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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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Generalized Characteristics of Electrical Conductance Film Thickness Gauges

Calibration of miniature electrical-conductivity gauges for measuring thickness of water film flowing along a wall as a function of position and time is discussed. The results for several gauges at various temperatures are combined into a generalized plot where the gauge geometry and water conductivity alone are variables. Experimental scatter of data from all gauges at all temperatures tested is reasonably small when presented in this format.

1. Introduction

The measurement of thin liquid film thickness is important in a number of practical applications. It allows us to study complex two-phase flow phenomena, which occur in a whole variety of industrial processes. Liquid films occur, for instance, on the surface of blades in the final stages of steam turbines and similarly on duct surfaces of a boiling channel. The measurement of liquid film thickness is of importance in a number of chemical engineering applications as well as in safety considerations of liquid-sodium or pressurized water-cooled reactors. Thus various methods have been discussed and presented in the literature, such as radioactive absorption methods, capacitance probes, needle contact devices, light absorption methods and electrical conductance probes [1, 2]. Usually, observation of thin liquid films must be accomplished without disturbing the film in any manner. For this reason the conductance probe, installed flush with the wall, is a most desirable method. This paper presents a simple theory for the circular conductance probe, a method for obtaining the general characteristics of such probes, as well as an example of our own calibration measurements.

2. Analysis

Consider the probe to consist of three concentric parts: a center conducting electrode of radius r_1, an insulating ring of which the outside radius is r_2, and finally a conducting infinite plate (Fig. 1). Electrical connections are made to the center electrode and the plate.

Assume a uniform electrically-conducting liquid film of thickness h covers the gauges. The insulating ring prevents any current flow other than through the liquid film.

The electrical potential distribution in the film is described by the Laplace equation

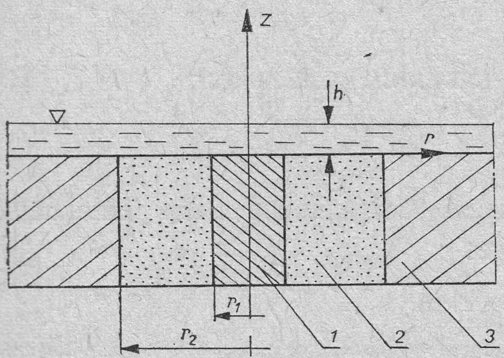

Fig. 1. Schematic representation of conductance probe

in cylindrical coordinates r, z (Fig. 1) i.e.,

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}=0, \tag{1}$$

where $u=u(r, z)$.

The boundary conditions are the following:

$$\begin{aligned} &\text{for} \quad z=0 \quad 0<r<r_1 && \frac{\partial u}{\partial z}=j/\sigma, \\ &\text{for} \quad z=h && \frac{\partial u}{\partial z}=0, \\ &\text{for} \quad r=0 && \frac{\partial u}{\partial r}=0, \\ &\qquad\quad r\to\infty && \frac{\partial u}{\partial r}=0, \end{aligned} \tag{2}$$

where σ – specific conductivity of liquid, j – density of electric current flow. The problem is two-dimensional, and the analytical solution of such a problem may be only approximate.

In order to simplify, we consider that the film is very thin, and the change of electric potential across the film is very small, so we can apply an average potential, $\bar{u}$

$$\bar{u}=\frac{1}{h}\int_0^h u\,dz. \tag{3}$$

If we apply operator $\bar{u}$ to the terms of eq. (1) we obtain

$$\frac{1}{h}\int_0^h \frac{\partial^2 u}{\partial r^2}\,dz=\frac{\partial^2}{\partial r^2}\left[\frac{1}{h}\int_0^h u dz\right]=\frac{\partial^2 \bar{u}}{\partial r^2},$$

$$\frac{1}{h}\int_0^h \frac{1}{r}\frac{\partial u}{\partial r}\,dz=\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{h}\int_0^h u dz\right]=\frac{1}{r}\frac{\partial \bar{u}}{\partial r}, \tag{4}$$

$$\frac{1}{h}\int_0^h \frac{\partial^2 u}{\partial z^2}\,dz=\frac{1}{h}\left[\frac{\partial u}{\partial z}\bigg|_h-\frac{\partial u}{\partial z}\bigg|_0\right]=\frac{-j}{h\sigma}.$$

Substituting eq. (4) into (1) and applying operator (3) and boundary conditions (2) for two coordinates, we obtain

$$\frac{\partial \bar{u}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{u}}{\partial r}-\frac{j}{h\sigma}=0. \tag{5}$$

Solving (5) for three domains $0<r<r_1$, $r_1<r<r_2$, $r>r_2$, and then combining solutions, regarding function as continuous, we obtain electric current i, flowing through the liquid film

$$i=\frac{4\pi\sigma h}{\left(\frac{r_2}{r_1}\right)^2-1}\,(u_2-u_1)=C_g h\sigma\Delta u, \tag{6}$$

where $i=\pi\sigma r_1^2 j$ – electric current, $u_1=u$ for $r=r_1$ – potential on first electrode, $u_2=u$ for $r=r_2$ – on second electrode, C_g – geometric constant of the gauge.

From eq. (6) for thin liquid films, the electric current, i, or the voltage output of the gauge, depends on film thickness gauge geometry, and liquid specific conductivity. Hewitt, et al. [1] found experimentally, for their conductance probe, that

$$C=2.918\sigma h, \tag{7}$$

where C is the film conductivity between the probe electrodes. Comparing eq. (6) with (7) we can find good qualitative agreement between simple theory and experiments.

The curve of conductance against film thickness is initially linear as shown theoretically (6) and experimentally (7). For higher film thicknesses, the conductance rises less rapidly with increasing thickness, as shown by our present experiments. We can assume the following relation for this range of thickness.

$$C=C_g\sigma f(h), \tag{8}$$

$f(h)$ – unknown function of film thickness.

The function of f can be easily established experimentally by calibration of the gauges. In the linear case we can obtain average factor C_g, which is equal to C/h. We can then apply this factor to eq. (8) in order to find $f(h)$. This method for finding characteristics of gauges will be explained using our present measurements.

3. Experiments

Experimental measurements of film thickness were performed using 4 gauges. Fig. 2 is typical with the diameter of the center electrode ~2.28 mm, and the outside diameter of insulating wire 4.76 mm. The water for the film thickness measurement was specially

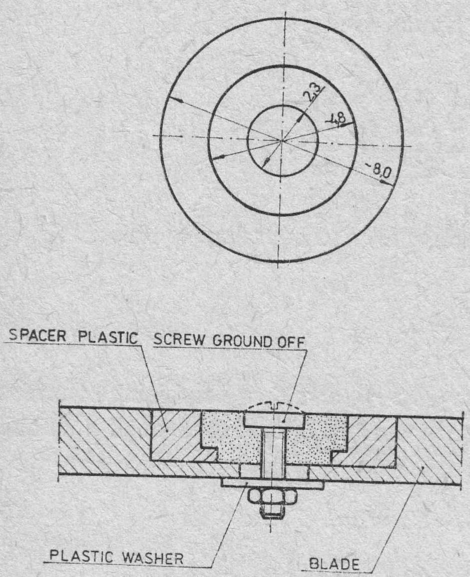

Fig. 2. Design drawing of gauge

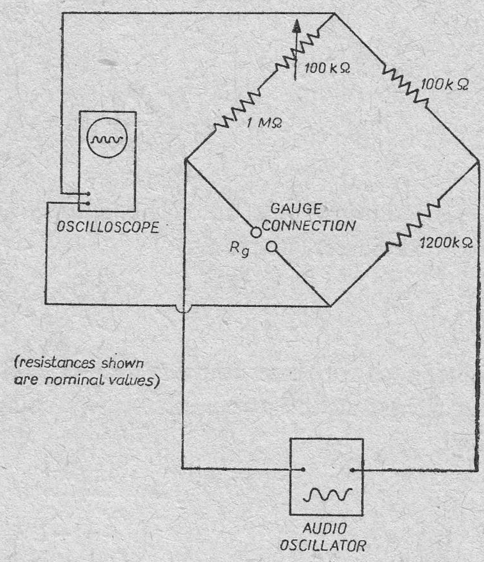

Fig. 3. Wheatstone bridge circuit and related equipment

prepared using a standard solution of distilled water with 0.5% salt (by mass) addition, to avoid variations in water conductivity.

Measurements were taken using an out-of-balance Wheatstone bridge, Fig. 3, excited by an A.C. audio-oscillator [3]. Frequency of 1 kHz was used, producing a sinusoidal wave amplitude of 10 volts. The bridge output voltage was fed to an oscilloscope. The output voltage is a function of the film resistance. Known film thicknesses were obtained by placing a pre-measured circular shim around the gauge [3], putting a few drops of water on the gauge, and finally setting a circular non-conducting plate over fixed shims, trapping the water over the gauge, and squeezing it to the shim thickness. Measurements were made with the standard water solution at different temperatures, ranging from ~21 to 65°C. Typical calibration curves are shown in Fig. 4. The function, f, depending only on film thickness for a given gauge, was found using values from the calibration curves (Figs. 4).

The resulting function $f(h)$, along with the dependence upon the factor $C_g\sigma$ versus temperature, are presented in Figs. 5 and 6. The small scatter of data points shown on Figs. 5 and 6 indicates the general precision level of the calibration, and the realism of the calculating procedure used. Function $f(h)$ is general for all gauges, but the factor $C_g\sigma$ is somewhat different for various gauges.

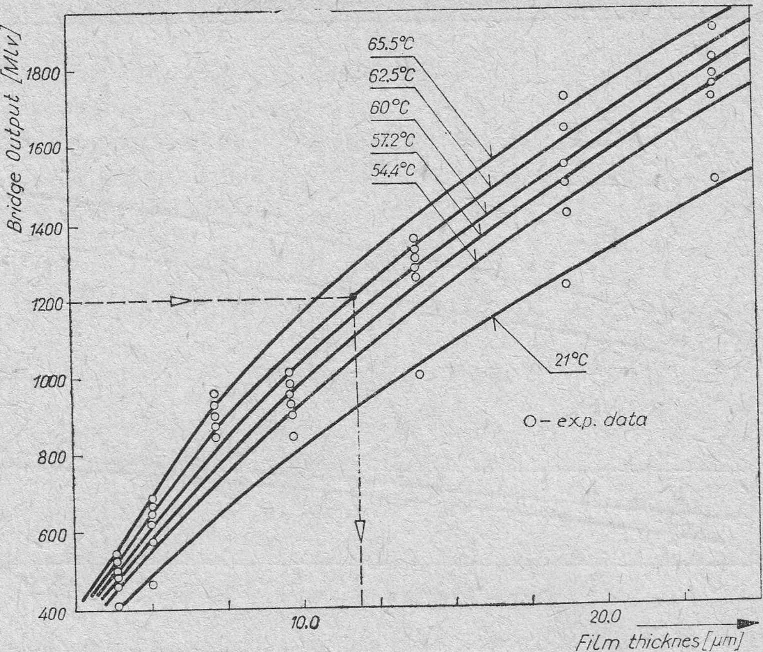

Fig. 4. Typical calibration curves (gauge No. 1)

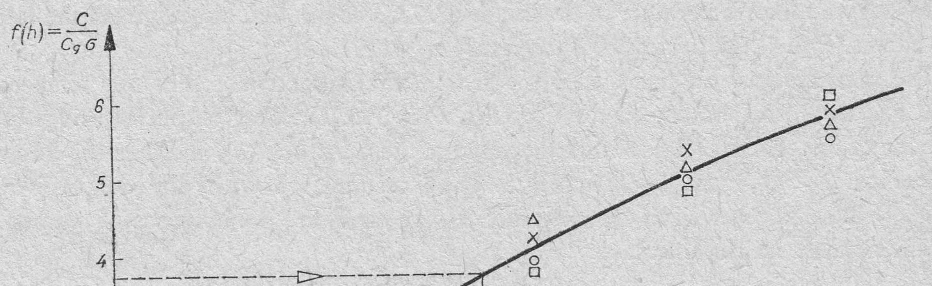

Fig. 5. Dependence of $f(h)$ on film thickness, h for gauges Nos. 1, 2, 3, 4

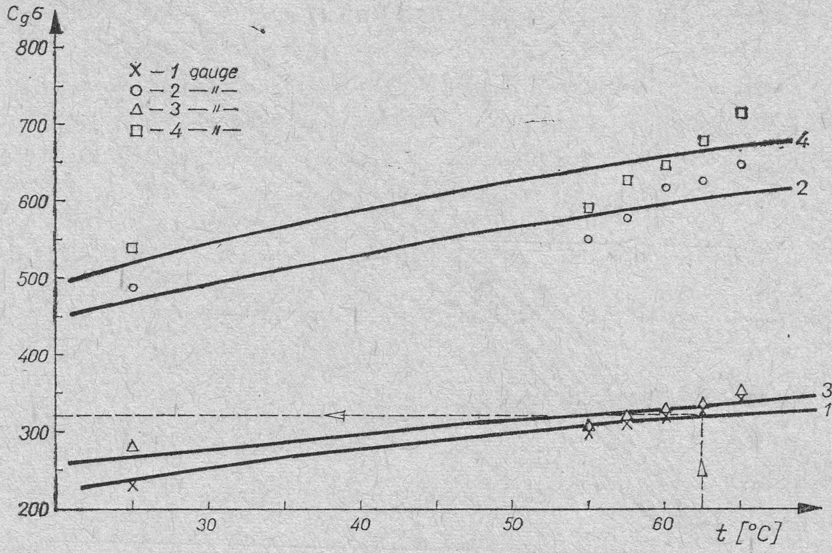

Fig. 6. Dependence of product of geometric factor and water conductance on temperature for gauges Nos. 1, 2, 3, 4

4. Method for using generalized characteristic

The following presents an example of how to apply the characteristic curves.

Using the generalized characteristic of (h) Fig. 5 along with the factor $C_g\sigma$ Fig. 6, one can obtain the thickness of liquid film. For instance, if the output of the Wheatstone bridge is 1200 mV and the temperature of the film is 62.5°C, we can find directly from the calibrating curves (Fig. 4) the thickness of the film, i.e. $\sim$120 µm.

The same result is obtained using the generalized characteristic. For film temperature of 62.5°C the corresponding factor $C_g\sigma$ (Fig. 6) is 320. Dividing output of bridge (1200) by value of factor $C_g\sigma$ (320), we find the value of the function $f(h)$, and then the measured thickness of the film, equal to $\sim$118µ (Fig. 5). The procedure is shown by dotted lines on Figs. 5 and 6. Using this generalized characteristic we can easily see the influence of temperature on the thickness measurement.

5. Conclusion

The advantage of this method of presenting the calibration measurements is that all the data for a given gauge for all temperatures is reflected in each curve. The method produces a general characteristic of a gauge, by using a small quantity of data to calibrate or even avoid calibrating for a given gauge geometry, when the characteristic for such a gauge is known. Determining all the factors in finding the characteristic allows a final general characteristic of the gauge, for a given gauge shape.

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References

[1] J. G. Collier and G. F. Hewitt, *Film Thickness Measurements.* ASME Paper No. 64-WA/HT-41, (1964).
[2] R. Puzyrewski, R. Jasiński, *Pomiar grubości cienkich warstewek wody metodą opornościową,* Prace IMP, z. 26, 1965.
[3] Peter Fenyes, *Liquid Film Thickness Measurement: Design and Use of a Conductance Probe.* ORA Report No. UMICH 012449-6-I, December 1974.

Appendix A

Error analysis

Let us consider first error of measurements, and then error of correlating experimental data for the characteristic of the gauges.

A. Systematic Error of Measurement

Function $f(h)$ is

$$f(h)=\frac{C}{C_g\sigma}, \tag{A-1}$$

where product $C_g\sigma$ was evaluated from data corresponding to linear dependent of output of bridge, so

$$C_g\sigma=\frac{C_{\text{lin}}}{h_{\text{lin}}}. \tag{A-2}$$

Substituting (A-2) for (A-1) we obtain

$$f(h)=\frac{Ch_{\text{lin}}}{C_{\text{lin}}} \tag{A-3}$$

The maximum possible error of dependent variable $f(x_i)$ when errors of independent variable x_i are all of the same sign, is

$$\Delta f=\sum_{1}^{i}\frac{\partial f}{\partial x_i} \tag{A-4}$$

with x_i independent variables.

Applying (A-4) to (A-3) we can find easily the relative error of function $f(h)$ as:

$$\frac{\Delta f(h)}{f(h)}=\left|\frac{\Delta C}{C}\right|+\left|\frac{\Delta C_{\text{lin}}}{C_{\text{lin}}}\right|+\left|\frac{\Delta h_{\text{lin}}}{h_{\text{lin}}}\right|. \tag{A-5}$$

Assuming: $\Delta h=1{,}8$, $\Delta C=\Delta C_{\text{lin}}=10$, $C=C_{\text{lin}}=28$ we obtain maximum possible systematic error

$$\left|\frac{\Delta f(h)}{f(h)}\right|\cong 10.4\%.$$

More probable is situation when errors of dependent variable are not of the same sign. In this case, most probable systematic error is

$$\Delta f=\sqrt{\Sigma\left(\frac{\partial f}{\partial x_i}\Delta x_i\right)^2}\,. \tag{A-6}$$

Applying (A-6) to (A-3) and rearranging we obtain most probable relative systematic error of function $f(h)$ as

$$\frac{f(h)_p}{f(h)}=\sqrt{\left(\frac{\Delta C}{C}\right)^2+\left(\frac{\Delta C_{\text{lin}}}{C_{\text{lin}}}\right)^2+\left(\frac{\Delta h}{h}\right)^2}\,. \tag{A-7}$$

Putting experimental data into (A-7) we find that this value ranges

$$1.15\%<\frac{\Delta f}{f(h)}<6.3\%.$$

B. Random error

This error is not possible to estimate because there are not enough experimental measurements for each condition to apply the statistical analysis required.

C. Error of correlating experimental data

This error is presented in the Table 1-A as a function of film thickness.

Table 1-A

The correlation against film thickness

Thickness	Predicted measured value of $f_t(h)$	Minimum measured value of $f_m(h)$	Maximum measured value of $f_m(h)$	Range of error $\frac{f_t-f_m}{f_t}\%$
39.37	39.37	38.1	40.64	± 3.1
50.80	50.80	48.26	53.34	± 5.0
73.66	69.85	69.85	78.74	0, + 12.6
99.06	86.36	76.20	88.90	− 11.8, + 14.6
140.97	107.95	96.52	116.84	− 10.6, + 8.3
191.77	132.08	121.92	137.16	− 7.7, + 3.8
240.03	147.32	142.24	154.94	− 3.4, + 5.2

Comparing error correlation with systematic error, we find that these errors are very close. This means that the characteristic of the gauges takes into account all factors and scatter is due only to method of measurement.

Appendix B

Simple theory of the conductance probe permits us to find the theoretical value of the geometrical factor of the gauge — C_g (form. 6)

$$C_g=\frac{4\pi}{\left(\frac{r_2}{r_1}\right)^2-1}\,. \tag{B-1}$$

For given dimensions of gauge, this factor becomes

$$C_g=\frac{4\pi}{\frac{0.1875^2}{0.090}\quad 1}\approx 3.7. \tag{B-2}$$

Experimental data also allows us to find this factor. An equation expressing the output voltage of our bridge as a function of the film resistance is

$$E_{out}=10\left(\frac{1160}{1160+\frac{1}{C}}-\frac{100}{1300000}\right). \tag{B-3}$$

Dropping negligible terms, we obtain from (B-3):

$$E_{out}\approx 11600C=11600C_g\sigma h \ [\text{mV}]. \tag{B-4}$$

From experiments, we know the product 11600 $C_g\sigma$. For gauges 2 and 4, this product is ~ 550 and for gauge 1, 3 it is 300 (see Fig. 6).

The theoretical value of the same product is $11{,}600\times 3.7\times\sigma=430{,}00\sigma$. If we assume typically that $\sigma\cong 0.183\ \frac{1}{\Omega\text{m}}$ (ref. [2]), we will find that the product ≈ 200. Comparing this value with the experimental value, we find a considerable difference, though they are of the same order of magnitude.

This difference can be explained only by fabrication tolerances of the gauges. Particularly important are the dimensions in the direction perpendicular to the working surface of the gauge. Any mismatching of these dimensions were not taken into account in the theoretical analyses. Using the calibration curves, we find that a small axial step between metallic electrodes and insulation of ~ 25 μm leads to an error $\sim 20\%$, but 50μm gives an error $\sim 50\%$, so that the geometrical factor can change by ~ 2 times. It is thus necessary to calibrate all gauges, even though supposedly identical, at least at one temperature.

Uogólniona charakterystyka czujników elektrycznych do pomiaru cienkich warstw cieczy

Streszczenie

W pracy przedstawiono metodę cechowania miniaturowych czujników elektrycznych dla pomiaru grubości cienkich warstw cieczy na powierzchni ciała stałego. Opracowano uproszczoną teorię tych czujników, która pozwoliła na opracowanie uogólnionych charakterystyk czujnika. Charakterystyki te pozwalają na uwzględnienie wpływu zmienności przewodności elektrycznej cieczy, temperatury oraz parametrów geometrycznych czujnika. Podano przykładowo charakterystyki dla kilku badanych czujników. Punkty eksperymentalne, przy proponowanym przedstawieniu czujników generują dość dobrze uogólnioną charakterystykę tego typu czujników.

Обобщенная характеристика электрических датчиков для измерения тонких слоев жидкости

Резюме

В работе представлен метод калибровки миниатюрных электрических датчиков для измерения толщины тонких слоев жидкости на поверхности твердого тела. Обработана упрощенная теория этих датчиков, которая позволила определить обобщенные характеристики датчика. Эти характеристики позволяют учесть влияние изменяемости электропроводности жидкости, температуры и геометрических параметров датчика.

Даны примерные характеристики нескольких исследуемых датчиков. Экспериментальные пункты, при предлагаемом представлении датчиков, довольно хорошо генерируют обобщенную характеристику датчиков такого типа.