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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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An Analysis of the Unsteady Discharge Process from a Cylinder

In this paper relationships are derived for the expansion of a high pressure fluid from a cylinder of constant or variable volume across a nozzle into a constant pressure atmosphere. This process has many applications in applied thermodynamics. For example the discharge of steam in feedheater spaces through the turbine when the turbine load falls. This is particularly important in large power station turbines where the energy of the steam causes overspeeding of the machine. The expressions developed allow for heat transfer and internal irreversibilities in the cylinder and in the nozzle. A comparison is then made of this unsteady flow process with a similar non flow equilibrium process in a cylinder and with a process of free unrestrained expansion in a cylinder. Using the concept of high grade energy, it is shown that all three processes may be analysed in the same manner.

Nomenclature

c – velocity,
F – force in the piston rod,
h – specific enthalpy,
HGE – high grade energy,
M – mass and original mass in cylinder,
M_a – mass discharged,
n – number of layers,
P – pressure and pressure in the cylinder,
P_a – constant pressure of the fluid surrounding the system,
P_e – a pressure lying between the initial pressure in the cylinder and P_a,
P_x – total restraining pressure at the piston,
q_e – external heat per unit mass,
q_f – internal or frictional heat per unit mass,
s – specific entropy,
T – temperature,
t – time,
u – specific internal energy,
V – total volume of the cylinder,
v – specific volume.

Additional suffixes

a – denotes conditions in the mass discharged from the nozzle,
c – denotes cylinder,
n – denotes nozzle,
o – denotes total head conditions.

1. Introduction

One function of this paper is to analyse the unsteady process which takes place when a high pressure system inside a vessel is allowed to expand and discharge mass across a nozzle fitted to the vessel. Amongst the practical applications of this problem are the

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discharge of gas through the ports of an engine cylinder and the discharge of steam from feedwater heaters in turbine plants when, on load shedding, the steam in the heaters passes through the turbine to the condenser causing overspeed of the turbine rotor. It is however also the intention of the authors to show clearly the relationships that exist between this unsteady flow process and similar steady flow and non flow processes with a view to illustrating the fact that the circumstances in which a system undergoes an expansion are incidental to the application of thermodynamic principles.

For an elemental part of a non flow process where a system contained within an engine cylinder is allowed to expand slowly, the energy equation may be written as

$$dq_e + dq_f = du + Pdv. \tag{a}$$

Here dq_e is the external heat crossing the system boundary, du is the change in internal energy and Pdv is the total work done by the system. During the process a part of the total work done is dissipated by friction forces and the energy thus destroyed is reabsorbed by the system as a frictional heat quantity dq_f so that the net work done by the system is

$$\text{Net}\, d(\text{WD}) = Pdv - dq_f. \tag{b}$$

Of the net work done, part of it appears as useful work done at the piston rod and the remainder is absorbed in displacing fluid which surrounds the system.

For the elemental process the change in entropy is given by

$$ds = \frac{dq_e + dq_f}{T}. \tag{c}$$

During this non flow process the system in the cylinder passes through a series of state points so fixing a process path. Suppose now that the same system passes through the same process path in a steady flow process. This expansion could take place in a static nozzle or in the rotating nozzles of a reaction turbine. For the flow process it may again be shown that equations (a) and (c) apply. With regard to equation (b) however a different interpretation must be given to the terms in the relationship. The equation may now be written as

$$\text{Net}\, d(\text{HGE}) = Pdv - dq_f, \tag{d}$$

where high grade energy (HGE) is the sum of all forms of work energy produced in the process together with the change in kinetic energy in the process. For a finite process from state point 1 to state point 2 in either the static or moving nozzles, the work done on surrounding fluid is $P_2v_2 - P_1v_1$ which for the elemental process is $d(Pv)$ and there will be a change in kinetic energy (cdc). For the rotating nozzles there will also be work done (WD) at the turbine shaft. Thus in general equation (d) may be written as

$$\text{Net}\, d(\text{HGE}) = Pdv - dq_f = d(\text{WD}) + cdc + d(Pv). \tag{e}$$

Rearranging equation (e) as

$$-vdP = d(\text{WD}) + cdc + dq_f \tag{f}$$

gives the familiar relationship known as the "momentum" equation in which the term

δq_f is the work done against friction forces and is at the same time the heat generated by friction.

We see therefore that the non flow process or its similar steady flow process may both be treated in exactly the same way using equations (a), (c) and either (b) or (e) and that it is the concept of high grade energy which allows one to treat the processes in a similar manner ignoring the circumstances of the process.

For further discussion of the need for a friction term in the energy equation, of the effect of friction in various processes and of the concept of high grade energy the reader is referred to references [1, 2] and [3].

2. The general case of the discharging process

Consider the piston-cylinder arrangement shown in Fig. 1 where the cylinder head is fitted with a nozzle. At a given time t a mass M occupies a volume V between the piston and the cylinder head and in this space the instantaneous values of pressure, specific volume, internal energy, enthalpy and velocity are P, v, u, h and c respectively. During time interval δt mass δM is discharged across the nozzle to a final constant pressure P_a and in the same time the volume V increases by δV while the piston motion is restrained by an external force corresponding to pressure P_x, where P_x is due to the combined effects of a force in the piston rod and external fluid pressure. During time δt let δq_{ec} be the external heat crossing the cylinder per unit mass inside the cylinder and let δq_{fc} be the corresponding frictional heat due to dissipation of HGE inside the cylinder. Once the mass

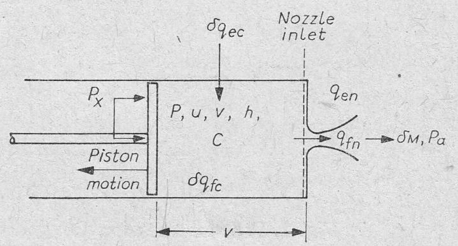

Fig. 1. Arrangement for the discharging process

to be discharged crosses the inlet section of the nozzle the time that it then takes to pass through the nozzle does not depend on δt but is a function amongst other things of the length of the nozzle. Thus, while δq_{ec} and δq_{fc} are in the limit differentials, the internal and external heat quantities involved during the passage of δM through the nozzle are finite. Let the external and internal heat quantities per unit mass discharged be q_{en} and q_{fn} respectively.

If we consider the process to which mass $M-\delta M$ remaining in the vessel is subjected, then for $M-\delta M$

$$\delta q_{ec}+\delta q_{fc}=\delta u+P\delta v=T\delta s, \tag{1}$$

where δs is the change in specific entropy and T — the temperature. Thus the specific

HGE produced in this elemental process is obtained as

$$\delta q_{ec} - \delta u = P\delta v - \delta q_{fc}. \tag{2}$$

If we now consider the process to which the discharged mass is subjected then for δM

$$\delta q_{ec} + \delta q_{fc} + q_{en} + q_{fn} = u_a - u + \int^a P dv, \tag{3}$$

where a is the final state after expansion to pressure P_a. Thus the specific HGE produced in this process is

$$\delta q_{ec} + q_{en} + u - u_a = \int^a P dv - (\delta q_{fc} + q_{fn}). \tag{4}$$

Pressure-specific volume diagrams for the two processes are shown in Fig. 2. If there were no frictional effects the sum of the areas under these process lines (multiplied by their respective masses) would give the total HGE produced. With friction the total HGE produced may be obtained from equations (2) and (4) as

$$(M - \delta M)(\delta q_{ec} - \delta u) + \delta M(\delta q_{ec} + q_{en} + u - u_a) =$$
$$= (M - \delta M)(P\delta v - \delta q_{fc}) + \delta M\left(\int^a PdV - (\delta q_{fc} + q_{fn})\right). \tag{5}$$

The total HGE produced may also be obtained as the total increase in kinetic energy plus all the displacement work done on the surroundings.

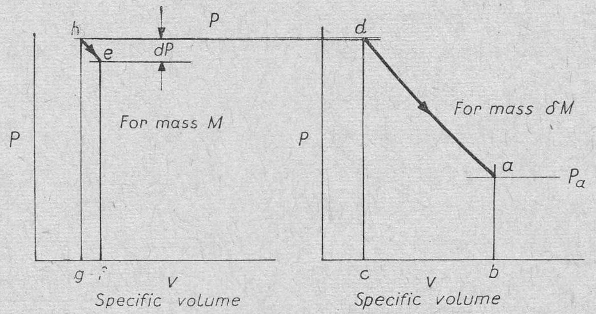

Fig. 2. Process lines for the mass in the cylinder and the discharged mass elemental cylinder process

The increase in kinetic energy is

$$(M - \delta M)\frac{(c + \delta c)^2}{2} + \frac{\delta M c_a^2}{2} - \frac{Mc^2}{2} = Mc\delta c - \frac{\delta M c^2}{2} + \frac{\delta M c_a^2}{2} \tag{6}$$

and the displacement work is

$$P_x \delta V + \delta M P_a v_a. \tag{7}$$

Thus from equations (5), (6) and (7) the total HGE output is

$$(M - \delta M)(\delta q_{ec} - \delta u) + \delta M(\delta q_{ec} + q_{en} + u - u_a) =$$
$$= Mc\delta c - \delta M\frac{c^2}{2} + \frac{\delta M c_a^2}{2} + P_x \delta V + \delta M P_a v_a. \tag{8}$$

Neglecting the products of small quantities equation (8) may be rearranged to give

$$\delta M\left(u_a+Pav_a+\frac{c_a^2}{2}\right)=\delta M\left(u+\frac{c^2}{2}+q_{en}\right)+M\delta q_{ec}-M\delta u-(Mc\delta c+P_x\delta V). \tag{9}$$

From equation (1), $M\delta u$ may be written as

$$M\delta u=M\delta q_{ec}+M\delta q_{fc}-MP\delta v \tag{10}$$

or

$$M\delta u=MT\delta s-MP\delta v. \tag{11}$$

Now

$$V=Mv \tag{12}$$

and

$$\delta V=(M-\delta M)(v+\delta v)-Mv \tag{13}$$

$$\therefore\ P\delta V=MP\delta v-\delta MPv \tag{14}$$

Substituting for $MP\delta v$ in equations (10) and (11) from equation (14) gives

$$M\delta u=M\delta q_{ec}+M\delta q_{fc}-\delta MPv-P\delta v \tag{15}$$

or

$$M\delta u=MT\delta s-\delta MPv-P\delta V. \tag{16}$$

Replacing $M\delta u$ in equation (9) from equation (15) gives

$$h_{oa}=h_o+q_{en}-\left[\frac{M\delta q_{fc}}{\delta M}+\frac{Mc\delta c}{\delta M}+\frac{(P_x-P)\delta V}{\delta M}\right] \tag{17}$$

and replacing $M\delta u$ in equation (9) from equation (16) gives

$$h_{oa}=h_o+q_{en}-\left[-\frac{M\delta q_{ec}}{\delta M}+\frac{MT\delta s}{\delta M}+\frac{Mc\delta c}{\delta M}+\frac{(P_x-P)\delta V}{\delta M}\right]. \tag{18}$$

Further analysis of equations (17) and (18) is possible if one now considers only the elemental process to which the system of mass M or $(M-\delta M)$ is subjected. For mass M the net HGE output for the elemental process is

$$MP\delta v-M\delta q_{fc}=P_x\delta V+Mc\delta c+\delta MPv, \tag{19}$$

where δMPv is the displacement work done by the system on the surroundings at the nozzle inlet section.

From equation (19)

$$MP\delta v-\delta MPv=P_x\delta V+Mc\delta c+M\delta q_{fc} \tag{20}$$

and from equations (14) and (20)

$$P\delta V=P_x\delta V+Mc\delta c+M\delta q_{fc}. \tag{21}$$

From equation (21) it will be seen that the expression inside the brackets in equation (17) is zero. The expression inside the brackets in equation (18) can also be shown to be zero. The relationships of equations (17) and (18) have been included as shown because the form of these equations is identical to corresponding relationships for the case where mass is charged into a vessel across a nozzle. For the charging case however it is not possible to reduce the terms inside the square brackets to zero. An analysis of the charging process is given in reference [4].

For the discharging process therefore equations (17), (18) and (21) give

$$h_{oa} = h_o + q_{en}\,. \tag{22}$$

Equation (22) shows that the steady flow energy equation may be applied to this unsteady flow problem provided the instantaneous values of the appropriate enthalpies etc. are used. The relationship may be applied with or without external heat transfer across the cylinder and with or without irreversibilities within the cylinder although these will affect the instantaneous values of enthalpy during the process. The equation may also of course be applied with or without irreversibilities in the expansion within the nozzle.

Equations (17), (18), (21) and (22) are alternative expressions for this general case where mass is discharged, where the piston moves and where there is external heating and irreversibilities in the cylinder and nozzle. These equations will now be applied to particular cases.

2.1. Discharge from a cylinder of constant volume

Where there is no movement of the piston the problem is one of discharge from a cylinder of constant volume. Here δV is zero so that equation (21) reduces to

$$M\delta q_{fc} = -Mc\delta c\,. \tag{23}$$

Where δV is zero there can be no question of the mass in the cylinder moving with a common bulk velocity c and local velocities would only be developed to any degree within the cylinder if the cylinder volume was small compared with the size of the nozzle. For a large cylinder and small volume therefore no kinetic energy will be developed and hence from equation (23) we see that there can be no question of irreversibilities in the container under these circumstances.

Thus where δV is zero, c and δq_{fc} are each also zero and equation (9) gives

$$\delta M h_{oa} = \delta M u - M\delta u + \delta M q_{en} + M\delta q_{ec}\,. \tag{24}$$

Equation (2) now becomes

$$h_{oa} = h + q_{en} \tag{25}$$

and may be written as

$$\delta M\left(P_a v_a + \frac{c_a^2}{2}\right) = \delta M(u - u_a + q_{en}) + \delta M P v\,. \tag{26}$$

Here $\delta M(P_a v_a + c_a^2/2)$ is the total HGE produced under these circumstances and the term $\delta M(u - u_a + q_{en})$ is equal to

$$\delta M \text{ (area } dabc \text{ in Fig. 2. } - \; q_{fn}) \tag{27}$$

and is thus the total HGE which could be expected from δM due to its "own" expansion. From equation (26) we see that this HGE is less than the total actually produced.

From equation (26) or equation (14) the extra HGE produced is

$$\delta MPv = MP\delta v = M \text{ (area } hefg \text{ in Fig. 2)}. \tag{28}$$

Thus the extra HGE produced is due to the expansion of M which provides the displacement work, sometimes called the flow work, done on mass δM, as δM crosses the inlet section of the nozzle.

2.2. Discharge from a cylinder of constant volume for internally reversible flow

Where q_{fn} is also zero the processes in the cylinder and nozzle are each internally reversible. The total HGE produced is then given by

$$\text{total HGE} = M \text{ (area } hefg \text{ in Fig. 2)} + \delta M \text{ (area } dabc \text{ in Fig. 2)}. \tag{29}$$

Here the effects of external heating in the container and nozzle are accounted for automatically by the way in which the heat supplied varies the appropriate specific volume.

The total kinetic energy produced at nozzle exit in a finite time interval may be obtained from equation (26) as

$$\int_P^{P_e} \frac{c_a^2}{2}\, dM = \int_P^{P_e} (h - u_a + q_{en})\, dM - \int_P^{P_e} P_a v_a\, dM, \tag{30}$$

where P_e is the final pressure in the cylinder and h the variable enthalpy in the cylinder. If external heating in the cylinder and nozzle are such that the process path is the same for the discharged mass and for the mass remaining in the cylinder then v_a in equation (30) is a constant. For these circumstances the appropriate pressure volume diagrams are shown in Fig. 3 and the total kinetic energy produced is

Total kinetic energy

$$= (M - M_d)(\text{area } hefg) + M_d(\text{area } dabc) - M_d(\text{area } abij), \tag{31}$$

where M is the original mass in the cylinder, M_d – the mass discharged and M_d (area $abij$) is the term $\int_P^{P_e} P_a v_a\, dM$ in equation (30) which is the total external displacement work which the discharged mass does on the surroundings as it exhausts from the nozzle.

Under the same circumstances let mass be discharged until the pressure inside the cylinder is P_a. The pressure-volume diagram for the mass remaining inside the cylinder and for the discharged mass is the same and is shown in Fig. 4. In addition the external heat per unit mass supplied to the discharged mass and to the mass remaining in the cylinder is the same and we shall call this heat quantity q_{en} though it will be noted that part of q_{en} is supplied to the discharged mass while the latter is inside the cylinder. Considering Fig. 4 and applying equation (31) under these circumstances, the total kinetic energy produced at nozzle exit is

$$\text{Total kinetic energy } = M(dabc) - M_d P_a v_a \tag{32}$$

and using the energy equation this gives

$$\text{Total kinetic energy } = M(u - u_a + q_{en}) - M_d P_a v_a, \tag{33}$$

where u is the initial internal energy in the cylinder.

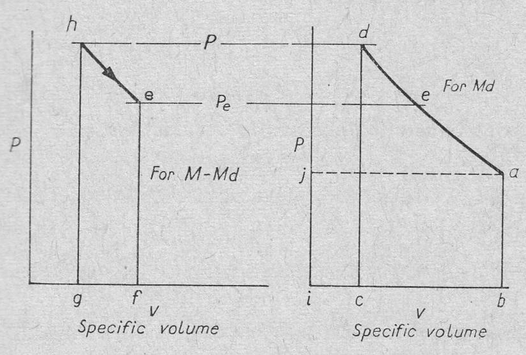

Fig. 3. Process lines for a finite process in the cylinder

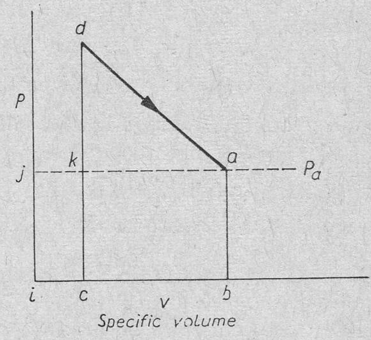

Fig. 4. Process line for expansion to atmospheric pressure in the cylinder

Now the mass remaining in the cylinder after expansion to P_a is

$$M - M_d = \frac{Mv}{v_a}, \tag{34}$$

$$\therefore \; M_d = M\left(1 - \frac{v}{v_a}\right). \tag{35}$$

Hence from equations (35) and (33) the total kinetic energy may also be expressed as

$$\text{total kinetic energy} = M(u - u_a + q_{en} - P_a(v_a - v)) = M \text{ (area } dak \text{ in Fig. 4)}. \tag{36}$$

This same result for the total kinetic energy produced may also be obtained using equation (25) i.e. using the "instantaneous" steady flow energy equation. Here in order to obtain the kinetic energy one must first calculate the instantaneous value of h by expressing h in terms of the initial conditions knowing the process line.

2.3. Equilibrium expansion of the mass in the cylinder

Consider the same mass M inside the cylinder at pressure P with the surroundings again at pressure P_a. Suppose as shown in Fig. 5 that the wall which is fitted with the nozzle becomes a piston and that the nozzle is closed so that no mass is discharged. A restraining force F is then needed on the piston rod to prevent the piston from moving down the cylinder. The mass M may then be allowed to expand down the cylinder by reducing the force F below its initial value. If this is done slowly by gradually reducing the force F by small amounts then equilibrium will be maintained in the cylinder and no kinetic energy will be developed. In the absence of internal irreversibilities in the expansion the work done against the restraining force F is given by area *dak* in Fig. 5. It will be seen that this work quantity is the same as the total kinetic energy developed at nozzle outlet in the previous section i.e. that given in equation (36).

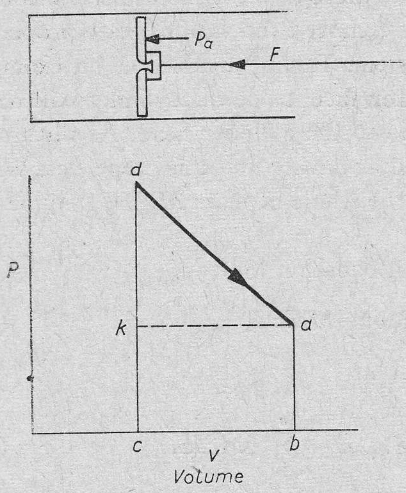

Fig. 5. The equilibrium expansion process

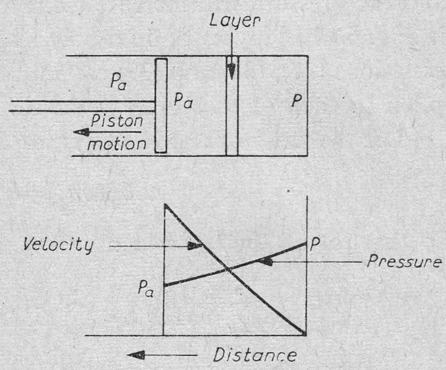

Fig. 6. The free expansion process

2.4. The free non-equilibrium expansion of the mass in the cylinder

If the piston in Fig. 1 is allowed to move but no nozzle is fitted to the cylinder head then δM is zero and equation (18) reduces to

$$\delta q_{ec} = T\delta s + c\delta c + (P_x - P)\,\delta v \tag{37}$$

or replacing $T\delta s$ by $\delta u + P\delta v$ this gives

$$\delta q_{ec} = \delta u + c\delta c + P_x\,\delta v \tag{38}$$

which is one form of the energy equation.

From equation (17)

$$\delta q_{fc} = P\delta v - (P_x\,\delta v + c\delta c), \tag{39}$$

where $P\delta v$ is the total specific HGE which is ever produced in the elemental process and $P_x\delta v + c\delta c$ is the net HGE produced.

It will be seen from equation (39) that if internal friction and velocity effects are negligible then $P_x = P$. Now P_x is due to the combined effects of a piston rod force F and external fluid pressure such as P_a. For $P_x = P$ then expansion is only possible if F is gradually reduced so that the expansion is slow, no kinetic energy is generated and equilibrium is maintained. Under these circumstances if expansion proceeds to pressure P_a then as shown in the last section of the work done at the piston rod is given by equation (36).

Consider now the case where $P_x = P_a$ so that there is no restraining force in the piston rod. Here before expansion begins a stop must be fitted to prevent the piston from moving down the cylinder. With the stop in position equilibrium exists inside the cylinder, the state properties are all uniform throughout mass M and there is no velocity. When the stop is removed the piston will move down the cylinder and particles in contact with the piston will acquire the speed of the piston, while those at the cylinder head will have zero velocity. At any given time t after the stop is removed the various state properties will each have different values throughout the system. Taking pressure as an example, then for zero piston mass, the pressure on the piston face inside the cylinder will be P_a and the pressure will increase from the piston face to the cylinder head. As illustrated in Fig. 6 small layers of fluid only may be regarded as having uniform properties. While equations (37), (38) and (39) cannot be applied to the whole of mass M they may be applied to each of the layers involved.

Let the mass of a given layer be δM then from equation (39)

$$\delta M \delta q_{fc} = \delta M P \delta v - \delta M P_x \delta v - M c \delta c \tag{40}$$

so that for all the layers comprising the total mass M

$$\sum_1^n \delta M \delta q_{fc} = \sum_1^n \delta M P \delta v - \sum_1^n \delta M P_x \delta v - \sum_1^n \delta M c \delta c, \tag{41}$$

where n is the number of layers.

Suppose now that expansion proceeds until the pressure in the cylinder is uniform at pressure P_a. Then, from equation (41),

$$\int_P^{P_a} \sum_1^n \delta M \delta q_{fc} = \int_P^{P_a} \sum_1^n \delta M P \delta v - \int_P^{P_a} \sum_1^n \delta M P_x \delta v - \int_P^{P_a} \sum_1^n \delta M c \delta c. \tag{42}$$

If heating effects are uniform throughout the system then each layer follows the same process path so that equation (42) may be written as

$$M q_{fc} = M \int_P^{P_a} P dv - \int_P^{P_a} \sum_1^n \delta M P_x \delta v - \frac{Mc^2}{2}. \tag{43}$$

Here $\delta M P_x \delta v$ is displacement work done by one layer on another layer during expansion and thus

$$\int_P^{P_a} \delta M P_x \delta v = \delta M P_a (v_a - v) \tag{44}$$

for the complete process. Substituting from equation (44) into equation (43) gives

$$M q_{fc} = M \int_P^{P_a} P dv - M P_a (v_a - v) - \frac{Mc^2}{2}. \tag{45}$$

Now if there is no internal friction (and since we are ignoring mechanical friction between the piston and the cylinder this means fluid friction) causing the degeneration of kinetic energy into heat, then from equation (45)

$$\frac{Mc^2}{2}=M\int_{P}^{P_a} Pdv-MP_a(v_a-v). \tag{46}$$

The pressure-volume diagram and the process path under these circumstances are shown in Fig. 7. Each layer passes through this process path though different layers take different times to do so. From equation (46) we see that the kinetic energy generated is

$$\text{Kinetic energy}=M(\text{area } dak \text{ in Fig. 7}). \tag{47}$$

This quantity of kinetic energy is the same as that produced at the nozzle exit for the case where the cylinder volume is constant and mass is discharged across a nozzle (see equation (36)) and is also equal to the work done against the piston rod for the case of equilibrium expansion of the mass in the cylinder (area *dak* in Fig. 5).

The fact that the kinetic energy generated for the free expansion is the same as that produced at nozzle exist for the case of discharge across a nozzle from a cylinder of constant volume is a result which would be expected. This is so because the removal of the restraining force in the piston rod in effect means that we open to the system a nozzle of area equal to the cylinder area. Since the kinetic energy generated is independent of the nozzle size the result with this large nozzle is the same as that with any other nozzle.

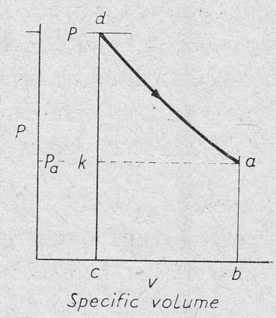

Fig. 7. The process line for the internally reversible free expansion process

When the pressure finally reaches P_a uniformly throughout the cylinder, if we then prevent further motion of the piston by inserting a stop, the kinetic energy which has been developed must be destroyed. A frictional heat quantity Mq_{fc} equal to $Mc^2/2$ in equation (46) would then heat mass M at constant volume. Alternatively we may allow the frictional heat to heat mass M at constant pressure P_a by first of all destroying the kinetic energy by inserting the stop and then removing the stop while the heating takes place.

We see then that such a process where the piston is allowed to freely expand down the cylinder only becomes internally irreversible if some or all of the kinetic energy generated during expansion is degenerated by viscous forces into heat. With no external heating the actual expansion process could be as nearly isentropic as that in a well designed nozzle. In addition if there were some means of removing from the different layers the kinetic energy generated during expansion then the process would be an efficient one. Since this is not a practical proposition, the system retains this kinetic energy which must all be degenerated to heat when the piston finally comes to rest and it is this fact which makes the free expansion in effect an irreversible process.

3. Concluding Remarks

The relationships derived above for the unsteady discharge across the nozzle, for the slow expansion in the cylinder and for the free expansion in the cylinder have one theme in common. This is that the process path of a given mass of a system, which is fixed by the quantities of external heat and internal heat of friction involved, determines the quantity of high grade energy made available in the process. This high grade energy may appear in a number of forms depending on the actual circumstances in which the expansion takes place. For the slow equilibrium expansion in the cylinder the HGE appears as work done at the piston rod and as displacement work done on surrounding fluid. For the expansion in the cylinder or for the case of discharge across a nozzle the HGE appears as kinetic energy and as displacement work on surrounding fluid. For the expansion across a nozzle the process path of the mass remaining in the vessel may be different from that of the mass which is discharged from the vessel but again when these process paths are known the total quantity of HGE made available from both masses is determined. For the slow equilibrium expansion in the cylinder the process would be internally reversible if there was no mechanical friction between the piston and the cylinder during the process. Similarly for the free expansion in the cylinder the process would again be internally reversible if during the process none of the kinetic energy developed was dissipated by friction. For the free expansion process it is only the fact that there is no means of removing the kinetic energy (remaining when the process ceases) from the system that makes the process inherently irreversible.

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References

[1] Hatton A. P., *Polytropic Processes with Irreversibility and Heat Transfer.* Bull. Mech. Engng. Educ. 1962, 1, 89 (Pergamon Press, London).

[2] Neilson J. H. and Crawford R. A., *The Effect of Friction on the Efficiencies of Various Steady Flow Processes.* Bull. Mech. Engng. Educ. 1970, 2, (Pergamon Press London).

[3] Neilson J. H. and Crawford R. A., *Losses in Power Cycles.* To be published in the International Journal of Mechanical Sciences.

[4] Neilson J. H. and Crawford R. A., *An Analysis of the Unsteady Charging Process in a Cylinder.* Awaiting publication.

[5] Horlock J. S. and Woods W. A., *The Thermodynamics of Charging and Discharging Processes.* Proc. Inst. Mech. E. 1965 - 66, 180.

Analiza procesu przejściowego wypływu z cylindra

Streszczenie

W niniejszej pracy wyprowadzono zależności opisujące ekspansję płynu pod wysokim ciśnieniem z cylindra, o stałej lub zmiennej objętości, przez dyszę do atmosfery o stałym ciśnieniu. Proces ten znajduje wiele zastosowań w termodynamice stosowanej. Jednym z nich jest na przykład wypływ pary z podgrzewacza pary przez turbinę przy spadku obciążenia turbiny. Jest to szczególnie ważne w dużych turbinach, gdzie energia pary powoduje rozwinięcie przez maszynę nadmiernej prędkości obrotowej. Wyprowadzone wyrażenia uwzględniają wymianę ciepła oraz wewnętrzne efekty nieodwracalne, w cylindrze i w dyszy. Dokonano następnie porównania tego przejściowego procesu przepływu z podobnym nie przepływowym procesem w równowadze w cylindrze oraz z procesem swobodnej nieograniczonej ekspansji w cylindrze. Wykorzystując ideę energii wysokiego rzędu pokazano, że wszystkie trzy te procesy można analizować w ten sam sposób.

Анализ переходного процесса истечения из цилиндра

Резюме

В настоящем труде выводятся зависимости, описывающие экспансию жидкости, находящейся под высоким давлением, из цилиндра постоянного или переменного объема, через сопло в атмосферу постоянного давления. Этот процесс многократно применяется в прикладной термодинамике. Одним из примеров применения может быть истечение рабочего пара из пароперегревателя через турбину при сбросе ее нагрузки. Это является особенно важной проблемой в больших паротурбинных установках, где энергия пара вызывает чрезмерно большой рост скорости вращения ротора турбины. В выведенных зависимостях учитывается теплообмен и внутренние неотвратимые эффекты в цилиндре и в сопле. Затем проводится сравнение этого переходного процесса течения с похожим непроточным равновесным процессом в цилиндре и с процессом свободной неограниченной экспансии в цилиндре. Пользуясь идеей энергии высокого порядка указывается, что все эти три процесса можно анализировать таким же способом.