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Further remarks on the pseudomomentum balance in hydro- and gasodynamics

Dalsze uwagi o bilansie pseudopędu w mechanice płynów

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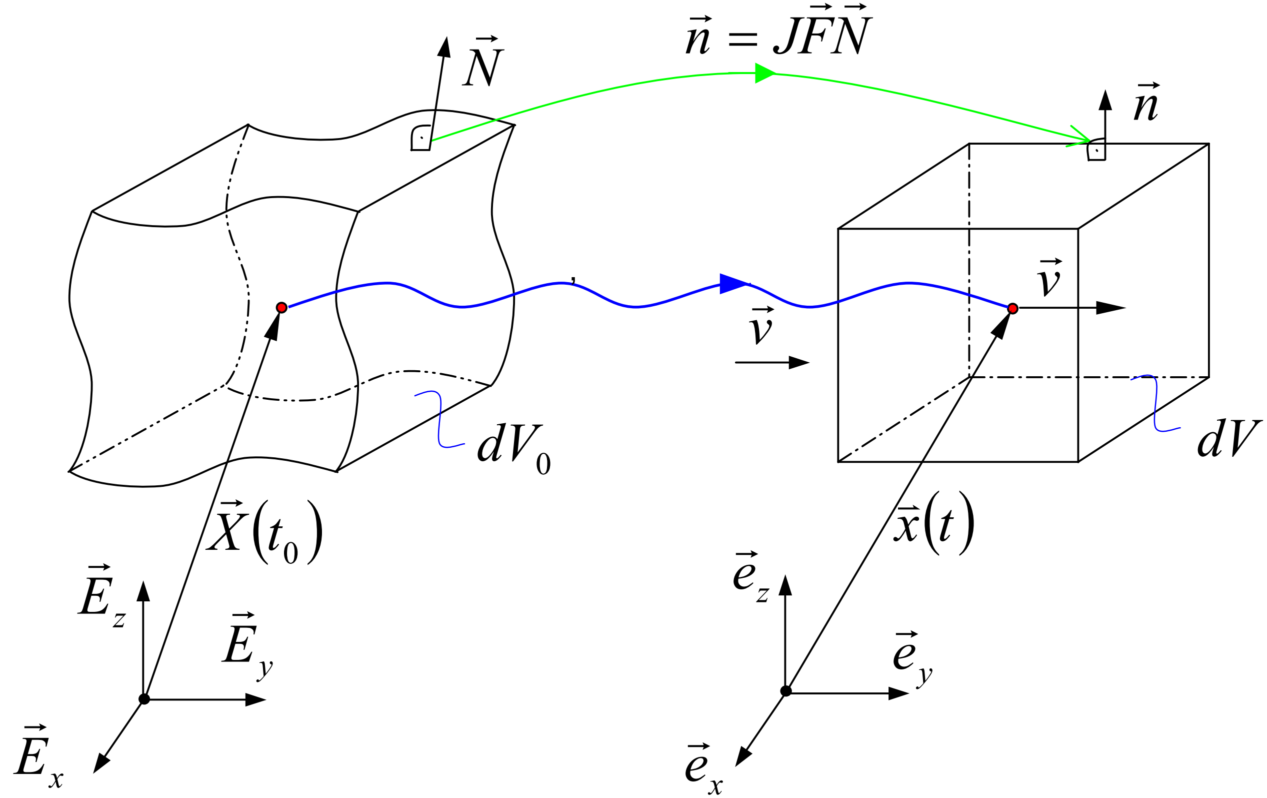
deformation gradient $f_{Ai} = \dfrac{\partial X_A}{\partial x_i}$ $F_{iA} = (f_{Ai})^{-1}$



momentum $$p_i = \rho v_i$$

$$i = x, y, z$$

specific momentum $$v_i$$

pseudomomentum $$P_A = \rho_0 v_i F_{iA} = \rho_0 u_A$$

$$A = X, Y, Z$$

specific pseudomomentum $$u_A = v_i F_{iA}$$

momentum flux

$$p_{ij} = -T_{ij} = p\delta_{ij} - (\tau_{ij} + R_{ij} + \tau_{ij}^{T} + \tau_{ij}^{dyf} + \tau_{ij}^{rad} + \ldots)$$

pseudomomentum flux

$$P_{AB} = \rho_0(\varepsilon + \Omega - T - \theta\eta)\,\delta_{AB} - F_{iA}T_{iB}$$ - solid

$$P_{AB} = \rho_0(\varepsilon + \frac{p}{\rho} + \Omega - T - \theta\eta)\,\delta_{AB} = \rho_0\mu\,\delta_{AB}$$ - fluid

newtonian force density

$$b_x = b_y = 0\,, \quad b_z = -g,$$

configurational (pseudo) force

$$F_A^{inh} = \left(\frac{\partial L}{\partial X_A} \right)_{\exp l}$$

momentum balance

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}(\rho v_i v_j + p_{ij}) = \rho b_i$$

pseudomomentum balance

$$\frac{\partial}{\partial t}(\rho_0 u_A) + \frac{\partial}{\partial X_B}(P_{AB}) = F_A^{inh}$$

Euler- Lagrange equation

$$E_i(L) = \frac{\partial L}{\partial x_i} - \frac{\partial}{\partial X_A}\left(\frac{\partial L}{\partial F_{iA}}\right) = 0$$

Transversality conditions

$$\frac{\partial}{\partial t}(\rho_0 u_A) + \frac{\partial}{\partial X_B}(P_{AB}) = F_A^{inh} + E_i F_{iA}$$

Pseudomomentum via Variational Principles of Thermomechanics

- Thomson – Tait , 1867
- Gibbs, 1877 (II part)
- Natanson, 1899
- Eckart , 1960

Extension of Gibbs' procedure

solid body 3D ← **interphase 2D** → **fluid body 3D**

↓

pseudomomentum conservation in solids 3D (Badur 1997) ← **Jump of Pseudomomentum Gibbs, 1877** → **pseudomomentum conservation in fluids (Badur 1997)**

↓

Jump extension (Badur 1997)

↓

$$\delta\mathrm{iv}\left[\overleftrightarrow{\sigma} - \vec{b}\,\overleftrightarrow{C} + \vec{N} \otimes (\overleftrightarrow{I} - \vec{N} \otimes \vec{N})\,\delta\mathrm{iv}(\overleftrightarrow{C} - \vec{b}\,\overleftrightarrow{K})\right] = (\overleftrightarrow{P}' - \overleftrightarrow{P}'')\vec{N}$$

J. Badur [in:] Z. Bilicki ed. *Współczesne kierunki w Termodynamice*, 129-146, Wyd IMP, Gdańsk

The Eckart variational principle

pseudoenergy conservation:

$$\frac{\partial}{\partial \tau}\rho_0(T+\Omega+\varepsilon+\theta\eta)+\frac{\partial}{\partial X_A}(-pV_A)=0$$

pseudomomentum conservation :

$$\frac{\partial}{\partial \tau}\rho_0 u_A+\frac{\partial}{\partial X_A}\rho_0\left(\varepsilon+\frac{p}{\rho}+\Omega-\theta\eta-T\right)=0$$

pseudo-vorticity conservation

$$w_i = \in_{ijk} w_{jk} \qquad w_{ij} = \frac{1}{2}(v_{j,k} - v_{k,j})$$

$$W_A = \in_{ABC} W_{BC} \qquad W_{BC} = \frac{1}{2}(u_{B,C} - u_{C.B})$$

$$\vec{\alpha} = \text{rot}\,\vec{a} = \vec{w}^t = \dot{\vec{v}} + \{tr\,\vec{\vec{d}}\,\vec{\vec{I}} - \vec{\vec{l}}\,)\vec{v}$$

Cauchy
(1815)

$$W_A = J\,f_{Ai} w_i$$ **rotor of velocity**

$$A_A = J\,f_{Ai}\,\alpha_i$$ **rotor of acceleration**

circulation

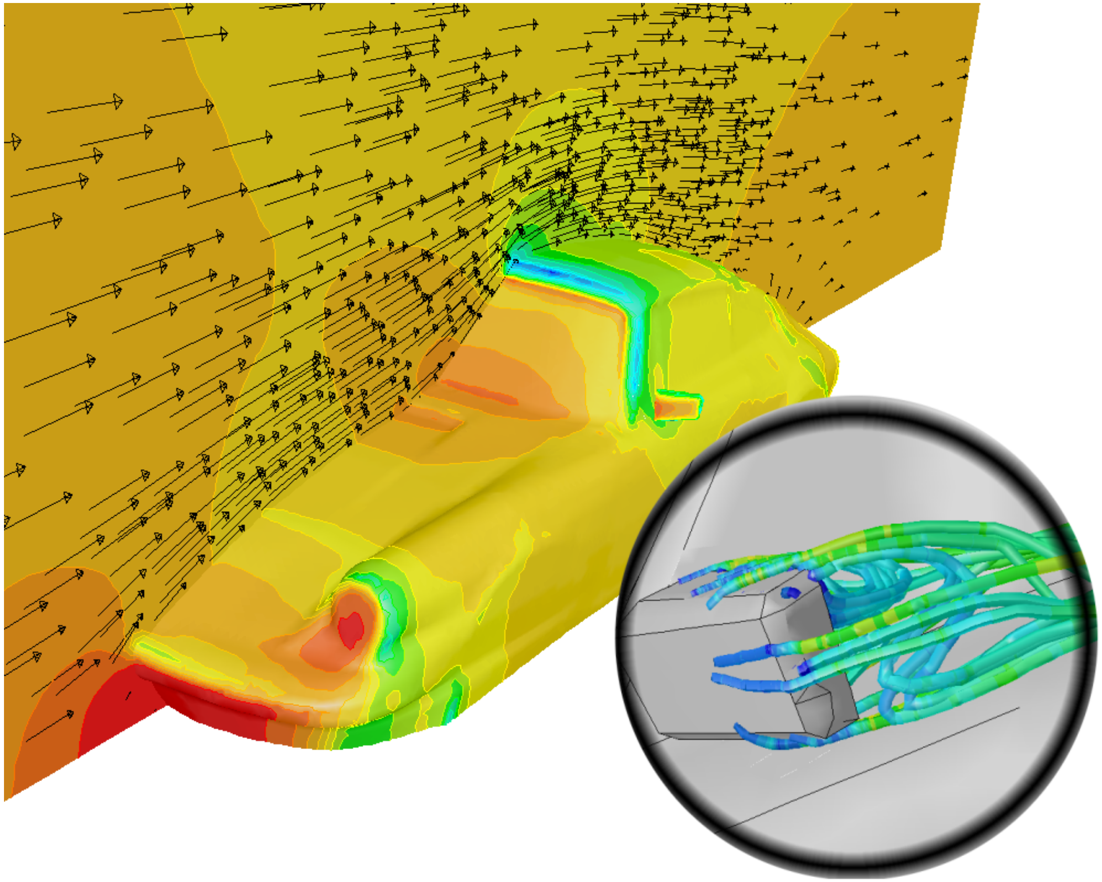
$$\Gamma = \oint_C v_i dx_i = \oiint_s w_i n_i\, da = \oint_{C_0} u_A dX_A = \oiint_{s_0} W_A N_A\, dA$$

Eulerian representation of pseudomomentum

Author	Pseudomomentum flux tensor		Pseudomomentum vector	
	Lagrangean	Eulerian	Lagrangean	Eulerian
Eckart, 1960	$\alpha_{\alpha\alpha}$	—	$u_\alpha, u_\beta, u_\gamma$ *	—
Eshelby, 1970	P_{ij}	Σ_{ij}	g_i	—
Rogula, 1977	$\overline{p}_{AB}$	p_{Ai}	$\overline{p}_A$	p_A
Andrews & McIntyre, 1978	B_{ij}	—	A_i *	—
Gołębiewska–Herman, 1981	b_{ij}	B_{ik}	b_i	B_i
Grinfeld, 1981	μ_{IJ}	χ_{ij} **	—	—
Peirles, 1988	—	—	κ_i ***	—
Duan, 1987	$B_{\mu\nu}$	$b_{i\nu}$	B_ν	b_ν
Maguin & Trimarco, 1992	b_{kl}	B_{ki}	P_k	jP_k
Gurtin, 1995	C_{AB}	—	—	—
present paper	P_{AB}	p_{Ai}	P_A	p_A

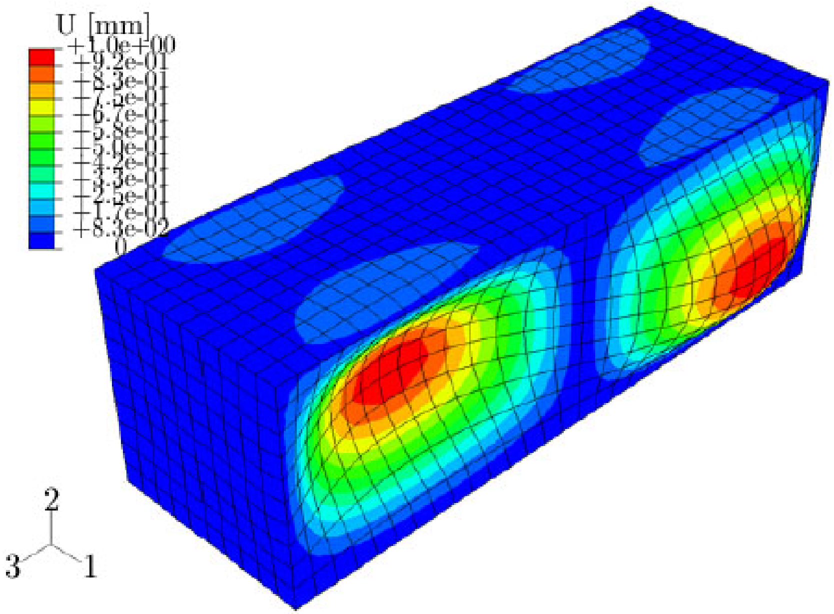
*) per unit mass **)symmetric — an analogue of the II[nd] Piola-Kirchhoff flux ***) in the whole volume

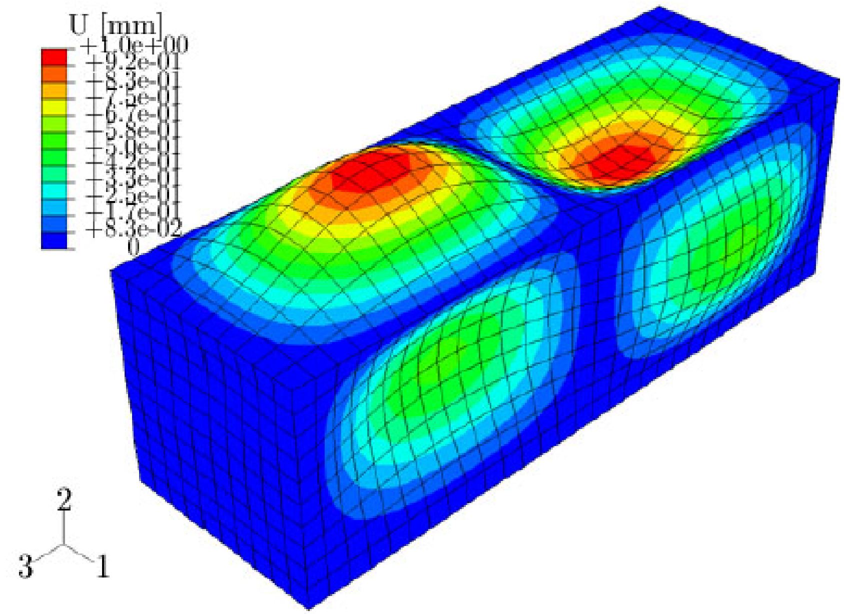
The acoustic pseudomomentum



from: M. Karcz, R. Kucharski, J. Badur, Matematyczne modelowanie dzwięków generowanych przepływem czynnika roboczego, pp 1-162, submitted to: *Zesz. Nauk. IMP,* 2004

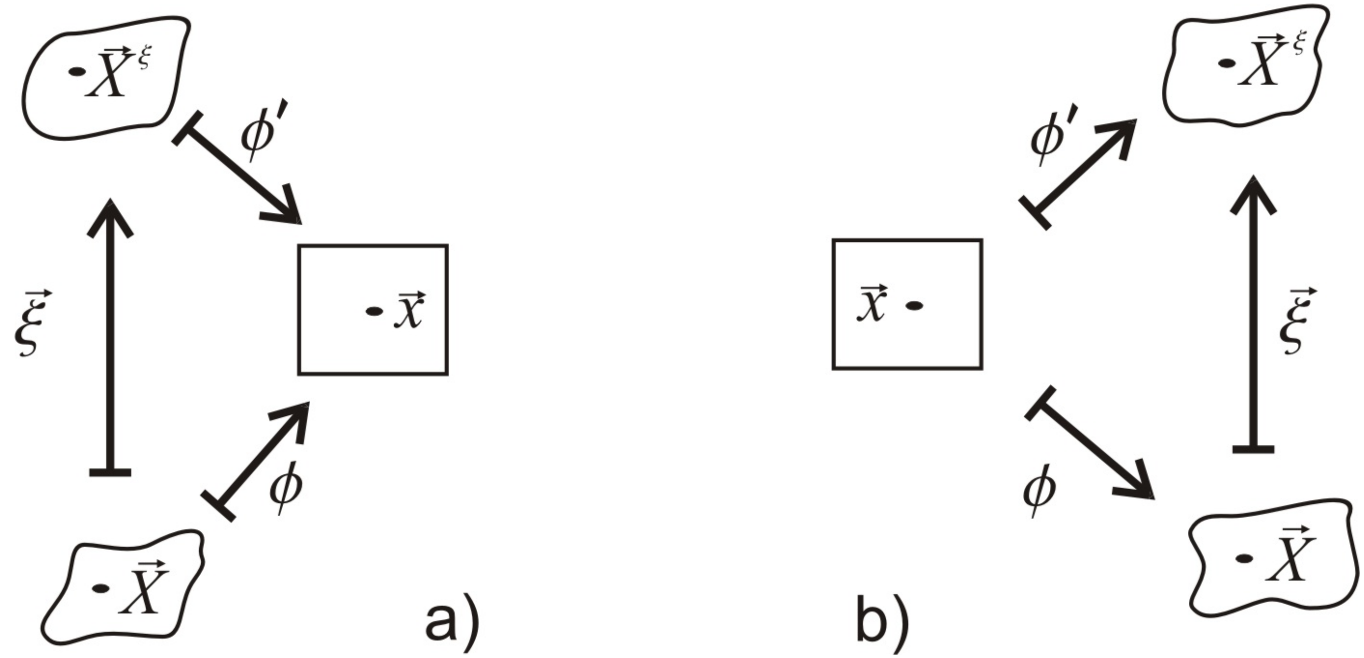
The acoustic pseudomomentum





from: R. Kucharski, A. Wiśniewski, J. Badur, Eigencharakteristics
of prestressed fluid filled tanks vie an extended
Eckart copuled approach, 2004, pp 1-20, submited to *Journal of Sound and Vibration.*

Generalized Lagrangian Mean approach



Two possible approaches for comparison of the excited motion superimposed on a normal motion of fluid element.

Andrews-McIntyre GLM (1978)

Wave displacement

$$[\vec{\xi}(\vec{x},t)]'' = 0$$

the mean Lagrangean velocity $\vec{v}^{\,L}$

the pseudomaterial derivative

$$\frac{D^L}{Dt}(\bullet) \equiv \frac{\partial}{\partial t}(\bullet) + [(\bullet)\otimes\nabla]\vec{v}^{\,L}$$

pseudomomentum balance:

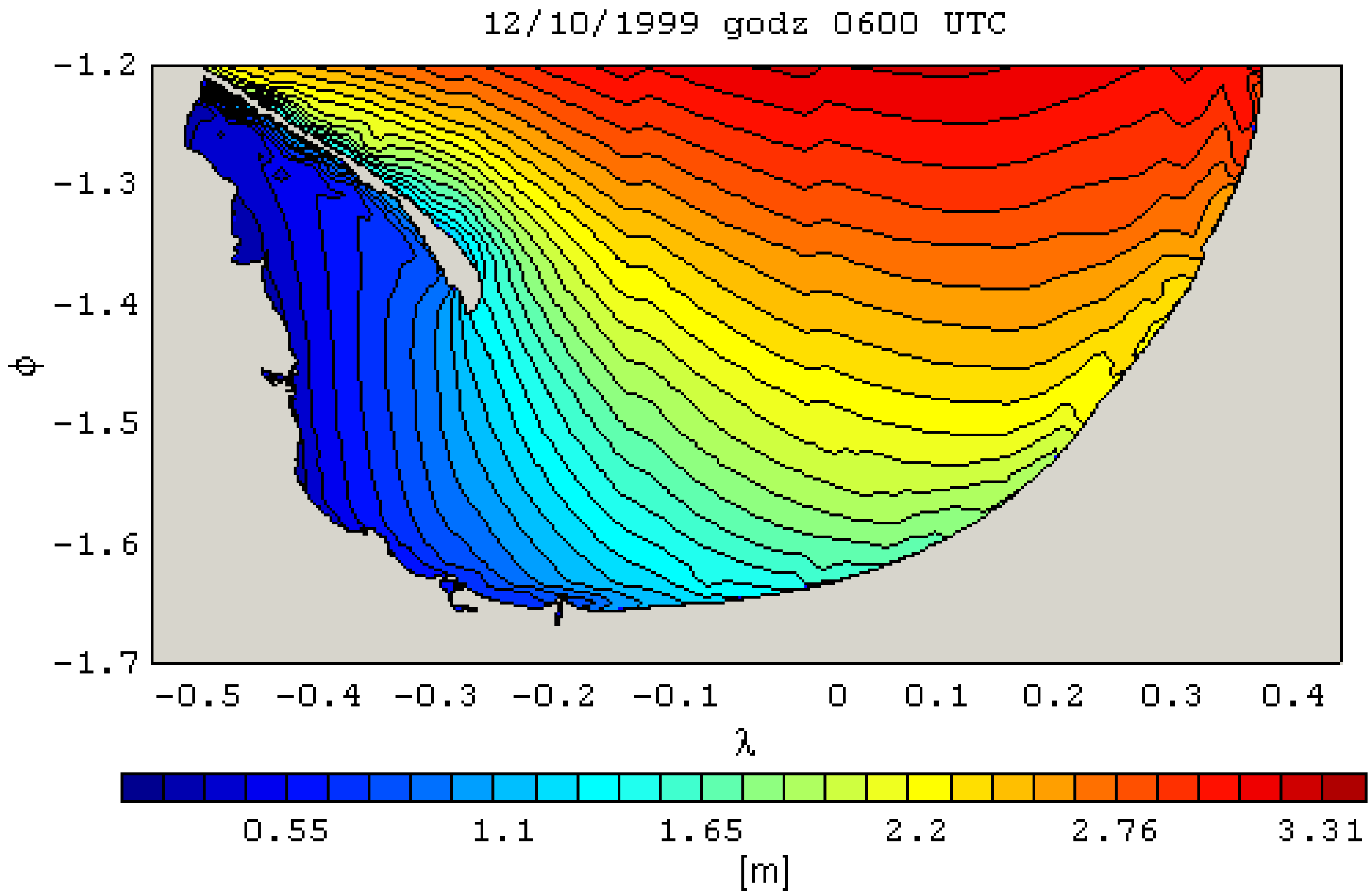
$$\frac{D^L}{Dt}(\vec{v}^{\,L} - \vec{u}^{\,L}) + (\vec{v}^{\,L} - \vec{u}^{\,L})(\vec{v}^{\,L}\otimes\nabla) = -\nabla\pi + \vec{F}^{\,L}$$

where pseudomomentum (per unit mass)

$$\bar{u}^{\,L} = -[\xi_{j,i}\,\xi_{j,t}]''\vec{e}_j$$

and

$$\pi = p^L/\rho^\xi + 1/2[\xi_{i,t}\,\xi_{i,t}]''$$



An example of wave action equation application in the SWAN model in the area of the Gulf of Gdańsk – Significant Wave Height H_s [m] field on the 12.10.1999

Badur J., Cieślikiewicz W. „Parametryczny model wysokości fali znacznej i średniego okresu fali na obszarze Zatoki Gdańskiej ”, Dokumentacja projektu KBN 6 P 04E 037 20

summary points

- reconstruction and revalorizatoion of
  - Thompson –Tait
  - Gibbs $\rightarrow$ extension of Laplace formulae
  - Natanson
  - Eckart
- role of pseudomomentum in acoustics
- role of pseudomomentum in turbulence