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A New Approach to Predicting Pressure Drop in Two-Phase Flows*

A method of deriving analytical formulas for local values of friction factor R and void fraction φ in two-phase flow has been presented. Use has been made of the experimental values of these coefficients, as given by Martinelli and Nelson for water flow. On the grounds of the theory of similitude a way has been proposed of applying the derived formulas to the calculation of the pressure drop in two-phase flows of media other than water.

Symbols

A – channel cross-section area,
$\dot{m}$ – mass flow rate,
ρ – density,
R – local value of friction factor for two-phase flow $R = \frac{(dp/dL)_{TPF}}{(dp/dL)_{l_0}}$,
X, χ – Martinelli's parameter,
$\dot{M}$ – mass velocity ($\dot{M} = \dot{m}/A$),
x – quality ($x = \dot{m}_g/\dot{m}_g + \dot{m}_l$),
φ – local value of void fraction ($\varphi = A_g/A$),
p – pressure,
L – length,
μ – absolute dynamic viscosity,
ϕ – parameter proposed by Martinelli.

Subscripts refer to:

g – gas (vapour),
g_0 – gas (vapour) flow of intensity equal to that of two-phase flow,
l – liquid,
tt – turbulent flow,
l_0 – liquid flow of intensity equal to that of two-phase flow,
TPF – two-phase friction.

1. Introduction

The research works concerning the pressure drop in two-phase vapour-liquid flows have been started long ago. A flow of this kind appears, for instance, in water-cooled thermonuclear reactors, in two-fluid vapour generators etc.

As a result of these researches, different analytical methods for determining the pressure drop in two-phase flow have been established. All existing methods of analysis can be divided into two types. The first type embraces all the methods based upon the relations resulting from a purely theoretical description of the two-phase flows. Due to complexity of the associated physical phenomena which affect the behavior of the two-phase medium

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only approximate solutions may be obtained by using these methods. The second group of methods, as based directly upon the results of various experiments, gives more accurate results. It has, however, a limited range of applicability, confined by particular conditions of the experiments. The empirical data to be found frequently in papers are those connected with the description of pressure drop effect in two-phase water flows. From the methods allowing to analyze the behaviour of the mentioned medium the Martinelli-Nelson's method is used more often. This method is based upon experimental data determined in a broad range of the pressure values, concerning the following two characteristic parameters:

R — the friction factor of two-phase flow,

φ — the void fraction in channel cross-section.

Relating both these parameters with pressure values the Martinelli-Nelson's methods offers better results than other analytical approaches. Its main defect consists in the fact that the relations between R and φ are given in the form of diagrams; moreover this method may be applied to the description of water flows only.

An attempt is made in this paper to generalize the different methods of analysis described previously in [1]. It's basic purpose consists in describing the pressure drop effect in arbitrary two-phase flows on the grounds of the experimental values of R and φ given by Martinelli and Nelson.

Two simple analytical relations are derived, allowing to determine the values of friction factor-R and void fraction-φ for all kinds of two-phase flow and sufficient for determining the corresponding pressure drop.

2. The derivation of analytical relations for friction factor and void fraction of a two-phase flow

The resultant value of pressure drop in a two-phase flow in heated vertical channel is given by the relation

$$\Delta p = \Delta p_{\mathrm{TPF}} + \Delta p_a + \Delta p_h, \tag{1}$$

where Δp_{TPF} — a pressure drop caused by friction effects, Δp_a — pressure drop caused by change of momentum and Δp_h — hydrostatic pressure drop.

Denoting the input channel cross-section as $x=0$ one can determine all the terms of sum (1) from the formulas:

$$\Delta p_{\mathrm{TPF}} = \bar{R} \cdot \Delta p_{lo}, \tag{2}$$

$$\Delta p_a = \dot{M}^2 V_l \left(\frac{(1-x)^2}{1-\varphi} - \frac{x^2}{\varphi} \frac{V_g}{V_l} - 1 \right), \tag{3}$$

$$\Delta p_h = \int_0^L g \left[(1-\varphi) \rho_L + \varphi \rho_g \right] dL, \tag{4}$$

where $\bar{R}$ is the average value of friction factor of two-phase flow defined by relation

$$\bar{R} = \frac{1}{L} \int_0^L R \, dL. \tag{5}$$

As the quality follows from the thermal calculation of the heat exchanger and the average friction factor $\bar{R}$ is, by virtue of (5), defined in terms of its local values, it becomes evident that for calculating the total pressure drop one ought to know two parameters, namely: R and φ.

Taking into account the set of experimental values of these parameters given by Martinelli and Nelson in [3], one can develop the following algorithm for analytical derivation of the both above factors.

The local value of friction factor R may be expressed in terms of Φ_{ltt} parameter values

$$R=(1-x)^{1.75}\,\Phi_{ltt}^2, \tag{6}$$

where Φ_{ltt} is dimensionless constant introduced by Lockhart and Martinelli in [2] and defined by the relation

$$\Phi_{ltt}=\frac{(dp/dL)_{\mathrm{TPF}}}{(dp/dL)_l}. \tag{7}$$

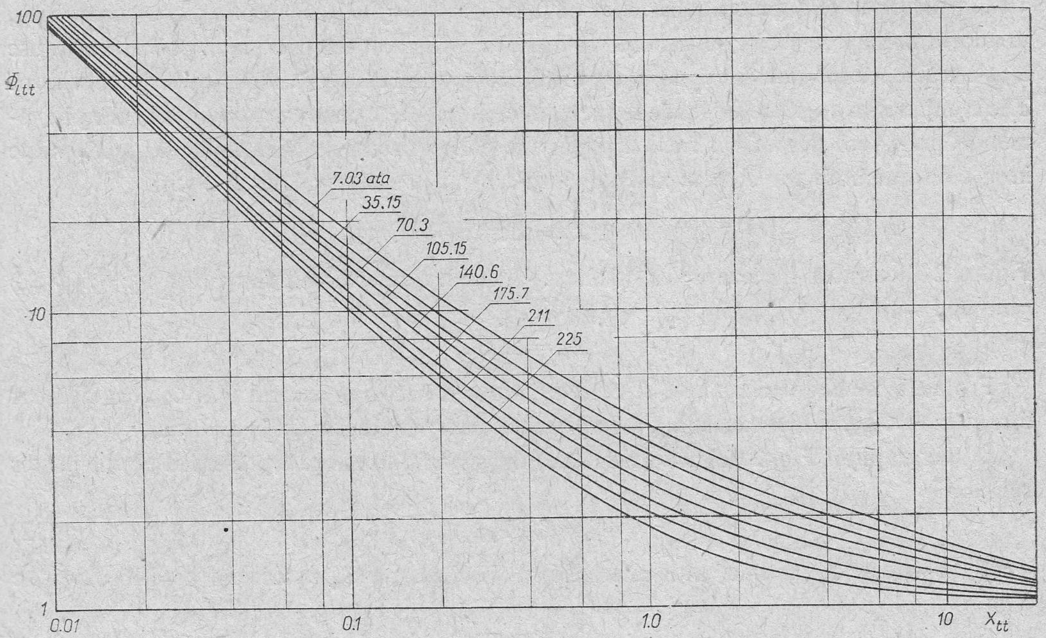

Fig. 1. Values of the parameter Φ_{ltt} versus X_{tt} for two-phase water-steam system at different pressures

With R known, it becomes possible to determine the relation between Φ_{ltt}, Martinelli's parameter X_{tt} and pressure. The last relation is illustrated by Fig. 1 plotted on the basis of the results described in [1]. The Martinelli's parameter used in [1] has the form

$$X_{tt}=\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{0.1}\left[\frac{(1-x)}{x}\right]^{0.9}. \tag{8}$$

From the set of the curves shown in Fig. 1 the lowest one is most important, corresponding to the critical pressure value. It's importance follows from the fact that it can be obtained analytically from eqs. (6) and (8). For critical pressure there is $R=1$, which permits to put down two following relations

$$\Phi_{ltt}=\sqrt{\left(\frac{1}{1-x}\right)^{1.75}}=\Phi_{lttkr}, \tag{9}$$

$$X_{tt}=\left(\frac{1-x}{x}\right)^{0.9}=X_{ttkr}. \tag{10}$$

Taking the value of quality from (10) and introducing it into (9), it can be easily shown that for critical pressure there is

$$\Phi_{lttkr}=\sqrt{\left(1+\frac{1}{X_{tt}^{1.111}}\right)^{1.75}}. \tag{11}$$

The relation (11) is of a general character, being independent of the type of the working mixture, whichever is considered, because there is always $R=1$ for pressure reaching its critical value.

On the basis of a general character of function $\Phi_{ltt}=f(X_{tt})$ for critical pressure which, in addition, may be easily determined in analytical form, it seems to be a good idea to use it as a reference relation for comparing with the Φ_{ltt} values for pressures lower than critical. The comparison ought to be made for certain characteristic conditions proper of the phenomenon under consideration. In this paper the comparison has been made with X_{tt} value kept constant both for critical and any sub-critical pressure values i.e., with

$$X_{tt}=\text{idem}.$$

Figure 2 illustrates the examples of the analytically determined functions $\Phi_{ltt}=f(\Phi_{lttkr})$ obtained with

$$X_{tt}=\text{idem}.$$

From a more detailed analysis of these relations it becomes exident that Φ_{ltt} has a linear form for all Φ_{lttkr} values exceeding certain ones corresponding to small qualities.

In the graph of Fig. 2 (logarithmic scale) the straight lines are represented by the power relation

$$\Phi_{ltt}=C_{\Phi}(\Phi_{lttkr})^{m}, \tag{12}$$

where m – slope of the line, with reference to abscissa and C_{Φ} coefficient described by the relation

$$C_{\Phi}=\frac{\Phi_{ltt}}{(\Phi_{lttkr})^{m}}. \tag{13}$$

The formula (12) is valid for all values of pressure starting from a certain point defined by the local values of quality. All these points lay approximately on a straight line (limit line). The quality value calculated along this line appeared to be equal to $x\cong 0.05$ approximately. It restricts the area of validity of eq. (12) to the cases when $x\geqslant 0.05$. Using formula (11) the relation (12) can be rewritten in the following form

$$\Phi_{ltt}=C_{\Phi}\sqrt{\left(1+\frac{1}{X_{tt}^{1.111}}\right)^{1.75m}}. \tag{14}$$

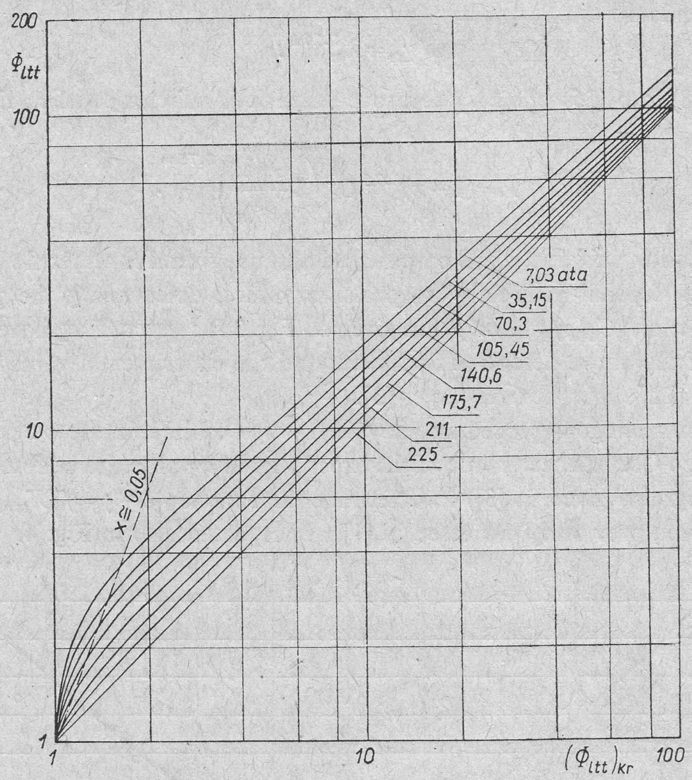

Fig. 2. Φ_{ltt} versus $(\Phi_{ltt})_{kr}$ for two-phase water-steam system at different pressures

Putting consequently (14) into eq. (6) the friction factor can be expressed directly as

$$R=(1-x)^{1.75}\,\Phi_{ltt}^2=(1-x)^{1.75}\,C_\Phi^2\left(1+\frac{1}{X_{tt}^{1.111}}\right)^{1.75m}. \tag{15}$$

In order to simplify the above formula it is possible to introduce the Martinelli's parameter in the form [4]

$$\chi=\left[\frac{(dp/dL)_l}{(dp/dL)_g}\right]^{\frac{1}{2-n}} \tag{16}$$

different from it's form given by (8) which was the consequence of the following defining relation

$$X=\left[\frac{(dp/dL)_l}{(dp/dL)_g}\right]^{\frac{1}{2}}.$$

Assuming that $n=0.2$ one obtains from eq. (16) the relation valid for turbulent flows

$$\chi_{tt}=\left(\frac{\rho_g}{\rho_l}\right)^{0.555}\left(\frac{\mu_l}{\mu_g}\right)^{0.111}\left(\frac{1-x}{x}\right). \tag{17}$$

Between the parameters X_{tt} after eq. (8) and their new values defined by (17) there is the following dependence

$$\chi_{tt} = X_{tt}^{1.111}. \tag{18}$$

Introducing (18) into (15) it is easy to prove that friction factor satisfies, finally, the relation,

$$R = (1-x)^{1.75} C_{\Phi}^2 \left(1 + \frac{1}{\chi_{tt}}\right)^{1.75m}. \tag{19}$$

The above equation substitutes the empirical diagrams given for water flow by Martinelli and Nelson. It depends indirectly on pressure values, determining the parameters C_{Φ} and m. The functions $C_{\Phi}(p)$ and $m(p)$ are given on Fig. 3 for water. They have been obtained with the use of the above described formalism and the experimental values of R given by Martinelli and Nelson in their paper [3].

By way of an analogous procedure the void fraction of the mixture φ can also be expressed analytically. Use has been made of the experimental φ values (to be found on Martinelli-Nelson's diagrams given in [3]) to determine the function $1-\varphi = f(X_{tt}, p)$. The results of the calculation, previously discussed in [1], are repeated also in Fig. 4.

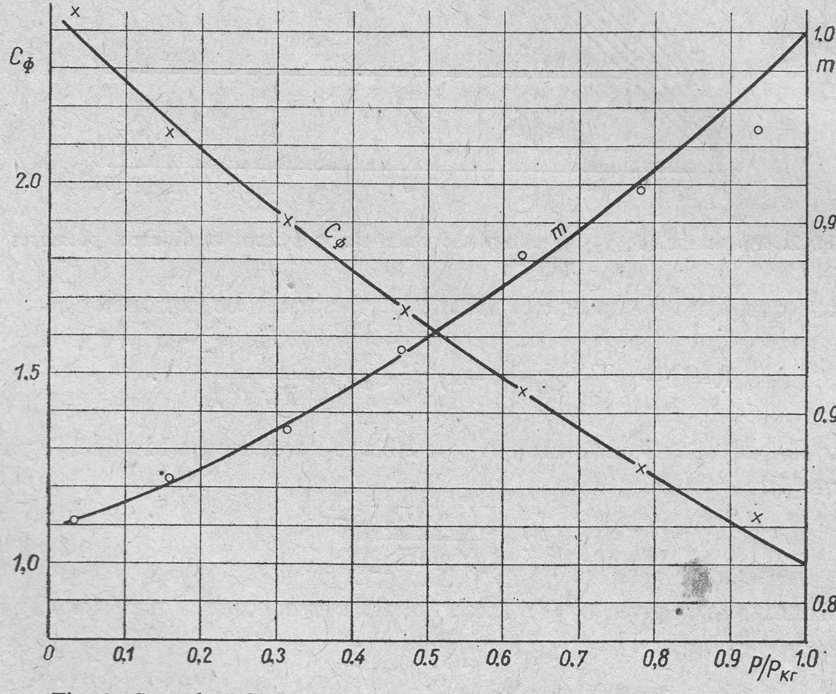

Fig. 3. C_{Φ} and m factors versus p/p_{kr} for two-phase water-steam system

For critical pressure the physical properties of vapour phase and liquid phase of the mixture are identical and the value of φ becomes equivalent to quality (for critical pressure value the two-phasity vanishes). Taking this into account it becomes easy to write down the analytical relation $1-\varphi = f(X_{tt})$, since there is

$$(1-\varphi)_{kr} = 1 - x. \tag{20}$$

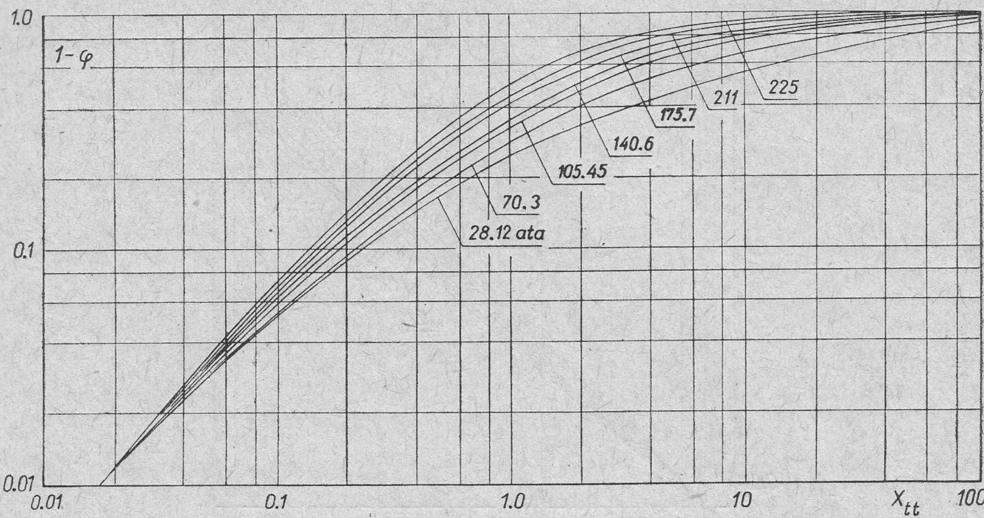

Fig. 4. $(1-\varphi)$ versus X_{tt} for two-phase water-steam system at different pressures

With the quality given by (10) and using the last relation (20) one can get the expression

$$(1-\varphi)_{kr}=\left(1+\frac{1}{X_{tt}^{1.111}}\right)^{-1} \tag{21}$$

which is sound for critical values of pressure.

Because the relation $(1-\varphi)=f(X_{tt})$ is, for critical pressure, a general one (independent of the kind of medium) it may constitute a reference relation to be used for comparison with the $(1-\varphi)$ values obtained for pressures lower than critical. The additional condition of X_{tt} being equal both for critical and sub-critical pressures is taken as previously.

With this condition and using Fig. 4 the function $1-\varphi=f(1-\varphi)_{kr}$ has been calculated; it's dependence on flow pressure being shown in Fig. 5. The relation $1-\varphi=f(1-\varphi)_{kr}$ is, beginning from a certain point, a linear one. That means — as the scale is logarithmic — that an exponentral dependence exists, which can be written down as follows

$$1-\varphi=C_{\varphi}(1-\varphi)_{kr}^{k}, \tag{22}$$

where k – slope of straight lines, and C_{φ} coefficient given by relation

$$C_{\varphi}=\frac{1-\varphi_{k}}{(1-\varphi)_{kr}^{k}}. \tag{23}$$

It is easy to notice that, as previously for R, the area of validity of the formula (23) is restricted to $x\geqslant 0.05$.

Transforming the equations (21) and (22) with the help of formula (18), it is possible to show that the analytical relation defining the void fraction φ has the following form

$$1-\varphi=C_{\varphi}\left(1+\frac{1}{\chi_{tt}}\right)^{-k}. \tag{24}$$

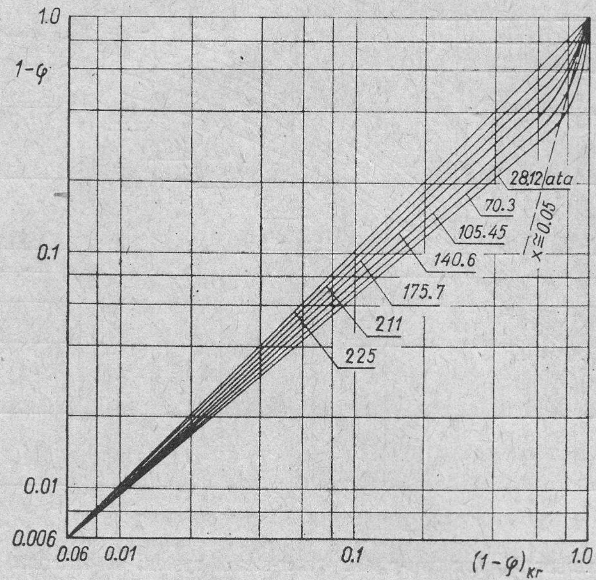

Fig. 5. $(1-\varphi)$ versus $(1-\varphi)_{kr}$ for two-phase water-steam system at different pressures

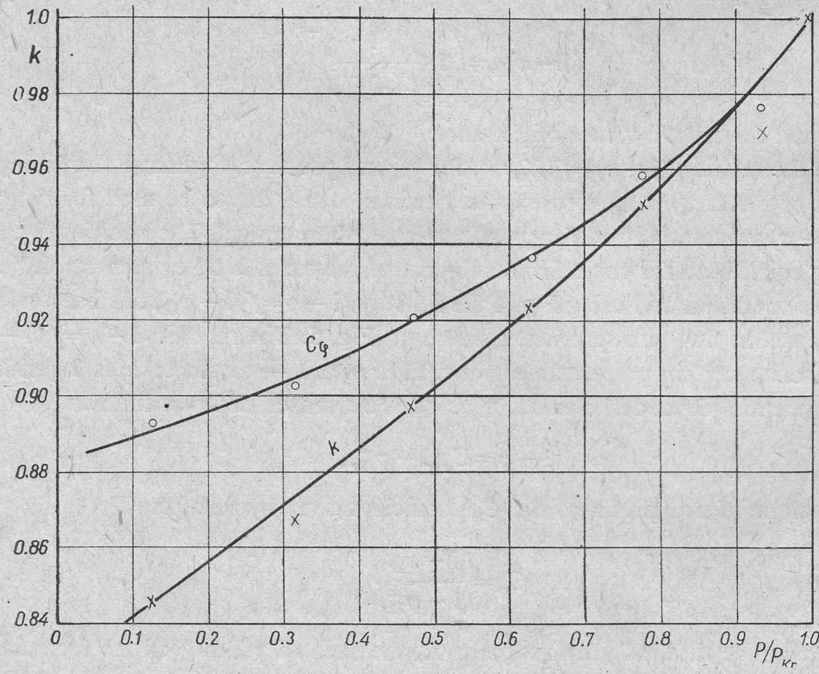

Fig. 6. C_φ and k factors versus p/p_{kr} for two-phase water-steam system

The relation (24) is equivalent to experimentally proved $\varphi = f(x, p)$ – Martinelli-Nelson's diagrams. The pressure value enters into the last formula by its influence on the parameters C_φ and k. The character of variation of these parameters with water pressure is shown in Fig. 6.

The Martinelli-Nelson method was not largely used in practice because of difficulties connected with the determination of R and φ values from their diagram representation. These practical difficulties can be avoided owing to the derivation of relations (19) and (24).

3. Comparison of the present results with some bibliographical data from literature

The main advantage of the relation (19) and (24) is that being, as they are, established on the basis of experimental values of friction factor R and void fraction φ they ensure correct results when used for predicting the pressure drop in two-phase flows.

This kind of relations linked to empirical data found in the literature refers, in the first place, to low values of pressure. There is, however, a set of analytical correlations thought for substituting the graphical relation $\Phi_{ltt}=f(X_{tt})$ given by Lockhart and Martinelli in [2] and applied to pressure near atmospheric. From among these correlations, the most known is the one given by Chisholm [5], which has the following form

$$\Phi_{ltt}^2=1+\frac{20}{X_{tt}}+\frac{1}{X_{tt}^2}. \tag{25}$$

This relation, by virtue of (6), defines the local friction factor as

$$R=(1-x)^{1.75}\left(1+\frac{20}{X_{tt}}+\frac{1}{X_{tt}^2}\right). \tag{26}$$

The pressure and its influence on the R values cannot be determined from the last relation, as the parameter X_{tt} appearing in eq. (26) does not depend on pressure values. The relation between pressure and friction factor R is given by Becker's formula [6]

$$R=1+2400\left(\frac{x}{p}\right)^{0.96} \tag{27}$$

which is valid only for water.

Besides the mentioned correlation there are two elementary relations for R obtained by Lottes and Levy as a final results of a purely theoretical study of two-phase flows.

The Lottes formula [7] has the simple form

$$R=\left(\frac{1-x}{1-\varphi}\right)^2 \tag{28}$$

whereas the Levy's theoretical formula for R is a little more complicated and reads

$$R=\frac{(1-x)^{1.75}}{(1-\varphi)^2}. \tag{29}$$

It seems interesting to make a comparison between eq. (19) and Levy's theoretical formula (29). Putting into the denominator of (29) the relation (24) one obtains

$$R=(1-x)^{1.75}\left(\frac{1}{1-\varphi}\right)^2=(1-x)^{1.75}\frac{1}{C_\varphi^2}\left(1+\frac{1}{\chi_{tt}}\right)^{2k} \tag{30}$$

but simultaneously eq. (19) gives

$$R=(1-x)^{1.75}\,C_{\Phi}^{2}\left(1+\frac{1}{\chi_{tt}}\right)^{1.75m}.$$

Comparing the last two results it is evident that they are equivalent only when

$$\frac{1}{C_{\varphi}^{2}}=C_{\Phi}^{2}\ \text{that is}\quad C_{\Phi}C_{\varphi}=1\,,$$
$$1.75m=2k\ \text{that is}\quad m/k=1.142\,. \tag{31}$$

Other way round the relation (19) corresponds to the Levy's formula only when the conditions (31) are rigorously met. The actual relations between C_{Φ}, C_{φ}, m and k obtained experimentally could be determined from the data summarized at Fig. 3 and Fig. 6. The results of this calculations are put together in the enclosed Table. From this Table it is

The value of C_{Φ}, C_{φ}, m and k as functions of the reduced water pressure

P/P_{kr}	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C_{Φ}	2.26	2.08	1.92	1.77	1.62	1.48	1.35	1.23	1.11	1.0
C_{φ}	0.445	0.48	0.52	0.56	0.61	0.67	0.73	0.80	0.88	1.0
m	0.876	0.885	0.895	0.907	0.92	0.933	0.948	0.963	0.981	1.0
k	0.842	0.857	0.872	0.877	0.902	0.919	0.936	0.955	0.976	1.0
$C_{\Phi}\cdot C_{\varphi}$	1.02	1.0	1.0	0.995	0.997	0.985	0.988	0.985	0.985	1.0
m/k	1.041	1.035	1.028	1.022	1.02	1.015	1.012	1.009	1.005	1.0

clear that the actual values of the product C_{Φ}, C_{φ} and quotient m/k do not depend on pressure being always equal to

$$C_{\Phi}C_{\varphi}\cong 1\,,\qquad \frac{m}{k}\cong 1\,. \tag{32}$$

The existence of relation (32) (independent of pressure) defines through eqs. (19) and (24) a correlation between friction factor R and void fraction φ in the two-phase flows. The same correlation is also given directly by the Lottes's and Levy's formulas. On the basis of (31) and (32) it is possible to calculate the difference between real and theoretical values of R (given by Levy's equation). This difference results from the value of m/k ratio which in the Levy's formula differs approximately by 15% from unity.

As to the correlations defining the void fraction φ one can use the following expression

$$l_n(1-\varphi)=\sum_{0}^{n=5} a_n(l_n X_{tt})^n \tag{33}$$

given by Hevitt [9] for low pressure values. In the analytical Hevitt's equation (33) the a_n coefficients form a set of constants which ought to be properly chosen allowing to replace the Lockhart-Martinelli's graphical relation $\varphi=f(X_{tt})$ [2].

For higher pressures there is a number of analytical relations which generally do not take account of the dependence between the void fraction φ and X_{tt} parameter. One of the best is the well known Thom's [10] relation established for water flows in the form

$$\varphi=\frac{\Theta x}{1+x(\Theta-1)}\,. \tag{34}$$

where Θ is a parameter which depends on pressure.

4. The similarity criterion for pressure drop in two-phase flows

The relations (19) and (24) introduced in previous paragraphs may be directly applied to two-phase water flows, since the values of C_{Φ}, C_{φ}, m and k coefficients were determined from the Martinelli and Nelson experiments for this kind of medium. The mentioned relations are the functions of dimensionless parameters x and X_{tt} describing the properties of the two-phase flows. The last observation suggests the virtual possibility of applying them to the calculation of the pressure drop in the media other than water if only C_{Φ}, C_{φ}, m and k could be defined independently of the kind of matter. Admitting that two-phase flows of different media are physically similar there is a possibility of applying the laws of the theory of similitude in describing them. A condition indispensable for making use of the similarity laws, is the possibility of describing the considered phenomena in terms of known analytical relations. This obvious condition has been fulfilled, because the pressure drop has been described analytically by relations (19) and (24) formulated in the course of previous considerations.

In view of the similarity laws the two physical phenomena are similar when:

1) the conditions of uniqueness are alike and the similarity numbers obtained on the basis of these conditions have the same numerical values,

2) the values of reduced quantities are equal when calculated in corresponding points.

In the light of above general statements, the similarity of different two-phase flows is, from the viewpoint of pressure drop, secured when for any given two media (I and II) the condition of equality of corresponding reduced quantities R_{I} and R_{II} is fulfilled

$$R_{\mathrm{I}}=R_{\mathrm{II}}\,. \tag{35}$$

Taking into account eq. (19), the relation (35) can be rewritten in the form

$$\left[(1-x)^{1.75}\,C_{\Phi}^{2}\left(1+\frac{1}{\chi_{tt}}\right)^{1.75m}\right]_{\mathrm{I}}=\left[(1-x)^{1.75}\,C_{\Phi}^{2}\left(1+\frac{1}{\chi_{tt}}\right)^{1.75m}\right]_{\mathrm{II}}. \tag{36}$$

Considering the uniqueness conditions, and particularly the boundary conditions, it is possible to come to the conclusion that the considered phenomena are similar as the set of relation

$$\begin{aligned} &\text{for} \quad x=0 \qquad R=1\\ &\qquad\;\; x=1 \qquad R=\frac{(dp/dL)_{g0}}{(dp/dL)_{l0}}\\ &\qquad\;\; p=p_{kr} \qquad R=1 \end{aligned} \tag{37}$$

must be always fulfilled.

Since relation (19) describing the pressure drop is a function of reduced quantities, it holds for all kinds of two-phase flows characterized by the quality and pressure within the limited range of variability of these parameters.

The relations (37) (sound for all media) make the uniqueness conditions similar for all media, which permits to extract from eq. (36) two equalities securing that the friction factor R_{I} and R_{II} will be the same for any given pair of two-phase flows. There is

$$\begin{aligned} C_{\Phi\mathrm{I}}(p_{\mathrm{I}}) &= C_{\Phi\mathrm{II}}(p_{\mathrm{II}}), \\ m_{\mathrm{I}}(p_{\mathrm{I}}) &= m_{\mathrm{II}}(p_{\mathrm{II}}). \end{aligned} \tag{38}$$

For the void fraction the similar relations result directly from (32), in the following form

$$\begin{aligned} C_{\varphi\mathrm{I}}(p_{\mathrm{I}}) &= C_{\varphi\mathrm{II}}(p_{\mathrm{II}}) \\ k_{\mathrm{I}}(p_{\mathrm{I}}) &= k_{\mathrm{II}}(p_{\mathrm{II}}) \end{aligned} \tag{39}$$

In the both above relations the coefficients C_{Φ}, C_{φ}, m and k are functions of pressure only.

The character of their dependence on pressure can be established from eq. (19) which states that the friction factor R is a function of two variables, namely

$$R = R(x, p) \tag{40}$$

as follows directly from the relations

$$\chi_{tt} = \chi_{tt}(x, p), \tag{41}$$

$$\begin{aligned} C_{\Phi} &= C_{\Phi}(p), \\ m &= m(p). \end{aligned} \tag{42}$$

For $x=1$ the friction factor R ought to depend on pressure only, since from the relation (19) it follows that

$$\lim_{x\to 1} R = \lim (1-x)^{1.75}\, C_{\Phi}^2 \left(1 + \frac{1}{\chi_{tt}}\right)^{1.75} = \frac{(dp/dL)_{go}}{(dp/dL)_{lo}} = K. \tag{43}$$

The relation (43) indicates that for any pressure lower than critical the friction factor $R(x=1)$ becomes equal to some constant value K.

This new constant K can be always determined analytically because the two-phase flow transforms itself into a single-phase vapour flow for $x=1$.

Using the Blasius formula for the single-phase flow friction factor one obtains from (43) the following expression for the sought values of K

$$K = \frac{\rho_l}{\rho_g} \left(\frac{\mu_g}{\mu_l}\right)^{0.25}. \tag{44}$$

For any two similar phenomena there is

$$R_{\mathrm{I}} = R_{\mathrm{II}}, \quad K_{\mathrm{I}} = K_{\mathrm{II}} \tag{45}$$

which is a direct result of K being (for $x=1$) equal to the local value of friction factor R.

At this stage of the analysis it is possible to replace relations (42) by two equations

$$C_\Phi = C_\Phi(K), \quad m = m(K) \tag{46}$$

which can be used in evaluating the friction factor R. Besides there is also

$$C_\varphi = C_\varphi(K), \quad k = k(K) \tag{47}$$

which suffices for calculating the void fraction φ.

From the all above remarks it seems to be clear that the quantity K may be regarded as an additional criterial number because, in view of (46) and (47), there is

$$K_{\mathrm{I}} = K_{\mathrm{II}} \tag{48}$$

if only the conditions of similarity (39) and (38) are fulfilled.

Recapitulating, it is possible to state that any given couple of two-phase flow are always similar from the point of view of the pressure drop, if only criterial numbers characterizing them are mutually equal.

Expressing the parameters C_Φ, C_φ, m and k as functions of similarity number K it becomes possible to apply the relations (19) and (24) for describing the pressure drop in flows of media different from water. The examples of this kind of calculations, made on the basis of eq. (44), Fig. 3 and Fig. 6 are given in Fig. 7.

In a recently published monograph [11] the results may be found, supporting the present considerations on the similarity criterion for two-phase flows. In [11] there is a diagram (quoted here in Fig. 8) obtained by Baroczy, which represents R as a function of the qua-

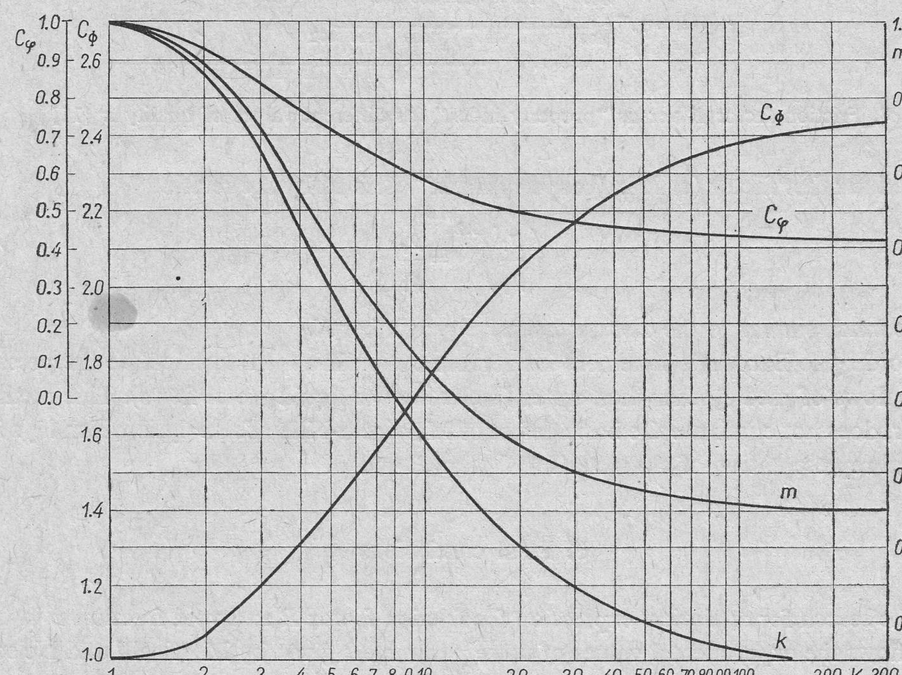

Fig. 7. Factors C_Φ, C_φ, m and k versus K parameter

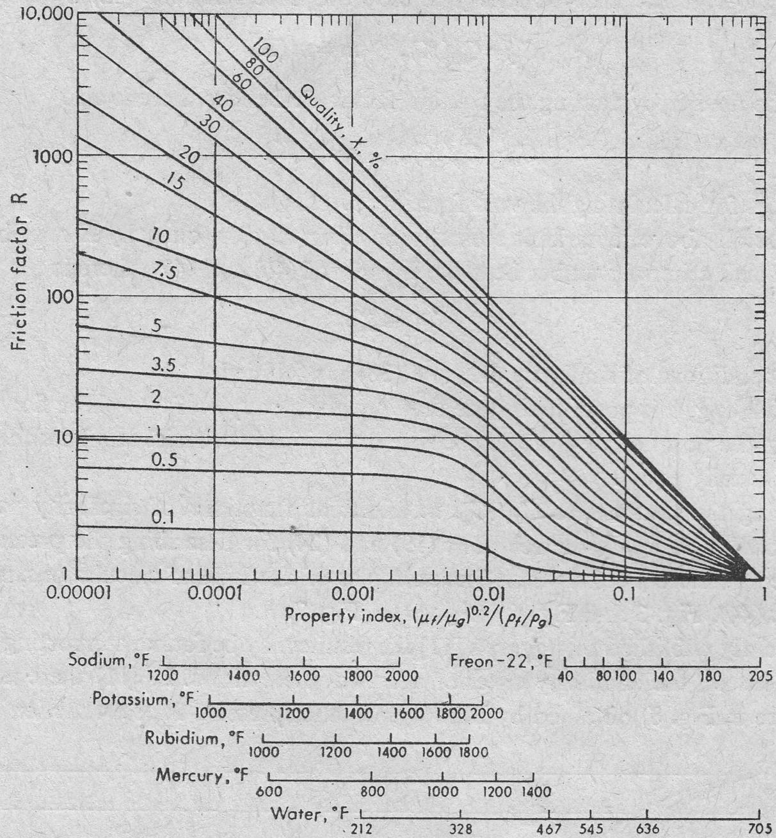

Fig. 8. Friction factor R versus "property index" for different values of quality x [11]

lity x and another factor (called: „property index") defined by the relation

$$\left(\frac{\mu_l}{\mu_g}\right)^{0.2} = \frac{\rho_g}{\rho_l} \tag{49}$$

which is much similar to the discussed similarity number K.

According to Baroczy's statement his representation R allows to calculate the pressure drop in flows of media other than water (including the liquid metal flows). This method, however, does not eliminate the basic defect of Martinelli-Nelson's analysis which lays in a graphical representation of characteristic factors R and φ.

5. Final conclusions

In the search for analytical expression for friction factor R and void fraction φ in two--phase flows the experimental values of these parameters given by Martinelli and Nelson's diagrams have been employed as a basic point of reference for further formal investigations.

In this way the final relations are directly supported by well proved experimental data, allowing to avoid the lack of practical exactness characteristic of all the purely theoretical approaches to the description of two-phase flows. Introducing the similarity conditions (formulated from the point of view of pressure drop effect) the possibility of applying the presented results to the analysis of two-phase flows of media other than water has been discussed and examplified.

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Proponowana metoda wyznaczania spadku ciśnienia w przepływie dwufazowym

Streszczenie

W pracy przedstawiono sposób wykorzystania współczynników tarcia przepływu dwufazowego R oraz zawartości pary φ, podanych wykreślnie dla wody przez Martinellego i Nelsona, do obliczania spadku ciśnienia przy przepływie dwufazowym innych niż woda czynników. Wykorzystując fakt, iż dla ciśnienia krytycznego przepływ dwufazowy przechodzi w przepływ jednofazowy, za pomocą parametru X_{tt} (8) oraz (6), uzyskano zależność między parametrem Φ_{ltt} dla dowolnego ciśnienia, a tym samym parametrem dla ciśnienia krytycznego (rys. 2). Przedstawione na rysunku 2 zależności dadzą się zapisać w postaci (12). Parametr Φ_{ltt} dla ciśnienia krytycznego daje się przedstawić analitycznie przez (9) i (11). Podstawiając (11) i (12) do (6) otrzymano zależność (15), która po prostych przekształceniach daje poszukiwaną zależność analityczną współczynnika tarcia R (19)

$$R=(1-x)^{1,75}\,C_{\Phi}^{2}\left(1+\frac{1}{\chi_{tt}}\right)^{1,75m}.$$

Wpływ ciśnienia na współczynnik tarcia w powyższej zależności uwzględniają współczynniki C_{Φ}, m, które dla wody zmieniają się według rys. 3.

W podobny sposób, wykorzystując rys. 5 oraz odpowiednie zależności dla ciśnienia krytycznego, otrzymano zależność (24) na podaną wykreślnie dla wody przez Martinellego i Nelsona zawartość pary φ,

$$1-\varphi=C_\varphi\left(1+\frac{1}{\chi_{tt}}\right)^{-k}.$$

Wpływ ciśnienia na zawartość pary φ w powyższej zależności uwzględniają współczynniki C_φ i k, które dla wody zmieniają się według rysunku 6.

Stwierdzono na podstawie obliczeń w załączonej tabeli, że między współczynnikami C_Φ, C_φ, m i k zachodzi relacja (32)

$$C_\Phi C_\varphi \cong 1, \qquad m/k \cong 1.$$

Następnie przeprowadzono porównanie otrzymanych zależności z innymi występującymi w literaturze. Stwierdzono, iż zależność Leviego (29) można doprowadzić do postaci bardzo zbliżonej do (19). Dla umożliwienia stosowania (19) i (24) do obliczania spadku ciśnienia w przepływach dwufazowych innych czynników niż woda, posłużono się teorią podobieństwa zjawisk fizycznych. W wyniku analizy otrzymano warunki podobieństwa spadku ciśnienia w przepływach dwufazowych (35), (38), (39). Z warunków tych wynika liczba podobieństwa spadku ciśnienia w przepływach dwufazowych K (44). Uzależniając współczynniki C_Φ, C_φ, m, k od liczby podobieństwa K (rys. 7) można za pomocą (19) i (24) obliczać spadek ciśnienia przy przepływach dwufazowych także dla innych niż woda czynników.

Предложение метода определения перепада давления в двухфазном течении

Резюме

В работе представлен способ использования коэффициентов трения двухфазного течения R и паросодержания φ, представленных графически для воды Мартинелли и Нельсоном, для расчета перепада давления при двухфазном течении неводяных сред. Используя факт, что для критического давления двухфазное течение становится однофазным течением, при помощи параметра X_{tt} (8) и (6) получена зависимость между параметром Φ_{ltt} для произвольного давления и тем самым параметром для критического давления (рис. 2). Представленные на рис. 2 зависимости можно записать в форме (12). Параметр Φ_{ltt} для критического давления можно представить аналитически формулами (9) и (11). Подставляя (11) и (12) в (6) получается зависимость (15), которая после простых превращений дает искомую аналитическую зависимость для коэффициента трения R (19)

$$R=(1-x)^{1,75}\,C_\Phi^2\left(1+\frac{1}{\chi_{tt}}\right)^{1,75m}.$$

Влияние давления на коэффициент трения в вышеуказанной зависимости учитывается коэффициентами C_Φ, m, которые для воды принимают значения как на рис. 3.

Похожим способом, используя рис. 5 и соответственные зависимости для критического давления, получена аналитическая формула (24) для графически представленной Мартинелли и Нельсоном зависимости паросодержания для воды φ:

$$1-\varphi=C_\varphi\left(1+\frac{1}{\chi_{tt}}\right)^{-k}.$$

Влияние давления на паросодержание φ в указанной зависимости учитывается коэффициентами C_φ, k, изменяемость которых для воды представлена на рис. 6.

Из расчетов, представленных в приложенной таблице, следует, что между коэффициентами

C_Φ, C_φ, m, k происходит зависимость (32):

$$C_\Phi C_\varphi \cong 1, \qquad m/k \cong 1.$$

После этого проведено сравнение полученных зависимостей с литературными данными. Констатировано, что зависимость Леви (29) можно привести к форме очень близкой к зависимости (19). Для приспособления зависимостей (19) и (24) к расчетам перепада давления в двухфазных течениях и других неводяных средах использована теория подобия физических явлений. В результате анализа получены условия подобия перепада давления в двухфазных течениях (35), (38), (39). Из этих условий следует число подобия перепада давления в двухфазных течениях K (44). Устанавливая зависимость коэффициентов C_Φ, C_φ, m, k от числа подобия K (рис. 7) можно при помощи (19) и (24) проводить расчет перепада давления при двухфазных течениях также и для неводяных сред.