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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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Instytut Maszyn Przepływowych PAN
ul. Gen. Józefa Fiszer 14, 80-952 Gdańsk, skr. pocztowa 621, tel. 41-12-71

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TADEUSZ KORONOWICZ

Gdańsk

Hydrodynamic Characteristics of Systems of Helical Vortices with Vortex Cores of Finite Dimensions*

Starting from the basic equations of the fluid mechanics relations between the velocity field induced far behind the propeller and external forces representing a screw propeller generating this field have been derived in the paper. The model adopted for calculations has been based on a system of concentrated vortices with cavitation singularities of a vortex core.

The relations derived make it possible to determine ideal hydrodynamic characteristics of a screw propeller. Exemplary characteristics for selected vortex systems in various coordinates systems have been presented in the paper.

Nomenclature

- | | |
|--|---|
| B_T — diameter-independent thrust loading coefficient, | R_∞ — radius of an axial cylinder on the side surface of which centres of the tip vortex singularity are distributed; assumed radius of the slipstream far behind the propeller, |
| B_N — diameter-independent power loading coefficient, | T — thrust of screw propeller, |
| C_T — thrust loading coefficient, | V_0 — undisturbed flow velocity, |
| C_N — power loading coefficient, | v_x, v_ρ, v_ϕ — axial, radial and circumferential components of induced velocities, |
| $J = \frac{V_0}{nD}$ — advance coefficient, | Δv — deviation from mean values, |
| k_∞ — coefficient of contraction of slipstream, | Z — number of tip vortices; also number of propeller blades, |
| K_T — thrust coefficient, | β — pitch angle of the helix of tip vortices, |
| $m_{v_\infty} = \frac{\Gamma}{4\pi V_0 R_\infty \tan \beta_\infty} \cdot 2$ — dimensionless circulation of helical vortices, related to the pitch of helix; also relative mean value of the axial component of velocities induced in the slipstream, | Γ — velocity circulation, |
| p — pressure, | $\delta_0 = \frac{m_{v_\infty} \tan \beta_\infty}{\sqrt{\sigma_0}}$ — parameter characterizing, cavitation singularities of vortex cores, |
| Q — screw propeller torque, | η — propeller efficiency, |
| $r = \frac{R}{R_\infty}$ — relative radius, | $\lambda = \frac{V_0}{\omega_0 R}$ — advance coefficient, |
| $r_r = \frac{R_r}{R_\infty}$ — relative radius of the axial vortex singularity, | $\mathfrak{z}, \rho, \mathfrak{z}$ — integral discriminants of the induced velocity field, |
| R_2 — propeller radius, | σ — cavitation number, |
| | ω_0 — angular velocity of screw propeller, |

subscript 1 — refers to a cross-section far ahead of the screw propeller,

subscript 2 — refers to the screw disk cross-section,

subscript 3 or ∞ — refers to a cross-section far behind the screw propeller.

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1. Introduction

Derivation of relations between external forces representing a propeller, and the velocity field induced by these forces constitutes one of principal problems to be solved by the vortex theory of screw propellers. To this end appropriate vortex models of the propeller blades (system of bound vortices) and the slipstream (system of free vortices) are being developed. For several tens of years a model with undeformable vortex surface has served as a dominating vortex model for the system of free vortices. As a rule, in this model semi-infinite vortex lines have been regarded as regular helices.

It follows from a number of theoretical works, and especially from experiments, that vortex sheets flowing off propeller blades are unstable, and wind up relatively quickly forming a concentrated form of vorticity. For practical purposes it can be assumed that already at a distance of one propeller radius behind the propeller a concentrated form of vorticity is representative for the vorticity in the slipstream. A system of Z tip vortices in the form of helices together with a single axial vortex with cavitation singularities of vortex cores may be taken as a model for calculations.

For several years theoretical as well as experimental investigations of such a vortex model of the propeller wake have been carried out at the Department of Ship Propellers of the Institute of Fluid-Flow Machinery, Polish Academy of Sciences. The velocity field induced by systems of concentrated vortices has been calculated and presented in a proper way [3, 4], a detailed analysis of this field has been made [6]. The present paper constitutes a final, so to say completing, part of the cycle. Formulae and diagrams presented in the paper render it possible to relate characteristics of a screw propeller to the vortex system, and hence to the velocity and pressure field induced far behind the propeller. This is a result especially important in connection with the design and analysis of tandem co-rotating and contra-rotating propeller sets.

The complete cycle of papers on the vortex model of the propeller wake having a form of concentrated vorticity constitutes a new quality in the propeller theory. It makes possible carrying out investigations of a vortex model of highly loaded screw propellers in the vicinity of the propeller blades and over the blades themselves.

2. Velocity and pressure fields

The theoretical models of flow as adopted in this paper for the determination of hydrodynamic characteristics of screw propeller can be characterized as follows:

- a) an open space filled with an inviscid and weightless liquid will be taken into account in the considerations; the liquid is incompressible for pressures equal to or higher than a certain value characterizing the critical pressure p_{kr} , while below this value it changes into gas producing cavitation bubbles;
- b) far ahead of the propeller the velocity field is a steady-state uniform and rectilinear one;
- c) the screw propeller represents all external forces acting on the liquid;
- d) the screw propeller rotates at a constant angular velocity $\vec{\omega}_0 = \text{const}$, with directions of vectors $\vec{\omega}_0$ and $\vec{V}_0$ coinciding;

e) far behind the propeller the whole vorticity of the flow is concentrated within a very narrow wake and distributed evenly around cavitation singularities of the concentrated form of vorticity;

f) the velocity and pressure fields far behind the propeller are represented by a vortex system in a concentrated form having cavitation singularities consisting of Z tip vortices forming regular helices and a straightlined axial vortex.

Basic equations of fluid mechanics in the integral form [1] will be used as a basis for the calculations of the resultant thrust and resultant torque of the propeller. In these calculations a control surface S will be selected, consisting of a surface S_p confining the

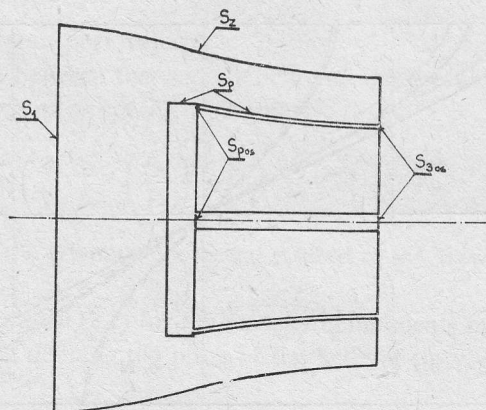


Fig. 1

set of external forces acting on the liquid (screw propeller), and a surface located sufficiently far from the propeller to allow neglecting a direct influence of the propeller over this surface. The latter surface will be composed of (Fig. 1):

- S_z — surface located far from the propeller axis of rotation, and forming a stream surface (stream tube),
- S_1 — part of a plane situated far ahead of the propeller, perpendicular to the axis of rotation and contained within the surface S_z ,
- S_3 — part of a plane situated far behind the propeller, perpendicular to the axis of rotation and contained within the surface S_z ; singularities of the concentrated form of vorticity excluded.

Calculations will be made for a cylindrical reference coordinate system moving with the rotating propeller. The x axis of this system is in line with the axis of rotation of the propeller.

In the cross-section far ahead of the propeller the components of induced velocities on the control surface S_1 are equal to zero in agreement with the assumption on the selection of this surface

$$\vec{v}_1 = \vec{V}_0$$

On the control surface S_3 in the cross-section far behind the propeller the velocity and pressure field will be characterized by a system of vortices in a concentrated form.

This system can be defined unequivocally (within the frames of the model adopted) using the following quantities:

- a) number Z of tip vortices,
- b) pitch $\text{tg}\beta_\infty$ of tip vortices,
- c) radius R_∞ of the axial cylinder on the side surface of which the tip vortices are distributed,
- d) circulation of the vortex system $\Gamma = Z \cdot \Gamma_z$.

Velocity field for the vortex system defined in such a way has been characterized and described in detail in References [3, 4, and 6]. Below will only be quoted some formulae

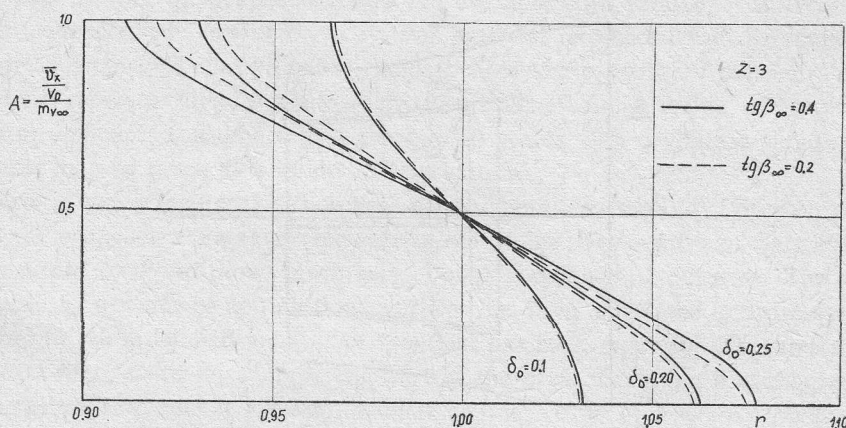


Fig. 2

describing the induced velocity field, which may be useful when calculating hydrodynamic characteristics.

The components of induced velocities can be presented conveniently in the form

$$\frac{v_x}{V_0} = m_{v\infty}(A + \Delta v), \quad (2.1a)$$

$$\frac{v_\varphi}{V_0} = -\frac{m_{v\infty} \text{tg}\beta_\infty}{r}(A + \Delta v), \quad (2.1b)$$

$$\frac{v_\rho}{V_0} = m_{v\infty} \Delta v_\rho, \quad (2.1c)$$

where Δv , Δv_ρ are deviations from mean values [6], and A is a quantity depending on the location of the point under consideration, having the following values:

- $A=1$ for radii below the singularities of the concentrated form of tip vortices,
- $A=0$ for radii above the singularities of the concentrated form of tip vortices,
- $A=1/2$ for $r=1$.

For radii crossing the singularities of the concentrated form of tip vortices A adopts values between 0 and 1. An example of function $A(r)$ for a vortex system of $Z=3$ and several values of pitch $\text{tg}\beta_\infty$ has been shown in Fig. 2.

According to the assumption adopted the whole vorticity in the flow far behind the propeller is concentrated in a restricted region around cavitation singularities of the concentrated form of the tip vortices and the axial vortex. A potential flow exists outside of this very confined region and therefore the relation between the velocity field and the pressure field in an absolute reference system is given by the Cauchy-Lagrange integral.

According to the hypothesis of Joukovski a steady-state flow picture will be obtained in a system associated with a propeller operating in a uniform and steady-state liquid inflow ($\vec{V}_0 = \text{const}$). In a moving system the Cauchy-Lagrange integral takes the form [2]

$$\frac{v^2}{2} - \vec{v} \cdot \vec{v}_e + \frac{p}{\rho} - u = \text{const},$$

where $\vec{v}_e = \vec{\omega}_0 \times \vec{R}$ is the transport velocity.

The following relation between the velocity field and the pressure field will be obtained on the ground of assumptions as specified previously

$$\frac{p_0 - p}{\frac{1}{2}\rho V_0^2} = \left(\frac{v_x}{V_0}\right)^2 + \left(\frac{v_\varphi}{V_0}\right)^2 + \left(\frac{v_r}{V_0}\right)^2 + 2\left(\frac{v_x}{V_0} - \frac{r}{\lambda_\infty} \frac{v_\varphi}{V_0}\right), \quad (2.2)$$

where $\lambda_\infty = V_0/(\omega_0 R_\infty)$ is the advance coefficient related to the cross-section of slipstream far behind the propeller.

After substitution of formulae (2.1) for the velocity components into the above formula, and basing on the formula defining the pitch of the helix of tip vortices [3]

$$\text{tg } \beta_\infty = \frac{1 + \frac{v_{x0}}{V_0}}{\frac{1}{\lambda_\infty} + \frac{v_{\varphi 0}}{V_0}} = \frac{1 + \frac{m_{v\infty}}{2}(1 + 2\Delta v_0)}{\frac{1}{\lambda_\infty} - \frac{m_{v\infty} \text{tg } \beta_\infty}{2}(1 + 2\Delta v_0)} \quad (2.3)$$

one obtains the final formula relating the pressure field to the field of velocities induced by the vortex system far behind the propeller

$$\frac{p_0 - p}{\frac{1}{2}\rho V_0^2} = m_{v\infty}^2 \left[(A + \Delta v)^2 \left(1 + \frac{\text{tg}^2 \beta_\infty}{r^2} \right) + \Delta v^2 + (A + \Delta v)(1 + \text{tg}^2 \beta_\infty)(1 + 2\Delta v_0) \right]. \quad (2.4)$$

In what follows a quantity characterizing the cavitation singularity of the axial vortex is used. This is a dimensionless singularity radius $r_r = R_r/R_\infty$. Making use of equation (2.3) one can associate this quantity with parameters characterizing the vortex system and with the cavitation number characterizing the undisturbed flow $\sigma_0 = (p - p_{kr})/(\frac{1}{2}\rho V_0^2)$. It follows from an analysis of the velocity field [4] that for radii $r < 0.3$ values of the deviations Δv and Δv_p are negligibly low, so that the pressure in the point of location of the axial vortex can be determined from a much simpler form of equation (2.4), namely

$$\frac{p_0 - p}{\frac{1}{2}\rho V_0^2} \cong m_{v\infty}^2 \text{tg}^2 \beta_\infty \left[\frac{1}{r^2} - 1 - \frac{2\Delta v_0}{\sin^2 \beta_\infty} \right]. \quad (2.5)$$

The radius of cavitation singularity of the axial vortex can be determined within the frames of the computational model under consideration from equation (2.5), if it is as-

sumed that for the radius equal to r_r the pressure p has the value of the critical pressure p_{kr} . The left-hand side of equation (2.5) becomes then equivalent to the cavitation number σ_0 .

After some transformations one arrives at a formula defining the radius of the axial vortex singularity

$$r_r = \frac{m_{v\infty}^2 \operatorname{tg}^2 \beta_\infty}{\sqrt{\sigma_0 + m_{v\infty}^2 \operatorname{tg}^2 \beta_\infty \left(1 + \frac{2\Delta v_0}{\sin^2 \beta_\infty}\right)}} = \frac{\delta_0}{\sqrt{1 + \delta_0^2 \left(1 + \frac{2\Delta v_0}{\sin^2 \beta_\infty}\right)}}, \quad (2.6)$$

where $\delta_0 = (m_{v\infty} \operatorname{tg} \beta_\infty) / \sqrt{\sigma_0}$ is a parameter giving a good characteristic of the vortex singularities in the concentrated form (δ_0 is the radius of cavitation singularity of an isolated straight-lined vortex).

3. Thrust and power of external forces

Based on an information on the velocity and pressure fields on the outer control surface ($S-S_p$) the total thrust and torque of external forces (screw propeller) confined within the outer control surface S_p can be determined from the momentum and angular momentum equations presented in the integral from [1].

For the assumptions made in Chapter 2 integral momentum and angular momentum equations will take the form

$$\oint_S \rho \vec{v} (\vec{v} \cdot \vec{n}) dS = - \oint_S p \vec{n} dS, \quad (3.1a)$$

$$\oint_S \rho (\vec{PO} \times \vec{v}) (\vec{v} \cdot \vec{n}) dS = - \oint_S p (\vec{PO} \times \vec{n}) dS, \quad (3.1b)$$

where $\vec{PO}$ is the radius vector, and $\vec{n}$ is a vector normal to the control surface and directed outwards.

The closed control surface S can be divided into two parts: S_p — a surface confining the set of external forces and the singularities of the concentrated form of vorticity, $S-S_p = S_z + S_1 + S_3$ — the remaining, outer part of the control surface S .

In resolved equations (3.1) integrals taken over the control surface S_p will represent in the momentum equation the resultant of external forces $\vec{S}_w$, and the resultant moment of external forces $\vec{M}_0$ in the angular momentum equation

$$\begin{aligned} \vec{S}_w &= \int_{S_p} \rho \vec{v} (\vec{v} \cdot \vec{n}) dS + \int_{S_p} p \vec{n} dS, \\ \vec{M}_0 &= \int_{S_p} \rho (\vec{PO} \times \vec{v}) (\vec{v} \cdot \vec{n}) dS + \int_{S_p} p (\vec{PO} \times \vec{n}) dS. \end{aligned}$$

To determine $\vec{S}_w$ and $\vec{M}_0$ from the above equations would be a very difficult task. However, the same quantities can be determined much simpler from the terms of equations (3.1) that concern the outer part of the control surface ($S-S_p$)

$$\begin{aligned} \vec{S}_w &= - \int_{S-S_p} \rho \vec{v} (\vec{v} \cdot \vec{n}) dS - \int_{S-S_p} p \vec{n} dS, \\ \vec{M}_0 &= - \int_{S-S_p} \rho (\vec{PO} \times \vec{v}) (\vec{v} \cdot \vec{n}) dS - \int_{S-S_p} p (\vec{PO} \times \vec{n}) dS. \end{aligned}$$

For the determination of hydrodynamic characteristics of a screw propeller it is essential that the thrust, that is the component of the external forces taken in the direction of the x axis, and the torque relative to the x axis (the axis of the propeller rotation) be known. Therefore in the momentum and angular momentum equations only the components of the respective terms of equations taken in the direction of the x axis will be taken into account

$$T_{xi} = \int_{S-S_p} \rho (\vec{v})_x \vec{v} \cdot \vec{n} dS + \int_{S-S_p} p (\vec{n})_x dS, \quad (3.2a)$$

$$Q_{xi} = \int_{S-S_p} \rho (\vec{PO} \times \vec{v})_x \vec{v} \cdot \vec{n} dS + \int_{S-S_p} p (\vec{PO} \times \vec{n})_x dS. \quad (3.2b)$$

An appropriate choice of the control surface makes it possible to reduce to zero a number of terms in equations (3.2) and to substantially simplify these equations.

Namely, the quantity $(\vec{PO} \times \vec{n})_x$ is equal to zero over the entire outer surface ($S-S_p$) consisting of a stream surface, which at the same time is a surface (S_z) and of two planes perpendicular to the axis of rotation. Thus the second term on the right-hand side of equation (3.2b) becomes equal to zero.

Also the second term of equation (3.2a) can be substantially simplified

$$\begin{aligned} \int_{S-S_p} p \vec{n} dS &= \int_{S_1} p_0 \vec{n} dS + \int_{S_z} p_0 \vec{n} dS + \int_{S_3} p \vec{n} dS = \int_{S-S_p+S_{3os}} p_0 \vec{n} dS + \int_{S_3} (p-p_0) \vec{n} dS - \\ &\quad - \int_{S_{3os}} p_0 \vec{n} dS, \\ \int_{S-S_p} p (\vec{n})_x dS &= \int_{S_z} (p-p_0) dS - \delta T. \end{aligned}$$

Over the surface S_z — which surface is a stream surface — all terms of equation (3.2) containing the scalar product $\vec{v} \cdot \vec{n}$ become equal to zero.

The following is valid on the surface S_1

$$\vec{v} \cdot \vec{n} = -V_0, \quad (\vec{PO} \times \vec{n})_x = 0, \quad (\vec{PO} \times \vec{v})_x = [PO \times (-\vec{n} \cdot V_0)]_x = 0,$$

while on the surface S_3

$$\vec{v} \cdot \vec{n} = (V_0 + v_x), \quad (\vec{PO} \times \vec{n})_x = 0, \quad (\vec{PO} \times \vec{v})_x = R \cdot v_\varphi,$$

where R is a radius vector beginning at the axis of rotation x .

Taking into account the above mentioned simplifications together with formula (2.2) and the continuity equation (for the stream tube formed by the surface S_z)

$$\int_{S_3} (V_0 + v_x) dS = \int_{S_1} V_0 dS = V_0 S_1$$

one arrives at a relatively simple formulae expressing the thrust and torque

$$T_{xi} = \rho V_0^2 \int_{S_3} \left\{ \frac{v_x}{V_0} \left(1 + \frac{v_x}{V_0} \right) - \frac{1}{2} \left[\left(\frac{v_x}{V_0} \right)^2 + \left(\frac{v_\varphi}{V_0} \right)^2 + \left(\frac{v_\rho}{V_0} \right)^2 \right] - \frac{v_x}{V_0} + \frac{r}{\lambda_3} \frac{v_\varphi}{V_0} \right\} dS + \delta T, \quad (3.3a)$$

$$Q_{xi} = \rho V_0^2 \int_{S_3} R \frac{v_\varphi}{V_0} \left(1 + \frac{v_x}{V_0} \right) dS, \quad (3.3b)$$

where $\delta T = - \int_{S_{3os}} p_0 dS$ (for the meaning of this quantity see Chapter 4).

After substitution of (2.1) and (2.3) into (3.3), and some transformations formulae for the thrust T_{xi} and the power absorbed by the propeller are obtained

$$T_{xi} = \rho V_0^2 \pi R_\infty^2 m_{v\infty} \left[(1 + m_{v\infty})(1 - r_r^2) + \frac{m_{v\infty} \operatorname{tg}^2 \beta_\infty}{2} \mathfrak{z} + (v + m_{v\infty} \mu) + \frac{m_{v\infty} \operatorname{tg}^2 \beta_\infty}{2} S_k \right] + \delta T, \quad (3.4a)$$

$$N_i = Q_{xi} \cdot \omega_0 = \rho V_0^3 \pi R_\infty^2 \frac{\operatorname{tg} \beta_\infty}{\lambda_3} m_{v\infty} \left[(1 + m_{v\infty})(1 - r_r^2) + \frac{m_{v\infty} \operatorname{tg}^2 \beta_\infty}{2} \mathfrak{z} + (v + m_{v\infty} \mu) \right],$$

where S_k is a quantity characterizing the pressure field induced by the vortex system far behind the propeller, given by the formula

$$S_k = 2 \ln r_r - \frac{1}{2} (\mathfrak{z} + \chi + \rho) + (1 - r_r^2) + \frac{2 \Delta v_0 (1 - r_r^2)}{\sin^2 \beta_\infty} + \frac{1 + 2 \Delta v_0}{\sin^2 \beta_\infty} v - \frac{1}{\operatorname{tg}^2 \beta_\infty} \mu - \xi. \quad (3.5)$$

Here $\mathfrak{z}$, χ , ρ are integral discriminants of the induced velocity field concerning the axial, circumferential and radial induced velocity component [6], respectively, and v , μ , ξ are

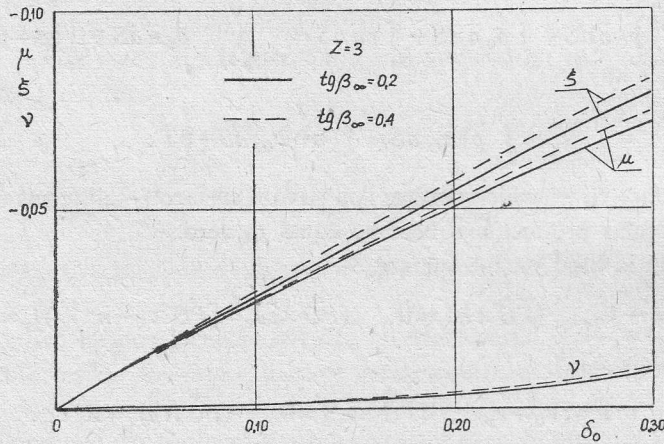


Fig. 3

quantities resulting from a substitution of the actual form of A for radii within the tip vortex singularity (Fig. 2).

These quantities are of the following form

$$v = 2 \int_1^{1+r_{w2}} A r dr + 2 \int_{1-r_{w1}}^1 (A-1) r dr, \quad (3.6a)$$

$$\mu = 2 \int_1^{1+r_{w2}} A^2 r dr + 2 \int_{1-r_{w1}}^1 (A^2-1) r dr, \quad (3.6b)$$

$$\xi = 2 \int_1^{1+r_{w2}} A^2 \frac{dr}{r} + 2 \int_{1-r_{w1}}^1 (A^2-1) \frac{dr}{r}, \quad (3.6c)$$

where r_{w1} , r_{w2} are radii of the tip vortex singularity as measured inwards and outwards, respectively (a vortex singularity has, in general, the cross-section differing from a circle).

An exemplary diagram of coefficients ν , μ and ζ for a vortex system consisting of $Z=3$ tip vortices has been presented in Fig. 3.

The subscript i in the symbols designating the thrust, torque and power has been added to emphasize that these quantities had been determined with viscous forces and a wake in the velocity and pressure field over the surface S_3 , associated with these forces, omitted.

4. Correction for the pressure field induced at the propeller hub

The propeller thrust as given by formulae (3.4a) and (3.5) is related to external forces outside of the singularities of the system of concentrated vortices. Therefore, if the screw propeller under consideration has a hub, at one end of which the pressure will be close to the undisturbed flow pressure p_0 , while at the other end a cavitation singularity of axial vortex is attached, having inside the pressure of critical value p_{kr} , than an additional force acting on the hub will appear. The force, which is not taken into account in formulae (3.4a) and (3.5) is opposite to the propeller thrust; it can be expressed as

$$\begin{aligned} \Delta T_1 &= - \int_{S_{pos}} p_{kr} dS + \delta T = \int_{S_{3os}} p_{kr} dS + \delta T = \int_{S_{3os}} (p_{kr} - p_0) dS, \\ \Delta T_1 &= -\pi R_r^2 (p_0 - p_{kr}) = -\rho V_0^2 \pi R_\infty^2 \frac{1}{2} \sigma_0 r_r^2. \end{aligned} \quad (4.1)$$

The term $\frac{1}{2} \sigma_0 r_r^2$ can be determined directly from formula (2.6). Hence the additional force acting on the hub can be determined with the aid of the same parameters that describe the propeller thrust and torque in formulae (3.4) and (3.5)

$$\Delta T_1 = -\rho V_0^2 \pi R_\infty^2 \frac{m_{v\infty}^2 \operatorname{tg}^2 \beta_\infty}{2} \left(1 - r_r^2 - \frac{2 \Delta v_0 r_r^2}{\sin^2 \beta_\infty} \right). \quad (4.2)$$

If the conditions are such that an axial vortex may develop (e.g. in model investigations of free propeller in a cavitation tunnel with the water inflow from the propeller shaft side i.e. with the propeller mounted as a pushing one), and the more so if such a vortex has been observed during investigations the above mentioned thrust correction must be applied.

If the water inflow is from the side opposite to the propeller shaft side (a pulling propeller — typical case of open water investigations in a tank) there are two possibilities:

a) the radius r_r of the axial vortex singularity is smaller than the propeller shaft radius $r_w = R_w/R_\infty$,

b) the radius r_r of the axial vortex singularity is larger than r_w .

If $r_w > r_r$ than the radius r_r of the axial vortex singularity should be substituted in formulae (3.4) and (3.5) by the propeller shaft radius r_w ; the additional sucking force ΔT at the propeller will be equal to zero.

If $r_w < r_r$ formulae (3.4) and (3.5) remain unchanged, and the additional sucking force at the hub will differ from ΔT_1 by a quantity taking into account the „dead” cross-section

area

$$\Delta T_2 = -\pi(R_r^2 - R_w^2)(p_0 - p_{kr}) = \Delta T_1 \left(1 - \frac{r_w^2}{r_r^2}\right). \quad (4.3)$$

Very often such bodies as rudders or streamlined hub extensions disturb, due to their location directly downstream of the propeller, the mechanism of formation of the system of vortices flowing off the propeller at the hub into a single axial having a concentrated form with a cavitation singularity.

If this is the case then a system of Z vortices distributed over a cylinder having the radius approximately equal to the hub radius would be a more adequate model describing the pressure field at the hub. As a rule the pitch of helices of the vortex system at a hub is very large. Therefore a system of straightlined vortices [5] can describe the pressure field for such system fairly well. The correction for an additional force acting on the hub can be suggested based on the results of analysis of the velocity and pressure fields for a straightlined vortex system [5]. Namely the radius r_r of the axial vortex singularity in formulae (3.4) and (3.5) should be substituted by the hub radius $r_p = R_p/R_\infty$; besides, the term $2\ln r_r$ in formula (3.5) should be substituted by

$$2\ln\left(r_p + \frac{\delta_0}{Z}\right) + \frac{2}{Z} \left[1 + \frac{\delta_0^2}{\left(r_p + \frac{\delta_0}{Z}\right)^2}\right] \ln \frac{\delta_0}{r_p + \frac{\delta_0}{Z}}.$$

The problem of accurate determination of the sucking force on the hub is of high importance in the propeller theory as well as in the domain of model investigations. This is evident especially in a comparative analysis of results of model investigations carried out for various positions of the propeller shaft relative to the direction of the water inflow to the propeller.

If a typical way of mounting a propeller model on shaft for open-water investigations in a tank (pulling propeller) is regarded as the standard propeller mounting than the results of investigations obtained in a cavitation tunnel with a model mounted in a different manner (pushing propeller) ought to be corrected. The difference of the propeller thrust will be obtained from a comparison of corresponding forces acting on the hub. For $r_r < r_w$ the correction is

$$\begin{aligned} \Delta T_3 = & \int_0^{2\pi R_p} \int_{R_r} (p_0 - p) R dR d\varphi + \pi R_r^2 (p_0 - p_{kr}) - \\ & - \int_0^{2\pi R_p} \int_{R_r} (p_0 - p) R dR d\varphi - \pi (R_r^2 - R_w^2) (p_0 - p_{kr}), \end{aligned} \quad (4.4)$$

$$\Delta T_3 = \pi R_w^2 (p_0 - p_{kr}) = \rho V_0^2 \pi R_\infty^2 \frac{r_w^2}{r_r} \frac{1}{2} \sigma_0 r_r^2 = \left(\frac{r_w}{r_r}\right)^2 \Delta T_1.$$

For $r > r_w$

$$\Delta T_4 = \rho V_0^2 \pi R_\infty^2 \frac{m_{v\infty}^2 \operatorname{tg}^2 \beta_\infty}{2} 2\ln \frac{r_w}{r_r} + \Delta T_3. \quad (4.5)$$

Comparative investigations have been carried out in order to determine how large are differences in the thrust value for different methods of mounting a model on a propeller shaft. Model propellers have been tested each for two different ways of mounting on the shaft (as a pulling propeller and as a pushing one). The same rig for open-water tests in tank has been used. In order to reduce possible disturbances of the water inflow to the propeller, and also to maximize the thrust difference value the tests have only been made for the inflow velocity equal to zero (bollard condition tests). For propellers having large pitch coefficients ($P/D=1.5$) differences up to 15 per cent of the value of thrust coefficient K_T have been observed, with the torque coefficient K_Q of practically constant value. Corrections as specified above proved to be adequate qualitatively as well as quantitatively.

It is evident that the value of the additional force at the hub depends strongly on the value of the radius r_r of the axial vortex singularity; thus it also depends indirectly on the cavitation number value.

Very often model investigations in a cavitation tunnel are being carried out at cavitation

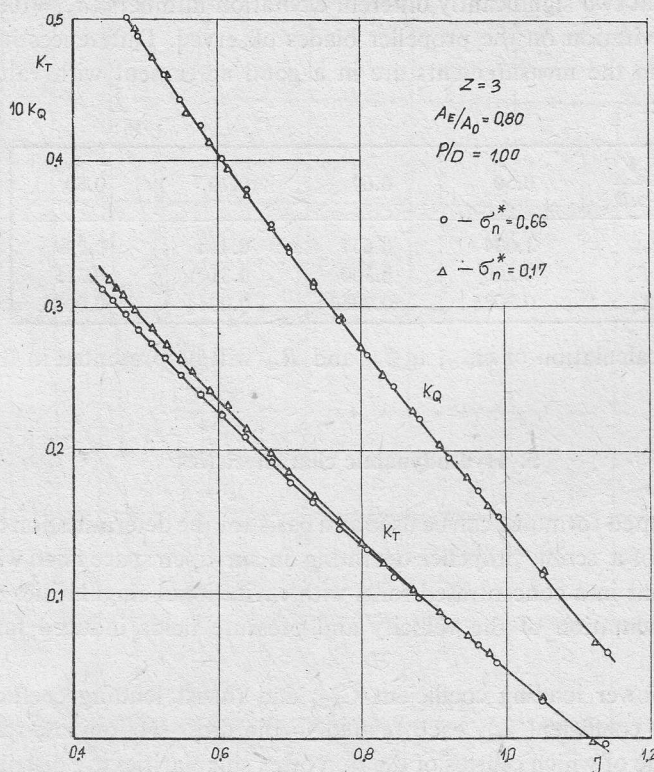


Fig. 4

numbers of largely different values without observing any signs of cavitation on the propeller blades, but with the size of the cavitation singularity of the axial vortex distinctly changed. A thrust correction corresponding to this effect can be determined basing on the difference of

the value of the force on the hub for two values of r_r

$$\begin{aligned} \Delta T_5 &= \int_0^{2\pi} \int_{R_{r1}}^{R_p} (p_0 - p) R dR d\varphi + \pi R_{r1}^2 (p_0 - p_{kr}) - \\ &\quad - \int_0^{2\pi} \int_{R_{r2}}^{R_p} (p_0 - p) R dR d\varphi - \pi R_{r2}^2 (p_0 - p_{kr}) = \\ &= \rho V_0^2 \pi R_\infty^2 \frac{m_{v\infty}^2 \operatorname{tg} \beta_\infty}{2} \left[2 \ln \frac{R_{r2}}{R_{r1}} + (r_{r2}^2 - r_{r1}^2) + \frac{2\Delta v_0}{\sin^2 \beta_\infty} \right], \quad (4.6) \\ \Delta T_5 &\cong \rho V_0^2 \pi R_\infty^2 \frac{m_{v\infty}^2 \operatorname{tg} \beta_\infty}{2} \ln \frac{\sigma_1}{\sigma_2}. \end{aligned}$$

It has been proved in many experiments that such correction must be applied to the results of tunnel investigations. Fig. 4 shows characteristics of a propeller tested in a cavitation tunnel at the Institute of Fluid-Flow Machinery, Polish Academy of Sciences (pushing propeller) at two significantly different cavitation numbers ($\sigma_{n1}^* = 0.66$, $\sigma_{n2}^* = 0.17$) but still with no cavitation on the propeller blades observed. Differences in the value of K_T as resulting from the measurements are in a good agreement with values calculated from formula (4.6):

$J = \frac{V}{nD}$	0.50	0.60	0.70	0.80
$m_{v\infty}$	1.004	0.657	0.444	0.239
$\operatorname{tg} \beta_\infty$	0.290	0.300	0.310	0.315
ΔK_T	0.0085	0.0060	0.0044	0.0025

The method of calculation of $m_{v\infty}$, $\operatorname{tg} \beta_\infty$ and R_∞ will be presented in the subsequent part of this paper.

5. Hydrodynamic characteristics

The above presented formulae can be used as a basis for the determination of hydrodynamic characteristics of a screw propeller operating in an open space filled with an inviscid liquid. Vortex systems in a concentrated form with cavitation singularities constitute the characteristic representation of the velocity and pressure fields induced far behind this propeller.

Dimensionless power loading coefficient C_{N3i} and thrust loading coefficient C_{T3i} , as well as the advance coefficient λ_∞ , each related to the cross-section πR_∞^2 of the circular cylinder, over the side of which centres of the tip vortex singularities are distributed, may be obtained directly from formulae (3.4), (3.5) and (2.3)

$$C_{N3i} = \frac{N_i}{\frac{1}{2} \rho V_0^3 \pi R_\infty^2} = 2 \frac{\operatorname{tg} \beta_\infty}{\lambda_\infty} m_{v\infty} \left[(1 + m_{v\infty})(1 - r_r^2) + \frac{m_{v\infty} \operatorname{tg}^2 \beta_\infty}{2} \mathcal{J} + (v + m_{v\infty} \mu) \right], \quad (5.1)$$

$$C_{T3i} = \frac{T_i}{\frac{1}{2} \rho V_0^3 \pi R_\infty^2} = \frac{\lambda_\infty}{\operatorname{tg} \beta_\infty} C_{N3i} + \frac{m_{v\infty}^2 \operatorname{tg}^2 \beta_\infty}{2} S_k, \quad (5.2)$$

$$\lambda_{\infty} = \frac{V_0}{\omega_0 R_{\infty}} = \frac{\operatorname{tg} \beta_{\infty}}{1 + \frac{m_{v\infty}}{2} (1 + \operatorname{tg}^2 \beta_{\infty}) (1 + 2\Delta v_0)}, \quad (5.3)$$

$$\eta_i = \frac{T_i V_0}{N_i} = \frac{C_{T3i}}{C_{N3i}} = \frac{\lambda_{\infty}}{\operatorname{tg} \beta_{\infty}} \left[1 + \frac{\frac{m_{v\infty} \operatorname{tg}^2 \beta_{\infty}}{2} S_k}{(1 + m_{v\infty})(1 - r_r^2) + \frac{m_{v\infty} \operatorname{tg}^2 \beta_{\infty}}{2} \beta + (v + m_{v\infty} \mu)} \right]. \quad (5.4)$$

The quantity S_k is given by (3.5) with a correction, if necessary, for an additional sucking force at the hub ΔT .

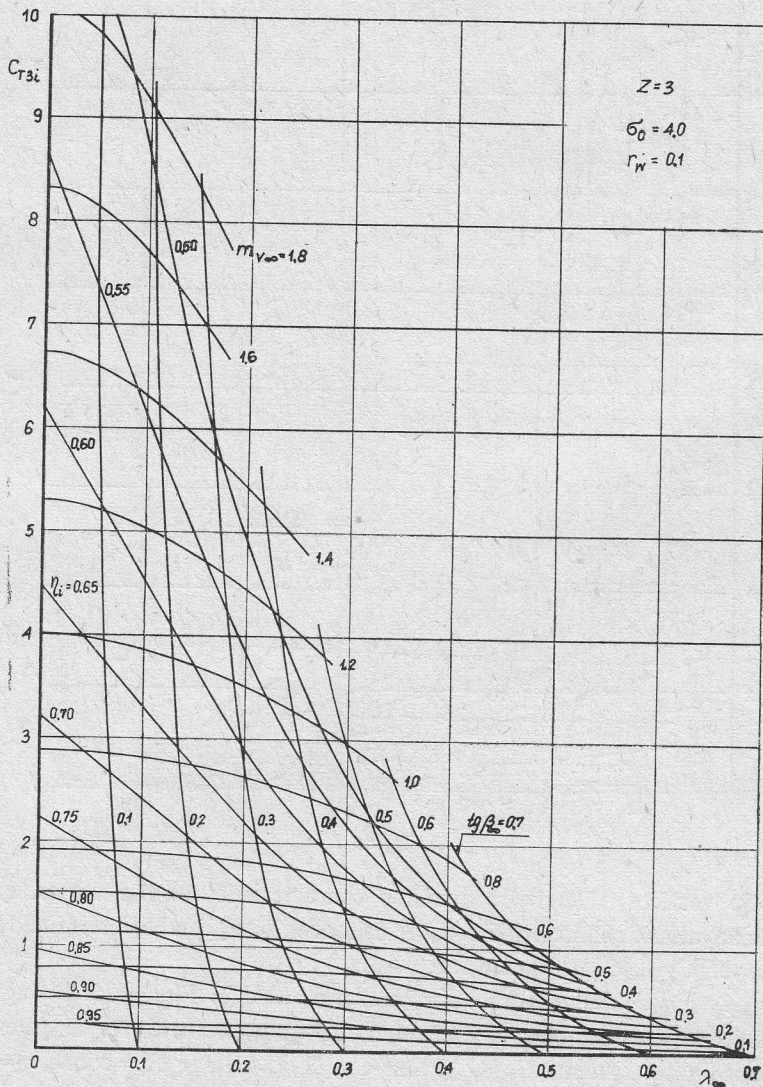


Fig. 5

When a system of concentrated vortices characterized by certain parameters (Z , $\text{tg } \beta_\infty$, R_∞ , $m_{v\infty}$) is defined, the cavitation number σ_0 characterizing the undisturbed flow is known, and known are coefficients, corresponding to this vortex system, characterizing nonuniformity of the induced velocity field (β , χ , ρ , ν , μ and ζ) as well as the proper velocities

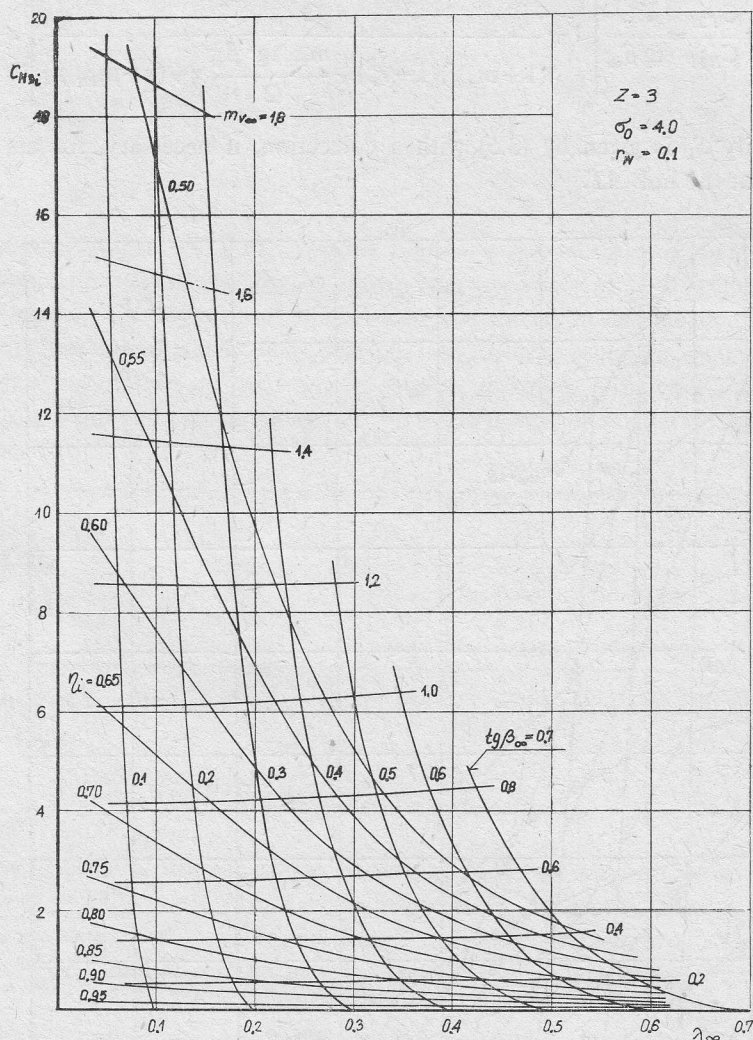


Fig. 6

Δv_0 of the tip vortices, then hydrodynamic characteristics of the vortex system, and indirectly hydrodynamic characteristics of the propeller itself, can be determined for various sets of values of C_{T3i} , C_{N3i} , η_i and λ_∞ .

Examples of such characteristics have been presented in Figures 5 through 8.

Figs. 5 and 6 show examples of hydrodynamic characteristics for a system of $Z=3$ vortices in coordinates systems $C_{T3i}(\lambda_\infty)$ and $C_{N3i}(\lambda_\infty)$, with $\text{tg } \beta_\infty = \text{const}$, $\eta_i = \text{const}$ and $m_{v\infty} = \text{const}$.

The same results have been presented in Figs. 7 and 8 in a logarithmic scale. This is the system in which, as a rule, functions $C_{Ti}(\lambda)$ and $C_{Ni}(\lambda)$ have been plotted, for $\eta_i = \text{const}$, for systems of free vortices developed into a helical surface of constant pitch and radial circulation distribution complying with the Betz postulate (Kramer diagrams).

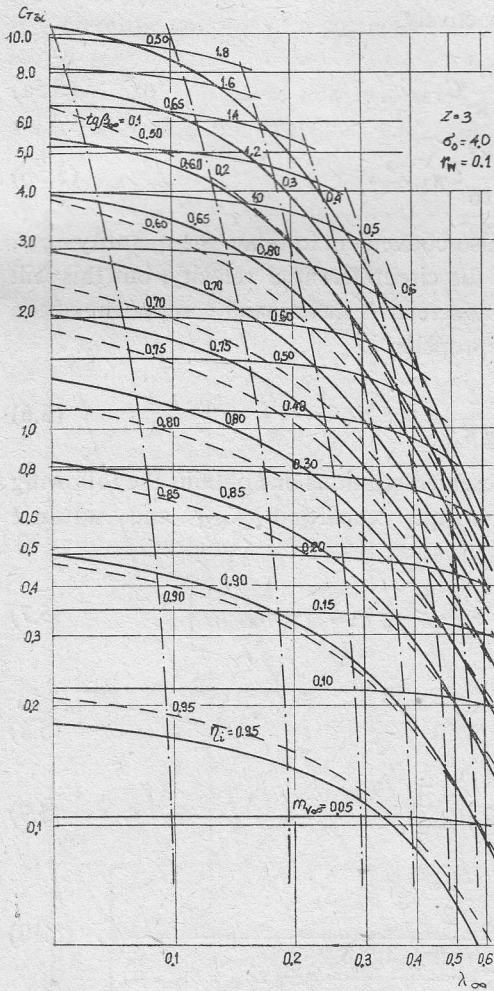


Fig. 7

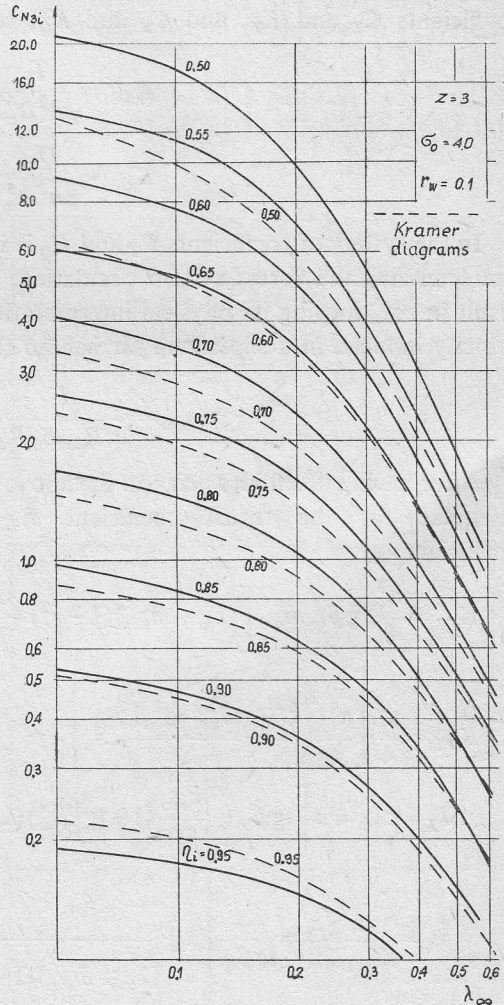


Fig. 8

In order to compare so different vortex systems from the standpoint of the efficiency diagrams of C_{Ni} plotted versus λ based on Kramer diagrams have been shown in Figs. 7 and 8. The comparison shows that vortex systems fulfilling the Betz postulate seem to have at higher loads poorer performance than a concentrated form of velocity. This fact calls for a revision of commonly adopted opinion as to the optimum load distribution over propeller blades.

The thrust loading coefficient C_{Ti} and the power loading coefficient C_{Ni} do not depend explicitly on the rotative speed of the propeller, ω_0 . Sometimes this may be convenient, however more natural coefficients characterizing the operation of a screw propeller are the thrust coefficient K_T and torque coefficient K_Q . Hydrodynamic characteristics of a propeller may be presented in terms of K_T and K_Q in the whole range of operating conditions of the propeller, including the bollard conditions tests. There is a strict relation between the coefficients C_T and K_T , and C_N and K_Q , respectively

$$K_{T3i} = \frac{T_i}{\rho n^2 D_\infty^4} = \frac{\pi^3}{8} C_{T3i} \lambda_\infty^2, \quad (5.5a)$$

$$K_{Q3i} = \frac{Q_{xi}}{\rho n^2 D_\infty^5} = \frac{\pi^3}{16} C_{N3i} \lambda_\infty^3. \quad (5.5b)$$

In the system of coefficients K_T and K_Q it is also convenient to relate the quantity $m_{v\infty}$, characterizing the vortex system circulation, to the circumferential velocity, but this will result in $m_{v\infty}$ loosing its physical interpretation as a relative mean axial component of the velocity induced in a slipstream far behind the propeller

$$m_{n\infty} = \frac{\Gamma}{4\pi R_\infty \omega_0 R_\infty \operatorname{tg} \beta_\infty} = m_{v\infty} \lambda_\infty. \quad (5.6)$$

After having taken into account the above presented relations one obtains the following formulae for the thrust coefficient K_{T3i} , torque coefficient K_{Q3i} and advance coefficient λ_∞

$$K_{Q3i} = \frac{\pi^3}{8} \operatorname{tg} \beta_\infty m_{n\infty} \left[(\lambda_\infty + m_{n\infty})(1 - r_r^2) + \frac{m_{n\infty} \operatorname{tg}^2 \beta_\infty}{2} \mathcal{J} + (v\lambda_\infty + m_{n\infty} \mu) \right], \quad (5.7)$$

$$K_{T3i} = \frac{\pi^3}{4} \left[\frac{8}{\pi^3} K_{Q3i} + \frac{m_{n\infty}^2 \operatorname{tg}^2 \beta_\infty}{2} S_k \right], \quad (5.8)$$

$$J_3 = \pi \lambda_\infty = \pi \left[\operatorname{tg} \beta_\infty - \frac{m_{n\infty}}{2} (1 + \operatorname{tg}^2 \beta_\infty) (1 + 2\Delta v_0) \right], \quad (5.9)$$

$$\eta_i = \frac{K_{T3i}}{K_{Q3i}} \frac{J_3}{2\pi} = \frac{\lambda_\infty}{\operatorname{tg} \beta_\infty} \left[1 + \frac{\frac{m_{n\infty}}{2} \operatorname{tg}^2 \beta_\infty S_k}{(\lambda_\infty + m_{n\infty})(1 - r_r^2) + \frac{m_{n\infty} \operatorname{tg}^2 \beta_\infty}{2} \mathcal{J} + (v\lambda_\infty + m_{n\infty} \mu)} \right]. \quad (5.10)$$

There also will be a slight change of the formula for the axial vortex singularity radius

$$r_r = \frac{m_{n\infty} \operatorname{tg} \beta_\infty}{\sqrt{\sigma_{n\infty}^* + m_{n\infty}^2 \operatorname{tg}^2 \beta_\infty \left(1 + \frac{2\Delta v_0}{\sin^2 \beta_\infty} \right)}} = \frac{\delta_0}{\sqrt{1 + \delta_0^2 \left(1 + \frac{2\Delta v_0}{\sin^2 \beta_\infty} \right)}}, \quad (5.11)$$

where

$$\sigma_{n\infty}^* = \frac{p_0 - p_{kr}}{\frac{1}{2}\rho (\omega_0 R_\infty)^2} = \sigma_0 \lambda_\infty^2. \quad (5.12)$$

Examples of hydrodynamic characteristics of vortex systems, plotted in the above reference system, have been presented in Fig. 9.

Nearly all of the above presented formulae have a slight shortcoming, consisting in a dependence on the dimensions of a cross-section of slipstream far behind the propeller. In order to fully make use of these formulae one should know the relation between the radii of cross-sections of slipstream far behind the propeller, R_∞ , and at the screw disk, R_0 . Here the concept of a coefficient of contraction (narrowing) of slipstream k_α [7] may be of use.

The coefficient k_α of contraction of slipstream is defined as a ratio of cross-section areas far behind the propeller, πR_∞^2 , and at the screw disk, πR_0^2 . With the aid of this

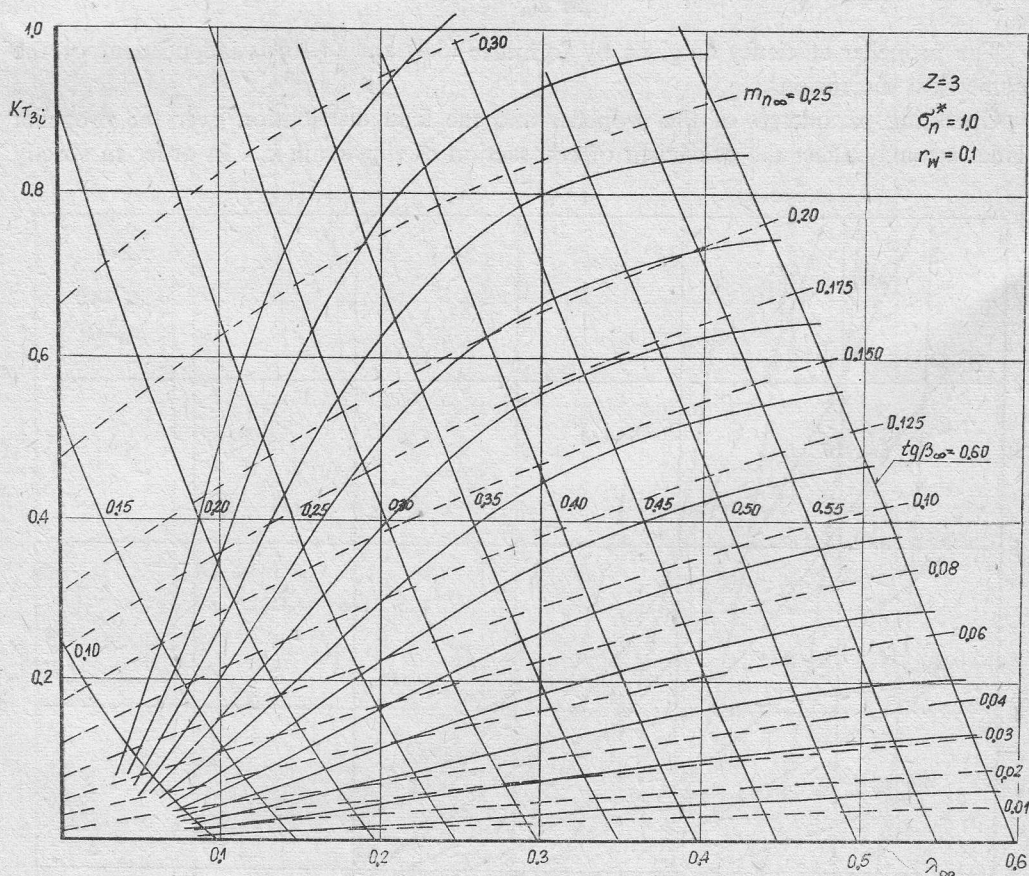


Fig. 9

coefficient one can obtain geometrical quantities characterizing the screw propeller itself, basing on those related to the size of slipstream far behind the propeller

$$k_\alpha = \frac{\pi R_\infty^2}{\pi R_0^2} = \left(\frac{R_\infty}{R_0} \right)^2, \quad (5.13)$$

$$C_{N2i} = \frac{N_i}{\frac{1}{2}\rho V_0^3 \pi R_0^2} = k_\alpha C_{N3i}, \quad (5.14)$$

$$C_{T2i} = \frac{T_i}{\frac{1}{2}\rho V_0^2 \pi R_0^2} = k_\alpha C_{T3i}, \quad (5.15)$$

$$J_2 = \sqrt{k_\alpha} J_3, \quad (5.16)$$

$$K_{T2i} = \frac{T_i}{\rho n^2 D_0^4} = k_\alpha^2 K_{T3i}, \quad (5.17)$$

$$K_{Q2i} = \frac{Q_i}{\rho n^2 D_0^5} = k_\alpha^2 \sqrt{k_\alpha} K_{Q3i}. \quad (5.18)$$

The propeller efficiency as given by formulae (5.4) and (5.10) is independent of the geometrical dimensions.

Operating parameters of the propeller and the load distribution over the propeller blades strongly affect the coefficient of contraction of slipstream k_α . In order to strictly

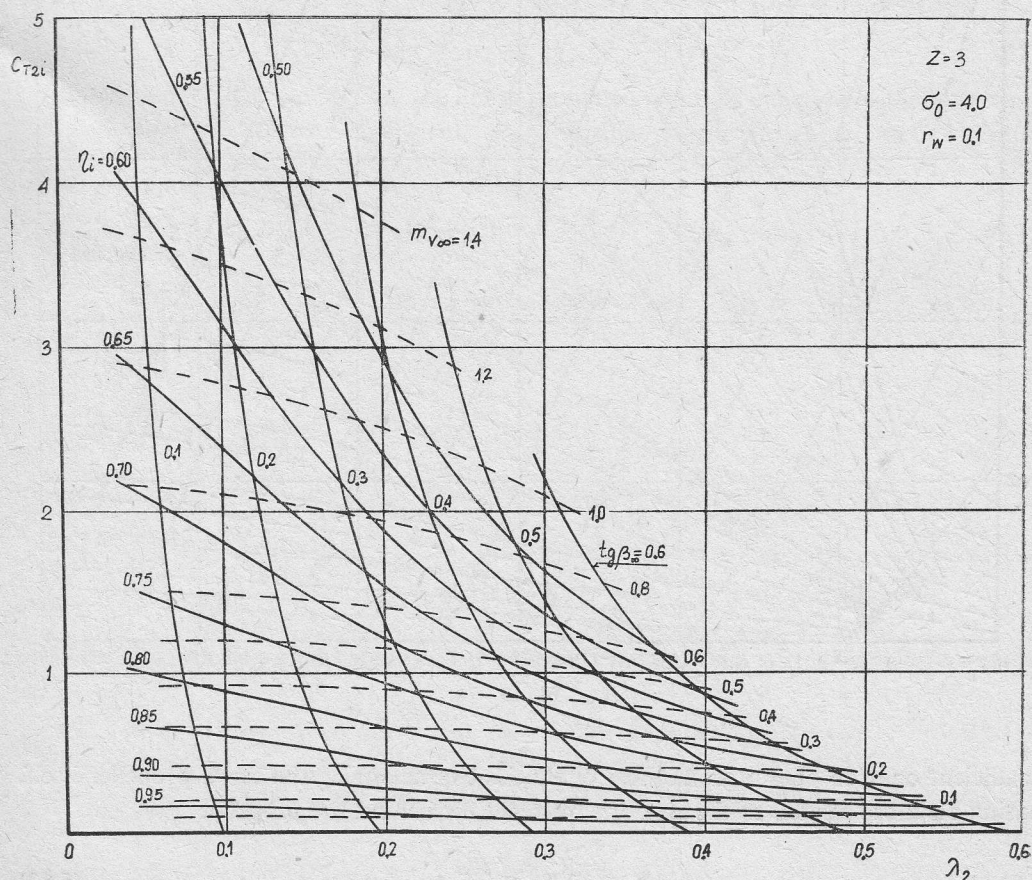


Fig. 10

determine k_α one should solve the problem of flow on the propeller itself and in its vicinity, taking into account the process of development of a concentrated form of vorticity.

The contraction coefficient may be, in general, presented in the form of a product [7]

$$k_{\alpha} = k_m k_{\gamma},$$

where k_m expresses the narrowing of an assumed boundary of the slipstream and is related to the rise of the axial component of induced velocities. It can be defined by means of

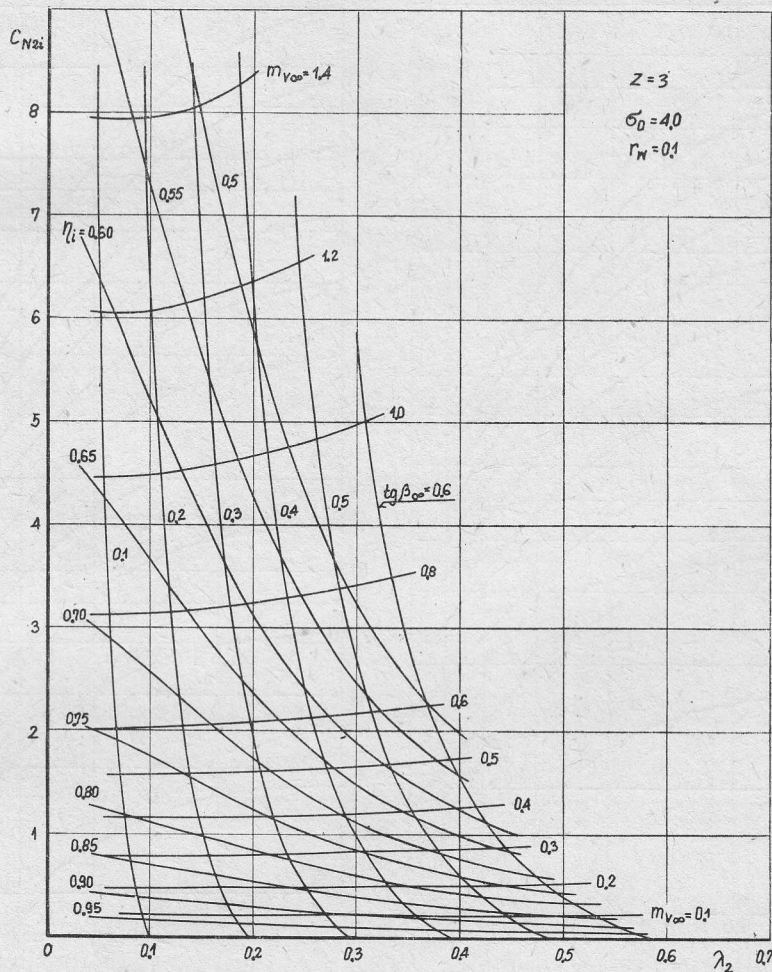


Fig. 11-

the same formula as used for an ideal propeller ($Z=\infty$) for which a radially constant circulation distribution is assumed

$$k_m^2 = \frac{\left(1 + \frac{m_{v\infty}}{2}\right)(1 - r_p^2)}{(1 + m_{p\infty})(1 - r_r^2)}. \quad (5.19)$$

The quantity k_γ expresses the narrowing of vortex lines caused by the deformation (coiling up) of the vortex surface and development of a concentrated form of vorticity. Difficulties arising in connection with attempts to determine k_γ are responsible for the difficulties in the determination of k_α .

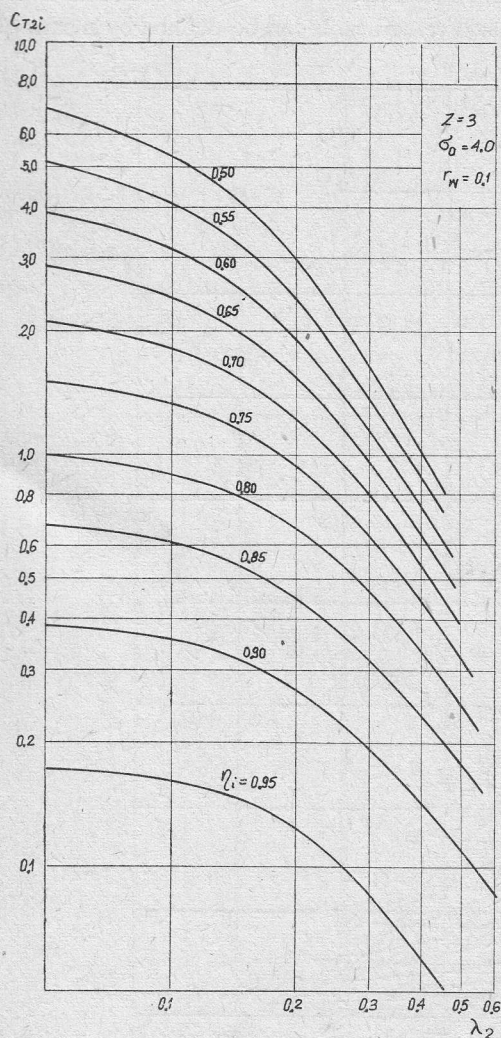


Fig. 12

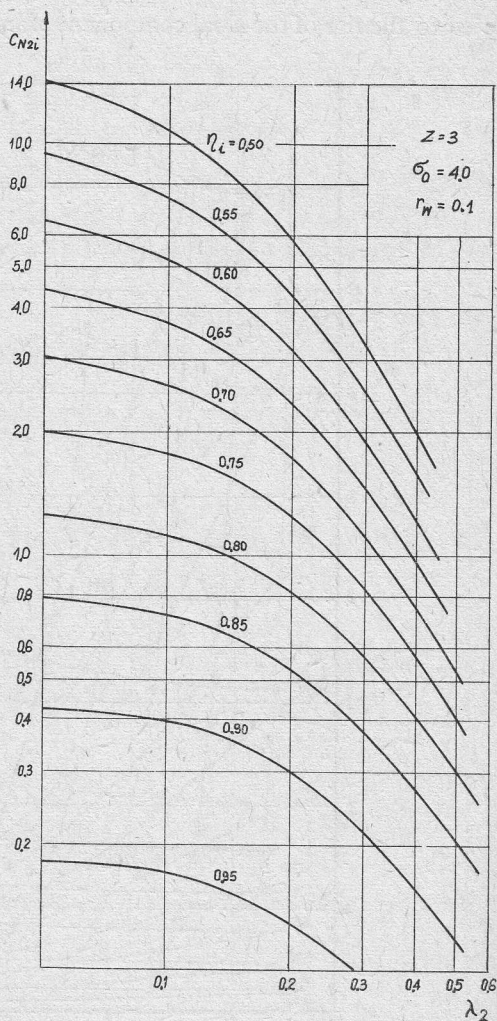


Fig. 13

Taking into account a contraction of the slipstream results in an alteration of hydrodynamic characteristics of a propeller. This may be observed by letting $k_\alpha = k_m$.

Figures 10 through 14 show hydrodynamic characteristics related to the screw disk cross-section area through the relation (5.19) that have been plotted for the same vortex systems as presented in Figs. 5 through 7. From these figures it is evident that leaving the

contraction of slipstream out of account (which is the common practice in the propeller theory) may lead to quite large errors in the evaluation of the induced velocity field as corresponding to external forces creating this field. Such errors will also affect the evaluation of relations between the respective coefficients characterizing the operation of a screw propeller. The errors and discrepancies would be even more clearly marked if the quantity k_γ has been taken into account.

The formulae and relations as presented above can not be fully utilized in the theory and for practical purposes because a complete quantitative information on k_γ is unavailable. However, in the theory of propellers there are quantities that characterize the propeller

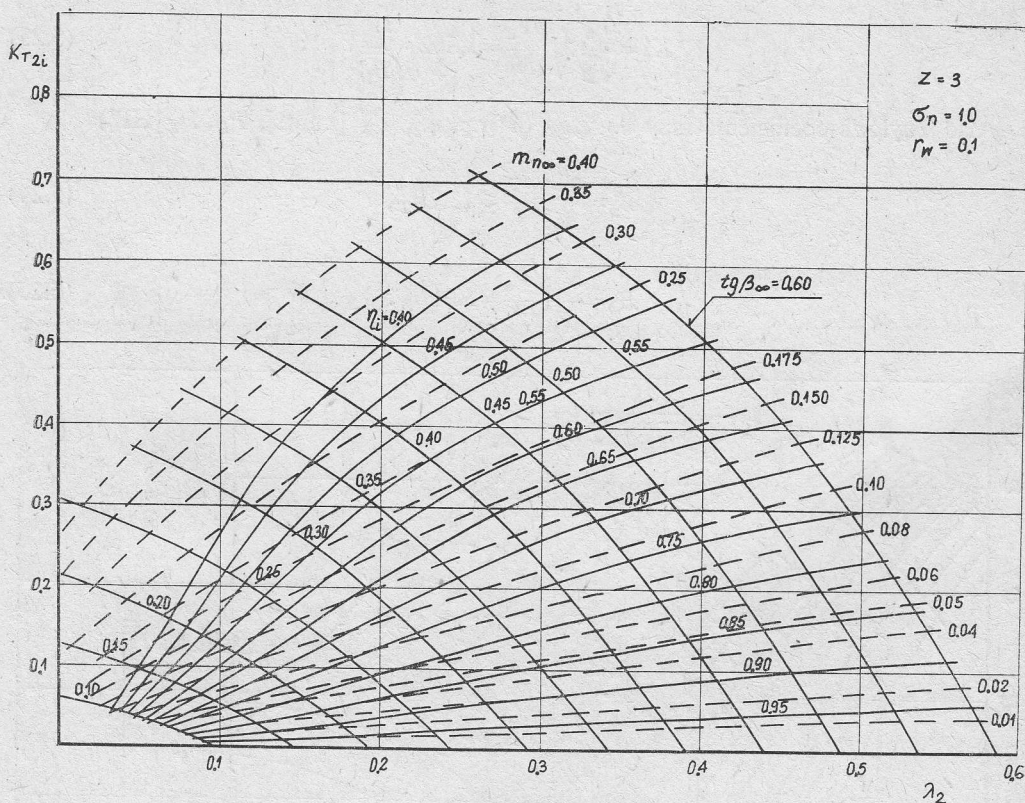


Fig. 14

loading but do not contain terms characterizing the geometry of propeller wake. These are so called diameter-independent power loading and thrust loading coefficients.

The coefficients may be derived from one of the sets of coefficients, K_T , K_Q and J , or C_T , C_N and λ , by elimination of the radius or diameter of propeller or slipstream

$$\frac{K_T}{J^4} = \frac{T n^2}{\rho V_0^4} = B_T^2, \quad (5.20)$$

or

$$\frac{C_T}{\lambda^2} = 8\pi \frac{T n^2}{\rho V_0^2} = 8\pi B_T^2, \quad (5.21)$$

$$\frac{K_Q}{J^5} = \frac{Qn^3}{\rho V_0^5} = B_N^2,$$

$$\frac{C_N}{\lambda^2} = 8\pi \frac{Qn^3}{\rho V_0^5} = 8\pi B_N^2.$$

Such coefficients, K_n and K'_n , used by authors from the Soviet Union have the form

$$K_n = \frac{V_0}{\sqrt{n}} \sqrt[4]{\frac{\rho}{T}} = \frac{1}{\sqrt{B_T}}, \quad (5.22)$$

$$K'_n = \frac{V_0}{\sqrt{n}} \sqrt[4]{\frac{\rho V_0}{N}} = 1.862 \frac{1}{\sqrt{B_N}}. \quad (5.23)$$

Also Taylor's coefficients may be used (n^* [r.p.m.], V_0^* [knots], P_D , P_T [HP])

$$B_u = \frac{n^* \sqrt{P_T}}{V_0^{*2.5}} = 13.2 B_T, \quad (5.24)$$

$$B_p = \frac{n^* \sqrt{P_n}}{V_0^{*2.5}} = 33.1 B_N. \quad (5.25)$$

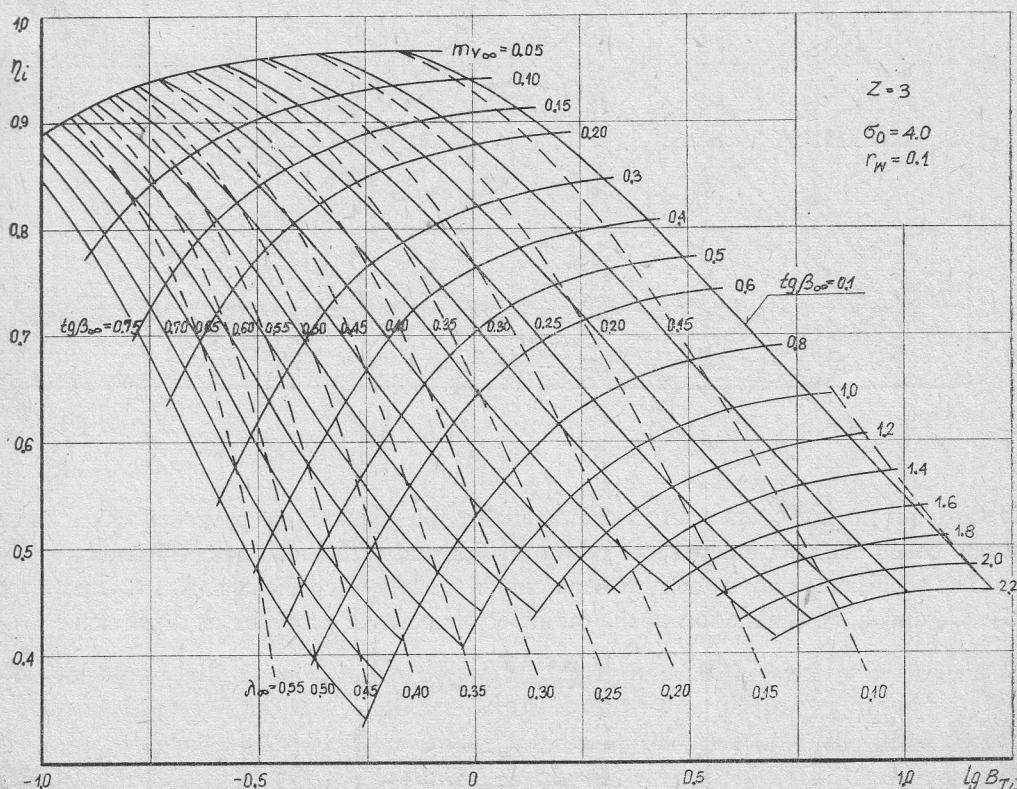


Fig. 15

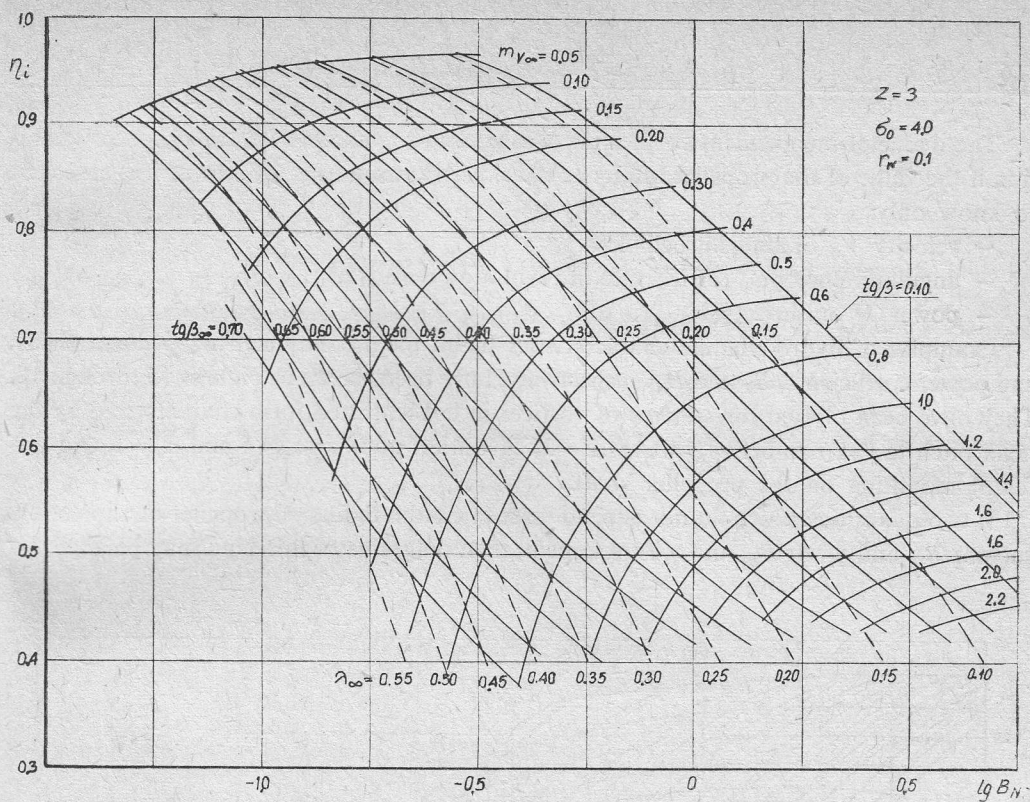


Fig. 16

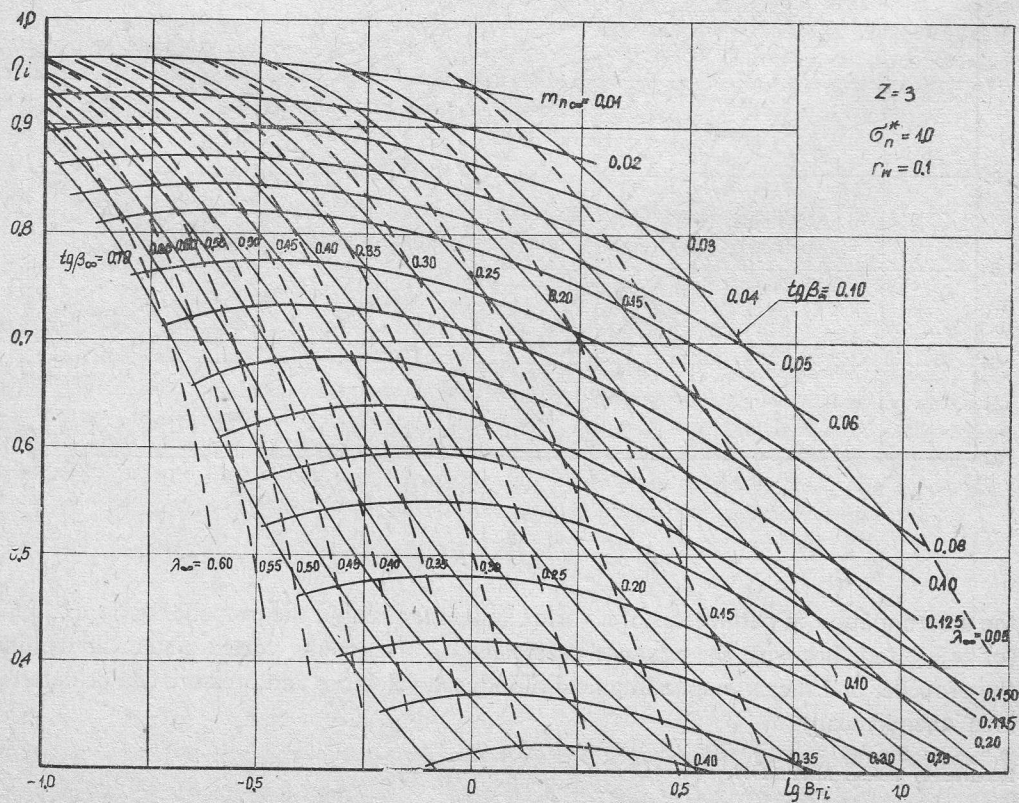


Fig. 17

The diameter-independent coefficients can be used to characterize the propeller operation if the value of the propeller radius or the slipstream radius is unknown; it is sufficient to know only:

- velocity V_0 of undisturbed flow,
- angular velocity ω_0 (or the rate of revolution $n = \omega_0/2\pi$),
- power N or thrust T .

Examples of hydrodynamic characteristics based on the diameter-independent thrust and power coefficients, B_T and B_N , respectively, have been shown in Figures 15 through 18. They have been plotted for systems of vortices in concentrated form characterized by the helix pitch $\tan \beta_\infty$, number of vortices $Z=3$, cavitation number $\sigma_0=4$ and $\sigma_n^*=1$, and different mounting on the propeller shaft.

If two quantities which do not depend directly on the radius of propeller or slipstream, such as B_{Ti} and η_i , or B_{Ni} and η_i , are known, then diagrams of this kind may be used for

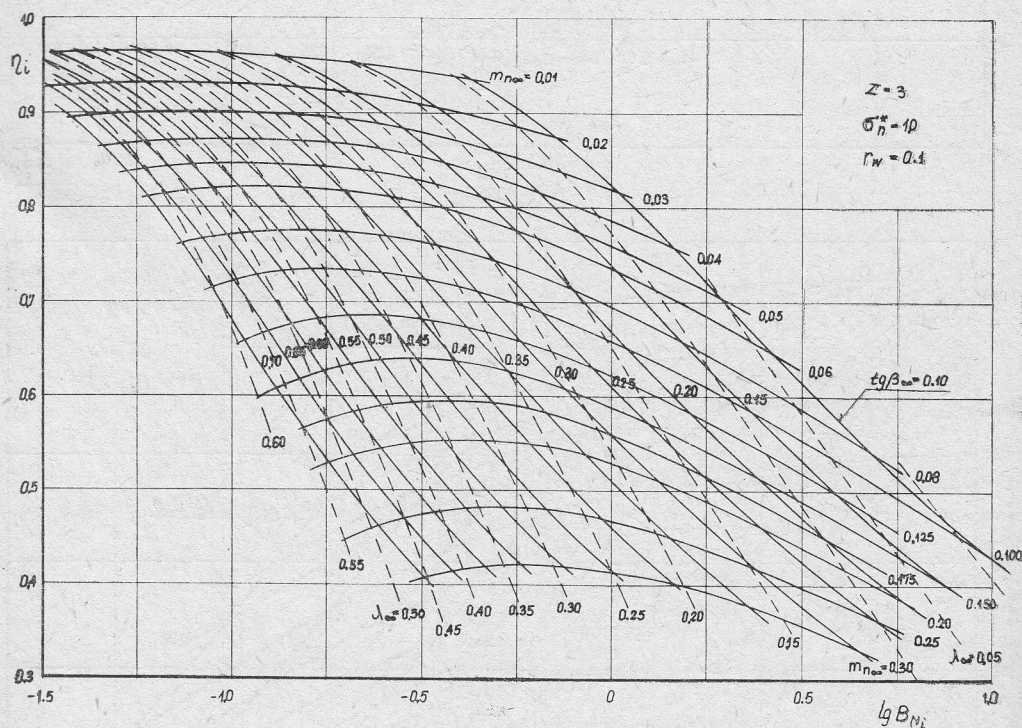


Fig. 18

the determination of parameters of a vortex system far behind the propeller $\tan \beta_\infty$ and $m_{v\infty}$ (or $m_{n\infty}$), together with the advance coefficient λ_∞ . Hence the vortex system far behind the propeller and the corresponding field of induced velocities and pressure can be determined unequivocally.

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Charakterystyki hydrodynamiczne układów wirów śrubowych o skończonych wymiarach jądra wirowego

Streszczenie

Na podstawie równań mechaniki płynów, określono zależności pomiędzy polem prędkości indukowanych daleko za pędnikiem, a siłami zewnętrznymi pola wywołanego pędnikiem śrubowym. Model obliczeniowy opiera się na układzie wirów skupionych z osobliwościami kawitacyjnymi jądra wirowego. Wyznaczone zależności umożliwiają określenie idealnych charakterystyk hydrodynamicznych pędnika śrubowego. W pracy podano przykłady takich charakterystyk dla wybranych układów wirów śrubowych. Charakterystyki przedstawiono w różnych układach odniesienia.

Гидродинамические характеристики винтовых вихрей характеризующихся конечными размерами вихревого ядра

Резюме

На основе уравнений механики жидкостей выведены зависимости между полем скоростей индуцированных далеко за винтом и внешними силами поля вызванного винтовым движителем. Расчетная модель основана на системе сосредоточенных вихрей с кавитационными особенностями вихревого ядра. Выведенные зависимости позволяют определить идеальные гидродинамические характеристики винтового движителя. В работе даны примерные характеристики для избранных систем винтовых вихрей. Характеристики представлены в различных системах отнесения.