the thermal properties of one of the components are used instead of the mean properties.

In this paper the heat transfer is calculated for two cases in a homogeneous flow. The two different cases are denoted by indices HL and HM, respectively:

a) HL – denotes the quantity calculated using the liquid properties as the slurry properties,

b) HM – denotes the quantity calculated using average (mean) properties (from liquid and solid phase).

The above means that in the first case the heat transfer is assumed to be determined by the liquid boundary layer, while in the second case – by the liquid-solid boundary layer. Under homogeneous flow model assumptions, heat transfer to the coal slurry in a turbulent flow may be calculated as for a single phase flow. So for the turbulent flow in pipes Dittus-Boelter equation may be used

$$Nu = 0.023 Re^{0.8} Pr^{0.4}. \tag{12}$$

A. For the first case (HL)

$$(Nu)_{HL} = 0.023 (Re)_{HL}^{0.8} (Pr)_{HL}^{0.4} \tag{13}$$

or developing this expression

$$\frac{h_{lsg} D}{k_l} = 0.023 \left(\frac{V_{lsg} D \rho_l}{\mu_l} \right)^{0.8} \left(\frac{C_{pl} \mu_l}{k_l} \right)^{0.4}. \tag{14}$$

For the homogeneous flow model, V_{lsg} is calculated from

$$V_{lsg} = \frac{\dot{Q}_l + \dot{Q}_s + \dot{Q}_g}{A} = V_{lso}(l+Z), \tag{15}$$

where V_{lso} – the liquid-solid velocity (the subscript „o" stands for „only", so *lso* means „liquid solid flow only").

It is practical to compare heat transfer coefficient for the three-phase coal slurry flow with heat transfer to the liquid flow only. Heat transfer in the liquid turbulent flow is given by

$$(Nu)_l = 0.023 (Re)_l^{0.8} (Pr)_l^{0.4} \tag{16}$$

or

$$\frac{h_{lo} D}{k_l} = 0.023 \left(\frac{V_{lo} \rho_l D}{\mu_l} \right)^{0.8} (Pr)_l^{0.4}. \tag{17}$$

If we divide (14) by (17) for the same liquid and slurry velocities e.g.

$$V_{lo} = V_{lso}$$

the result will be

$$\left(\frac{h_{lsg}}{h_{lo}} \right)_{HL} = \left(\frac{V_{lsg}}{V_{lo}} \right)^{0.8} = (l+Z)^{0.8}. \tag{18}$$

B. For the second case HM

$$(Nu)_{HM} = 0.023 (Re)_{HM}^{0.8} (Pr)_{HM}^{0.4} \tag{19}$$

or

$$\frac{h_{lsg} D}{k_{HM}} = 0.023 \left(\frac{V_{lsg} \rho_{HM} D}{\mu_{HM}} \right)^{0.8} \left(\frac{C_{pHM} \mu_{HM}}{k_{HM}} \right)^{0.4}. \tag{20}$$

If the assumption $V_{lo}=V_{lso}$ still holds one gets

$$\left(\frac{h_{lsg}}{h_{lo}}\right)\left(\frac{k_l}{k_{HM}}\right)=\left(\frac{V_{lsg}}{V_{lo}}\right)^{0.8}\left(\frac{\rho_{HM}\mu_l}{\rho_l\mu_{HM}}\right)^{0.8}\left(\frac{Pr_{HM}}{Pr_l}\right)^{0.4} \tag{21}$$

or

$$\left(\frac{h_{lsg}}{h_{lo}}\right)_{HM}=\left(\frac{\rho_{HM}\mu_l}{\rho_l\mu_{HM}}\right)^{0.8}\left(\frac{Pr_{HM}}{Pr_l}\right)^{0.4}\left(\frac{k_{HM}}{k_l}\right)\left(\frac{V_{lsg}}{V_{lo}}\right)^{0.8}=B(l+Z)^{0.8}, \tag{22}$$

where B – stands for the three terms on the right side of the relationship (22).

Comparing now (18) with (22) one gets

$$\frac{(h_{lsg})_{HM}}{(h_{lsg})_{HL}}=\left(\frac{\rho_{HM}\mu_l}{\rho_l\mu_{HM}}\right)^{0.8}\left(\frac{Pr_{HM}}{Pr_{HL}}\right)^{0.4}\frac{k_{HM}}{k_l}=B. \tag{23}$$

It is seen that factor B indicates the discrepancies in the heat transfer due to the different thermal properties of the homogeneous fluid. This factor is also equal to the ratio $(h_{ls})_{HM}/h_{lo}$ at $V_{lso}=V_{lo}$ because

$$\frac{(h_{ls})_{HM}}{h_{lo}}=\frac{0.023Re_{HM}^{0.8}\,Pr_{HM}^{0.4}\,k_{HM}}{0.023Re_l^{0.8}\,Pr_l^{0.4}\,k_l}=B. \tag{24}$$

In order to calculate B one must determine quantities ρ_{HM}, C_{pHM} and μ_{HM}. They were defined as weighted averages. But for solid-liquid mixtures it is advised to calculate μ_{HM} in another way [2]. For example reference [2] gives the equations to calculate the viscosity of solid-liquid mixtures. One of the equat ons is

$$\mu_{HM}/\mu_l=1+4.5\alpha_s \tag{25}$$

valid for

$$\alpha_s<0.4.$$

The average density ρ_{HM} may be calculated as follows

$$\dot{W}_l=V_{lo}A\rho_l=j_lA\rho_l, \qquad \dot{W}_s=V_{so}A\rho_s=j_sA\rho_s \tag{26}$$

so

$$\dot{W}_{ls}=\dot{W}_l+\dot{W}_s=\rho_{HM}AV_{lso}=j_lA\rho_l+j_sA\rho_s \tag{27}$$

because in homogeneous flow

$$V_{lso}=j_l+j_s \tag{28}$$

then from (27) one gets

$$\rho_{HM}=\frac{j_lA\rho_l+j_sA\rho_s}{A(j_l+j_s)}=\frac{j_l\rho_l+j_s\rho_s}{j_l+j_s}. \tag{29}$$

In the same way one can calculate C_{pHM}

$$\dot{W}_{ls}C_{pHM}=\dot{W}_lC_{pl}+\dot{W}_sC_{ps} \tag{30}$$

and

$$C_{pHM}=\frac{j_l\rho_lC_{pl}+j_s\rho_sC_{ps}}{\rho_{HM}(j_l+j_s)}. \tag{31}$$

In order to compare heat transfer coefficients resulting from the above derived formulas, the following problem will be considered.

Problem. Calculate heat transfer coefficient to a three-phase (naphthalene, coal, nitrogen) flow in $6.35 \cdot 10^{-3}$ m tube at a temperature of 100°C and ambient pressure.

The coal slurry velocity (without nitrogen) is $V_{lso}=0.5$ m/s. The coal amount in the feed is 25 wt %. Volumetric factor $Z=1$ through 10.

Solution. Thermal properties of coal from [4] are:

$$\rho_s=1098 \text{ kg/m}^3,$$

$$C_{ps}=1.25 \text{ kJ/kg K},$$

$$k_s=0.25 \text{ W/m K}.$$

Thermal properties of naphthalene from Figs 1 - 4 are:

$$\mu_l=0.58\times10^{-3} \text{ kg/ms},$$

$$\rho_l=960 \text{ kg/m}^3,$$

$$k_l=0.123 \text{ W/m K},$$

$$C_{pl}=1.80 \text{ kJ/kg K}.$$

Thermal properties of nitrogen from [3] are:

$$\mu_g=20.86\times10^{-6} \text{ kg/ms},$$

$$\rho_g=0.93 \text{ kg/m}^3.$$

According to (17) the heat transfer coefficient for the oil flow is

$$h_{lo}=0.023Re_l^{0.8}Pr_l^{0.4}\frac{k_l}{D}=994 \text{ W/m}^2 \text{ K}.$$

Slurry mass flow rate

$$\dot{W}_{ls}=\dot{W}_l+\dot{W}_s=j_l A\rho_l+j_s A\rho_s$$

slurry velocity

$$V_{lso}=j=j_l+j_s=0.5 \text{ m/s}. \tag{32}$$

The calculation of j_l and j_s needs additional information. Coal amount in the feed may be expressed as:

$$p=\frac{\dot{W}_s}{\dot{W}_s+\dot{W}_l}=\frac{V_{lso}A_s\rho_s}{V_{lso}(A_s\rho_s+A_l\rho_l)}=\frac{1}{1+\frac{j_l}{j_s}\frac{\rho_l}{\rho_s}} \tag{33}$$

from which

$$\frac{j_l}{j_s}=\left(\frac{1}{p}-1\right)\frac{\rho_s}{\rho_l} \tag{34}$$

and finally considering (32) and (34) the superficial velocities are

$$j_l = 0.387 \text{ m/s}, \quad j_s = 0.113 \text{ m/s}.$$

Thus the average slurry density is

$$\rho_{HM} = \frac{j_l \rho_l + j_s \rho_s}{j_l + j_s} = 991.2 \text{ kg/m}^3.$$

The ratio of viscosities μ_{HM}/μ_l is

$$\mu_{HM}/\mu_l = 1 + 4.5\alpha_s = 1 + 4.5\left(\frac{j_s}{V_{lso}}\right) = 2.017.$$

The conductivity of the solid-liquid slurry may be evaluated from (35) which is based on [7]

$$\frac{k_{HM}}{k_l} = \frac{2k_l + k_s - 2\alpha_s(k_l - k_s)}{2k_l + k_s - \alpha_s(j_l - k_s)} = 1.057. \tag{35}$$

From (31) the ratio C_{pHM}/C_{pl} is

$$\frac{C_{pHM}}{C_{pl}} = \frac{\rho_l j_l + j_s \rho_s \dfrac{C_{ps}}{C_{pl}}}{\rho_{HM} j} = 0.9243. \tag{36}$$

Putting together the above calculated values we have

$$\frac{\mu_{HM}}{\mu_l} = 2.017, \quad \frac{\rho_{HM}}{\rho_l} = 1.0325, \quad \frac{k_{HM}}{k_l} = 1.0574, \quad \frac{C_{pHM}}{C_{pl}} = 0.9243.$$

These values enable us to calculate B factor which may be rearranged to

$$B = \left[\frac{\rho_{HM}}{\rho_l}\frac{\mu_l}{\mu_{HM}}\right]^{0.8}\left[\frac{C_{pHM}}{C_{pl}}\frac{\mu_{HM}}{\mu_l}\right]^{0.4}\left[\frac{k_{HM}}{k_l}\right]^{0.6} = 0.7764. \tag{37}$$

Basing on the above data, the ratios $(h_{lsg}/h_{lo})_{HL}$ and $(h_{lsg}/h_{lo})_{HM}$ are calculated and shown in Fig. 6. The results indicate that using different definition for the slurry properties (HL or HM) one can get discreapancies of about 25% for the homogeneous flow model.

2.4. Heat Transfer to the Coal Slurry – Separated Flow Model

Separated flow model considers the phases to be artificially segregated into three streams, one of liquid, one of solid and one of gas. In the simplest model form every stream is assumed to travel at a certain mean velocity [5, 6]. A separate flow model has been often used in two-phase flow (liquid-gas). Good results are obtained at higher values of mass fraction X_g when the flow pattern is annular, because this flow may be considered as separated one.

The basic premises upon which the separated flow model is based are the following assumptions:

a) constant but different velocities for each phase,

b) the attainment of thermodynamic equilibrium between the phase,

c) the use of empirical correlation for the hold-up. In the case under consideration (coal slurry stream) it is reasonable to assume that velocities of coal and oil are equal because of the same densities so we have got the next assumption,

d) the velocities V_l and V_s are equal ($V_l = V_s = V_{ls}$).

The above assumptions implicate that the heat transfer to the coal slurry will be determined by the liquid-solid annulus on the pipe wall. The heat resistance of the annulus depends on the hydrodynamic conditions and thermal properties of the boundary layer. Thermal properties of the coal slurry are the same as in the former model so the desired quantity is the velocity V_{ls} of the liquid-solid annulus. If the liquid-solid stream is turbulent, the heat transfer may be described by Dittus-Boelter equation using the proper value of the annulus velocity V_{ls}.

A. Assuming that slurry properties are represented by liquid (index SL) we can write

$$(Nu)_{SL} = 0.023 (Re)_{SL}^{0.8} Pr_{SL}^{0.4} \tag{38}$$

or

$$\left(\frac{h_{lsg} D}{k_l}\right)_{SL} = 0.023 \left(\frac{V_{ls} \rho_l D}{\mu_l}\right)^{0.8} Pr_l^{0.4}. \tag{39}$$

For the single-phase flow, heat transfer is expressed by (17) so if $V_{lo} = V_{lso}$ the ratio $(h_{lsg}/h_{lo})_{SL}$ becomes

$$\left(\frac{h_{lsg}}{h_{lo}}\right)_{SL} = \left(\frac{V_{ls}}{V_{lso}}\right)^{0.8}. \tag{40}$$

In order to calculate heat transfer one must know the velocity of solid-liquid annulus. The velocity may be found if one knows the hold-up of the gaseous phase. From relations (2) and (4) the hold-up is expressed as

$$\alpha_l + \alpha_s = \alpha_{ls} = 1 - \alpha_g, \tag{41}$$

$$\alpha_l = \frac{A_l}{A} = \frac{\dot{W}_l}{\rho_l A V_l}, \qquad \alpha_s = \frac{A_s}{A} = \frac{\dot{W}_s}{\rho_s A V_s},$$

$$\alpha_{ls} = \alpha_l + \alpha_s = (1 - \alpha_g) = \frac{1}{A V_{ls}} \left(\frac{\dot{W}_l}{\rho_l} + \frac{\dot{W}_s}{\rho_s}\right). \tag{42}$$

From (42) the annulus velocity

$$V_{ls} = \frac{1}{A(1-\alpha_g)} \left(\frac{\dot{W}_l}{\rho_l} + \frac{\dot{W}_s}{\rho_s}\right) = \frac{1}{A(1-\alpha_g)} (\dot{Q}_l + \dot{Q}_s) = \frac{V_{lso} A}{(1-\alpha_g) A} \tag{43}$$

and

$$\frac{V_{ls}}{V_{lso}} = \frac{1}{1-\alpha_g}. \tag{44}$$

Putting the last result into (40) one gets

$$\left(\frac{h_{lsg}}{h_{lo}}\right)_{SL}=\left(\frac{V_{ls}}{V_{lso}}\right)^{0.8}=\left(\frac{1}{1-\alpha_g}\right)^{0.8}. \tag{45}$$

To find heat transfer coefficient in the separated flow model one must get the hold-up of the gaseous phase. There are many empirical correlations for α_g for different

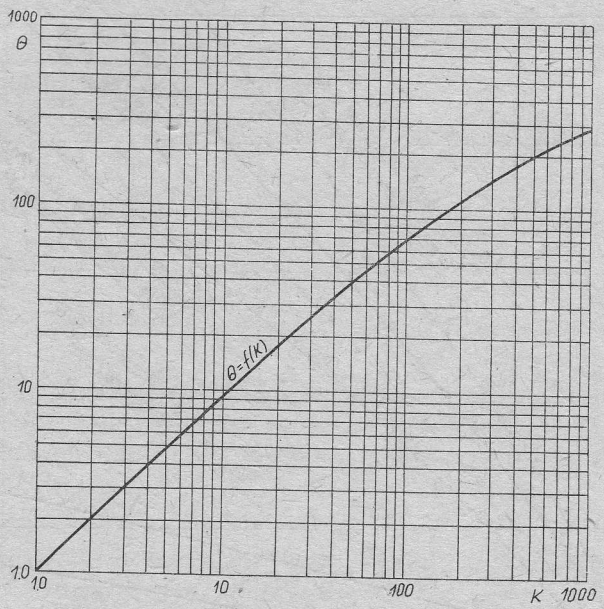

Fig. 5. Parameter θ versus property index K

conditions and different fluids. One of the best is Thom's correlation developed for the water-vapor flow in the whole pressure range [8]. Thom's correlation is given as

$$\alpha_g=\frac{\theta X_g}{1+X_g(\theta-1)}, \tag{46}$$

where θ – parameter determined experimentally for water-vapor system as the function of pressure (the θ values may be found in [8]).

Thom's correlation may be used directly for a water-vapor system only, because θ is given for it. One may use this correlation for other fluids if θ is expressed as a function of „property index K". This conclusion may be drawn out from [9] or [10].
Property index K is equal to

$$K=\left(\frac{\rho_l}{\rho_g}\right)\left(\frac{\mu_g}{\mu_l}\right)^{0.25}. \tag{47}$$

The relationship $\theta=\theta(K)$ was calculated in [10] and is shown in Fig. 5.

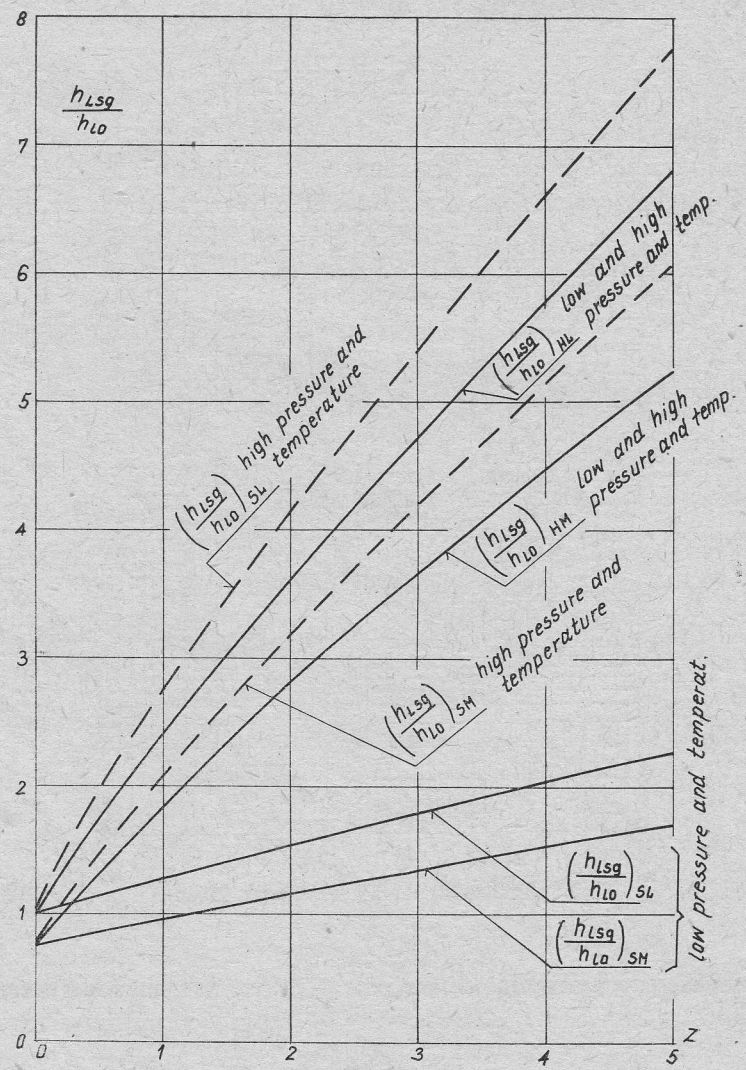

Fig. 6. Results of the heat transfer calculations

B. For the second case SM, assuming that slurry properties are weighted averages (from oil and solid), we can write

$$(Nu)_{SM}=0.023(Re)_{SM}^{0.8}Pr_{SM}^{0.4} \tag{48}$$

or

$$\left(\frac{h_{lsg}D}{k_{SM}}\right)_{SM}=0.023\left(\frac{V_{ls}\rho_{SM}D}{\mu_{SM}}\right)^{0.8}\left(\frac{C_{pSM}\mu_{SM}}{k_{SM}}\right)^{0.4}. \tag{49}$$

The ratio $(h_{lsg}/h_{lo})_{SM}$ may be rearranged to

$$\left(\frac{h_{lsg}}{h_{lo}}\right)_{SM}=B\left(\frac{V_{ls}}{V_{lo}}\right)^{0.8}. \tag{50}$$

Calculating now the ratio $(h_{lsg})_{SM}/(h_{lsg})_{SL}$ we have

$$\frac{(h_{lsg})_{SM}}{(h_{lsg})_{SL}} = B\frac{\left(\frac{V_{ls}}{V_{lo}}\right)^{0.8}_{SM}}{\left(\frac{V_{ls}}{V_{lo}}\right)^{0.8}_{SL}} = B\left[\frac{(1-\alpha_g)_{SL}}{(1-\alpha_g)_{SM}}\right]^{0.8}. \tag{51}$$

It is evident from the above formula that the difference in the heat transfer coefficient between the case A and case B is given by factor B and different values of gaseous hold-up α_g. In order to find the new values of α_g the new values of K and θ must be determined. From (47)

$$K = \left(\frac{\rho_{SM}}{\rho_g}\right)\left(\frac{\mu_g}{\mu_{SM}}\right)^{0.25} = \left(\frac{991}{0.93}\right)\left(\frac{20.86\times10^{-6}}{1.1688\times10^{-3}}\right)^{0.25} = 389 \tag{52}$$

and from Fig. 5 $\theta = 175$. The results of the heat transfer calculations for both homogeneous and separated flow model are represented in Fig. 6.

2.5. Estimation of the Effect of Pressure and Temperature Upon Heat Transfer in Both Models

So far heat transfer coefficients for both homogeneous and separated flow models have been calculated at ambient pressure and low temperature ($p \cong 1$ bar, $t = 100°C$). It is interesting to investigate the effect of both parameters on the heat transfer.

Homogeneous flow model

A. Let us assume that the new parameters are: $p = 120$ bar, $T = 700$ K (427°C). If we consider the homogeneous flow model for the case when slurry properties are expressed by oil properties then we see that the ratio $(h_{lsg}/h_{lo})_{HL}$ is independent of the temperature and pressure.

B. In order to estimate the dependence $(h_{lsg}/h_{lo})_{HM}$ on the pressure and temperature one must consider the change of the factor B resulting from the change of these two parameters.

Liquid density ρ_l depends on the pressure and temperature, but this dependence is very weak compared to that for gases so it will be neglected in this consideration. If ρ_l is constant then according to (29) ρ_{HM} is constant because superficial velocities are constant. By the same token it may be proved that other quantities are constant. So it is seen that if we neglected the change of ρ_l resulting from the pressure and temperature change then B is constant and the ratios

$$\left(\frac{h_{lsg}}{h_{lo}}\right)_{HL} \quad \text{and} \quad \left(\frac{h_{lsg}}{h_{lo}}\right)_{HM}$$

are independent of the temperature and pressure.

Separated flow model. To estimate the influence of the pressure and temperature on the heat transfer one needs to follow the whole preceding procedure starting with the calculation of the index property K.

A. Slurry properties are expressed by liquid

New value of K may be obtained considering the change in the gas density, the liquid and the gas viscosities.

The gaseous density may be calculated from the equation of state for perfect gases

$$\rho_{g2}=\rho_{g1}\frac{P_2}{P_1}\frac{T_1}{T_2}. \tag{53}$$

Nitrogen viscosity in the new conditions may be evaluated by using the Theory of Corresponding States [2]. From the chart Figure 3.14 in [2] it was estimated that in the new conditions nitrogen viscosity increases by a factor of 1.57.

Oil viscosity in the new conditions may be roughly estimated from Fig. 1 to be $\mu_l = 0.1 \times 10^{-3}$ kg/ms.

The obtained values enable us to calculate the property index K

$$K_{SL}=\left(\frac{\rho_l}{\rho_g}\right)\left(\frac{\mu_g}{\mu_l}\right)^{0.25}=22.$$

According to Fig. 5, $\theta=20$. Having θ one can calculate the ratio $(h_{lsg}/h_{lo})_{SL}$ for $p=120$ bar and $T=700$ K (427°C).

B. Slurry properties are weighted averages

For this case it may be proved that the influence of pressure and temperature on the heat transfer is expressed by the formula

$$\left(\frac{h_{lsg}}{h_{lo}}\right)_{SM}\cong B\left(\frac{h_{lsg}}{h_{lo}}\right)_{SL},$$

where B is practically independent of the temperature and pressure.

2.6. Discussion of the Results

The heat transfer coefficients calculated for the both models for low and high pressures and temperatures are shown in Fig. 6.

It is seen that for the separated flow model all values except those for a low pressure and temperature are in the region where maximum values differ by about 45%.

The deviations from the values calculated for the separated flow model at a low pressure and temperature should by expected if one remembered the assumptions of this model, which prefer it to be used at higher mass quality fraction X_g when flow pattern is annular.

The assumptions of the homogeneous model are better fulfilled at very low and very high mass quality fraction X_g, when the gaseous phase is dispersed in the solid-liquid phase or vice versa.

The other reason for deviations of the results for the separated flow model at a low pressure is the large error in the estimation of the hold-up α_g at a very low value of X_g. Usually the prediction fromulae for α_g are good if $X_g > 0.05$. In our case the values of X_g are below this limit.

In the coal liquefaction process one must expect low values of the mass quality fraction X_g ($X_g=2$ to 3% of the total three-phase mass flow rate) so the homogeneous flow model should be preferred in the calculations of heat transfer to a coal slurry flow. It seems that this conclusion should be valid also for non-Newtonian slurry.

3. Prediction of the Heat Transfer to a Non-Newtonian Coal Slurry

Heat transfer to a non-Newtonian turbulent flow has been investigated very poorly. No data were found in the literature about the heat transfer to a non-Newtonian three-phase flow. In such a situation a general approach to the problem will be suggested.

For the non-Newtonian turbulent flow the heat transfer may be calculated through the analogy between heat and momentum transfer [7]. This procedure is common for a Newtonian fluid and was extended to a non-Newtonian one by Metzner [7, 11].

If we replace the three-phase flow by a simple phase (homogeneous model) then we can follow the procedure proposed by Metzner. Metzner [11] has obtained a relationship between the heat transfer and momentum transfer in non-Newtonian fluids, which in the simplest form is given by

$$St=\frac{h}{C_p V\rho}=\frac{f/2}{1.20+11.8\sqrt{f/2}}, \tag{54}$$

where f – Fanning friction factor which may be obtained from Fig. 7 representing the general friction factor versus Reynolds number diagram,

$$Re=\frac{D^{n'}V^{2-n'}\rho}{K'8^{n'-1}}, \tag{55}$$

n' – flow behaviour index,

$$n'=\frac{d\ln\tau_w}{d\ln\left(\frac{8V}{D}\right)}, \tag{56}$$

τ_w – shear stress at the tube wall,

K' – consistency index which is defined by relation,

$$\tau_w=K'\left(\frac{8V}{D}\right)^{n'}. \tag{57}$$

The values of n' and K' may be obtained from a tube viscometer, measuring the tube shear stress in the laminar flow versus the rate of strain dV/dr [7]. The shear stress and the rate of strain may be expressed as

$$\tau_w=\frac{\Delta pD}{4L}, \quad \frac{dV}{dr}=\frac{8V}{D}, \tag{58}$$

where Δp – pressure drop in tube viscometer, L – distance over which the pressure is measured.

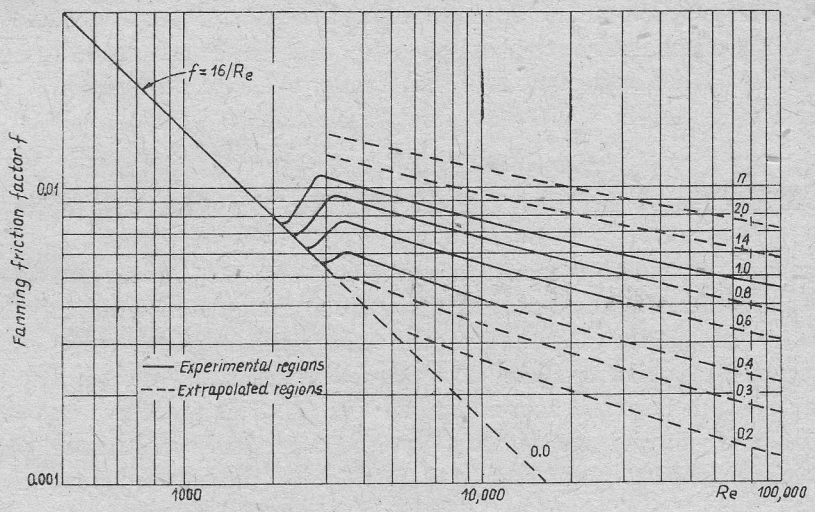

Fig. 7. Friction factor versus generalized Reynolds number

Having relations (58), the values of n' and K' may be found from (56) and (57), and friction factor f from Fig. 7. The value of friction factor enable us to get Stanton number and heat transfer coefficient from the equation (54). In this equation the density and the specific heat should be weighted averages. Comparing the proposed method for non-Newtonian fluids with the methods for Newtonian fluids one can see that a use of this method must be preceded by experimental investigations to estimate the flow behaviour indices: n' and K'. It must be noted here that these indices depend on the shear stress τ_w so in order to calculate heat transfer one must first know the pressure drop during the flow.

4. Concluding Remarks

A method for the heat transfer calculations in a Newtonian and non-Newtonian coal slurry flow has been proposed. It has been adopted from the homogeneous flow model as used commonly for two-phase flows.

This method should be verified experimentally for various kinds of the coal slurry. It should be expected that the coal slurry with a high amount of coal will exhibit very strong non-Newtonian behaviour. For this case the use of the method as presented in this paper must be preceded by measurements of the flow behaviour indices n' and K'. For the ambient pressure and temperature this may be done very easy in a tube viscometer or a pipe.

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Określenie wymiany ciepła podczas przepływu pasty węglowej

Streszczenie

Złożony mechanizm wymiany ciepła podczas przepływu trzyfazowej pasty węglowej (węgiel, olej, gaz) rozważono analitycznie. Zakładając, że pasta węglowa posiada własności cieczy newtonowskiej zaadaptowano do tego przypadku dwa modele obliczeniowe, stosowane dotychczas w przepływach dwufazowych gaz–ciecz, a mianowicie jednorodny oraz poślizgowy model przepływu. Obliczenia wykonane w pracy wskazują, iż zaadaptowanie jednorodnego modelu przepływu będzie bardziej uzasadnione z uwagi na małą zawartość fazy gazowej.

Ten sam model przepływu może być również użyty dla nienewtonowskiej pasty węglowej. W tym przypadku jednak dwa parametry charakteryzujące pastę węglową, tzn. n' i K' zdefiniowane przez zależności (56) i (57), muszą być przedtem wyznaczone na drodze eksperymentalnej. Mając wartości n' i K' oraz stosując analogię między wymianą ciepła i pędu przy założeniach jednorodnego modelu przepływu, otrzymuje się zależności na współczynnik wymiany ciepła w przepływie pasty węglowej.

Определение теплообмена при течении угольной пасты

Резюме

Очень сложный механизм теплообмена при течении трехфазной угольной пасты (уголь, масло, газ) рассуждается аналитически. Предпринимая, что угольная паста обладает свойствами ньютоновской жидкости, присваиваются в этом случае две расчетных модели, принимаемые до сих пор относительностью двухфазных течений газ–жидкость.

Это есть: однородная и скользящая модели течения. Расчеты проведенные в этой работе показывают, что присвоение однородной модели течения является более обоснованным с учета на низкое содержание газовой фазы.

Эта же самая модель течения может быть применена также и относительно неньютоновской угольной пасты. В этом однако случае два параметры, характеризующие угольную пасту, т.е. n' и K', определенные зависимостями (56) и (57), должны быть раньше определены экспериментально. Зная значения n' и K' и пользуясь аналогией между теплообменом и обменом импульса, при предположении однородной модели течения, получается зависимость на коэффициент теплообмена в течении угольной пасты.