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# The Mathematical Model of Condensing Turbine for Saturated Steam Installed in a Nuclear Power Station

## Introduction

Properties and characteristics of the steam turbo-set at the operating conditions differing from the nominal ones should be known both when designing and operating a power plant. The reasonable way of investigating the turbo-set and determining

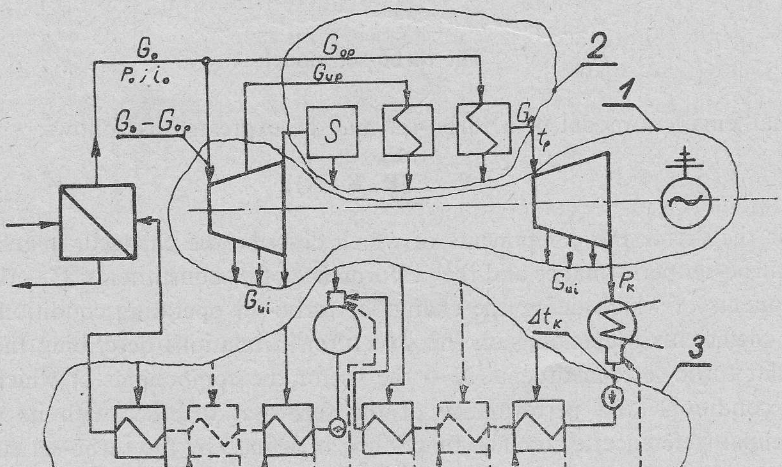


Fig. 1. Scheme of the analysed turbo-set with the decomposition
*1* – turbine with generator, *2* – external separator and steam reheaters, *3* – regenerative feed-heating system

its characteristics, apart from the measurements, is to use an adequately formulated mathematical model for simulation of changed operating conditions on a digital computer [10, 11].

The subject analyzed herewith is the condensing turbine of great output for saturated or low superheated steam, installed in nuclear power stations with water-cooled reactors

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[11]. The turbine is equipped with internal moisture removal devices, an external moisture separator and a steam-to-steam reheat (Fig. 1). The purpose of the investigations is to present the structure principles and to formulate the general static mathematical model of the turbine. The above mentioned model should allow determining on an electronic digital computer the performances and the turbine characteristics in the steady-state operating conditions changed in relation to design conditions. The accuracy of the obtained results as high as possible is required. The above mentioned model of the turbine is used mainly as one of the basic components of the turbo-set model when analysing the characteristics of a turbo-set operating in changed conditions.

The model of the turbine under consideration is based on the results obtained in the paper [7], where the exact requirements and structure principles were given and the general mathematical model of the condensing steam turbine of great output for the conventional power plant was formulated. For this reason only necessary completions and modifications of this model, resulting from the individual characteristics of the analysed turbine and its thermal cycle, have been presented in the paper. The peculiarities of the thermal cycle (mainly the steam-to-steam reheat) require different formulation of the input quantities (set for computation) and the output ones (determined as a result of computation) of the turbine model. This is connected with the organization of the computation in the turbo-set model.

## The turbo-set model

The mathematical model of a turbo-set may be expressed as follows:

$$\mathbf{F}=F(\mathbf{P},\mathbf{K},\mathbf{N}), \tag{1}$$

where $\mathbf{F}$ is the vector the components of which describe the changed, in relation to the nominal, turbo-set performance and the performance of its constituents, $\mathbf{P}$ – the set vector the components of which define the change of turbo-set operating conditions, $\mathbf{K}$ – the vector of coefficients (depending on the structure) in relations describing the properties of particular turbo-set constituents, $\mathbf{N}$ – the vector the components of which define the operating conditions and performance of the turbo-set together with its constituents in the nominal (reference) state. The function $F$ is defined by the turbo-set structure and when adequately generalized may be common for the chosen class of turbo-sets.

In the case under consideration the parameters $P_0$, $i_0$, the rate of flow $G_0$ of the inlet steam, the pressure $P_k$ in the condenser and supercooling of the condensate $\Delta t_k$ are the components of the vector $\mathbf{P}$:

$$\mathbf{P}=[P_0, i_0, G_0, P_k, \Delta t_k]. \tag{2}$$

These quantities should be regarded in the computation of the turbo-set model as known ones, as they are mainly connected with the functioning of other constituents of the power plant, the ones that the turbo-set co-operates with. Variable operating conditions of the primary circuit together with a steam-raiser (if present) and cooling system with the condenser are analyzed separately outside the turbo-set model.

To define the function $F$ and use it for the determination of the searched for vector **F** in case of the turbo-set scheme as in Fig. 1, a set of several hundred nonlinear equations and inequalities should be formulated and solved. In that case application of the decomposition principle is advisable both when the mathematical model of a turbo-set is formulated and solved [10]. Analyses showed [10] that the division into the following constituents is profitable:

turbine model

$$\mathbf{Ft} = Ft(\mathbf{Pt}, \mathbf{Kt}, \mathbf{Nt}),$$

separator and steam reheater model

$$\mathbf{Fs} = Fs(\mathbf{Ps}, \mathbf{Ks}, \mathbf{Ns})$$

regenerative feed heating system model

$$\mathbf{Fr} = Fr(\mathbf{Pr}, \mathbf{Kr}, \mathbf{Nr})$$

while

$$\mathbf{F} = [\mathbf{Ft}, \mathbf{Fs}, \mathbf{Fr}]. \tag{3}$$

The solution of the turbo-set model when a decomposition has been made can be found by a two-stage iterative procedure in which the quantities being the connections between the models of turbo-set constituents (interactions) are determined. The division of the quantities into the input and output ones in respective models results both from the specificity of computation for these models and from the possibility of solving the turbo-set model.

Quantities set for computation in the turbine model can be divided into three groups

$$\mathbf{Pt} = [\mathbf{P}, \mathbf{Pts}, \mathbf{Ptr}], \tag{4}$$

where **Pts** – the vector of quantities dependent on the separator and steam reheater activity (the rate of flow $G_p$ and the temperature $t_p$ of steam after the reheaters, the loss of pressure $\Delta p$ in the separator and reheaters, the consumption of the live steam $G_{0p}$ and the extraction steam $G_{up}$ by the reheaters), **Ptr** – the vector of quantities resulting from the activity of the regenerative feed-heating system (the steam flows $G_{ui}$ from the extractions of the turbine).

In the model of a turbine only a part of components $(P_0, i_0, p_k)$ of the vector **P** is utilized. The vectors **Pts** = **Fts** and **Ptr** = **Frt** are the constituents of the vectors **Fs** and **Fr** of the output quantities of separator steam reheaters and regenerative feed-heating system models.

The computation process in the turbo-set model after the decomposition has been made is shown in Fig. 2. On heaving read the data quantities (items *1*, *2*), the unknown at the moment constituents **Pts** and **Ptr** of the vector **Pt** of quantities set for computation of a turbine (item *3*) are estimated. Then the vector is obtained

$$\mathbf{Pt}' = [\mathbf{P}, \mathbf{Pts}', \mathbf{Ptr}']. \tag{5}$$

When adequate approximate relations are used **Pts**′ and **Ptr**′ are not too distant from the searched for solution **Fst**, **Frt**.

Then the computations in the model of a turbine (item *4*) are made and vector **Ft** = = [**Fts**, **Ftr**, **Ftt**] is defined. The components of the above vector are the input quantities

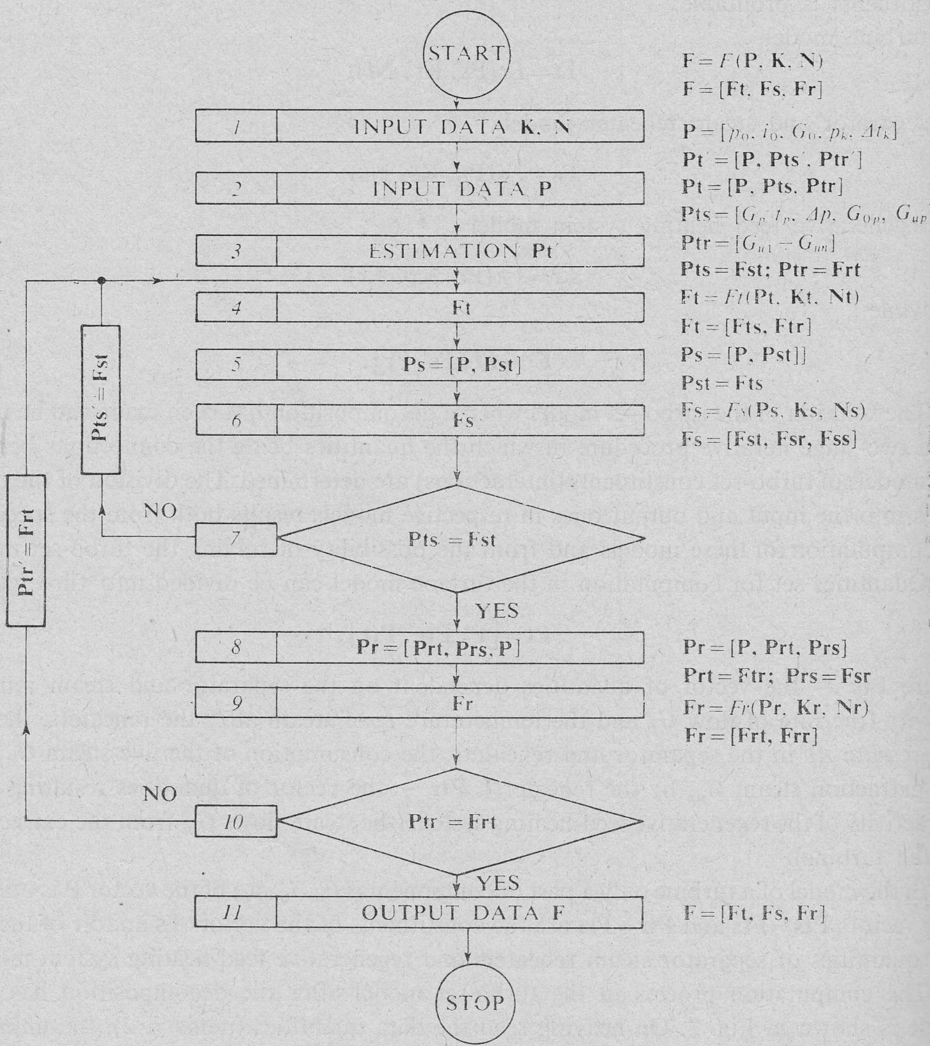


Fig. 2. Flow diagram of computations in the turbo-set model after decomposition

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of the separator and steam reheaters model **Fts**=**Pst**, of feed-heating system **Ftr**=**Prt** (rates of flow and steam conditions in all streams of steam flowing into and out of the turbine excluding the input quantities **Pt**). The vector **Ftt** describes the remaining quantities that characterize the work of a turbine itself and its components.

Computations in the separator and reheaters model (item *6*) can be made now, as the vector **Ps** of input quantities in this model is (item *5*) defined

$$\mathbf{Ps}=[\mathbf{P}, \mathbf{Pst}], \qquad \mathbf{Pst}=\mathbf{Fts}. \tag{6}$$

As a result, the vector of output quantities **Fs**=[**Fst**, **Fsr**, **Fss**] is obtained. One of its components is the vector **Fst**=**Pts**, estimated preliminarily in item *3*.

Now the accuracy of estimation of the vector **Pts**′ is being checked. In case of inadmissible differences the computations according to the items *4*, *5*, *6* are repeated on having substituted **Pts**′=**Fst**.

When computations in a turbine, separator and steam reheater model are performed, the vector **Pr** of input quantities of the feed-heating system model can be determined (item *8*)

$$\mathbf{Pr}=[\mathbf{P}, \mathbf{Prt}, \mathbf{Prs}], \qquad \mathbf{Prt}=\mathbf{Ftr}, \qquad \mathbf{Prs}=\mathbf{Fsr}. \tag{7}$$

Then computations in the feed-heating system model are performed and the vector **Fr**=[**Frt**, **Frr**] is fixed. Its component is the vector **Frt**=**Ptr**, estimated preliminarily in item *3*. That again makes the checking (in item *10*) of the accuracy of this estimation possible. Now, repeating of computations according to items *4*÷*9* can prove necessary, until the needed agreement has been obtained.

The solution of the turbo-set model for a conventional power station [9] is obtained by a one-stage iterative procedure as the system does not include a separator and steam reheaters.

## Models of turbine components

Performance of a turbine in changed operating conditions is mainly defined by properties of its flow part. To construct the mathematical model of a turbine the adequate division of its flow part into constituents common for different types [7] of turbines is made. Then the universal models of these constituents are defined and the completions with relations describing their mutual connections and co-operation are made. This allows for setting up a model of a turbine under consideration, practically irrespective of its structure [7]. These requirements can be met by dividing the flow part of the analysed turbine into the steam distribution system, governing stage, stages and groups of non-governed stages. The above division seems to be most reasonable from the viewpoint of determining the searched for quantities, accuracy, speed of calculations, general character and universality of the model, as well as the possibility of reducing the range of necessary detailed data concerning the turbine.

While elaborating the models of components of the flow part the models worked out for conventional condensing turbine of great output [7, 8] were used. Completions and modifications connected with the individual characteristics of the analyzed turbine were made.

Completions dealt mainly with these constituents of the flow part which operate within the region of high wetness.

The analyzed turbines for nuclear power plants are similarly equipped as the turbines for conventional plants in a nozzle control governing or throttle governing steam distribution set. In that case the model of steam distribution set presented in the papers [7] and [8] has been used. Here the condition has been introduced $i_z = i_0$ instead of $p_z v_z = p_0 v_0$ ($i_0, p_0, v_0$ — inlet steam conditions, $i_z, p_z, v_z$ — steam conditions after the valve) as to wet steam throttle.

In the governing stage model, relations for the conventional turbine [7, 10] have been also utilized. Here the losses connected with steam wetness have been taken into account. The internal output of the governing stage has been determined from the relation:

$$N_r = \sum_{i=1}^{m} G_{zi} H_{ri} [\eta_{rui} - a_1 k_{ri} (1 - x_i)] - \Delta N_r, \tag{8}$$

where $G_{zi}$, $H_{ri}$, $\eta_{rui}$, $k_{ri}$, $x_i$ denote, respectively, the rate of flow, isentropic heat drop, blade efficiency, velocity ratio and the final dryness fraction of steam in the steam stream flowing through the $i$-th governing valve, $a_1$ is a coefficient constant for the given stage, $\Delta N_r$ denotes output losses not included in $\eta_{ru}$ efficiency. The coefficient $a_1$ is to be estimated on the base of experimental results, eg. [2, 3, 6].

Both single non-governed stages and groups consisting of several stages should be distinguished in the flow part when setting up a model of the analyzed turbine. Here the use of the flow equation is needed in a form that takes into account the critical pressure ratio. This equation is therefore true both for the groups of stages and separate stages [9]. To determine the pressure $p_{\rm I}$ before the stage (group) at the set rate of flow $g$, and pressure $p_{\rm II}$ behind the stage, the following relations have been used:

— in the range of subcritical flows for

$$g = \frac{G_g}{G_{gn}} \geqslant \frac{p_{\rm II}}{p_{\rm In} \sigma \sqrt{\tau} B}, \qquad \tau = \frac{p_{\rm I} v_{\rm I}}{(p_{\rm I} v_{\rm I})_n},$$

$$p_{\rm I} = \frac{-\sigma p_{\rm II} + \sqrt{p_{\rm II}(1-\sigma)^2 + (1-2\sigma) g^2 \tau A}}{1 - 2\sigma}, \tag{9a}$$

— in the range of supercritical flows for

$$g < \frac{p_{\rm II}}{p_{\rm In} \sigma \sqrt{\tau} B}, \qquad p_{\rm I} = g p_{\rm In} \sqrt{\tau} B \tag{9b}$$

and if

$$\pi_n = \left(\frac{p_{\rm II}}{p_{\rm I}}\right)_n > \sigma$$

then

$$A = p_{\rm In}^2 [(1-\sigma)^2 - (\pi_n - \sigma)^2, \qquad B = \frac{\sqrt{(1-\sigma)^2 - (\pi_n - \sigma)^2}}{1 - \sigma}.$$

On the other hand at $\pi_n \leqslant \sigma$:

$$A = p_{\rm In}^2 (1-\sigma)^2; \qquad B = 1$$

where $G_g$ – rate of flow through the stage or a group of stages considered, $p_I$, $v_I$ – steam conditions at an inlet to the stage (group of stages), $p_{II}$ – outlet steam pressure, $\sigma$ – critical pressure ratio of the stage (group of stages), index $n$ deals with the values in nominal (reference) conditions. When estimating the value of the ratio $\sigma$, the results of the paper [9] can be used.

Essential supplements have been introduced to the non-governed stage model, operating in the region of wet steam. The influence of changes of dryness fraction of steam over the efficiency of the stage has been taken into account by the relation:

$$\eta=\eta'-a_2 k(1-x_1)-a_3 k(x_I-x_{II}), \tag{10}$$

where $\eta'$ – stage efficiency with no wetness losses considered, $k$ – velocity ratio, $x_1$, $x_{II}$ – dryness fraction of steam at the inlet and outlet of the stage, respectively (Fig. 3), $a_2$, $a_3$ – coefficients.

Coefficients $a_2$ and $a_3$ are to be estimated on the basis of the experimental results [2, 3, 6] while in general $a_2 \approx 2a_3$, in face of a much greater influence of the original wetness over the efficiency stage decrease.

When describing the internal moisture removal process (Fig. 3) it has been assumed that the total quantity of separated water $G_x$ is extracted from the flow part at the outlet

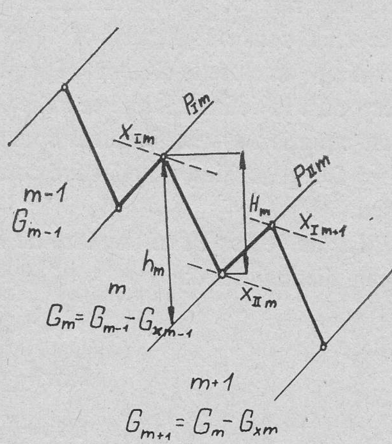


Fig. 3. Scheme of a process of vapour expansion for stages with internal moisture removal devices

$m$ – number of a stage, $h$, $H$ – enthalpy drop isentropic and utilized in a stage, respectively

of the analyzed stage [1]. As a result a sudden increase of dryness fraction of steam takes place before the next stage (Fig. 3), from the value $x_{IIm}$ (with no separation considered) to values $x_{Im+1}$.

The quantity of water $G_x$ extracted from the blade system as a result of internal moisture removal was determined from the dependence:

$$G_x=G(1-x_I)\alpha, \quad \alpha=f(x_I, k), \tag{11}$$

where $G$ – rate of flow at the inlet stage, $x_I$ – dryness fraction of steam at the inlet, $\alpha$ – rate of separation, $k$ – velocity ratio.

The coefficient $\alpha$ is to be estimated on the basis of the experimental results, e.g. [3, 4, 5]. The dryness fraction of steam $x_{Im+1}$ before the next stage results from the equation (Fig. 3):

$$x_{Im+1} = x_{IIm} + (1 + x_{IIm})\,\alpha\,. \tag{12}$$

## The turbine model

The formulation of a mathematical model of an actual turbine analysed requires, as in case of a turbine for conventional power station [7], the division of its non-governing stages into groups that have the same steam flow rate. The division is defined by the extraction points, the arrangement of stages in casings and the set of devices for separation and extraction of water from the flow part of a turbine. In the analysed turbines for nuclear power plants the groups of stages possessing the same flow rate often reduce themselves to single stages. These processes are different from the ones taking place in turbines for conventional power stations [7]. In such division the models of steam distribution set, governing stage (if there is any), non-governed stages and groups of such stages are utilized. There are however, in several cases, some differences between the models, but only in values of parameters of a model (vectors **Kt**, **Nt**). Additional relations, defining the connections between the constituents of the flow part, include balance equations for defining the rate of flow, pressure losses and turboset output. The mathematical model like this consists usually of several hundreds of relations (mainly non-linear ones) forming a closed system of equations, and their solution when the input quantity vector **Pt** is defined is the searched for vector of output quantities **Ft**. The solution of that equation system involves complex numerical calculations. For determining the searched for quantities, an effective combined sequence-iterative calculation method has been used.

In the vector **Pt** of the turbine model input quantities the following components have been distinguished:

$$\mathbf{Pt} = [\mathbf{Ptw}, \mathbf{Ptp}, \mathbf{Ptz}]\,, \tag{13}$$

where

$$\mathbf{Ptw} = [G_0 - G_{0p}\,;\, G_{up}\,;\, G_{u1} \div G_{un}]\,,$$

$$\mathbf{Ptp} = [p_k\,, \Delta p\,, t_p]\,,$$

$$\mathbf{Ptz} = [p_0\,, i_0]\,.$$

The suggested method of obtaining the solution takes the advantage of the possibility to group the relations forming the turbine model in such a way that the number of iteration levels will be limited to two. On the first level the iterations are conducted within a few or more relations describing the work of separate turbine constituents. Iteration on the second level includes much higher number of relations dealing with many flow part components. Within the second level iteration the sequential calculation is possible. The division of the searched for quantities **Ft** into the ones determined as a result of sequential calculations, and those obtained from the iteration on the first and the second level, has a fundamental influence over convergence of iteration processes and speed of calculations.

The following groups of relations have been distinguished in the turbine model as a result of the analysis:

I – Balance equations defining the rate of the flow vector **Fw**

$$\mathbf{Fw}=Fw(\mathbf{Pw},\mathbf{Kt},\mathbf{Nt}),\qquad \mathbf{Pw}=[\mathbf{Ptw},\mathbf{Pwb}]. \tag{14}$$

The quantities (rates of flow $G_x$ of separated water) determined in the V group of relations comprise the components of vector **Pwb**=**Fbw**. In the vector of output quantities **Fw** the following components have been distinguished:

$$\mathbf{Fw}=[\mathbf{Fwz},\ \mathbf{Fwb},\ \mathbf{Fwt}]. \tag{15}$$

They are in turn the components of the input quantities vector in the III (**Fwz**), II and V (**Fwb**) groups of relations or of the output quantities vector **Ft** of the model of turbine (**Fwt**).

II – Relations defining the pressure distribution (vector **Fp**) in the flow part of a turbine

$$\mathbf{Fp}=Fp(\mathbf{Pp},\mathbf{Kt},\mathbf{Nt}),\qquad \mathbf{Pp}=[\mathbf{Ptp},\mathbf{Ppw},\mathbf{Ppb},\mathbf{Ppd}], \tag{16}$$

where **Ppw**=**Fwb** and **Ppb**=**Fbp** and **Ppd**=**Fdp** form the components of the output quantities vector from the IV and V groups of the relations, respectively ($\tau$ ratios in the equations of flow for a stage or group of stages). Two components of the vector **Fp** have been distinguished:

$$\mathbf{Fp}=[\mathbf{Fpz},\mathbf{Fpb}], \tag{17}$$

where **Fpz** and **Fpb** are the components of the input quantity vector in the III and the V group of relations, respectively.

III – Relations defining the quantities, which characterize the work of steam distribution set (vector **Fz**)

$$\mathbf{Fz}=Fz(\ \mathbf{z},\mathbf{Kt},\mathbf{Nt}),\qquad \mathbf{Pz}=[\mathbf{Ptz},\mathbf{Pzw},\mathbf{Pzp}], \tag{18}$$

$$\mathbf{Pzw}=\mathbf{Fwz},\qquad \mathbf{Pzp}=\mathbf{Fpz}.$$

The vector of output quantities is split here into two components **Fz**=[**Fzd**, **Fzt**], where **Fzd** is the part of the input quantities vector in the IV group of relations and **Fzt** is the component of the vector **Ft**.

IV – Relations defining quantities which characterize the work of the governing stage in a turbine (vector **Fd**)

$$\mathbf{Fd}=Fd(\mathbf{Pd},\mathbf{Kt},\mathbf{Nt}),\qquad \mathbf{Pd}=[\mathbf{Pdz},\mathbf{Ptw},\mathbf{Pdp}], \tag{19}$$

$$\mathbf{Pdz}=\mathbf{Fzd},\ \mathbf{Pdw}=\mathbf{Fwz},\ \mathbf{Pdp}=\mathbf{Fpz}.$$

In the vector **Fd** four components have been distinguished

$$\mathbf{Fd}=[\mathbf{Fdp},\mathbf{Fdb},\mathbf{Fdg},\mathbf{Fdt}],\qquad \mathbf{Fdp}=\mathbf{Ppd}, \tag{20}$$

where **Fdb** is the component of the vector of input quantities for the V group whereas **Fdg** – for the VI group of relations, **Fdt** is the component of the vector **Ft** (**Fdp**=**Ppd**).

V – Relations defining quantities which characterize the work of non-governing stages and groups of such stages (vector **Fb**)

$$\mathbf{Fb}=Fb(\mathbf{Pb},\mathbf{Kt},\mathbf{Nt}),\quad \mathbf{Pb}=[\mathbf{Ptb},\mathbf{Pbd},\mathbf{Pbp},\mathbf{Pbw}],$$
$$\mathbf{Ptb}=[\mathbf{Ptw},\mathbf{Ptp}],\quad \mathbf{Pbd}=\mathbf{Fdb},\quad \mathbf{Pbp}=\mathbf{Fpb},\quad \mathbf{Pbw}=\mathbf{Fwb}. \tag{21}$$

Four components have been distinguished in the vector **Fb**:

$$\mathbf{Fb}=[\mathbf{Fbw},\mathbf{Fbp},\mathbf{Fbg},\mathbf{Fbt}]. \tag{22}$$

Here **Fbg** forms the component of the input quantities vector of the VI group of dependences, and **Fbt** is the component of **Ft** (**Fbw**=**Pwb** and **Fbp**=**Pob**, explained above).

VI – Relations for determining the output power at the generator terminals and losses of the output power (vector **Fg**)

$$\mathbf{Fg}=\mathbf{Fg}(\mathbf{Pg},\mathbf{Kt},\mathbf{Nt}),\quad \mathbf{Pg}=[\mathbf{Pgd},\mathbf{Pgb}],$$
$$\mathbf{Pgd}=\mathbf{Ftg},\quad \mathbf{Pgb}=\mathbf{Fbg}. \tag{23}$$

With the above division of the turbine model into groups of relations the searched for vector of output quantities has the form:

$$\mathbf{Ft}=[\mathbf{Fwt},\mathbf{Fp},\mathbf{Fzt},\mathbf{Fdt},\mathbf{Fbt},\mathbf{Fg}]. \tag{24}$$

A flow diagram of solution of the mathematical model for the turbine with the above division into groups of relations is shown in Fig. 4. The diagram contains only the iteration on the second level.

Iterations of the first level are contained in blocks *9*, *11*, *13*, *16*, which was not shown for the sake of simplicity. To start the calculations in the I and II groups of relations it is necessary first to estimate the unknown vectors **Pwb** (rate of flow $G_x$ of water separated in the flow part) and **Ppb** and **Ppd** (ratios $\tau$ in the equations of flow for stages and groups of stages).

When we use adequate approximate relations [7] (item *3*) the estimations **P′wb**, **P′pb**, **P′pd** obtained are not too distant from the searched for solutions **Fbw**, **Fbp**, **Fdp**, determined in the process of iteration on the second level (item *5* through *14*). The preliminary estimation makes the calculations shorter, reducing the number of required iterations. The choice of rates of flow $G_x$ and ratios $\tau$ as quantities determined in the iteration on the second level is justified, among others, by relatively small changes of these quantities in changed working conditions of a turbine. This makes the preliminary estimation easier. In the case of solving the model of a turbine for conventional power plant [7] defining the vector **Fw** of the rate of flow by iteration is usually not necessary.

A mathematical model of a 220 MW turbine output has been formulated according to the presented above principles. Its steam conditions are 44 ata, 250°C with the external

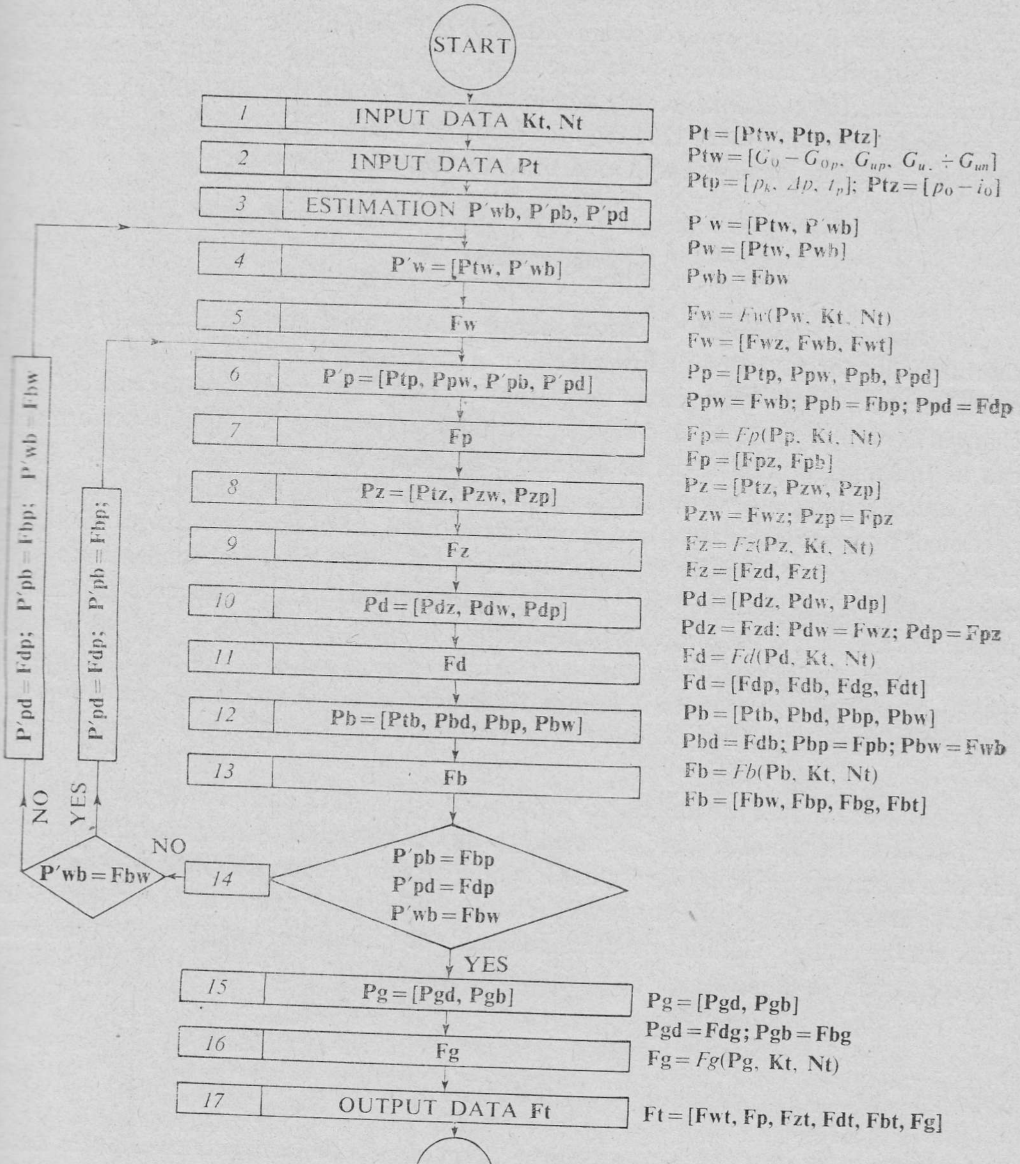


Fig. 4. Flow diagram of computations in a turbine model

moisture separator, and a single two-stage steam-to-steam reheater up to 240°C [12]. The turbine has a nozzle-control steam distribution system and its blade system consists of a governing stage and 10 non-governing stages. The mathematical model of that turbine included about 180 equations, mostly non-linear ones. Finding the solution for this system on the digital computer Odra 1204 required a couple of minutes; computations on a CDC computer CYBER 72 took about one third of second.

## Final remarks

The mathematical model discussed above allows quick determination of turbine characteristics for saturated or low superheated steam over the wide range of operating conditions and an arbitrary structure of the turbine. The influence of the rate of flow changes as well as the inlet and reheated steam conditions on the turbine performance can be investigated. The model allows also considering the changes of extraction steam flows caused, among others, by the changes in the thermal system of a turbo-set (e.g. disconnection of some of feed-heaters) and examining the influence of pressure changes of exhaust steam. This can be utilized for example when the optimum vacuum and condensing system are selected. When changing the values of model parameters, the influence of changes in efficiency of single stages and groups of stages can be investigated as well as changes of the effectiveness of the internal moisture extraction or gland leakage flows resulting for instance from the wear of parts. A change of the steam distribution model permits to compare the turbine performance with the nozzle control and throttle governing set and so forth.

In comparison with the traditional ways of determining the turbine characteristics by computation, the use of the above model greatly diminishes the labour consumption and the time necessary to obtain the results. Generally speaking, the model (within the turbo-set model) permits to investigate the results (changes of heat consumption) of changes of the turbo-set operating conditions over the wide range of values. The model can be used both for the turbine and power plant design and operation.

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## Model matematyczny turbiny kondensacyjnej na parę nasyconą dla elektrowni jądrowej

### Streszczenie

Znajomość osiągów turbozespołu parowego w innych warunkach pracy niż obliczeniowe jest nieodzowna, zarówno przy projektowaniu jak i w eksploatacji. Racjonalnym sposobem badania i wyznaczania charakterystyk turbozespołu, obok metody opartej na pomiarach, jest wykorzystanie modelu matematycznego do symulacji zmiennych warunków pracy na elektronicznej maszynie cyfrowej.

Przedmiotem rozważań jest kondensacyjna turbina dużej mocy na parę nasyconą lub lekko przegrzaną, instalowana w elektrowniach jądrowych. Podano ogólny model matematyczny takiej turbiny, pozwalający na wyznaczenie na elektronicznej maszynie cyfrowej osiągów i charakterystyk turbiny w ustalonych warunkach pracy, różniących się od warunków obliczeniowych.

Model rozpatrywanej turbiny jest oparty na wynikach pracy [7], gdzie podano zasady budowy i sformułowano taki model dla przypadku kondensacyjnej turbiny dużej mocy instalowanej w elektrowniach klasycznych. W referacie podano uzupełnienia i modyfikacje tego modelu, wynikające z osobliwości rozpatrywanej turbiny i jej układu cieplnego.

Odmienność układu cieplnego (głównie międzystopniowy przegrzew parą) wymagała innego określenia wielkości wejściowych i wyjściowych w modelu turbiny. Omówiono budowę i sposób rozwiązania modelu turbozespołu, z zastosowaniem zasady dekompozycji. Schemat blokowy obliczeń podano na rys. 2.

Aby zbudować model matematyczny turbiny dokonuje się odpowiedniego podziału jej części przepływowej na elementy wspólne dla turbin różnych typów i określa ogólne modele tych elementów, uzupełnione zależnościami opisującymi powiązania i współpracę elementów. Pozwala to na zestawienie z tych modeli cząstkowych modelu konkretnej turbiny. Wykazano, że racjonalnie jest wyróżnić tu układ rozrządu pary, stopień regulacyjny oraz stopnie i grupy stopni nieregulowanych. Modele tych elementów części przepływowej oparto na wynikach pracy [7]. Modele rozrządu pary stopnia regulacyjnego oraz stopnia pracującego w obszarze pary wilgotnej wymagały uzupełnień i modyfikacji.

Model matematyczny turbiny, zestawiony z modeli elementów, stanowi układ kilkuset nieliniowych równań i nierówności, którego rozwiązanie jest złożonym zadaniem numerycznym. Opracowano sekwencyjno-iteracyjną metodę obliczeń (schemat blokowy rys. 4).

Przedstawiony model matematyczny pozwala na szybkie wyznaczenie charakterystyk turbiny na parę nasyconą lub lekko przegrzaną o praktycznie dowolnej konstrukcji w szerokim zakresie ogólnie rozumianych zmian warunków pracy. Omówiono sposoby wykorzystania i zastosowania modelu.

Według tych zasad zbudowano przykładowy model matematyczny turbiny o mocy 220 MW [11] na parametry pary 44 ata, 250/240°C. Model tej turbiny zawierał ok. 180 równań i był rozwiązywany na maszynie cyfrowej Odra 1204 w ciągu kilku minut.

## Математическая модель конденсационной турбины на насыщенный пар для ядерной электростанции

### Резюме

Знание параметров, достигаемых паровым турбоагрегатом в условиях работы отличающихся от расчетных, необходимо как на этапе проектирования, так и во время эксплуатации. Рациональным способом исследования и определения характеристик турбоагрегата, рядом с методом, основанным на измерениях, является использование математической модели для симуляции переменных условий работы на ЭВМ.

Предметом рассуждений является конденсационная турбина большой мощности на насыщенный или легко перегретый пар, предназначенная для ядерной электростанции. Целью работы является определение принципов постройки и сформулирование общей математической модели такой турбины, позволяющей определять на ЭВМ достигаемые параметры и характеристики турбины в установившихся условиях работы, отличающихся от математически определенных условий.

Модель рассматриваемой турбины основана на результатах работы [7], где представлены принципы постройки и сформулирована такая модель для конденсационной турбины большой мощности, предназначенной для классических электростанций. В настоящей работе даны дополнения и выяснения, касающиеся модификации этой модели, вытекающие из особенностей рассматриваемой турбины и ее тепловой схемы.

Отличие тепловой схемы (в главной мере межступенчатый перегрев пара паром) требует другого определения входных и выходных величин в модели турбины, что связано с организацией расчетов в модели турбоагрегата. Обсуждается постройка и способ решения модели турбоагрегата с применением принципа декомпозиции. Блочная схема расчетов представлена на рис. 2.

Для постройки математической модели турбины проводится соответственный раздел ее проточной части на элементы общие для турбин различных типов и определяются общие модели этих элементов, пополненные зависимостями, описывающими взаимные связи и сотрудничество элементов. Это позволяет составить из этих моделей элементов модель конкретной турбины, практически независимо от ее конструкции. Доказывается, что обосновано здесь выделение системы парораспределения, регулирующей ступени, а также нерегулируемых ступеней и групп ступеней. Модели этих элементов проточных частей основаны на результатах работы [7]. Модели парораспределения регулирующей ступени и ступени, работающей во влажном паре требовали пополнений и модификации.

Математическая модель турбины, составленная из моделей элементов, образует систему нескольких сотен нелинейных уравнений и неравенств, решение которой является сложной цифровой задачей. Разработан секвенционно-итерационный метод расчета (блочная схема на рис. 4).

Представленная математическая модель позволяет быстро определять характеристики турбины на насыщенный или легко перегретый пар, практически произвольной конструкции, в широком диапазоне общепонимаемых изменений условий работы. Обсуждаются способы использования и применения модели.

По указанным принципам построена примерная математическая модель турбины мощностью 220 Мвт [11] на параметры пара 44 ата, 250/240°С. Модель этой турбины состояла из ок. 180 уравнений и решалась на ЭВМ Одра 1204 в течение нескольких минут.