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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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STEAM TURBINES OF GREAT OUTPUT

Gdańsk, September 24 - 27, 1974

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III НАУЧНАЯ КОНФЕРЕНЦИЯ

на тему

ПАРОВЫЕ ТУРБИНЫ БОЛЬШОЙ МОЩНОСТИ

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A Method of Analysis of the Axi-Symmetrical Flow in the Blade Channels of Turbines

Nomenclature

- A, B, C – functions (eq. (18)),
- a – velocity of sound,
- c – absolute velocity,
- c_p – specific heat,
- D – quantity defined by the formula (27),
- $\bar{F}_2$ – mass force generated by the blade action,
- $\bar{f}$ – mean friction force,
- h^* – total rothalpy (eq. (7)),
- i – enthalpy,
- $\dot{m}$ – mass flow,
- p – pressure,
- R – individual gas constant,
- r, ϑ, z – cylindrical system co-ordinates,
- r_k – radius of the curvature of the meridional stream-line,
- s – entropy,
- T – temperature,
- u – circumferential velocity, circumference co-ordinate,
- w – relative velocity,
- α, β – position of absolute velocity angles vector and position of relative velocities angles vector,
- $\alpha_\gamma, \beta_\gamma$ – surface angles for $r=$const,
- δ – angle of S_2' surface inclination relative to the radius direction,
- ζ – coefficient of the flow losses,
- κ – isentropic exponent,
- ρ – density,
- σ – coefficient of the perfectness of process
- τ – contraction of flow coefficient (contraction of section),
- ω – angular velocity.

Indexing

- i – pertaining of the consecutive point of the stream-line,
- j – pertaining of the consecutive stream-line,
- m – pertaining to the direction along the meridional stream-line,
- 0 – pertains to the section with known parameters i.e. to the rim inlet,
- r, u, z – pertains to components in the system of co-ordinates r, ϑ, z,
- $*$ – (above sign) pertains to stagnation conditions.

1. Introduction

The constant growth of the heat turbines output and the increasing requirements laid upon their efficiency make the problem of the adequate flow description an immensely urgent one. Along with comparatively high efficiences obtainable at present, the further

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increase of the efficiency is at present possible only through a deep comprehension of the nature of phenomena taking place in the blade channels. One of the factors enabling such a comprehension is a constant development of calculating techniques.

In a situation where the special structure of the stream in a turbine depends on a vast number of geometrical and kinetic factors as well as on the flow properties, only the application of digital computers enables the researcher to carry out a full analysis of all the processes involved. Apart of the experiments the numerical experiments become at present an important means of carrying out research. The method presented in this paper may be in practice utilized fully only with the help of a digital computer.

The flow through a turbine stage is largely indetermined one having besides a three-dimensional character. Hence the problem of its theoretical description is usually set forth with some simplifying assumptions taken. In the majority of works the basic simplification concerns the restriction of the theory of the flow in the turbines to the description of the motion of the liquid within the inter-rim areas. One may point out the methods based on the assumption of the form of stream-lines in the meridional cross-section. In the reference papers [17, 20], for instance, the solution is achieved by choosing the stream-lines adequately to the geometry of the surface limiting the flow channel.

Among various possibilities the hypothesis of the sinusoidal shape of the stream-lines as exemplified in the works of Bammert and Fiedler [2, 3] has proved to be a very useful one.

The basic shortcoming of these methods lies in determining the stream-line shape on the basis of the distribution of flow parameters in the inter-rim areas while it is known that a greater part of the turbine flow developes within the blade rim areas. The blade influence has been partly taken into account in papers [4, 19].

The contemporary methods of analysis of the flow in blade channels of the fluid-flow machines have been originated by Wu [23] whose concept is based on reducing a three-dimensional problem to a set of two two-dimensional ones. A similar point has been more precisely presented by Sirotkin [16]. Another method based on the three-dimensional character of flows, which describes however the incompressible fluids only, is the singular-points method [14]. Despite the attempts to apply [11] the Wu's method in practice, it may be stated that the suggested procedures are very complex and ineffective, even with the application of contemporary digital computers. Therefore, the problem of preparing adequate and fast methods of analysis that would take into account all factors characteristic for the real flow – is still actual. The present considerations aim lies in solving the above problem.

There are known contemporarily the methods of solving the so called quasi three-dimensional problem [7, 18, 20, 22], where the flow description is being obtained as a result of the two-step solution:

a) assuming that the flow is axially symmetrical the flow on the surface S_2' is being determined,

b) the flow round the palisade profiles, is analyzed on the stream rotational surfaces (surface S_1').

The presented work leads to the solution of the so called primary problem of the aero-hydrodynamics, consisting in solving the first problem of the quasi three-dimensional flow analysis i.e. the axial-symmetrical flow on the surface S_2' (Fig. 1). A model of a generalized axial-symmetrical flow suitable to the case of a great blades number can be obtained with a considerable accuracy [18, 20].

The present work concerns a general stream structure. Hence the flow is generally a vortex-flow and the presence of the enthalpy and entropy gradients is accounted for. The averaged effects of fluid viscosity have been introduced and therefore the real character of the flow has been indirectly considered.

2. The formulation of axial-symmetrical flow through the blade array

In what follows a certain axial-symmetrical flow along a mean surface S_2' of the blade channel rotating with a constant angular speed ω will be considered (Fig. 1). In the first approximation, with a practically sufficient accuracy, the mean geometrical blade surface is regarded as a surface S_2'. The consecutive approximations require an earlier solution of the second problem of the quasi-three-dimensional flow.

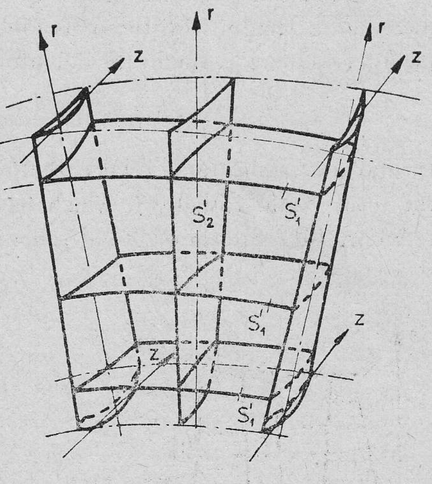

Fig. 1. Stream surfaces S_1' and S_2' in the quasi-three-dimensional flow models

Under the assumption that in a cylindrical rotating system of co-ordinates $(r, \vartheta\ z)$ the surface equation S_2' may be represented by the function $\vartheta = f_2\ (r, z)$ and assuming subsequently that $z = \text{const}$ and $r = \text{const}$, the geometry of the surface is determined by the angles on the two mutually perpendicular surfaces (Fig. 2)

$$\operatorname{tg}\delta = r\,\frac{d\vartheta}{dr}, \qquad \operatorname{ctg}\beta_p = r\,\frac{d\vartheta}{dz}.$$

The same angles may be determined by components of the vector $\bar{n}(n_r, n_u, n_z)$ normal to S_2':

$$\operatorname{tg}\delta = -\frac{n_r}{n_u}, \qquad \operatorname{ctg}\beta_p = -\frac{n_z}{n_u}. \tag{1}$$

The S_2' surface is the stream surface, therefore the vector of the relative flow velocity $\bar{w}$ must lie on a tangent plane i.e.

$$\bar{w}\cdot\bar{n}=0. \tag{2}$$

The components of velocity vectors have been presented in Fig. 3. Angles of inclination of the stream-lines as well as of the flux on the axial-symmetrical stream surface are described as follows:

$$\operatorname{tg}\gamma=\frac{w_r}{w_z}, \qquad \operatorname{ctg}\beta=\frac{w_u}{w_m}. \tag{3}$$

Using relations (1) and (3) in the equation (2) one obtains

$$w_u=w_r\operatorname{tg}\delta+w_z\operatorname{ctg}\beta_p \tag{4}$$

and consequently

$$\operatorname{ctg}\beta=\operatorname{ctg}\beta_p\cos\gamma+\operatorname{tg}\delta\sin\gamma. \tag{5}$$

After averaging [18] the surface forces of the blade action change homogeneously (on the circumference) into the field of the mass forces $\bar{F}_2$. A mean friction force $\bar{f}$ is also being introduced. Hence, with the omission of the terms referring to the pulsation of the parameters (assumed as small) the equation of fluid motion can be written in the Crocco form

$$-\bar{w}\times\operatorname{rot}\bar{w}+2\bar{\omega}\times\bar{w}=T\nabla s-\nabla h^*+\bar{F}_2+\bar{f}. \tag{6}$$

In above h^* indicates a total enthalpy of the relative motion. Wu [24] suggested for h^* the term "total rothalpy", which has been accepted for the present considerations. In reference [6] the term of "total generalized enthalpy" has been used.

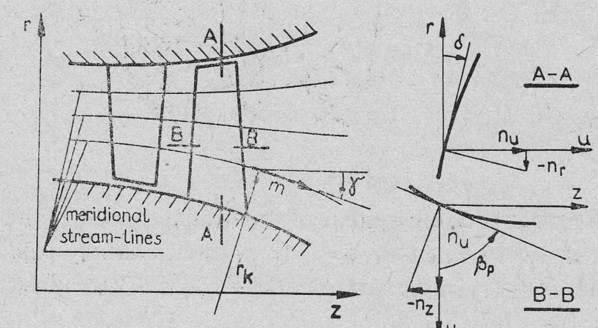

Fig. 2. Typical meridional section of a stage. Blade geometry

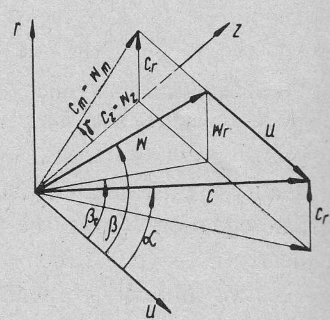

Fig. 3. Velocity vectors

Under the assumption of the flow adiabaticity (i.e. when there is no heat exchange between the fluxes) the total rothalpy does not change on the streamline:

$$h^*=i+\frac{w^2}{2}-\frac{u^2}{2}=\operatorname{const}(\psi), \tag{7}$$

where $\psi = \dot{m}(r)/\dot{m}_0$. The formula (7) expresses the principle of the energy conservation along the line of stream ($dh^*/dt = 0$). The flow continuity equation has a form

$$\operatorname{div}(\tau\rho\bar{w}) = 0, \tag{8}$$

where τ is the quantity characterizing the contraction of the flow channel caused by the thickness of the blades. The condition that the vector $\bar{F}_2$ is ortogonal to S_2' may be written as:

$$\bar{n} \times \bar{F}_2 = 0. \tag{9}$$

Multiplying equation (6) by the vector of relative velocity $\bar{w}$ and taking into account that $\bar{w} \cdot \nabla h^* = 0$ as well as that $\bar{w} \cdot \bar{F}_2 = 0$, one obtains the relation which determins the friction force

$$\bar{f} = -\bar{w} \frac{T}{w^2} \frac{ds}{dt}. \tag{10}$$

It is obvious that regarding the flow through the blading system as a flow during which certain losses occur; it becomes necessary to take into account the term defined in equation (6) as a force $\bar{f}$, since otherwise the equation could not be fulfilled. It is necessary to emphasize that the friction force $\bar{f}$ is here a consequence of introducing the flow losses into the consideration.

The changes of the entropy along the streamlines result from taking into account mean effects of the real fluid viscosity which combine the dissipation of the energy resulting from the friction and mixing. For a quantitative formulation of this process a certain function is being introduced. It represents the entropy increase i.e. the so called coefficient of the perfectness of process σ.

From the equation of the first principle of thermodynamics there is

$$T\,ds = di - \frac{1}{\rho}\,dp \tag{11}$$

and the equation of the state

$$p = \rho R T \tag{12}$$

one obtains the equation of process

$$\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} \frac{T_0}{T} = \exp\left(-\frac{s-s_0}{c_p}\right) = \sigma. \tag{13}$$

In case of an isentropic process ($s = \text{const}$) the coefficient σ is equal to 1.

Equations (2), (6), (8), (9), (11) and (12) constitute a closed set of the equations (with nine unknowns) describing the introductory basic problem of the aerodynamics ($\bar{w}$, $\bar{F}_2$, p, ρ, T). Usually, instead of the equation of motion (6), equation (7) is considered, whereas equation (11) is substituted by an equivalent equation (13).

3. Equations of the axial-symmetrical flow through the blade system of the heat turbines

Because of the axial symmetry of the stream surface only flow cross-sections parallel to a meridional plane will be considered in the further part of this paper. Indicating the co-ordinate along the meridional stream-line by m (Fig. 2), the following set of the relations can be written:

$$dm : dr : dz = c_m : c_r : c_z, \tag{14}$$

$$\frac{\partial}{\partial m} = \sin\gamma \frac{\partial}{\partial r} + \cos\gamma \frac{\partial}{\partial z}, \tag{15}$$

$$\frac{d}{dt} = c_r \frac{\partial}{\partial r} + c_z \frac{\partial}{\partial z} = c_m \frac{\partial}{\partial m}. \tag{16}$$

For analytical purposes a number of characteristic areas in the flow system of a turbine (Fig. 4) may be distinguished. These are: the area of the stator blades (*II*) and runner blades (*I*), the inter-rim area (*III*) and the inlet and outlet areas (*IV*) of the flow system. Further considerations will include equations describing the axial-symmetrical flow in the respective areas and the method of solving the equations.

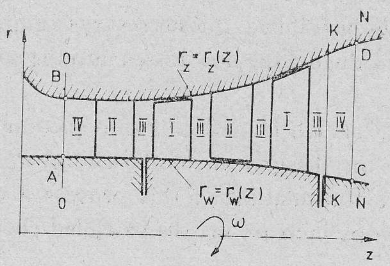

Fig. 4. Turbines flowing system–scheme

The presented equations of flow are expressed in the form based on the notion of the curvature of the metridional stream-line [8, 13]. Although the method is known for a long time, its practical application became possible after contemporary data processing systems have been developed. Particularly difficult were problems concerning the determination of the radius of the curvature of the meridional line of the stream (Fig. 2). The magnitude of the curvature is described by the expression

$$\frac{1}{r_k} = -\frac{\partial\gamma}{\partial m}. \tag{17}$$

Taking into account the specific character of the calculations carried out by a computer, the basic unknown will be c_m, i.e. the meridional component of the velocity.

This quantity appears on the meridional plane of all the specified areas which allows a more universal preparation of calculating programs. Because of the complex character of the flow equations the following notation has been accepted for the axial-symmetric flow:

$$c_m \frac{\partial c_m}{\partial r} + A c_m^2 + B c_m + C = 0, \tag{18}$$

where A, B, C indicate certain functional expressions.

If the equations of the fluid motion were written in a scalar form the following would be obtained:

$$-w_u \frac{1}{r}\frac{\partial(rw_u)}{\partial r}+w_z\left(\frac{\partial w_r}{\partial z}-\frac{\partial w_z}{\partial r}\right)-2\omega w_u=T\frac{\partial s}{\partial r}-\frac{\partial h^*}{\partial r}+F_{2r}+f_r. \tag{19}$$

On the basis of the formulae (15) and (17), the angular component of the rotation can be written as

$$\frac{\partial w_r}{\partial z}-\frac{\partial w_z}{\partial r}=\frac{1}{c_m\cos\gamma}\left[c_m^2\left(\frac{\sin\gamma}{c_m}\frac{\partial c_m}{\partial m}-\frac{\cos\gamma}{r_k}\right)-\frac{1}{2}\frac{\partial c_m^2}{\partial r}\right].$$

The above expression together with a relation $w_u=c_m\operatorname{ctg}\beta$ allows to transform the equation (19), giving

$$\frac{1}{2\sin^2\beta}\frac{\partial c_m^2}{\partial r}+c_m^2\left(-\frac{\sin\gamma}{c_m}\frac{\partial c_m}{\partial m}+\frac{\cos\gamma}{r_k}+\frac{1}{2}\frac{\partial\operatorname{ctg}^2\beta}{\partial r}+\frac{\operatorname{ctg}^2\beta}{r}\right)+$$

$$+2\omega c_m\operatorname{ctg}\beta=-T\frac{\partial s}{\partial r}+\frac{\partial h^*}{\partial r}-F_{2r}-f_r. \tag{20}$$

Transforming the equation of motion (6) in the analogous way one obtains

$$c_m^2\left(\frac{\partial\operatorname{ctg}\beta}{\partial m}+\frac{\operatorname{ctg}\beta\sin\gamma}{r}\right)+c_m\left(\operatorname{ctg}\beta\frac{\partial c_m}{\partial m}+2\omega\sin\gamma\right)=F_{2u}+f_u. \tag{21}$$

The relation between the components of the mass force $\bar{F}_2$ can be obtained from the condition of ortogonality (9):

$$F_{2r}=-\operatorname{tg}\delta\, F_{2u}. \tag{22}$$

In view of (13) the equation describing the change of entropy along the stream-line can be written as

$$s=s_0-c_p\ln\sigma. \tag{23}$$

Taking into account the expression (7)

$$i=h^*+\frac{u^2}{2}-\frac{c_m^2}{2\sin^2\beta} \tag{24}$$

one may determine the friction force in terms of the kinematic quantities obtaining

$$\bar{f}=\bar{w}\frac{\sin^2\beta}{c_m}\left(h^*+\frac{u^2}{2}-\frac{c_m^2}{2\sin^2\beta}\right)\frac{\partial\ln\sigma}{\partial m}. \tag{25}$$

Using the relation (22) and considering simultaneously the formulae (5), (23), (25) the basic equation for the axial-symmetrical flow in the blade chamber (area I on Fig. 4) can be obtained from equations (20) and (21). In accordance with the accepted notations this equation remains given by (18) where:

$$\left.\begin{aligned}
A \equiv \sin^2\beta\Bigg[&D(\sin\gamma+\operatorname{tg}\delta\operatorname{ctg}\beta)+\frac{\cos\gamma}{r_k}+\frac{1}{2}\frac{\partial\operatorname{ctg}^2\beta}{\partial r}+\frac{\operatorname{ctg}\beta_p\operatorname{ctg}\beta\cos\gamma}{r}-\\
&-\operatorname{tg}\delta\frac{\partial\operatorname{ctg}\beta}{\partial m}-\frac{1}{2\sin^2\beta}\cdot\left(\frac{1}{c_p}\frac{\partial s_0}{\partial r}-\frac{\partial\ln\sigma}{r}\right)-\frac{1}{2}(\sin\gamma+\operatorname{tg}\delta\operatorname{ctg}\beta)\frac{\partial\ln\sigma}{\partial m}\Bigg],\\
B \equiv{}& 2\omega\sin^2\beta\operatorname{ctg}\beta_p\cos\gamma,\\
C \equiv \sin^2\beta\Bigg\{&-\frac{\partial h^*}{\partial r}+\left(h^*+\frac{r^2\omega^2}{2}\right)\left[\frac{1}{c_p}\frac{\partial s_0}{\partial r}-\frac{\partial\ln\sigma}{\partial r}+\sin^2\beta(\sin\gamma+\operatorname{tg}\delta\operatorname{ctg}\beta)\frac{\partial\ln\sigma}{\partial m}\right]\Bigg\}.
\end{aligned}\right\}\quad(26)$$

In above

$$D\equiv-\frac{1}{c_m}\frac{\partial c_m}{\partial m}.\tag{27}$$

Taking into account the relations (15) and (17) one obtains from equations (8) and (27)

$$D\equiv\frac{\operatorname{tg}\gamma}{r_k}+\frac{\cos\gamma}{r}\frac{\partial(r\operatorname{tg}\gamma)}{\partial r}+\frac{\partial\ln\tau}{\partial m}+\frac{\partial\ln\rho}{\partial m}.\tag{28}$$

The form of the $\partial\ln\rho/\partial$ function can be recovered from the equation of the process (13), giving

$$\frac{\partial\ln\rho}{\partial m}=\frac{\kappa}{\kappa-1}\frac{\partial\ln\sigma}{\partial m}-a^{-2}\left(\frac{1}{2}c_m^2\frac{\partial\operatorname{ctg}^2\beta}{\partial m}+\frac{c_m}{\sin^2\beta}\frac{\partial c_m}{\partial m}-\omega^2 r\sin\gamma\right),$$

where the local velocity of sound is determined by the formula

$$a=\sqrt{\kappa RT}.\tag{29}$$

The final form of expression (28) becomes:

$$\begin{aligned}
D\equiv\left(1-\frac{c_m^2}{a^2\sin^2\beta}\right)^{-1}\Bigg[&\frac{\operatorname{tg}\gamma}{r_k}+\frac{\cos\gamma}{r}\frac{\partial(r\operatorname{tg}\gamma)}{\partial r}+\frac{\partial\ln\tau}{\partial m}+\frac{\kappa}{\kappa-1}\frac{\partial\ln\sigma}{\partial m}-\\
&-a^{-2}\left(\frac{1}{2}c_m^2\frac{\partial\operatorname{ctg}^2\beta}{\partial m}-\omega^2 r\sin\gamma\right)\Bigg].
\end{aligned}\tag{30}$$

The equation (18) and the expressions (26) contain two unknowns: the velocity c_m, and the angle γ. The angle β is determined by the relation (5); the remaining functions are assigned.

The closing equation of the system is the equation of continuity rewritten in the integral form

$$\dot{m}_0=2\pi\int_{r_w}^{r_z}\tau\rho c_m\cos\gamma\,dr.\tag{31}$$

Equation (26) is consistent with the blade channel rotating with the speed ω. To determine the sought for quantities in a system of rest also, it is necessary to have adequate formulae of "transition". The transformation formula for determining energy in a system at rest can be obtained by putting into the rothalpy equation (7) the known relation $w^2=c^2+u^2-2u\,c_u$. There is

$$h^*=i+\frac{c^2}{2}-uc_u.$$

Considering that the total enthalpy is determined by the formula

$$i^*+i+\frac{c^2}{2}$$

one obtains the final form of the transformation formula

$$h^*=i^*-uc_u. \tag{32}$$

Using the above relations it is easy to formulate the equation determining the speed of sound

$$\frac{a^2}{a_0^2}=\frac{\kappa RT}{\kappa RT_0^*}=\frac{i}{i_0^*}.$$

Considering (32), the following relation can be reached

$$a=a_0\left\{1+\frac{\kappa-1}{2a_0^2}\left[\omega^2r^2-\frac{c_m^2}{\sin^2\beta}-2(uc_u)_0\right]\right\}^{\frac{1}{2}}. \tag{33}$$

Analogously the relation determining the medium density can be obtained

$$\frac{\rho}{\rho_0^*}=\sigma^{\frac{\kappa}{\kappa-1}}\left\{1+\frac{\kappa-1}{2a_0^2}\left[\omega^2r^2-\frac{c_m^2}{\sin^2\beta}-2(uc_u)_0\right]\right\}^{\frac{1}{\kappa-1}}. \tag{34}$$

To obtain equations describing the axi-symmetrical flow through stator rims (area *II* on Fig. 4) it is necessary to put $\omega=0$ in the equations (26), (30), (33) and (34), and change the indexing putting $\beta\to\alpha$, $\beta_p\to\alpha_p$, $h^*\to i^*$.

Considering the range of the validity of the first approximation, there arises the problem of determining the stream angles while passing from the areas *I* and *II*, to the zone *III*. In reality the blade angles β' and the stream angles β differ from each other by a so-called deviation angles δ_2. In this situation there are two ways of treating the problem:

1. To assume that at the blade outlet i.e. during passing from the areas *I* (or *II*) to *III*, the stream angle is equal to the blade angle. The velocity component c_u will thus be:

$$c'_{u2}=c'_m\operatorname{ctg}\beta'+u'.$$

In the area *III* with the assumption $\bar{F}_2=0$, $\tau=1$, $\bar{f}=0$, $\sigma=1$, one obtains the equation

$$\frac{\partial(rc_u)}{\partial m}=0 \tag{35}$$

instead of equation (2). In the case however the work performed by rotor blades will differ from the actually performed work.

2. To assume that at certain distance behind the rotor, the value of circumferential velocity is in agreement with the actual value, given by a relation

$$c_{u2}=c_m \operatorname{ctg}\beta+u=c_m \operatorname{ctg}(\beta'-\delta_2)+u\,. \tag{36}$$

In this case the value of the performed work is also in agreement with reality. As a consequence of the flow angle discontinuity along the stream-line the discontinuity of the energy will apear. This is understandable if one considers the enthalpy at the point of the areas joining using the equation (32). This enthalpy discontinuity occurs only on the boundaries of particular areas, where its values can be considered as the boundary conditions for the area. One cannot then speak about mathematical inaccuracy. In the present paper the problem of the axi-symmetrical flow in the space between the blade rows is being solved in the second formulation i.e. with the stream angles β determined either by empirical equations or on the basis of palisade investigation. In passing from one area to the other, the circumferential component of absolute velocity can be determined by the equation (36) applying the transformational formula (32). The inaccuracies, presented above, connected with the angle discintinuity along the stream-line, disappear while solving the flow on the surfaces S_2''.

Equations for the axi-symmetrical flow in the inter-rim areas (area *III*) are being obtained from the systems of equations (6), (8), (12) and (13) with the assumption that $\bar{F}_2=0$ and $\tau=1$.

For the area beyond the runner rim for example the flow equation (18) have the following shape parameters

$$\left.\begin{aligned} A\equiv\sin^2\beta\Big[D\sin\gamma+\frac{\cos\gamma}{r_k}+\frac{1}{2}\frac{\partial \operatorname{ctg}^2\beta}{\partial r}+\frac{\operatorname{ctg}^2\beta}{r}-\frac{1}{2\sin^2\beta}\left(\frac{1}{c_p}\frac{\partial s_0}{\partial r}\right)+\\ +\frac{1}{2\sin^2\beta}\frac{\partial\ln\sigma}{\partial r}-\frac{1}{2}\sin\gamma\frac{\partial\ln\sigma}{\partial m}\Big],\\ B\equiv 2\omega\sin\beta\cos\beta,\\ C\equiv\sin^2\beta\left[-\frac{\partial h^*}{\partial r}+\left(h^*+\frac{r^2\omega^2}{2}\right)\left(\frac{1}{c_p}\frac{\partial s_0}{\partial r}-\frac{\partial\ln\sigma}{\partial r}+\sin^2\beta\sin\gamma\frac{\partial\ln\sigma}{\partial m}\right)\right]. \end{aligned}\right\} \tag{37}$$

For determining the D quantity the derivative ∂ ctg $\beta/\partial m$ has been used in a form resulting from the equation (21)

$$\begin{aligned} D\equiv\left(1-\frac{c_m^2}{a^2}\right)^{-1}\Big[\frac{\operatorname{tg}\gamma}{r_k}+\frac{\cos\gamma}{r}\frac{\partial(r\operatorname{tg}\gamma)}{\partial r}+\frac{\kappa}{\kappa-1}\frac{\partial\ln\sigma}{\partial m}-a^{-2}\cos^2\beta\left(h^*+\frac{u^2}{2}-\frac{c_m^2}{2\sin^2\beta}\right)\times\\ \times\frac{\partial\ln\sigma}{\partial m}+a^{-2}\sin\gamma\left(\frac{c_m^2}{r}\operatorname{ctg}^2\beta+c_m 2\omega\operatorname{ctg}\beta+\omega^2 r\right)\Big]. \end{aligned} \tag{38}$$

The axi-symmetrical flow of fluid in areas *IV* (no blades) (Fig. 4) depends mainly on the edge condition at the inlet or outlet cross-section, and on the character of the flow in the neighbouring runner rim. In these areas the flow equations are being obtained, from the general system under assumption that $\bar{F}_2=0, \bar{f}=0$, $\tau=1$, $\sigma=1$. An adequate equations for the outlet area will read:

$$\left.\begin{aligned}
&A\equiv\sin^2\beta\left[D\sin\gamma+\frac{\cos\gamma}{r_k}+\frac{1}{2}\frac{\partial\operatorname{ctg}^2\beta}{\partial r}+\frac{\operatorname{ctg}^2\beta}{r}-\frac{1}{2\sin^2\beta}\left(\frac{1}{c_p}\frac{\partial s_0}{\partial r}\right)\right],\\
&B\equiv 2\omega\sin\beta\cos\beta\,,\\
&C\equiv\sin^2\beta\left[-\frac{\partial h^*}{\partial r}+\left(h^*+\frac{r^2\omega^2}{2}\right)\frac{1}{c_p}\frac{\partial s_0}{\partial r}\right],
\end{aligned}\right\}\tag{39}$$

$$D\equiv\left(1-\frac{c_m^2}{a^2}\right)^{-1}\left[\frac{\operatorname{tg}\gamma}{r_k}+\frac{\cos\gamma}{r}\frac{\partial(r\operatorname{tg}\gamma)}{\partial r}+a^{-2}\sin\gamma\cdot\left(\frac{c_m^2\operatorname{ctg}^2\beta}{r}+2\omega c_m\operatorname{ctg}\beta+\omega^2 r\right)\right].\tag{40}$$

The angle β can be determined from the condition (35), which, for a definite line of stream (taking the intersection points of the stream-line with straight lines K-K and N-N (Fig. 4)) becomes

$$(rc_m\operatorname{ctg}\beta)_N=(rc_m\operatorname{ctg}\beta)_K+\omega(r_K^2-r_N^2),$$

where the parameters indicated by the subscript K are known from the solution of the flow in the preceding area.

The equation for the axi-symmetrical flow on the inlet (where the angle usually has the value $\alpha=90°$) will have the following form:

$$\left.\begin{aligned}
&A\equiv D\sin\gamma+\frac{\cos\gamma}{r_k}-\frac{1}{2}\left(\frac{1}{c_p}\frac{\partial s_0}{\partial r}\right),\\
&B\equiv 0,\\
&C\equiv-\frac{\partial i^*}{\partial r}+i^*\left(\frac{1}{c_p}\frac{\partial s_0}{\partial r}\right),
\end{aligned}\right\}\tag{41}$$

$$D\equiv\left(1-\frac{c_m^2}{a^2}\right)^{-1}\left[\frac{\operatorname{tg}\gamma}{r_k}+\frac{\cos\gamma}{r}\frac{\partial(r\operatorname{tg}\gamma)}{\partial r}\right].\tag{42}$$

The egde conditions on the inlet and outlet are set along the straight lines $O-O$ and $N-N$ (Fig. 4). The layout of these edges is (in the area IV) determined while calculations are being carried out, and the limit position is determined as a position, beyond which the change of flow parameters is negligible. In the practice these straight lines are being chosen at the distance of at least one scale unit from the blade rim, which results from the fact that flow disturbancies fade rapidly with the increasing of the distance from the source approximately like the function e^{-z} [18]). The edge conditions in equations for the whole flow area as shown in Fig. 4 are as follows:

- Mass flux $\dot{m}_0$ is given and it is smaller from the limit flux.
- Surfaces AC and BD are stream surface.
- All flow parameters along AB are assigned. They must fullfil the equation of continuity (31) and the equation of equilibrium of the radius (41).
- Along CD the distribution of angles γ and the curvature of the stream-line are assigned.

– In areas I and II the functions α_p, β_p, δ, τ, σ, ω are assigned.

– In areas III the functions α, β, σ are assigned.

In combining the solutions on the edges of the particular areas it is necessary to ensure their continuity and the continuity of their first derivativas. The above presented axi-symmetrical flow equations are derived with the assumption of a total ideality of the gas. Spreading the validity of those equations upon the real gases (steam) description does not present any serious obstacles. In the case of the semi-ideal gases, it is possible to introduce, in accordance with suggestions contained in papers [17] and [21] the averaged exponents of the isentrope. With greater inaccuracy the model of a real gas (steam) can be introduced fulfilling, in the place of equation (12), the equation of state in the form [20, 21]

$$p = z\rho RT\,, \tag{43}$$

where z is the compressibility number (coefficient of gas ideality [21]). The application of digital computers enables to consider the more precise equations of the state of the gas (steam) [1].

4. The problem of determining the flow losses

In the frame of the accepted model of a non-viscuous fluid, the considering of its real properties reduces to determining the mean effects of viscosity of the real fluid in every particular flux. These effects (in the form of entropy increase) result from all kinds of flow losses. For the assessment of the characteristics of a flat cascades, the profile loss coefficient ζ_p is generally accepted. It presents the ratio of the waste energy in the cascade and the real energy of the flux at the outlet [10]. In the turbine blade rims the special character of the flow causes that the radial loss gradient is being influenced not only by profile losses but also by those occurring outside of the profile.

The choice of an adequate distribution of the losses (particularly across the axi-symmetrical flow channels) is very troublesome as it is impossible to assume similar losses for a non-viscous fluid under consideration which allow for much greater gradients. Because of the necessity of limiting them it is suggested to accept a loss gradient of a radial character for the profile losses simultaneously considering all remaining losses as distributed evenly at the height of the blades. Hence, the coefficient of enthalpy losses in the flow may be written in a form:

$$\zeta(r) = \zeta_p(r) + \overline{\sum\zeta}\,, \tag{44}$$

where $\overline{\sum\zeta}$ is the mean value of the outside profile losses. Taking into account that the enthalpy losses in the flow through the rim are determined by the formula

$$\Delta i = \zeta \tfrac{1}{2} w^2\,, \tag{45}$$

the entropy growth along the stream-line will amount to ([2, 8])

$$\Delta s = \zeta \frac{1}{2T} w^2\,, \tag{46}$$

whereas the coefficient of the perfectness of the process may be calculated from the formula

$$\sigma=\exp\left(-\zeta\frac{1}{2i}w^2\right). \tag{47}$$

Because of the fact that the values of the loss coefficients are determined on the basis of the measurements carried out at a given distance beyond the cascade (e.g. 0.5 of the length of the profile chord [5]) it is necessary to assume that the quantities ζ, w, i, in formula (47) are to be regarded as referring to a certain section of the rim outlet. In the multi-stage systems it may be assumed that this section coincides with the inlet edges of the subsequent blade rim. In the process of looking for the solution the flow parameters w and i ought to be determined upon the basis of the previous approximation. To obtain a relation describing the changes of the entropy along the stream-line between the inlet edges of the consecutive rims (distance m_w in Fig. 5) it is enough to assign the distribution of the loss coefficient along the stream-line segment under consideration.

Indicating the distance from the inlet rim by m, the last exigence can be satisfied by assuming that

$$\zeta(r,m)=\zeta(r)\left(\frac{m}{m_w}\right)^n, \tag{48}$$

where the exponent n must be adequately chosen (most often $n=1\div 2$). Assuming that when the "impact" losses at the inlet are omitted the greatest amount of the waste of the energy results from the outlet losses, Sirotkin suggested that n have to be equal to 2. That problem however requires further investigation.

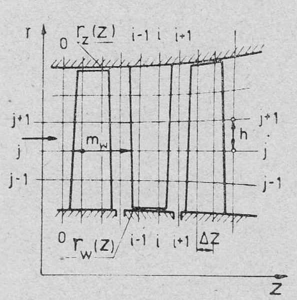

Fig. 5. Turbine blading with co-ordinates network

It is necessary to point out that the assumption that the flow is an adiabatic one is fully justified for the systems of a limited number of stages. The existence of the phenomenon of the energy levelling may lead to some discrepancy of the results particularly at the rim outlets.

5. The method of solving the flow equations

The presented axi-symmetrical flow equations constitute a non-linear system of differential equations. When chosing the solution method such factors as: brief time of calculations, result simplicity, high precision of the results and the capacity of the available computers, have been considered. The proposed method is based upon the concept of solving differential equations using the "straight lines" [12] method.

The nucleus of this method lies in substituting the derivatives with respect to one of the variables by the approximating finite differences.

The system of co-ordinates for the axi-symmetrical flow is constituted by the radial straights and the meridional lines of stream. Expressing the derivatives with respect to the co-ordinates directed along the meridional stream-line by the quantity determined on the basis of the previous approximation, it is possible to proceed from equations with partial derivatas to a system of ordinary differential equations. The functional expressions in equation (18) become then the functions of the radial co-ordinates only. The equation (18) itself becomes an ordinary Abel's differential equation of the second kind [9] which can be solved numerically. The process of solving this equation requires a procedure of iteration.

Within this procedure it is particularly important to determine the angle of the inclination and the curvature of the stream-line standing upon the data obtained in the course of the former approximation. Among many existing and ascertained methods [15] the most advantageous one seems to be the mean quadratic approximation. The approximate polynomials obtained by this method smooth well all errors, giving the solution in the form of an even curve. For a polynomial of the third order (determined for the five points of the j – stream-line r_{-2j}, r_{-1j}, r_{0j}, r_{1j}, r_{2j} obtained from the previous approximation at a constant distance $\Delta z = \text{const}$ (Fig. 5)) the value of derivatives in the middle point follows the formulae:

$$r'_0 = \frac{1}{\Delta z}\left[\frac{1}{10}(r_{1j} - r_{-1j}) + \frac{1}{5}(r_{2j} - r_{-2j})\right], \tag{49}$$

$$r''_0 = \frac{1}{\Delta z^2}\left[-\frac{2}{7} r_{0j} - \frac{1}{7}(r_{1j} + r_{-1j}) + \frac{2}{7}(r_{2j} + r_{-2j})\right]. \tag{50}$$

With the number of points increasing the polynomials of higher orders can be obtained and used.

The inclination angle and the curvature (meridional) of the stream-lines can be calculated after the formulae

$$\gamma = \operatorname{arctg}(r'_0), \qquad \frac{1}{r_k} = \frac{r''_0}{(1 + r_0'^2)^{1.5}} \tag{51}$$

and, therefore are known when the derivatives (49) and (50) are determined.

Some singularities entering the calculation of D (27) from the formulae (30), (38) (40) and (42) require the additional explanation. These singularities arise when velocity w or c in the blade rim area reaches the velocity of sound. The system of equations should be then changed from elliptic into hiperbolic one. The same is true for c_m in the inter-rim areas. In the range of the transonic velocities (i.e. when the stated formulae fail) one may calculate derivative $\partial c_m/\partial m$ by interpolation formulae or using the assymptoting polynomials.

Equation (18) has the form suitable for numerical calculations:

$$\frac{dc_m}{dr} = -\left[A(r)\, c_m + B(r) + \frac{C(r)}{c_m}\right] \equiv F(r, c_m). \tag{52}$$

The above equation will be solved by means of the trapezium method [12]. Because of various reasons, however, it may be more profitable to use other numerical methods (e.g. the Runge-Kutta method). Integrating procedure is carried out in such a way that the particular nodes $\dots r_{j-1}, r_j, r_{j+1} \dots$ are placed on the consecutive streamlines $\dots j-1, j, j+1, \dots$ (Fig. 5). As a result of integrating the equation (52) between nodes r_j and r_{j+1} one obtains

$$c_{mj+1} = c_{mj} + \frac{h}{2}\left[F(r_j, c_{mj}) + F(r_{j+1}, c_{mj+1})\right], \tag{53}$$

where h is the step of the integration

$$h = r_{j+1} - r_j. \tag{54}$$

The h value determined from the equation of continuity by dividing all the mass flow $\dot{m}_0$ into adequate streams $\Delta\dot{m}$

$$\Delta\dot{m} = \pi h\left[(\rho\tau c_m \cos\gamma r)_j + (\rho\tau c_m \cos\gamma r)_{j+1}\right]. \tag{55}$$

Taking the above into consideration, the process of consecutive approximations proceeds, giving

$$c_{mj+1}^{(\mu+1)} = c_{mj} + 0.5\, h^{(\mu)}\left[F(r_j, c_{mj}) + F(r_{j+1}^{(\mu)}, c_{mj+1}^{(\mu)})\right] \tag{56}$$

up to the point when

$$\left|c_{mj+1}^{(\mu+1)} - c_{mj+1}^{(\mu)}\right| \leqslant \varepsilon_1,$$

where μ is the number of the steps needed for the quantity c_m to achieve the degree of the exactness equal to ε_1 whose small magnitude is assigned in the frame of the approximation.

Determining at every step, c_{mj+1} and h and calculating from the relation (54) the co-ordinate r_{j+1} on the subsequent stream-line one can proceed analogously for all the lines. The meridional velocity c_{m0} on the r_0 radius (which is the integrating constant of integration), is being chosen from the condition, that the co-ordinate of the last line should coincide with a required accuracy with the edge r_z, i.e.

$$\left|r_0 + (\sum_j h) - r_z\right| \leqslant \varepsilon_2(r_z).$$

In the similar way one has to proceed along all the chosen straight lines in all of the flow. In blade – rim areas it is recommended to accept at least two straight lines. The calculations are to be carried out in the direction of flow.

Having acquired the solution, the new co-ordinates of the stream-lines can be determined and consequently the next approximation can begin. The course of the streamlines in I approximation is obtained by dividing the field of the flow in the transverse sections into rings of the equal areas. This is the consequence of assuming a constant meridional velocity in each section.

This procedure continues up to the point when the co-ordinates of the streamlines and the meridional velocities from two consecutive iterations will fullfil the following inequalities:

$$\left|c_{mij}^{(\nu+1)} - c_{mij}^{(\nu)}\right| \leqslant \varepsilon_3(c_m),$$

$$\left|r_{ij}^{(\nu+1)} - r_{ij}^{(\nu)}\right| \leqslant \varepsilon_4(r_z),$$

where quantities ε_3, ε_4 determine the assigned accuracy of the convergence and ν indicates the number of the approximation.

In the cases in which the acquiring of an adequate convergence of the iteration process is difficult, it is advisable to determine of the quantities pertaining to the next approximation, employing the so called relaxation coefficients. An adequate formula for determining the new co-ordinates of the streamlines has a form

$$r_j = r_j^{(\nu)} + \lambda_i (r_j^{(\nu+1)} - r_j^{(\nu)}), \tag{57}$$

where λ_i is the relaxation coefficient (which value is determined by a numerical experiment) and ν is the approximation number.

6. Concluding remarks

The presented equations permit to analyze the axi-symmetrical flows in the blade channels and in the inter-rim areas of heat turbines. These equations can be regarded as general ones when the non-viscuous fluid is considered. They take indirectly into the account the viscosity of the medium, by considering the entropy gradient along the stream-line and hence the resulting mean friction force.

In posing the theoretical problem together with the mathematical difficulties there appear also many physical problems connected with determining the edge conditions; to overcome them the adequate empirical data are needed. Hence aiming at the acquisition of great accuracy of the solution is justified only when the sufficiently exact edge conditions can be determined.

A further development of the presented method of the analysis of the axi-symmetrical flows in the blade system of the heat turbines may be expected after an iterative coupling of the quasi three-dimensional flow solutions with the solution resulting from the boundary layer theory will be achieved.

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Metoda analizy osiowo-symetrycznego przepływu w kanałach łopatkowych turbin cieplnych

Streszczenie

Wykorzystując założenia teorii quasi-trójwymiarowego przepływu, w pracy przedstawiono metodę analizy osiowo-symetrycznego przepływu czynnika ściśliwego i nielepkiego w układach łopatkowych turbin cieplnych.

Przejście od zagadnienia przestrzennego do zagadnienia osiowo-symetrycznego uzyskano przez zastosowanie operacji uśrednienia wszystkich funkcji względem czasu i współrzędnej obwodowej. W wyniku tego, oddziaływanie łopatek na strumień płynu zamienia się na jednorodne wzdłuż obwodu pole masowych sił $\bar{F}_2$.

Celem przybliżonego uwzględnienia rzeczywistych własności płynów, w ramach modelu płynu idealnego, określono w kanałach łopatkowych uśrednione efekty lepkości w postaci gradientu entropii wzdłuż linii prądu. *W równaniach ruchu pojawia się przez to uśredniona siła tarcia.*

W pracy uwzględniono najbardziej ogólną strukturę strumienia. Zgodnie z tym przepływ płynu jest w ogólnym przypadku wirowy, charakteryzujący się gradientami entalpii i entropii. Stosunkowo zwięzłe ujęcie tak postawionego zagadnienia było możliwe dzięki zastosowaniu „metody krzywizny linii prądu”, która należy do najdokładniejszych spośród metod obliczeń przepływów.

W pracy podano równania osiowo-symetrycznego przepływu dla wszystkich charakterystycznych elementów łopatkowych turbiny, to jest dla obszarów wieńców łopatkowych i dla przestrzeni bezłopatkowych. Do ich rozwiązania zaproponowano metodę opartą na koncepcji rozwiązywania równań różniczkowych „metodą prostych”. Metoda ta pozwala przejść od równania o pochodnych cząstkowych

do układu równań różniczkowych zwyczajnych. Ogólna forma zapisu tych równań przedstawia się następująco:

$$c_m \frac{\partial c_m}{\partial r} + Ac_m^2 + Bc_m + C = 0\,.$$

Rozwiązanie tych równań uzyskuje się na drodze kolejnych przybliżeń. Opierając się na formule „trapezów", w pracy przedstawiono procedurę obliczeń, łatwą do zaprogramowania na maszyny cyfrowe.

Метод анализа осесимметрического течения в лопаточных каналах тепловых турбин

Резюме

Используя предположения теории квазитрехразмерного течения предъявляется метод анализа осесимметрического течения сжимаемой и невязкой среды в лопаточных системах тепловых турбин.

Переход от пространственной задачи к осесимметрической достигнут путем применения операции осреднения всех функций относительно времени и окружной координаты. В результате этой операции воздействие лопаток на поток среды заменяется на однородное по окружности поле массовых сил $\bar{F}_2$.

С целью приближенного учета действительных свойств сред, в рамках модели идеальной жидкости, в лопаточных каналах определяются осредненные эффекты вязкости в форме градиента энтропии вдоль линии потока. Таким образом в уравнениях движения появляется осредненная сила трения.

В работе учитывается наиболее общая структура потока. Согласно с этим течение жидкости в общем случае является вихревым, характеризующимся градиентами энтальпии и энтропии. Относительно простое представление поставленной задачи было возможно благодаря применению „метода кривизны линии тока", который принадлежит к наиболее точным из существующих методов расчета течений.

В работе представлены уравнения осесимметрического течения для всех характерных областей турбинной лопаточной системы, т.е. в областях лопаточных венцов и в безлопаточных пространствах. Для их решения предлагается метод, основанный на концепции решения дифференциальных уравнений „методом прямых". Этот метод позволяет перейти от уравнения с частными производными к системе обыкновенных дифференциальных уравнений. Общая форма записи этих уравнений следующая:

$$c_m \frac{\partial c_m}{\partial r} + Ac_m^2 + Bc_m + C = 0\,.$$

Решение этих уравнений происходит путем очередных приближений. В работе представлена процедура расчетов, основанная на формуле „трапеций", легко программируемая на ЭВМ.