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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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Gdańsk, Newark

Breakdown of a Shear Driven Liquid Film**

The paper presents an analysis of the breakdown of a shear-driven liquid film into stable rivulets. The criteria for such breakdown include equality of mechanical energy of the film and the resulting rivulets as well as a minimum of energy of the latter.

List of symbols

E – energy,
e – energy per unit width of flow,
F_1 – total area per unit width of unbroken film,
F_2 – wetted area per unit width of unbroken film,
g – function of wetting angle defined by eqn. (10),
G – mass flow per unit width of flow,
h – film thickness,
h^+ – dimensionless film thickness, defined by eqn. (15),
p – pressure,
R – radius of rivulet,
w – flow velocity,
x – transverse coordinate or wetting parameter,
y, z – coordinates,
ζ'' – friction coefficient,
θ – polar coordinate,
θ_0 – contact angle,
λ – width of film,
μ – viscosity,
ρ – density,
σ – surface tension,
τ – shear stress,
φ – function of contact angle defined by eqn. (7).

Subscripts

f – film,
s – rivulets,
1 – solid surface,
2 – liquid,
3 – vapor.

1. Introduction

Breakdown of liquid films driven by shear stresses acting at the film boundary between the liquid and gas phases, is important in a number of practical applications. It occurs for example on the surfaces of stator blades in the final stages of steam turbines. The structure of the droplet stream leaving the liquid phase and impinging on the rotor blades depends

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to a large extent on the flow regime of the liquid phase on the surfaces of the stator blades. Similarly, the occurence of a boiling crisis in a flow is related to the breakdown of the liquid film on the duct surface. The problem is of importance in a number of chemical engineering applications as well as in safety considerations of liquid-sodium or pressurised-water cooled reactors. Thus, several aspects of the problem have been considered in the literature. One group of authors studied the stability of the film surface with the aid of small perturbation theory [1, 2, 3]. Such an analysis yields necessarily approximate results, concerning the criterion for film breakdown, since it completely neglects the very important contact angle which has considerable influence on the formation of rivulets. An analysis allowing for the effect of the contact angle has been presented by Hartley and Murgatroyd [4]. They considered the equilibrium of forces at the stagnation point in the already torn film. Hartly and Murgatroyd also presented an alternative stability criterion postulating that the flow of kinetic and surface energy in the broken film must be a minimum. Their „minimum power" criterion again omits any influence of the contact angle and the predicted results differ substantially from their experimental data.

Subsequent analyses based on the work of Hartley and Murgatroyd consider such further factors as the variation of surface tension with temperature [5], the pressure on the surface caused by the thrust of evaporating molecules [6] etc.

An independent, and in our opinion more convincing, formulation of the minimum energy principle has been given by Hobler [7]. His analysis does allow for the effect of the contact angle but his mathematical description is not quite complete. In order to obtain final results he assumes the ratio of the wetted surface to the total surface to be unity which allows only an estimate of the critical film thickness or the critical mass flow density. The parameter x is one of the calculated results of the present analysis.

An alternate approach is offered by Bankoff [8]. He also asumes the above area ratio to be equal to unity and for a criterion for film breakup he assumes the equality of mechanical (kinetic plus surface) energy in the film about to break up and in the resulting rivulets. Unfortunately an arithmetic error occured in the integration leading to his equation (15). When this error is corrected, the resulting calculated minimum film thicknesses are imaginary. This is to be expected as under the assumptions of the paper equality of mechanical energy in the film and in rivulets does not appear possible.

The present paper presents yet another approach to the problem and constitutes in a sense an extension of the work of Hobler.

2. Analysis

We consider a uniform liquid film of thickness h_0 and unit length in the streamwise direction being driven by a shear stress τ caused by the motion of the vapor phase relative to the liquid. Assuming the liquid to be incompressible and to have temperature independent properties and neglecting inertia and gravity forces, the velocity distribution in the film is given by

$$w=\frac{1}{\mu}\frac{dp}{dz}y^2+\left(\frac{\tau}{\mu}-\frac{2}{\mu}\frac{dp}{dz}h_0\right)y. \tag{1}$$

For simplicity we assume $(dp/dz)=0$, so that

$$w=\frac{\tau}{\mu}y. \tag{2}$$

The total mechanical energy per unit width of the unbroken film consists of kinetic and surface energies (c.f. Fig. 1) and with the aid of eqn. (2) can be written:

$$e_f(h_0)=\frac{E}{\lambda}=\frac{\rho'}{2}\int_0^{h_0} w^2 dy+\sigma_{2-3}+\sigma_{1-2}=\frac{\rho'}{6}\frac{\tau^2}{\mu^2}h_0^3+\sigma_{2-3}+\sigma_{1+2}. \tag{3}$$

Here σ_{2-3} and σ_{1-2} are respectively the surface tensions between the liquid and vapor and between the liquid and solid.

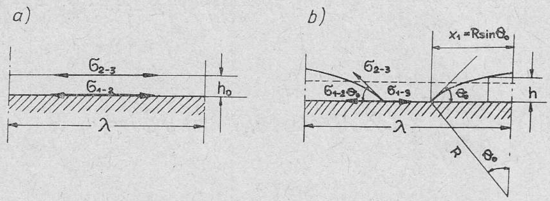

Fig. 1. a) Unbroken film, b) ruptured film

The mass flow per unit width fo film is given by

$$G_f=\bar{w}_f h_0=\frac{\tau}{2\mu}h_0^2. \tag{4}$$

Let us now consider the flow in rivulets formed as a result of the break up of the film, as shown in Fig. 1b. It is easy to show that if gravity and temperature dependence of surface tension are neglected, the rivulets are in the shape of circular segments of radius R. Let the liquid height be given by $h(x)$, where x is measured from the center of the rivulet. In terms of polar coordinates R and θ

$$h(x)=R(\cos\theta-\cos\theta_0) \qquad 0\leqslant x\leqslant x_1, \tag{5}$$

where θ_0 is the contact angle $(0<\theta<\theta_0)$.

We assume next, in a manner similar to Bankoff [8], that at any point x in the rivulet, where the thickness is $h(x)$, the velocity profile is the same as that which would prevail in a uniform film of thickness $h(x)$. With this assumption the mass flow in rivulets, per unit width of film is given by

$$G_s=\frac{2}{\lambda}\int_0^{x_1}\int_0^{h(x)} w(x,y)\,dy\,dx=\frac{\tau}{\mu}\frac{\phi}{\lambda}R^3, \tag{6}$$

where equations (2) and (5) have been used and we have put $x_1=R\sin\theta_0$. The function ϕ in eqn. (6) is given by

$$\phi(\theta_0)=\sin\theta_0-\tfrac{1}{3}\sin^3\theta_0-\theta_0\cos\theta_0. \tag{7}$$

The total mechanical energy in rivulets per unit width is

$$e_s=\frac{E}{\lambda}=\frac{2}{\lambda}\left[\frac{1}{2}\int_0^{x_1}\int_0^{h(x)}\rho' w^2(x,y)\,dx\,dy\right]+$$

$$+\frac{1}{\lambda}\left[(2R\theta_0+\lambda\cos\theta_0+R\sin 2\theta_0)\sigma_{2-3}+\lambda\sigma_{1-2}\right]. \tag{8}$$

With the aid of eqn. (4) and (6), eqn. (8) yields

$$e_s=\frac{\rho}{6}\frac{\tau^2}{\mu^2}\,g(\theta_0)\,h_0^3\left(\frac{\sin\theta_0}{\phi}\right)^{\frac{3}{2}}x^{-\frac{1}{2}}+$$

$$+\left(x\,\frac{\theta_0}{\sin\theta_0}+\cos\theta_0-x\cos\theta_0\right)\sigma_{2-3}+\sigma_{1-2}, \tag{9}$$

where

$$g(\theta_0)=\frac{1}{\sin\theta_0}\int_0^{\theta_0}(\cos\theta-\cos\theta_0)^3\cos\theta\,d\theta=$$

$$=-\frac{1}{4}\cos^3\theta_0-\frac{13}{8}\cos\theta_0+\frac{15\theta_0}{8\sin\theta_0}-\frac{3}{2}\theta_0\sin\theta_0, \tag{10}$$

$$x=\frac{2R\sin\theta_0}{\lambda}=\frac{F_2}{F_1}.$$

At the instant of break-up of the film, the mass flow must be the same in the film and in the rivulets, and so must the energy. Comparison of eqn. (4) and (6) and of (3) and (9) yields

$$R=\left(\frac{\sin\theta_0}{\phi x}\right)^{\frac{1}{2}}h_0 \tag{11}$$

and

$$\frac{\rho'\tau^2}{6\mu^2}\,g(\theta_0)\left(\frac{\sin\theta_0}{\phi}\right)^{\frac{3}{2}}h_0^3x^{-\frac{1}{2}}+\left(\frac{\theta_0}{\sin\theta_0}-\cos\theta_0\right)\sigma_{2-3}x-(1-\cos\theta_0)\sigma_{2-3}-\frac{\rho'\tau^2}{6\mu^2}\,h_0^3=0. \tag{12}$$

If the newly formed rivulets are to be stable the energy of the system should be a minimum i.e.

$$\frac{de_s}{dx}=0 \tag{13}$$

and hence

$$\frac{1}{2}\frac{\rho'}{6}\frac{\tau^2}{\mu^2}\,g(\theta_0)\left(\frac{\sin\theta_0}{\phi}\right)^{\frac{3}{2}}h_0^3x^{-\frac{3}{2}}=\left(\frac{\theta_0}{\sin\theta_0}-\cos\theta_0\right)\sigma_{2-3}. \tag{14}$$

The system of equations (11), (12) and (14) allows the determination of h_0, x and R and subsequently from the definition of x the rivulet spacing λ may be calculated. Solving for x from eqn. (14) and substituting in eqn. (12), one obtains with the aid of eqn. (11)

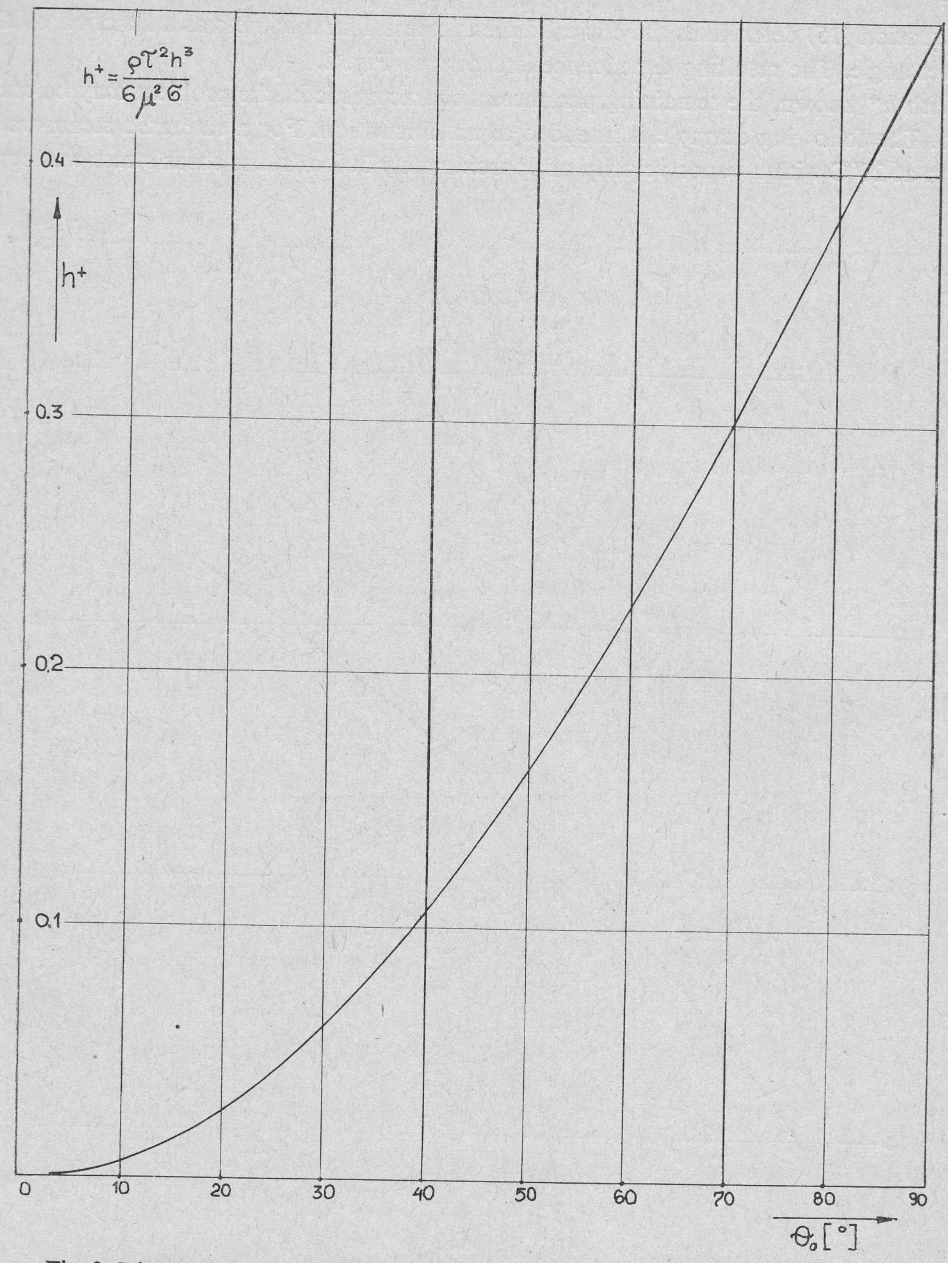

Fig. 2. Dimensionless minimum film thickness as a function of the contact angle

$$h^+ = 3\cdot 2^{-\frac{2}{3}} \left(\frac{\theta_0}{\sin\theta_0} - \cos\theta_0\right)^{\frac{1}{3}} h^{+\frac{2}{3}} g(\theta_0)^{\frac{2}{3}} \frac{\sin\theta_0}{\phi} - (1 - \cos\theta_0), \tag{15}$$

where

$$h^+ = \frac{\rho\tau^2 h_0^3}{6\mu^2\sigma_{2-3}} .$$

Equation (15) determines the critical dimensionless film thickness as a function of the contact angle. The resulting dependence is shown in Fig. 2.

With h^+ known, the remaining parameters characteristic of the problem may be computed. The following example is intended as an illustration. For a saturated steam-water system at 20°C with a vapor velocity of approximately 300 m/s, the shear stress was com-

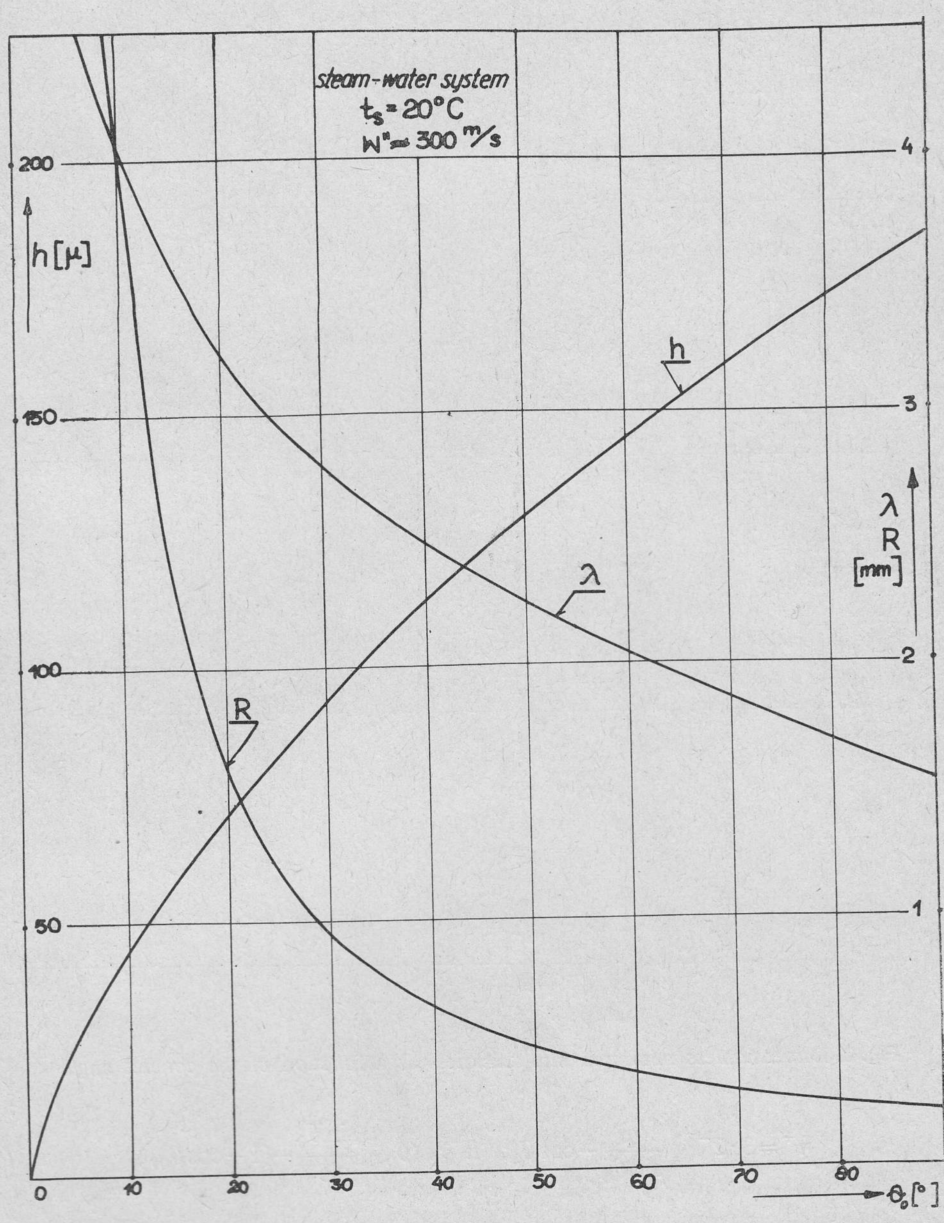

Fig. 3. Minimum films thickness, rivulet radius and rivulet spacing for saturated steam-water system at 20°C and steam velocity 300 m/s

puted from the simplified relation

$$\tau = \frac{\zeta''}{8} \rho'' \bar{w}^2 \tag{16}$$

Subsequently, the critical film thickness at break-up, the rivulet radius R and the wetted area ratio x have been calculated from the equations established above. The results of these calculations are shown in Fig. 3.

3. Comparison with experiment

Experimental measurements of minimum film mass-flow rates in two-phase annular flow were performed by Hewitt and Lacey [9] and by the team of the present authors*). Hewitt and Lacey investigated the break-up of a thin water film driven by air flowing in an annulus formed by two glass tubes 0.5 in. and 1.125 in. respectively in diameter. The film

Table 1

Comparison of experimental data of Hewitt and Lacey [9] with theory

G_g [lb/hr]	200	300	400	500
$\bar{w}_g$ [m/s]	30	45	60	75
τ_{TP} [kg/ms²]	5,7	12,6	22,7	35,5
G_f [lb/hr]	18	14	10	8
h_e [μ]	64	38	23,8	17
h_{teor} [μ]	66	39	26	19

Theoretical values computed for $\zeta''=0.01$, $\theta_0=15°$.

was formed on the inner surface of the outer tube. The results were presented in the form of a minimum film mass flow rate versus the air mass flow. For air mass flows of 200, 300, 400 and 500 lb/hr the respective minimum film mass flow ratios were 18, 14, 10 and 8 lb/hr.

Table 2

Comparison of experimental data of S. Gumkowski and E. Ihnatowicz with theory

t_s [°C]	39,9	39,5	34	32,6
G_g [kg/h]	178	164	119	104
$\bar{w}_g$ [m/s]	152	142	137	129
τ_{TP} [kg/ms²]	1,12	1,01	0,82	0,72
G_f [kg/h]	20,5	23	24	27
h_e [μ]	245	272	300	378
h_{teor} [μ]	249	270	338	371

Theoretical values computed for $\zeta''=8{,}3\cdot10^{-3}$, $\theta_0=40°$. This water film was driven by steam flowing in an annulus formed by two tubes 35 mm and 98 mm, respectively, in diameter. The film was formed on the outer surface of the inner tube made of copper.

* The actual measurements were performed by S. Gumkowski and E. Ihnatowicz.

Assuming a friction coefficient $\zeta''=0.01$ for air flowing over a water film, the shear stresses could be calculated from eqn. (16) and subsequently the minimum film thicknesses. The latter were compared with calculated values based on an assumed contact angle of 15°. The resulting comparison is shown in table 1, which shows excellent qualitative agreement. The actual quantitative agreement is due to a fortunate choice of the friction coefficient ζ'' and the contact angle θ_0. A thorough comparison of theory and experiment is only possible if these two parameters are known. No experimental investigations providing such information have been found in the literature.

In the authors' own measurements shown in table 2, these quantities have as yet also not been obtained. The trend of experimental data is very similar to that of Hevitt and Lacey and with a suitable choice of ζ'' and θ_0 good agreement with theory can be obtained. The somewhat higher minimum mass flow rates reflect the lower steam shear stresses employed by the authors.

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Bibliography

[1] S. G. Bankoff, *Stability of Liquid Flow Down a Heated Inclined Plane.* Int. J. Heat Mass Transfer, Vol. 14, pp. 377 - 385, (1971).

[2] V. A. Hallet, *Surface Phenomena Causing Breakdown of Falling Liquid Films During Heat Transfer.* Int. J. Heat Mass Transfer, Vol. 9, pp. 283 - 294, (1966).

[3] J. Mikielewicz, *Evaporation and Stability of Water Films on Heated Stator Blades of Steam Turbines* (in Polish). Transactions of the Instute of Fluid–Flow Machinery, Polish Acad. of Science, in print.

[4] D. E. Hartley, W. Murgatroyd, *Criteria for the Break up of thin Liquid Layers Flowing Isothermally over Solid Surface.* Int. J. Heat Mass Transfer, Vol. 7, pp. 1003 - 1015, 1964.

[5] A. Orell, S. G. Bankoff, *Formation of a Dry Spot in a Horizontal Liquid Film Heated From Below.* Int. J. Heat Mass Transfer, Vol. 14, pp. 1835 - 1842, 1971.

[6] N. Zuber, F. W. Staub, *Stability of Dry Patches Forming in Liquid Films Flowing over Heated Surface.* Int. J. Heat Mass Transfer, Vol. 9, pp. 897 - 905, 1966.

[7] T. Hobler, *Minimum Surface Wetting Rates* (in Polish). Chemia Stosowana v. 2B, pp. 145 - 159, (1964).

[8] S. G. Bankoff, *Minimum Thickness of a Draining Liquid Film.* Int. J. Heat Mass Transfer, Vol. 14, pp. 2143 - 2146 (1971).

[9] G. F. Hewitt, *Annular Two-Phase Flow.* Pergamon Press, 1970.

Analiza warunków rozrywania się filmu cieczowego na strugi

Streszczenie

Analizie poddano jednorodny film cieczy płynący pod wpływem naprężeń stycznych na granicy rozdziału pomiędzy fazą ciekłą i parową. Ustalono zależności na energię mechaniczną filmu i powstałych strug przypadającą na jednostkę szerokości filmu. Do rozważań przyjęto energię kinetyczną oraz energię powierzchniową na granicy rozdziału faz: ciało stałe – ciecz, ciecz – para oraz w przypadku strug dodatkowo ciało stałe – para. Jako warunki rozrywania się filmu na strugi przyjęto równość energii mechanicznej w filmie i strugach przy tym samym natężeniu przepływu, przypadającym na jednostkę szerokości filmu oraz minimum energii w strugach. Ten ostatni warunek wynika z warunku stabilności strug po rozerwaniu. W wyniku rozważań otrzymano bezwymiarową grubość rozrywającego się filmu w zależności od kąta zwilżania cieczy. Otrzymany rezultat porównywano z badaniami eksperymentalnymi Hewitta i Racey'a [9]. Wynik porównań jest zadowalający.

Анализ условий разрыва пленки жидкости на струи

Резюме

Анализируется однородная пленка жидкости, протекающая под влиянием касательных напряжений на границе раздела жидкой и паровой фаз. Устанавливаются зависимости на механическую энергию пленки и образовавшихся струй, приходящуюся на единицу ширины пленки. В рассуждениях принимаются кинетическая и поверхностная энергии на границе раздела фаз: твердое тело—жидкость, жидкость—пар, а в случае струй добавочно твердое тело—пар. За условия разрыва пленки на струи принято равенство механической энергии в пленке и струях при одинаковом расходе среды на единицу ширины пленки, а также минимум энергии в струях. Последнее условие вытекает из условия устойчивости струй после разрыва. В результате рассуждений получена зависимость безразмерной толщины разрывающейся пленки от угла смачивания жидкости. Полученный результат сравнивался с результатами экспериментальных исследований Хевитта и Расея [9]. Результат сравнений оказался удовлетворительным.