Modeling of vibration of pre-stressed liquid filled tank

J. Badur, M. Karcz, M. Lemański, R. Kucharski, A. Wiśniewski,
W. Zakrzewski, D. Sławiński & L. Nastałek
Energy Conversion Department, Institute of Fluid-Flow Machinery PAS, Gdańsk, Poland

ABSTRACT: Thin-walled liquid storage tanks are the main elements of many types of engineering structures. Therefore, understanding of their dynamic and vibration problems is indispensable. In the paper, an extended symmetrical coupled approach has been developed for the fluid-solid vibration analysis. A symmetric acoustic tensor has been defined for the pre-deformed and pre-stressed thin-walled tank. Several tests have been performed and their results are discussed. Finally, the proposed model has been used for modeling of a full scale, double cylindrical reactor filled by a reacting vacuum residues.

1 MOTIVATIONS

Gas and liquid storage tanks are the main elements of many types of engineering structures. Their dynamic and vibration characteristics are very important. For this reason, many research works have been devoted to the free-vibration problem, see Morland & Ohayon (1995), Howe (1998), Chen (1972). A separate fluid-solid modal analysis technique called mass-adding has been discussed in Howe (1998). A coupled fluid-structure vibration analysis approach has been developed by Morland & Ohayon (1995), Howe (1998), Chen (1972), Zienkiewicz & Betters (1978), Ding & Chen (2001). This approach is based on the so-called acoustic approximation limited to fluid media.

In the present paper we attempt to extend a similar approach also to solid medium. It gives an opportunity to analyze the tank which has been pre-stressed and pre-deformed due to stationary creep, plasticity, etc. However, the coupled structure-acoustic formulation needs some symmetrization. The previous formulation proposed by Zienkiewicz & Betters (1978) has been based on the acoustic pressure, while Chen (1972) and Everstine (1981) have considered the velocity potential. However, both formulations are inherently bound to asymmetric or complex matrix equations, see Cho & Song (2001). Everstine (1981) has suggested introducing the acoustic velocity potential, in addition to the sound pressure, as an independent variable in the fluid domain. The effective formulation based on mixed acoustic pressure and displacement potentials has been introduced by Morland & Ohayon (1995) and also by Sandverg & Goranson (1988). Nevertheless, the latter model cannot take excitation of acoustic source into account. Extending the above formulations, Cho & Lee (2004) used finite elements based on some kind of displacement vectors in solid and fluid.

In this paper, we intend to introduce over-displacement vectors in solid and fluid media that describe small motion superimposed on the finite state of the pre-stressed fluid-solid continuum. Such kind of superimposed small displacement is well known in structural analysis of vibrational acoustics and stability. In fluid mechanics this kind of fluid displacement has been proposed and analytically motivated by Eckart (1963). The great advantage of Eckart's approach is that the small motion can be superimposed on an arbitrary finite state of fluid. In the presented paper, the coupled free-vibration analysis has been examined and tested using author's implementation into the commercial FEM software. The parametric study of the natural frequency has been examined in various combinations of pre-stressed state of a thin-walled structure and fluid, Wiśniewski & Kucharski (2006).

In numerous formulations of the coupled symmetric structure-fluid problem, the free-vibration problem has been approximated with degenerated 8 or 6–node 2D shell elements and 3-D fluid elements, Ding & Chen (2001), Cho et al. (2002). Degenerated shell elements require the reduced integration technique for preventing the shear and membrane locking in statics, Cho & Oden (1997), Badur (1984), Zienkiewicz et al. (1971) and similar mass locking effects in free-vibration, Kekana & Badur (2000). In the case of the fluid finite elements, reduced integration is also required for elements of low approximation. On the other hand, it follows from previous studies by Badur (1984), Badur & Chróścielewski (1985), Chróścielewski et al. (2003) that every pre-stressed background finite state minimizes the effect of reduced integration. Therefore, in the present paper 3D finite elements will be tested simultaneously for the solid and the fluid medium and the effect of the reduced integration will be verified.

2 EXTENSION OF ECKART'S APPROACH TO THE SYMMETRICAL STRUCTURE-FLUID SUPERIMPOSED SMALL DISTURBANCES

Let us assume that a thin-walled tank filled with fluid in arbitrary state can, in general, undergo finite state deformation. Initially pre-stressed and pre-deformed structures possess an additional stiffness and much higher frequencies than stress-free and deformation-free structures. In the problem of modeling small disturbances superimposed on a finite state of solid/fluid continuum, it is assumed that the disturbances will propagate as small amplitude elastic waves. Therefore, even if the finite state possesses irreversible contributions, like plastic deformations, viscous stresses, and turbulent losses, we omit any viscous and irreversible contributions when modeling the superimposed motion. Similarly, the superimposed small disturbances, such as vibrational displacements, density and entropy fluctuations will be treated as purely elastic. Then, the recoverable energy of a superimposed motion in the solid–fluid coupled medium is defined as the difference between internal, kinetic and potential energies, Morland & Ohayon (1995): Howe (1998):

$$L = L_{\mathrm{fluid}} + L_{\mathrm{solid}} + L_{\mathrm{coupling}} , \quad (1)$$

where:

$$L_{fluid} = \iiint_{V_f} \rho_f(\varepsilon_f - \frac{1}{2} g^f_{ij} v_i v_j + V(\bar{x}))dV_f + D , \quad (2)$$

$$L_{solid} = \iiint_{V_s} \rho_s(\varepsilon_s - \frac{1}{2} g^s_{ij} \dot{u}_i \dot{u}_j + V(\bar{x}))dV_s , \quad (3)$$

$$L_{\mathrm{coupling}} = \iiint_{surface} p_f u_i n_i dA . \quad (4)$$

In the above relations ρ_s and ρ_f denote densities of solid and fluid, respectively, $\bar{v} = v_i \bar{e}_i = \dot{\bar{u}}$ the superimposed fluid velocity, $\bar{u} = u_i \bar{e}_i$ the superimposed displacement of solid, ε_f and ε_s the solid and fluid specific densities of internal energy in superimposed motion, $V(\bar{x}) = zg$ a specific body force potential at the point $\bar{x} \in V_f$ or $\bar{x} \in V_s$, p_f superimposed fluid pressure on the surface of a solid body and $\bar{n} = n_i \bar{e}_i$ a normal vector orienting the surface of ∂V_s. Finally, D denotes a dissipation functional.

The calculation of kinetic energy of small disturbances superimposed on the finite state is not an obvious problem and, in some cases, cannot be expressed simply as a half of velocity square, i.e. $\frac{1}{2} v_i v_i$. The problem of acoustic vibration was analyzed by Eckart (1963) who introduced a kinetic metric of acoustic fluid g^f_{ij}. Its mathematical interpretation is a symmetric metric tensor that describes dynamics of fluid in a finite state. In the classical case it is the kinetic energy $g^f_{ij} = \delta_{ij}$ that physically has the same meaning as, for instance, the kinetic energy of a stone thrown into an empty space. Following Eckart, a kinetic metric of solid body g^s_{ij} have to be introduced. Both objects g^f_{ij} and g^s_{ij} are symmetric, positive definite covariant metrics determined as functions of parameters of the finite, prestressed state, Wiśniewski & Kucharski (2006). We denote these parameters by additional index $(.)^0$. There are the state parameters : p^0, T^0, v^0, s^0 – in a fluid medium, and σ^0_{ij}, T^0_{ij}, ε^0_{ij}, s^0_{ij} – in a solid medium.

Table 1. Natural frequencies of empty and air filled steel tank at $p = 0.1$ [MPa].

	Empty tank		Filled tank	
Mode no.	8 node (2D element)	20 node (3D element)	8 node (2D element)	20 node (3D element)
1	35.55	35.07	36.41	35.97
2	43.89	43.78	44.67	44.60
3	49.62	48.98	50.14	49.55
4	56.44	56.24	57.00	56.84
5	57.81	58.19	58.49	58.91
6	59.41	59.02	60.15	59.80
7	63.79	63.81	64.46	64.52
8	68.09	68.51	68.65	69.10
9	69.72	70.12	70.30	70.73
10	74.17	74.75	74.75	75.37

3 NUMERICAL ANALYSIS OF VIRTUAL STIFFNESS INFLUENCE

3.1 *Natural frequencies of empty tank and air filled one*

Natural frequencies of the empty tank and the air-filled one are presented in Table 1. In the latter case the pressure of air was about 0.1 MPa.

From the results presented in Table 1 one can see that the three-dimensional element does not show the locking effect and gives results which are comparable with those obtained from the shell element. One also could see that influence of the air is very small. The difference between eigenvalues are less than 1%. It is the result of a small acoustic stiffness of the air. During calculations of gas filled tanks under atmospheric pressure, the modeling of fluid can be neglected. Calculations at high stressed walls of tank show that locking of 3D elements is small. Similar results were presented in the paper by Morland & Ohayon (1995), where only the linear state without high loads is susceptible for locking.

3.2 *Frequencies of water filled tank*

The natural frequencies of a water filled tank are listed in Table 3. Two types of elements were used, 8 node brick element and face-centered second-order 20 node element. Calculations were performed for

Table 2. Natural frequencies of water filled tank in [Hz].

Mode no.	Nastran 8 node (2D element)	Nastran 20 node (3D element)	Abaqus 8 node (2D element)	Abaqus 20 node (3D element)
1	106.08	106.25	105.96	113.87
2	161.78	163.67	162.85	178.96
3	175.23	176.95	176.78	193.7
4	188.84	189.68	188.32	207.8
5	193.24	195.86	195.11	213.38
6	197.64	200.84	199.87	222.69
7	210.53	213.86	212.34	235
8	218.87	223.27	221.86	245.85
9	227.02	230.95	229.52	250.73
10	228.26	231.87	230.54	263.54

Table 3. Natural frequencies of air and water filled tank.

Mode no.	Air filled tank 8 node (2D element)	Air filled tank 20 node (3D element)	Water filled tank 8 node (2D element)	Water filled tank 20 node (3D element)
1	36.41	35.97	106.08	106.25
2	44.67	44.60	161.78	163.67
3	50.14	49.55	175.23	176.95
4	57.00	56.84	188.84	189.68
5	58.49	58.91	193.24	195.86
6	60.15	59.80	197.64	200.84
7	64.46	64.52	210.53	213.86
8	68.65	69.10	218.87	223.27
9	70.30	70.73	227.02	230.95
10	74.75	75.37	228.26	231.87

the acoustic modulus $\lambda^0 = 200$ [MPa] and density $\rho_f = 1000$ [kg/m^3].

20 node element gives results which are similar to those received for the shell element. 8 node shell element leads to locking, which results from low level shape functions. In the case of unsteady calculations or/and eigenvalue analysis it is better to use at least second-order elements.

3.3 *Comparison with air*

The natural frequencies of air and water filled tank are listed in Table 3. The natural frequencies of water filled tank are higher than air filled ones. The reason of this fact is that the fully water filled tank vibrates like a solid beam. In the air filled tank every wall vibrates independently, as it can be observed in Figure 1. Modes of the empty tank are not presented here, because they are identical with those obtained for the air filled tank at atmospheric pressure.

3.4 *First natural frequency versus λ^0 (calculations for water)*

Acoustic elastic modulus of water is a free model parameter which needs calibrations. To present influence of λ^0 on vibrations of the tank we have made calculations in a wide range of λ^0 values from 50 [MPa] to 1000 [MPa]. Calculations contain also an influence of stresses coming from hydrostatic pressure. The first natural frequencies versus λ^0 are listed in Table 4.

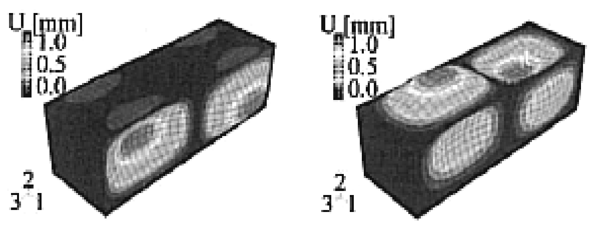

Figure 1. Second natural frequency of water filled tank (left) – 161,78 Hz and air filled one (right) – 44.76 Hz.

Table 4. First natural frequency versus λ^0. Bold fonts mean parameters for tap water.

Acoustic modulus λ^0[MPa]	First natural frequency [Hz]
50	70.40
100	87.30
200	**108**
400	129.60
700	154.96
1000	174.96

Table 5. First natural frequency versus λ^0 for air filled tank.

Pressure [MPa] = λ^0	Density [kg/m^3] at 20°C	First natural frequency [Hz]
0.1	1.20	36.42
0.3	3.56	38.08
0.5	5.94	39.66
0.7	8.32	41.16
1.0	11.89	43.28
1.5	17.85	46.52
2.0	23.82	49.47
5.0	59.80	63.05
10.0	120.42	76.53

3.5 *First natural frequency of tank versus pressure of air*

For air (gas) λ^0 modulus equals to pressure. Data of calculations and first natural frequencies of air filled tank at different pressures are listed in Table 5. With increasing pressure the values of first natural frequency is also increasing, and for high pressures results are close to ones obtained for the water filled tank.

4 EXAMPLE OF A FULL SCALE, BITUMEN FILLED, CYLINDRICAL REACTORS

Finally, our symmetrical coupled technique has been employed for calculation of a real industrial tank. The

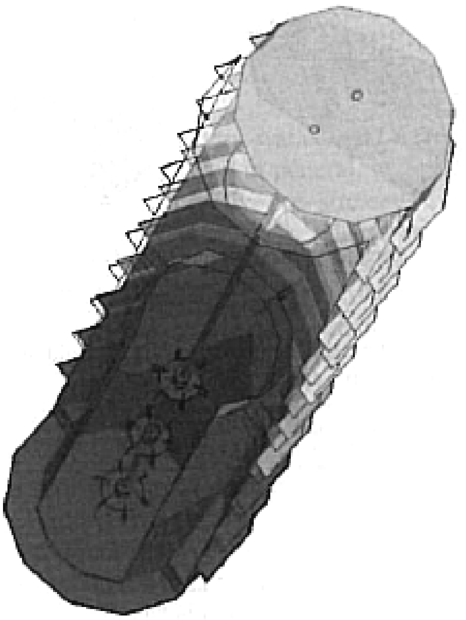

Figure 2. Scheme of geometry of cylinder and form of vibration for frequency f4.

Table 6. Comparisons of free frequencies of empty and filled cylinder.

No of frequency	Empty reactor	Filled reactor
1	2.528	3.091
2	2.528	4.060
3	5.356	4.941
4	5.360	4.942
5	7.459	7.811
6	7.465	7.811
7	8.951	8.761
8	9.458	10.700
9	13.013	15.510
10	13.190	15.520

subject of our investigation has been a cylindrical reactor employed for a modification of vacuum residue, Figure 2.

We divide volume of the reactor into two parts – the lower part filled by the bitumen and the upper part filled by gas under elevated pressure. We assume the fluid to be motionless. Small pre-deformations has been excluded from considerations, however, prestresses have been taken into account precisely. In Table 6 the frequencies of the empty and the partially bitumen filled cylinder are listed. The above simulations clearly indicate how interactions between the tank and its content influence the real dynamical state of the structure.

REFERENCES

Badur, J. 1984. *Numerical shell analysis via a second order approximation to the internal energy*. Ph.D. thesis Institute of Fluid-Flow Machinery Gdańsk (in Polish).

Badur, J. & Chróścielewski, J. 1985. Reduced integration technique in nonlinear shell theory. In *Proceedings of Fifth Conference on Computational Methods in Structure Mechanics, Gdynia, 1985* 12–17.

Chen, S. S. 1972. Free vibration of a coupled fluid/structural system. *Journal of Sound and Vibration* 21: 387–398.

Cho, J. R. & Lee, H. W. 2004. Numerical study on liquid sloshing in baffled tank by nonlinear element method. *Computational Methods in Applied Mechanics and Engineering* 193: 2581–2598.

Cho, J. R., Lee, H. W. & Kim, K. W. 2002. Free vibration analysis of baffled liquid-storage tanks by the structural-acoustic finite element formulation. *Journal of Sound and Vibration* 258: 847–866.

Cho, J. R. & Oden, J. T. 1997. Locking and boundary layer in hierarchical models for thin elastic structures. *Computational Methods in Applied Mechanics and Engineering* 149: 33–48.

Cho, J. R. & Song, J. M. 2001. Assessment of classical numerical models for the separate fluid-structure modal analysis. *Journal of Sound and Vibration* 239: 995–1012.

Chróścielewski, J., Lubowiecka, I. & Pietraszkiewicz, W. 2003. FEM and time stepping procedures in non-linear dynamics of flexible shell structures. In R. e. a. Kienzler (ed.) *Theories of Plates and Shells, Euromech Colloquium 444* 21–28.

Ding, W. P. & Chen, H. L. 2001. A symmetrical finite element model for structure-acoustic coupling analysis of an elastic, thin-walled cavity. *Journal of Sound and Vibration* 243: 547–559.

Eckart, C. 1963. Some transformations of the hydrodynamic equations. *Physics of Fluids* 6: 1037–1041.

Everstine, G. C. 1981. A symmetric potential formulation for fluid-structure interaction. *Journal of Sound and Vibration* 79: 157–160.

Howe, M. S. 1998. *Acoustics of Fluid Structure Interaction*. Cambridge: John Wiley & Sons.

Kekana, M. & Badur, J. 2000. Modeling of beams using the reduced integration technique – statics and free vibration. *Research & Development Journal* 16: 9–15.

Morland, H. J. P. & Ohayon, R. 1995. *Fluid Structure Interaction – Applied Numerical Methods*. New York: John Wiley & Sons.

Sandberg, G. & Goransson, P. 1988. A symmetric finite element formulation for acoustic fluid-structure interaction analysis. *Journal of Sound and Vibration* 123: 507–515.

Wiśniewski, A. & Kucharski, R. 2006. Eigencharacteristics of fluid filled tanks: An extended symmetrical coupled approach. *TASK Quarterly* 10: 455–468.

Zienkiewicz, O. C. & Betters, P. 1978. Fluid-structure dynamic interaction and wave forces. an introduction to numerical treatment. *International Numerical Methods in Engineering* 13: 1–16.

Zienkiewicz, O. C., Too, J. & Taylor, R. L. 1971. Reduced integration technique in general analysis of plates and shells. *International Journal for Numerical Methods in Engineering* 3: 275–290.