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Gdańsk

Flow Choking and Maximal Flow Rate Calculation of Non-Equilibrium Wet Steam Expansions*

The phenomenon of flow choking and its influence on the maximal flow rate of non-equilibrium expanding wet steam flows has been analysed. Calculation methods allowing various degrees of approximation have been proposed. A numerical analysis of choking flows has been carried out and the effects of various factors on the choking parameters and on the maximal flow rate have been discussed.

Nomenclature

a – local sound velocity in the gas phase,
c – critical flow velocity of the gas phase (formal value),
c_p – specific heat at constant pressure,
g – mass flow content of the gas phase,
g' – mass flow content of the liquid phase,
h – latent heat of phase change,
i, i' – specific enthalpy (of the gaseous and liquid phase, respectively),
m, m' – mass transfer coefficients in eq. (2.13),
n, n' – heat transfer coefficients in eq. (2.13),
P – interphase momentum transfer flux (mass force) per unit volume,
p – static pressure,
$\dot{p}=dp/dt$ – pressure increase velocity (=negative expansion rate),
Q – interphase heat transfer flux per unit volume,
R – gas constant of gaseous vapour
r – drag coefficient,
T, T' – absolute temperature of the gaseous and liquid phase, respectively,
t – time,
u – interphase velocity deviation („slip"),
v, v' – specific volume (of the gaseous and liquid phase, respectively),
w, w' – flow velocity (of the gaseous and liquid phase, respectively),
Y – interphase mass transfer flux per unit volume,
$\alpha=c/a$ – dimensionless critical gas phase velocity,
Γ – mass density of the two-phase medium,
Δ – indication of a supplemental quantity in Chapt. 3.4,
δ, δ' – deviation from equilibrium temperature (in the gaseous and liquid phase, respectively),

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κ – isentropic exponent of the gas phase,

$\mu = \Gamma_k / \Gamma_{0k}$ – maximal flow rate coefficient,

ρ – droplet radius,

Φ – total droplet surface per unit volume of the medium.

Indices

0 – reference equilibrium flow,

k – critical conditions in non-equilibrium flow,

$0k$ – critical conditions in equilibrium flow.

1. Introduction

The phenomenon of flow choking determines the maximal flow rate and the outlet parameters of flow ducts; therefore the knowledge of choking flow data is of importance for designing and dimensioning purposes.

Choking conditions of single-phase gaseous media are described by the well known gasdynamic theory, relating the choking flow parameters directly to those of sound wave propagation in the medium. The nature of two-phase flows is essentially more complex due to the heterogeneity of the medium, to the forming of various (not always fully predictable) flow patterns and to interphase effects. Research studies on this subject, based both on experimental work and analytical considerations, have been devoted for the most part to problems of evaporating liquid–vapour mixture flows through orifices or pipes in connection with various flow patterns (e.g. [2, 3, 4, 9]).

In some of the recent papers generalized relations for several types of two-phase flows have been analyzed and calculation methods presented [1, 10, 12, 13].

The present paper is concerned with analytical prediction and numerical investigation of choking conditions in expanding wet steam flows occurring e.g. in flow ducts of steam turbines. Special attention is paid to effects of interphase non-equilibrium states caused by rapid expansion. It should be noted that a decisive direct experimental investigation on interphase processes seems at present extremely improbable in view of the highly dispersed liquid phase (tiny droplets).

The aim of the paper is to develop calculation methods for the determination of choking flow parameters and maximal flow rates and to present numerical analysis results of non-equilibrium choking flow data. The general approach is based on analytical derivations and models developed in previous papers of one of the authors [6, 7, 8].

In order to reveal the most essential features of the phenomena at hand and to obtain feasible procedures a certain degree of idealization is inevitable, which on the other hand allows a more distinct relation to the framework of general thermogasdynamics.

2. General Relations

The medium under consideration is wet steam consisting of a continuous gas phase and a liquid phase in the form of small droplets assumed to be spherical and incompressible, their volume negligible as compared to the volume of the gaseous vapour. The droplets are distributed irregularly in the medium; due to the high degree of dispersion of the

liquid the influence of the droplets upon the gas phase may be described by space-continuous functions.

Let us consider a one-dimensional, stationary expanding flow of wet steam. The expansion-rate (i.e. the velocity of static pressure decrease) may be given as an arbitrarily chosen function of time. Any wall effects will be neglected, thus the flow of the medium as a whole may be considered adiabatic, the pressure variation being the only external forcing working upon the medium and causing changes of its state.

The pressure variation causes primarily changes of thermal and kinematical parameters of the gas phase; the liquid droplets adjust themselves to the state of the surrounding vapour by means of interphase heat, mass and momentum transfer. Thus finite time-rates of pressure variation bring about states of internal non-equilibrium between the phases. The behaviour of the two-phase medium is that of a medium with internal relaxation undergoing a follow-up process and the momentary values of its state parameters become history-dependent, displaying the property of „fading memory".

In a non-equilibrium flow each of the phases flowing through a given cross-section of a duct has different velocities and thermal parameters, neither of these parameters being equal to the value of the respective parameter in an equilibrium flow. Using the saturation temperature corresponding to the static pressure of the medium as the equilibrium temperature, the state of non-equilibrium in the medium may be described by means of deviation parameters

$$\delta = T - T_0, \quad \delta' = T' - T_0, \quad u = w - w'. \tag{2.1}$$

In the following the values of the deviations from equilibrium will be regarded as small compared to the respective reference values

$$\left|\frac{\delta}{T_0}\right| \ll 1, \quad \left|\frac{\delta'}{T_0}\right| \ll 1, \quad \left|\frac{u}{w}\right| \ll 1. \tag{2.2}$$

This assumption excludes formation of new droplets by way of spontaneous condensation; thus the interphase transfer processes take place on the surface of existing drops and a change of moisture content in the course of the process brings about a respective change of the droplet size.

In accordance to the assumptions made above the flow of the two-phase medium may also be regarded as a flow of the sole gaseous component in which heat, mass and momentum sources are acting. These internal sources represent the action of interphase transfer processes upon the gaseous phase, hence they are described by space-continuous functions and their values are equal to the values of the respective transfer fluxes from the liquid droplets entrained in a given volume of the fluid. Provided the functions describing the internal sources are known, the equations of continuity, momentum and first law of thermodynamics of such a flow can be written as follows (including simplifications stemming from the assumptions already made):

$$\frac{dw}{w} - \frac{d\Gamma}{\Gamma} + \frac{dv}{v} - v \cdot Y \cdot dt = 0, \tag{2.3}$$

$$w \cdot dw + v \cdot dp - v \cdot w \cdot P\, dt = 0, \tag{2.4}$$

$$di - v \cdot dp - v \cdot Q \cdot dt = 0 \tag{2.5}$$

and the equation of state of the gaseous vapour may be assumed approximately as that of a perfect gas

$$pv = RT. \tag{2.6}$$

The elimination of some of the variables from the equations (2.3) to (2.6) yields after simple transformations the equation governing the flow conditions

$$\left(1 - \frac{w^2}{a^2}\right)\frac{dw}{w} = \frac{d\Gamma}{\Gamma} + \left(Y + \frac{\kappa - 1}{a^2} Q - \frac{P \cdot w}{a^2}\right) \cdot v \cdot dt, \tag{2.7}$$

where

$$a = \sqrt{\kappa_p v} \tag{2.8}$$

is the sound velocity of the gaseous vapour (considered as a perfect gas).

Equation (2.7) may be brought into the form:

$$\left(1 - \frac{w^2}{c^2}\right)\frac{dw}{w} = \frac{d\Gamma}{\Gamma} \tag{2.9}$$

with the aid of the expression

$$c^2 = a^2 \frac{\dot{p} - P \cdot w}{\dot{p} - (\kappa - 1) Q - a^2 Y}. \tag{2.10}$$

The flow conditions in a given cross-section of the duct will be defined as critical if the mass flow density in this cross-section reaches its maximal value (with non-zero derivative of the flow velocity). It may be seen that such a situation takes place when the flow velocity of the gaseous vapour becomes equal to the value c determined by (2.10), which in itself is a function of the local parameters of state of the two-phase medium (including the values of interphase transfer fluxes ascribed to this state) and of the locally existing expansion rate. Hence, with

$$w = c \equiv c_k, \qquad w = w_k(p_k) \tag{2.11}$$

the critical velocity i.e. the velocity of the gas phase at critical conditions of the two-phase flow follows from:

$$\frac{c_k}{a} = \frac{-aP \pm \sqrt{a^2 P^2 + 4\dot{p}\left[\dot{p} - (\kappa - 1) Q - a^2 Y\right]}}{2\left[\dot{p} - (\kappa - 1) Q - a^2 Y\right]} \tag{2.12}$$

with the sign „+" for the positive flow direction.

On the basis of the relations derived above it becomes possible to outline a numerical calculation method of critical flow parameters; in general such a method will consist in determining the location of the cross-section of the duct (or the respective pressure value), where the gas phase velocity reaches the value c_k given by eq. (2.12).

Detailed derivation of the method is contained in previous papers of one of the authors [7, 8]. The dependence of transfer fluxes on the deviations from equilibrium is introduced in the form

$$\left.\begin{aligned} Y(t) &= \Phi \cdot [m \cdot \delta(t) + m' \delta'(t)], \\ Q(t) &= \Phi \cdot [n \cdot \delta(t) = n' \delta'(t)], \\ P(t) &= -r \cdot u(t) \end{aligned}\right\} \tag{2.13}$$

with the appropriate coefficients depending on the state of the medium and on the content and dispersion of the liquid phase. A set of differential equations derived in [7, 8] describes

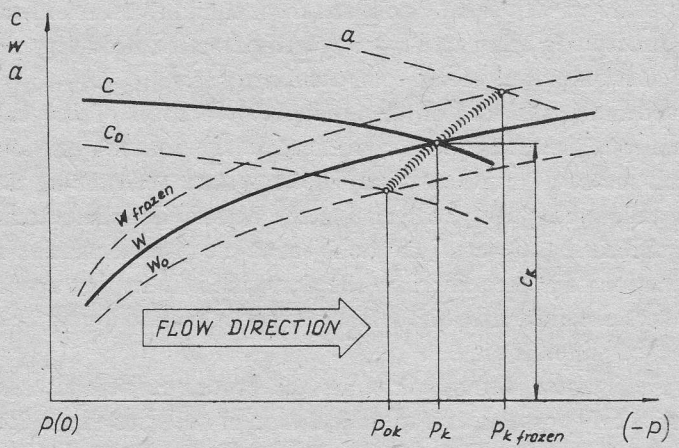

Fig. 1. Determination of critical conditions (schematic sketch)

the run of all the thermal and kinematical parameters of the medium in the course of the expansion process. The idea of the numerical procedure (using the value of pressure as the independent variable) is explained in Fig. 1 *).

Having determined the critical flow parameters, the maximal mass flow density of the two-phase flow may be found as:

$$\Gamma_k = \left(\frac{w}{gv}\right)_k . \tag{2.14}$$

In the absence of droplets in the medium eq. (2.12) reduces itself to the well known case of a perfect gas flow. The same result may be obtained by a formal introduction of an extremely high expansion-rate, thus creating a border-case of „frozen" interphase transfer; in reality, however, this case cannot be attained even approximately in view of former assumptions (for non-zero moisture flow content).

Another border-case is the equilibrium flow which will be described in the next chapter.

*) The equations derived in [7, 8] take into account the presence of droplets having various sizes and histories (multi-fraction liquid phase) in the medium. In this paper, for the sake of simplicity, the liquid phase is regarded as containing uniform droplets i.e. all the droples flowing through a given cross-section are of the same size. This limitation is not substantial for further considerations.

3. The Indirect Method of Critical Flow Determination

3.1. Main Features of the Method

The method outlined in the previous chapter (which may be called a „direct" method) considers the gas phase flow as the reference process, affected additionally by interphase transfer fluxes. As the interphase processes – according to their physical nature – are evoked by the deviations from equilibrium, the case of equilibrium expansion (which occurs at extremely low expansion-rates) appears as a singularity and can not be solved using the general set of equations; this impedes the accurate calculation of technically most important cases of flows with small deviations from equilibrium. Another disadvantage is that numerical procedures using step-by-step methods often need very small step values, which leads to uneconomically long computation times.

In this chapter a method based on the „indirect" procedure of flow calculation will be derived; the „indirect" method has been outlined in the author's previous papers [6, 8]. According to this method the calculation is split up into determining the run of the reference process and, subsequently, calculating the non-equilibrium effects. The reference process is an equilibrium (saturated) flow, subject to the given expansion-rate $\dot{p}(t)$.

The determination of non-equilibrium effects, which is in its first approximation based on state parameters obtained from the reference flow calculation, may be carried out with a chosen degree of simplification.

The use of the indirect method simplifies the procedure without decreasing the accuracy. It is particularly convenient if the calculation of the same expansion process has to be repeated for various assumptions. The reference process appears as a simple border case of the flow calculations.

In the following the pressure variation will be used as the main independent variable in the calculation procedures. As an auxiliary variable the value of time will be used.

3.2. The Equilibrium Reference Process

The set of equations governing the flow of saturated wet steam may be written down following simple derivations from well known formulae as in [6, 8]:

$$\frac{dT_0(t)}{dt} = \frac{v_0 T_0}{h} \cdot \dot{p}(t), \tag{3.1}$$

$$\frac{dw_0(t)}{dt} = \frac{-g_0 v_0}{w_0} \cdot \dot{p}(t), \tag{3.2}$$

$$\frac{dg_0'(t)}{dt} = \left[\frac{c' T_0}{h} - g_0\left(1 - \frac{T_0}{h} \cdot \frac{\partial h}{\partial T_0}\right)\right] \frac{v_0}{h} \dot{p}(t), \tag{3.3}$$

$$\frac{dg_0(t)}{dt} = -\frac{dg_0'(t)}{dt}, \tag{3.4}$$

$$\frac{dv_0(t)}{dt} = \left(1 - \frac{h}{pv_0}\right) \cdot \frac{v_0^2}{h} \cdot \dot{p}(t). \tag{3.5}$$

The flow of saturated wet steam may be also regarded as a gas phase flow with internal sources – in accordance with the pattern used in Chapter 2 (eqs. (2.3) to (2.5)). Then the appropriate source terms are given by the expressions:

$$Y_0(t)=\left[1-\frac{T_0}{h}\left(\frac{c'_p}{g_0}+\frac{\partial h}{\partial T_0}\right)\right]\cdot\frac{1}{h}\cdot\dot{p}(t), \tag{3.6}$$

$$Q_0(t)=\left(\frac{c_p T_0}{h}-1\right)\cdot\dot{p}(t), \tag{3.7}$$

$$P_0(t)=\frac{g'_0}{w_0}\cdot\dot{p}(t). \tag{3.8}$$

The critical velocity of the equilibrium reference flow may be found immediately with the use of (2.12):

$$c_0^2=\frac{g_0 h}{\frac{T_0}{h}\left(\frac{c'_p}{g_0}+\frac{\partial h}{\partial T_0}\right)+\frac{h}{pv_0}-2}. \tag{3.9}$$

The above value is equal to the velocity of sound of extremely low frequency in the two-phase medium.

The procedure of finding the location of the critical cross-section (respectively the critical pressure value) remains the same as described in Chapter 2 (Fig. 1); the condition (2.11) will be written here as:

$$w_0=c_0\equiv c_{0k}, \qquad W_0=w_{0k}(p_{0k}). \tag{3.10}$$

Hence the maximal flow density of the saturated wet steam flow is:

$$\Gamma_{0k}=\left(\frac{w}{gv}\right)_{0k}. \tag{3.11}$$

3.3. Calculation of the Deviation Parameters

The determination of the run of deviations from equilibrium is based on the assumption that their relative values are small (eq. 2.2).

Hence simplified procedures allowing a separate calculation of thermal and kinematical deviations (as a result of decoupling heat and mass transfer from momentum transfer processes) may be adopted.

The run of deviations from the equilibrium temperature is controlled by the set of differential equations ([6, 8]):

$$\begin{aligned}\frac{d\delta'(t)}{dt}&=k_{11}\delta'(t)+k_{12}\delta(t)-\frac{v_0 T_0}{h}\dot{p}(t),\\ \frac{d\delta(t)}{dt}&=k_{21}\delta'(t)+k_{22}\delta(t)+\left(1-\frac{c_p T_0}{h}\right)\frac{v_0}{c_p}\dot{p}(t)\end{aligned} \tag{3.12}$$

which can be solved numerically with the use of coefficients based on state parameters of the equilibrium process:

$$\left.\begin{aligned} k_{11}&=\frac{-3v_0'}{\rho_0}\frac{m'h+n'}{c_p'}, & k_{12}&=\frac{-3v_0'}{\rho_0}\frac{mh+n}{c_p'},\\ k_{21}&=\frac{3v_0'}{\rho_0 c_p}\frac{g_0'}{g_0}n', & k_{22}&=\frac{3v_0'}{\rho_0 c_p}\frac{g_0'}{g_0}n. \end{aligned}\right\} \tag{3.13}$$

A further simplification may consist in assuming the values of coefficients and all parameters of state appearing in the above equations as approximately constant. If the expansion process starts from an initial state of equilibrium the resulting solutions appear in the form:

$$\left.\begin{aligned} \delta'(t)&=\sigma_1 H_1 G_1(t)\exp(\sigma_1 t)+\sigma_2 H_2 G_2(t)\exp(\sigma_2 t),\\ \delta(t)&=b_1\sigma_1 H_1 G_1(t)\exp(\sigma_1 t)+b_2\sigma_2 H_2 G_2(t)\exp(\sigma_2 t), \end{aligned}\right\} \tag{3.14}$$

where the forcing functions are:

$$G_m(t)=\int_0^t \dot{p}(t)\exp(-\sigma_m t)\,dt, \quad \text{(with } m=1,2) \tag{3.15}$$

and the constants:

$$\sigma_{1,2}=\frac{k_{11}+k_{12}}{2}\pm\frac{1}{2}\sqrt{(k_{11}+k_{22})^2-4(k_{11}k_{22}-k_{12}k_{21})}, \tag{3.16}$$

$$b_1=\frac{\sigma_1-k_{11}}{k_{12}}, \quad b_2=\frac{\sigma_2-k_{11}}{k_{12}}, \tag{3.17}$$

$$\left.\begin{aligned} H_1&=\frac{-v_0}{c_p\sigma_1(b_2-b_1)}\left[1-(1-b_2)\cdot\frac{c_p T_0}{h}\right],\\ H_2&=\frac{v_0}{c_p\sigma_2(b_2-b_1)}\left[1-(1-b_1)\cdot\frac{c_p T_0}{h}\right]. \end{aligned}\right\} \tag{3.18}$$

In a similar way the differential equation governing the run of the velocity deviation („slip") may be written:

$$\frac{du(t)}{dt}=-D\cdot u(t)+\frac{dw_0(t)}{dt}, \tag{3.19}$$

where the coefficient D results from the adopted law of the droplet drag resistance. The simplified solution, valid for processes starting from the initial equilibrium, is:

$$u(t)=-\exp(-Dt)\int_0^t \frac{g_0 v_0}{w_0}\dot{p}(t)\exp(Dt)\,dt. \tag{3.20}$$

3.4. Application of the Indirect Method to the Calculation Procedure

The course of the expansion process is calculated by subsequently solving the reference equilibrium process (Chapt. 3.2) and then defining the run of the deviation parameters (Chapt. 3.3). In order to apply this method to the pattern of determining the critical flow conditions (as shown in Fig. 1) it is necessary to derive appropriate expressions for the source terms in eq. (2.12).

Let us consider an element of the two-phase medium going over from a state of non-equilibrium into a corresponding equilibrium state of the same pressure by means of adiabatic-isobaric relaxation.

A description of such a change will yield relations between a momentary state of the medium undergoing a real non-equilibrium process and the corresponding state of the reference equilibrium process.

Writing down the equations of the thermal energy balance and of the balance of total flow energy between the two corresponding states, the supplementary increases of the gas phase flow content and of the gas phase velocity will be obtained (with the use of simplifying assumptions already made):

$$\Delta g \equiv g - g_0 = \frac{-1}{h} (c_p g_0 \delta + c'_p g'_0 \delta'), \tag{3.21}$$

$$\Delta w \equiv w - w_0 = g'_0 u. \tag{3.22}$$

The source functions in eq. (2.12) will be considered as sums of the respective reference source functions defined by eqs. (3.6) to (3.8) and supplemental terms representing non-equilibrium effects in the flow:

$$\left.\begin{aligned} Y(t) &= Y_0(t) + \Delta Y(t), \\ Q(t) &= Q_0(t) + \Delta Q(t), \\ P(t) &= P_0(t) + \Delta P(t). \end{aligned}\right\} \tag{3.23}$$

The supplemental functions have to be calculated on the basis of the already determined run of the deviation parameters.

The equations of the first law of thermodynamics of each of the components contained in a two-phase medium element are:

$$\frac{1}{v}\frac{di}{dt} = \dot{p} + Q, \qquad \frac{g'}{g}\frac{1}{v}\frac{di'}{dt} = -(Q + Y h). \tag{3.24}$$

Writing down the same equations for the reference equilibrium process and subtracting them respectively from (3.24) the supplemental expressions ΔY and ΔQ may be obtained. Their first approximations are:

$$\Delta Y(t) = \frac{-1}{g_0 v_0 h}\left[c_p g_0 \frac{d\delta(t)}{dt} + c'_p g'_0 \frac{d\delta'(t)}{dt}\right], \tag{3.25}$$

$$\Delta Q(t) = \frac{c_p}{v_0}\frac{d\delta(t)}{dt}. \tag{3.26}$$

Similarly the momentum source function (i. e. mass force working upon the gas phase) may be expressed by

$$P=-\frac{g'}{gv}\frac{dw'}{dt}. \tag{3.27}$$

Subtracting from (3.27) the same expression written for the reference process and bearing in mind that

$$w_0-w'=u-\Delta w \tag{3.28}$$

one arrives at the supplemental source term in the form:

$$\Delta P(t)=\frac{g'_0}{v_0}\frac{du(t)}{dt}. \tag{3.29}$$

4. Numerical Investigations

4.1. Description of the Numerical Calculation Arrangement

The numerical analysis of critical two-phase flow conditions is based on the derivations presented above. The particular aim is to define the influence of non-equilibrium effects upon the values of critical parameters and upon the maximal flow-rate.

For the evaluation of results the following characteristic parameters are introduced: the ratio of the critical velocity to the local sound velocity in the gaseous phase

$$\alpha=\frac{c_k}{a} \tag{4.1}$$

and the ratio of maximal mass-flow density of the real non-equilibrium flow to the maxima mass flow density of the reference equilibrium process

$$\mu=\frac{\Gamma_k}{\Gamma_{0k}}=\frac{\Gamma_k(p_k)}{\Gamma_{0k}(p_{0k})}. \tag{4.2}$$

It should be noted that each of these two maximal flow densities takes place when the critical pressure value of its respective flow process is reached (i.e. in a duct flow the critical conditions will be reached at different cross-sections of the duct).

In order to facilitate the assessment of the main factors causing non-equilibrium effects additional assumptions are adopted. Thus the calculated cases of expansion processes are assumed to take place at constant expansion rates:

$$\dot{p}(t)=\frac{dp}{dt}\equiv\dot{p}=\text{const}. \tag{4.3}$$

As the finite expansion rate is primarily responsible for causing interphase non-equilibrium the analysis of a given process at various expansion-rates is most essential for the investigation of flow choking conditions.

The initial state of the medium is assumed to be at interphase equilibrium. It is also assumed that the expansion process starts at zero flow velocity (practically, allowing for a small inlet velocity, the thermal parameters have to be expressed in terms of the total pressure as the initial pressure value). Thus the initial conditions are:

$$\left.\begin{aligned} &\text{at } p=p(0); \quad w(0)=0, \\ &\qquad\qquad\quad \delta(0)=\delta'(0)=0, \\ &\qquad\qquad\quad u(0)=0. \end{aligned}\right\} \tag{4.4}$$

For detailed calculations 18 sets of parameters marking the initial state of the medium have been chosen, these parameters being: the initial pressure, the initial moisture mass-flow content and the initial radius of droplets of which the liquid phase is made up.

The chosen initial values are:

$$p(0)=0{,}1\ ;\ 1{,}0\ ;\ 10 \text{ bar},$$

$$g'(0)=0{,}05\ ;\ 0{,}10\ ;\ 0{,}15,$$

$$\rho(0)=10^{-8}\ ;\ 10^{-7} \text{ m}.$$

For each of these 18 initial situations process calculations have been carried out on the assumption of different expansion rates, covering the range:

$$-10>\dot{p}>-10^{8} \text{ bar/s}.$$

The calculations have been carried out after the method described in Chapt. 3, using step-by-step procedures to follow the course of the expansion.

4.2. Results of the Numerical Analysis

The analysis is preceded by critical parameter calculations of reference equilibrium processes in order to obtain comparative values. Figs. 2 and 3 show values of the critical pressure ratio and dimensionless critical velocity of saturated wet steam expansions from various initial conditions; as is obvious the results are independent of the expansion rate (which is meant to be extremely slow). The curves for zero initial moisture content have no physical significance since real expansion from an initial dry state cannot be considered as an equilibrium process; they appear in the diagrams as an indication of theoretical border cases.

The accuracy of deviation parameter calculations has been checked by comparing results obtained with the help of various calculation methods. In Figs. 4 and 5 the run of temperature deviations for a chosen expansion process is depicted; the calculations have been carried out using the method proposed in this paper, with and without additional simplification, and compared with results given by the „direct" method calculation. A slight shift of the initial points of the curves showing direct method results is due to the use of a special starting procedure for this method.

Values of the non-dimensional critical velocity resulting from calculations of various expansion processes are shown in Fig. 6, plotted as functions of the expansion rate for

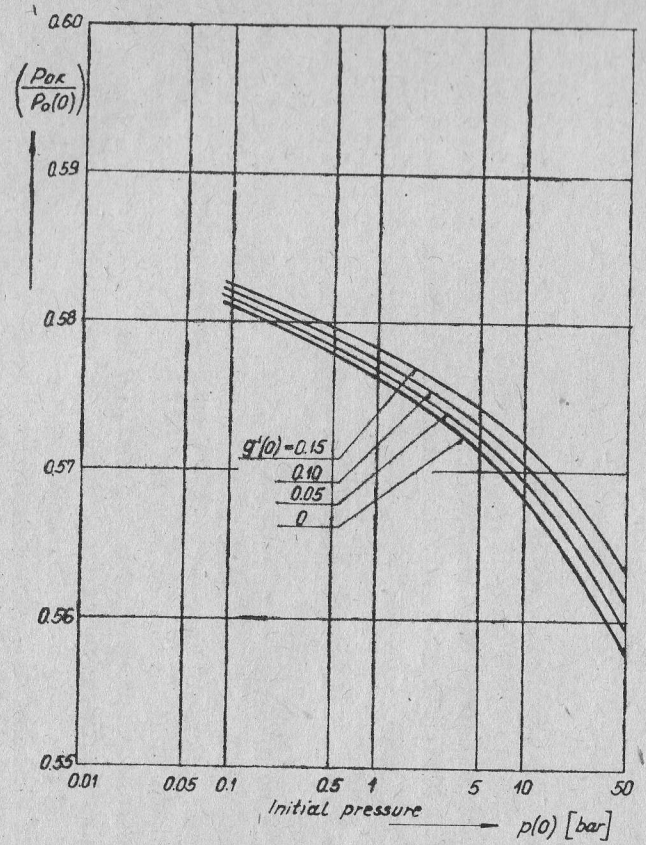

Fig. 2. Critical pressure ratio of equilibrium (saturated) expansion

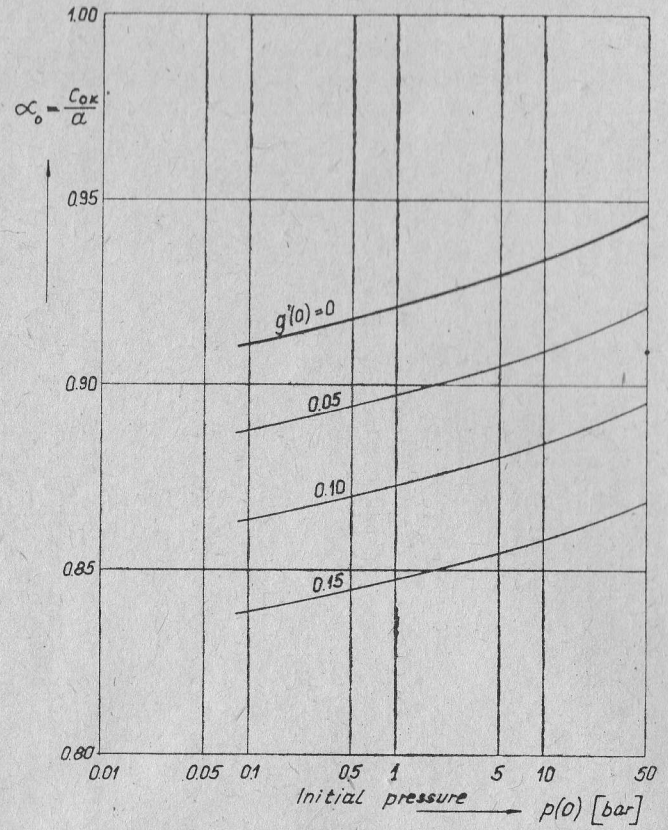

Fig. 3. Dimensionless critical velocity of equilibrium (saturated) expansion

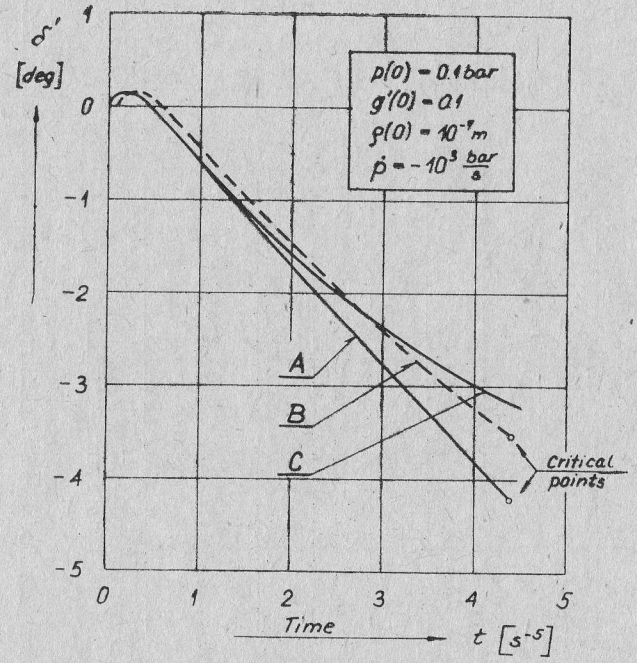

Fig. 4. Run of the liquid phase temperature deviation during an expansion process

A – numerical solution of eqs (3.12), *B* – direct method calculation results, *C* – approximate calculation after eqs (3.14)

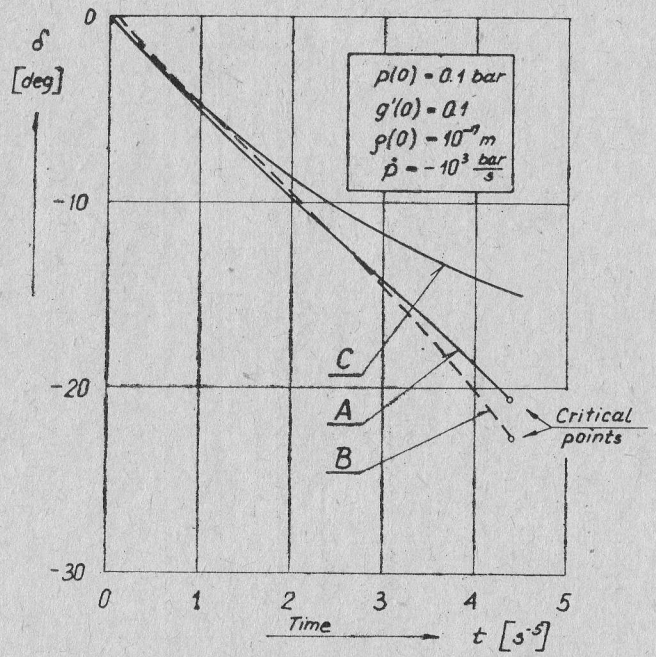

Fig. 5. Run of the gas phase temperature deviation during an expansion process

A – numerical solution of eqs (3.12), *B* – direct method calculation results, *C* – approximate calculation after eqs (3.14)

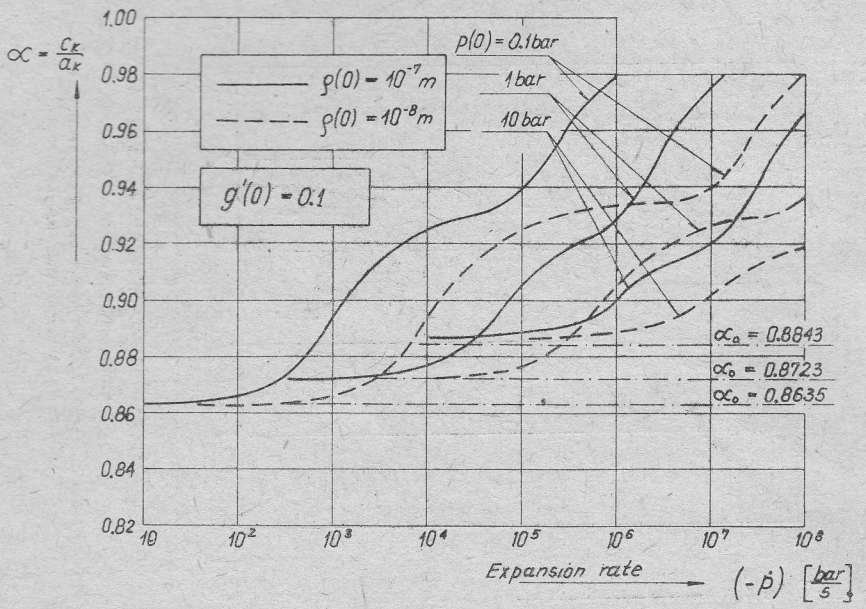

Fig. 6. Dimensionless critical gas-phase velocity (initial moisture flow content 0.1)

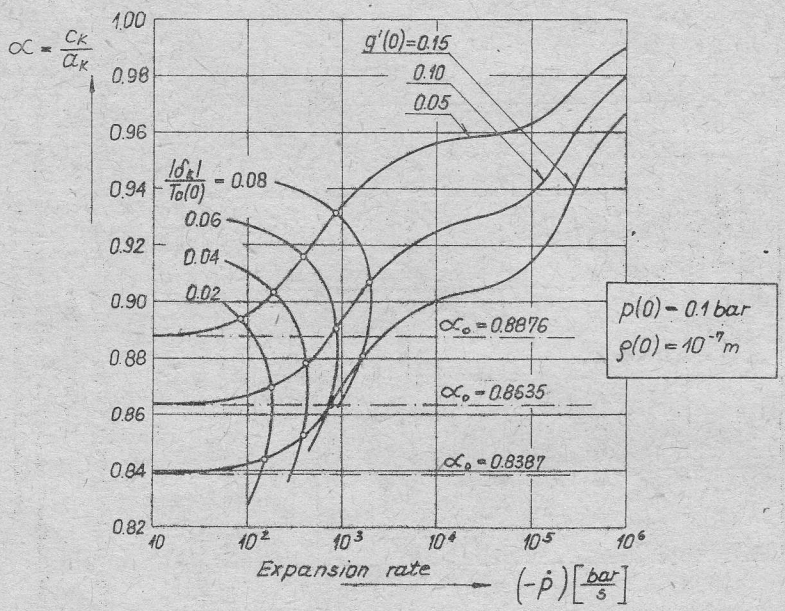

Fig. 7. Dimensionless critical gas-phase velocity (initial pressure 0.1 bar, initial droplet radius 10^{-7} m)

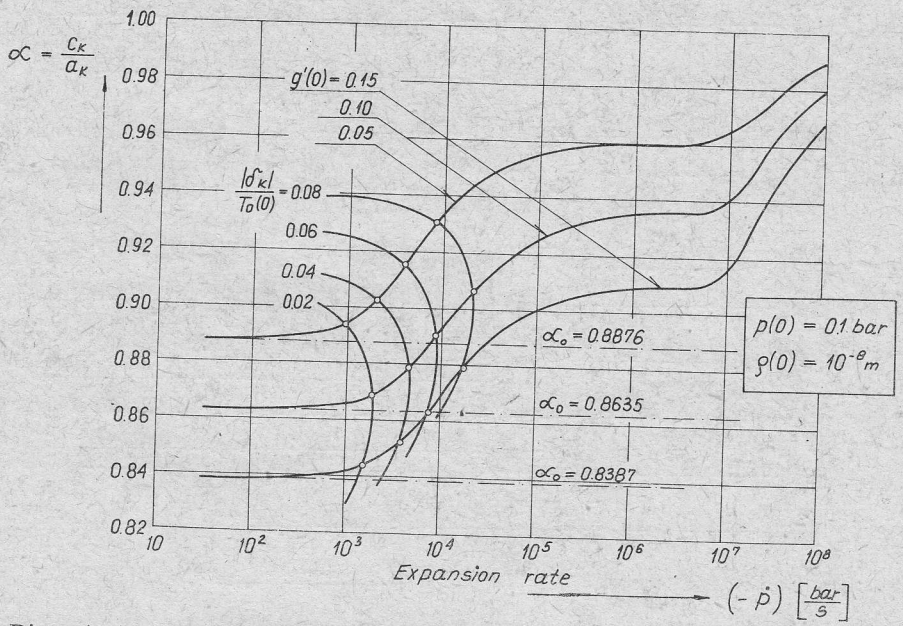

Fig. 8. Dimensionless critical gas-phase velocity (initial pressure 0.1 bar, initial droplet radius 10^{8-} m)

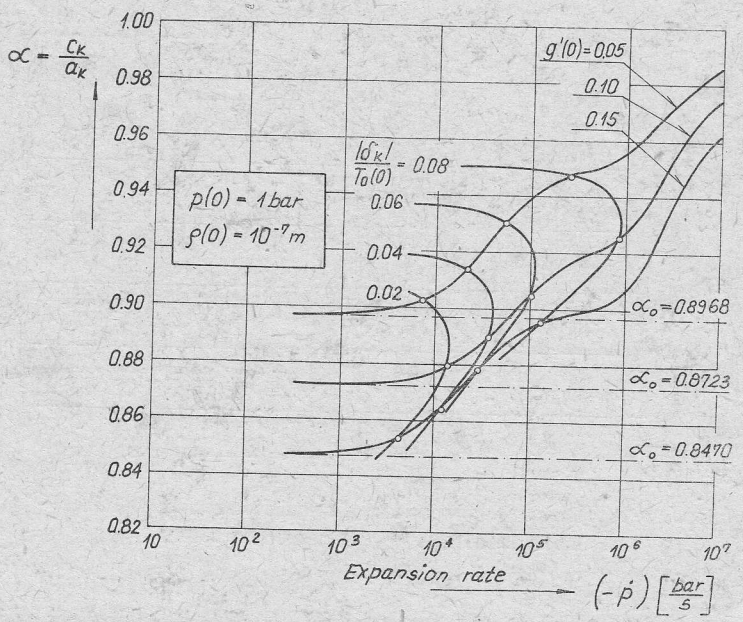

Fig. 9. Dimensionless critical gas-phase velocity (initial pressure 1 bar, initial droplet radius 10^{-7} m)

various initial conditions. The respective values of the equilibrium processes are shown as asymptotic lines, corresponding to $\dot{p} \approx 0$.

Further critical velocity values are contained in Figs. 7 to 10. As the feasibility of the indirect calculation method is limited to small deviations from equilibrium, the rela-

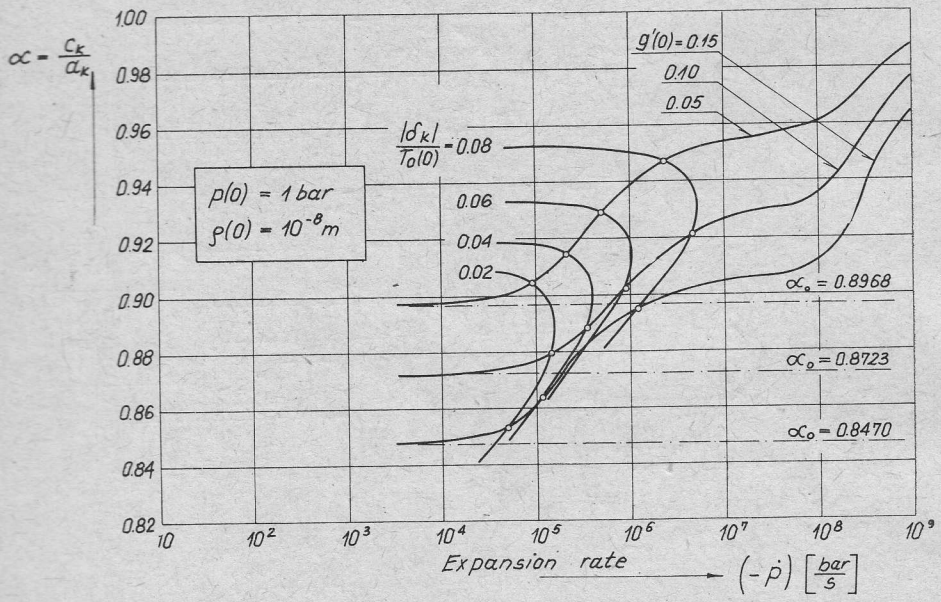

Fig. 10. Dimensionless critical gas-phase velocity (initial pressure 1 bar, initial droplet radius 10^{-8} m)

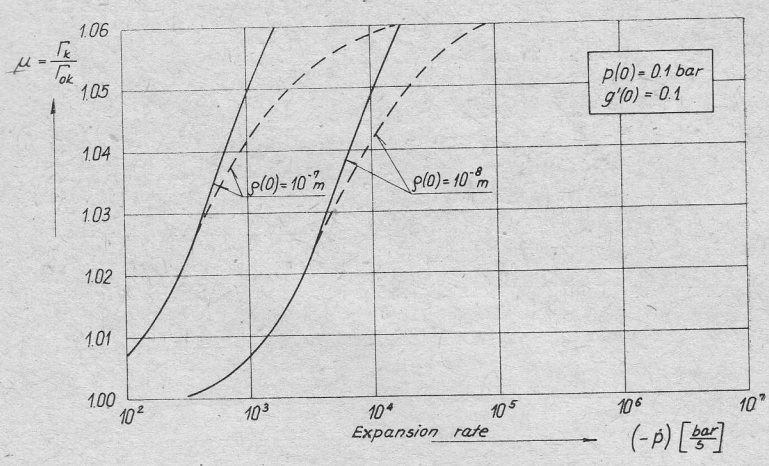

Fig. 11. Maximal flow rate coefficients – comparison of calculation results (full lines – indirect method results, dotted lines – direct method results)

tive deviation parameters have been checked for every calculated example; it has been found that the temperature deviation of the gas phase is most substantially responsible for the non-equilibrium effects. The lines of constant relative value of this deviation are depicted in the diagrams as an approximate criterion for the expected accuracy range.

Results of comparative calculations of the maximal flow rate coefficient, obtained with the use of the direct and the indirect method are shown in Fig. 11. It can be seen that

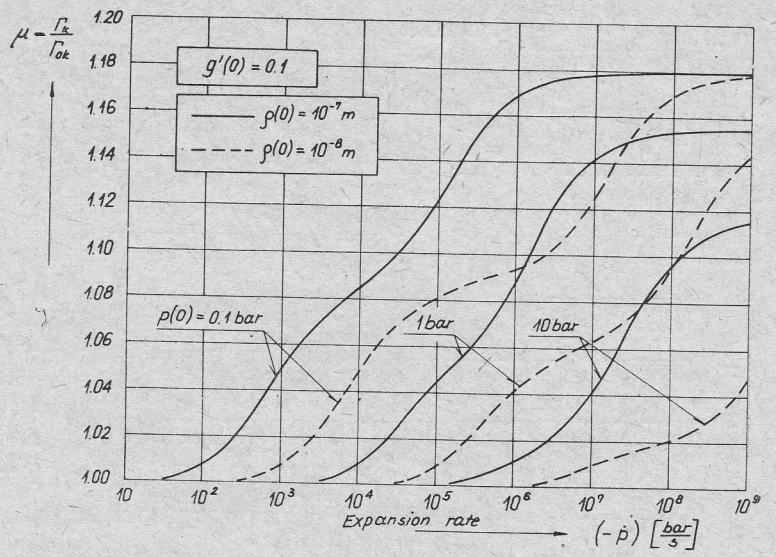

Fig. 12. Maximal flow rate coefficients (initial moisture flow content 0.1)

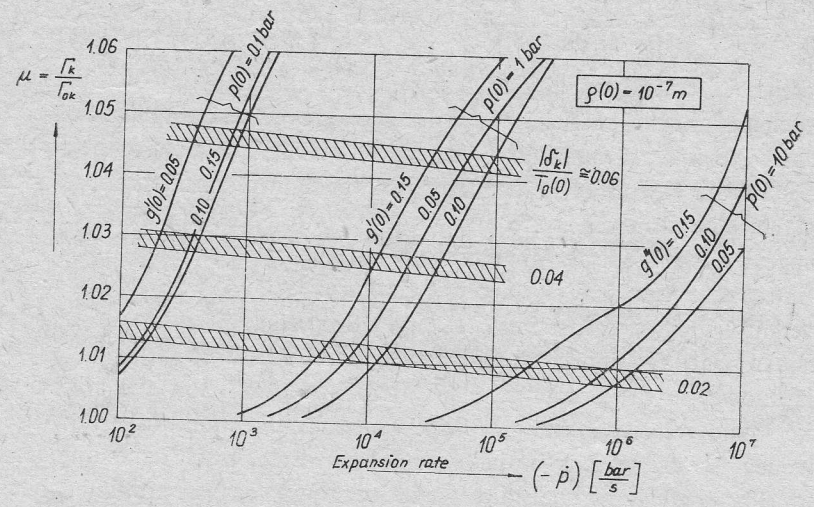

Fig. 13. Maximal flow rate coefficients (initial droplet radius 10^{-7} m)

the difference of results increases substantially with the increase of the expansion rate i.e. corresponding to increasing deviation from equilibrium.

Calculated values of maximal flow rate coefficients are shown in Figs 12 to 14, plotted as functions of the expansion rate for various initial conditions. Into the last two diagrams the quantities of relative temperature deviations have been inserted; for reasons of clarity they are shown as zones of approximately constant values (instead of drawing bundles of curves), thus marking areas of expected degrees of accuracy.

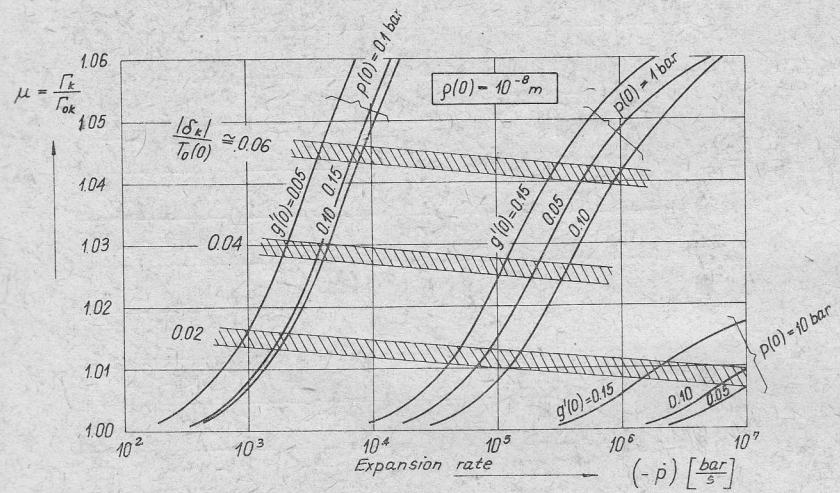

Fig. 14. Maximal flow rate coefficients (initial droplet radius 10^{-8} m)

5. Conclusions

The appearance of non-equilibrium effects in wet steam flows causes an increase in both critical velocity and maximal flow rate above the level of their respective reference flow values.

The increase of critical velocity or maximal flow rate depends quantitatively on the value of the expansion rate (which is the external forcing evoking interphase non-equilibrium) and on the medium internal inertia (which resists a quick adjustment to the changes caused by the external forcing). It may be seen from both analytical considerations and numerical results that the internal inertia is stronger (and thus the non-equilibrium effects are more likely to appear even at slower expansion-rates) when the medium displays properties impeding interphase transfer. Such properties are:

– coarser liquid phase dispersion (i.e. larger droplets),

– lower gas phase density (cases with lower initial pressure).

As it is known, in the border case of „frozen-transfer" flow the critical velocity is equal to the sound velocity in the gas phase whilst for the equilibrium flow it equals the „zero-frequency" sound velocity in the two-phase medium. In all remaining cases of non-equilibrium flow there is no such direct relationship, as has been pointed out in [7, 8].

The expansion rate values in steam turbine guide vanes lie between 10^2 and 10^3 bar/s. As the numerical results indicate, within this range a substantial increase of the maximal flow rate is to be expected rather for flow processes having low initial pressure values; therefore this phenomenon is essential for flow conditions which may be found in last stages of condensing turbines. The degree of liquid phase dispersion is also a decisive parameter; it can be seen from Figs 12 to 14 that the droplet radius has a strong influence on the maximal flow rate.

Higher expansion rates may be expected in blow-down processes where the influence of the state of the medium and of the structure of its liquid phase may be even more substantial.

Within the range of practically applied expansion rate values the accuracy of the indirect method calculations seems satisfactory as compared with the more exact „direct" method calculations. It is to be noted that calculation errors have a stronger effect on the maximal flow rate coefficient than on the critical velocity value.

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Obliczanie maksymalnej przelotności i krytycznych warunków w nierównowagowych przepływach par wilgotnych

Streszczenie

Praca poświęcona jest określaniu krytycznych parametrów przepływu i maksymalnej przelotności w nierównowagowych ekspandujących przepływach par wilgotnych.

Rozpatruje się jednowymiarowe ustalone przepływy rozprężające czynnika dwufazowego, w którym faza ciekła ma postać małych kropel.

Na skutek skończonej szybkości rozprężania i nienadążania zmian stanu cieplnego i szybkości kropel za odpowiednimi zmianami stanu fazy gazowej powstaje stan braku równowagi wewnętrznej w czynniku; wywołuje on procesy wymiany ciepła, substancji i pędu między fazami, przebiegające w kierunku przywrócenia równowagi wewnętrznej.

W rozdziale 2 przeprowadzono analizę powstawania krytycznych warunków przepływu przy użyciu modelu przepływu fazy gazowej traktowanego jako przepływ gazu ze „źródłami wewnętrznymi" i otrzymano wzór (2.12) określający krytyczną prędkość przepływu. Opisano również zasady określania krytycznych warunków przepływu (rys. 1).

W rozdziale 3 została wyprowadzona metoda obliczania krytycznych warunków przepływu i ma-

ksymalnej przelotności, oparta na tzw. „pośrednim" sposobie wyznaczania przebiegu parametrów odchylenia od równowagi. Metoda ta umożliwia wyznaczenie różnic wartości odnośnych parametrów w stosunku do ich wartości w porównawczym przepływie równowagowym.

Stosowalność proponowanej metody obejmuje zakres technicznie ważnych przypadków przepływów z niewielkimi odchyleniami od równowagi wewnętrznej.

W rozdziale 4 przedstawiono wyniki analizy numerycznej dla szerokiego zakresu przypadków krytycznych warunków przepływu.

Wyniki obliczeń bezwymiarowej prędkości krytycznej w funkcji prędkości rozprężania przedstawiono na rysunkach 6 do 10 dla różnych przyjęć dotyczących stanów początkowych czynnika i struktury fazy ciekłej.

Rysunki 11 do 14 przedstawiają wyniki obliczeń stosunku maksymalnej przelotności rozpatrywanego przepływu do maksymalnej przelotności przepływu równowagowego.

Jak wynika z przeprowadzonej analizy, powstawanie stanów braku równowagi termodynamicznej powoduje zwiększenie zarówno prędkości krytycznej, jak i maksymalnej przelotności.

Zwiększenie to występuje silniej w przypadku większych rozrzedzeń fazy gazowej i większych promieni kropel, z których składa się faza ciekła i związane jest z faktem skończonej szybkości rozprężania.

Расчет максимальной пропускной способности и критических условий в неравновесных течениях влажных паров

Резюме

Работа посвящена определению критических параметров течения и максимальной пропускной способности в неравновесных экспандирующих течениях влажных паров. Рассматриваются одномерные установившиеся течения с расширением двухфазной среды, в которой жидкая фаза имеет форму мелких капель.

Ввиду ограниченной скорости расширения и неуспеваемости изменений теплового состояния и скорости капель за соответствующими изменениями состояния газовой фазы возникает состояние отсутствия внутреннего равновесия в среде. Это состояние вызывает процессы обмена тепла, массы и импульса между фазами, направленные на отыскание внутреннего равновесия.

В разд. 2 проведен анализ возникновения критических условий течения при использовании модели течения газовой фазы, принимаемой в виде течения газа с „внутренними источниками". Получена формула (2.12), определяющая критическую скорость течения. Описаны также принципы определения критических условий течения (рис. 1).

В разд. 3 выработан метод расчета критических условий течения и максимальной пропускной способности, основанный на т. наз. „посредственном" способе определения распределения параметров отклонения от равновесия. Этот метод позволяет определять разницы значений соответствующих параметров по отношению к их значениям в сравниваемом течении. Применяемость предлагаемого метода возможна в пределах технически важных случаев течений с небольшими отклонениями от внутреннего равновесия.

В разд. 4 представлены результаты численного анализа для широкого предела случаев критических условий течения. Результаты расчетов безразмерной критической скорости в функции скорости расширения представлены на рис. 6 до 10 для различных приемов касающихся начальных состояний среды и структуры жидкой фазы. На рис. 11 до 14 представлены результаты расчетов отношения максимальной пропускной способности рассматриваемого течения к максимальной пропускной способности равновесного течения. Как следует из проведенного анализа, возникновение состояний отсутствия термодинамического равновесия вызывает увеличение как критической, так и максимальной пропускной способности. Это увеличение объявляется сильнее в случае больших разрежений газовой фазы и больших радиусов капель, из которых состоит жидкая фаза и связано с фактом ограниченной скорости расширения.