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ZENON ZAKRZEWSKI

Gdańsk*

Phase-Sensitive Microwave Reflectometer for Plasma Diagnostics

Part I — The Reflectometer

1. Introduction

The measurement of complex reflection coefficient is one of the most common measurements in the microwave frequency region. The methods of measurement of the absolute value and argument of the complex reflection coefficient in stady state conditions are known and widely used [1]. The conventional microwave reflectometers enable simple and direct measurements of the absolute value of reflection coefficient, also in dynamic conditions.

It happens, however, that a dynamic measurement of the argument of complex reflection coefficient is to be made in conditions where the changes in its absolute value are but slight. The diagnostic of time-varying plasma is here a typical example. The amplitude of an electromagnetic wave propagating in the cold, low-pressure plasma does not change noticeably; while the changes of phase-shift are explicity related to the electron density. This calls for the necessity of measurement of the phase shift. In view of continuously growing interest in the physics of nonstationary plasma, these measurements are very often made in dynamic conditions. The interferometric methods of measurement of the phase-shift are commonly used in plasma diagnostics [2 - 5]. In most cases they require a complicated electronic circuitry.

Recently Glover [6] has described a reflectometer system that enables the dynamic measurement of complex reflection coefficient in the microwave range. Unfortunately this system exhibits some fundamental disadvantages in many applications. The results of measurements require further numerical treatment and the precision of determination of the phase-shift is low in the region of small angles.

In this paper a simple phase-sensitive microwave reflectometer is described which allows to measure the complex reflection coefficient in a waveguide. In a common situation where there are small changes in the absolute value of this coefficient, the direct reading of the argument is assured on the real time-basis. The reflectometer circuit can be set up of standard waveguide components.

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In the first part of the paper the working principle, analysis of errors and calibration experiments are described. The second part [7] comprises a description of application of the reflectometer to the dynamic measurement of electron density in a discharge plasma.

2. Principle of operation

The simplified diagram of the reflectometer circuit is presented in Fig. 1. The microwave power is fed from a generator into the main path of the reflectometer, terminated with the load under test. The two directional couplers C_I and C_R allow to direct specified parts of power of the incident wave as well as of the reflected wave into the reference and test paths, respectively, and farther into the E and H arms of the hybrid junction ("magic tee"). The symmetric arms A and B of this junction are terminated with crystal detectors DA and DB, respectively. The reflectometer requires initial adjustment for a given load,

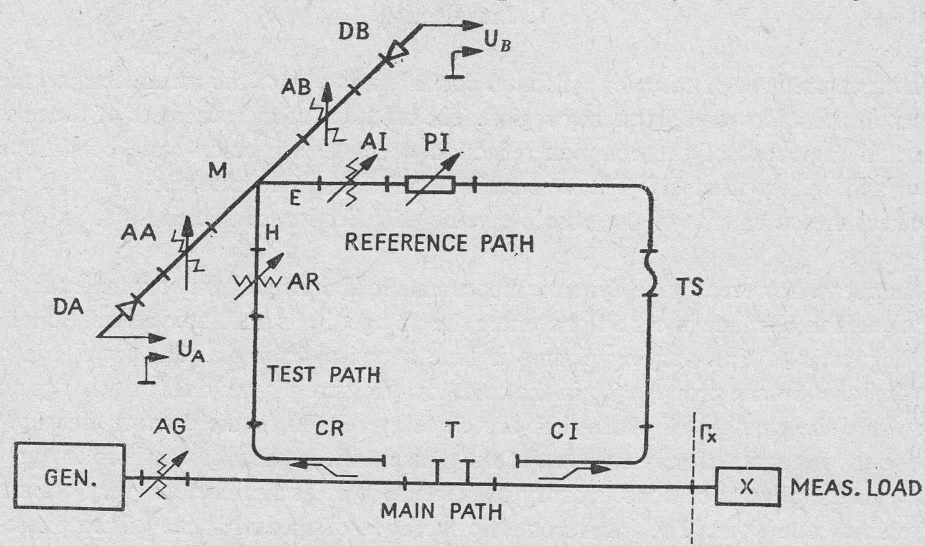

Fig. 1. Simplified diagram of the reflectometer circuit

as it will be described below. With the adjustment executed properly, the detector output voltages U_A and U_B contain full information about the absolute value and argument of reflection coefficient being measured. Such a detection system has been applied by Hotston and Seidl [8] in their interferometer. It's main advantage consist in the possibility of the direct and dynamic measurement of the argument in presence of small changes in the absolute value of the reflection coefficient.

The geometrical arrangement of the directional couplers C_I and C_R and also of the hybrid junction M and the measured load has been chosen in purpose as shown in Fig. 1. In this way approximately equal paths of the incident and reflected waves have been secured, which consequently yields a significant reduction of the errors caused by frequency fluctuations of the generator signal.

Let us consider the reflectometer system consisting of ideal elements. The reflection coefficient in all input planes of the multiports are equal zero. The directivity of both couplers, the symmetry of A and B arms of the hybrid junction and the isolation between E and H arms are ideal. The crystal detectors are identical and obey the square law characteristics. In such a circuit the ratio of complex amplitudes of the electric fields E_H and E_E in the input planes H and E, respectively, is

$$\frac{E_H}{E_E}=\frac{C_R}{C_I}\Gamma_x \exp(j\phi), \quad \text{where} \quad \Gamma_x=|\Gamma_x|\exp(j\gamma_x)$$

denotes reflection coefficient of the load under test, ϕ – phase-angle resulting from the difference in the electrical length of the incident and reflected wave paths, C_I and C_R – complex coupling coefficients of both directional couplers $[|C_I|, |C_R| \leqslant 1)$.

The initial adjustment is an important part of the measuring procedure. It is performed with the reflectometer being terminated with a load of initial value of reflection coefficient $\Gamma_0=|\Gamma_0| \exp(j\gamma_0)$. First the incident signal reaching the arm E of hybrid junction M is cut off by the attenuator AI. The output voltages $U'_A=U'_B$ of the crystal detectors DA and DB are caused solely by the microwave signal reaching arm H. Next, after cutting off the power flow in the test branch using AR, the attenuation of AI is chosen to obtain the output voltages $U''_A=U''_B$ equal to $U'_A=U'_B$. Then the initial value of the attenuation AR is restored. Finally the phase shifter PS is adjusted so that output voltages of both detectors are equal and obey the relationship

$$U_0=U_{A0}=U_{B0}=U'_A+U''_A=U'_B+U''_B. \tag{2.1}$$

Fulfilment of the conditions (2.1) may occur to be impossible in the case of imperfect reflectometer elements. Influence of these imperfections on the reflectometer operation will be dealt with in the next part of the paper.

After the first stage of initial adjustment the requirement $U'_A=U'_B=U''_A=U''_B$ is fulfiled and the sensitivities of detectors are equalized by means of AA and AB. The absolute values of amplitude of the electric field in the both input parts of the hybrid junction are equal. The output voltages of the detectors are

$$U_A=k|E_H-E_E|^2=k|E_0|^2|1-\exp j(\gamma_0+\phi)|^2, \tag{2.2}$$

$$U_B=k|E_H+E_E|^2=k|E_0|^2|1+\exp j(\gamma_0+\phi)|^2, \tag{2.3}$$

where k is a constant depending on the sensitivity of detection.

During the second stage of adjustment the condition (2.1) is fulfiled with proper setting of PS, which takes place when $\gamma_0+\phi=(2n+1)\pi/2$, where n – integer, and $U_0=U_{A0}=$ $=U_{B0}=2k|E_0|^2$.

Change of the reflection coefficient from Γ_0 to Γ_x causes the following change in the output voltages of detectors:

$$\frac{U_A}{k|E_0|^2}=\left(1+\left|\frac{\Gamma_x}{\Gamma_0}\right|\sin\theta\right)^2+\left|\frac{\Gamma_x}{\Gamma_0}\right|^2\cos^2\theta, \tag{2.4}$$

$$\frac{U_B}{k|E_0|^2}=\left(1-\left|\frac{\Gamma_x}{\Gamma_0}\right|\sin\theta\right)^2+\left|\frac{\Gamma_x}{\Gamma_0}\right|^2\cos^2\theta, \tag{2.5}$$

where $\theta=\gamma_x-\gamma_0$ is a change in the argument of reflection coefficient.

From (2.4) and (2.5) it follows directly that

$$\sin\theta=\frac{U_A-U_B}{U_A+U_B}\cdot\frac{1+\left|\frac{\Gamma_x}{\Gamma_0}\right|^2}{2\left|\frac{\Gamma_x}{\Gamma_0}\right|}=\frac{U_A-U_B}{2U_0}\left|\frac{\Gamma_x}{\Gamma_0}\right|^{-1} \tag{2.6}$$

$$\frac{|\Gamma_x|}{|\Gamma_0|}=\left(\frac{U_A+U_B}{U_0}-1\right)^{\frac{1}{2}}. \tag{2.7}$$

The system is particularly suitable for dynamic measurements of the argument of reflection coefficient with constant or insignificantly varying absolute value. In this case the sum of the both output voltages is constant and equal $U_A+U_B=2U_0$. The measured difference of output voltages (U_A-U_B) is proportional to $\sin\theta$. For $|\Gamma_x/\Gamma_0|=1$

$$\sin\theta=\frac{U_A-U_B}{2U_0} \tag{2.8}$$

and

$$\theta\cong\frac{U_A-U_B}{2U_0}, \tag{2.9}$$

when $\theta\leqslant 30°$.

A certain advantage of the proposed reflectometer system consists in a high sensitivity and linearity of display in the region of small measured phase angles. It would be inadvisable to use this reflectometer for measurements of phase angles in vicinity of 90°. In this region even small errors in measurement of (U_A-U_B) can lead to much greater errors in determination of θ. This disadvantage may be avoided by introducing an appropriate additional phase shift (i.e. using a calibrated phase shifter *PS*) in the reference branch of the reflectometer, after the initial adjustment.

3. Main sources of errors

Only these sources of errors which are directly related to the particular reflectometer system will be discussed. They are mainly the imperfections of the reflectometer elements and the variation of the absolute values of reflection coefficient. The possibilities of reduction or elimination of the discussed errors are indicated.

3.1. Variation of absolute value of reflection coefficient

In the case of direct display of the measured value of phase angle, the relation (2.8) is assumed to be valid. A change in the absolute value of reflection coefficient such that $|\Gamma_x|=(1+\kappa)|\Gamma_0|$ causes a relative error κ. Simultaneous observation of U_A+U_B makes possible to estimate the value of this error. For small changes

$$\kappa\cong\frac{U_A+U_B}{2U_0}-1. \tag{3.1}$$

When this error is too large it is necessary to measure the sum and the difference of the output voltages simultaneously. Then

$$\sin\theta=\frac{U_A-U_B}{U_A+U_B}. \tag{3.2}$$

It follows from (2.6) and (3.2) that the relative error in this case is $\kappa^2/2$, which is much less than κ.

3.2. Imperfect initial adjustment of reflectometer

Imperfect adjustment can arise as a result of imperfection of elements or of faults in the adjustment procedure itself. This kind of errors have been calculated [9] using the results of the analysis by Hotston and Seidl [8]. Two coefficients

$$\delta_A=\frac{U'_A+U''_A-U_0}{2U_0}, \qquad \delta_B=\frac{U'_B+U''_B-U_0}{2U_0} \tag{3.3}$$

can be selected as a measure of misadjustment. For small phase angles θ the relative error is

$$\frac{\delta\theta}{\theta}\cong\delta_A+\delta_B. \tag{3.4}$$

3.3. Imperfect directional couplers

Two waveguide directional couplers, C_R and C_I, are elements of the reflectometer system. For standard couplers the reflections in both input planes of the main branch can be neglected. We will also assume a weak coupling and neglect the change in the amplitude of the signal passing the main branch. Such a coupler can be described by two parameters: coupling C and directivity D. Both of them are complex numbers of absolute value much smaller than unity.

The ratio of the complex amplitudes of electric field in the output planes of the couplers [10] is

$$\frac{E_R}{E_I}=\frac{C_R}{C_I}\,\frac{D_R+\Gamma_x}{1+D_I\Gamma_x}. \tag{3.5}$$

The mismatch at the generator side does not affect this ratio. If there are no other sources of error, the measured phase change $\theta=\gamma_x-\gamma_0$ is equal to the changes of the argument of E_R/E_I. For $|\Gamma_x|=|\Gamma_0|$ and small θ one obtains

$$\left(\frac{E_R}{E_I}\right)_x\bigg/\left(\frac{E_R}{E_I}\right)_0\cong\exp j\theta\left(1-j\frac{D_R}{\Gamma_0}\theta-jD_I\Gamma_0\theta\right). \tag{3.6}$$

It follows directly from (3.6) that the maximum value of relative error in the measured phase shift is

$$\left(\frac{\delta\theta}{\theta}\right)_{\max}=\left|\frac{D_R}{\Gamma_0}\right|+|D_I\Gamma_0|. \tag{3.7}$$

Usually $|D_R|$, $|D_I| \ll 1$ and the error is small (max 2% for a directivity of 40 dB) so long as there simultaneously is $|\Gamma_0| \sim 1$. For large ratios $|D_R/\Gamma_0|$ this error may be unacceptable. We will show below that the application of a tuner placed in the main path as shown in Fig. 1, practically enables a complete elimination of this error.

The scattering matrix

$$\hat{T} = \begin{vmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{vmatrix}$$

of a tuner is symmetric and unitary. The elements of this matrix satisfy the following conditions

$$\sum_i T_{ij} T_{ij}^* = \begin{cases} 1 & \text{for} \quad j=i \\ 0 & \text{for} \quad j \neq i. \end{cases} \tag{3.8}$$

The input and output planes of the tuner can be selected, without restrictions in generality of the analysis, so that $T_{12} = |T_{12}|$ and $T_{11} = j|T_{11}|$. Then $T_{12}^2 - T_{11}^2 = 1$ and only one of the scattering coefficients i.e. T_{11} describes the tuner univocally.

It can be shown that in presence of the tuner the (3.5) becomes

$$\frac{E_R}{E_I} \cong \frac{C_R}{C_I} \frac{1}{1 - T_{11}\Gamma_G} \frac{D_R + T_{11} + \Gamma_x}{1 + D_I \Gamma_x}. \tag{3.9}$$

Tuning T for minimum power in the reference path with the reflectometer terminated with a matched load leads to

$$\frac{E_R}{E_I} \cong \frac{C_R}{C_I} \frac{1}{1 + D_R \Gamma_G} \frac{1}{1 + D_I \Gamma_x} \cdot \Gamma_x, \qquad T_{11} = -D_R. \tag{3.10}$$

In the formulas (3.9) and (3.10) Γ_G denotes the reflection coefficient at the generator end of the main path. Now the maksimum value of the relative error is

$$\left(\frac{\delta\theta}{\theta}\right)_{\max} = |D_I \Gamma_0|. \tag{3.11}$$

This error, for small $|\Gamma_0|$, is much smaller than (3.7). The result of measurement also in this case practically does not depend on Γ_G.

3.4. Departure from the square-law characteristics of the detectors

The actual characteristics of the crystal detectors may differ from the square-law characteristics assumed in the analysis of the reflectometer operation. Let us consider the case when the exponent of the characteristics is $2(1-\delta)$ instead of 2 and $|\Gamma_x| = |\Gamma_0|$. The formulas (2.4) and (2.5) become

$$\frac{U_A}{k|E_0|^2} = 2^{1-\delta}[1 + \sin\theta]^{1-\delta}, \tag{3.12}$$

$$\frac{U_B}{k|E_0|^2} = 2^{1-\delta}[1 - \sin\theta]^{1-\delta}, \tag{3.13}$$

in this case.

For $\delta \sin^2 \theta \ll 1$, as a first approximation we have

$$\sin \theta \cong (1+\delta) \frac{U_A - U_B}{U_A + U_B}. \tag{3.14}$$

The relative error in the determination of $\sin \theta$ is δ. This error can be easily eliminated in the case of operation on the part of characteristic with known and constant exponent.

4. X-band reflectometer

Fig. 2 shows a diagram of the X-band waveguide reflectometer system being described in this section. The reflectometer has been constructed of standard waveguide elements (Polish standard FP22×10, equivalent to WR90). All measurements have been carried out at the center frequency of X-band, $f = 9375$ MHz.

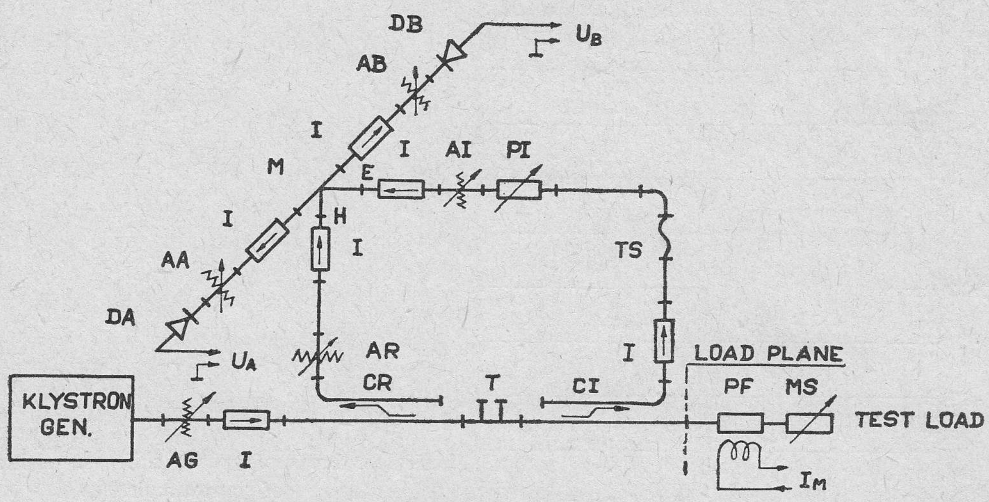

Fig. 2. Diagram of the X-band waveguide reflectometer system

Two kinds of calibration experiments have been performed. First, the reflectometer has been terminated with a precision movable short circuit. The dependence of the measured phase shift θ on a position l of the short has been examined. The results of this test are presented in Fig. 3. Reflectometer has been initialy adjusted for $l = 10$ mm. The full line indicates the phase shift resulting from the change in the position of short circuit. The circles denote the measured values of θ, determined as $\theta = \arcsin [U_A - U_B/(U_A - U_B)_{90°}]$. There $(U_A - U_B)_{90°}$ is the value of the differential output voltage corresponding to the introduced phase shift equal 90°. In the ideal case the sum of the both output voltages is constant $U_A + U_B = (U_A - U_B)_{90°}$. In practice this sum may vary as a result of misadjustment of the reflectometer or imperfection of its elements. For the described reflectometer those fluctuations do not exceed 5%. The employed method of determination of the measured phase shift θ allows for estimating the errors occuring during the direct display of results. This follows the fact that for given conditions $(U_A - U_B)_{90°}$ remains constant and the difference $(U_A - U_B)$ is directly a measure of the phase shift. The error in measurement of θ was less than 10% during the described experiment.

In the second calibration experiment the reflectometer was terminated with a test load consisting of current controled ferrite phase shifter backed with a fixed short (Fig. 2). The magnetizing current I_M of the shifter had a constant and sinusoidal (low frequency) components. This enabled to vary periodicaly the introduced phase shift in the choosen region of the characteristic of the shifter. The angle θ was measured directly by displaying $(U_A - U_B)$ on the oscilloscope screen with assumption $\sin\theta = U_A - U_B/(U_A - U_B)_{90°}$.

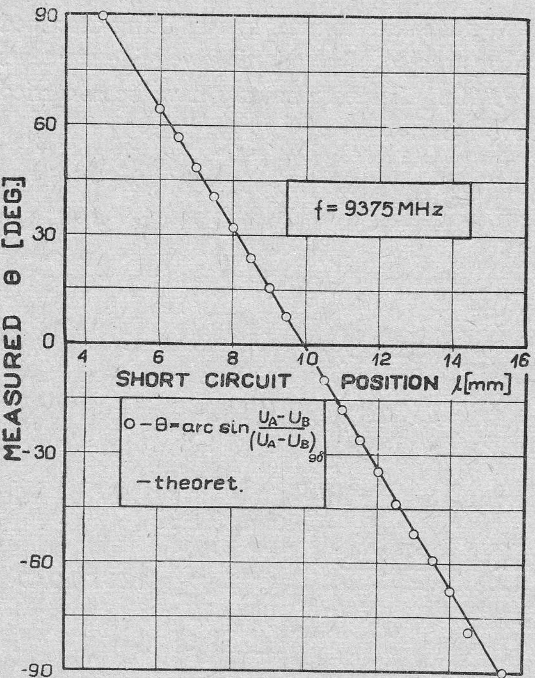

Fig. 3. Measured phase shift vs. position of short circuit

Fig. 4 shows typical results of measurements together with the changes of magnetizing current. Fig. 4a shows the measured output voltage in the case of large changes of θ. The asymmetry of $\sin\theta$ results from the non-linearity of the phase shifter for angles exceeding $\pm 90°$ (compare with the scale of introduced phase shift).

For small changes of θ the characteristic of phase shifter is linear and the displayed output voltage is directly proportional to I_M (Fig. 4b).

Because of the performance of the ferrite phase shifter those calibration experiments have been carried out at low frequencies just to show the possibility of the measurements on the real time basis. The time resolution of the reflectometer system itself allows to measure much more rapid variation. This resolution depends on parameters of the crystal detectors and the differential amplifier. It has been estimated that the time resolution of the presented X-band reflectometer is better than 1 μs.

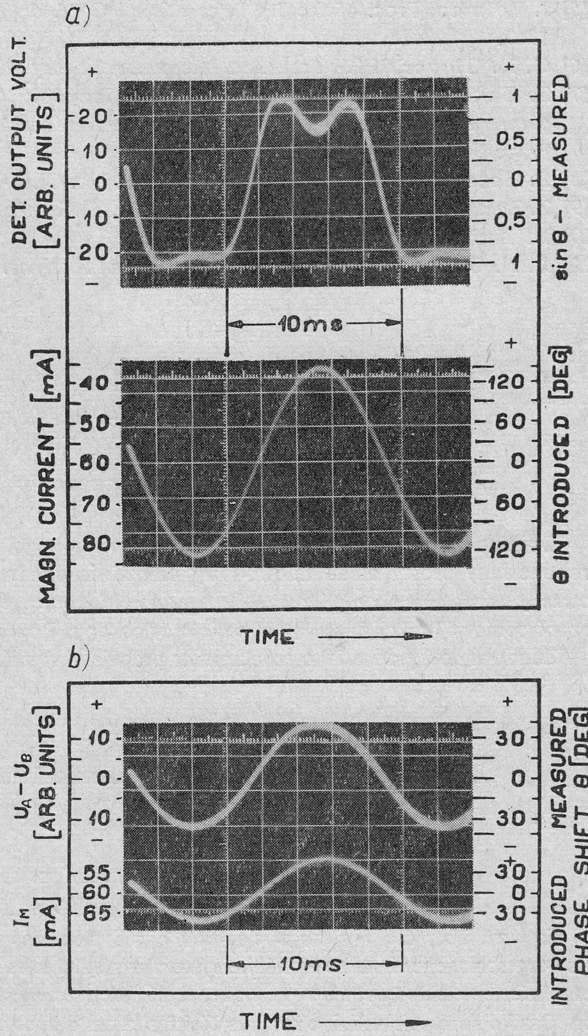

Fig. 4. Results of measurement of time – varying phase shift; a) large shifts, b) small shifts

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Fazoczuły reflektometr mikrofalowy dla potrzeb diagnostyki plazmy Część I — Reflektometr

Streszczenie

Opisano zasadę działania oraz układ mikrofalowego reflektometru z bezpośrednim odczytem. Proponowany prosty układ falowodowy umożliwia dynamiczny pomiar przesunięcia fazowego, bez potrzeby stosowania skomplikowanej aparatury elektronicznej. Wyniki pomiaru mogą być przedstawione bezpośrednio w funkcji czasu na ekranie oscyloskopu. Rozdzielność czasowa wynosi około 0,5°.

Zbudowano i przebadano reflektometr pracujący w mikrofalowym pasmie X.

Phasenempfindliches Mikrowellenreflektometer zur Plasmadiagnostik Teil I — Reflektometer

Zusammenfassung

Es wird über die Wirkungsweise und das System des Mikrowellen — Reflektometers berichtet. Das vorgeschlagene einfache Hohlleitersystem ermöglichte es die Phasenverschiebung, ohne Anwendung komplizierter elektronischer Apparatur, dynamisch zu messen. Die Meßergebnisse können in Form von Oszillogrammen unmittelbar als Zeitfunktion dargestellt werden. Das Zeitauflösungsvermögen beträgt 0,5°.

Es wurde ein, im Mikrowellenbereich X, arbeitendes Reflektometer gebaut und geprüft.

Фазочувствительный микроволновой рефлектометр для диагностики плазмы Часть I — Рефлектометр

Резюме

Описываются принципы действия и система микроволнового рефлектометра с непосредственным отчетом. Предлагаемая простая волноводная система позволяет производить динамический замер фазового перемещения, без необходимости применения сложной электронной аппаратуры. Результаты замера могут быть представлены непосредственно в функции времени на экране осциллоскопа. Временное распределение равно ок. 0,5°.

Построен и обследован рефлектометр, работающий в микроволновой полосе X.