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III НАУЧНАЯ КОНФЕРЕНЦИЯ

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ПАРОВЫЕ ТУРБИНЫ БОЛЬШОЙ МОЩНОСТИ

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A Critical Review of the Outlet Flow of Supersonic Turbine Nozzles

Nomenclature

a – sonic velocity,	w – velocity,
c_F – force coefficient (s. eq. (4)),	β – flow angle (measured against cascade front),
F – blade force,	κ – ratio of specific heats,
M – Mach number,	ν – Prandtl-Meyer angle,
M^* – dimensionless velocity w/a^* (related to the critical velocity of sound)=critical Mach number,	φ – velocity coefficient (s. eq. (10)).
N – blade force component perpendicular to the rear suction surface,	
p – pressure,	
S – blade force component in axial direction,	
s – entropy,	
T – blade force component in direction of the rear suction surface,	
t – pitch,	
U – blade force component in circumferential direction,	

Suffixes

1 – uniform inlet condition,
2 – uniform outlet condition,
A – condition behind trailing edge,
B – uniform condition at nozzle inlet,
b – base condition,
h – homentropic,
tot – total condition.

1. Introduction

Within stators of steam turbines the thermal and pressure energy of the steam is converted into kinetic energy, and transformed in the following rotor into mechanical work on the turbine shaft. This energy transfer in the rotating wheel can be accompanied by a further static pressure reduction (reaction-type) or can take place at the same static pressure level (action or Curtis type). To achieve a high work output per stage it is necessary to operate with a high pressure drop across both wheels. Parallel to the augmented pressure ratio the steam velocity increases rapidly, too, and therefore it is the highly loaded lattice row where the supersonic velocities occur for the first time.

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Already at the beginning of this century, supersonic velocities were observed in operating steam turbines. At the very beginning they occurred at the exit of the first high pressure turbine nozzles and velocity stages, used to lower the flow temperature and to reduce the thermal charge of the following stages. But soon afterwards velocities greater than the speed of sound were found in the outer part of the last low pressure stages of high output condensation steam turbines too. In the latter, the relative supersonic velocities are commonly entailed by the great outer diameters of the blades, necessary to get high performance, if the rotational speed is limited by a direct coupled generator. The great radii then lead to a high circumferential and therefore high relative inlet velocity. Beside the application of supersonic velocities in steam turbines, they are also used in turbines and supersonic compressors of air breathing engines, in the outer part of propellers, in compressors of refrigerating systems and in small turbines of very high rotational speed used for example as power unit of rocket fuel pumps.

In steam turbine applications especially the transonic turbine flow is of interest, accelerating the incoming subsonic flow to supersonic velocities at the exit. The behaviour of the supersonic downstream part of such turbines is therefore of major interest here, at design and off-design conditions. In this paper the associated phenomena are analysed using an inviscid flow model and an idealised straight oblique nozzle as supersonic part of the blade row. In the final part of this contribution a critical review is given, as much complete as possible, showing the history of the tremendous effort of scientists to understand the complex nature of the different flow configurations in transonic turbines.

2. Idealized flow in supersonic turbine nozzles

In the discussion of the supersonic flow out of turbine nozzles, some simplifying assumptions are made, as this has also been done in most of the so far published papers, dealing with this subject.

1. The classical gas dynamics shall be valid, that means the flow is inviscid, without heat convection and radiation and also external forces are not present. On the other hand shock losses and entropy rises are included. From this follows that the Reynolds number does not occur as a parameter. Besides this the flow shall be almost homoenergetic which means that the total temperature T_{tot} is constant in the whole flow field (energy equation).

2. The flow shall be plane or at least two-dimensional. In turbine rotors and stators this leads to an axisymmetric flow on cylindrical stream surfaces.

3. The flow shall be stationary.

4. The fluid shall be a caloric and therefore thermal ideal gas of constant specific heats.

Besides these principal simplifications it is further proposed that in the cross-section ($A-B$) of the blade passage, normal to the suction surface contour at point B (see Fig. 1), a uniform supersonic flow with respect to Mach number ($M_B = \text{const}$) and flow direction ($\beta_B = \text{const}$) exists. There the flow direction β_B shall be identical to the suction surface contour angle at point B and the suction surface downstream of this point up to the trailing edge E shall be straight with the same direction β_B .

In this paper only the flow field in the triangular area ($A-B-E-A$) and downstream

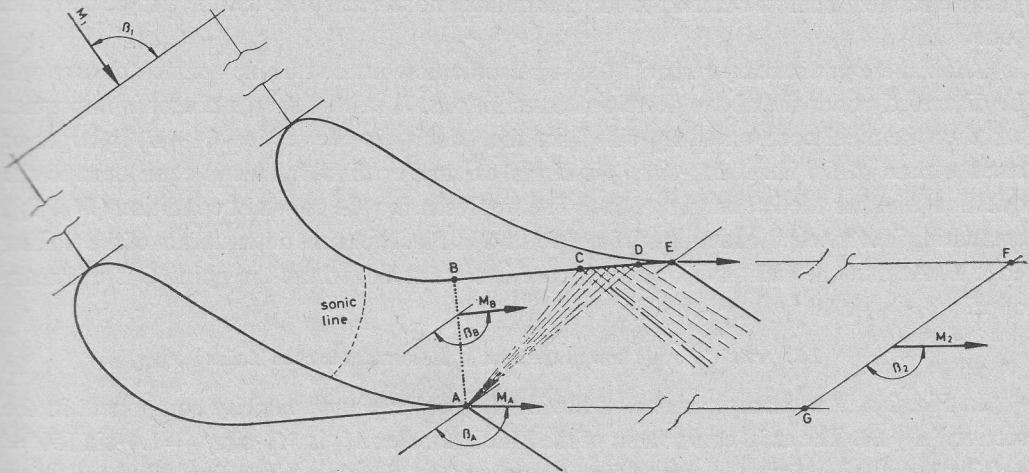


Fig. 1. Cascade nomenclature

of the cascade is discussed. The inlet flow conditions (M_1, β_1) far ahead of the cascade are of no interest here, because it is not relevant in what manner the uniform flow field (M_B, β_B) at B has been generated. All exit flow quantities are therefore related to the flow conditions in the section $(A - B)$. In the following this triangular part of the nozzle is normally called straight oblique nozzle or straight backed blades and the flow conditions (M_B, β_B, p_B) at the inlet of this area therefore are sometimes called nozzle inlet state. The supersonic flow at the nozzle inlet may be generated for instance from a subsonic inlet condition (transonic turbine) by a blade passage shaped as a Laval nozzle (see e.g. Ohlsson [43] and Neilson *et al.* [42]). However, it must be pointed out that the exact calculation of these transonic blades is difficult especially in the transitional flow region (see e.g. Griepentrog [18], Lawaczek [30], Rhomberg [46], Rigaut [47], Sampson [48], Sieverding [49]). Besides this, supersonic exit flow conditions out of cascades can also occur if the inlet flow is already supersonic (supersonic turbine or compressor).

The restriction of this paper on the nozzle exit flow with a uniform condition along $(A - B)$ and a straight suction surface along $(B - E)$ leads to an essential simplification of the flow calculation and understanding of the principle flow phenomena. The differences in relation to the general case with nonuniform inlet flow and curved suction surface are only of quantitative nature. On the other hand, the assumption of inviscid flow is much more restricting. In reality very often local or complete flow separations from the blade surfaces can be observed, especially if strong shock waves are present. Under these circumstances the real flow may be completely different from the inviscid solution. But at the moment a universally valid method is not available to solve also these real flow phenomena and therefore the presented results may be taken as first approximation only, if strong separations cannot be avoided.

Normally the two-dimensional cascade flow model is used to describe the flow in stators and rotors of axial turbomachines. Thereby the two-dimensional cascade must be infinitely long and the flow has to be periodic with respect to the blade spacing. Hence, to get com-

In gas dynamics it is well known that it is possible in supersonic flows, for instance in a Laval nozzle, to vary the exit flow without any change of the inlet condition. This independence of the inlet from the exit flow is naturally limited to a certain range and it is called the choked condition of the nozzle. In accordance to this, also the cascade flow shall be called choked if the exit flow can be varied without any influence on the inlet flow; that is why the back pressure is a free parameter. (This definition of cascade choking, see Lawaczeck [32], goes beyond the usual definition of subsonic cascade choking).

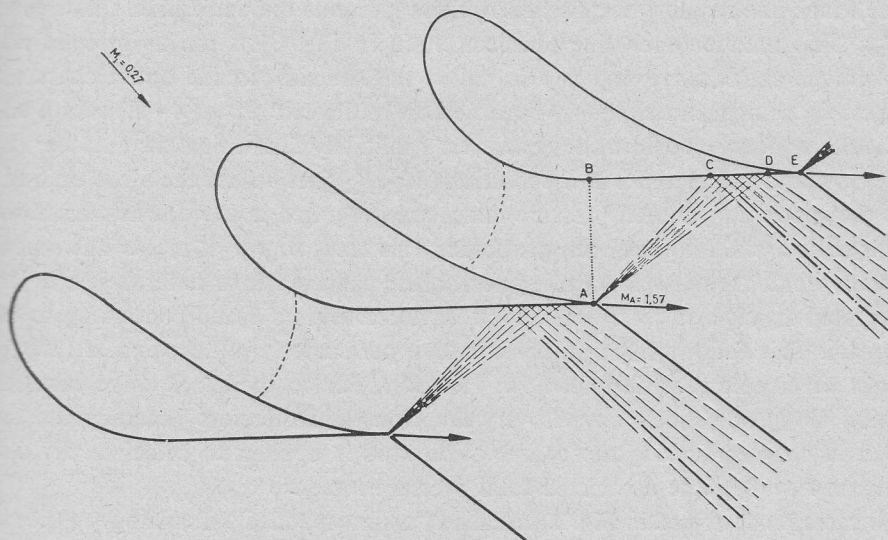


Fig. 3. Underexpanded flow pattern

$$M_B = 1.4, \beta_B = 150^\circ$$

$$M_2 = 1.6, \beta_2 = 145.7^\circ$$

When now in the flow case of Fig. 2 the back pressure or the flow angle β_A just behind the trailing edge are changed (both methods are equivalent), then the associated disturbances reach the nozzle area ($A-B-E$). In Fig. 3 such a flow configuration is shown with a reduced back pressure and therefore a reduced trailing edge flow angle β_A , too. The flow at the entry ($A-B$) of the nozzle area ($A-B-E$) remains unchanged because the flow downstream of the left running Mach line ($A-C$) through A only can be influenced from behind. Due to the flow turning from β_B to β_A , a centered Prandtl-Meyer expansion occurs at the trailing edge pressure side. Thereby the first left running wave is identical to the Mach line ($A-C$), which belongs to the flow condition in the section ($A-B$). The last expansion wave of the fan is marked as ($A-D$). With the exception of the first one all the Mach lines of the expansion fan are partly curved, because near the suction surface they are crossed by the reflected waves starting from the straight surface along ($C-D$). Only when this surface would have been shaped like a streamline in the expansion fan, these reflections could have been avoided.

In the proposed case, however, the flow at the trailing edge suction-side has still the direction β_B . On the other hand the flow downstream of the trailing edge has a reduced flow angle β_A due to the expansion fan. As a result an oblique right running shock wave is generated which starts at the trailing edge E and propagates downstream of the cascade.

Downstream of E the flow angle must be equal to β_A . This condition follows from the periodicity requirement, that is, the flow must be completely periodic with respect to one spacing. In case of single nozzles or limited numbers of blades this condition is not always satisfied in the whole backward flow field.

Downstream of the cascade exit plane ($A-E$) the right running oblique shock waves are attenuated by the reflected expansion fans, whereby the nonuniformity of the flow field also decreases with increasing distance away from the cascade. Infinitely far behind only one Mach line remains uncanceled, which represents the uniform exit flow condition (M_2, β_2). This unique Mach line (dashed-dotted in Fig. 3) is the asymptotic position of the oblique shock wave and is also called the neutral cascade outlet characteristic.

It must be pointed out explicitly that the flow in the exit plane ($A-E$) is non-uniform and that the uniform exit flow does occur only infinitely far behind the cascade whereby the latter is not identical to the flow condition (M_A, β_A) just behind the blade trailing edge. Due to the shock waves, total pressure losses are generated in the downstream flow field although an inviscid fluid has been proposed. This leads to $p_{\text{tot } 2} \leq p_{\text{tot } B}$. The amount of total losses, which the mass flow through one blade passage has to suffer, is just as large as the integrated shock loss along one shock wave starting in E and continuing to infinity downstream. This follows from an investigation performed by Klapproth [29] for the analogous supersonic cascade inlet flow. However, due to the shock wave curvature an integration along one shock wave is very laborious and therefore generally the method described in chapter 4 using the conservation laws is applied to calculate the uniform downstream flow and the associated total pressure losses.

Flow cases, as described in Fig. 3 are called "underexpanded", in analogy to the normal Laval nozzle flow. The static pressure p_B at the upstream side of the trailing edge, corresponding to the Mach number M_B , is higher than the pressure p_A at the trailing edge downstream side, so that there the flow expands further in an expansion fan. (The flow case shown in the preceding example is called an adapted flow condition, presuming that neither a compression nor an expansion occurs at the trailing edge pressure side).

One important fact of the example under consideration is, that the last Mach line of the expansion fan is still within the nozzle, i.e. point D is in front of E . On the other hand, all the following left running characteristics have to come again from the uniform outlet condition infinitely far downstream. Hence the axial outlet flow component must be again subsonic for the same reasons as in the preceding example.

It is therefore possible to reduce further the back pressure or the flow angle β_A . However, a special singular point is reached, when the last Mach line of the expansion fan just passes through the trailing edge E of the adjacent blade and therefore point D coincides with E (limit loading). The corresponding flow pattern is shown in Fig. 4. Again the Mach number M_B remains unchanged. When in this example the uniform outlet flow is calculated, it results that the axial velocity (component) is just sonic, i.e. $M_2 \cdot \sin \beta_2 = 1$. This is also the physical reason, why no further reduction of the back pressure is possible, because no disturbances can travel upstream in the axial direction and can influence the nozzle flow. This is also obvious from the position of the Mach lines in Fig. 4, where the expansion fan would move behind the cascade at any lower back pressure, but the blade pressure distribution remains unchanged. Above all, such expansion fan behind the cascade would

violate the periodicity condition. The exit flow pattern of the axial sonic flow case is principally the same as in the preceding example, only the shock losses are still higher. In chapter 4 it is shown that the losses have in this limiting case a relative maximum.

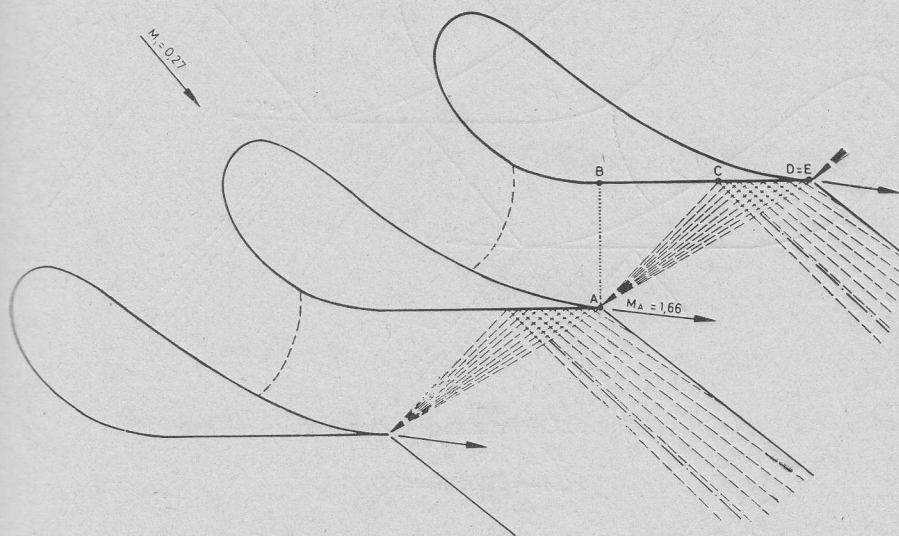


Fig. 4. Flow pattern at limit loading

$$M_B = 1.4, \quad \beta_B = 150^\circ$$

$$M_2 = 1.68, \quad \beta_2 = 143.4^\circ$$

In all examples discussed so far the back pressure setting has been lower than the pressure in the nozzle section ($A-B$). However, if in ($A-B$) supersonic velocity exists, also flow configurations are possible where $p_2 > p_B$ or $\beta_A > \beta_B$. Such a flow is presented in Fig. 5 again for the same condition (M_B, β_B) in the nozzle. Upstream of the trailing edge A the flow angle at the pressure surface is equal to β_B , whereas downstream of A the increased flow angle β_A shall occur at the pressure side. Therefore an oblique left-running shock wave starts in A , is reflected at the suction surface of the adjacent blade, and runs downstream out of the cascade. Due to the periodicity condition, a centered Prandtl-Meyer expansion with right running Mach lines is necessary at the suction surface trailing edge, which attenuates the shock wave until far downstream the uniform outlet flow (M_2, β_2) is achieved. There the shock wave is completely cancelled and is changed into the neutral characteristic as asymptotic position. Fig. 3 and 5 can be considered as equivalent flow cases where expansion fan and shock wave are interchangeable. Because now the back pressure p_2 is larger than p_B and owing to the shock wave this flow case is called "overexpanded" (again in analogy to the usual Laval nozzle flow).

If the back pressure is further increased, at a certain point sonic velocity occurs locally

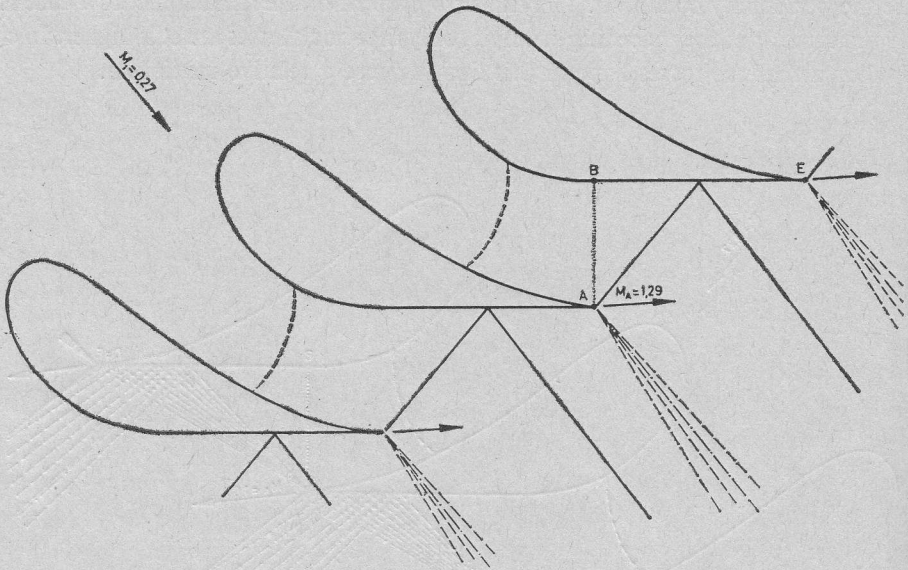


Fig. 5. Overexpanded flow pattern

$$M_B = 1.4, \quad \beta_B = 150^\circ$$

$$M_2 = 1.25, \quad \beta_2 = 151.9^\circ$$

in the triangular nozzle section. This occurs first at the suction surface where the oblique shock wave is reflected. At any further increased back pressure a regular shock reflection is impossible and the so called Mach type reflection occurs (Fig. 6). Consequently the

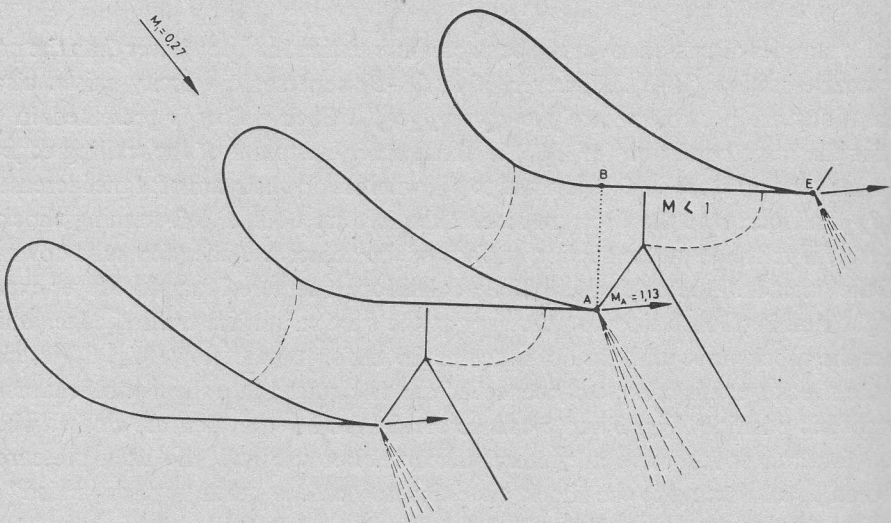


Fig. 6. Principal flow configuration with local subsonic velocities in the exit region

areas of the subsonic flow are formed within the nozzle. A qualitative flow pattern of this type is shown in Fig. 6. With increasing back pressure this local subsonic flow field enlarges and the associated normal shock is extended. The maximum pressure rise is obviously reached when the normal shock wave crosses the whole nozzle section and decelerates the

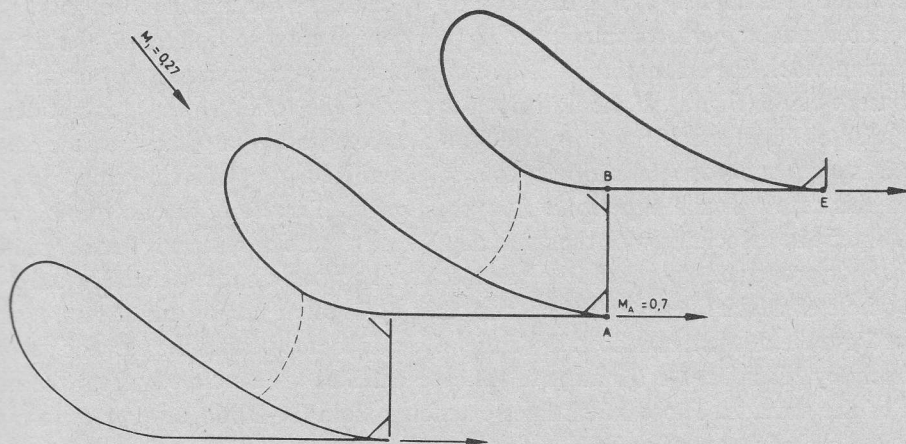


Fig. 7. Flow pattern at maximum back pressure (normal shock)

$$M_B = 1.4, \beta_B = 150^\circ$$

$$M_2 = 0.7, \beta_2 = 150^\circ$$

flow (M_B, β_B) without turning, that is $\beta_2 = \beta_B$, down to subsonic velocity (see Fig. 7). Because in the adapted flow case (Fig. 2) as well as at maximum back pressure the flow turning $\delta = \beta_B - \beta_2$ between the nozzle inlet ($A-B$) and infinitely far down is equal to zero, whereas in the intermediate range negative values are obtained, there must be a maximum negative turning somewhere in between (see also chapter 4). In the flow configuration with positive turning (Fig. 3 and 4) a positive maximum value is reached at the limiting axial sonic condition. Negative turning angles are of course only possible if $M_B > 1$. If this condition is not satisfied, for instance at $M_B = 1$ in section ($A-B$) any small increase in back pressure would reach the sonic line, thereby altering the inlet flow, and the sonic condition in the throat would disappear.

After the discussion of the characteristic nature of the supersonic flow in the triangular exit region of a turbine nozzle under axial subsonic conditions ($M_B \cdot \sin \beta_B < 1$), in the following chapter the axial supersonic flow case is covered.

2.2. Axial supersonic flow at the inlet of the straight backed blades

In Fig. 8 a cascade is shown which is characterized by a larger geometric area ratio between ($A-B$) and the throat. Because of the proposed subsonic inlet flow velocity this is the only way to increase the Mach number in ($A-B$). In other words, there exists only one possible supersonic flow condition (M_B, β_B) for each cascade geometry. (In this considera-

tion the small amount of sonic line variation is neglected, occurring if the subsonic inlet flow angle β_1 and Mach number M_1 are varied along the choking line. In this case also the uniform flow assumption in $(A-B)$ is no longer possible because each supersonic parallel flow requires a different shaped Laval nozzle.) If, however, the inlet flow velocity is already supersonic (as for instance in compressor cascades) then the flow Mach number in $(A-B)$ can be varied continuously at a fixed geometry by changing the inlet flow velocity. Due to the proposed axial supersonic velocity component in the cross section $(A-B)$ the respective left running Mach line emanating in A leaves the blade passage and does not interfere with the adjacent blade (Fig. 8). If the flow downstream of the trailing edge A is not deflected at all (adapted flow) a uniform flow field within and downstream of the cascade develops which is surely one possible periodic flow solution. Also the uniform outlet flow must have in this case an axial supersonic flow component. Therefore, the flow configuration of Fig. 8 must be associated with the minimum back pressure because any further reduction of p_2 cannot propagate upstream into the cascade. This can also be realized at once from Fig. 8, where all Mach lines in the blade exit plane do begin at suction and pressure surfaces and not from the downstream flow. Even the whole flow area behind the cascade is covered completely by Mach lines originating in the cascade. This results directly from the infinite length of the cascade. Those Mach lines extending from $(A-E)$ do cover a certain range behind the cascade, repeating itself periodically in the tangential direction at the distance of pitch t .

Although it is now impossible to decrease the back pressure below the value of the above example, an increase is possible because shock waves can propagate upstream against an axial supersonic flow. This leads to back pressures p_2 and trailing edge flow angles β_A (just behind A) higher than those in Fig. 8. Such a case is shown in Fig. 9. A great similarity exists between this flow configuration and that of Fig. 5, especially all left running Mach lines behind the trailing edge shock wave $(A-D)$ do come from infinity downstream, which means that the related outlet flow must have an axial subsonic velocity component.

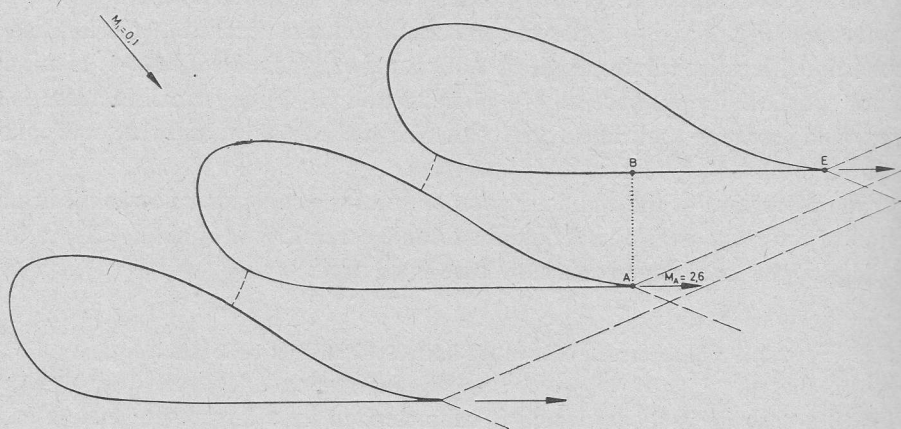


Fig. 8. Adapted flow configuration at axial supersonic outlet velocity (unique deviation)

$$M_B = 2.6, \beta_B = 150^\circ$$

$$M_2 = 2.6, \beta_2 = 150^\circ$$

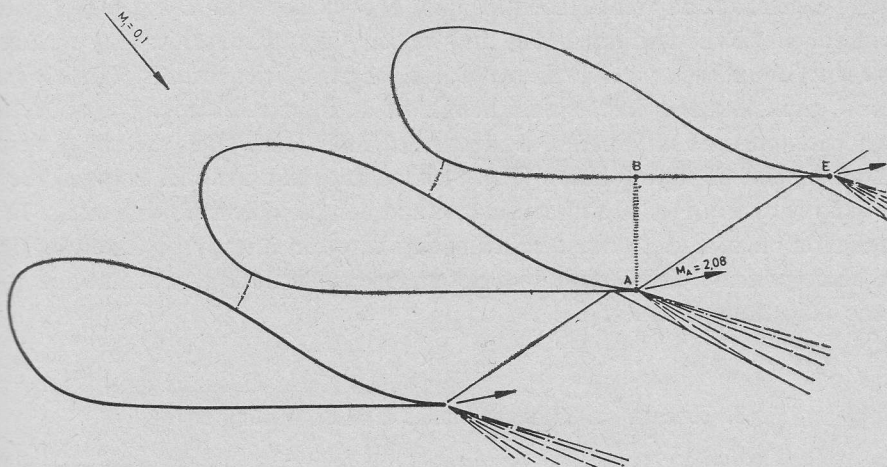


Fig. 9. Overexpanded flow pattern

$$M_B = 2.6, \quad \beta_B = 150^\circ$$

$$M_2 = 2.03, \quad \beta_2 = 161.0^\circ$$

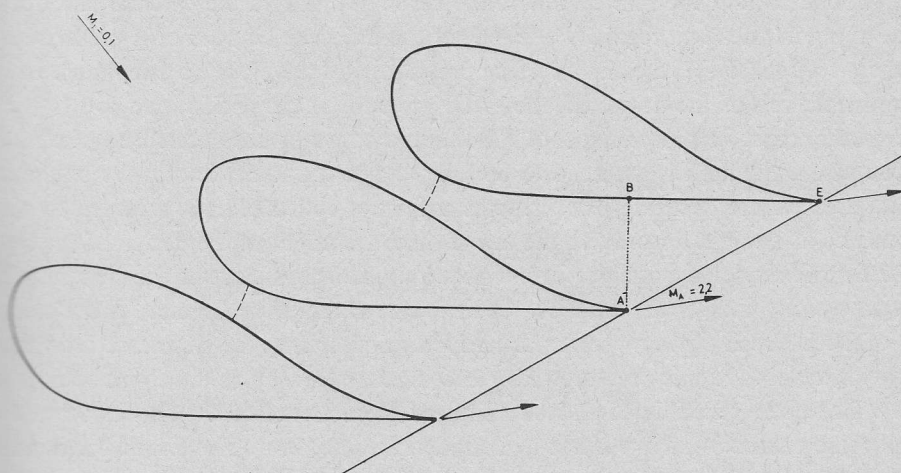


Fig. 10. Flow pattern at limit loading (axial subsonic limit)

$$M_B = 2.6, \quad \beta_B = 150^\circ$$

$$M_2 = 2.2, \quad \beta_2 = 159.2^\circ$$

At any further increased back pressure there is in principle no difference to the above described flow cases of Fig. 6 and 7. Again sonic velocity occurs first downstream of point B at a certain back pressure setting. At higher static downstream pressure local subsonic areas with Mach reflections do occur and finally a normal shock wave crosses the whole section (A-B). On the other hand the flow behaviour at a reduced back pressure is completely different because at a certain value of p_2 the left running trailing edge shock wave just meets

the trailing edge of the adjacent blade (i.e. point D coincides with E , Fig. 10). The related axial component of the uniform outlet flow velocity is still subsonic. Any further continuous reduction in back pressure is impossible for periodicity reasons. The left running shock wave cannot extend downstream in case of an infinite cascade. The above shown bound in back pressure is therefore again named limit load. Only the above described single axial supersonic flow solution (Fig. 8) is additionally possible. Between the latter and the limiting case of Fig. 10 a discontinuous and non periodic transition exists. That the flow configuration of Fig. 10 must be a limiting case can already be stated by the fact that the blade pressure distribution does not change at all during this transition to axial supersonic outlet flow velocities.

3. Cascade exit flow for blades of thick trailing edges

In the preceeding chapter the nozzle flow has been idealized assuming a partly straight suction surface and a chamfered trailing edge. These assumptions shall be dropped now and a qualitative description of the flow field in the vicinity of trailing edges of finite thickness is given.

In the flow configuration shown in Fig. 11 the Mach number at the edge point P shall be high enough so that the related Mach line through P leaves the cascade and does not cross the triangular dead air region ($P-E-S$) of the adjacent blade. For a prescribed flow in the cross-section ($A-B$) the blade surface pressures can therefore be calculated. In order to determine the rest of the flow field, the static pressure in the dead air region ($P-E-S$) (base pressure) must be prescribed too. Assuming a constant base pressure p_b in the dead air region where the fluid is assumed to be fully at rest the upper and lower boundary of this finite area can be accurately constructed as lines of constant static pressure (free jet) or approximated as straight lines. With the aid of these assumptions the design of the cascade exit flow field is rendered possible. Two oblique shock waves start at the closure point E and extent downstream of the cascade. At the point E the static pressure is the same upstream of both shock waves whereas the Mach number may be different. Downstream of the shock waves the static pressures have to be again equal (p_E) and a common flow angle β_E exists. However, generally the Mach numbers downstream of the shocks are different and a slip-line emanating from E has to be taken into account. The shock waves and expansion waves cancel each other downstream of the cascade and at infinity a uniform flow condition (M_2, β_2) exists. Because in this case no Mach lines of the uniform flow (M_2, β_2) do reach the plane ($E-E$) the axial velocity component of the outlet flow has to be supersonic. The quantitative results of M_2 and β_2 depend essentially on the magnitude of the base pressure p_b . In the above description this pressure was chosen arbitrarily, but, due to the axial supersonic outlet flow it cannot be influenced from behind, so that it seems to exist a contradiction. Regarding the supersonic flow around single profiles or axisymmetric bodies of finite trailing edge thickness this ostensible difficulty vanishes at once. There it is well known that the base pressure is defined completely by the inlet flow and the profile geometry and cannot be influenced from backwards. Transferring this result to the cascade flow with supersonic axial outlet velocity components, also there the amount of base

pressure cannot be chosen arbitrarily, but is prescribed by the inlet condition and the blade shape.

Unfortunately a broad and satisfactory quantitative solution is not available so far even in the flow cases of single profiles and bodies.

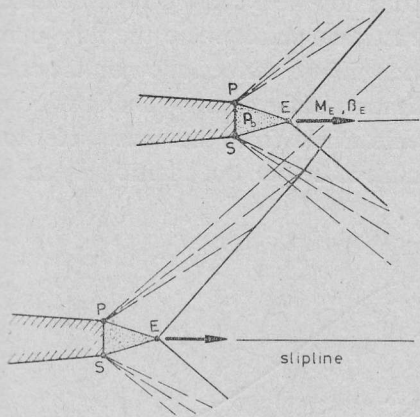


Fig. 11. Principal wave pattern at blunt trailing edges for axial supersonic outlet velocity

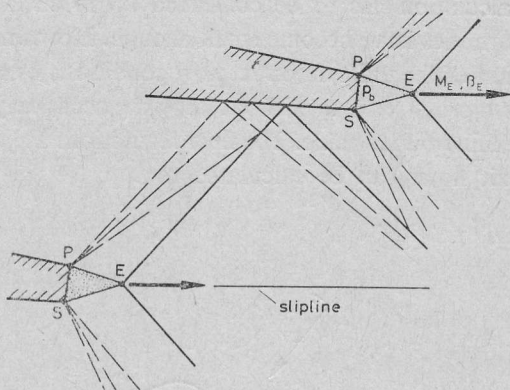


Fig. 12. Principal wave pattern at blunt trailing edges for axial subsonic outlet velocity

If the axial velocity component of the outlet flow is subsonic left running Mach lines emanating in this uniform flow field cross the plane ($E-E$) (Fig. 12). In these cases the back pressure p_2 or the flow angle β_E , which mainly determines this back pressure, is a free parameter (s. chapter 2). But in contrast to the axial supersonic flow now a chosen base pressure p_b does not fix the flow angle β_E (s. Curtis *et al.* [9], Meyer [40a]), so that the determination of the unknown base pressure is again not possible within the inviscid flow model. The detailed calculation of the nozzle exit flow itself has to be performed iteratively. This is necessary because the shape of the triangular dead air region depends on the suction surface Mach number distribution, which itself is influenced by the base pressure and the position of the point E . Again the conditions of the same static pressure p_E and same flow direction β_E on both sides of the slip line emanating from E have to be fulfilled.

The further computation of the downstream flow field does not impose any principle difficulty and therefore the exit flow (M_2, β_2) can be calculated as a function of the proposed flow angle β_E and the associated base pressure p_b . The limiting condition at minimum back pressure at axial subsonic velocity (limit loading) is reached, when the left running shock wave starting in E just passes through the point E of the adjacent blade.

The description of the flow along rounded trailing edges can be performed in quite the same way as for the blunt blades, only the centered Prandtl-Meyer expansions have to be substituted by an expansion around the trailing edges down to the profile coordinates where the base pressure p_b occurs.

4. Calculation of the downstream flow by application of the conservation laws

In section 2 the method of characteristics has been used to determine the supersonic flow field within and behind the nozzle. By this treatment a detailed description of the flow field is obtained. On the other hand the determination of the uniform outlet conditions infinitely far downstream is very tedious in using this flow field construction, because the calculation has to be continued up to such downstream plane where the flow nonuniformities have become small enough. Now a method will be described allowing the calculation of the uniform outlet flow conditions by application of the conservation laws between two cross-sections without knowing accurately the flow field from the nozzle inlet to the infinite outlet plane. The flow restrictions are the same as before, but additionally subsonic exit flows can be calculated too.

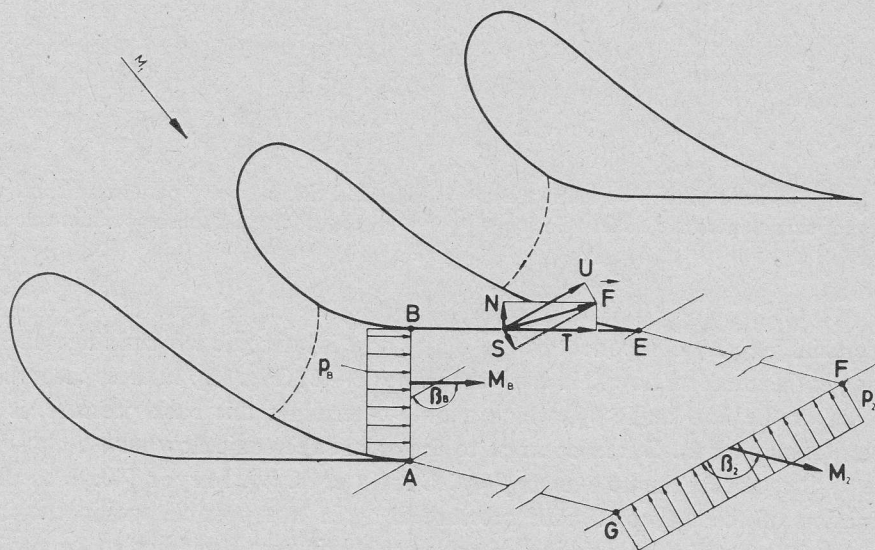


Fig. 13. Scheme defining the control volume and the blade force components for the method of conservation laws

The conservation laws are applied to a control surface as shown in Fig. 13. The upstream bound coincides with the cross-section $(A-B)$ perpendicular to the straight suction surface $(B-E)$. The flow conditions at this place are uniform and indicated by the letter (M_B, β_B, p_B) , where β_B is the angle of the suction surface $(B-E)$ measured against the exit front $(A-E)$. The upper and lower boundaries are formed by the periodic stagnation streamlines through the trailing edges A and E . At infinity downstream that part $(G-F)$ of a parallel to the exit front acts as control surface which is cut off by the two boundary streamlines. At this plane again a uniform flow condition exists (M_2, β_2, p_2) . The forces acting on this control volume are:

- the static pressure p_B normal to the upstream boundary,
- the blade force $\vec{F}$ exerted by the flow on the rear part $(B-E)$ of the suction surface,

c) the uniform static pressure p_2 normal to the downstream plane ($G-F$). The pressure forces at both of the two streamlines ($E-F$) and ($A-G$) are cancelled, due to periodicity. Under these assumptions the continuity equation can be written as follows:

$$\frac{p_{\text{tot } 2}}{p_{\text{tot } B}} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right]^{1/(\kappa-1)} M_2^* \sin \beta_2 = \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right]^{1/(\kappa-1)} M_B^* \sin \beta_B, \quad (1)$$

M^* , named as critical Mach number, is a dimensionless velocity. But due to the assumed homoenergetic flow ($T_{\text{tot}} = \text{const}$), here it is a unique relation of the Mach number.

The application of the momentum equation in axial direction (perpendicular to the cascade front) yields:

$$\begin{aligned} \frac{p_{\text{tot } 2}}{p_{\text{tot } B}} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right]^{1/(\kappa-1)} \left\{ \frac{\kappa-1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right] + M_2^{*2} \sin^2 \beta_2 \right\} = \\ = \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right]^{1/(\kappa-1)} \left\{ \frac{\kappa-1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right] + M_B^{*2} \right\} \sin^2 \beta_B + c_S \end{aligned} \quad (2)$$

and applying the momentum equation in a direction parallel to the cascade front yields:

$$\begin{aligned} \frac{p_{\text{tot } 2}}{p_{\text{tot } B}} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right]^{1/(\kappa-1)} M_2^{*2} \sin \beta_2 \cos \beta_2 = \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right]^{1/(\kappa-1)} \times \\ \times \left\{ \frac{\kappa-1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right] + M_B^{*2} \right\} \cos \beta_B \sin \beta_B - c_U. \end{aligned} \quad (3)$$

The first parts within the parentheses designate the pressure forces acting on the up- and downstream boundaries, whereas c_S and c_U are dimensionless coefficients of the axial part S and the circumferential component U of the blade force $\vec{F}$. The definition of the force coefficients reads:

$$c_F = \frac{\kappa+1}{2\kappa} \left(\frac{\kappa-1}{\kappa+1} \right)^{1/(\kappa-1)} \frac{F}{B t p_{\text{tot } B}} \quad (4)$$

with t as blade pitch and B as width normal to the considered flow plane.

Between the force components U and S and the components N perpendicular and T parallel to the suction surface ($B-E$) the following geometrical relation holds:

$$c_N = c_U \sin \beta_B + c_S \cos \beta_B, \quad c_T = -c_U \cos \beta_B + c_S \sin \beta_B. \quad (5)$$

Because in this paper inviscid flow is presumed, the force component T , due to frictional effects, in direction of the suction surface has to vanish. Hence:

$$c_T = 0. \quad (6)$$

Introducing equation (6) and (5) into equation (2) and (3) and combining them in such a way, that the force component c_N is eliminated, the following equation can be found:

$$\frac{p_{\text{tot}2}}{p_{\text{tot}B}} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right]^{1/(\kappa-1)} \left\{ \frac{\kappa-1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right] \sin \beta_B + M_2^{*2} \sin \beta_2 \cos(\beta_B - \beta_2) \right\} =$$

$$= \frac{\kappa+1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right]^{1/(\kappa-1)} [M_B^{*2} + 1] \sin \beta_B. \quad (7)$$

This equation could be derived also directly applying the momentum equation in direction of the suction side ($B-E$).

Eliminating $p_{\text{tot}2}/p_{\text{tot}B}$ in equation (1) and (7) yields after some conversions (s. Lawa-zek [31]):

$$M_2^\kappa = \frac{\kappa+1}{4\kappa} \left\{ M_B^* \left[\sin \beta_2 \cos(\beta_B - \beta_2) - \frac{\kappa-1}{2\kappa} \sin \beta_B \right] \right\}^{-1} \left\{ (M_B^{*2} + 1) \sin \beta_2 \pm \right.$$

$$\left. \pm \sqrt{(M_B^{*2} + 1)^2 \sin^2 \beta_2 - \frac{8\kappa}{\kappa+1} M_B^{*2} \sin \beta_B \left[\sin \beta_2 \cos(\beta_B - \beta_2) - \frac{\kappa-1}{2\kappa} \sin \beta_B \right]} \right\}. \quad (8)$$

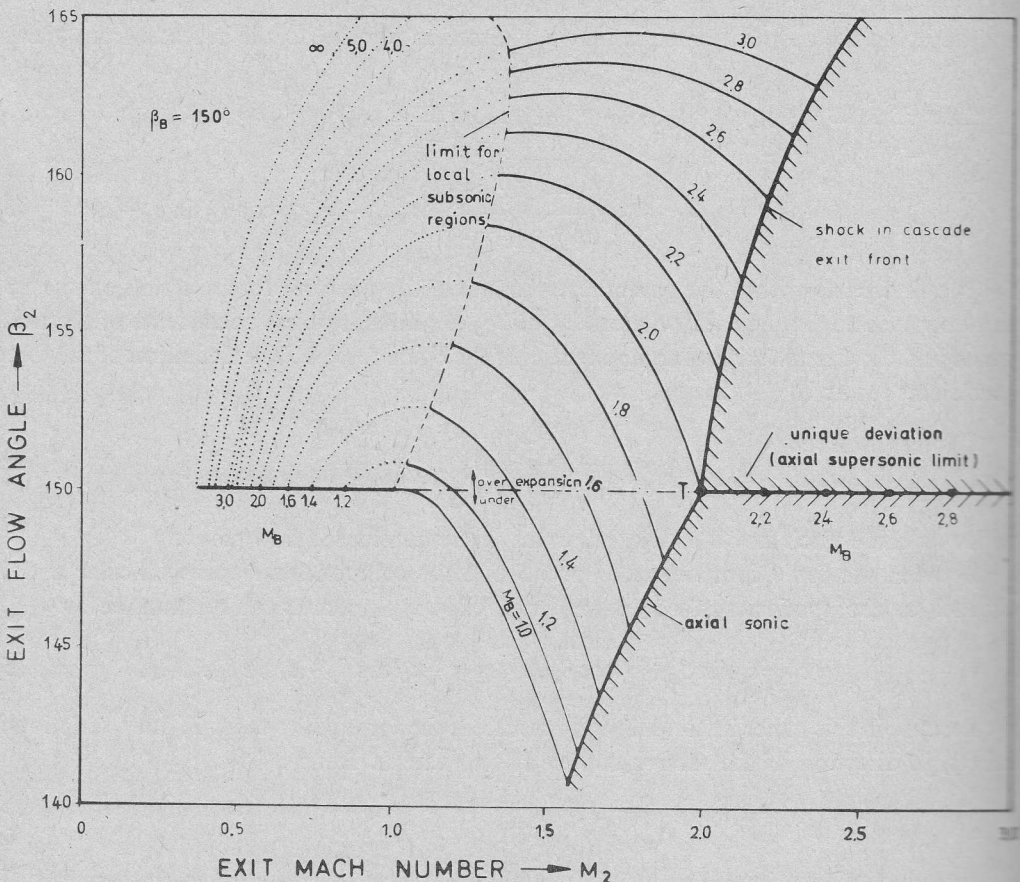


Fig. 14. Outlet flow angle in dependency of the outlet flow Mach number

If the geometrical angle β_B and the critical flow Mach number M_B^* are prescribed, equation (8) allows the calculation of the critical outlet Mach number M_2^* for each chosen outlet flow angle β_2 . Inserting this computed M_2^* into equation (1) or (7) finally also yields the total pressure ratio $p_{\text{tot}2}/p_{\text{tot}B}$, a measure of the shock losses occurring between the inlet and outlet control plane. The free selection of the far downstream flow angle β_2 is justified at supersonic inlet conditions because the back pressure is an independent parameter which can be varied without alteration of the inlet condition. It is possible, of course, to prescribe β_2 instead of p_2 , as for each flow angle also the critical Mach number and the total pressure ratio can be determined by equation (8) and (1) and so the associated back pressure is fixed by β_2 , too. Due to the double square root sign, two solutions M_2^* are calculated for each outlet flow direction β_2 . Both of them can be physically relevant. On the other hand the decision which of the two actual solutions is valid, cannot be given by means of the above equations expressing the conservation principle, but only by a fully two-dimensional flow consideration. To get a correct solution it is therefore necessary to combine the above solutions with the limiting conditions derived in chapter 2.

In Fig. 14 the results given by equation (8) are plotted for one nozzle inlet flow angle β_B but different Mach numbers M_B . The four strong lines through the point T are the limiting conditions for minimum or maximum back pressure and therefore equation (8) will

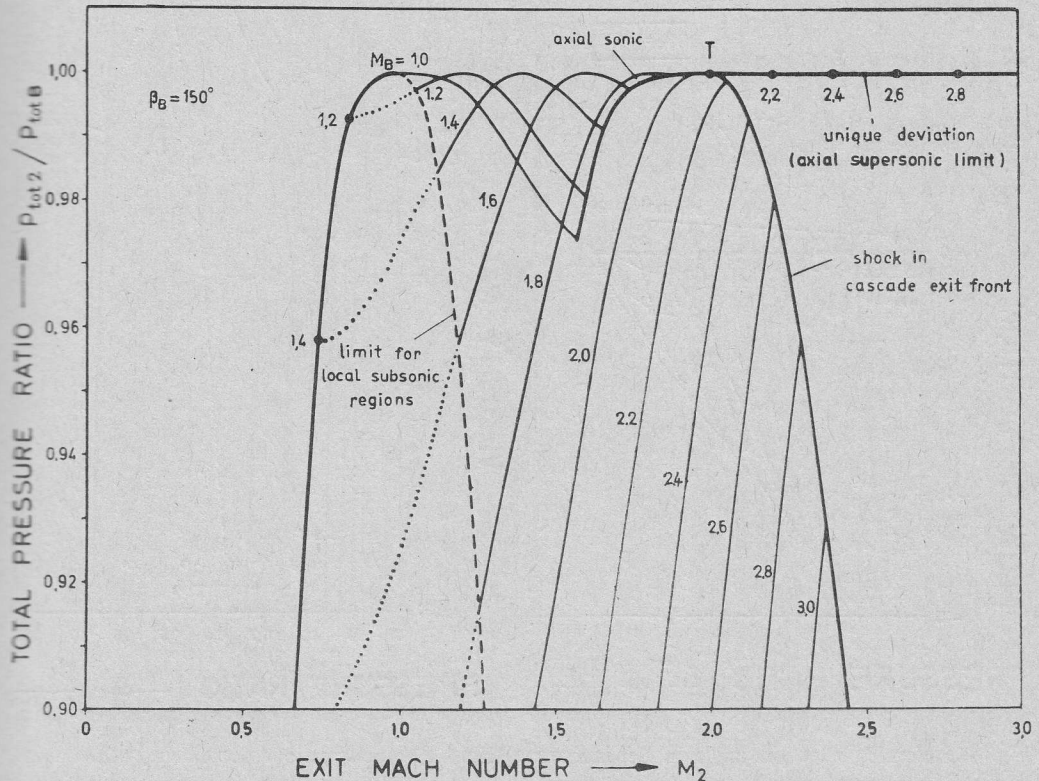


Fig. 15. Nozzle total pressure ratio in dependency of the outlet flow Mach number

lead to physically relevant results within the above mentioned bounds. At the lower limit line the flow conditions lead to an axial sonic condition. All throttle curves ($M_B = \text{const}$) ending on this line belong to flow configurations having an axial subsonic component ($M_B \cdot \sin \beta_B \leq 1$) in the control plane ($A-B$), and at the boundary itself the last Mach line of an expansion fan just connects two adjacent trailing edges (Fig. 4). The upper limiting line can be calculated, applying the oblique shock relations to such configurations, at which the flow conditions in front of the discontinuity have the uniform values (M_B, β_B), and additionally the shock wave itself coincides with the cascade exit front. An example of such an operational mode is shown in Fig. 10. This limit represents the lowest possible back pressure at axial subsonic outlet conditions. The limiting line going to the right represents the single flow configurations with axial supersonic outlet components. Here, in the case of simple straight oblique nozzles with constant conditions (M_B, β_B), the respective limit conditions are identical with the inlet conditions of the control volume (trivial solution $M_2 = M_B$ and $\beta_2 = \beta_B$). To be in accordance with the supersonic compressor or turbine nomenclature this single solution is called unique deviation. Between the upper and the

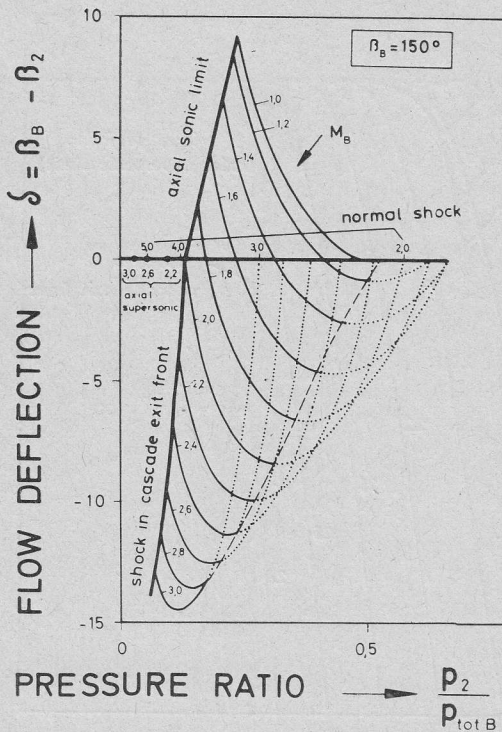


Fig. 16. Nozzle deflection angle in dependency of the nozzle static outlet to total inlet pressure ratio

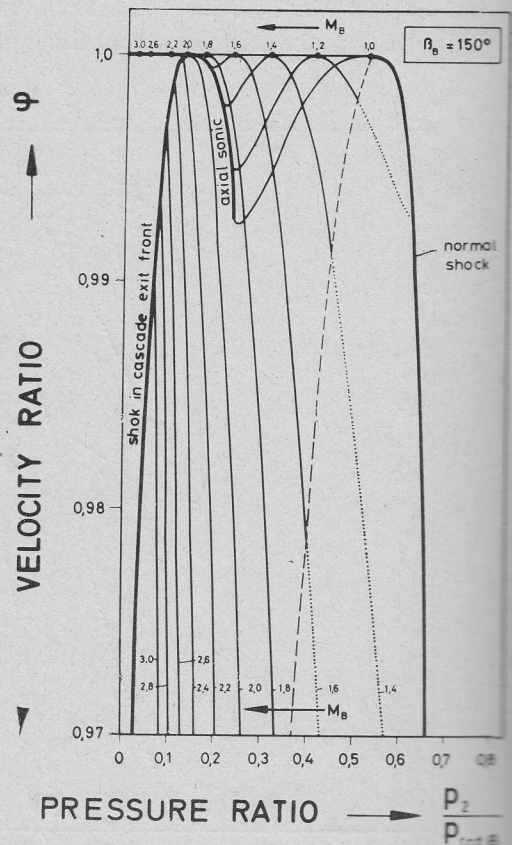


Fig. 17. Velocity ratio in dependency of the nozzle static outlet to total inlet pressure ratio

right boundary no periodic solutions are possible. The left running boundary-line results from the relations of a normal shock, with $\beta_2 = \beta_B$ and represents the maximum possible back pressure as long as M_B is constant.

Using equation (8) it can be shown that the axial sonic line $M_2 \cdot \sin \beta_2 = 1$ (lower limit) at the same time is a line of maximum circumferential velocity or critical Mach number $-M_2^* \cdot \cos \beta_2$. Moreover by application of equation (1) and (8) it is possible to prove, that the total pressure ratio $p_{\text{tot } 2}/p_{\text{tot } B}$ will become an extremum, if

$$\sin(\beta_B - \beta_2) \left[1 - \frac{2}{\kappa - 1} \frac{M_2^{*2} \sin^2 \beta_2}{\kappa + 1} - M_2^{*2} \right] = 0$$

is fulfilled. This condition holds if either

a) $2/(\kappa - 1) M_2^{*2} \sin^2 \beta_2 / ((\kappa + 1)/(\kappa - 1) - M_2^{*2}) = 1$

or if

b) $\beta_2 = \beta_B$.

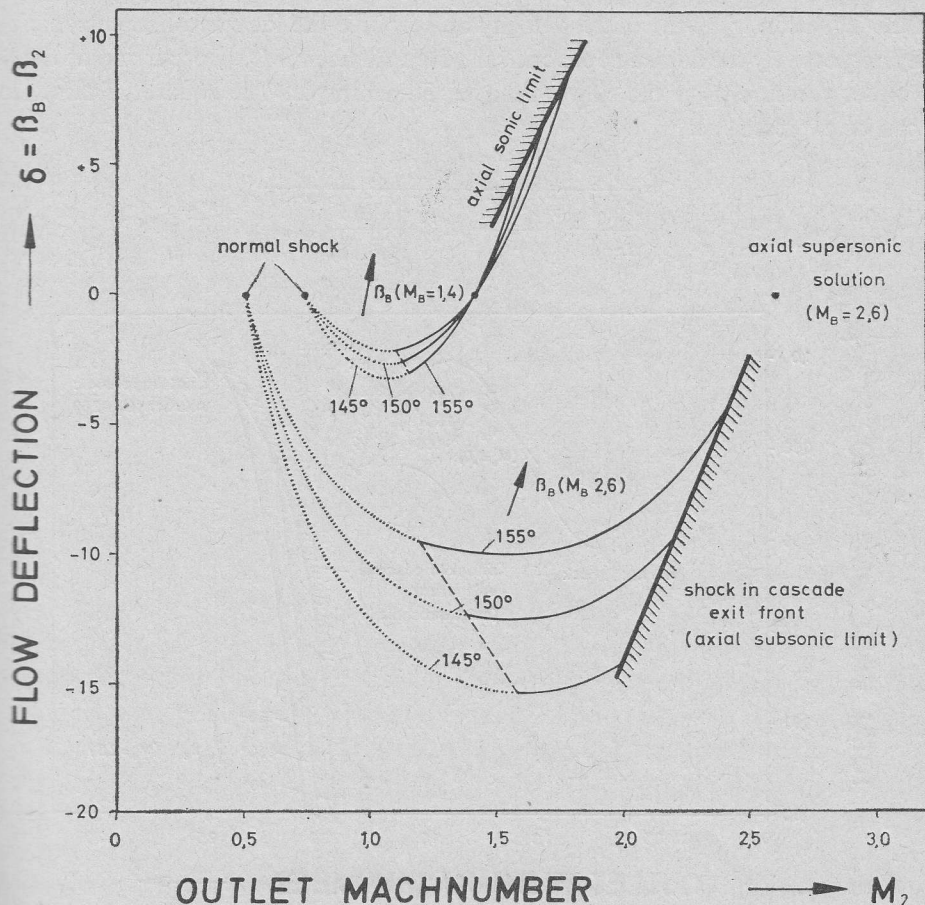


Fig. 18. Nozzle deflection angle for three different nozzle inlet flow angles β_B

The relation a) again means axial sonic condition, so that the lower limiting line is also an extremum of the total pressure ratio, whereas the relation b) inserted into equation (8) yields the two solutions $M_2^* = M_B^*$ and $M_2^* = 1/M_B^*$. The first is fulfilled for the trivial axial supersonic limiting line and the second holds for the normal shock wave, hence on the left running boundary line.

In Fig. 15 the total pressure ratios $p_{\text{tot } 2}/p_{\text{tot } B}$, associated to the flow cases of Fig. 14, are plotted as function of the outlet Mach number. Additionally the above discussed limiting lines are included in the figure. In Figs. 16 and 17 the same results are shown for other variables. In these diagrams $\delta = \beta_B - \beta_2$ is the flow deflection of the oblique nozzle part; $p_2/p_{\text{tot } B}$ is the ratio of the static outlet to the total inlet pressure and can be expressed as:

$$p_2/p_{\text{tot } B} = \left(\frac{\kappa - 1}{\kappa + 1} \right)^{\kappa/(\kappa - 1)} \left(\frac{p_{\text{tot } 2}}{p_{\text{tot } B}} \right) \left[\frac{\kappa + 1}{\kappa - 1} - M_2^{*2} \right]^{\kappa/(\kappa - 1)} \quad (9)$$

The velocity ratio is defined as:

$$\varphi = w_2/(w_2)_h \text{ and is here equal to } \varphi = M_2^*/(M_2^*)_h.$$

In this equation w_2 is the outlet velocity and $(w_2)_h$ is the idealized homentropic outlet velocity, respectively the homentropic critical Mach number $(M_2^*)_h$, occurring at the same static outlet pressure p_2 if the flow would be homentropic. The square of this velocity ratio can be expressed as:

$$\varphi^2 = \frac{M_2^{*2}}{\frac{\kappa + 1}{\kappa - 1} - \left[\frac{\kappa + 1}{\kappa - 1} - M_2^{*2} \right] \left(\frac{p_{\text{tot } 2}}{p_{\text{tot } B}} \right)^{(\kappa - 1)/\kappa}} \quad (10)$$

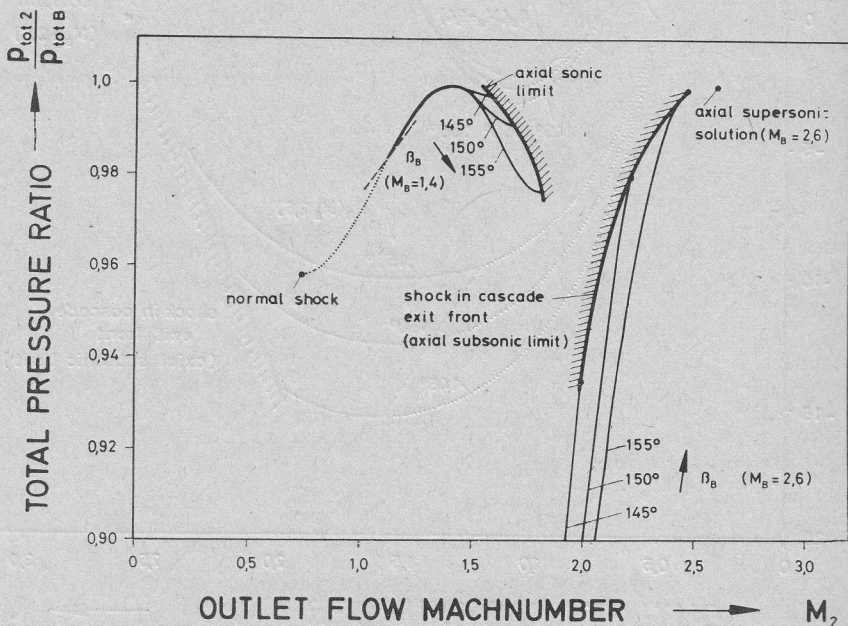


Fig. 19. Nozzle total pressure ratio for three different nozzle inlet flow angles β_B

In the following Figs. 18 and 19 the deflection δ and the total pressure ratio $p_{\text{tot } 2}/p_{\text{tot } B}$ are shown as functions of the outlet Mach number. Now also the suction side angle β_B is varied, but only two different Mach numbers M_B are included.

5. Historical review of the different steps in the development of supersonic turbine nozzle flow calculation methods

By the foregoing considerations the fundamental statements of the flow within and behind the oblique part of a turbine nozzle are concluded and in this final part the historical development shall be shown in its essential feature. Of course this review cannot be complete and therewith objective, because the diverse contributions will not be included proportionally to the effort of the different countries.

The supersonic flow deflection in straight-oblique turbine nozzles at exit to inlet pressure ratios lower than the critical one were probably first observed by Lewicki [33]. Owing to this discovery the interest of numerous engineers and scientists in many countries awoke and is still present to study in detail the complex nature of all the associated flow phenomena. In the following a short and critical compendium will be given, showing the historical development of this special part of the general fluid dynamic research.

A first detailed experimental investigation on various single and multiple nozzles was carried out by Josse [26] and Christlein [7]. Both used a test apparatus in which the nozzle parameters are deduced by measuring the reaction force (momentum flux) of the nozzle and the mass flow rate (in regard to some measuring errors see Stodola [51]). Experiments with similar testing machines were carried out later on also by Warren and Keenan [57] and Wewerka *et al.* [59].

The first theoretical determination of the deflection angle $\delta = \beta_B - \beta_2$ was given by Baer [3], using the continuity equation in conjunction with the homentropic ($p_{\text{tot } 2} = p_{\text{tot } B}$) approximation, but he did not see any limits in the expansion process (i.e. $p_2 \rightarrow 0$). In a modern form his relation is given by equation (1).

At the same time Loschge [38] calculated the same problem by means of the simple Prandtl-Meyer corner expansion in which one of the two Mach line families are straight lines of constant flow condition. Due to the fact, that he took the flow properties of the last characteristic of the expansion fan as outlet value (index 2) he could use the following relation, in which ν represents the Prandtl-Meyer angle:

$$\beta_B - \beta_2 = \nu_2 - \nu_B. \quad (11)$$

The influence of the nonhomentropic nature of the real flow he incorporated in using a polytropic exponent n instead of κ in the homentropic relation of the ideal gas. He was the first who recognized the existence of a lower bound in the outlet static pressure in the case of two-dimensional flow. Within his approximation (straight Mach lines of the fan) he stated this limit correctly as that condition, where the last straight expansion characteristic (here also a line of const pressure) coincides with the nozzle exit plane. In his paper some additional remarks are included, concerning the qualitative description of the flow if the back pressure is lower than the above mentioned limit and if the flow is no longer two-dimensional behind the nozzle exit.

In a next step Flügel [14] calculated the deflection δ and the velocity coefficient φ (expressing irreversibility) by means of the equation of continuity and both momentum equations (equivalent to equations (1) - (3)). He did not neglect frictional effects, so that he had to introduce an approximate relation for the friction force T acting in the direction of the nozzle surface ($B-E$). To simplify the calculation a uniform static pressure p_2 was assumed in the exit plane ($A-E$). He called as limit condition the minimum pressure p_2 at which in his method the above condition can be fulfilled together with the continuity equation. Also the difference between the arrangement of a single and of multiple nozzles were described qualitatively but supposed to be of small influence.

To determine the expansion flow in a turbine nozzle Zerkowitz [60] used both momentum equations, but made no use of the continuity equation. Because he did not eliminate the unknown pressure force N normal on the straight nozzle surface, he could only give qualitative results. He also used a uniform pressure p_2 in the exit area as approximation and his limit condition coincides with that of Loschge [38]. If the pressure behind the nozzle is lower than this limit one, an additional expansion (still 2-dimensional) occurs outside the nozzle and he calculated this also by the momentum equations. The given results show that for this outer expansion a divergence of the two stagnation streamlines through A and E occurs. For this reason periodicity is no longer possible and the two-dimensional outer expansion is therefore only feasible for a single or at least finite nozzle arrangement. To avoid the failure, introduced by the assumption of straight Mach lines in the expansion fan, he proposed a special contoured nozzle, in which the suction side ($B-E$) is formed by the respective streamline of the simple Prandtl-Meyer corner expansion. He also mentioned the possibility of pressure ratios p_2/p_B greater than 1, if at the inlet ($A-B$) of the oblique part of the nozzle supersonic velocities appear.

In two short contributions Flügel [15] and Zerkowitz [61] disputed the applicability of the above mentioned contoured nozzle in a blade row.

A summarizing description of the supersonic outlet flow in oblique nozzles was given by Wewerka [58]. He also made use of both momentum equations without eliminating the blade force N , which he has to take from experiments. He used for the first time a dimensionless velocity (equivalent to the critical Mach number M^*) to become independent of absolute values and dimensions and he also first recognized the equivalence between an axial sonic outlet velocity and the condition that the last Mach line of the expansion fan falls into the exit plane ($A-E$). If the exhaust pressure is lower than this limit, (here called $(p_2)_{\min}$) another expansion is allowed for. But to ensure periodicity Wewerka has to give up the assumption of two dimensional flow. Behind the exit plane, in which a uniform flow of axial sonic condition (i.e. $(M_2)_{\min} \cdot \sin(\beta_2)_{\min} = 1$) is assumed, the stream spreads out symmetrically normal to the considered flow plane surrounded by the ambient pressure p_2 , (s. Fig. 20). In this outer region then the following relation holds:

$$M_2^* \cos \beta_2 = (M_2^*)_{\max} \cos(\beta_2)_{\min}. \quad (12)$$

The final flow angle β_2 is given as:

$$\operatorname{tg} \beta_2 = \operatorname{tg}(\beta_2)_{\min} \frac{1}{\kappa} \left[(\kappa + 1) - \frac{p_2}{(p_2)_{\min}} \right] \quad (13)$$

with an associated area ratio of:

$$\frac{B}{B_2} = \frac{p_{\text{tot } 2}}{(p_{\text{tot } 2})_{\min}} \left[\frac{\frac{\kappa+1}{\kappa-1} - M_2^{*2}}{\frac{\kappa+1}{\kappa-1} - (M_2^*)_{\min}^2} \right]^{1/(\kappa-1)} \frac{\text{tg } \beta_2}{\text{tg } (\beta_2)_{\min}} \quad (14)$$

The maximum expansion is finally reached for vacuum as exhaust pressure (i.e. $p_2=0$ or $M_2^{*2} = \frac{\kappa+1}{\kappa-1}$), yielding:

$$\text{tg } \beta_2 = \frac{\kappa+1}{\kappa} \text{tg } (\beta_2)_{\min} \quad (15)$$

$$\frac{B}{B_2} = 0. \quad (16)$$

In a third paper Zerkowitz [62] made use of the continuity equation (as Baer had done before), but he now interpreted these results by using centered straight isobares. At limit loading ($p_2 = \text{minimum}$), that condition is indicated where the last isobare coincides with the exit plane ($A-E$).

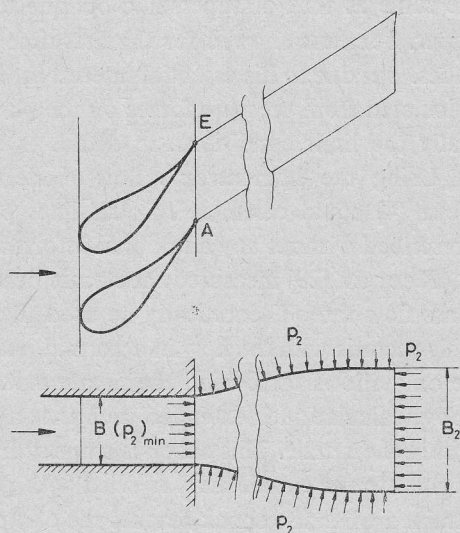


Fig. 20. Scheme defining the flow configuration at increasing flow area normal to the flow plane (after-expansion)

A great deal of these early investigations is summarized, supplemented and compared with own and other experiments in the famous and well-known book of Stodola [51].

Busemann [6] determined the supersonic nozzle flow (not necessarily straight nozzles) with the aid of the method of characteristics, developed by himself. He only calculated a single nozzle and assumed a constant ambient pressure outside of the nozzle (i.e. behind of the exit ($A-E$)). Under these circumstances the periodicity condition can not be ful-

filled, so that these results can not be applied to blade rows. The same method was also proposed later by Ferri [13] and Shapiro [50].

To calculate the supersonic homentropic flow of a straight oblique turbine nozzle Szewalski [55] made use of the momentum equation applied in the suction surface direction ($B-E$) in combination with the continuity equation. Using the variables of this paper, a combination of equation (1) and (7) (both with $p_{\text{tot } 2} = p_{\text{tot } B}$) yields:

$$\cos(\beta_B - \beta_2) = \frac{\frac{\kappa+1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right]^{1/(\kappa-1)} [M_B^{*2} + 1] - \frac{\kappa-1}{2\kappa} \left[\frac{\kappa+1}{\kappa-1} - M_2^{*2} \right]^{\kappa/(\kappa-1)}}{M_2^* M_B^* \left[\frac{\kappa+1}{\kappa-1} - M_B^{*2} \right]^{1/(\kappa-1)}}. \quad (17)$$

In this equation the flow deviation $\delta = \beta_B - \beta_2$ is independent of the nozzle direction β_B (also Loschge's simple Prandtl-Meyer relation (11) does not depend on β_B). On the other hand there is no reason why this equation (17) should have more relevance than the continuity equation (1) or the momentum equation (7) alone, because both equations are also sufficient (due to the assumption $p_{\text{tot}} = \text{const}$) to determine a relation $\beta_2 = f(M_2^*)$ between the outlet parameters β_2 and M_2^* .

Strauss [53] (see also Oswatitsch [44]) investigated the purely supersonic flow through infinitely thin flat plate cascades. The outlet flow of these cascades is completely the same as that of supersonic oblique turbine nozzles, under the scope of view in this paper. Strauss showed for the first time the fully periodic flow pattern behind such cascades. But due to his essential interest in wave drag and lift-to-drag ratio (gliding number), calculated from pressure forces on the plates, he did not study the downstream flow, especially the limit conditions, in detail.

Using the homentropic flow model in conjunction with the continuity equation Betz [4] could calculate throttle lines $\beta_2 = (M_2^*)$, if the cascade inlet conditions were prescribed without any additional information of the blade geometry.

Keenan [28] measured the nozzle behaviour in picking-up the momentum flux and the mass flow rate. Theoretically he used the fully one-dimensional homentropic flow model (this accords with the flow in a normal nozzle), without regard to any flow deflection which he only allows far behind the nozzle exit. In case of a compression within the nozzle the above calculation is extended such that a normal shock (no deflection) can be established within the nozzle. This shock is situated in such a manner, that the whole exit region (here normal to the nozzle axis) is filled with gas at the prescribed exhaust pressure. A limit is attained when the shock reaches the exit plane. At lower exhaust pressures the flow within the nozzle is assumed homentropic. A next bound is attained when the lowest exhaust pressure occurs at which the exit can be fully filled with flow. Beyond this pressure the conditions within the nozzle remain constant.

Hauser *et al.* [22] compared cascade measurements with six different calculation methods:

1. The deviation of the homentropic flow is calculated by the continuity equation.
2. It is assumed, that the circumferential velocity component $-w \cdot \cos \beta$ remains unchanged between the nozzle inlet ($A-B$) and exit ($A-E$), and the flow is homentropic.

But in this case the continuity equation shows that an area increase normal to the flow plane has to occur,

3. They made the same assumption ($-w \cdot \cos \beta = \text{const}$) as under 2, but instead of homentropic flow the losses were chosen such that continuity is fulfilled.

4. The expansion takes place in a simple centered Prandtl-Meyer expansion. Where the first Mach line of the fan hits the suction surface (point *C*) a flow separation occurs with a separation bound formed by a stream line of the expansion fan through point *C*.

5. The supersonic flow is calculated by the method of characteristics with the assumption that everywhere in the wake the exhaust pressure acts. This method is therefore identical to that of Busemann and Shapiro who used it for single nozzles only. Nevertheless Hauser applied it to cascade flows where the periodicity condition is valid. For low exhaust pressures he recognized crossing of streamlines behind the cascade which he could not interpret physically.

6. As done by Zerkowitz [60] both momentum equations are used to determine the outlet flow, but again the blade force *N* is unknown. For this reason Hauser applied this method only for the data reduction of his experiments.

As limit condition he stated the condition of axial sonic outlet velocity though he found in his experiments a continuous transition to axial supersonic values. For real turbines he stated that this would only be possible if the hub-to-tip diameter ratio is increased.

In a second paper Hauser *et al.* [23] investigated, above all, the supersonic cascade outlet flow at limit loading (i.e. $p_2 = \text{minimum}$). Applying continuity and momentum equation in tangential direction he deduced that this limit with maximum circumferential velocity occurs if the trailing edge shock hits the trailing edge of the adjacent blade. In his flow construction for blades having finite trailing edges, he assumed the final trailing edge shock to start at the same point *P* (see Fig. 12) in which the expansion fan develops.

In a contribution, concerning mostly the flow adjustment of several stages, Flügel [16] described qualitatively the supersonic outlet flow of straight backed blades up to a limit loading with an axial sonic outlet velocity component. Additionally he allowed an after expansion beyond limit loading. For this flow condition he held the circumferential velocity component constant. But it is shown above (see discussion of Wewerka's paper) that these flow configurations are only possible if the downstream annular area can increase.

Meyer H. [41] describes the flow pattern within and behind the nozzle by means of characteristics and shock wave pattern, but made the wrong assumption that already just behind the trailing edge (point *A*) the uniform downstream pressure is attained. In such downstream flow field therefore shock waves vanish inside of the flow. In a comparison between the application of continuity and simple Prandtl-Meyer expansion he could show, that the latter will lead to flow deflection angles greater than observed in reality.

A very easy applicable and exact method, valid for the inviscid flow in a straight oblique nozzle at uniform supersonic inlet velocities was developed by Traupel [56]. Using the equation of continuity and momentum in the direction of the straight nozzle part he could calculate the uniform downstream flow values including the entropy rise produced by the compression shock fronts within and behind the cascade. Due to the one-dimensional character of his derivation he could not estimate the boundaries up to which his method

is useful. This can only be decided by a two-dimensional consideration, but he supposed that a limit occurs if the losses show a relative maximum at low back pressure. Except of this restriction and the fact that he could solve the equations only in an iterative manner, the calculation method used in section 4 of this paper is identical with Traupel's ideas. In detailed papers about cascade flow Deich *et al.* [10, 12] also described the supersonic outlet flow of turbine nozzles. As simplest kind he used the equation of continuity in order to calculate the deflection of a homentropic flow. Thereby the maximum expansion takes place if the last Mach line hits the nozzle trailing edge. A second method, originating from Stepanov, also includes shock losses and is quite the same as the Traupel method. With the aid of Schlieren photos and associated characteristic flow pattern constructions he explained the different operational modes. As limit loading for cascades of finite trailing edges he described the configuration with an oblique shock wave connecting the trailing edges of the blades, however, he allowed a further continuous acceleration to various axial supersonic outlet velocities. The respective base pressure for axial sub- and supersonic velocities he took from experimental results. One of his examples shows a centred expansion at an infinitely thin trailing edge, which is limited by a shock wave emanating from the expansion origin, which is not a quite correct assumption. Linnecken [37] compared in his paper the methods of Baer, Zerkowitz, Szewalski and Busemann. The latter he found to be too complicated for practical applications and in Zerkowitz's method the unknown pressure force makes it useless, so he preferred the use of continuity introduced firstly by Baer. On the other hand the formula of Szewalski seemed questionable to him because a flow deflection can be computed by it also for the case of a normal nozzle, which is indeed impossible. But his contradiction vanishes at once if the formula is applied in combination with the necessary limit conditions, as it is necessary for all the one-dimensional methods.

The work of Strauss was continued later on by Brieden [5] who investigated the influence of the entropy rise due to shock waves. Additionally he calculated the outlet conditions far downstream by means of the continuity and momentum equations, applied between the inlet plane far upstream and the outlet plane far behind the blade row. The blade force introduced in the momentum equation, he had to calculate separately at each back pressure setting from the static pressure on both sides of the blades.

Using the method of characteristics Hausenblas [19, 20, 21] calculated the flow in a straight oblique nozzle and in addition also in a special designed curved one using a vortex-source as basis. Especially he described the attenuation of the nonuniformities behind the cascade, without being able to calculate them. He had also difficulties in determining the maximum possible deflection. For this he referred to Wewerka but used the homentropic formula (17) of Szewalski up to $p_2 = 0$. On the other hand he stated, that the results of Wewerka had only been proven for single nozzles and that for cascades another boundary may exist.

Gretler [17] compared shallow-water experiments with calculations of the expanding nozzle flow. As limit-loading he stated such configuration where the last fan Mach line or in case of a finite trailing edge, the trailing edge shock hits the rear point of the following profile. Axial supersonic velocities he allowed for but only in semi-infinite cascades. In rearranging the Traupel equations he could solve them explicitly for the deflection angle

$= \delta\beta_B - \beta_2$. Using these formulae he proved that the condition of maximum losses $(d(p_{\text{tot } 2})/d(p_2))=0$ leads to axial sonic outlet conditions, so that for these flow cases the assumption of Traupel is correct. Without proof he also supposed that the maximum circumferential velocity $-w_2 \cdot \cos \beta_2$ is attained at the same point.

Independently, using the simple centered Prandtl-Meyer expansion, Cordes [8] could show that the assumption of Traupel leads approximately to the flow configuration where a Mach line connects two adjacent trailing edges. He reduced the inaccuracy, introduced by the straight Mach lines, combining Prandtl-Meyer formula with continuity. Calculating the outer expansion behind the nozzle exit, he derived quite similar ideas and formulae as Wewerka.

The axial supersonic flow behind flat plate cascades was treated by Albring [1], assuming homentropic flow and small disturbances.

Stratford *et al.* [52] (see also Horlock [24]) investigated, experimentally and theoretically, the cascade outlet flow at axial subsonic velocities especially he verified the flow deflection by changing back pressure. The uniform outlet conditions he calculated by means of simplified conservation laws. He noticed the disturbances of the periodicity in his experiments due to the lower endwall but he assumed them to be small in comparison to the throttle effect, at least near design conditions.

In a further contribution Deich *et al.* [11] showed experimental results of supersonic turbine cascades. In this article viscous effects and boundary layer separation at moderate Mach numbers are placed in the foreground. By observation he found the following marked boundary layer effects: a) The generation of a shock wave at the transition point of the laminar boundary layer on the suction surface, b) local laminar separation on the suction side induced by the trailing edge shock, and c) turbulent suction side separation due to this latter shock front. To avoid the local laminar separation he proposed a special profile, convex kinked at the suction surface. The experiments verified these ideas but showed a strong influence of kink position and operational Mach number.

The difficulties in realizing a periodic supersonic flow pattern behind the blades in a cascade wind-tunnel is shown by Sieverding [49], who also investigated the transition from subsonic to supersonic flow, using side wall static pressure taps.

Lawaczeck [30, 31, 32] analyzed in detail the flow in transonic turbine cascades of uncambered, sharp but very thick profiles. Besides the accompanied experiments he calculated the supersonic flow part. The sonic line is established in a simplified manner and the annexed supersonic field up to the cascade exit is constructed, using the method of characteristics. In order to get the uniform outlet conditions he made use of the four conservation laws, established between the cascade exit front and infinitely far downstream. The explicit solution of this method, exact in an inviscid, homoenergetic flow was developed by Amecke [2].

Rhomberg [46] utilized the shallow-water flow through cascades for the determination of the sonic line which he is obliged to know as initial condition for calculating the supersonic flow region. Moreover he compared the results of computation and water table measurement. Within the nozzle exit region he allowed a maximum expansion up to the axial sonic outlet condition. But due to a dead-water region at the profile rear an additional expansion to axial supersonic conditions takes place. His principal flow pattern

of this configuration is quite the same as in section 3 of this paper, but he admits various axial supersonic outlet conditions in changing continuously the base pressure, which is in contrast to the fact, that at axial supersonic exit flow this base pressure is completely fixed by the inlet condition. Shadowgraphs of tests in a rotating cascade test bank showed similar shock wave pattern as the hydraulic analogy and theory. In his rotating cascade wind-tunnel he could observe a ring shock behind the cascade, parallel to the cascade exit front. The stated equivalence of this ring shock to the shock wave, emanating from the conjunction of the lowest trailing edge with the lower end-wall in a plane cascade wind-tunnel, cannot be accepted because the latter is only a result of the finite dimension of such tunnels and the lowest trailing edge shock is in general neither parallel to the blade front nor is the flow periodic behind it, see for instance Lichtfuss and Starken [34].

Another description of the transonic turbine flow is given by Käppeli [27] who sustained his considerations mostly on those of Rhomberg.

For simple geometric blades Paulon *et al.* [45] treated the fully supersonic flow through a cascade, theoretically and experimentally. Throttling could be achieved either by changing the inlet condition or by rotating the whole cascade relative to the nozzle flow. However, this latter setting also varies the infinite cascade inlet flow condition.

Meyer J. B. [39, 40] measured the flow behaviour in different distances behind the cascade and compared these results and the associated uniform outlet conditions with those obtained by the method of characteristics. Additionally he could explain the so called overpressure (increase in total pressure) as error in the measurement of the static pressure, which has to be determined very accurately to get a correct conversion from measured pitot into total pressure.

By the method of characteristics Lichtfuss and Starken [34] calculated the unseparated supersonic flow through cascades of sharp cambered profiles. He also showed the existence of the discontinuous transition from axial subsonic to the single axial supersonic outlet condition. In addition he described in detail the difficulties to obtain a periodic supersonic exit flow and established a method to get, at least theoretically, a periodic flow area also at axial subsonic outlet flow components.

The supersonic flow through a lattice row of infinitely thin flat plate cascades is treated by Lichtfuss and Starken [35]. The rear part of this flow is quite the same as the supersonic flow of the straight oblique turbine nozzle as discussed in section 2.

The axial subsonic outlet flow of cascades having a finite trailing edge thickness was investigated in detail by Curtis *et al.* [9]. The ideas leading to the flow pattern of Fig. 12 are based on this paper.

Surugue and Fabri [54] described in a simplified manner the supersonic flow behind the flat plate cascade and compared the qualitative results with those obtained in an annular cascade wind tunnel.

In a survey of the supersonic cascade flow, Lichtfuss and Starken [36] used the method of Traupel also for the subsonic outlet conditions of transonic turbine nozzles, though the special mode of a normal shock was realized already by Gretler.

Janssens and Guyot [25] investigated the supersonic exit flow of flat plate cascades, too, but especially under the view to establish a uniform supersonic inlet flow for a following test blade row in an annular cascade tunnel.

6. Summary

In order to describe the supersonic outlet flow of transonic turbine cascades the gas flow in a simple oblique nozzle, bounded by straight lines is investigated. The fluid is assumed to be inviscid but shock waves yielding entropy rises are allowed for. Additionally the flow at the nozzle inlet has to be uniform.

Already with this simple model all the different flow configurations at the exit and behind transonic turbine cascades can be explained, except of those effects of course, which are generated by viscous effects. It is shown that the axial velocity component at the inlet of the nozzle are one essential parameter. At axial subsonic components both expansion or compression is possible within the nozzle. For such operational mode the highest expansion occurs if the last Mach line of the expansion fan connects both nozzle end-points and the associated uniform downstream velocity just reaches a sonic axial component. If the inlet flow possesses an axial supersonic component no expansion at all is possible and the minimum back pressure at axial subsonic outlet flow occurs if the trailing edge shock falls within the exit front. Only one single lower back pressure with an axial supersonic outlet component is additionally possible.

The supersonic flow in case of a trailing edge of finite trailing edge thickness is described qualitatively. Again the outlet flow behaviour is different for sub- and supersonic axial exit components. By application of the conservation laws it is possible to calculate the uniform downstream flow directly without information of the flow pattern inlet and outlet, and also the associated subsonic solutions can be calculated. On the other hand, the limit conditions up to which the above method is valid have to be established by consideration of the two-dimensional nature of the flow.

Last, but not, least, a critical and historical review shows the transonic turbine and nozzle investigations carried out since the beginning of our century.

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Krytyczny przegląd zagadnień przepływu na wylocie naddźwiękowych dysz w turbinach

Streszczenie

Celem opisanie naddźwiękowego przepływu na wylocie z transsonicznych palisad łopatkowych w turbinach, zbadano przepływ gazu w dyszy skośnej, ograniczonej liniami prostymi. Założono, że przepływający płyn jest nielepki, dopuszczając istnienie fal uderzeniowych powodujących wzrost entropii. Dodatkowo założono, że przepływ na wlocie dyszy jest jednorodny.

Już ten prosty model pozwolił na wyjaśnienie różnorodnych konfiguracji przepływu na wylocie i za transsonicznymi palisadami łopatkowymi turbiny, z wyjątkiem oczywiście tych efektów, które wynikają z istnienia lepkości. Pokazano, że istotną rolę odgrywa osiowa składowa prędkości na wlocie dyszy. Stwierdzono, że dla poddźwiękowych składowych osiowych możliwe są w dyszy albo ekspansja, albo sprężanie. Najsilniejsza ekspansja zachodzi wówczas, gdy ostatnia linia Macha w fali rozrzedzeniowej łączy oba końcowe punkty dyszy, a związana z nią jednorodna prędkość w kierunku przepływu zaledwie osiąga w kierunku osiowym składową prędkości równą prędkości dźwięku. Jeżeli w przepływie na wlocie występuje naddźwiękowa osiowa składowa prędkości, to ekspansja w ogóle nie jest możliwa. Minimalne przeciwcisnienie dla osiowego poddźwiękowego przepływu na wylocie występuje wówczas, gdy fala uderzeniowa przy krawędzi spływu znajduje się na wysokości frontu wylotu. Wówczas możliwe jest tylko jedno niższe przeciwcisnienie dla osiowej składowej naddźwiękowej na wylocie.

W dalszym ciągu podano jakościowy opis przepływu naddźwiękowego dla skończonej grubości krawędzi spływu. W tym przypadku własności przepływu na wylocie są różne dla pod- i naddźwiękowych składowych osiowych na wyjściu. Wskazano, że stosując prawa zachowania można bezpośrednio obliczyć jednorodny przepływ na wylocie, bez informacji o obrazie przepływu pomiędzy wlotem a wylotem; można również znaleźć rozwiązania poddźwiękowe. Ustalenie granicznych warunków ważności tej metody wymaga rozważenia dwuwymiarowego charakteru przepływu.

Na zakończenie dokonano krytycznego i historycznego przeglądu badań transsonicznych turbin i dysz od początku naszego stulecia.

Критический просмотр вопросов течения на выходе из сверхзвуковых сопел в турбинах

Резюме

С целью описания сверхзвукового течения на выходе из сверхзвуковых решеток лопаток в турбинах, проведены исследования течения газа в сопле с косым срезом, ограниченном прямыми линиями. Принимается, что протекающая жидкость является невязкой, однако, допускается существо-

зование ударных волн, вызывающих рост энтропии. Добавочно принимается, что на входе в сопло течение однородное.

Уже эта простая модель позволяет выяснить все разнообразные конфигурации течения на выходе и за сверхзвуковыми решетками турбинных лопаток, конечно, за исключением тех эффектов, которые следуют из наличия вязкости. Показано, что существенным параметром является аксиальная составляющая скорости на входе в сопло. При дозвуковых значениях аксиальных составляющих в сопле может произойти расширение или сжатие. В таком режиме работы наиболее сильное расширение происходит тогда, когда последняя линия Маха в разрежающей волне связывает обе краевые точки сопла, а связанная с ней однородная скорость в направлении течения едва достигает в аксиальном направлении значения составляющей скорости, равняющейся скорости звука. Если в потоке на входе имеется сверхзвуковая аксиальная составляющая скорости, тогда расширение вообще невозможно, а минимальное противодавление для аксиального сверхзвукового течения на выходе имеется тогда, когда ударная волна у выходной кромки находится на уровне фронта выхода. Возможно только одно противодавление для аксиальной сверхзвуковой составляющей на выходе.

Представлено качественное описание сверхзвукового течения для случая конечной толщины выходной кромки. Здесь также свойства течения на выходе различаются для дозвуковых и сверхзвуковых аксиальных составляющих на выходе. Применяя закон сохранения можно непосредственно рассчитать однородное течение непосредственно за выходом, без информации о структуре течения между входом и выходом. Можно также произвести расчет дозвукового течения. С другой стороны, необходимо здесь определить предельные условия правильности этого метода. Это определение должно основываться на анализе двухразмерного характера течения.

В заключении производится критический и исторический просмотр исследований сверхзвуковых турбин и сопел, проведенных от начала нашего столетия.