POLSKA AKADEMIA NAUK
INSTYTUT MASZYN PRZEPŁYWOWYCH

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

TRANSACTIONS
OF THE INSTITUTE OF FLUID-FLOW MACHINERY

70-72

WARSZAWA–POZNAŃ 1976

PAŃSTWOWE WYDAWNICTWO NAUKOWE

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

REDAKCJA – EDITORIAL OFFICE
Instytut Maszyn Przepływowych PAN
ul. Gen. Józefa Fiszera 14, 80-952 Gdańsk, skr. pocztowa 621, tel. 41-12-71

Printed in Poland

PAŃSTWOWE WYDAWNICTWO NAUKOWE – ODDZIAŁ W POZNANIU

Wydanie I. Nakład 630+90 egz. Ark. wyd. 63,25 Ark. druk. 49,25 Papier druk. sat. kl. V. 65 g., 70×100. Oddano do składania 4 VII 1975 r. Podpisano do druku w czerwcu 1976 r. Druk ukończono w czerwcu 1976 r. Zam. 528/118. H-16/245.
Cena zł 190,–

DRUKARNIA UNIWERSYTETU IM. ADAMA MICKIEWICZA W POZNANIU

III KONFERENCJA NAUKOWA

na temat

TURBINY PAROWE WIELKIEJ MOCY

Gdańsk, 24 - 27 września 1974 r.

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IIIrd SCIENTIFIC CONFERENCE

on

STEAM TURBINES OF GREAT OUTPUT

Gdańsk, September 24 - 27, 1974

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III НАУЧНАЯ КОНФЕРЕНЦИЯ

на тему

ПАРОВЫЕ ТУРБИНЫ БОЛЬШОЙ МОЩНОСТИ

Гданьск, 24 - 27 сентября 1974 г.

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH
1976 Zeszyt 70 - 72

DIETER BARSCHDORFF, GOTTFRIED HAUSMANN, ARNOLD LUDWIG

Federal Republic of Germany*

Flow and Drop Size Investigations of Wet Steam at Sub- and Supersonic Velocities with the Theory of Homogeneous Condensation**

List of symbols

A – cross-section area of the nozzle [m²],
a – frozen local sound speed [m/s], thermal accomod. coefficient,
C – concentration [m^{-3}],
c_p – specific heat capacity at constant pressure [J/kg K],
D – diffusion coefficient between vapor and carrier gas [m²/s],
dg' – increase of condensate mass fraction due to nucleation,
dg'' – increase of condensate mass fraction due to droplet growth,
g – condensate mass fraction, eq. (15, 16) [kg/kg],
J – nucleation rate [$m^{-3}s^{-1}$],
k – Boltzmann's constant [J/K],
Kn – Knudsen number,
L – latent heat of condensation or sublimation [J/kg],
$\bar{l}$ – mean free path of molecules [m],
M – local Mach number,
m – molecular mass [kg],
$\dot{m}$ – mass flow (in eq. (4): from environment to cluster; elsewhere: through nozzle) [kg/s],
n – number of molecules in cluster; total number of droplet classes existing at present,
p – pressure [N/m², bar],
R – universal gas constant, $R=8314.3$ J/kmol K,
r – droplet radius [m],
s – supersaturation ($s=p/p_s$),
T – temperature [K, °C],
t – time [s],
v – molecular volume [m³],
x – co-ordinate along the nozzle axis [m],
y^* – nozzle height at throat [m],
Z – number of clusters within a droplet class,
α – heat transfer coefficient [W/m²K],
β – mass transfer coefficient [kg/N s],
γ – isentropic exponent,
η – dynamic viscosity [kg/ms],
θ – preexponential factor in non-isothermal nucleation theory,
λ – thermal conductivity [W/m K],
μ – mole mass [kg/kmol],
ρ – mass density [kg/m³],
σ – surface tension [N/m, dyn/cm].

Superscripts

$\bar{\ }$ – averaged the values of vapor and carrier gas by mixture rule (but for $\bar{l}$ see list of symbols),
$*$ – at the throat; critical, i.e. corresponding to Kelvin-Helmholtz equation, eq. (1).

Subscripts

c – condensate,
e – origin position of clusters,

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** Dedicated to Professor Dr.-Ing. Rudolf Friedrich on the occasion of his 65th birthday in September 1974.

i – index of droplet class,
n – forming nucleus,
s – saturation, i.e. state at the vapor-liquid coexistence line,
v – vapor,
0 – supply parameters; initial,
∞ – flat interface ($r = \infty$).

Introduction

Spontaneous condensation of water vapor in rapid expansions of superheated or saturated steam is still under investigation. While the droplet growth and the gasdynamics of nozzle flow with condensation are well understood, fundamental uncertainties exist in the physical conception of nucleation theories in their classical treatment [1, 2] or the statistical mechanical formulation [3]. Briefly, a possible motion of critical clusters, the concept of curvature dependence of the surface tension and the mass accomodation coefficient should be recalled.

We do not attempt to verify the one or the other condensation theory. Rather, the purpose of this investigation is to show, that consistently applied Volmer's nucleation rate equation together with suitable droplet growth and gasdynamic equations describes the expansion of condensing water vapor without any further empirical factors like friction or sticking coefficients or even a curvature dependence of the surface tension of small clusters, where statements of investigators diverge [4 - 7]. A detailed analysis of the wetness problem, blade erosion, efficiency and flow stability in turbine stages implies the knowledge of a calculation algorithm, which shows the practical relevance of this work.

Nonisothermal nucleation rate equation

The classical nucleation theory [1, 2] and [8] treats the equilibrium distribution of motionless critical clusters with radius r^*,

$$r^* = \frac{2\sigma(T)\,v_c}{kT \ln\left(\dfrac{p}{p_s(T)}\right)}, \tag{1}$$

within a vapor phase, multiplied by the collision rate of monomers from the kinetic theory of gases. The results of several authors differ slightly, but with the formulation of Volmer the well known result is

$$J = \frac{(2m\sigma(T)/\pi)^{\frac{1}{2}}}{\rho_c}\left(\frac{p}{kT}\right)^2 \exp\left\{-\frac{4\pi r^{*2}\sigma(T)}{3kT}\right\}. \tag{2}$$

This expression is derived assuming macroscopic, motionless droplets of critical radius r^*, which are in thermodynamic equilibrium with the vapor, i.e. an isothermal process is considered.

However, critical clusters containing only few water molecules cannot be assumed to stay motionless in space because of their Brownian motion, and recent treatments of nucleation theory use as a model a cluster, which has rotational and translational degrees of freedom and which is free to move in a certain volume, a concept, which may correspond to the flow process in nozzles. So far, no correlation between the statistical mechanical theories and experimental observations in steam condensation could be achieved [9].

Moreover, the isothermal model assumption does not apply for the spontaneous condensation in a pure vapor. Here the latent heat of condensation from growing nuclei has to be removed by collisions of vapor molecules with clusters and the nucleation rate is decreased with respect to eq. (2). A correction term was derived by Feder *et al.* [10], which yields for steam condensation

$$\theta=\frac{\gamma+1}{(\gamma+1)+2(\gamma-1)\{L/((R/\mu)T)-0.5-(1-\partial\ln(\sigma)/\partial\ln(T))\ln(s)\}^2} \tag{3}$$

and it was shown [11] that uncertainties in steam nozzle condensation are reduced by using eq. (2) together with eq. (3). This will be discussed here systematically.

Droplet growth and droplet spectra

Once clusters of the critical size have been established within a subcooled vapor, their further behaviour as a function of supersaturation, droplet size and thermodynamic properties can be described by suitable droplet growth laws. The size of droplets with critical radius after eq. (1) at nozzle flow conditions usually are much smaller than the mean free path of the vapor molecules (Knudsen numbers $Kn=\bar{l}/2r\gg1$). Therefore, their growth to equilibrium conditions with the vapor can only be ascertained by considering molecular and macroscopic transport processes, depending on the droplet size.

Difficulties with the choise of suitable condensation coefficients [12] can be avoided by using Gyarmathy's [13] droplet growth law, which takes into account diffusion of vapor molecules through the surrounding gas as well as heat and mass transfer, and the influence of capillarity. This law covers continuum and free molecular flow conditions.

With

$$\dot{m}=\rho_c 4\pi r^2\,\frac{dr}{dt} \tag{4}$$

and

$$\frac{dr}{dx}=\frac{dr}{dt}\,\frac{dt}{dx}=\frac{1}{M\left[\gamma\dfrac{R}{\mu}T\right]^{\frac{1}{2}}}\,\frac{dr}{dt} \tag{5}$$

we use a more detailed form of this droplet growth equation* for steady, one-dimensional nozzle flow of steam

$$\frac{dr}{dx}=\frac{1-\dfrac{r^*}{r}}{M\left[\gamma\dfrac{R}{\mu}T\right]^{\frac{1}{2}}}\,\frac{1}{\rho_c(T)}\,\frac{\ln\dfrac{p}{p_s(T)}}{\dfrac{L^2(T)}{\alpha\dfrac{R}{\mu_v}T^2}+\dfrac{1}{\beta p_v}}, \tag{6}$$

where

$$\alpha = \frac{\bar{\lambda}}{r + \frac{2\sqrt{2\pi}}{a} \frac{\gamma}{\gamma+1} \frac{\bar{\lambda}}{1.5\,\bar{\eta}\,\bar{c}_p} \bar{l}}, \tag{7}$$

$$\beta = \frac{\frac{D}{(R/\mu_v)\,T} \frac{p}{p-p_v}}{r + \frac{\sqrt{2\pi}}{a_v} \frac{\mu_v}{\bar{\mu}} \frac{p}{p-p_v} \frac{\rho D}{1.5\,\bar{\eta}} \bar{l}} \tag{8}$$

and

$$l \simeq \frac{1.5\bar{\eta}\sqrt{(R/\bar{\mu})\,T}}{p}. \tag{9}$$

The diffusion term $1/\beta \cdot p_v$ in eq. (6) vanishes for $p = p_v$ i.e. pure water vapor.

For numerical solutions of eq. (6) an initial or starting radius r_{init} is needed and different model assumptions are suggested by authors. A droplet of critical radius r^* from eq. (1) is in equilibrium with the surrounding vapor and $dr^*/dt = 0$ per definitionem, see eq.

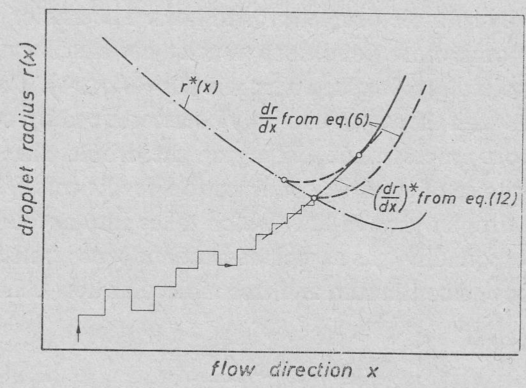

Fig. 1. Growth of critical clusters

(6) for $r = r^*$. The underlying assumption is shown in Fig. 1. A zero growth rate of critical nuclei can be avoided by empirically chosing $r_{\text{init}} = r^* + \Delta r$. This again is a very delicate parameter for calculations of steam condensation, because droplet growth starts far ahead of the actual "condensation zone" within the nozzle near saturation condition, $s \approx 1$. Condensate mass fraction increase due to nucleation, dg', first dominates but is soon outstripped by mass fraction increase due to growth, dg''. The avoidance of empirical factors for initial radii or accomodation factors in the droplet growth equation therefore decreases uncertainties.

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* From a private communication with Dr. G. Gyarmathy, Brown, Boveri & Cie, Baden, Switzerland, we learned, that for detailed calculations the factor 1.59 in eq. (51) of his original paper has to be replaced by two expressions, derived from kinetic theory and implicit given in eq. (7) and (8).

Nucleation as given by eq. (2) is a growth process through droplet classes from monomers, dimers etc. until critical nuclei are reached at r^*, containing n^* molecules. The critical droplets still have a "growing tendency" [9] and the critical growth rate can be derived from eq. (2).

Therefore the growth rate of critical clusters in the steady state of formation corresponds to the rate, at which clusters grow from $r(n^*) \to r(n^*+1)$. This yields

$$r_{n^*+1} = r_{n^*} + dr_n^*. \tag{10}$$

The rate of formation then is given by

$$\frac{J}{C(n^*)} = \frac{4\pi r_n^2}{v_c} \frac{dr^*}{dt}, \tag{11}$$

where $C(n^*)$ is the equilibrium distribution of critical clusters.

The growth rate of critical nuclei evaluated from eqs. (2) and (3) is therewith

$$\frac{dr^*}{dx} = \frac{\theta}{M\left(\gamma \frac{R}{\mu} T\right)^{\frac{1}{2}}} \left(\frac{m}{\rho_c}\right)^2 \left(\frac{p}{kT}\right) \sqrt{\frac{2\sigma_\infty}{\pi m}} \frac{1}{4\pi r^2}. \tag{12}$$

Instead of eq. (6), which yields $dr^*/dx = 0$, the growth of just born nuclei is calculated from eq. (12) as far as this gives a greater dr/dx.

To get closer to the physical reality, instead of an "average droplet size" a spectral distribution of droplet sizes is taken into account. Herewith the slower rate of condensation on larger drops originated at a position earlier within the nozzle and more rapid rates on smaller clusters at downstream positions are regarded explicitly.

For this purpose, all droplets originating in an interval x_e, $x_e + \Delta x_e$ are collected in a "droplet class" with an initial size averaged over the small range Δx_e only, where $\Delta x_e/y^* \ll 1$ (y^* is the height of the nozzle throat). An averaging algorithm is used for the product $J(x_e) \cdot A(x_e) \cdot \Delta x_e$, which signifies the number of nuclei originating in the volume $A(x_e) \cdot \Delta x_e$ per unit time, and for the initial radius r_0, which corresponds to the critical radius. The algorithms are

$$Z_i = \{J(x_{ei}) A(x_{ei}) \Delta x_{ei}\}_{\text{average}} =$$
$$= \tfrac{1}{4} \{J(x_e) A(x_e) + 2J(x_e + \tfrac{1}{2}\Delta x_e) A(x_e + \tfrac{1}{2}\Delta x_e) + J(x_e + \Delta x_e) A(x_e + \Delta x_e)\}_i \Delta x_{ei} \tag{13}$$

and

$$r_{0i} = \frac{1}{4Z_i} \{r^*(x_e) J(x_e) A(x_e) +$$
$$+ 2r^*(x_e + \tfrac{1}{2}\Delta x_e) J(x_e + \tfrac{1}{2}\Delta x_e) A(x_e + \tfrac{1}{2}\Delta x_e) +$$
$$+ r^*(x_e + \Delta x_e) J(x_e + \Delta x_e) A(x_e + \Delta x_e)\}_i \Delta x_{ei}, \tag{14}$$

where $i = 1, 2 \ldots, n$ is the index of the droplet class, and n is the total number of classes. The radius of each of these classes is integrated separately, and its contribution is added to the increasing mass of condensate.

Calculation of flow parameters

The calculation of flow parameters within the nozzle follows the basic ideas outlined by Oswatitsch [14]. The equations are derived under the usual assumptions for one-dimensional nozzle flow:

- the vapor phase is considered as perfect gas,
- the volume and compressibility of the condensate is neglected,
- the variations of state proceed in a quasi-static manner, friction and coagulation of droplets are insignificant.

From the basic equations of state, continuity, momentum and energy, we derive a system of differential equations which is applicable for a mixture of a vapor and an inert gas and for a pure vapor as well

$$\frac{dg}{dx}=\frac{4\pi\rho_c(T)}{\dot{m}}\left\{\sum_{i=1}^{n}\left(Z_i r_i(x)^2\frac{dr_i}{dx}\right)+\tfrac{1}{3}r_0(x)^3 J(x)\,A(x)\right\}, \tag{15}$$

$$\frac{dp}{dx}=p\,\frac{\gamma M^2}{(1-g)-\gamma M^2+(\gamma-1)\,M^2\dfrac{(1-g)}{\left(1-\dfrac{g}{\bar{c}_p}\dfrac{dL}{dT}\right)}}\left\{\frac{1}{A}\frac{dA}{dx}+\right.$$

$$\left.+\left(\frac{\bar{\mu}}{\mu_v}-\frac{(1-g)}{\left(1-\dfrac{g}{\bar{c}_p}\dfrac{dL}{dT}\right)}\frac{L}{\bar{c}_p T}+\frac{\left(1-\dfrac{c_{pv}}{\bar{c}_p}\right)}{\left(1-\dfrac{g}{\bar{c}_p}\dfrac{dL}{dT}\right)}\right)\frac{1}{(1-g)}\frac{dg}{dx}\right\}, \tag{16}$$

$$\frac{dT}{dx}=T\,\frac{(1-g)}{\left(1-\dfrac{g}{\bar{c}_p}\dfrac{dL}{dT}\right)}\left\{\frac{(\gamma-1)}{\gamma}\frac{1}{p}\frac{dp}{dx}+\left(\frac{L}{\bar{c}_p T}+\frac{\left(1-\dfrac{c_{pv}}{\bar{c}_p}\right)}{(1-g)}\right)\frac{1}{(1-g)}\frac{dg}{dx}\right\}, \tag{17}$$

$$\frac{d\rho}{dx}=\rho\left\{\frac{1}{p}\frac{dp}{dx}+\frac{\bar{\mu}}{\mu_v}\frac{1}{(1-g)}\frac{dg}{dx}-\frac{1}{T}\frac{dT}{dx}\right\}, \tag{18}$$

and

$$\frac{dM}{dx}=-M\left\{\frac{(1-g)}{\gamma M^2}\frac{1}{p}\frac{dp}{dx}+\frac{1}{2}\frac{1}{T}\frac{dT}{dx}+\frac{1}{2}\left(1+(\gamma-1)\frac{c_{pv}}{\bar{c}_p}-\gamma\frac{\bar{\mu}}{\mu_v}\right)\frac{1}{(1-g)}\frac{dg}{dx}\right\}. \tag{19}$$

Here ρ is the density of the two phase mixture with $\rho(1-g)$, the density of the gas phase only. For a vapor-inert gas mixture, $1/\bar{\mu}$ and $\bar{c}_p$ of the gas phase have to be evaluated from mixing laws.

The condensate mass fraction $g=\dot{m}_c/\dot{m}$ and the derivation $dg(x_e, x)$ depends on the position x_e in the nozzle, where a droplet was born with initial s ize $r_0(x_e)$, and the calculation coordinate x. Therefore evaluation of dg requires an integration of number and size of all droplets for all values of $x_e = x_{e1}$ until $x_e = x$, where x_{e1} is the nozzle location, where nucleation occurs first. Isentropic expansion of the steam is assumed ahead of the initial point of calculations upstream of the actual condensation zone in the nozzle.

Operating modes of steam nozzle

Convergent-divergent steam flow channels can operate in different flow modes [15]. In experiments with steam condensation zones usually appear in the supersonic part of the nozzle, where flow without and with a steady shock wave is observed. Unstable flow has been reported [15, 18] and operating modes with subsonic throat conditions will be discussed later.

1. Subcritical heat addition

If all flow parameters vary continuously in the zone of spontaneous condensation, we have subcritical heat addition by the condensation process. We will discuss the application of the above derived calculation algorithm on the results of several authors and on our own pressure measurements on two different nozzles (Appendix).

The calculation of nozzle flow parameters requires the knowledge of an effective area distribution rather than the geometrical nozzle contour. We obtain $A = A(x)$ from the measured pressure distribution for an expansion of superheated steam without condensation in the test section, assuming isentropic flow.

For the comparison of calculations with experimental results an empirical factor Γ is inserted in eq. (2)

$$J_{\text{exp}} = \Gamma\, \theta\, J_{\text{Volmer}}. \tag{20}$$

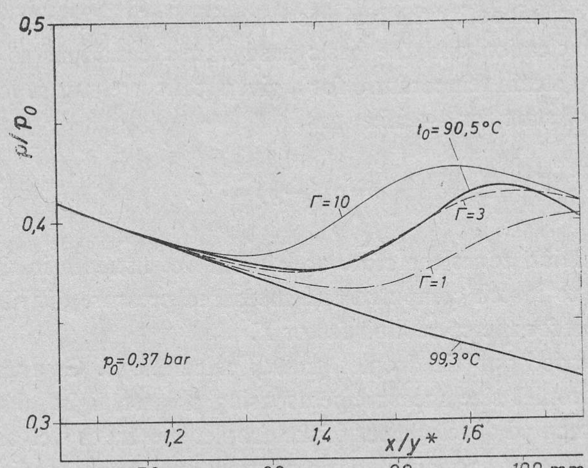
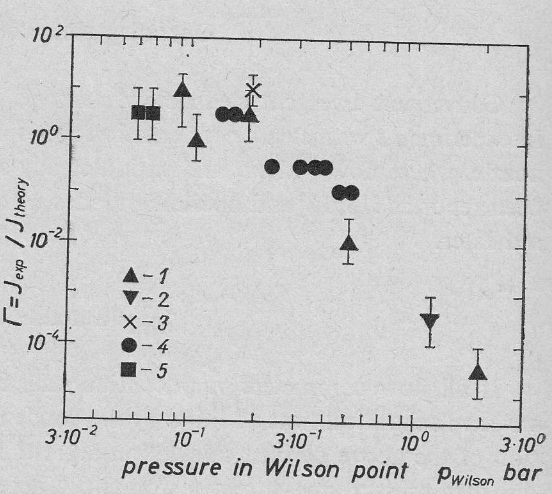


Fig. 2. Variation of matching factor Γ to achieve best correspondence between calculation and experiment. (Exp. 2 - 19 and 2 - 20 in nozzle 2, Barschdorff and Ludwig 1972)

$\Gamma = J_{exp}/J_{theory}$ is varied until best agreement between the experimental and the theoretical pressure distribution is obtained, Fig. 2. By applying the here described computation algorithm, a rather small correction, $\Gamma = 1$ to 10, to eq. (2) yields excellent agreement to measured pressure curves.



1 – Gyarmathy and Meyer [19], different nozzles,
2 – Petr [20],
3 – Stein and Moses [21],
4 – Barschdorff and Ludwig, nozzles 1 and 2,
5 – Barschdorff, shock tube [22]

Fig. 3. Correction factor $\Gamma = J_{exp}/J_{theory}$ versus static pressure in the Wilson point, corresponding to the maximum in supersaturation. (J_{exp}: nucleation rate from eq. (20), where theory and experiment are matched, Fig. 2. J_{theory}: nonisothermal nucleation rate from eq. (2) and eq. (4))

However, this good agreement is obtained only for low initial stagnation pressures $p_0 < 0.5$ bar or for Wilson pressures $p_{Wilson} < 0.3$ bar, Fig. 3 and 7. Here, an analysis of steam nozzle expansions of three other authors [19 - 21] and data from shock tube experiments [22] show a systematical deviation of J_{exp}/J_{theory} for higher pressures and temperatures. This can be explained by the real gas behaviour of the steam, assuming the theory of homogeneous nucleation and the basic gasdynamic equations to be valid*. At high pressures, the actual steam temperature for a given static pressure is lower than computed from the equation of state for a perfect gas

$$p = \rho (1 - g) \frac{R}{\mu} T . \tag{21}$$

Calculated condensation zones therefore are shifted downstream in the nozzle, requiring larger values for Γ to match computed and measured pressure distributions. This points in the right direction for the correction factor Γ.

A second test is carried out via droplet growth calculations. Gyarmathy, Lesch [18] reported detailed droplet size measurements using a laser light absorption technique. Fig. 4 shows the computed development of the droplet spectra as well as the mass averaged droplet radius for the corresponding nozzle contour and initial conditions. The spectrum

* High pressure condensation experiments qualitatively show the same pressure distributions as low pressure steam nozzle flow [26].

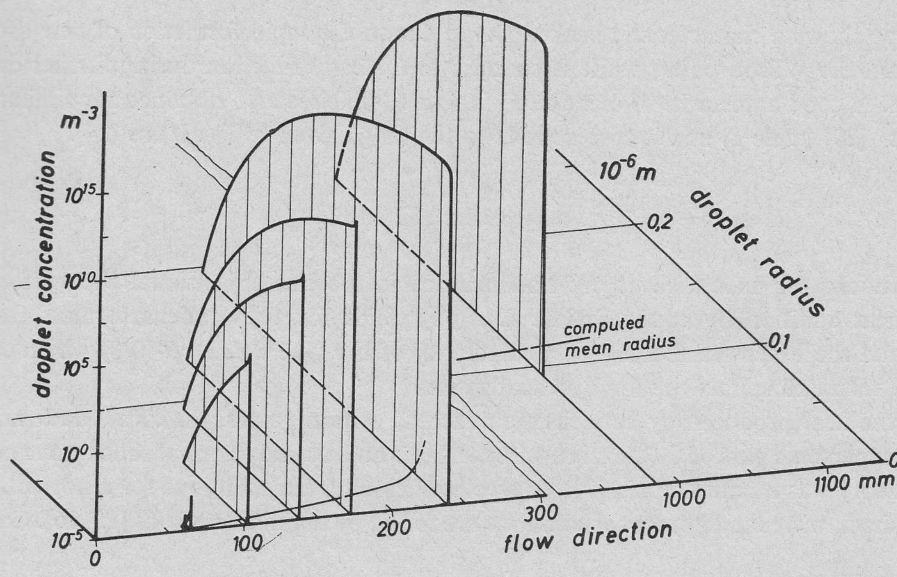


Fig. 4. Computed development of drop size distribution in a Laval nozzle. (Experimental data from Gyarmathy, Lesch [23]. Measured mean radius was approximately $0.14 \cdot 10^{-6}$ m)

Table 1

Selected data of analysed experiments

Author	Reference	Number of experiments	p_0 [bar]	t_0 [°C]	p_{Wilson} [bar]	t_{Wilson} [°C]	Mass average droplet radius (10^{-6} m) at the end of the nozzle: measured	calculated
Petr	[20]	—	3.5	170.0	1.19	66.7	0.1	0.095
Gyarmathy and Lesch	[23]		1.03	125.0	0.38	38.6	0.12 - 0.15	0.077 - 0.113
Stein	[21]	258	0.537	99.65	0.194	18.0	—	0.005
Gyarmathy and Meyer	[19]	119	4.91	169.0	1.87	77.5	0.033 - 0.046	0.055
		108	0.621	119.0	0.185	17.4	0.014 - 0.02	0.02
		95	2.05	175.0	0.515	47.5	0.1 - 0.15	0.093 - 0.115
Barschdorff and Ludwig		1 - 03	0.78	123.0	0.236	21.4	—	0.012
		1 - 16	1.16	119.8	0.465	41.4	—	0.027
		2 - 36	1.45	134.2	0.539	46.6	—	0.035
		2 - 08	0.78	108.0	0.318	32.9	—	0.026
		2 - 26	0.41	93.3	0.157	16.3	—	0.012
		2 - 19	0.37	90.5	0.141	13.7	—	0.011

envelope shows a characteristic peaked shape due to the rapid formation of new droplets upstream the Wilson point, while it changes to a steady function during further droplet growth. A good agreement of mean radii is found, which is also obtained for similar data by Petr [20] (table 1) and it corresponds to the range given by Walters [24].

2. Supercritical heat addition

Solutions for shock waves are not included in the one-dimensional theory for flow with heat addition by condensation. Eq. (15) yields for the condensate mass fraction $g=0$ and the Mach number $M\to 1$ a singularity of the type $dp/dx=p\cdot(0/0)$ which can be solved with l'Hospitals rule or by linearization.

If the heat produced by condensation exceeds certain values, the local Mach number in the supersonic part of the nozzle again decreases and a steady normal shock wave occurs. The theory was modified for this flow type by including the equations for a normal, adiabatic shock wave, and numerical solutions are found by iterative methods [15, 16]. Calcula-

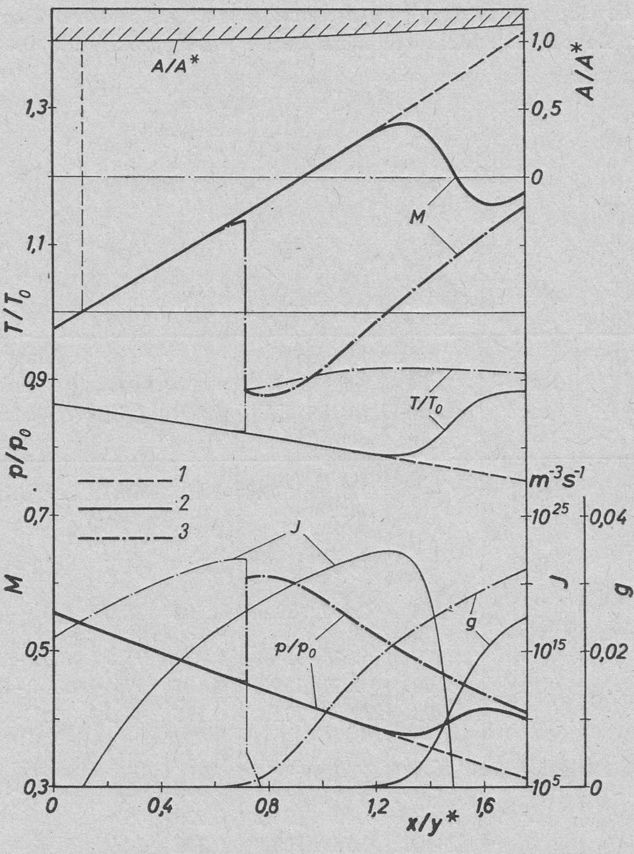


Fig. 5. Calculated flow parameters along the nozzle axis; condensation at supersonic speed
1 – isentropic flow, $p_0=0.41$ bar, $t_0=104.6°C$ (Exp. 2-24), *2* – subcritical heat addition by condensation, $p_0=0.41$ bar, $t_0=93.3°C$ (Exp. 2-26), *3* – supercritical heat addition by condensation, $p_0=0.41$ bar, $t_0=82.7°C$ (Exp. 2-31)

ted shock positions are found to be close to the measured ones. Fig. 5 shows the computed flow parameters for typical experiments with sub- and supercritical heat addition*. It can be seen, that due to the temperature, pressure and density increase in the shock wave the nucleation rate J nearly drops out and only few new droplets are born downstream of the aerodynamic shock wave.

3. Unstable flow with condensation

For nearly saturated initial conditions of the steam unstable flow patterns are observed. Shock waves are formed and move upstream through the nozzle throat in a strict periodic manner. Frequencies between 580 and 903 Hz were measured in a slender nozzle with circular profils [25]. Problems related to this flow process are the stability limit for pulsating flow and the frequencies observed for different initial conditions and curvatures of the nozzle contour. Resonant vibration phenomena could cause dangerous operating conditions for turbine rotor and stator blades. Also, nozzle mass flow is reduced in this operating mode, which should be avoided in turbine cascades.

4. Condensation with subsonic throat conditions

In turbine cascades, homogeneous nucleation is assumed to occur at subsonic velocities. Due to the work performed at the rotor blades steam attains highly supersaturated conditions without being accelerated to supersonic speeds. Condensation then may take place in convergent flow channels and the arising questions deal with flow stability, feedback of condensation on nozzle flow, erosion and efficiency of the flow process. Theoretical solutions of this flow regime are discussed now, which however have not yet been successfully simulated in steam flow channels**. If the calculations are started out with initially supercooled conditions, the condensation zone occurs upstream of the effective nozzle throat. There the flow speed is subsonic but increases to supersonic values in the divergent nozzle section due to further heat addition by condensation.

In the numerical solutions, however, this yields a singularity of eq. (15) like in the solutions for the adiabatic shock wave. It is of the type $dp/dx=p\cdot 0/0$ for $g>0$, $M<1$ and eq. (15) can be linearized, if numerator and denominator approach zero simultaneously. Two typical solutions exist which do not represent the real behaviour of a Laval-type nozzle flow at supercritical pressure ratios:

– if the numerator faster tends to zero, not sufficient heat is produced to fully expand the flow. Supersonic flow is not achieved;

– if the denominator faster approaches zero, this leads to a solution $dp/dx\rightarrow\infty$, which is physically not meaningful.

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* Numerical calculations were performed on he UNIVAC 1108 computer, computer center University of Karlsruhe.

** In our laboratory nozzle experiments with initially supercooled steam are planned for the near future.

Therefore, at a fixed nozzle location upstream the condensation zone the Mach number and the other depending flow parameters iteratively are changed and the computation is repeated. The procedure quickly converges against the one and only solution for flow with condensation at subsonic velocities with given supercooled initial conditions. Two typical examples are shown in Fig. 6.

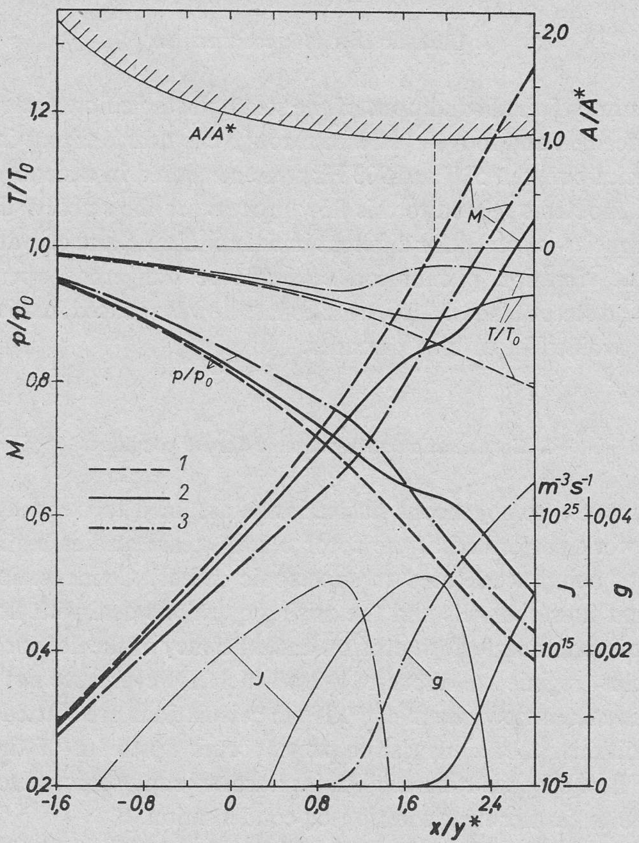


Fig. 6. Calculated flow parameters along the nozzle axis; condensation onset at subsonic speed
1 – isentropic flow, $p_0 = 1$ bar, $t_0 = 190°C$, *2* – slightly supercooled stagnation state, $p_0 = 1$ bar, $t_0 = 95°C$, *3* – largely supercooled stagnation state, $p_0 = 1$ bar, $t_0 = 85°C$

Different from condensation at supersonic speeds here pressure, temperature and Mach number deviate from isentropic values upstream from the actual condensation zone and initial steam nozzle conditions, not stagnation values, are altered. In general, the whole flow field in a nozzle is influenced and the mass flow rate is decreased if condensation occurs at subsonic velocities. The condensation zone is not characterized by marked pressure disturbances especially for a larger subcooling. At locations ahead of the condensation zone, deviations from isentropic values become more significant with increasing supercooling. Less supercooling produces deviations near the throat similar those well known from supersonic experiments.

Conclusions

Nucleation rate calculations and their application to steam expansions in different environments prove, that the nonisothermal theory of homogeneous nucleation by Feder *et al.* [10] quantitatively predicts supersonic nozzle flow of condensing steam at low pressures. By properly taking into account the growing tendency of critical clusters as well as the spectral distribution of droplet sizes and their different growth rates, uncertainties in computed nucleation rates are reduced from the often cited 10^{17} to one order of magnitude or less without the aid of corrections for a unknown curvature dependence of surface tension, friction factors or mass accomodation coefficients.

For low pressure steam flow in supersonic nozzles, the computed parameter of state variations, the mass averaged droplet radii and stability limits for pulsating flow can be predicted with adequate accuracy, compared with measurements. However, the analysis shows that great care has to be taken when high pressure steam flow with homogeneous condensation is treated as a perfect gas. The identical data reduction and computational scheme, applied to steam expansions of different authors in a pressure range 0.1 bar $\leqslant$ $\leqslant p_0 \leqslant$ 4.9 bar (40 bar) clearly points out, that the application of the perfect gas equation of state is limited to pressures $p_{\text{Wilson}} \leqslant 0.5$ bar. This might not be relevant for purely fluid-mechanics calculations for turbine stage designing, not taking into account the condensation process.

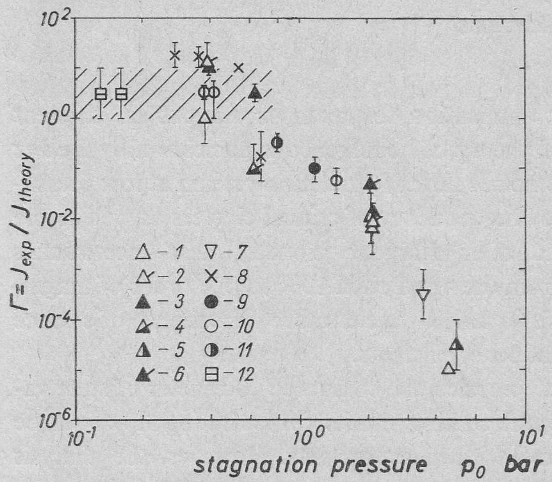
Steam expansions with saturated or slightly supercooled initial conditions show strong pressure and Mach number gradients in the transonic nozzle section and this leads to unstable flow patterns. These are dangerous operating modes, which can induce resonant blade vibrations. Highly supercooled steam however expands "smoothly" without major pressure disturbances and the flow should stabilize again.

The described theory treats sub- and supersonic flow of condensing steam at low pressures and can be included in turbine stage designing. It yields informations on questions concerned with the spectral distribution of drop sizes, thermodynamic losses and aerodynamic feedback of the condensation process on flow parameters.

Appendix A

Matching factor to nucleation theory

A series of experiments in a given nozzle, performed with constant initial stagnation pressure p_0 but different temperatures can be correlated to computed pressure distributions with one value for the correction factor Γ within a small error limit. This is relevant for practical analysis of steam expansions, where initial conditions are given, but the static pressure at which condensation will start is not known a priori. Fig. 7 shows the correction factor Γ versus the stagnation pressure for a series of experiments of different authors.



Nozzle experiments:
1 – Gyarmathy, Meyer [19] large nozzle
2 – ,, ,, medium nozzle
3 – ,, ,, small nozzle
4 – ,, ,, large and medium nozzle
5 – ,, ,, large and small nozzle
6 – ,, ,, medium and small nozzle
7 – Petr [20]
8 – Stein, Moses [21]
9 – Barschdorff, Ludwig nozzle 1
10 – ,, ,, nozzle 2
11 – ,, ,, nozzles 1 and 2
Shock tube experiments:
12 – Barschdorff [22]

Fig. 7. Correction factor $\Gamma = J_{\mathrm{exp}}/J_{\mathrm{theory}}$ versus initial stagnation pressure p_0
Dashed area represents the pressure range where perfect gas state equation may be applied

Appendix B

Experimental procedure

Nozzle experiments were performed in the steam tunnel of the Institut für Thermische Strömungsmaschinen, Universität Karlsruhe, W. -Germany. The tunnel provides facilities for the regulation of initial pressures and temperatures, maximum stagnation pressure is 2.0 bar. The test section is a two-dimensional flow channel with a width of 50 mm. As nozzle contour we used symmetrical circular profiles with a radius of $R_1 = 584$ mm for nozzle 1 and $R_2 = 1027$ mm for nozzle 2. The height in the throat was $y^* = 60$ mm for both nozzles.

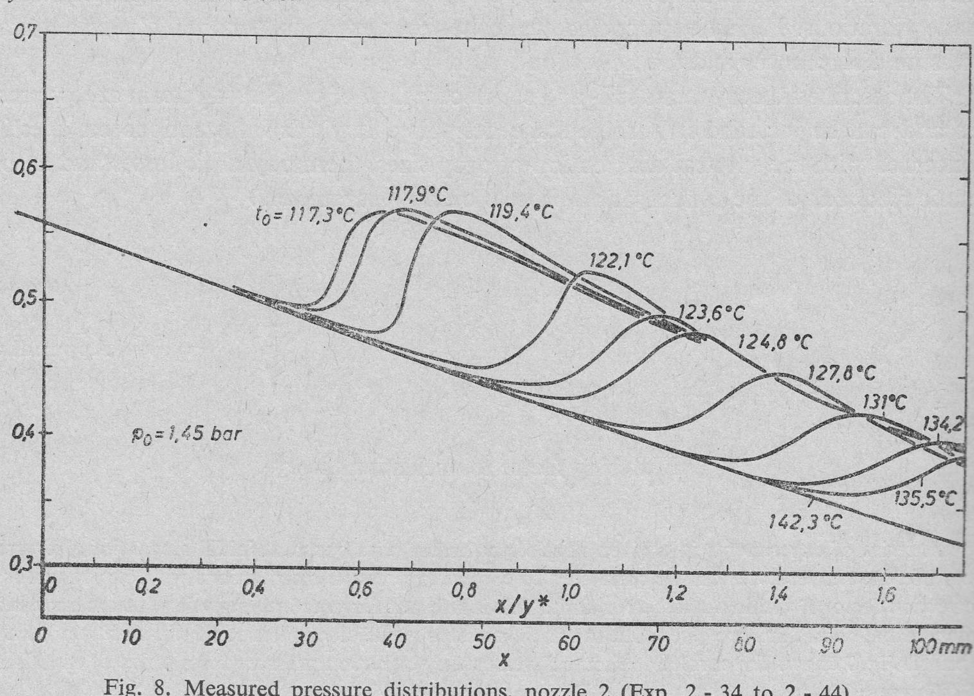


Fig. 8. Measured pressure distributions, nozzle 2 (Exp. 2 - 34 to 2 - 44)

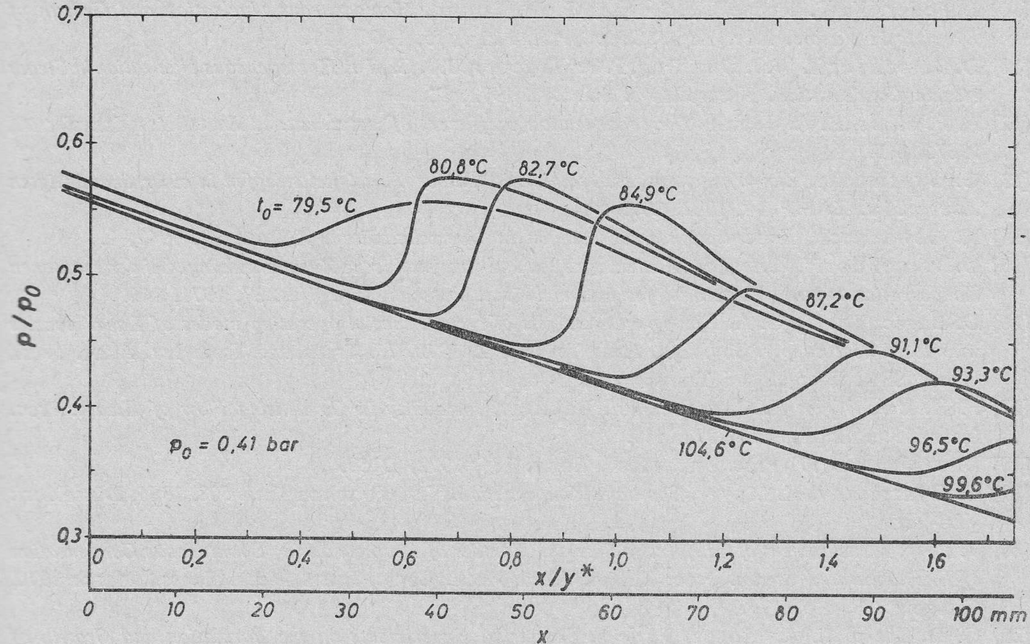


Fig. 9. Measured pressure distributions, nozzle 2 (Exp. 2 - 23 to 2 - 33)

The static pressure distribution is measured with a movable centerline pressure probe, reaching from the low subsonic into the supersonic part of the nozzles. An electric pressure transducer and a displacement pickup was used to record the pressure traces on a x-y-recorder as a continuous slope. Simultaneously, the flow field was observed by a differential interferometer to obtain quantitative density results and to determine the exact position of shock waves, not marked from pressure measurements alone. The measured static pressure distributions for nozzle 2 and constant stagnation pressures $p_0 = 1.45$ bar and $p_0 = 0.41$ bar, respectively are given in Fig. 8 and 9. The pressure distribution for the experiment $p_0 = 0.41$ bar, $t_0 = 79.5$ °C (Fig. 9) indicates unstable flow. Measured pressure distributions for nozzle 1 are given in [15].

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Badanie przepływu i wielkości kropel w mokrej parze wodnej przy prędkościach pod- i naddźwiękowych za pomocą teorii jednorodnej kondensacji

Streszczenie

Szczegółowe obliczenia przepływu pary mokrej przy projektowaniu stopni turbinowych bardzo rzadko uwzględniają teorię nukleacji w procesie ekspansji, a to ze względu na niepewny wynik jej zastosowania. Istnieje jednak możliwość posługiwania się teorią potwierdzoną eksperymentem. W ten sposób można określić nieizotermiczną homogeniczną nukleację, wzrost kropel, ciśnienie, gęstość i funkcję rozkładu kropel zarówno w stacjonarnych, jak i niestacjonarnych reżimach kondensującej się pary przy przepływie przez dyszę de Lavala.

Badania wykazały, iż równanie stanu gazu doskonałego może być zastosowane do pary mokrej bez większych błędów jedynie dla bardzo niskich ciśnień statycznych $p \leqslant 0{,}5$ bar. Ekspansja nasyconej lub nieznacznie przechłodzonej pary prowadzi do silnych gradientów parametrów w obszarze transso-

nicznym co powoduje niestacjonarność przepływu. Obliczenia wykazują, że dla wyższych przechłodzeń zmiana parametrów stanu powinna być łagodna, a przepływ winien się stabilizować przy kondensacji zachodzącej w poddźwiękowej części.

Исследование течения и размеров капель влажного водяного пара при дозвуковых и сверхзвуковых скоростях при помоши теории однородной конденсации

Резюме

Неуверенность применения теории возникновения зародышей конденсации и экспансии водяного пара является причиной того, что в методах проектирования турбинных ступеней почти не учитываются расчеты для влажного пара. Однако последовательное применение неизотермической теории однородного возникновения зародышей конденсации и алгоритма возрастания капель, позволяет определить давления, плотности и распределение капель, а также как устойчивых, так и неустойчивых рабочих модов течения конденсирующегося водяного пара в соплах Лаваля. Результаты расчетов сходны с результатами экспериментов. В результате проведенных исследований доказано, что применение уравнения состояния для идеального газа, без значительных расчетных ошибок, допустимо относительно влажности пара, однако только в диапазоне очень малых локальных значений статического давления $P \leqslant 0{,}5$ бар. Расширение насыщенного или незначительно переохлажденного водяного пара характеризуется большими градиентами параметров течения в сверхзвуковых условиях, что приводит к развитию неустойчивого течения. Расчеты, выполненные для сильно переохлажденного пара, указывают на снисходительные изменения параметров состояния. При конденсации, происходящей при дозвуковых скоростях, течение должно стабилизироваться.