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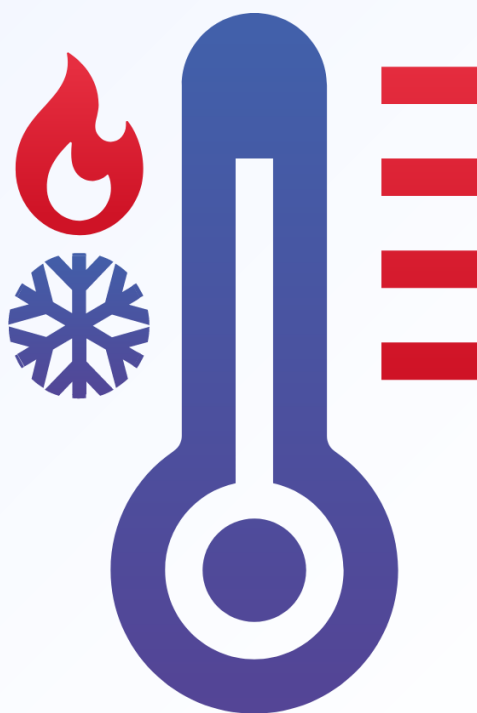
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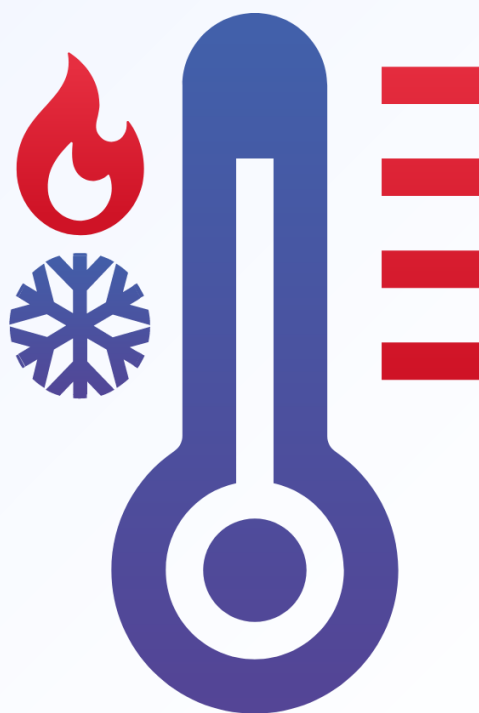
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Experimental verification of the effectiveness of CO₂ capture from flue gas after pulverised coal combustion

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Abstract

This article presents experimental verification of CO₂ capture from flue gases generated in a pulverised coal-fired power boiler. The tests were carried out using a pilot-scale moving bed temperature swing adsorption system installed in an operating coal-fired power plant (Skawina Power Plant). Activated carbon in the form of beads was used as the adsorbent due to its developed microporous structure, large specific surface area and excellent mechanical resistance under conditions of continuous circulation. The installation enables treatment of a flue gas stream with a volumetric flow rate of up to 50 m³/h. The supplied flue gas, composed mainly of N₂ and CO₂, was extracted downstream of the plant's gas-cleaning system. Two recent measurement campaigns were analysed under variable inlet CO₂ concentrations (4.81–7.16 vol.%; it depends on the operation of the power plant, so it cannot be controlled by researchers), different flue gas flow rates and varying sorbent mass flow rates. Under quasi-steady-state operating conditions, a capture efficiency of 50–60% was achieved at an inlet CO₂ concentration of 6–7% by volume, with an average energy consumption of 55.3 MJ/kgCO₂. A second series of measurements confirmed the high sensitivity of capture efficiency to the ratio of sorbent mass flow rate to flue gas flow rate and to regeneration conditions. The results demonstrate the technical feasibility of moving bed temperature swing adsorption operation under actual coal-fired power plant conditions; however, significant optimisation of heat integration, process configuration and sorbent management is necessary to reduce energy requirements to a level compatible with industrial implementation.

Keywords: Experimental tests; CO₂ capture; Research facility; Moving bed temperature swing adsorption

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1. Introduction

Anthropogenic carbon dioxide (CO₂) emissions remain a primary driver of climate change and continue to pose a direct challenge for the power sector and other energy intensive industries. Even under accelerated deployment of renewable energy and electrification, large stationary point sources such as coal and gas fired power plants, cement kilns, steelworks and refineries are expected to contribute substantially to residual emissions over the coming decades. For this reason, carbon capture, utili-

sation and storage (CCUS) is widely recognised as a key bridging technology to enable deep decarbonisation while maintaining security of supply and industrial competitiveness. A comprehensive review by Markewitz et al. [1] emphasised that CCUS deployment must be assessed not only in terms of capture efficiency, but also in terms of system integration, energy penalty and the implications for the entire energy chain.

From an engineering and policy perspective, the major barrier to large-scale CCUS implementation has consistently been the cost and energy demand of capture, particularly for post-

Nomenclature

CO_2 – CO_2 concentration in the flue gas, %

m_{CC} – mass of CO_2 captured from the flue gas, kgCO_2

Q – thermal energy supplied via steam for sorbent regeneration, MJ

Greek symbols

ζ – energy consumption of CO_2 capture, MJ/kgCO_2

η – CO_2 capture efficiency, %

Subscripts and Superscripts

in – before contact with adsorbent

out – after contact with the adsorbent

Abbreviations and Acronyms

BAC – ball-shaped activated carbon

CCS – carbon capture and storage

CCUS – carbon capture, utilisation and storage

CO_2 – carbon dioxide

IGCC – integrated gasification combined cycle

MBTSA – moving bed temperature swing adsorption

MEA – monoethanolamine

MOF – metal-organic frameworks

PSA – pressure swing adsorption

SCADA – supervisory control and data acquisition

TSA – temperature swing adsorption

VSA – vacuum swing adsorption

VTSA – vacuum temperature swing adsorption

combustion flue gas streams where CO_2 is diluted in nitrogen, and where the treated gas flow rates are large. Rubin et al. [2] showed that capture dominates the levelised cost of carbon capture and storage (CCS) for power generation and that cost reductions depend strongly on both process intensification and integration opportunities at the plant level. For pulverised coal units, retrofitting post-combustion capture remains an attractive route because it can, in principle, be implemented without radical modifications to the boilers and turbines system.

Aqueous amine absorption (most commonly monoethanolamine (MEA) based) remains the benchmark technology for post-combustion CO_2 capture due to its technological maturity and demonstrated performance. Rochelle's [3] widely cited review pointed out that amine scrubbing is robust and industrially proven, but also highlighted the fundamental thermodynamic minimum work and the practical need to reduce energy consumption through solvent and process improvements. In practice, the regeneration duty and the associated steam extraction reduce power output and may complicate part-load operation. These limitations motivate continuous research into alternative capture concepts that could reduce the energy penalty, mitigate solvent degradation, and simplify operation and maintenance.

At the industrial scale, the Shell CANSOLV process provides an important reference point for post-combustion CO_2 capture based on regenerable amine absorption. The first commercial applications of this technology demonstrated high CO_2 capture performance and high product purity [4].

Post-combustion capture options include chemical absorption, physical adsorption, membrane separation, and hybrid configurations combining several mechanisms. In a broad review of post-combustion carbon capture, Chao et al. [5] underlined that no single technology is universally optimal; instead, performance depends on flue gas composition, required CO_2 purity and recovery, integration constraints, and site-specific economics. Within this landscape, adsorption-based approaches have attracted increasing attention because regeneration can be driven by temperature and/or pressure changes, and because solid sorbents can potentially avoid some drawbacks of liquid solvents (corrosion, solvent losses, and oxidative/thermal degradation).

The viability of adsorption-based CO_2 capture depends on two coupled elements: the intrinsic properties of the sorbent (capacity, selectivity, kinetics, stability) and the process design

(contactor configuration and regeneration strategy). Choi et al. [6] provided an early, influential review of sorbent classes for large point sources, discussing activated carbons, zeolites, alkaline oxides, hydrotalcites, amine-functionalised solids and metal-organic frameworks (MOFs), and highlighted that process design constraints often dominate material selection when moving from laboratory to industrial scales. In a later review focused specifically on post-combustion capture using solid sorbents, Samanta et al. [7] stressed the importance of cyclic stability, tolerance to moisture and impurities, and the practical challenges of scaling fixed-bed systems, including pressure drop and heat management. More recently, Raganati et al. [8] summarised the state of adsorption-based post-combustion capture, emphasising that regeneration strategy and heat integration determine much of the overall energy penalty and capture cost.

The development of advanced sorbents remains an active area of research, especially for low CO_2 partial pressures where high selectivity is required. D'Alessandro et al. [9] discussed the prospects and limitations of new porous materials for CO_2 separation, noting that improved materials must be evaluated under realistic operating conditions, including temperature, humidity and exposure to contaminants. In a recent review, Kong et al. [10] compared carbon sorbents, zeolites and MOFs for post-combustion capture, highlighting recurring trade-offs: zeolites and some MOFs exhibit high efficiency and selectivity but can be sensitive to water, while carbonaceous materials are generally more resistant to moisture and contamination but may exhibit lower selectivity and operating efficiency under flue gas conditions.

From a process standpoint, adsorption cycles can be broadly classified as pressure swing adsorption (PSA), vacuum swing adsorption (VSA), temperature swing adsorption (TSA), and combined VTSA cycles. TSA is particularly relevant for near-atmospheric flue gas because it avoids deep vacuum operation and can allow us to use waste heat; yet, TSA can suffer from slow heating and cooling rates, large thermal inertia and high heat demands. Clausse et al. [11] performed a numerical parametric study on indirect TSA for post-combustion capture, and demonstrated that cycle performance is strongly affected by the regeneration temperature, purge conditions and heat transfer characteristics, with energy consumption being a central constraint for industrial relevance. Earlier, Bonjour et al. [12] investigated a TSA process with indirect heating and cooling, and dis-

cussed how internal heat exchanger design and scaling influence specific heat consumption, providing a process-intensification perspective directly relevant to pilot- and industrial-scale systems.

Thermodynamic analyses of integrated gasification combined cycle (IGCC) systems with pre-combustion, post-combustion and oxy-fuel configurations make a significant contribution to the assessment of the efficiency of coal-fired power plants integrated with CCS systems. Kindra's et al. [13] work provides a comparative analysis of these variants using simulation models, showing that the highest net efficiency (47.2%) can be achieved in an IGCC system with pre-combustion capture, assuming effective thermal integration of the synthesis gas preparation section, while the oxy-fuel configuration minimises CO₂ emissions (approx. 9 g/kWh), but at the cost of a significant drop in efficiency to 36.3%. These results clearly indicate that the competitiveness of CCS technology is primarily determined by the energy balance of the system and the extent of heat recovery. However, it should be emphasised that the presented analyses are model-based and refer to fixed operating conditions, which justifies the need for experimental research conducted under real conditions, allowing for the verification of the assumptions made and the assessment of operational limitations, in particular with regard to adsorption processes.

Beyond the choice of the cycle, it is increasingly recognised that meaningful comparisons between the capture technologies require consistent performance metrics: capture rate, product purity, productivity and specific energy consumption (thermal and/or electrical), ideally evaluated under comparable boundary conditions. Zhao et al. [14] proposed performance analysis frameworks for TSA systems and emphasised the role of thermodynamic benchmarking when interpreting reported energy demands. In a comprehensive evaluation of TSA for post-combustion capture, Zhao et al. [15] further highlighted that the apparent energy penalty depends not only on the heat of adsorption but also on the heat capacity of the sorbent and the overall heat management of the contactor and auxiliaries. This is particularly important for systems aiming at continuous or quasi-continuous operation, where thermal integration is an explicit design objective.

Moving bed temperature swing adsorption (MBTSA) has been proposed as an approach to overcome some intrinsic limitations of fixed-bed TSA. The key concept is continuous circulation of the adsorbent through spatially separated sections for adsorption, heating (desorption) and cooling, which can reduce pressure drop in the adsorption zone and potentially improve heat transfer by decoupling the thermal front from the gas-phase pressure constraints. Mondino et al. [16] developed and analysed MBTSA configurations for post-combustion capture in a coal-fired power plant context and showed that process modifications, particularly gas recirculation from the desorption section, can substantially improve CO₂ purity and capture rate compared to simple baseline configurations. In a later study, Mondino et al. [17] investigated MBTSA as an alternative to absorption for natural gas combined cycle plants, where the CO₂ partial pressure is typically low, demonstrating that MBTSA can be competitive when integrated at the plant level and when the cap-

ture process auxiliaries are fully accounted for. More recently, MBTSA was assessed for a waste-to-energy application, with a clear focus on internal heat recovery and experimentally informed heat-transfer correlations, reinforcing that heat transfer and thermal integration remain decisive factors for the practical feasibility of the concept [18].

A concept utilising a fixed sorbent in continuous mode for CO₂ capture has also been developed by Kawasaki; in this system, the process relies on a porous, amine-impregnated material regenerated with low-temperature steam, which differs significantly from the spherical activated carbon used in the current MBTSA plant. In early moving-bed tests, CO₂ was captured from coal-combustion exhaust gas using steam at approximately 60°C, with a reported CO₂ product concentration of at least 95%, whereas later parametric tests were performed on a 5 t_{CO₂}/day moving-bed unit [19,20]. These studies indicate that the moving-bed configuration itself can be technically feasible, but they also show that process performance is strongly dependent on sorbent chemistry, regeneration strategy and heat integration.

While modelling studies provide critical insights into process design space, experimental evidence under real flue gas conditions remains essential. Flue gas contains water vapour and trace contaminants, exhibits temporal variability and imposes operational constraints that are often simplified or ignored in simulations. Pilot scale adsorption capture using multi-layer VSA/VPSA or MBTSA configuration (described in the next chapters) has been demonstrated at coal-fired power plants, providing valuable benchmarks for actual performance. Liu et al. [21] reported on onsite CO₂ capture from coal-fired flue gas using an adsorption process after dehydration, demonstrating high CO₂ purity and recovery in a multi-bed cycle and reporting experimentally measured energy consumption under plant integrated conditions. In a related short communication, Wang et al. [22] experimentally evaluated adsorption capture in an existing coal-fired power plant and showed that dehumidification and real-gas operation are central to stable performance and energy consumption. These studies underscore the importance of plant-level constraints and provide context for interpreting energy metrics obtained in pilot systems.

For adsorption-based capture from flue gas, moisture is a dominant factor affecting cycle performance. Activated carbons are often considered advantageous in wet conditions due to their relatively hydrophobic character compared to many zeolites. Xu et al. [23] investigated the effects of water vapour on CO₂ capture using activated carbon in VSA operation and demonstrated that, even when CO₂/N₂ separation remains feasible, moisture can increase energy consumption and complicate cycle optimisation. Consequently, pilot-scale evaluation under real flue gas conditions should clearly consider humidity management, thermal inertia and the stability of the sorbent under repeated cycling.

Another central but frequently underestimated contributor to energy penalty in TSA/MBTSA systems is the sensible heat required to raise and lower the temperature of the circulating solid phase. Even when the heat of adsorption is moderate, a high sorbent heat capacity or excessive thermal mass in the contactor can lead to large regeneration duties. Querejeta et al. [24] sys-

tematically measured heat capacities of activated carbons in the temperature range relevant for TSA-based CO₂ capture and concluded that selecting and/or modifying carbon materials to reduce heat capacity can be an effective strategy to lower regeneration energy demand. This insight is directly relevant for moving bed concepts, where continuous sorbent circulation amplifies the impact of solid-phase thermal properties.

In addition to the international studies discussed above, Polish pilot and demonstration projects provide important quantitative benchmarks for assessing post-combustion CO₂ capture processes under real-world flue gas conditions. Pilot tests based on the amine process, carried out at the Łaziska Power Station, enabled the removal of over 19 000 kg of CO₂ over approximately 550 hours of operation and demonstrated the practical significance of mobile pilot installations integrated with coal-fired units [25]. Earlier Polish studies on amine-based post-combustion CO₂ capture reported a pilot plant capacity of up to 200 Nm³/h of flue gas [26].

These studies are complemented by national energy and economic analyses of CO₂ capture installations using monoethanolamine and ammonia, which indicate that the choice of absorbent significantly affects the heat demand for regeneration and the cost of the process [27]. In the case of adsorption-based capture methods, Polish research into DR-VPSCA was carried out using actual flue gases, a four-column adsorption system and a complete gas pre-treatment line, comprising dust removal, cooling, desulphurisation, denitrification, drying and compression [28]. The long-term performance of activated carbon in a pilot DR-VPSCA plant was also analysed over 808 hours under real flue gas conditions [29]. These results are relevant to the present MBTSA study, as they show that capture efficiency must be interpreted in conjunction with gas conditioning, sorbent durability, regeneration stability, pressure drops, heat recovery and auxiliary energy requirements.

In the next chapters, this paper presents an experimental performance of CO₂ capture using a pilot MBTSA system installed in an operating pulverised coal-fired power plant (Skawina Power Plant, Poland). The installation processes actual flue gas taken from the end of the existing purification installation and uses ball-shaped activated carbon (BAC) as a mechanically robust sorbent suitable for continuous circulation. The scope of the experiment focuses on determining the capture efficiency at different CO₂ inlet concentrations and flow conditions, and calculating the energy consumption associated with thermal regeneration using a heat source integrated with the power plant and operating data recorded by the SCADA system. This experimental analysis complements previous work comparing MBTSA simulations with experimental observations for the same installation [30].

2. Description of the research facility

As part of the conducted research, a specially designed pilot installation for CO₂ capture from flue gas was employed. The system enables treatment of a flue gas stream with a volumetric flow rate of up to 50 m³/h. The process was implemented using the MBTSA technology. The installation was connected to the flue gas duct of a pulverised coal-fired boiler downstream of the

existing gas-cleaning units. As a result, the supplied gas stream consisted predominantly of N₂ and CO₂. A specially prepared ball-shaped activated carbon (BAC), manufactured by Kureha [31], was used as the adsorbent. The sorbent circulates continuously between three main process sections: the adsorption section, where CO₂ and partially N₂ are captured; the regeneration section, in which the sorbent undergoes thermal desorption; and the cooling section. Material transport between the columns is carried out by insulated belt conveyors, which receive the sorbent after its passage through each process section.

2.1. Description and purpose of individual sections of the facility

The pilot installation consists of three main process sections enabling continuous CO₂ capture from flue gas using the temperature swing adsorption concept. Each section is equipped with a rotary valve that ensures controlled sorbent flow by allowing gradual discharge at the bottom of the column. The combination of bed level sensors and frequency inverters enables automatic adjustment of the rotational speed of the rotary valves. Since the belt conveyors operate at constant speed, regulation of sorbent dosing via the rotary valves makes it possible to vary the sorbent mass flow rate while maintaining predefined filling levels in each process section.

The three principal sections of the installation (Fig. 1) are described below in the order corresponding to sorbent circulation, excluding mechanical transport elements:

1. Adsorption section (reactor).

The adsorption section consists of a sealed, cylindrical column with plastic walls and filled with Mellapak™ structured packing. Within the column, upward-flowing flue gas comes into contact with adsorbent particles descending under gravity, resulting in adsorption of CO₂ onto the sorbent surface. The flue gas is introduced at the bottom of the column and, under slight overpressure, exits at the top. The sorbent is supplied from the top and flows downward to the bottom of the column. The application of Mellapak™ structured packing ensures uniform distribution of both gas and solid phases and extends gas–solid contact time.

2. Regeneration/heating section.

The regeneration section consists of a thermally insulated column in which the sorbent undergoes thermal desorption at a temperature of approximately 180°C, required for effective regeneration of the activated carbon. Three shell and tube heat exchangers are installed inside the column. The upper heat exchanger forms part of a closed internal heat recovery loop. It transfers energy between the regeneration and cooling sections, enabling preheating of the incoming sorbent while simultaneously recovering heat from the cooling column. The remaining two heat exchangers operate in series. Saturated steam extracted from the turbine, with parameters of approximately 190°C and 1.3 MPa, is supplied to the lower exchanger. The steam condenses while transferring heat to the sorbent. The resulting condensate is then directed to the middle exchanger, where its temperature decreases to approximately 121°C. The desorbed gas stream, consisting primarily of CO₂ and

N₂, is removed through an outlet equipped with a fan generating slight underpressure, ensuring stable evacuation of the desorption gases from the system.

3. Cooling section.

The final section of the installation is a thermally insulated column designed to reduce the sorbent temperature to restore its adsorption capacity prior to re-entry into the adsorption section. Two shell and tube heat exchangers are installed within this column. The upper exchanger is integrated into the previously described internal heat recovery loop, while the lower exchanger provides direct cooling of the sorbent. Cooling water at approximately 15°C flows through the lower heat exchanger, enabling reduction of the sorbent temperature to below 25°C before it returns to the adsorption section.

Figure 1 presents the pilot installation viewed from the conveyor connecting the cooling section, visible as the insulated column on the left, with the adsorption section, visible as the column on the right. The regeneration section is located on the opposite side of the facility and is only partly visible in the photograph. One belt conveyor transfers the activated carbon from the adsorption section to the regeneration section, while another conveyor transports the regenerated sorbent from the regeneration section to the cooling section. The silver-coloured cladding visible on the regeneration section, the cooling section and the conveyor between these sections protects and stabilises the mineral-wool insulation, ensuring continuity and high effectiveness of thermal insulation. Due to the presence of supporting structures and other power plant equipment, capturing a complete view of the installation in a single photograph is not feasible.

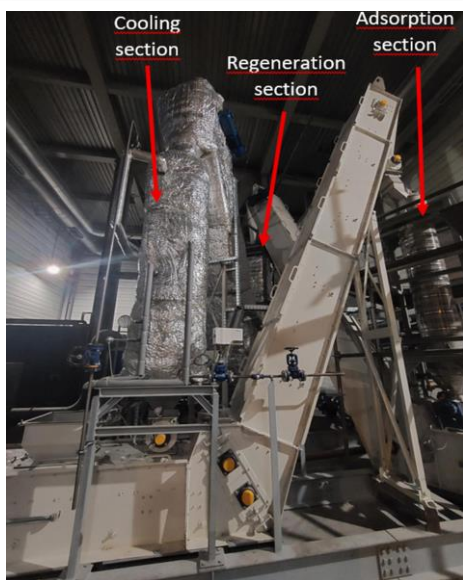


Fig. 1. Illustrative photograph of research facility.

A simplified process scheme of the installation is shown in Fig. 2. The scheme illustrates the main circuits present in the CO₂ capture system, including the flue gas path, sorbent circulation loop, heating medium circuit, cooling-water circuit and CO₂-rich gas outlet. The sections connecting the bottom of one column with the top of the next column represent belt conveyors

used for transporting the activated carbon between the process sections.

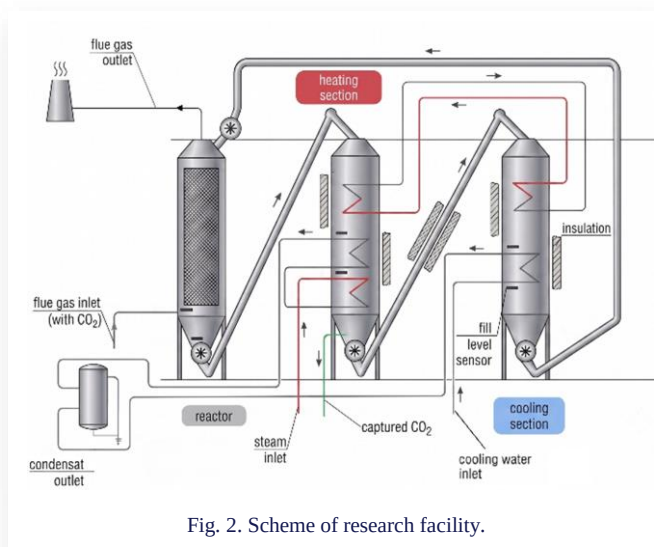


Fig. 2. Scheme of research facility.

The flue gas is introduced into the lower part of the adsorption reactor and is withdrawn from the upper part of the reactor under a slight negative pressure. In the upper parts of the regeneration and cooling sections, heat exchangers filled with circulating water form an internal heat-recovery loop between these two sections. In the middle and lower parts of the regeneration section, additional heat exchangers are installed. Saturated steam extracted from the turbine is supplied to the inlet of the lower heat exchanger, where it condenses while transferring heat to the activated carbon. The condensate then flows through the middle heat exchanger and is subsequently discharged to the condensate tank, where it is cooled before being released to the drainage system. The CO₂-rich gas released from the activated carbon during thermal desorption is extracted by a small fan and discharged from the installation. In the lower part of the cooling section, a heat exchanger supplied with cold water is used to reduce the temperature of the regenerated sorbent before it returns to the adsorption section. After leaving the cooling section, the water is directed to the condensate tank, where it assists in reducing the condensate temperature before discharge to the drainage system.

2.2. Structural layout of the research facility

The pilot installation was designed in a triangular configuration, in which the three main process sections (adsorption, regeneration and cooling) are located at the vertices of a triangle, while the sorbent transport system forms its sides. This arrangement enabled efficient utilisation of the available space and ensured compliance with structural constraints associated with installing the system inside an existing building. The adopted layout also provides protection of the installation components against adverse environmental conditions and facilitates operation and maintenance. A vertically stacked configuration was initially considered, as it would have theoretically reduced the number of required belt conveyors. However, this option exceeded the available floor height and permissible structural load limits, and was therefore rejected at the design stage.

2.3. Characteristics of flue gases feeding the system

The flue gas supplied to the adsorption section originated from a power boiler operating under real plant conditions. Consequently, its chemical composition exhibited continuous variability, resulting in significant fluctuations in the carbon dioxide (CO₂) concentration within the analysed stream. The recorded volumetric CO₂ concentrations ranged from 4.81% to 7.16%, which is considerably lower than the values assumed at the design stage of the pilot installation. The CO₂ concentration in the flue gas was measured using a Vaisala CARBOCAP® GMP251 probe based on infrared (IR) absorption technology.

2.4. Information about the adsorbent

Activated carbon was selected as the sorbent material primarily due to its favourable thermo-physical properties, in particular its relatively high heat transfer coefficient on the solid side, reported in the range of 69–117 W/(m²·K) [17]. An additional factor influencing this choice was its superior mechanical strength compared with alternative sorbents such as zeolite 13X.

Although zeolites exhibit more favourable adsorption characteristics, resulting in lower specific energy consumption in MBTSA processes, their practical applicability remains limited. Simulation studies indicate that the specific energy demand for zeolite 13X may reach approximately 2.3 MJ/kg CO₂ [16], whereas for activated carbon it is estimated at approximately 5.7 MJ/kg CO₂ [17]. However, commercially available zeolites are characterised by relatively low mechanical strength and high brittleness, leading to rapid degradation under continuous circulation conditions. This makes their application in MBTSA systems currently impractical.

The thermo-physical properties of the activated carbon have been reported in the literature [17,31]. Its principal characteristics are summarised in Table 1, while a photograph of the material in its original packaging is presented in Fig. 3.

Table 1. Selected parameters of activated carbon [32].

Shape	Specific surface area	Average particle diameter	Hardness
Spherical	1100-1300 m ² /g	≈0,7 mm	≥95%



Fig. 3. BAC-type activated carbon: a) in its original packaging; b) spilled out on a paper.

3. Analysis of experimental results

The experimental results are presented in graphical form and include both direct measurements obtained from the pilot installation and values calculated on the basis of these measurements. All data were acquired and archived using a SCADA (supervisory control and data acquisition) system, which enables continuous monitoring of process parameters based on signals from sensors installed throughout the installation.

Figures 4 and 5 present selected measurement data recorded during operation of the pilot unit, including:

- CO₂ concentration in the flue gas at the inlet and outlet of the adsorption section (Figs. 4a and 5a),
- mass flow rate of the activated carbon (Figs. 4b and 5b),
- volumetric flow rate of the supplied flue gas (Figs. 4b and 5b).

In addition, two key performance indicators were determined based on the recorded data:

- CO₂ capture efficiency (η), defined as the relative reduction in CO₂ concentration in the flue gas after contact with the sorbent. This parameter was calculated according to the following equation:

$$\eta = 1 - \frac{CO_{2,out}}{CO_{2,in}}, \quad (1)$$

where: η – CO₂ capture efficiency [%], $CO_{2,out}$ – CO₂ concentration in flue gas after contact with the adsorbent [%], $CO_{2,in}$ – CO₂ concentration in flue gas before contact with the adsorbent [%],

- energy consumption of the CO₂ capture process (ζ), defined as the ratio of the total heat supplied for sorbent regeneration to the mass of captured CO₂:

$$\zeta = \frac{Q}{m_{CC}}, \quad (2)$$

where: ζ – energy consumption of CO₂ capture [MJ/kgCO₂], Q – thermal energy supplied via steam for sorbent regeneration [MJ], m_{CC} – mass of CO₂ captured from the flue gas [kgCO₂].

Figure 4 presents the results of measurements and calculations performed on 8 March 2024. Although these are not the earliest available datasets, it should be noted that the activated carbon used during this campaign had already been subjected to significant prior operation (several dozen hours of circulation within the system). Consequently, partial sorbent degradation and a reduction in adsorption performance were expected.

The results presented correspond to an approximately 30-minute segment of measurement data selected from the analysed day. This period was chosen because it represented the longest uninterrupted operation of the installation under a constant activated carbon mass flow rate and a relatively stable flue gas volumetric flow rate (Fig. 4b). This approach limited the influence of transient operating conditions on CO₂ capture efficiency and specific energy consumption. As shown in Fig. 4a, the CO₂ concentration in the flue gas entering the adsorption column varied within the range of 6–7%, while the outlet concentration averaged approximately 3%. This corresponded to a capture effi-

ciency of 50–60% during most of the analysed interval. At the same time, a high specific energy consumption was observed (Fig. 4c), with a median value of 55.3 MJ/kgCO₂, which must

be considered unfavourable from the standpoint of energy performance.

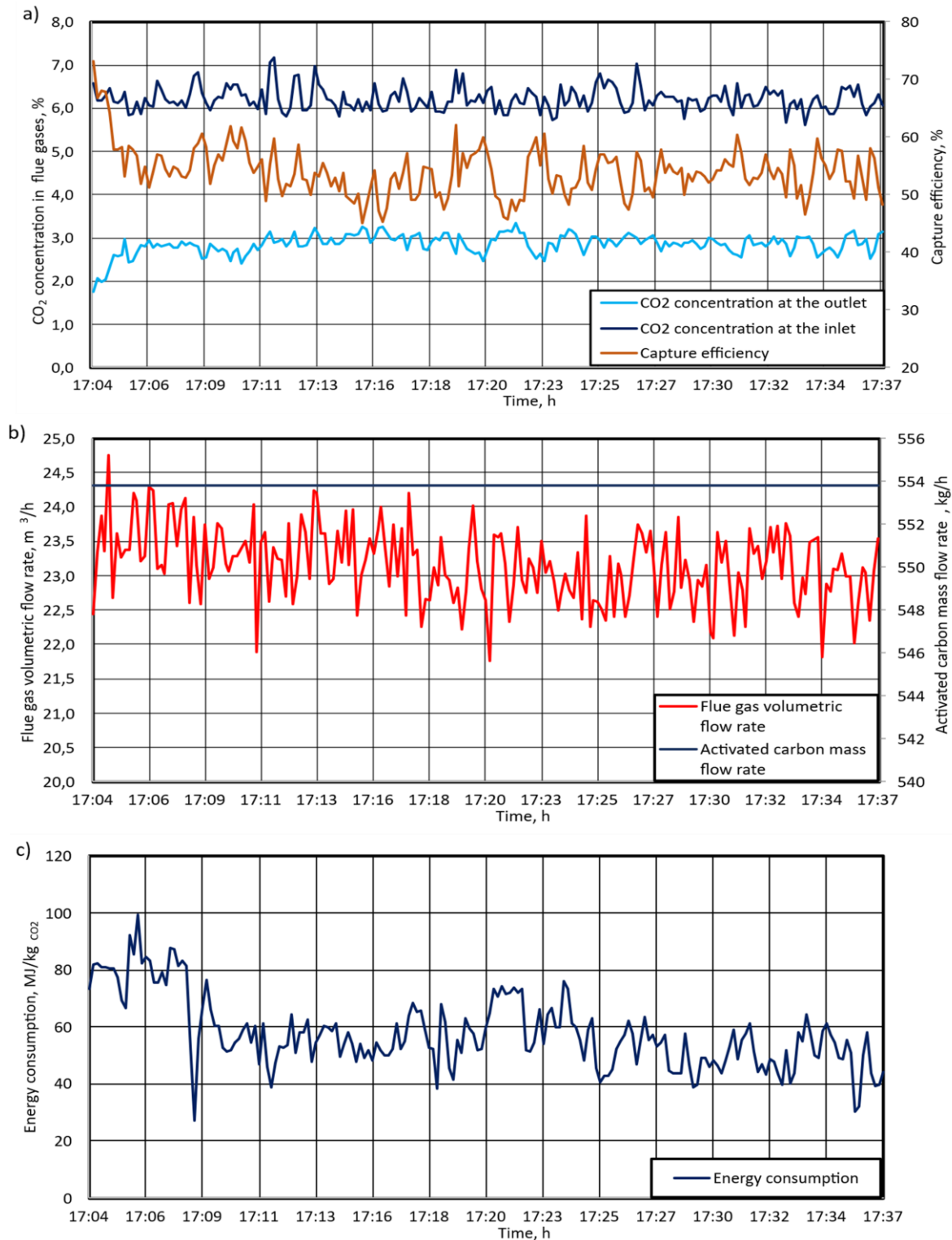


Fig. 4. Results obtained on 8 March 2024: a) CO₂ concentration in the inlet and outlet flue gas streams and corresponding capture efficiency; b) volumetric flow rate of the flue gas and mass flow rate of the activated carbon; c) specific energy consumption of the capture process.

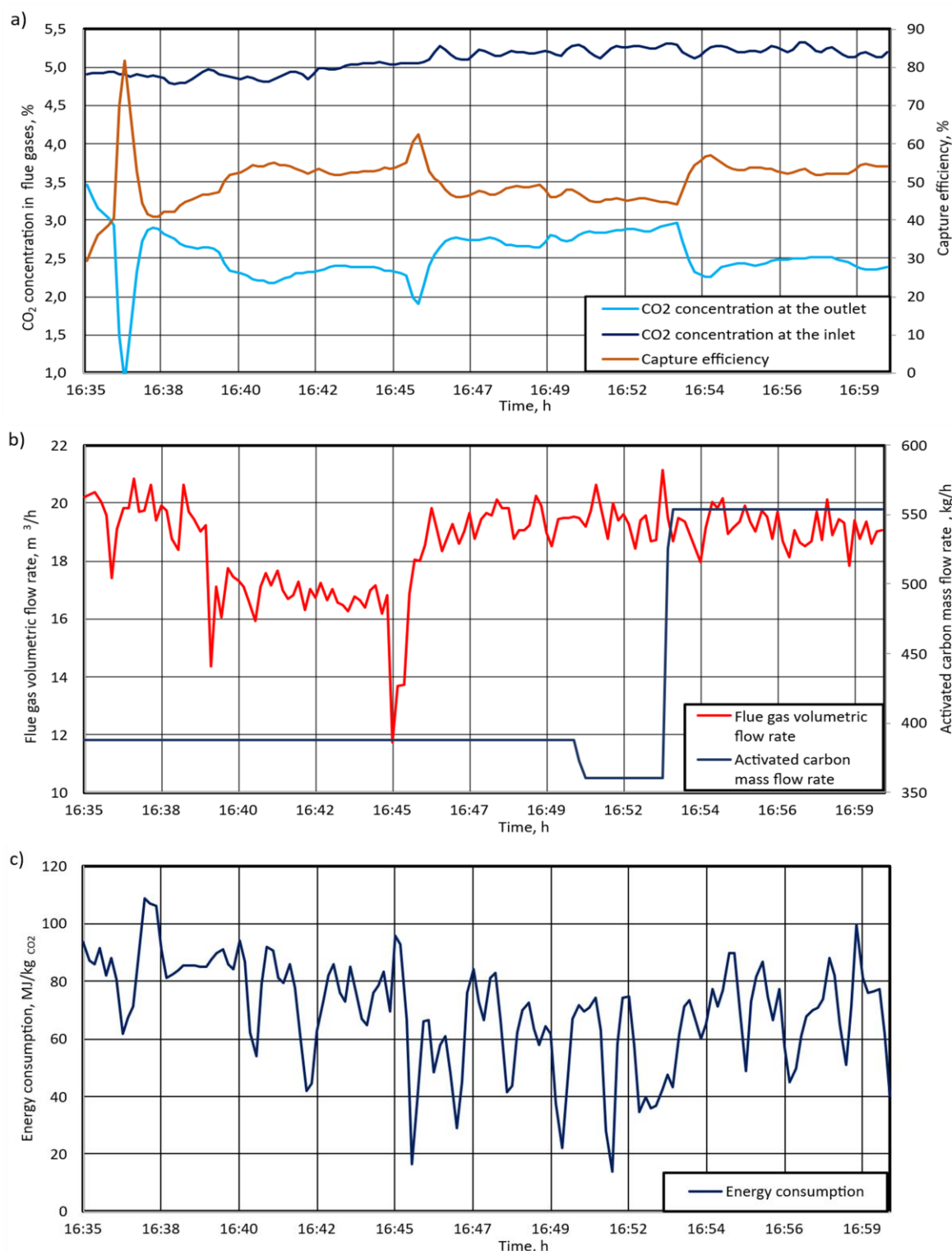
Figure 5 presents the results obtained on 17 April 2024. During the period between the two measurement campaigns, the installation remained inactive, and no fresh sorbent was intro-

duced into the circulation loop, implying further physico-chemical degradation of the activated carbon. In contrast to the relatively stable conditions shown in Fig. 5, the time interval pre-

sented in Fig. 5b exhibits noticeable variability in both the activated carbon mass flow rate and the flue gas volumetric flow rate. These variations directly affected the CO₂ concentration at the reactor outlet. A decrease in flue gas flow rate or an increase in sorbent feed resulted in improved capture efficiency, whereas the opposite changes led to a reduction in performance.

Pronounced changes in specific energy consumption were

also observed (Fig. 5c); however, its response was less immediate. The specific energy demand exhibited a delayed reaction to variations in the sorbent mass flow rate, which can be attributed to system inertia, including gradual sorbent dosing through the rotary valve and the dynamic behaviour of the internal heat recovery loop.



Significant variations in the operating parameters of the installation may substantially affect data interpretation, particu-

larly when datasets covering periods longer than two hours are analysed with a time resolution of 30 s. In contrast, results

obtained over shorter intervals with a 15 s sampling step allow for more detailed analysis of transient effects occurring after changes in key operating conditions.

4. Conclusions

The analysis of results obtained from the two measurement series clearly indicates a significant influence of the activated carbon mass flow rate on the efficiency of the CO₂ adsorption process. Although increasing the sorbent feed appears to be a straightforward method for improving capture efficiency, it may simultaneously lead to higher specific energy consumption and an increased risk of insufficient heating of the sorbent to the required regeneration temperature. Under the investigated experimental conditions, the specific energy demand reached very high values, representing a major barrier to potential industrial implementation of the technology. For this method to become attractive from both economic and environmental perspectives, a substantial reduction in regeneration energy consumption is essential. Inadequate heating of the activated carbon results not only in incomplete sorbent regeneration, leading to reduced adsorption capacity, but also in gradual cooling of the internal heat recovery circuit. This effect may further deteriorate the overall energy balance of the installation and hinder the achievement of stable operating conditions.

It should also be pointed out that the presented results were obtained using the sorbent that had already operated in the circulation loop for several hours, implying a reduction in its initial adsorption performance. Previous studies [31] have reported significantly higher CO₂ capture efficiencies, reaching approximately 76%, together with lower specific energy consumption of about 25 MJ/kgCO₂, when partial replacement of activated carbon was implemented during operation.

In light of these observations, further optimisation of the MBTSA technology appears justified, both in terms of installation design and sorbent selection. One potential development pathway is the reconstruction of the system into a vertically stacked configuration, in which the individual process sections would be arranged one above another. Such a layout could reduce heat losses associated with sorbent transport between sections and decrease the risk of leakage through flexible components (e.g. seals), which may allow ingress of cold and humid ambient air. In parallel, further investigations of alternative commercial sorbents are recommended, particularly those offering lower regeneration energy demand, higher CO₂ selectivity, and mechanical durability comparable to that of activated carbon.

The obtained results should also be considered in the broader context of Polish pilot and demonstration experience. Compared with Polish pilot tests based on amine processes, in which CO₂ removal above 90% and large cumulative amounts of captured CO₂ were reported [25], the results obtained in the present MBTSA study confirm the technical feasibility of conducting an adsorption-based CO₂ capture process using real flue gas from coal combustion. At the same time, however, they indicate much stronger limitations related to regeneration energy and solid-phase heat management. The measured capture efficiency of approximately 50–60% at an inlet CO₂ concentration of 6–7 % vol.

and the median specific energy consumption of 55.3 MJ/kgCO₂ indicate that the investigated configuration is not yet energetically competitive with mature absorption-based processes. At the same time, the previous result of approximately 76% capture efficiency and about 25 MJ/kgCO₂ after partial replacement of the activated carbon shows that this technology is highly sensitive to sorbent condition, heat recovery efficiency and operating parameters. Therefore, the practical assessment of MBTSA technology should include not only instantaneous capture efficiency, but also sorbent ageing, regeneration completeness, steam parameters, cooling demand, heat losses during sorbent transport, internal heat recovery, leaks leading to the intake of cold and humid ambient air, and long-term operational stability [30].

Against the background of the international literature, it should only be noted that mature amine absorption processes, such as Shell CANSOLV [4] and moving-bed systems developed by Kawasaki [19,20], confirm the key importance of sorbent type, regeneration strategy and thermal integration for the achieved CO₂ capture performance.

At the same time, the results obtained in the present MBTSA installation cannot be directly compared on a one-to-one basis with results reported for other pilot and demonstration CO₂ capture installations [25–30]. These installations were based on different capture methods, including primarily amine absorption and cyclic adsorption processes, and they also differed in process configuration, scale, operating pressure, regeneration strategy, heat source, gas pretreatment requirements and performance assessment indicators. Moreover, the experimental campaign described in this study revealed a number of design and construction imperfections in the investigated installation that may have adversely affected the measured capture efficiency and specific energy consumption. These include excessive heat losses during sorbent transport, possible air intake through flexible connections and sealing elements, non-uniform heating of the circulating activated carbon, incomplete regeneration, limitations of the internal heat recovery system and process sensitivity to sorbent ageing and dosing stability. Therefore, before the obtained results can be used for a reliable quantitative comparison with other demonstration studies, the identified design and construction problems should be eliminated, and the installation should be retested under repeatable operating conditions with clearly defined boundary parameters.

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Experimental study of a reversible ventilation – heat pump system with a ceramic regenerator

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Abstract

Mechanical supply-exhaust ventilation systems are increasingly required in newly constructed residential buildings, yet their higher primary energy consumption remains a key challenge, particularly given the growing demands of energy efficiency and unchanged fresh air requirements. Existing solutions based on recuperators achieve heat recovery efficiencies of 40–70%, whereas regenerative systems can exceed 90%, but their performance under real operating conditions and integration with heat pump technology has not been sufficiently investigated. This study presents an experimental analysis of a compact ventilation unit, dedicated to classrooms, operating in reversible mode with a rotary regenerator and an integrated R290 (propane) air-to-air heat pump. Measurements were conducted across an outdoor temperature range from 0.6°C to 8.6°C. The regenerator-only mode achieved heat recovery efficiency exceeding 90% throughout the entire tested temperature range. When the R290 heat pump was activated to sustain operation during periods of lower outdoor temperatures without an auxiliary heat source, the system yielded COP values between 3.70 and 4.31, substantially higher than 2.0–2.5 values typically reported for conventional air-to-air heat pumps. These results demonstrate that the proposed integration of regenerative heat recovery with a dedicated R290 heat pump constitutes a technically viable and energy-efficient solution for both heating and ventilation needs.

Keywords: Ventilation; Ceramic regenerator; Heat recovery; Heat pump; Energy efficiency

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1. Introduction

All buildings occupied by people must be equipped with a ventilation system, whose primary function is to supply fresh air and remove used one [1,2]. However, the requirements for ventilation performance vary depending on building function and occupancy characteristics [3]. Providing adequate indoor air quality (IAQ) represents one of the central challenges of contemporary building engineering. High occupant density combined with extended occupancy periods leads to rapid deterioration of indoor air quality, manifested primarily through elevated concentrations of CO₂ and particulate matter. Transitioning to mechan-

ical supply-and-exhaust ventilation resolves the IAQ problem but introduces substantial heating energy penalties, which can reach 35–40 kWh/m² annually at the building level [4]. Among available heat recovery technologies, fixed bed regenerators (FBR) based on ceramic fill represent an alternative to the widely used rotary and crossflow plate heat exchangers [5,6]. The effectiveness of heat exchangers in heat recovery systems depends on airflow rate, temperature differences, and the moisture content of the air streams [7]. Figures 1 and 2 show the actual heat recovery effectiveness for crossflow and rotary heat exchangers, respectively, for the full ranges of available products [8].

Nomenclature

- i – specific enthalpy, J/kg
 $\dot{m}$ – air mass flow rate, kg/s
 p – gauge pressure, bar
 P – electrical power, W
 q – air mass flow rate
 Q – heat, J
 $\dot{Q}$ – heat capacity, W
 RH – relative humidity, %
 t – time, s
 W – electrical energy, J

Greek symbols

- η – regenerator effectiveness,
 θ – air temperature, °C
 τ – time period, s

Subscripts and Superscripts

- 1,2,3,4 – process periods
11 – exhaust at inlet of regenerator
21 – supply at inlet of regenerator
22 – supply at outlet of regenerator
I – indoor air
II – condenser air side
III – outdoor air
IV – evaporator air side
cycle – amount for one cycle
el – electric
n – number of cycles

Abbreviations and Acronyms

- IAQ – indoor air quality
COP – coefficient of performance
FBR – fixed bed regenerators
R290 – propane refrigerant

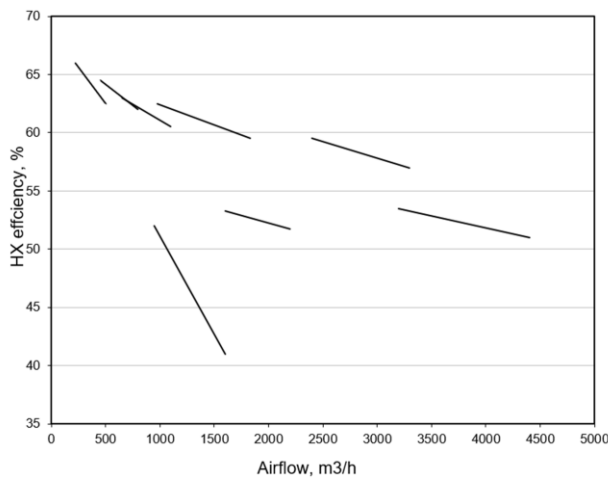


Fig. 1. Crossflow heat exchangers effectiveness (each line corresponds to one heat exchanger from the product range) [18].

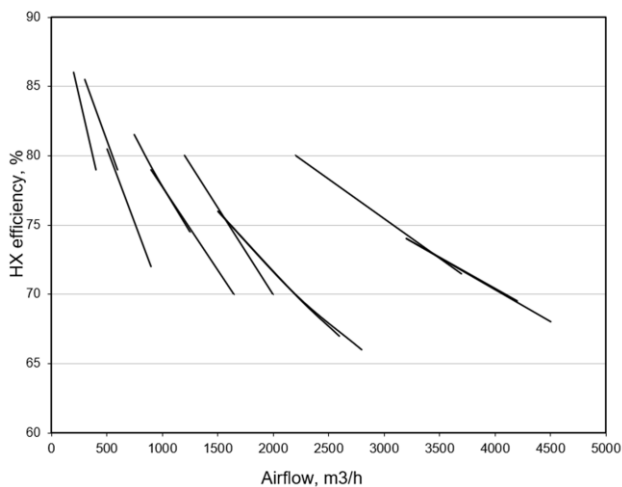


Fig. 2. Rotary heat exchangers effectiveness (each line corresponds to one heat exchanger from the product range) [18].

Each line corresponds to one heat exchanger from the product range. Results show that for rotary heat exchangers, effectiveness ranges from 40% to 66%. For each heat exchanger in the product range, effectiveness decreases when airflow increases (thus airflow speed). Similar results are obtained for rotary heat exchangers, but with higher effectiveness levels ranging from 65% to 86%.

Ceramic regenerators are characterised by a significantly greater heat transfer surface area per unit volume compared with recuperators, combined with the ability to recover both sensible and latent heat through adsorption and desorption of moisture on the porous ceramic surface [9]. The reversible operation of such devices, in which a single duct alternately serves as supply and exhaust, driven by a reversing fan, allows deployment without the need for separate supply and exhaust duct networks, making them particularly well-suited for retrofit applications in existing buildings. Commercial implementations of ceramic FBR ventilation units are currently offered by many manufacturers; however, systematic experimental characterisation of their performance under controlled conditions remains limited in the open literature.

A further improvement in energy efficiency can be achieved by integrating the ceramic regenerator with a heat pump. Exhaust air heat pumps (EAHP), which utilise the warm exhaust air stream as the heat source for the refrigeration circuit, represent an emerging class of ventilation and heating systems capable of simultaneously fulfilling the ventilation and space heating functions of a building [10]. The use of exhaust air at approximately 20°C as the heat source, rather than cold outdoor air, results in significantly more favourable evaporator operating conditions compared to conventional air-source heat pumps (ASHP), which translates directly into higher coefficients of performance [11]. The refrigerant R290 (propane) is an increasingly preferred working fluid for such applications due to its favourable thermodynamic properties and near-zero global warming potential (GWP) [12].

Despite the growing commercial interest in integrated ventilation – heat pump systems, published experimental data on systems combining a ceramic FBR regenerator with a heat pump

circuit are scarce. Niemierska and Jadwiszczak [5] investigated a ceramic FBR regenerator in a decentralised reversible ventilation system, reporting temperature effectiveness values above 80%, but without integration with a heat pump. Aktershev et al. [6] conducted a parametric numerical study of regenerative heat exchangers with periodic flow reversal. To the authors' knowledge, no experimental study has so far reported the combined performance of a ceramic fixed-bed regenerator integrated with an R290 air-to-air heat pump. The present paper addresses this gap. A laboratory test stand was constructed at the Institute of Heat Engineering, Warsaw University of Technology. The system was tested in both regenerator-only mode and heat pump mode. The primary objectives of this study were to develop a method for determining the temperature and total effectiveness of the ceramic FBR regenerator that accounts for the cyclic nature of its operation, and to experimentally characterise the heating capacity and coefficient of performance of the integrated system under varying outdoor conditions.

2. Materials and methods

Figure 3 shows the experimental laboratory setup and the locations of the temperature and relative humidity sensors. The purpose of the stand was to test the heat regeneration effectiveness of ceramic blocks and the operation of a heat pump with an incorporated regenerator. Propane (R290) was used as the working refrigerant. The total charge of propane was equal to 800 g. Although R290 is classified as a class A3 flammable refrigerant according to the PN-EN 378 standard, this charge quantity is permissible for rooms that meet the minimum space requirements or are equipped with a leak detection system that activates additional ventilation [13].

The stand is composed of:

- ventilation duct, inside of which is a reversible fan (which

allows for the airflow to alternate between directions using a specialised guide vane);

- heat regenerator in a form of ceramic blocks;
- two heat exchangers associated with a heat pump;
- compressor with an inverter;
- 4-way valve;
- an expansion valve;
- refrigerant receiver;
- air filters at the inlet and outlet of the duct.

The test stand is equipped with an automation system that enables changing airflow direction, airflow rate, compressor rotational speed, heat exchangers operating mode (evaporator/condenser) and storing the refrigerant in a receiver.

Inside the device, there are two rows of ceramic blocks, 30 in total. The top row consists of 15 blocks with dimensions of $150 \times 150 \times 300$ mm, whereas the bottom row has 15 shorter blocks measuring $150 \times 150 \times 72$ mm. The masses are equal to 6.15 kg and 1.39 kg, respectively. The blocks are supported by thin-walled metal tubes. The total mass of the regenerator amounts to 113.1 kg, whereas the theoretical thermal capacity amounts to 99 528 J/K. The open flow area through each ceramic block is equal to approximately 0.01778 m^2 . The physical layout of the ceramic blocks is presented in Fig. 4, while a photograph of the ceramic regenerator is shown in Fig. 5.

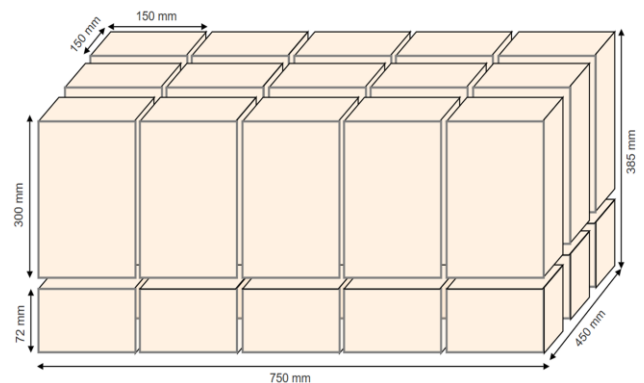


Fig. 4. Physical layout of ceramic blocks.



Fig. 5. Ceramic regenerator.

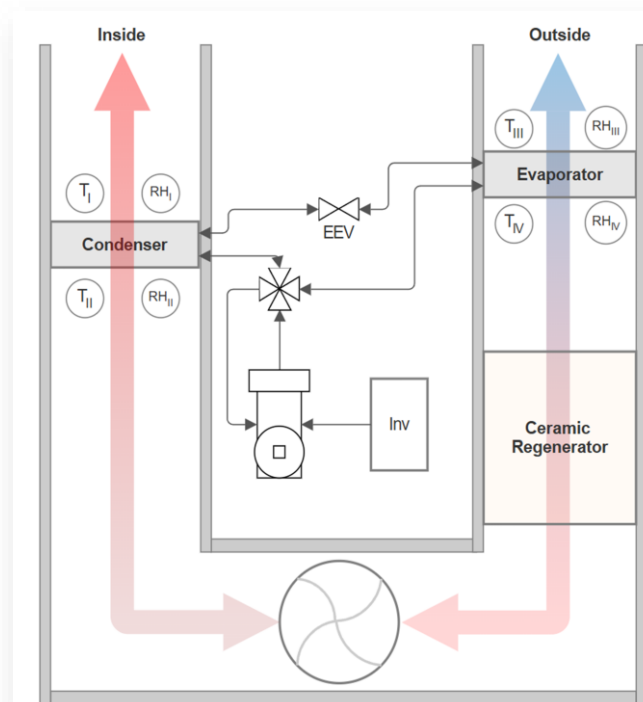


Fig. 3. Schematic diagram of the laboratory setup.

The device operates in two alternating modes. In an air supply phase, outdoor air flows through the ceramic regenerator, previously heated by the exhaust air removed from the room. As a result, the air supplied to the indoor space is preheated. During the air exhaust phase, indoor air is discharged to the outside while flowing through the said regenerator, which results in charging the heat storage. Additionally, incorporating a four-way valve allows the heat pump to operate in cooling mode during summer months.

NTC thermistors of accuracy class B were used for measuring temperature. In accordance with IEC 60539-1, the measurement tolerance of these sensors does not exceed $\pm 0.425^\circ\text{C}$ over the temperature range investigated in the device (from -10°C to 25°C) [14]. The relative humidity measurement accuracy is $\pm 3\%$ over the range of 20–80%, increasing linearly to $\pm 4.5\%$ for values outside this interval. An Iris-type damper (IRIS 250) was used to measure the airflow rate. The pressure difference was defined by two HK Instruments DPT250-R8 differential pressure transmitters, for which the measurement error is $\pm 1\%$, corresponding to a maximum of ± 8 Pa. The device also incorporated two Carel SPKT0041-DO pressure transducers on the refrigerant side, positioned at the discharge and suction ports of the compressor, corresponding to the condensation and evaporation pressures, respectively. Knowledge of the phase-transition pressures allows unambiguous determination of the condensation and evaporation temperatures. For this type of transducer, the measurement error does not exceed $\pm 1\%$, which translates to a deviation of ± 0.182 bar. To quantify the energy consumption of the control system, a Lumel N27P energy meter was employed, offering a measurement accuracy of 0.2% of the measuring range for active power, corresponding to approximately 2–3 W. The measurement instruments used in the experimental setup and their associated uncertainties are presented in Table 1.

Table 1. Measurement instruments and associated uncertainties used in the experimental setup.

Quantity	Instrument	Limit error	Standard uncertainty
Temperature	NTC class B	$\pm 0.425^\circ\text{C}$	0.245°C
Relative humidity	SHT20	$\pm 3\% \text{ RH}$ (20–80%), $\pm 4.5\%$ outside this range	1.732% RH
Airflow rate	IRIS250	$\pm 5\%$	2.887%
Air differential pressure	DPT250-R8	$\pm 1\%$ of range	± 8 Pa
Refrigerant pressure	Carel SPKT0041-DO	$\pm 1\%$	± 0.182 bar
Electrical power	Lumel N27P	$\pm 0.2\%$ of range	1.732 W

2.1. Ceramic regenerator effectiveness

In the first step, regenerator's effectiveness was tested for different temperature levels. Research was conducted in accordance with the norm: PN-EN 13141-8:2014-09 „Ventilation of buildings — Testing of performance of components/products for residential ventilation — Part 8: Testing of performance of ductless mechanical supply and exhaust units (including heat recovery) for mechanical ventilation systems” [10]. The device works periodically (supply/exhaust modes), and its cycle stages are shown in Fig. 6, where: τ_1 – time of air flow from the building to the outside, [s]; τ_2 – dwell time, [s]; τ_3 – time of air flow from the outside into the building, [s]; τ_4 – dwell time, [s]; $\dot{m}_{22}$ – mass flow rate of supply air at the outlet, [kg/s]; $\dot{m}_{11}$ – mass flow rate of exhaust air at the inlet, [kg/s].

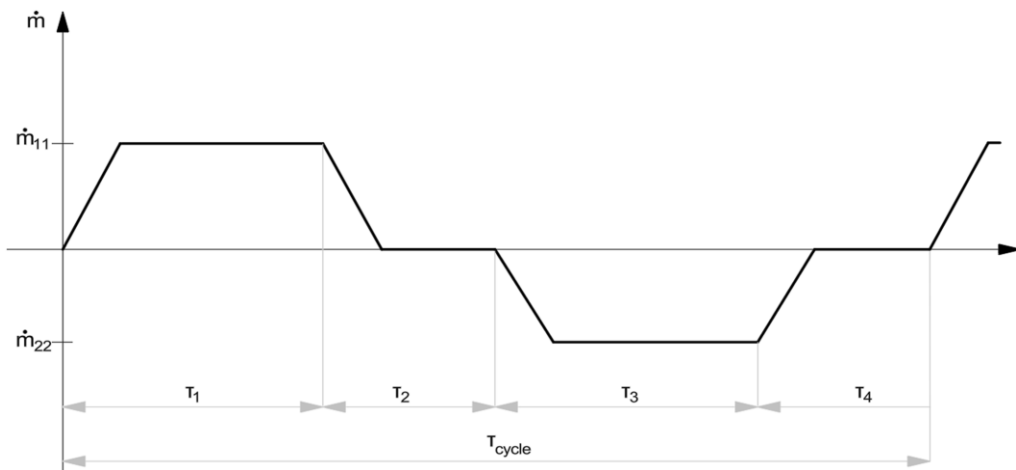


Fig. 6. Changes in the air mass flow rate as a function of time during one full cycle [19].

Times τ_1 and τ_3 correspond to the half time required for the fan guide vane to turn. The measured dwell time is equal to 4 s, therefore: $\tau_2 = 2.0$ s and $\tau_4 = 2.0$ s. This is the result of the device's design. The fan runs continuously, while the direction of airflow is changed by a guide vane; the fan is not turned off.

Therefore, the time to complete one full cycle is expressed as:

$$\tau_{cycle} = \tau_1 + \tau_2 + \tau_3 + \tau_4, [\text{s}]. \quad (1)$$

For this type of device, regenerator's effectiveness is defined by [15]:

$$\eta_\theta = \frac{\int (\theta_{22} - \theta_{21}) dt}{\int (\theta_{11} - \theta_{21}) dt} \cdot \frac{\dot{m}_{22}}{\dot{m}_{11}}, \quad (2)$$

where: θ_{22} – supply air temperature at the outlet of the heat recovery component, [°C]; θ_{21} – supply air temperature at the inlet of the heat recovery component, [°C]; θ_{11} – exhaust air temperature at the inlet of the heat recovery component, [°C].

Figure 7 presents regenerator's effectiveness results obtained during laboratory tests in accordance with Eq. (2) for the outside temperatures: -15°C , -7°C , $+2^{\circ}\text{C}$, $+7^{\circ}\text{C}$, and inside temperature equal to $+20^{\circ}\text{C}$.

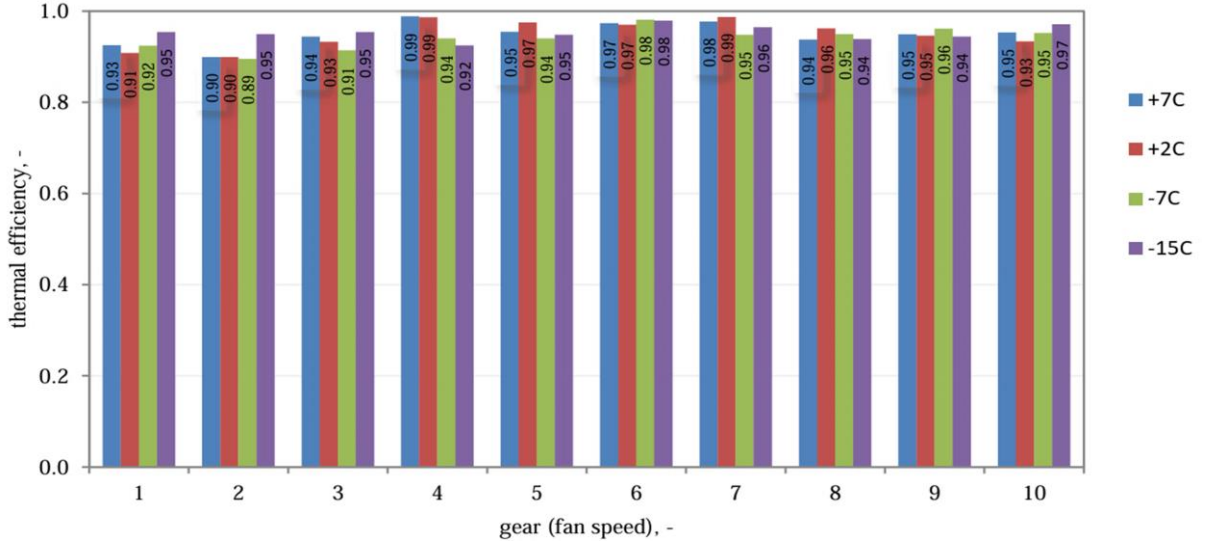


Fig. 7. Obtained ceramic regenerator's effectiveness.

In all analysed cases, the regenerator effectiveness exceeded 90%. The energy efficiency is influenced by the cycle duration and the length of the regenerator. In this article, only the results for the optimal cycle duration of 240 s and the optimal regenerator length are presented. Importantly, no water condensation occurred in the regenerator, which is a fundamental problem in the case of rotary and crossflow heat exchangers. In recuperators, electric preheaters are often used to preheat air to prevent condensation and water freezing. This solution significantly decreases heat recovery efficiency [16]. The absence of water vapour condensation in the ceramic regenerator can be attributed to two main factors. First, due to the short switching cycle in reversible operation mode, the ceramic mass is alternately heated and cooled at brief intervals, preventing its temperature from dropping to the dew point of the exhaust air. Second, the hygroscopic nature of the ceramic material allows moisture to be captured through adsorption rather than condensation, meaning that water vapour is retained within the material structure without forming a liquid phase.

2.2. Heat pump operation and performance

The heat supplied to the air stream was calculated from the difference in specific enthalpy between the supply and exhaust air. The specific enthalpy was calculated based on the temperature and relative humidity. The volumetric flow rate was measured using an ISA-standard nozzle equipped with pitot-tube measurements. The maximum supply air volumetric flow rate reached $1380 \text{ m}^3/\text{h}$. The total heat supplied to the room during one cycle is given by:

$$Q_{\text{cycle}} = \int_{t=0}^{t=t_{\text{half cycle}}} [\dot{m}(t)(i_2(t) - i_3(t))] dt, \quad [\text{J}], \quad (3)$$

where $t = 0$ [s] and $t_{\text{half cycle}}$ [s] are, respectively, the time of beginning and ending of the half-cycle, $\dot{m}(t)$ [kg/s] is an air mass flow rate, $i_2(t)$ [kJ/kg] and $i_3(t)$ [kJ/kg] are, respectively, instantaneous specific enthalpies of the air supplied and removed.

The electrical energy supplied in one cycle consists of the energy used by the compressor, fan and automation system:

$$W_{\text{cycle}} = \int_{t=0}^{t=t_{\text{cycle}}} P_{\text{el}}(t) dt, \quad [\text{J}], \quad (4)$$

where $t = 0$ [s] and t_{cycle} [s] are, respectively, the beginning and ending times of the entire cycle, and $P_{\text{el}}(t)$ is the total electrical power supplied at a given moment.

This allows us to calculate the coefficient of performance (COP) during one cycle:

$$\text{COP} = \frac{Q_{\text{cycle}}}{W_{\text{cycle}}}. \quad (5)$$

To determine the average heating capacity and electrical power, the total heat supplied and total electrical energy supplied were divided by the analysed time period:

$$\dot{Q}_{\text{cycle}} = \frac{Q_{\text{cycle}}}{\tau_1 - \tau_0}, \quad [\text{W}], \quad (6)$$

$$P_{\text{cycle}} = \frac{W_{\text{cycle}}}{\tau_2 - \tau_0}, \quad [\text{W}]. \quad (7)$$

3. Results and discussion

Studies of the vapour-compression heat pump with a regenerator were carried out for different outside temperatures in a range of 0.6°C to 8.6°C . The indoor air temperature was maintained approximately at 22°C . Each half-cycle of air supply and exhaust lasted 60 s, giving a full cycle duration of 120 s. Figure 8 shows the temperature changes over 30 full cycles (equivalent to 1 hour) at an outdoor temperature of 1.1°C . A close-up of tem-

perature changes during one cycle is shown in Fig. 9. The measured temperatures are: θ_I – indoor air temperature, [°C];

θ_{II} – condenser air-side temperature, [°C]; θ_{III} – outdoor air temperature, [°C]; θ_{IV} – evaporator air-side temperature, [°C].

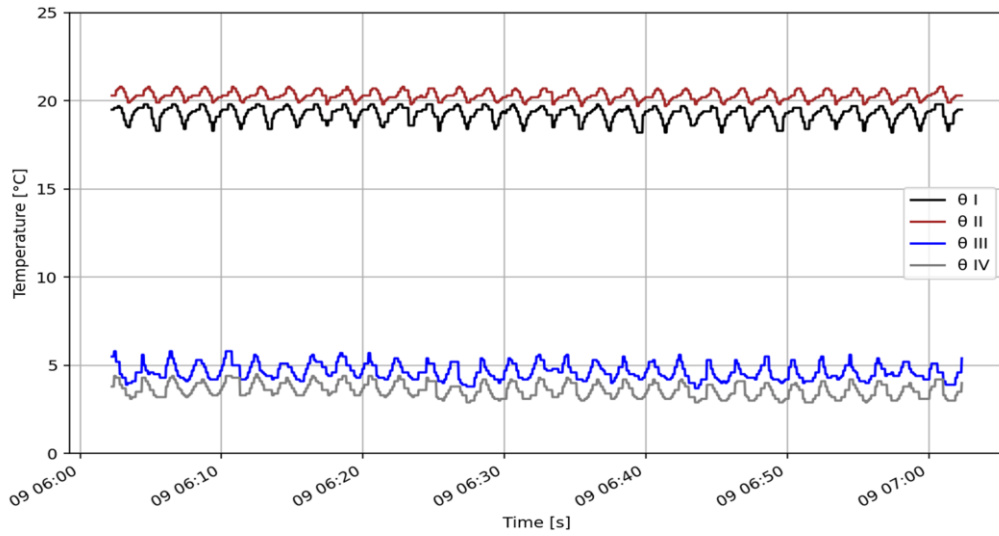


Fig. 8. Temperature changes during 30 full cycles.

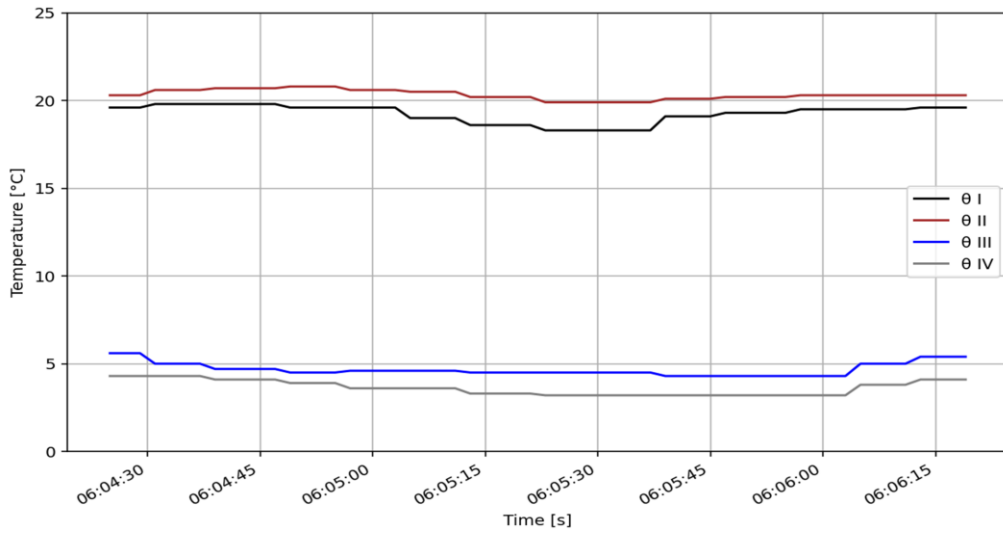


Fig. 9. Temperature changes during one full cycle.

Figure 10 presents changes in relative humidity during the same cycle as shown in Fig. 8. Measured relative humidities are: RH_I – indoor air relative humidity, [%]; RH_{II} – condenser air-side relative humidity, [%]; RH_{III} – outdoor air relative humidity, [%]; RH_{IV} – evaporator air-side relative humidity, [%]. The measuring points for temperature and relative humidity are shown in Fig. 3.

Figure 11 shows the changes in heating capacity and electrical power consumption during one full cycle for the outside temperature of 0.6°C. The electric power consumption varied only slightly, from 1150 W to 1220 W. The average heating capacity of this cycle was equal to 5306 W with a peak value exceeding 11 000 W, which is a result of accumulating heat in a ceramic regenerator. During the air exhaust phase, the heating capacity is equal to zero.

The experimental investigation of the device operating in heat pump mode was carried out under outdoor air temperature

conditions ranging from 0.6°C to 8.6°C. The indoor temperature was equal to approximately 20°C, while the indoor relative humidity remained approximately 30% across all measurement series. The relatively low indoor relative humidity recorded throughout the experiments reflects the conditions of the laboratory facility in which the device was installed, rather than typical residential indoor climate conditions. The relative humidity of the outdoor air for the investigated measurement series as a function of outdoor temperature is presented in Fig. 12.

Refrigerant parameters remained stable across the tested outside temperature range. The system was operated to keep the discharge pressure at 15 bar. Refrigerant condensation and evaporation pressures are shown in Fig. 13. As the outdoor temperature increases, the evaporation pressure rises. This reduces the pressure difference between the suction and discharge sides of the compressor, resulting in lower power consumption and a higher COP.

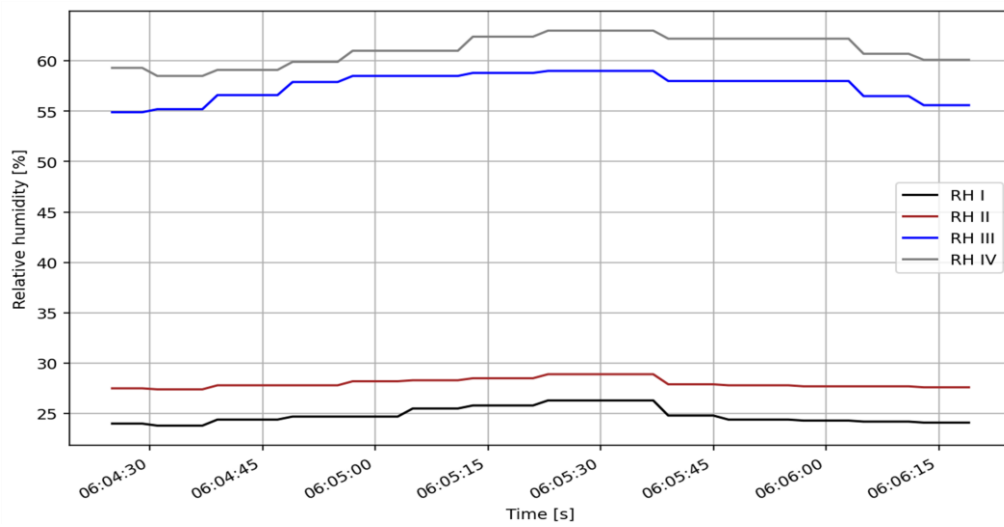


Fig. 10. Relative humidity changes during one full cycle.

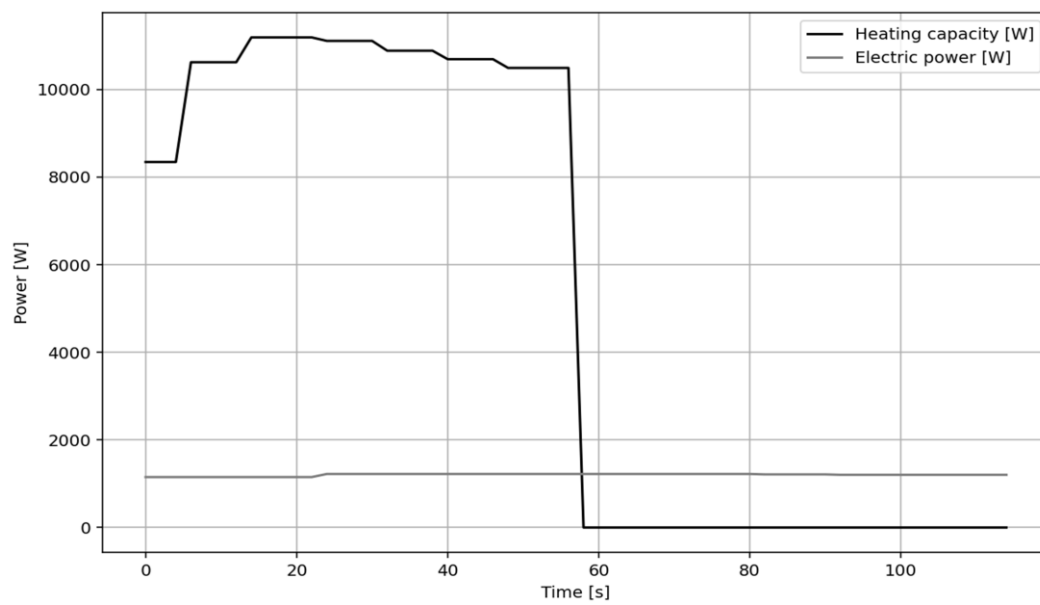


Fig. 11. Heating capacity and electrical power changes during one full cycle.

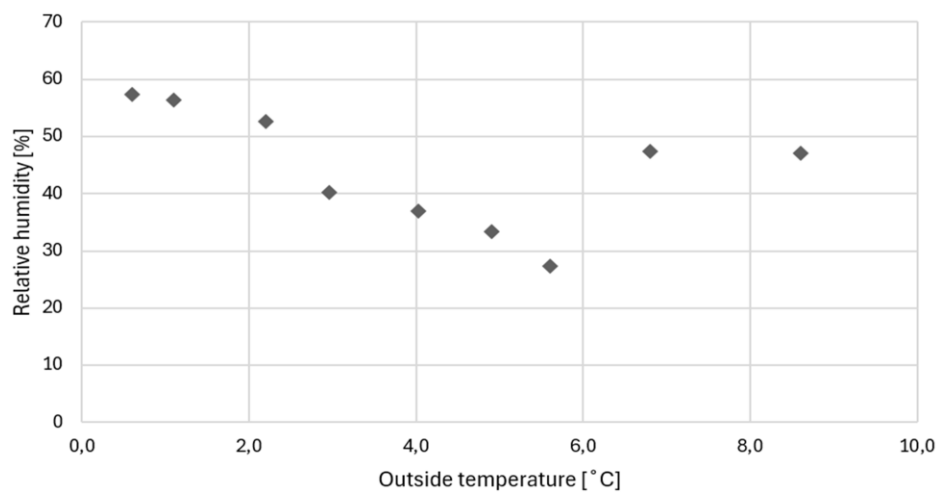


Fig. 12. Relative humidity vs. outside temperature.

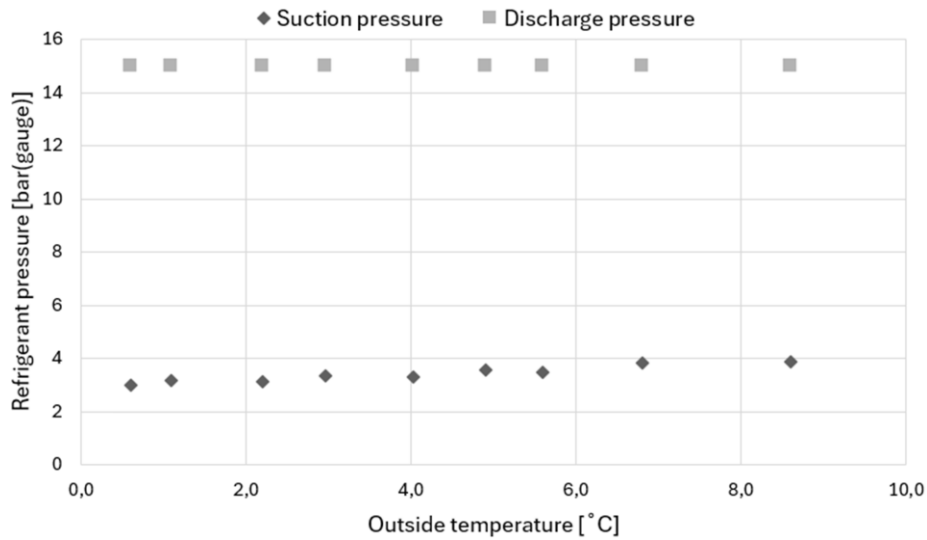


Fig. 13. Suction and discharge refrigerant gauge pressures vs. outside temperature.

The device's electrical power demand ranges from 1039 W to 1387 W. The electricity consumption of the rotary fan and automation system remained unchanged across the entire range of outside temperatures, amounting to 460 W. The heating capacity supplied to the room ranges from 4180 W to 5735 W. The uncertainty of the electrical power measurement is practically negligible, as it is equal to $\pm 0.45\%$. The calculated expanded uncertainty of heating capacity, determined in accordance with the Guide to Uncertainty in Measurement (GUM) methodology [17], amounts to $\pm 8.9\%$ at a coverage factor of $k = 2$, corresponding to a confidence level of approximately 95%.

Heating capacity exhibited an increasing trend with decreasing outdoor air temperature. This phenomenon is a consequence of the decreasing temperature difference between the indoor and outdoor air as the outdoor temperature rises, which, at a constant mass flow rate and under maintained constant condensation pressure, results in a reduced heat flux delivered to the conditioned space. The electrical power consumption exhibits a slight decreasing trend with the increasing outdoor air temperature,

driven by the gradual improvement in evaporator operating conditions: as the outdoor temperature rises, the evaporation temperature increases, reducing the pressure differential between the suction and discharge sides of the compressor, and thereby lowering the specific compression work.

Figure 14 shows the heating capacity of the device and its electrical power consumption at different outdoor temperatures.

Since both the heating capacity and the electrical power consumption decrease with increasing outdoor temperature at comparable rates, their ratio (COP) remains relatively stable across the investigated temperature range as shown in Fig 15. However, given the limited number of measurement series available, no statistically robust trend in COP as a function of outdoor temperature can be established from the present dataset, and this relationship warrants further investigation over a wider range of operating conditions. The COP of the heat pump with a ceramic regenerator was on average 3.70, and it reached the maximum value of 4.31 for the outside temperature of 1.1°C.

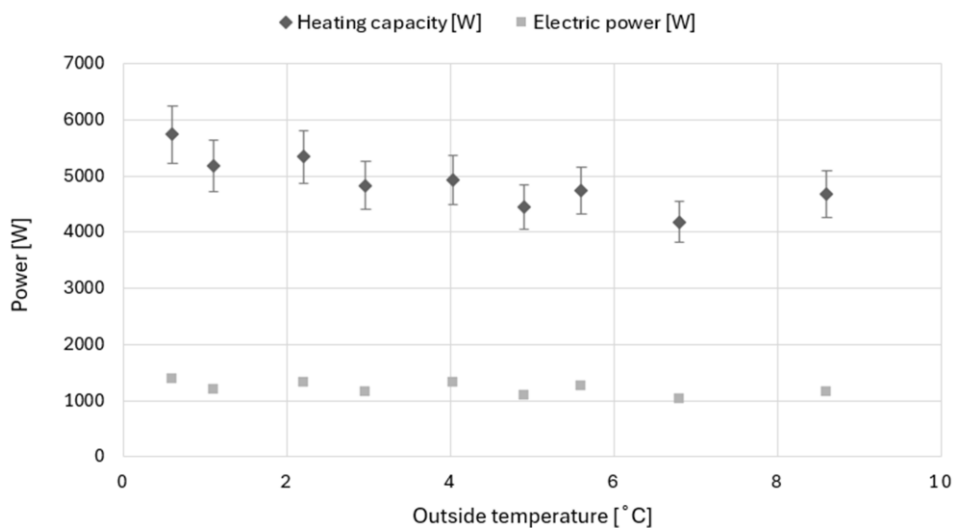


Fig. 14. Heating capacity and electric power vs. outside temperature.

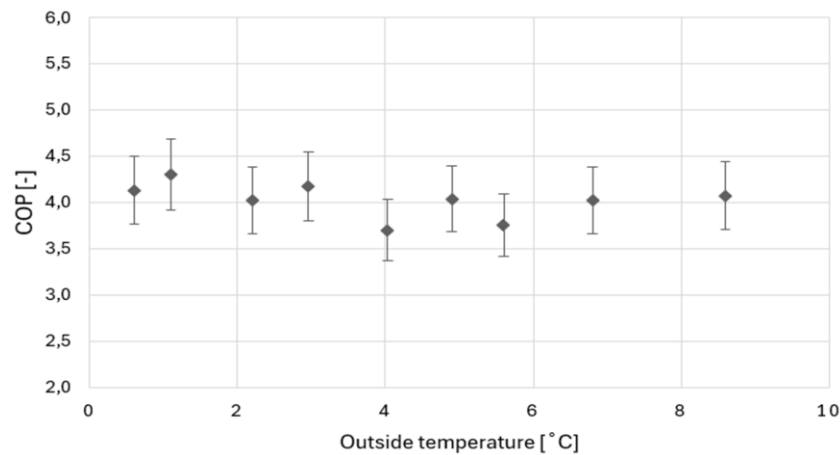


Fig. 15. COP vs. outside temperature.

4. Conclusions

This study experimentally characterised the performance of a compact reversible ventilation unit integrating a ceramic fixed-bed regenerator with an R290 vapour-compression heat pump. The following conclusions can be drawn from this study:

- Integration with an R290 vapour-compression heat pump enabled operation under the tested winter conditions without an auxiliary heat source. COP values of 3.70–4.31 were achieved at outdoor temperatures ranging from 0.6°C to 8.6°C, substantially exceeding 2.0–2.5 of conventional air-to-air heat pumps. The favourable evaporator conditions provided by warm exhaust air (~20°C) directly account for this elevated performance.
- Electrical power demand ranged from 1039 W to 1387 W, with the fan and automation contributing a constant 460 W. The average heating capacity ranged from 4180 to 5735 W, with characteristic double-capacity peaks during the supply phase from simultaneous regenerator and heat pump heat release.
- No water vapour condensation was observed in the ceramic matrix under any of the tested conditions, indicating that no electric pre-heating or defrosting was required during the experiments.
- The expanded COP uncertainty of $\pm 8.9\%$ ($k = 2$) is dominated by airflow (IRIS 250, $\pm 5\%$) and humidity (SHT20, $\pm 3\% RH$) measurements. Upgrading to a class 1 flowmeter and a $\pm 1.5\% RH$ sensor would reduce this to $\sim \pm 4\%$. Future work should extend on testing below 0°C and investigating variable compressor speed effects on COP.

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Phase change material for thermal management of high-temperature proton exchange membrane fuel cells in electric vehicle

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Abstract

The current work examines the possibility of reducing the radiator size in high-temperature proton exchange membrane fuel cell vehicles by incorporating phase change material. Using pinch analysis optimisation, the research demonstrates that the radiator area in a Toyota Mirai car can be reduced by 4.7 times with the integration of D-mannitol (melting point 170°C) phase change material. The required quantity of D-mannitol accounts for only 0.62% of the vehicle's curb mass. Additionally, the work investigates the impact of operating temperature on radiator size, revealing that lower coolant inlet temperatures increase the minimum radiator area needed for sufficient heat dissipation. A 40°C increase in the inlet coolant temperature results in a 44% reduction in the radiator area. Consequently, a vehicle powered by low-temperature fuel cells (60–80°C) versus high-temperature fuel cells (120–180°C), under the same driving conditions, would require minimum radiator areas of 1.07 m² and 0.26 m², respectively. These results are expected to aid in the development of techno-economically feasible cooling systems in fuel cell vehicles.

Keywords: Thermal management; PEMFC; PCM; Radiator; Electric vehicle

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1. Introduction

Today, one of the main techniques for reducing carbon emissions in the transportation sector is automobile electrification [1]. To align with the International Energy Agency's agenda to limit the global temperature rise, it is required that the cumulative sale of electric vehicles (EVs) reach 1.8 billion by the year 2060 [2]. The escalating growth in the production and use of electrified vehicles is largely determined by energy storage technologies powering them like batteries and fuel cells. Compared to internal combustion engine (ICE) vehicles, which are characterised by low power-

train efficiency (with a maximum of 40% due to intermediate conversion into heat and associated thermodynamic losses) and significant emissions, fuel cell vehicles have superior efficiencies (with a minimum of 50% due to the direct conversion of chemical energy into electrical energy) and produce zero local emissions [3]. Fuel cell-based EVs also have the advantages of shorter refuelling time and longer driving ranges compared to battery EVs. However, currently, fuel cell vehicles require research efforts to develop less costly catalyst and electrode materials, better humidifiers, heat exchangers, etc., to improve the efficiency and durability of the system while reducing costs [4].

Nomenclature

a	– acceleration, m/s ²
A	– frontal area, m ²
C_d	– drag coefficient
f_r	– rolling friction
g	– gravitational acceleration, m/s ²
m	– vehicle curb mass, kg
v	– vehicle speed, m/s

Greek symbols

α	– road slope, °
----------	-----------------

ρ	– air density, kg/m ³
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Abbreviations and Acronyms

EV	– electric vehicle
FCEV	– fuel cell electric vehicle
ICE	– internal combustion engine
PCM	– phase change material
PEMFC	– proton exchange membrane fuel cell
TDR	– turn-down ratio
TMS	– thermal management system

Effective thermal management of fuel cells in vehicles is important to operate the system at maximum efficiency. Literature reports that fuel cells generate a considerable amount of thermal energy, proportional to their electrical output. For example, a fuel cell with an efficiency of 50% and a capacity of 10 kW will produce 10 kW of electrical energy and 10 kW of thermal energy by electrochemical reactions [5]. This makes temperature regulation essential to prevent performance degradation in fuel cells that may happen due to the breakdown of the membrane and catalyst at elevated temperatures [6].

A proper thermal management system dissipates the generated heat and establishes a uniform temperature distribution across the fuel cell stack. Liquid cooling, typically with water and excess heat rejection via radiators, is the most widely adopted cooling strategy in fuel cell electric vehicles (FCEVs) [7,8]. However, compared to ICE vehicles for the same vehicle platform, fuel cell vehicles require a radiator with a 2–4 times larger frontal area [9,10]. A 60 kW Ford fuel cell vehicle powered by a proton exchange membrane fuel cell (PEMFC) is reported to require 1.5 times higher radiator area than a conventional vehicle to maintain the fuel cell within its optimum operating temperature range [11]. Similarly, Ahluwalia and group [12] reports the need for 125% higher frontal area in fuel cell vehicles as a result of their study on heat rejection from the vehicle with respect to stack temperature. The need for large radiators in fuel cell vehicles is primarily because of the minimal heat carried away by exhaust gases.

A large radiator necessitates a large frontal area, which increases the overall vehicle power demand. Various recent studies have researched improving the thermal management system compactness without compromising its efficiency. Prabakaran et al. [13,14], Su et al. [15] and Mohamed et al. [16] have proposed to improve the heat transfer via radiators by coupling spray cooling. Modifications in radiator design, such as improving the shape of the radiator intake grill and alternate fin structure have also been proposed by researchers [17,18]. However, the majority of these works report results of computational evaluations and require further optimisations and experiments before being commercialised.

Recent works have introduced the concept of phase change material (PCM) radiator thermal management system for fuel cell vehicles, where a PCM of suitable mass serves as a thermal reservoir, allowing for radiator size reduction [19,20]. PCMs can absorb and release large amounts of thermal energy

(150–250 kJ/kg) during phase change. They are broadly categorised into three: organic PCMs, inorganic PCMs, and eutectic PCMs. Organic PCMs (e.g. paraffins, fatty acids) are biodegradable with low environmental impact, have good chemical stability as well as low toxicity, but exhibit low thermal conductivity (0.1–0.3 W/(m K)) and high flammability (limiting oxygen index ~ 17%). Meanwhile, inorganic PCMs (e.g. salt hydrates, metallic PCMs) have higher latent heat and thermal conductivity, non-flammability (limiting oxygen index ~60 %), and are 3–10 times cheaper compared to organic PCMs, but they show corrosiveness, supercooling issues, and potential environmental issues (harm soil/water if leaked, eutrophication risks). Eutectic PCMs are organic-organic, organic-inorganic, or inorganic-inorganic combinations that offer tailored melting points for specific applications [21,22].

The selection of PCMs for EV applications depends on key properties such as high latent heat, high thermal conductivity, high specific heat, melting point in the desired temperature range, minimum volume change during phase transition, stability, non-flammability, cost-effectiveness, etc. [13]. PCMs can easily integrate into existing vehicular thermal management systems (TMS) via encapsulation in polymer/aluminium modules, panels or shells with no major redesign. The high volumetric energy density of PCMs (180–300 MJ/m³) allows compact placement in space constrained areas. Their near-isothermal operation allows rapid, effective buffering of transient heat loads, and their passive operation ensures minimal parasitic power requirement. The stored thermal energy has the potential to improve the energy efficiency of the system in scenarios such as cold start.

The current work investigates the potential for size reduction in radiators by implementing a PCM-based TMS in a high-temperature PEMFC-powered electric vehicle. High-temperature PEMFC vehicles are advantageous owing to enhanced electrochemical kinetics, higher carbon monoxide (CO) tolerance and simplified water management. Additionally, these systems also expel waste heat at a higher temperature, which can be recovered and utilised for direct heating [22,23].

In this study, the size optimisation of the PCM-radiator system components is carried out using pinch analysis. The key contributions of this research are: (i) determining the minimum radiator size for dissipating the heat from a high-temperature PEMFC stack, (ii) identifying appropriate commercial PCM and the mass necessary to compensate for the reduced radiator area,

(iii) defining a design space of feasible radiator sizes and corresponding PCM masses, allowing users to select the most suitable combination based on specific requirements, and (iv) analysing the effect of operating temperature on the TMS size.

2. Materials and methods

The study considers a light-duty 5-seater vehicle (Toyota Mirai) operated by a high-temperature PEMFC. For this work, it is assumed that PEMFC is the only power source in the vehicle. The vehicle specifications are given in Table 1. The vehicle follows a standard driving cycle (worldwide harmonised light duty test cycle – WLTC), so the analysis's results would also apply to real-world driving conditions. The power demanded by the light-duty vehicle can be estimated using the following formula [24]:

$$P_v = (0.5\rho C_d A v^2 + f_r m \cos\alpha + m a + m g \sin\alpha) v, \quad (1)$$

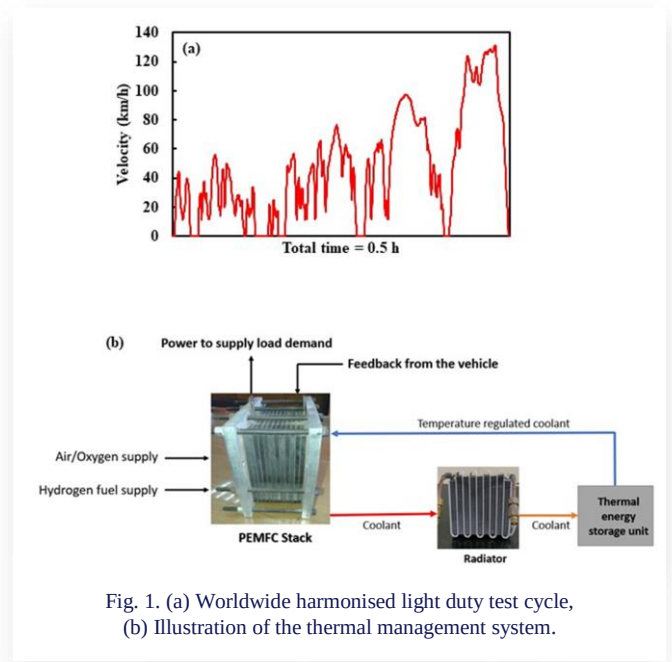
where: A – frontal area, a – acceleration, C_d – drag coefficient, f_r – rolling friction, g – gravitational acceleration, m – vehicle curb mass, v – vehicle speed, α – road slope, ρ – air density. To calculate the load demand by the vehicle, a road slope of zero is assumed.

Table 1. Specifications of system.

Parameter	Value
Mass of vehicle (kg)	1930
Frontal area of vehicle (m ²)	2.35
Coefficient of drag	0.29
Rolling friction coefficient	0.01
Coolant temperature at the inlet of fuel cell (°C)	170
Maximum rate of flow of a coolant (kg/s)	0.20
Specific heat of a coolant (kJ/(kg °C))	4.20
Environmental temperature (°C)	50
Allowed temperature rise in the stack (°C)	10
Overall heat transfer coefficient for radiator (kW/(m ² °C))	0.30
Fuel cell TDR (%)	20

Estimation of the power demand by the automobile gives an insight into the thermal energy generated by the fuel cell. The proposed thermal management system primarily consists of the PEMFC, coolant, radiator and heat exchanger containing PCM. The coolant exiting the PEMFC stack passes into a radiator and then through the PCM system to dissipate the thermal energy generated in the PEMFC stack. This ensures a uniform temperature for the coolant entering back into the stack. The drive cycle considered in this work and a schematic of the TMS are depicted in Fig. 1.

The pinch analysis technique adopted in this study is an algebraic optimisation method, by which the thermal management needs of the vehicle can be met with minimum resources. For this, the thermal energy generation by the PEMFC stack is compared with the heat rejected by the radiator size of the user's choice, to obtain the minimum PCM mass required for satisfactory thermal management. By using PCM, the excess thermal energy can be stored and reused at later periods to ensure the



thermal regulation of the coolant. This maintains the coolant temperature entering the PEMFC stack in the range of the desired operating temperature of the fuel cell. The PCM considered for this study is D-mannitol, which has a melting point of 170°C and a latent heat capacity of 297 kJ/kg [25].

The optimisation is subject to the constraints listed below:

- 1) The energy in the PCM unit at the start and finish of the driving cycle should be the same.
- 2) The minimum state of charge in the PCM unit at any given time should not be less than 0.
- 3) The overall heat transfer coefficient of the radiator unit lies between a lower limit (influenced by radiator turn-down ratio) and an upper limit.

The implementation of pinch analysis involves three major steps: calculation of thermal energy generated by the PEMFC stack, calculation of the cooling capacity of the radiator, and calculation of PCM mass. The parameters to be calculated in each step are summarised in Fig. 2 [19]. The software used for optimisation was the GRG Non-Linear Engine in Microsoft Excel.

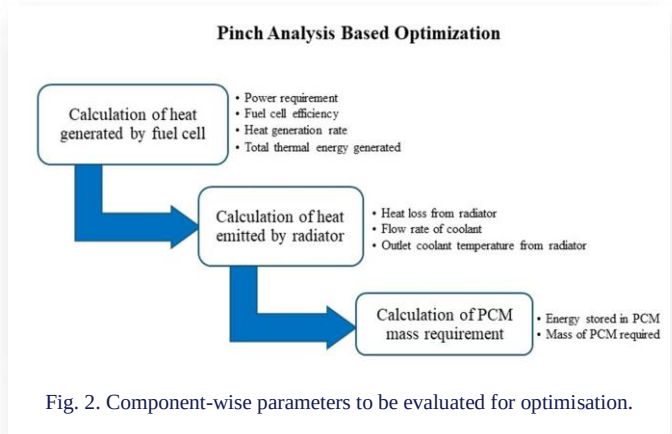


Fig. 2. Component-wise parameters to be evaluated for optimisation.

3. Results and discussion

The present study uses a PCM-based TMS to reduce the radiator size in fuel cell-powered vehicles. For this, a pinch analysis-based optimisation algorithm was adopted and applied to a standard drive cycle to study the effectiveness of the PCM system when used in a FCEV. A 1980 kg light-duty vehicle that follows a WLTC class-3 drive cycle (duration: 0.5 h) is considered here.

3.1. Sizing curve for the thermal management system

Sizing curves that depict the radiator area and corresponding PCM requirement are plotted to illustrate the feasible and infeasible operating regions. Each point on the sizing curve (Fig. 3) is the result of pinch analysis optimisation performed for a particular value of minimum radiator area. In each case, the mismatch between the cumulative thermal energy generated and the cumulative thermal energy dissipation by the chosen radiator size represents the cumulative thermal energy to be stored in the PCM unit. The points on the sizing curve and the region above it represent the feasible region. However, the region on the left and lower part of the sizing curve represents infeasible combinations of the radiator area and PCM mass either because of insufficient radiator size or due to limited PCM mass. It is seen that with respect to the constraints considered in this study, to ensure adequate heat dissipation through the radiator-PCM system, the radiator area should not be less than 0.26 m^2 . Hence, the results show the possibility of decreasing the radiator area in

a high-temperature powered PEMFC by 4.7 times when complemented with a suitable thermal energy storage system.

Pinch analysis results show that to dissipate the thermal energy generated by the stack with a radiator of area 0.26 m^2 , 11.97 kg of D-mannitol PCM is essential. The required PCM mass corresponds to only about 0.62% of the vehicle's curb mass. The higher the radiator area, the lower the thermal energy to be stored and hence, the lower the PCM mass required. On investigating the sizing graphs for various radiator turn-down ratios (TDR), it is also evident that after a local minimum, there is a need for higher PCM mass despite the high radiator areas. The TDR constraint necessitates using additional PCM to store additional heat and ensure appropriate temperature regulation of the coolant.

3.2. Effect of operating temperature on the size of thermal management system

To understand the influence of the operating temperature of the fuel cell on the TMS size of the vehicle, it is required to perform a pinch analysis with different coolant temperatures. The TMS sizing is studied for cases of different inlet coolant temperatures entering the fuel cell unit. The analysis shows that the lower the inlet coolant temperature, the higher the minimum required radiator area for adequate heat dissipation. Furthermore, an increase in the inlet coolant temperature from 120°C to 180°C produces a drastic drop in the minimum required radiator area by 44.18%, while the required PCM mass declines by about 2%. Figure 4 shows the sizing curve for the radiator-PCM system for

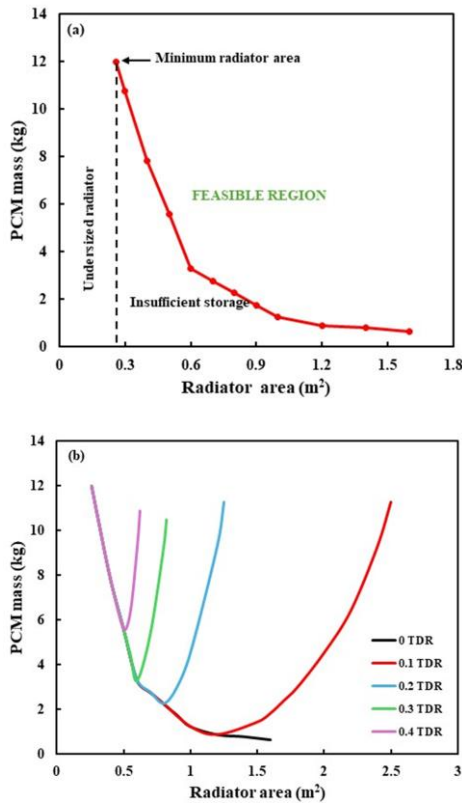


Fig. 3. (a) Sizing curve; (b) Sizing curves for various radiator turn-down ratios.

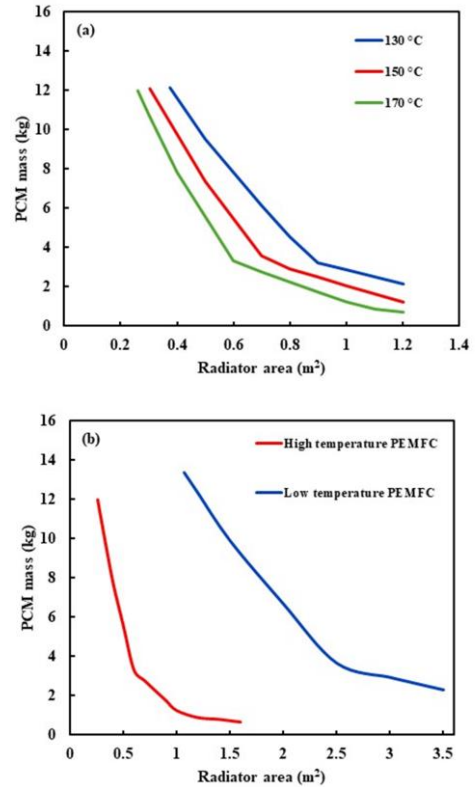


Fig. 4. Sizing curves for TMS in (a) high temperature PEMFC at various inlet coolant temperatures, (b) in high and low temperature PEMFCs.

the inlet coolant temperatures of 130°C, 150°C and 170°C. An increase in the coolant temperature by 40°C decreases the required radiator area by 0.12 m². This is because a greater temperature difference between the coolant and ambient air allows the same heat rejection with a smaller radiator area. Also, for a constant radiator area, PCM requirement increases with a reduction in the inlet coolant temperature. For instance, if we consider the TMS with the radiator areas of 0.5 m² and 1.0 m², it is evident that PCM requirement increases by ~2.0 kg (for 0.5 m²) and ~0.80 kg (for 1.0 m²) for every 20°C reduction in inlet coolant temperature.

In the same context, a comparison between the TMS (radiator-PCM) sizes for proton exchange membrane fuel cells, which work at low temperatures (60°C to 80°C) and high temperatures (120°C to 180°C) is also presented (Fig 4b). Due to the low difference between fuel cell operating temperature and ambient temperature, low-temperature PEMFCs require higher radiator areas when compared to high-temperature PEMFCs. To reduce the radiator size in these systems, paraffin wax and D-mannitol PCMs were selected for the low-temperature and high-temperature systems, respectively. The temperatures of the coolant entering the fuel cell were taken as 70°C (low-temperature PEMFC) and 170°C (high-temperature PEMFC). Pinch analysis optimisation was performed for the two systems for the same vehicle and driving conditions and the results are shown in Fig. 4b. From the figure, it is seen that while the minimum area requirement for the radiator in a high-temperature PEMFC system is 0.26 m², the low-temperature PEMFC system requires a radiator with at least 4.12 times larger area (1.07 m²), even with the usage of maximum PCM. However, the mass of PCM required for the low-temperature PEMFC is only 11.70% higher than the mass of PCM required for the high-temperature PEMFC. Hence, irrespective of the type of PEMFC chosen for the FCEV, the mass of PCM required for maximum radiator area reduction does not exceed 0.70% of the vehicular curb mass.

4. Conclusions

The study explores the potential for reducing the size of the radiator in Toyota Mirai cars powered by a high-temperature proton exchange membrane fuel cell by incorporating a phase-change material into its thermal management system. This research makes a key contribution by using pinch analysis to determine the minimum size of radiator required to dissipate heat from a high-temperature PEMFC stack and defining a design space comprising radiator sizes and the corresponding masses of phase change material. The results show that by integrating 11.97 kg of D-mannitol PCM, the radiator area in the high-temperature fuel cell-powered vehicle can be reduced by 4.7 times. Additionally, the impact of fuel cell operating temperature on the vehicle's TMS size was also assessed. The analysis reveals that lower inlet coolant temperatures necessitate a larger minimum radiator area for effective heat dissipation. For instance, on increasing the inlet coolant temperature from 120°C to 180°C, there is a 44% reduction in the radiator area and a 2% decrease in the required PCM mass. Furthermore, if we analyse the same vehicle powered by a low-temperature fuel cell, the thermal management system would need a radiator with an area

4.12 times larger at minimum and a PCM mass 11.70% greater for appropriate heat dissipation.

The present study assesses the advantage of adding a PCM-based thermal energy storage unit in the TMS of a fuel cell vehicle. In addition to eliminating the disadvantages of using large radiators such as larger frontal area and increased fuel consumption, the usage of PCM can also improve the energy efficiency of the system through the utilisation of stored heat. The findings of this study are expected to contribute to the development of techno-economically feasible cooling systems for fuel cell vehicles, enabling more efficient and compact thermal management solutions.

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Comparative experimental studies of the specific heat of Fe and Ni using scanning calorimetry methods

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Abstract

This study presents comparative experimental investigations of the specific heat of materials undergoing phase transitions, with the aim of assessing the capabilities and metrological limitations of differential scanning calorimetry methods and instrumentation. Nickel and iron were selected as reference materials due to the availability of reliable literature data for verification. Two approaches to determining heat flux density in differential scanning calorimetry were compared using three calorimeters: Pyris 1 (PerkinElmer), DSC 3 (Mettler Toledo), and DSC 404 F1 Pegasus (Netzsch). Measurements were conducted in a protective atmosphere over a temperature range up to 600°C on samples that had been thermally stabilised prior to testing. For the Pyris 1 calorimeter, a proprietary measurement procedure based on heating and cooling cycles with isothermal holds was applied, enabling reliable determination of specific heat during heating and cooling. In contrast, for the DSC 3 and DSC 404 F1 instruments, the analysis was primarily based on heating data. The obtained results allowed analysis of the second-order phase transition in nickel, including determination of the transition end temperature and the temperature corresponding to the maximum differential scanning calorimetry peak. The phase transition in iron was characterised solely using data from the DSC 404 F1 instrument, due to measurement limitations of the remaining equipment. Additionally, three independent temperature-dependent specific heat characteristics were compared, obtained using a relative method with identical reference standards and standard differential scanning calorimetry signal conversion procedures. The results contribute to improved understanding of calorimetry methodology and measurement uncertainty in materials undergoing phase transitions.

Keywords: Specific heat capacity; Ferromagnetic phase transition; Heat-flow calorimetry; Power-compensation calorimetry

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1. Introduction

The specific heat of materials is one of the basic thermo-physical properties, important both from the point of view of scientific research and engineering applications [1,2]. Accurate knowledge of this quantity allows for effective modelling of thermal processes, optimisation of technological parameters, and design of materials and devices operating in variable temperature conditions. In the case of metals undergoing phase transitions, such as iron (Fe) and nickel (Ni), it is particularly important to take

into account changes in specific heat as a function of temperature and the characteristics of these transitions, and research has been conducted for almost 100 years [3,4].

Differential scanning calorimetry (DSC) is one of the most commonly used methods for testing specific heat [5–9]. It is characterised by high sensitivity, the ability to perform measurements over a wide temperature range, and relatively simple laboratory procedures with an error of less than 3% [8,10]. Despite the popularity of this method, the results obtained may vary depending on the apparatus used, the measurement methodology

Nomenclature

c_p – specific heat capacity, J/(kg·K)
 f – thermogram data
 m – mass, kg
 t – time, s
 T – temperature, °C

Subscripts and Superscripts

base – baseline data
 C – Curie
 ref – reference
 sp – specimen

Abbreviations and Acronyms

DSC – differential scanning calorimetry

and the method of data processing, in this case, thermograms. It is therefore advisable to conduct comparative studies to assess the consistency and precision of specific heat measurements performed with different devices and techniques within the DSC method.

This article presents the results of comparative experimental studies of the specific heat of iron and nickel, conducted using three different scanning calorimeters: PerkinElmer Pyris 1, Mettler Toledo DSC 3 and Netzsch DSC 404 F1 Pegasus. Iron and nickel were selected due to their ferromagnetic properties at room temperature and their loss in the second type of phase transition after exceeding the characteristic Curie temperature. Iron is additionally characterised by the occurrence of a first-order transition. These phenomena make it possible to use the characteristic temperature as an additional metrological parameter. The analysis examined the impact of measurement conditions and declared temperature change programmes, in particular the use of isothermal stops, on the measurement results. In addition to mutual comparison, the consistency of the obtained results with the literature data was also checked. Particular attention was paid to the characteristics of second-order phase transitions and the assessment of the metrological capabilities and limitations of individual devices.

Regardless of the specific objectives mentioned above, the research project is based on general methodological principles that focus on key issues across the entire field of modern thermal analysis. Very often, the measurement accuracy reported by authors is simply a repetition of data provided by manufacturers. The problem is that this information mainly concerns the repeatability of measurements and compliance with reference data obtained under specific experimental conditions. Not all of these conditions can be maintained during routine experiments. The results of the round-robin tests described in this article are intended to indicate the reliability of measurement results obtained under typical conditions, in terms of both qualitative and quantitative data.

2. Research equipment and subject of study

Although one might initially think that the research issue described above is limited solely to the problem of equipment, it should be noted at the outset that the measuring equipment was selected in such a way as to represent three different types of methods for implementing the idea of scanning calorimetry [11]. The Pyris 1 calorimeter from PerkinElmer is an example of the use of the heat flux compensation method (power compensated). The other two, Mettler Toledo DSC 3 and Netzsch DSC 404 F1 Pegasus, are heat flux calorimeters with different heat flux

measurement methods. On the one hand, the design differences affect the diversity of measurement procedures, but on the other hand, they create the possibility of highlighting a different parameter or property determined in the measurements.

In this case, a comparison was made of the capabilities of three selected scanning calorimeters in terms of their suitability for testing the specific heat of metals undergoing second-order phase transitions. The analysis covered the effectiveness and limitations of individual devices in determining the thermal characteristics of nickel (Ni) and iron (Fe), with particular emphasis on the temperature range, DSC signal resolution and repeatability of results.

2.1. PerkinElmer Pyris 1 power compensated DSC

PerkinElmer Pyris 1 DSC is a power compensated differential scanning calorimeter designed for thermal analyses and precise determination of the thermal properties of materials, in particular specific heat (c_p), in a temperature range up to 710°C. This limitation results from the use of resistive temperature sensors and compensating resistance heaters in the measuring head to implement the idea of power compensation. The apparatus enables quantitative determination of specific heat based on the analysis of the heat flow signal recorded during controlled heating of the sample relative to a reference material with known thermal parameters according to a so-called three-curve method [12]. This method involves using data from three test series – a baseline series, a series using a standard, and a series using the test sample – to determine the specific heat based on the equation:

$$c_{p,sp}(T) = c_{p,ref}(T) \frac{m_{ref} f_{sp}(T) - f_{base}(T)}{m f_{ref}(T) - f_{base}(T)}, \quad (1)$$

where: sp – indicates a specimen, ref – reference, and $base$ – baseline data.

The function f represents thermograms signal data that could be recorded in mW in the case of an enthalpy calibrated instrument or in arbitrary units in the case of an only temperature calibrated apparatus. Typical measurements are performed on samples weighing from 5 mg to 100 mg, placed in standard aluminium encapsulated crucibles with lids. A single crystal of sapphire (Al_2O_3), characterised by a well-defined c_p value, is usually used as a reference standard. The mass of both the test sample and the standard is determined with a resolution of 0.01 mg and recorded in the Pyris software.

The measurement is carried out in a protective atmosphere, most often purified nitrogen, with a controlled carrier gas flow

in the range of 20–50 ml/min. The standard heating rate is 5–20 K/min, but it is possible to use non-standard temperature profiles, including procedures with isothermal stops, which allow the elimination of dynamic effects such as the thermal inertia of the system.

The calorimeter's measuring range covers temperatures from room temperature to approximately 600°C, depending on the configuration of the apparatus and the crucibles used. As part of extended testing procedures, it is possible to use a cyclic heating and cooling regime with short isotherms between temperature steps. This approach allows for:

- improved accuracy of the determined c_p values,
- reduction of the impact of irreversible phenomena and thermal instabilities,
- independent analysis of the DSC signal in heating and cooling cycles, which increases the reliability and repeatability of the results.

The specific heat values are calculated using dedicated Pyris software. The programme compares the heat flow signals from the sample and the standard, taking into account their masses, and allows direct determination of c_p as a function of temperature. The data obtained are presented in the form of graphs and tabular summaries of $c_p(T)$ values, which can be exported for further analysis.

Baseline correction and signal smoothing functions are available as part of the data analysis. In the case of materials undergoing phase transitions, it is also possible to determine the characteristic transition temperatures and calculate the enthalpy of heat transitions.

2.2. Mettler Toledo DSC 3

The Mettler Toledo DSC 3 differential calorimeter is an advanced heat-flux DSC device used, among other things, for the precise determination of the thermal properties of materials, in particular specific heat capacity (c_p) as a function of temperature. The measurement is based on the analysis of the difference in heat flow signals recorded during the controlled heating of the sample and a reference standard with known thermal properties. The samples for analysis usually weigh from a few to several milligrams and are placed in standard aluminium crucibles, usually with a lid, without a seal. A sapphire disc (Al_2O_3) is usually used as a reference standard, as its thermal properties are well known and documented. Both the sample and the standard are weighed accurately (with a resolution of 0.01 mg and an accuracy depending on the type of balance) and then placed in two identical measuring chambers of the calorimeter. The test procedure data are as follows:

- the measurement is carried out in a controlled protective atmosphere, usually in a stream of nitrogen or argon, with a fixed gas flow,
- the standard heating rate is 10 K/min, although it can be adjusted to the specific characteristics of the material being tested,
- the measurement range covers temperatures from room temperature to approximately 600–700°C, depending on the configuration of the apparatus and the crucibles used.

The device determines the difference in heat flow between the sample and the reference standard as a function of temperature. Based on this data, the dedicated STARE software calculates the direct value of the sample's specific heat, taking into account the previously performed calorimetric calibrations. The results are presented in the form of a $c_p(T)$ graph and in tabular form, taking into account measurement uncertainty. The software allows baseline correction to be applied to compensate for any DSC signal drift. The results obtained can be compared with literature data or used for further analysis, such as calculating the enthalpy of phase transitions, analysing the thermal properties of composites, or modelling thermal processes in engineering systems.

2.3. Netzsch DSC 404 F1 Pegasus

The Netzsch DSC 404 F1 Pegasus differential scanning calorimeter is a high-temperature heat-flux DSC device designed for precise study of the thermal properties of substances, including specific heat capacity (c_p) determination, in a wide temperature range – up to 1600°C. The apparatus allows the use of the three-run method (the three-curve method presented in paragraph 2.1 [12]), which is the standard procedure for determining c_p in relation to a sapphire standard.

The analysis is carried out on samples weighing typically from a few to several milligrams, placed in cylindrical crucibles made of a suitable material. Platinum-rhodium (Pt/Rh), aluminium oxide (Al_2O_3) or graphite crucibles are used for specific heat measurements, depending on the expected temperature range and chemical properties of the sample material. A certified sapphire disc (Al_2O_3) with a well-defined specific heat, supplied by the manufacturer (Netzsch), is used as a standard. The mass of all measuring elements is determined with a resolution of 0.01 mg and entered into the Proteus® Thermal Analysis software.

Determination of specific heat using the three-measurement method involves the following steps:

- baseline measurement – recording the thermal background of the system with empty crucibles in the sample (S) and reference (R) positions,
- standard test – recording the reference thermogram for the standard material sample, most often sapphire – crystalline aluminium trioxide, in crucible S and reference crucible R,
- sample testing – measurement with the tested material in crucible S and the reference crucible in position R, which allows the recording of a thermogram that forms the basis for determining the specific heat capacity of the tested material.

All three measurements must be carried out under identical conditions, which means:

- using the same measuring vessels – crucibles,
- maintaining the same heating rate (e.g., 10 K/min),
- ensuring an identical protective atmosphere (usually nitrogen or argon) and the same volume flow rate, typically 50–70 ml/min,

- application of the same temperature change programme typically defined in the range from -170°C or 20°C to 1000°C or 1600°C , depending on the system equipment.

Specific heat calculations are performed by Proteus® Thermal Analysis software, which uses signal differences obtained from three measurements (baseline, reference, sample) applying appropriate calorimetric relationships. The results are presented in the form of $c_p(T)$ graphs and tables, with the possibility of further data processing: smoothing, mathematical model fitting and export to external analytical formats. The software enables the assessment of the repeatability and precision of measurements, as well as advanced thermal analysis of materials undergoing phase transitions, including the determination of transition enthalpy and characteristic temperatures.

The system is particularly suitable for testing construction materials, ceramics, metal alloys and functional materials used at elevated temperatures.

2.4. Subject of research

Given the methodological objective of the study, it was decided to attempt to determine the specific heat of pure elements for which reliable literature data on specific heat are available – samples of pure metals: nickel (Ni) and iron (Fe) [13]. These materials were selected not only due to their well-documented thermo-physical properties, including specific heat, but also with regard to a second-order phase transition within the temperature range available in standard DSC calorimetry. This allows for the comparison of experimental results with literature data and the assessment of the accuracy and sensitivity of the measurement methods used. The following metals were therefore selected for the study:

- Nickel (Ni) – the material used had a purity of $\geq 99.9\%$. Nickel is a metal undergoing a second-order phase transition – ferromagnetic-paramagnetic ordering – at a Curie temperature ranging from approximately 355°C (628 K) [14], through 357°C (630 K) [15], 358°C (631 K) [16,17] to 360°C (633 K) [13] (Fig. 1). Due to its relatively sharp peak, this transition is particularly useful as a reference signal in DSC measurements. The presented reference positions of the Curie point (T_C) of nickel, both in terms of the abscissa, i.e. the temperature T_C , as shown – both along the x-axis (i.e., the temperature) and the y-axis (i.e., the corresponding specific heat value) – define the range to which the position of this point, as determined in these experiments, should be referenced. In fact, almost all T_C position test results fall within the range of $355^{\circ}\text{C} \leq T_C \leq 360^{\circ}\text{C}$.
- Iron (Fe): the iron used was also characterised by high purity ($\geq 99.5\%$). In the analysed temperature range, Fe undergoes a ferromagnetic-paramagnetic phase transition with a Curie temperature close to 770°C (1043 K) [13,18]. Due to the temperature limitations of some devices, full parameterisation of this transformation was only possible using the Netzsch DSC 404 F1 Pegasus high-temperature device. However, the measurement range of the tests performed with this device also covered the first order α - γ phase transformation.

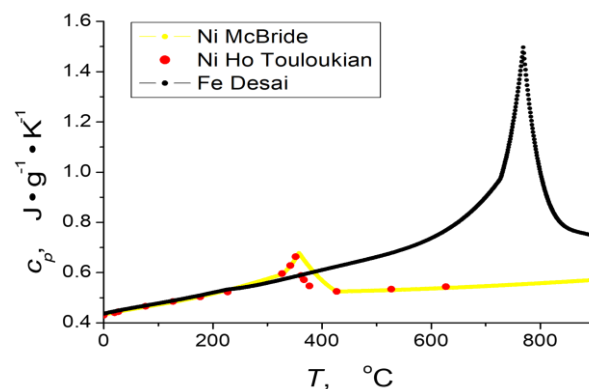


Fig. 1. Dependence of specific heat on temperature, showing the position of the Curie point as the peak of the second-order phase transition – measurements of specific heat for Ni [13,17] and Fe [18].

The nickel and iron samples were taken from a larger sample used for precise measurements of thermal expansion as a material with established thermal properties [19]. They were designed to ensure that the largest possible surface area of the bottom makes good contact with the bottom of the sample crucibles. The sample weights were selected to ensure compliance with the instruments' sensitivity requirements. All samples used in comparative tests are shown in Fig. 2.

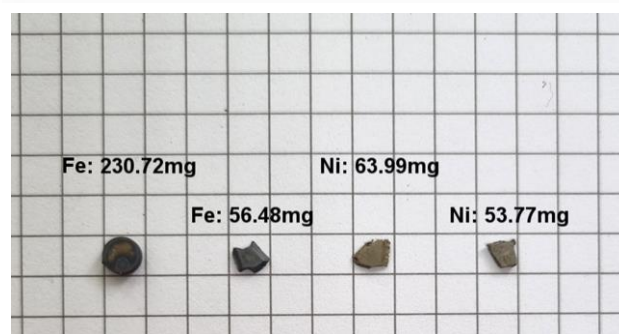


Fig. 2. View of the samples used in the comparative tests.

Prior to measurement, the materials were subjected to thermal stabilisation (preliminary annealing) in a protective atmosphere in order to remove residual moisture and surface contaminants and to reduce the effects of history (e.g. stresses or oxidation). The temperature change programmes used are presented later in this paper.

3. Experimental research

3.1. Research methodology

The experiments using the three different differential calorimeters mentioned above were conducted using two temperature-change programs, which were compared in this study. The first program, developed for the PerkinElmer Pyris 1 DSC, utilised heating and cooling rate of 20 K/min applied in the temperature range from -40°C to 570°C in the range of 570 – 600°C , the rate was changed to 10 K/min in order to ensure stable conditions of

transition between the heating and cooling phases, in accordance with the dynamic characteristics of the apparatus (Fig. 3).

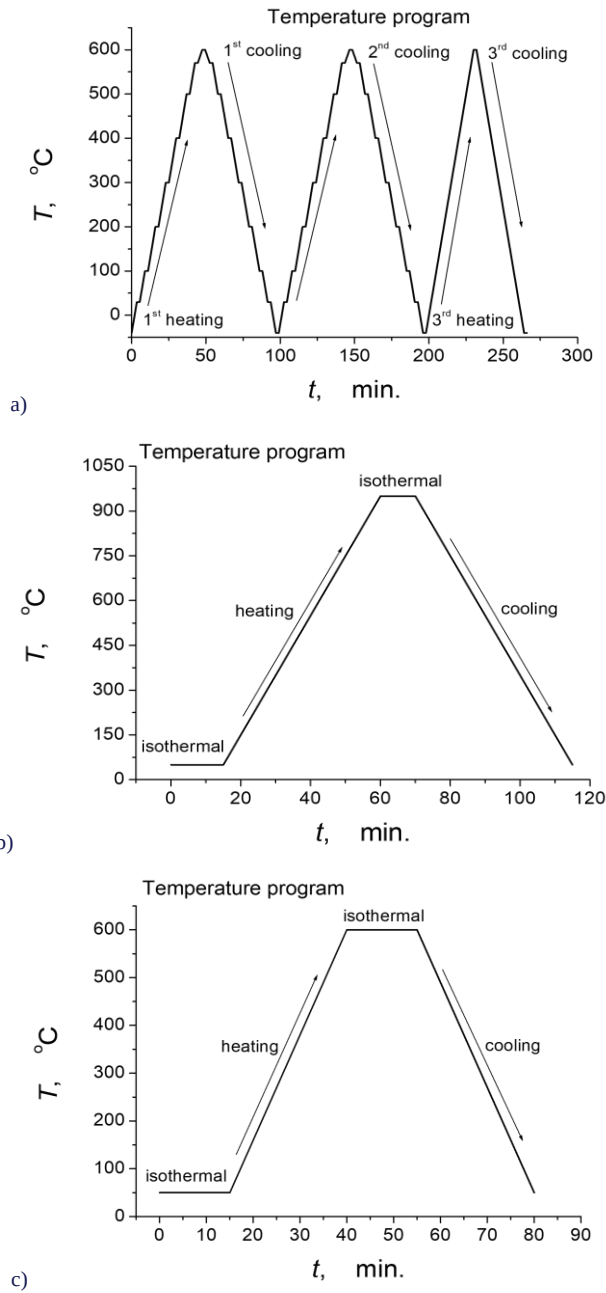


Fig. 3. Sample temperature programme for testing Ni (a, b) and Fe (c) materials on individual devices.

The temperature programme included two minutes of isotherm at the end of each heating and cooling stage in order to stabilise the temperature conditions and limit the impact of instrument equilibration time. The use of isotherms is an author's modification of the measurement procedure, aimed at improving the accuracy of determining the specific heat at constant pressure c_p [20,21]. The same program was used to conduct part of the research using the DSC 3 Mettler Toledo instrument. The measurements were carried out in a protective argon atmosphere at a constant flow rate of 20 ml/min (PerkinElmer) and in a protective nitrogen atmosphere at a constant flow rate of 150 ml/min (DSC 3 Mettler Toledo). The study also employed

a standard heating and cooling program with a single ramp of linear temperature changes at a rate of 20 K/min in the range from room temperature to 600 °C (Ni: DSC F1 Pegasus 404 Netzsch and DSC 3 Mettler Toledo) or to 950 °C (Fe: DSC F1 Pegasus 404 Netzsch; Fig. 3). When taking measurements with the Pegasus instrument, an argon flow rate was set to 70 ml/min. Data analysis was based on a standard calibration procedure using the so-called three-curve method and, additionally, the direct method (Mettler Toledo) including measurements for the reference system (empty aluminium vessels), reference material (sapphire) and the tested Ni and Fe samples. In accordance with the adopted methodology, the final results were obtained on the basis of DSC signals from the second or third heating and cooling cycle, i.e. after reaching a steady state in the sample-vessel-atmosphere system, as well as for the entire temperature range in order to present the discrepancies between individual temperature cycles.

3.2. Specific heat determination results

For calorimetric tests using DSC aimed at determining specific heat at constant pressure, two samples of Fe: with a mass of 230.72 mg in the shape of a cylinder with a diameter of approximately 4 mm (DSC 3 Mettler-Toledo tests), and an irregular in shape of 56.48 mg (PerkinElmer Pyris 1 and Netzsch DSC 4 F1 Pegasus studies) were applied. Two studied Ni samples were irregular in shape. The masses were equal to 63.90 mg (DSC 3) and 53.77 mg (Pyris 1 and DSC 4 F1 Pegasus). In both material cases, the investigated specimens were separated from a larger fragment of iron and nickel samples, respectively. It should be mentioned that the use of irregularly shaped samples is acceptable in DSC analysis, provided that the influence of the shape on the measurement uncertainty is taken into account and remains within the acceptable range of this uncertainty [22]. In the case of investigation of bulk metallic samples, these effects are negligible.

First, the temperature-dependent specific heat of the Fe sample, as determined in the experiments, will be analysed. The results are shown in Fig. 4.

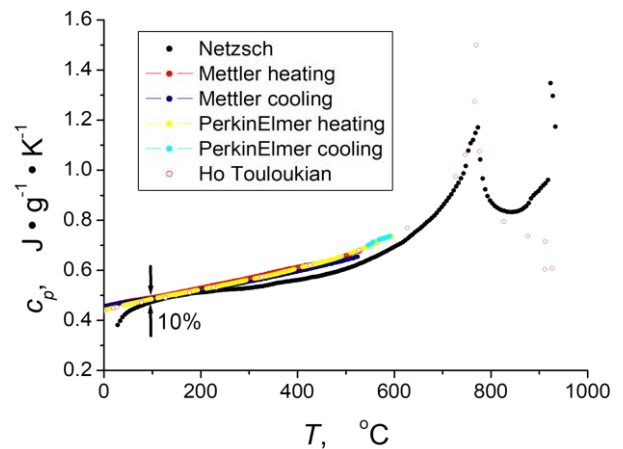


Fig. 4. Temperature characteristics of the specific heat of iron (Fe) determined using three different calorimeters, compared with reference data [13].

In addition to the measurement data collected within a temperature range limited to 600°C, the figure shows the results of measurements conducted using a DSC F1 Pegasus 404 calorimeter at temperatures up to about 950°C. When analysing these data, it is worth noting the significant deviation of the c_p values from those reported in the literature at temperatures above 850°C, which may indicate equipment limitations in this temperature range. In the ferromagnetic-paramagnetic transition region ($T_C = 770^\circ\text{C}$ [13]), the data consistency is more than satisfactory. Only this device made it possible to obtain data within the temperature range in discussion. For comparison, the results obtained using other calorimeters (Pyrus 1, DSC 3) in the range up to 600°C are also presented, which show good agreement with the reference values.

For a more detailed analysis of the measurement results, Fig. 5 shows the variation of specific heat as a function of temperature over a narrower temperature range. The graph shows a high degree of consistency between the results obtained using Pyris 1 and DSC 3 calorimeters and the reference data. The data obtained using the F1 Pegasus 404 apparatus show a systematic underestimation of values compared to the other measurements. The results presented come from the third heating and cooling cycle, i.e. after the measurement conditions in the sample-ves-sel-atmosphere system had stabilised.

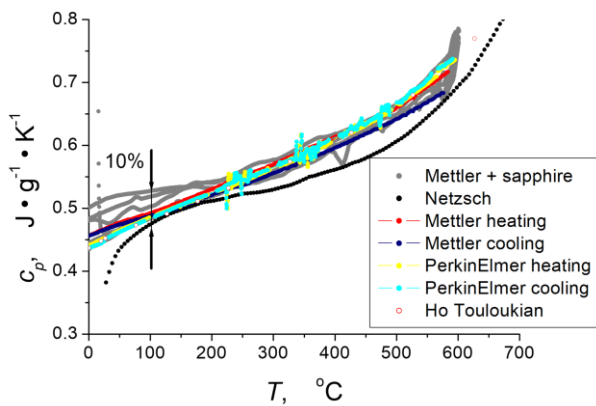


Fig. 5. The temperature characteristics of the specific heat of iron (Fe), determined using three different calorimeters, compared with reference data available in the literature [13].

The results of comparative studies on the specific heat of nickel as a function of temperature are shown in Fig. 6. The graph shows the results of measurements taken using three different calorimeters in the temperature range up to 600°C. It is important to note the significant deviation of the c_p value from the literature data in the case of measurements obtained using the DSC 3 calorimeter with the direct calculation method. The reason for this effect is difficult to identify, but the result itself illustrates the scale of problems that can be encountered in experimental studies of specific heat even for such well-defined materials as metallic elements.

Greater consistency between our test results and the data in the literature was achieved by using the three-curve procedure, referred to in the instrument's user manual as the "sapphire reference". This is illustrated by the comparison of the results of both types of measurements shown in Fig. 7. It should be noted

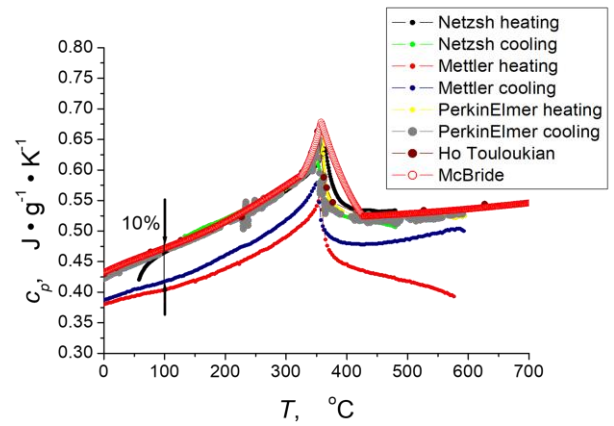


Fig. 6. The temperature characteristics of the specific heat of nickel (Ni), determined using three different calorimeters, compared with reference data available in the literature [13].

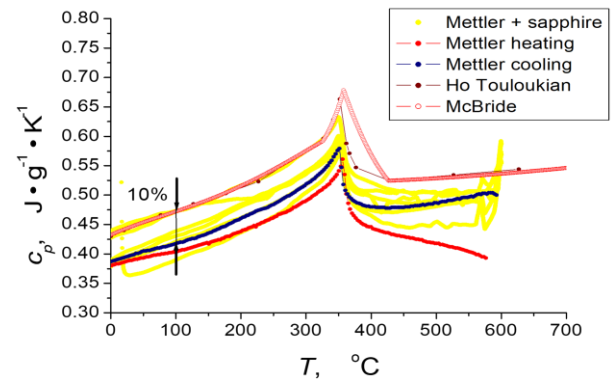


Fig. 7. Temperature characteristics of the specific heat of nickel (Ni) determined using a DSC 3 Mettler Toledo calorimeter by the three-curve method in comparison with reference data [13,18].

that the graph presents raw experimental data, which exhibit an artefact caused by irregular heating regime heat transfer in the form of apparent peaks at the extremes of the temperature range.

Based on the results of the specific heat measurements of nickel, the Curie temperature can be determined ($T_C = 352^\circ\text{C}$ [13]). The ferromagnetic transition is manifested by a characteristic lambda-shaped peak. The results of determining the position of the maximum of the temperature dependence of specific heat for the test results obtained using Pyris 1 and F1 Pegasus 404 devices are shown in Fig. 8a, and for the DSC 3 device in Fig. 8b.

The raw test results were further processed using a special computational procedure for approximation with B-spline curves [23] for cases where the range of the approximated data contained discontinuities in the first derivative of the temperature characteristic c_p . The test data for iron up to 600°C were approximated using a third-degree polynomial. This allowed us to determine representative specific heat characteristics as a function of temperature for each individual measurement. At the same time, the characteristic features of the peaks of the ferromagnetic phase transition were preserved. The results of the approximation, providing a summary illustration of the presented results in the context of their possible discrepancies, are shown in Fig. 9.

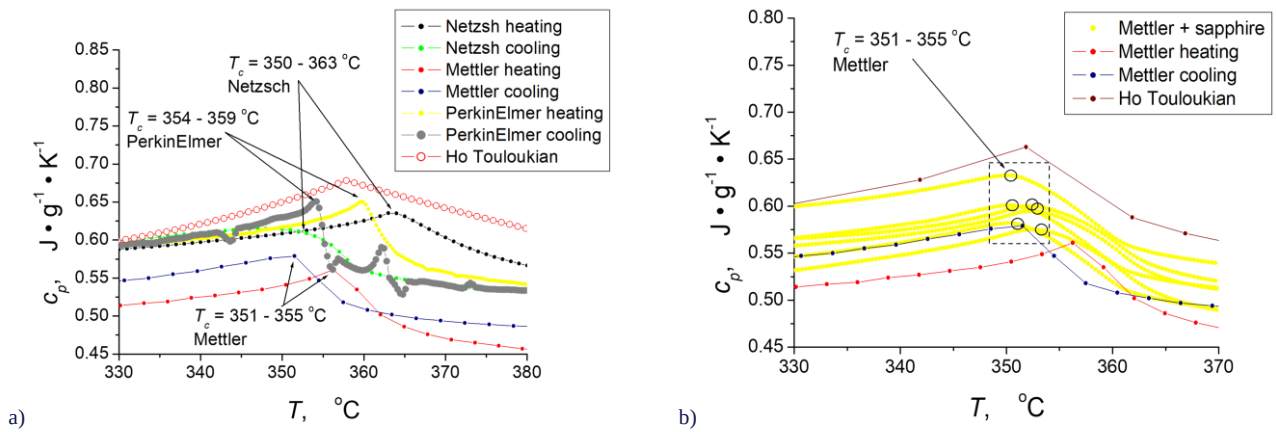


Fig. 8. Determination of the Curie temperature T_C for nickel: a) from measurements taken with Pyris 1 and DSC 404 F1 Pegasus devices, b) from calorimetric tests performed with a Mettler Toledo DSC 3 calorimeter.

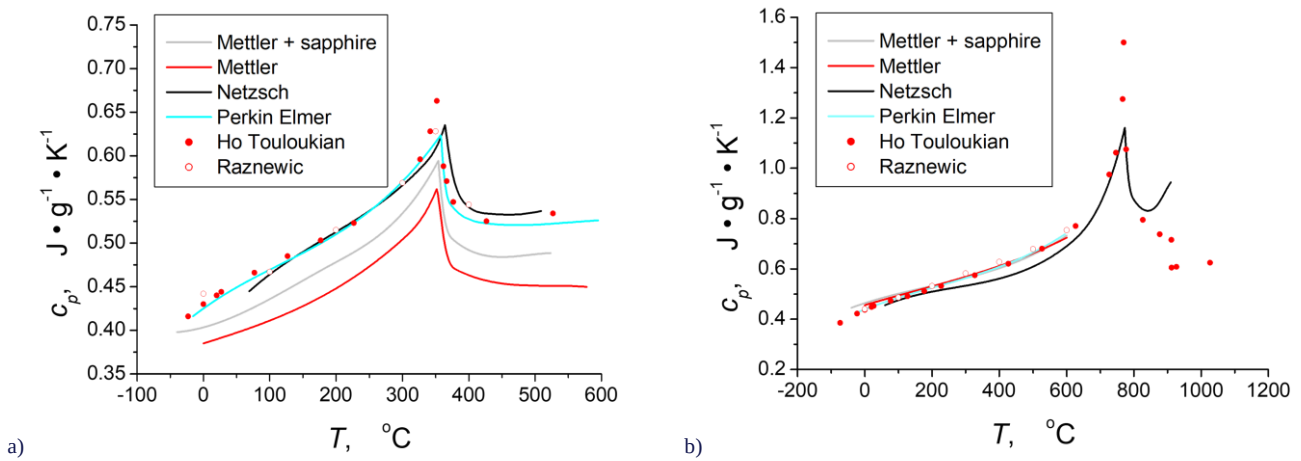


Fig. 9. Temperature-dependent specific heat at constant pressure characteristics for nickel (a) and iron (b) – B-spline [23] approximation results.

At the same time, these data provide an indirect answer to the question of measurement uncertainty. In thermal analysis studies, the emphasis is placed on the reproducibility of results and, where applicable, on agreement with data from reference materials [11]. It is possible to achieve better agreement than that shown in the data presented here. The results discussed above were obtained using standard procedures. Repeating measurements multiple times, using several samples, statistical averaging, and other such measures will yield a more satisfactory final result. However, this requires more effort and time. Therefore, the actual needs should be taken into account in each individual case.

It is also possible to estimate measurement uncertainty more accurately without resorting to interlaboratory comparisons. The problem, however, is that the necessary calculations must be performed for each individual case. And this time, it is not about, for example, the individual data of the tested samples, but rather about a set of data hidden within the calibration parameters. And when, for example, temperature calibration is involved, accounting for drift would require checking the calibration data each time. A case where temperature calibration data is critical is that of high-temperature-resolution measurements in the vicinity of

phase transitions, as characterised in the monograph [20] and publication [24].

4. Conclusions

The aim of the study was to compare the metrological capabilities of selected methods and devices for determining the specific heat of materials undergoing second-order phase transitions, with particular emphasis on nickel and iron. The data obtained using three different differential scanning calorimeters (DSC), using both standard procedures and proprietary measurement methods, were analysed. In the course of the experiments, the influence of the test procedures used and the declared temperature change programmes on the specific heat values obtained and the results of the Curie point location identification were assessed. Particular attention was paid to comparing the results with available literature data and analysing the course of phase transitions. Based on the results obtained, the following conclusions were drawn:

- The tests showed that it is possible to obtain reliable specific heat measurements using the DSC method, even with irregularly shaped samples, provided that the measurement

conditions and uncertainty analysis are repeated. This is important from the point of view of industrial applications, where it is not always possible to prepare samples that comply with standard requirements.

- A comparison of three independent calorimeters (PerkinElmer Pyris 1, Mettler Toledo DSC 3 and Netzsch DSC 404 F1 Pegasus) allowed for an assessment of their metrological capabilities and the impact of device design and conversion algorithms on data quality. The Pyris 1 device ensures the highest repeatability and accuracy of results; the DSC 404 F1 Pegasus device allows satisfactory measurement results to be obtained in the high temperature range, while the DSC 3 device, despite the observed differences in results compared to the literature data, proved to be a device ensuring repeatable results of Curie point identification.
- The introduction of a proprietary measurement procedure with isothermal stops in the Pyris 1 apparatus made it possible to obtain consistent results in both heating and cooling regimes. This procedure can be a useful tool in further studies of phase transition materials, increasing the amount of available reference data.
- Analysis of the $c_p(T)$ data allowed the determination of characteristic parameters of the second type of phase transition for nickel. The convergence of the peak maximum temperature and the end of the transition in various measurements confirms the correctness of the methods used. This, of course, applies to the repeatability and accuracy of the measurement results that can be obtained.
- For iron, reliable mapping of the phase transition curve was only possible when using the DSC 404 F1 Pegasus apparatus. Slightly larger deviations in the low-temperature test results obtained with this device confirm the need for complementary tests.
- The specific heat values obtained were in satisfactory agreement with the literature data in the range up to 600°C. The test results obtained with the DSC 404 F1 Pegasus device show large differences from the reference data in the range above 850°C. Considering the difficulties in accurately determining the temperature characteristics of all thermo-physical quantities in the vicinity of phase transitions, especially of the first kind [19–21], this effect cannot be treated as a basis for questioning the correctness of the apparatus or the correctness of the measurement procedures. On the contrary, it is necessary to consider continuing this type of research using comprehensive complementary testing procedures. This will allow data to be obtained in the form of temperature dependencies of other thermo-physical quantities [24], which will enable accurate identification of the issue and determination of reliable reference data.

The results of this research contribute to the methodology of calorimetry, providing both comparative data and practical guidelines for interpreting and verifying measurement results for metallic materials undergoing phase transitions. The results of the single-sample tests presented in this study provide detailed information regarding the selection of measurement pro-

cedures. This applies to the measurement procedure and the selection of the method for determining specific heat. Indirectly, they also provide guidance on the possible need to expand the project – by including a larger number of samples and cycles – in order to achieve statistical representativeness.

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Rheological and porous effects on mixed convection: Power-law fluid flow in a finned annular configuration

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Abstract

This study numerically investigates mixed-convection heat transfer of non-Newtonian power-law fluids in the annular space between a rotating outer cylinder and a stationary finned inner cylinder filled with a porous medium. The outer cylinder rotates at constant angular velocity and is maintained at a temperature lower than the finned inner cylinder, producing combined forced and buoyancy-driven flow. The governing equations were solved using the finite volume method implemented in Ansys CFX, with the fluid's rheology modelled by the Ostwald's law model. Parametric ranges include: Reynolds numbers from 5 to 35, Richardson numbers from 0 to 3, Darcy numbers from 10^{-5} to 10^{-2} , and power-law indices of 0.6, 1.0 and 1.6. The results show a transitional behaviour at a Richardson number of approximately 2.6 (for a Darcy number of 10^{-2}), beyond which the average Nusselt number becomes almost independent of the power-law index. Increasing rotational forcing enhances circulation and heat transfer, and under strong buoyancy the shear-thinning fluid (power-law index 0.6) yields up to 15–20% higher average heat transfer compared with the Newtonian case. The novelty of this work lies in the joint investigation of internal fins, outer-wall rotation and porous resistance in non-Newtonian media, which produces distinct flow regimes and offers practical guidance for porous heat exchanger and rotating-machine cooling designs.

Keywords: Complex fluid; Mixed convection; Porous media; Numerical investigation; Nusselt number; CFD

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1. Introduction

In recent years, the study of mixed convection in non-Newtonian or rheologically complex fluids has attracted considerable attention due to its importance in numerous engineering and industrial applications, including metal casting, enhanced oil recovery, glass manufacturing, biomedical devices and food processing. While many investigations have concentrated on forced convection in power-law fluids where flow is predominantly driven by external forces, researchers have also examined their thermal and flow behaviour around confined geometries, such as cylinders and ducts with different cross-sectional shapes. These studies often incorpo-

rate enclosures with adiabatic walls, structured inlets and outlets, or partially porous regions to better mimic real systems [1–3].

Complementary work has been devoted to natural convection, which arises solely from buoyancy forces induced by temperature gradients. Such investigations have considered power-law fluids within trapezoidal or wavy enclosures, typically featuring heated bases and either cooled or insulated boundaries [4,5].

Other studies have focused on rectangular and shallow cavities to evaluate how non-Newtonian rheology affects the onset of flow, velocity distribution, temperature fields and overall heat transfer efficiency under various thermal conditions [6,7].

Nomenclature

A – surface area of the inner cylinder, m^2
 C_p – heat capacity of fluid, $\text{J}/(\text{kg K})$
 d – diameter of the inner cylinder, m
 Gr – Grashof number
 g – gravitational acceleration, m/s^2
 I_2 – second invariant function of strain rate tensor
 K – permeability, m^2
 m – consistency index, Pa s^n
 n – power-law index
 Nu – Nusselt number (average value)
 Nu_L – Nusselt number (local value)
 p – pressure, Pa
 Pr – Prandtl number
 Re – Reynolds number
 Ri – Richardson number
 T – temperature, K

u, v – velocity components along x - and y -directions, m/s
 x, y – Cartesian coordinates, m

Greek symbols

β_T – coefficient of volume expansion
 η – dynamic viscosity, Pa s
 ρ – density of fluid, kg/m^3
 τ – stress tensor, Pa
 Ω – rotation speed, $1/\text{s}$

Subscripts and Superscripts

c – cold
 h – hot
 $*$ – dimensionless quantity

Abbreviations and Acronyms

ANN – artificial neural network
 CFD – computational fluid dynamics

The interplay between forced and natural convection, known as mixed convection in power-law fluids, has also been widely analysed using numerical simulations. These studies emphasise the key influence of dimensionless parameters, such as the Reynolds number (Re), Richardson number (Ri) and power-law index (n) on both flow structures and heat transfer mechanisms. Investigations have been carried out in different geometries, including lid-driven and wall-driven cavities with isothermal, adiabatic or combined boundary conditions [8–13]. Configurations featuring internally heated blocks or asymmetric heating have revealed how geometric and thermal non-uniformities can modify flow circulation and enhance heat transfer [14–21].

Another aspect of growing interest is internal rotation within non-Newtonian domains. The presence of rotating components, such as cylinders, significantly affects streamlines, isotherms and the overall heat transfer rate, often characterised through the Nusselt number [22–27]. Further complexity has been introduced in studies involving triangular or inclined cavities filled with power-law nanofluids subjected to magnetic fields, where changes in parameters, such as Re , Ri , n and magnetic inclination, were found to strongly influence both heat and mass transport [28,29].

Recent numerical and experimental works have also explored complex geometries and non-uniform heating conditions, such as lid-driven cavities with embedded obstacles [30], compound cavity-channel systems incorporating multiple heat sources [31,32], and enclosures containing hemispherical heaters [33]. Investigations on micropolar and magnetised fluids have further demonstrated that localised magnetic effects can generate secondary vortices, enhancing flow mixing and thermal regulation [34]. Moreover, studies of double-lid cubic enclosures with distributed heat sources have highlighted the sensitivity of heat transfer and flow structures to variations in Re , Ri and nanoparticle concentration [35].

Recent developments in reliability engineering have emphasised the integration of machine learning with statistical modelling to improve predictive accuracy and system assessment. In [36,37], reliability analyses were conducted for the generalised

exponential and generalised Rayleigh distributions, respectively, by incorporating the inverse power-law model within an artificial neural network (ANN) framework using Bayesian regularisation. These studies demonstrated that the combination of ANN-based learning and Bayesian regularisation significantly enhances the reliability prediction capability by minimising overfitting and improving model generalisation. The results confirmed that this hybrid approach provides more accurate lifetime estimation and better captures the complex reliability behaviour of engineering systems. Overall, [36,37] illustrate the growing potential of intelligent computational methods for advancing reliability modelling and analysis in modern industrial applications.

Despite this extensive literature, there remains a notable research gap concerning mixed convection in configurations that integrate rotating enclosures, internal fins and porous media. Most previous studies have addressed simpler geometries or neglected the presence of internal fins, even though fins play a vital role in modifying thermal performance by increasing the heat transfer surface area and disrupting flow symmetry. In addition, the effects of porous media on convective transport in non-Newtonian fluids have often been overlooked, limiting the practical relevance of existing models.

To address these gaps, the present study investigates mixed convection of power-law fluids in a complex annular configuration consisting of a rotating outer cylinder and a stationary finned inner cylinder embedded in a porous medium. This configuration is of particular interest for applications, such as rotating machinery, advanced cooling systems and compact heat exchangers. By systematically varying the Reynolds, Richardson, and Darcy numbers, as well as the power-law index, this work aims to clarify the coupled effects of rotation, non-Newtonian rheology, and porous resistance on fluid flow and heat transfer characteristics.

Numerical simulations are conducted using the finite volume method, providing detailed insights into the thermo-fluid dynamics within the enclosure. The results presented herein aim to

extend existing knowledge and offer a comprehensive framework for analysing non-Newtonian mixed convection in geometrically and physically complex systems relevant to modern thermal engineering applications.

2. Physical problem and mathematical formulations

Figure 1 presents the physical and computational configuration of the problem, consisting of a horizontal finned inner cylinder enclosed within a concentric outer cylinder. The inner cylinder, featuring uniformly distributed radial fins, is maintained at a higher temperature (T_h), while the outer boundary is held at a lower temperature (T_c), ensuring a consistent thermal gradient ($T_h > T_c$). The outer cylinder rotates counterclockwise with a constant angular velocity (Ω). The space between the two cylinders is filled with a porous medium saturated with a non-Newtonian power-law fluid. Three distinct power-law indices ($n = 0.6, 1.0$, and 1.6) are investigated to represent shear-thinning, Newtonian and shear-thickening behaviours, respectively.

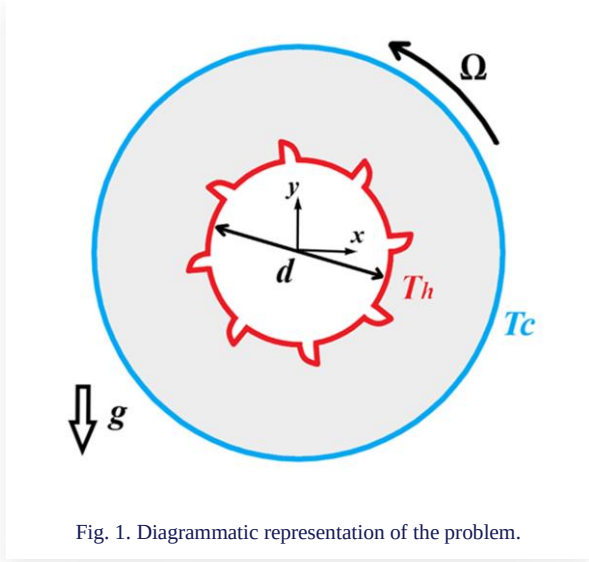


Fig. 1. Diagrammatic representation of the problem.

Mixed convection arises when both natural and forced convection mechanisms contribute to heat transfer. In this study, forced convection is induced by the rotation of the outer cylinder, while natural convection is driven by the temperature difference between the surfaces of the two cylinders. The Reynolds number determines the cylinder's rotational velocity value, while the Richardson number quantifies the balance between forced and natural convection effects. Four Richardson numbers are analysed: 0, 1, 2 and 3. The medium's permeability, represented by the Darcy number (Da), is investigated within the range of 10^{-5} to 10^{-2} .

The assumptions of laminar and non-Newtonian flow, steady state, mixed convection and constant fluid properties are adopted in this study. The equations of continuity, momentum and energy, subject to the Boussinesq approximation and the effect of negligible dissipation, are written in their dimensionless form as follows:

– the equation of continuity:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \quad (1)$$

– the equation of momentum along the x -direction:

$$u^* \left(\frac{\partial u^*}{\partial x^*} \right) + v^* \left(\frac{\partial v^*}{\partial y^*} \right) = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\text{Re}} \left(\frac{\partial \tau_{xx}^*}{\partial x^*} + \frac{\partial \tau_{yx}^*}{\partial y^*} \right), \quad (2)$$

– the equation of momentum along the y -direction:

$$u^* \left(\frac{\partial v^*}{\partial x^*} \right) + v^* \left(\frac{\partial v^*}{\partial y^*} \right) = -\frac{\partial p^*}{\partial y^*} + \frac{1}{\text{Re}} \left(\frac{\partial \tau_{xy}^*}{\partial x^*} + \frac{\partial \tau_{yy}^*}{\partial y^*} \right) + \text{Ri } T^*, \quad (3)$$

– the equation of energy:

$$u^* \left(\frac{\partial T^*}{\partial x^*} \right) + v^* \left(\frac{\partial T^*}{\partial y^*} \right) = \frac{1}{\text{Pe}} \left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} \right), \quad (4)$$

where Re and Pe are the Reynolds and Peclet numbers, Ri is the Richardson number, u^* and v^* are the fluid dimensionless velocities in the x^* - and y^* -directions, and dimensionless pressure and temperature are represented by p^* and T^* , respectively. The following represent the dimensionless variables:

$$x^* = \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad (5)$$

$$u^* = \frac{u}{\Omega d}, \quad v^* = \frac{v}{\Omega d}, \quad (6)$$

$$p^* = \frac{p}{\rho(\Omega d)^2}, \quad T^* = \frac{T - T_c}{T_h - T_c}, \quad (7)$$

$$\text{Pe} = \text{RePr}. \quad (8)$$

where Pr is the Prandtl number and d is the inner cylinder's diameter.

The following equation represents the power-law fluid's behaviour:

$$\tau_{ij} = 2\eta \varepsilon_{ij}, \quad (9)$$

where the viscous stress tensors and the rate of deformation are represented, respectively, by ε_{ij} and τ_{ij} . ($i, j = x, y$). Furthermore, for power-law fluids, the fluid viscosity, denoted by η , is defined (in dimensional form) as follows:

$$\eta = m \left(\frac{I_2}{2} \right)^{\frac{n-1}{2}}, \quad (10)$$

where I_2 is the second invariant of the rate of deformation tensor, m is the consistency index, and n is the power-law index. A fluid that is shear-thinning is represented by $n < 1$, a Newtonian limit by a value of $n = 1$, and a shear-thickening fluid by $n > 1$. The following equation gives I_2 in Cartesian coordinates:

$$\left(\frac{I_2}{2} \right) = 2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left[\left(\frac{\partial u}{\partial y} \right) + \left(\frac{\partial v}{\partial x} \right) \right]^2. \quad (11)$$

Generally, Grashof number, Richardson number and Darcy number for power-law fluids are computed as follows:

$$\text{Gr} = g\beta_T \Delta T d^3 \left(\frac{\rho}{m} \Omega^{1-n} \right)^2, \quad (12)$$

$$\text{Ri} = \frac{g\beta_T \Delta T d^3}{(\Omega d)^2} = \frac{\text{Gr}}{\text{Re}^2}, \quad (13)$$

$$Da = \frac{K}{d^2}, \quad (14)$$

where ρ , g , β_T , and K are the density of the fluid, the gravitational acceleration, volumetric expansion coefficient and permeability, respectively.

The Nusselt number, or average Nusselt number, is calculated by integrating local values along the inner cylinder's surface. The following are the local and average values of Nu, respectively:

$$Nu_L = \left(\frac{\partial T^*}{\partial n} \right)_{wall}, \quad Nu = \frac{1}{A} \int_S Nu_L dA, \quad (15)$$

where $\mathbf{n}$ is the normal vector to the wall and S represents the surface of the heated inner cylinder.

3. Numerical procedure

The numerical simulations in this study were carried out using Ansys-CFX, a commercial computational fluid dynamics (CFD) tool that transforms the governing equations of fluid flow and heat transfer into algebraic form. These equations were discretised using the finite volume method, with boundary conditions applied throughout the computational domain. To ensure high solution accuracy, a convergence criterion of 10^{-6} was adopted for both the momentum and energy equations. The high-resolution scheme was utilised to accurately capture convection effects, while the SIMPLEC (semi-implicit method for pressure linked equations-consistent) algorithm was implemented to efficiently couple pressure and velocity fields, offering enhanced stability for steady-state analyses.

To validate the accuracy of the numerical results, a grid independence study was conducted, acknowledging the significant influence of mesh resolution on simulation precision. The computational domain was discretised using a hybrid mesh structure with refined quadrilateral elements in the vicinity of the inner and outer walls to better resolve boundary layer gradients. As illustrated in Fig. 2 and detailed in Table 1, three progressively refined mesh configurations were examined: M1 (coarse), M2 (medium) and M3 (fine). The average Nusselt number along the heated cylindrical surface was calculated

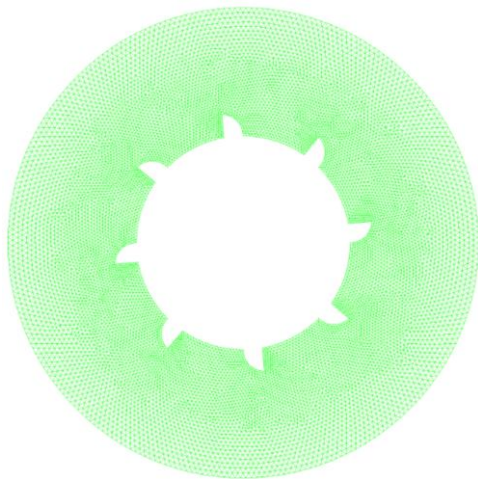


Fig. 2. The structure of the grid used for the calculations.

Table 1. Grid independency test for $n = 1.6$, $Re = 25$, $Ri = 3$ and $Da = 10^{-3}$.

Mesh	Elements	Nusselt number	Difference, %
M1	122 000	1.50581	1.53
M2	244 000	1.48278	0.0112
M3	488 000	1.48294	–

under representative conditions: $n = 1.6$, $Re = 25$, $Ri = 3$ and $Da = 10^{-3}$. The comparison showed that the medium-resolution mesh (M2) produced results nearly identical to those of the finest mesh (M3), confirming that M2 offers an optimal compromise between computational efficiency and solution accuracy. Hence, M2 was selected for all subsequent simulations, balancing computational cost and numerical precision.

The accuracy and robustness of the present numerical model were verified through comparisons with established benchmark studies. The first validation involved comparing results with the classical work of Kuehn and Goldstein [38] and the study by Matin and Khan [39], both of which examined buoyancy-driven flow in annular enclosures. For this case, the working fluid was considered Newtonian ($n = 1$) with properties of air ($Pr = 0.71$), and the Rayleigh number was varied from 10^3 to 10^6 . As shown in Fig. 3A, the predicted variation of the average Nusselt number with Rayleigh number ($Ra = Pr Gr$) for $Pr = 0.71$ and $n = 1$ closely matches the reference data, confirming the model's ability to capture natural convection phenomena accurately.

A second comparison assessed the model's capability to represent non-Newtonian fluid behaviour, focusing on the influence of the power-law index. Results were compared to those reported by Matin et al. [40] under conditions of $Pr = 100$ and a blockage ratio of 0.25. The present model reproduced the same trend of heat transfer enhancement for shear-thinning fluids ($n < 1$) and suppression for shear-thickening fluids ($n > 1$), consistent with theoretical expectations. Figure 3B illustrates strong agreement in the Nusselt number variation with respect to n , validating the model's treatment of fluid rheology and ensuring accurate representation of non-linear viscosity effects.

Lastly, validation was extended to the study by Sheikholeslami [41], which investigated natural convection between concentric cylinders filled with a porous medium and subjected to a temperature difference between the inner and outer walls. The comparison was made at $Da = 10^{-3}$ and $Ra = 10^5$. As seen in Fig. 3C, the present results show excellent consistency with the benchmark, further supporting the reliability of the adopted numerical approach. The close match across multiple validation cases confirms that the current CFD model is robust and capable of accurately resolving complex coupled effects of buoyancy, rotation and non-Newtonian fluid rheology in porous media.

4. Results and discussion

This study investigates the heat transfer and flow behaviour of a power-law fluid in a porous medium confined between a stationary finned inner cylinder and a rotating outer cylinder. The analysis focuses on the influence of key parameters: the Reynolds number ($Re = 5, 15, 25$ and 35), Richardson number ($Ri = 0, 1, 2$

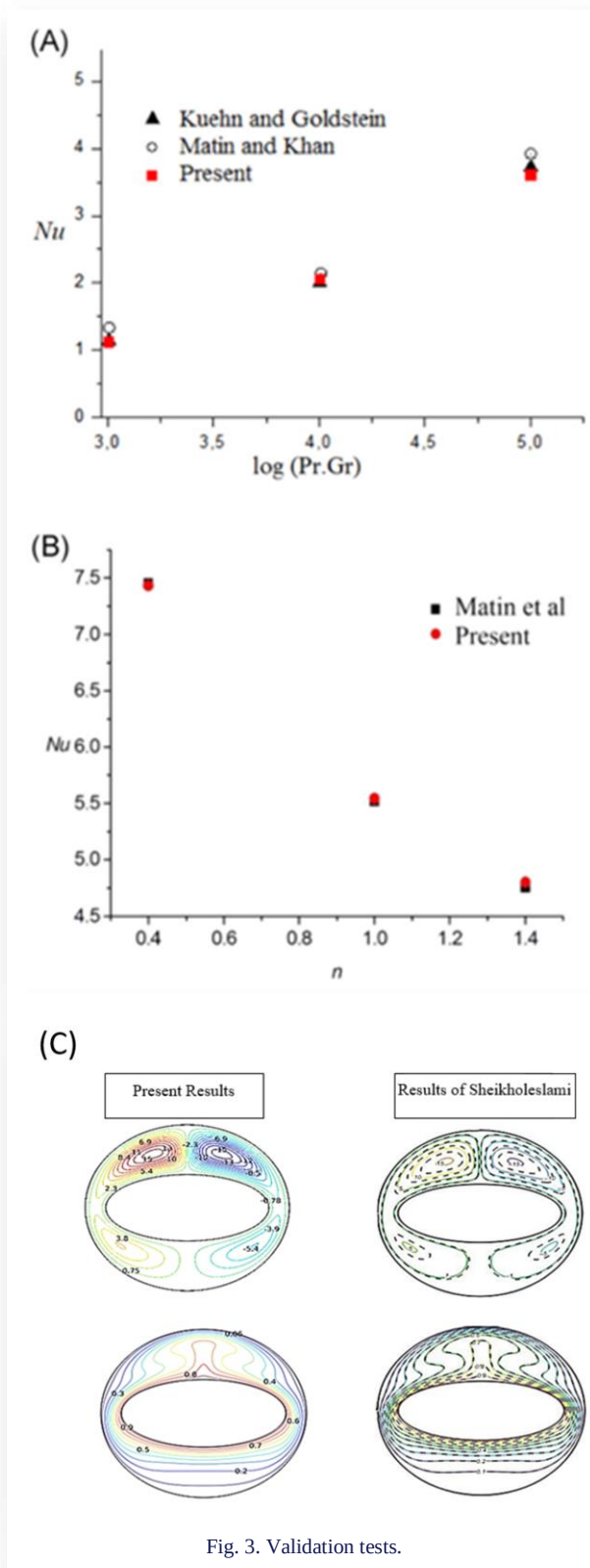


Fig. 3. Validation tests.

and 3), Darcy number ($Da = 10^{-5}$ to 10^{-2}) and power-law index ($n = 0.6, 1$ and 1.6), with a fixed Prandtl number ($Pr = 50$). Flow dynamics and thermal performance are explored using streamlines, isotherms and the Nusselt number. Some parameter values are selected based on a previous study [42].

Mixed convection arises when both forced and natural convection mechanisms coexist. In this study, natural convection is driven by the temperature difference between the two cylinders,

while forced convection results from the rotation of the outer cylinder. When $Ri = 0$, the flow is purely governed by forced convection. As Ri increases, buoyancy forces (representing natural convection) gradually become more significant. Simultaneously, the rotational speed of the outer cylinder, which governs the magnitude of forced convection, increases with Reynolds number.

Figure 4 illustrates the flow patterns represented by streamlines for various Reynolds numbers and power-law indices. At low Re , viscous forces dominate, resulting in smooth, symmetric streamlines around the finned inner cylinder. The flow remains relatively weak, with minimal recirculation. As Re increases, inertial effects become more pronounced, leading to the formation of a closed counter-rotating vortex on the left side of the domain. The streamlines extend further outward and form larger vortices, reflecting intensified flow activity around the fins.

At higher Re , the centrifugal force generated by the rotating outer cylinder significantly enhances momentum transfer, resulting in asymmetric vortical structures that extend toward the upper region of the enclosure. The secondary vortices near the fin tips indicate localised shear-layer separations, which enhance local mixing and amplify heat exchange along the finned surfaces.

For shear-thinning fluids ($n = 0.6$), the apparent viscosity decreases with increasing shear rate, which enhances fluid motion and results in more elongated and active recirculation zones, especially at higher Re . In contrast, as the power-law index increases ($n = 1.6$, shear-thickening behaviour), viscosity increases with shear rate, damping the fluid motion. This suppresses vortex formation, leading to smaller and less intense recirculation zones compared to the shear-thinning case.

Figure 5 displays the thermal fields (isotherms) under the same parameter conditions. The temperature distribution closely mirrors the fluid flow patterns. At low Re , nearly concentric isotherms are observed around the heated inner cylinder, indicating weak convective effects and localised thermal gradients. As Re increases, convection dominates, distorting the isotherms and pushing them outward due to intensified fluid motion.

This outward displacement signifies a transition from conduction-controlled to convection-dominated transport. The stretching of isotherms in the direction of rotation reflects enhanced azimuthal heat transport, while compression near stagnation zones reveals localised temperature build-up due to limited mixing.

The rotational direction of the outer cylinder enhances the effect of buoyancy on the right side of the domain, promoting heat transfer. On the upper part of the inner cylinder, the isotherms are further stretched and distorted, indicating stronger convective mixing. On the left side, however, the presence of recirculating flow thickens the isotherms and reduces thermal transport efficiency. A more detailed observation shows that the region downstream of the fin tip experiences a local thermal wake, where thermal diffusion dominates due to reduced local velocity magnitude. A notable feature is the tongue-like structure in the thermal boundary layer at the top of the inner cylinder. This structure reflects reduced fluid mixing, which limits

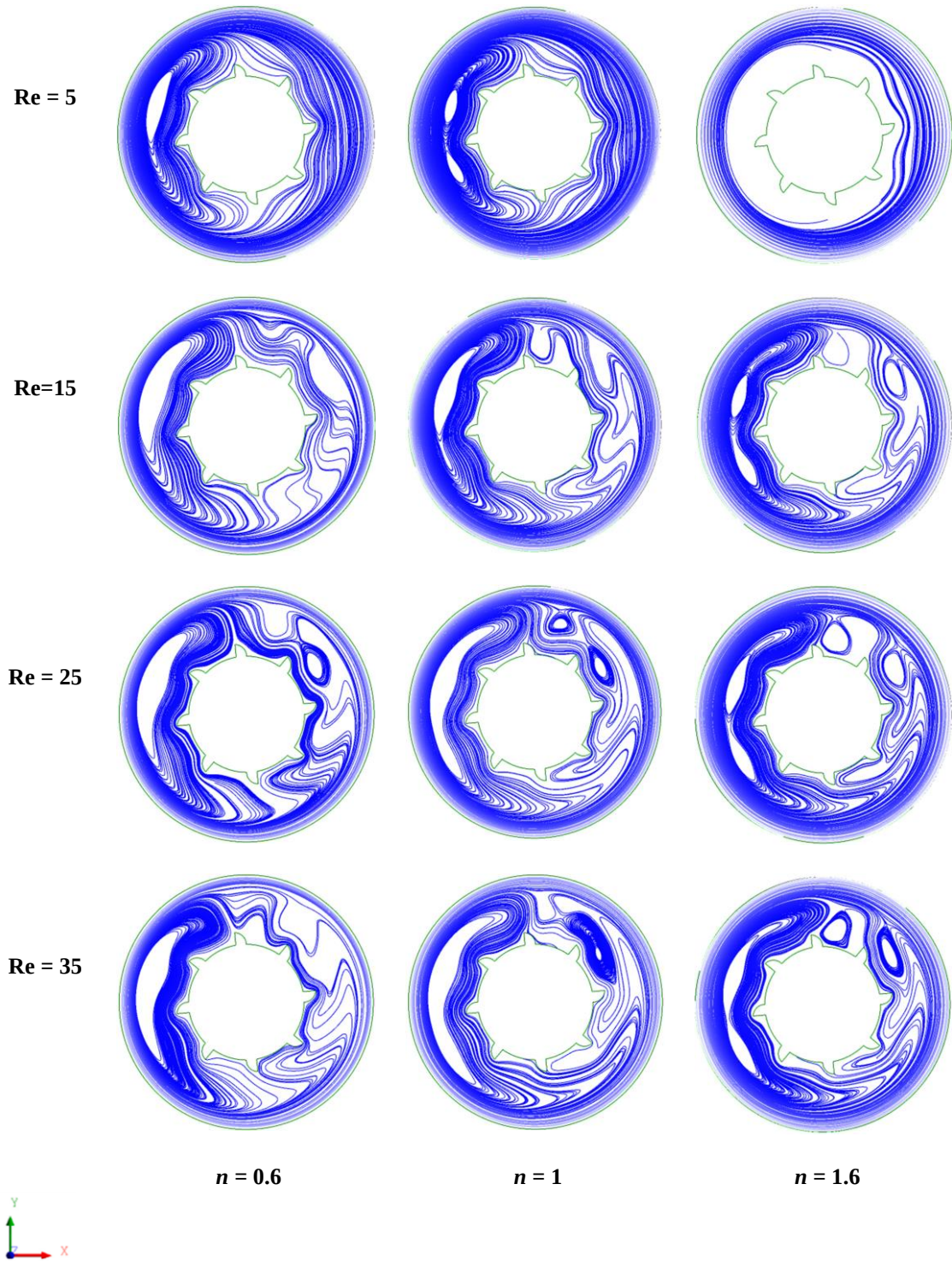


Fig. 4. Streamlines for various values of Re and n at $Da = 10^{-2}$ and $Ri = 3$.

convective heat dispersion. As n increases, this tongue splits into two branches due to altered flow patterns. This bifurcation represents a competition between viscous damping (which stabilises the thermal layer) and centrifugal enhancement (which destabilises it). The higher viscosity of shear-thickening fluids suppresses upward motion above the finned cylinder, leading to localised heat accumulation and bifurcated thermal zones around the fins.

Figure 6 illustrates the flow patterns represented by streamlines for various Richardson numbers and power-law indices, at fixed values of Reynolds number ($Re = 25$) and Darcy number ($Da = 10^{-2}$). When Ri is low, forced convection is dominant, resulting in a stable rotational flow between the cylinders. As Ri increases, buoyancy forces become more significant, giving rise to vertically oriented flow structures and the development of counter-rotating vortices. At intermediate Ri ($= 1-2$), the flow field exhibits mixed characteristics; the

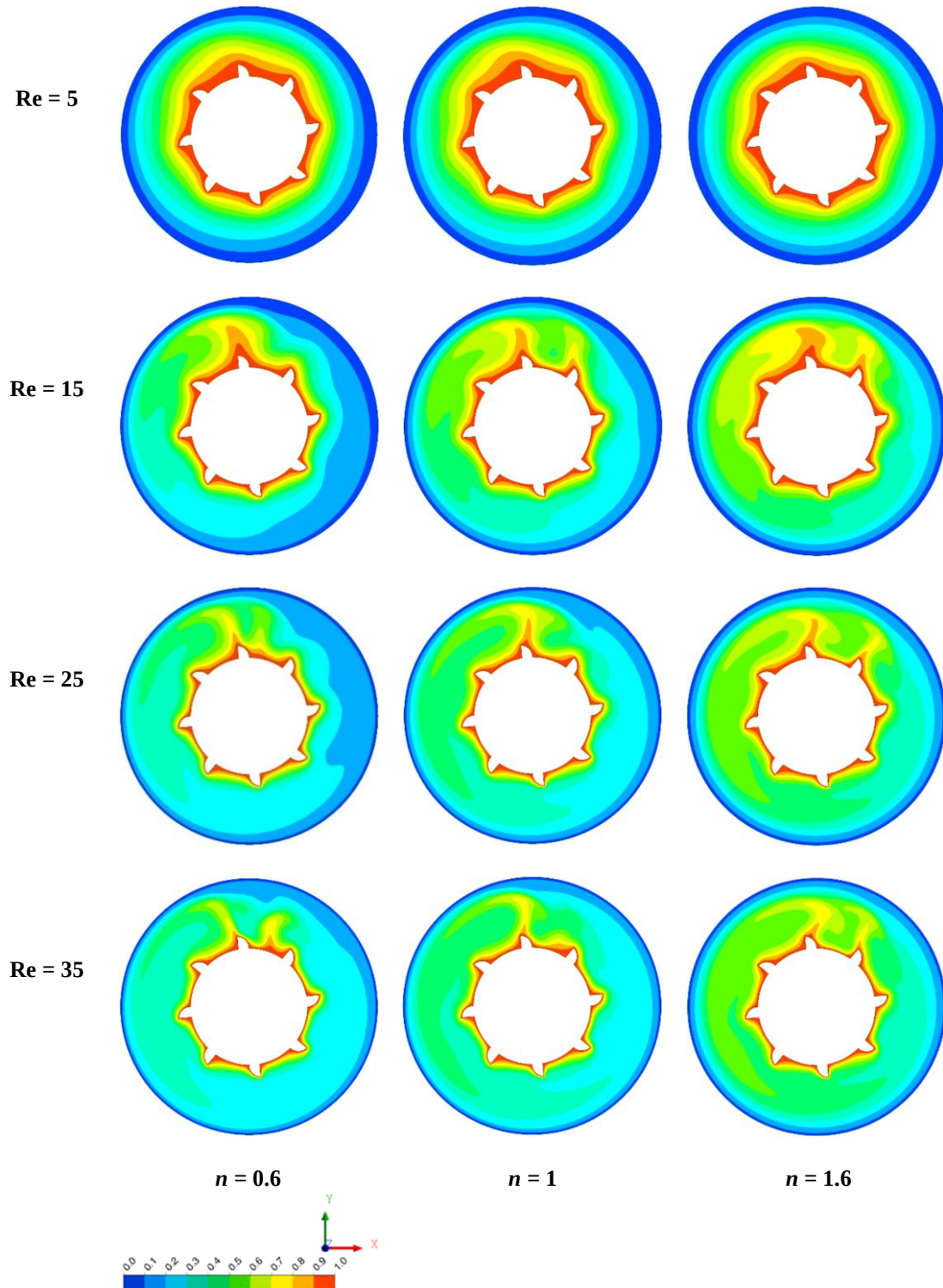


Fig. 5. Isotherms for various values of Re and n at $Da = 10^{-2}$ and $Ri = 3$.

upper vortex strengthens as buoyancy lifts hot fluid upward, while the lower region remains rotation-driven. This dual mechanism introduces asymmetry in the streamline topology, redistributing shear stresses along the finned surface.

For the shear-thinning fluid ($n = 0.6$), the enhanced flow mobility leads to more elongated and active recirculation zones. In contrast, increasing n suppresses these features: the

shear-thickening fluid ($n = 1.6$) exhibits higher apparent viscosity, which dampens the flow and yields smaller, weaker vortices. This demonstrates that buoyancy–viscosity coupling is nonlinear: buoyancy-induced motion is amplified in low-viscosity regions (shear-thinning), but strongly suppressed in high-viscosity zones (shear-thickening), leading to distinct thermal stratifications.

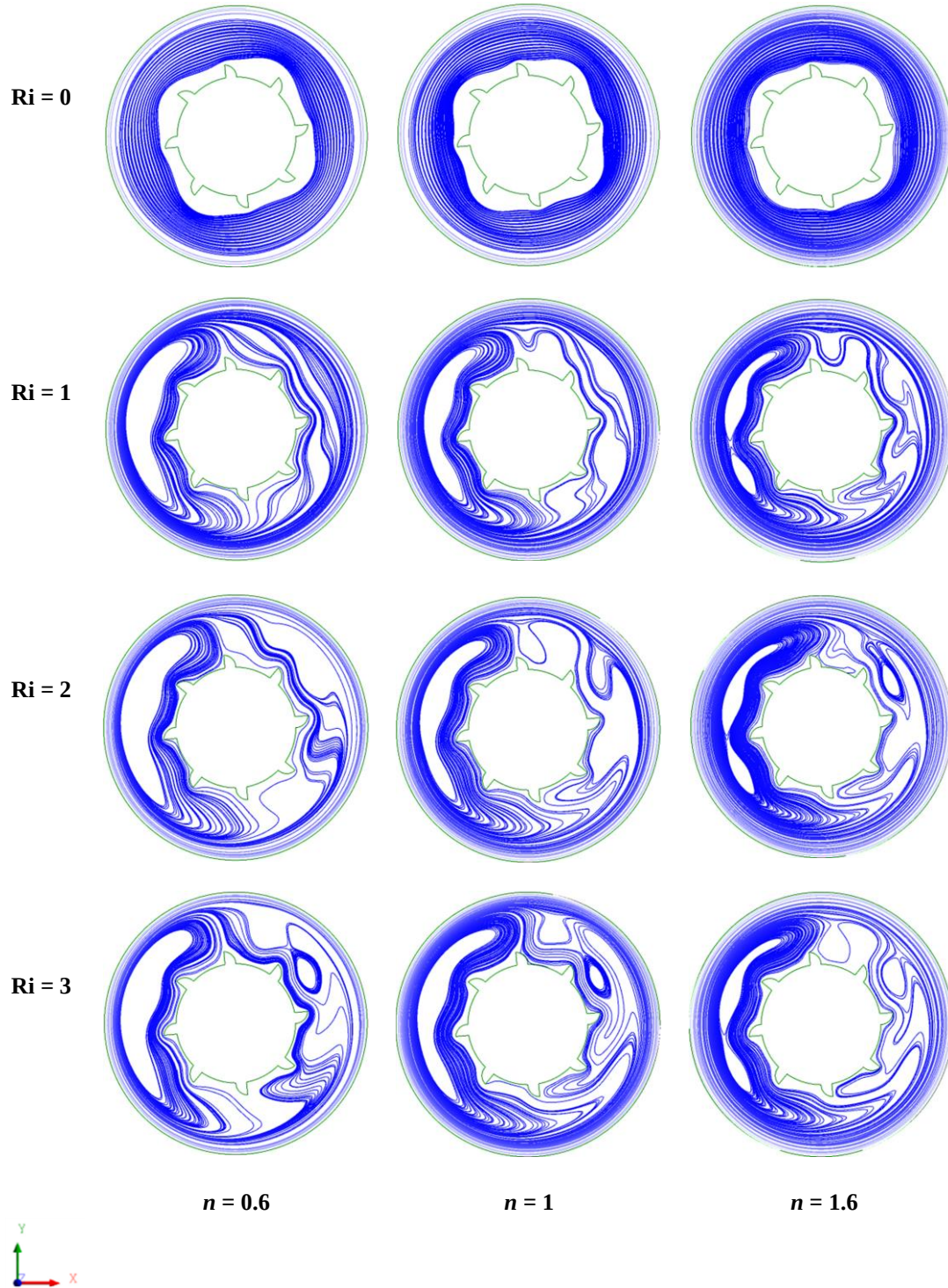


Fig. 6. Streamlines for various values of Ri and n at $Da = 10^{-2}$ and $Re = 25$.

Figure 7 presents the corresponding isotherms under the same conditions. At $Ri = 0$, the isotherms remain nearly parallel, showing that forced convection dominates. As Ri increases, the isotherms distort vertically, indicating a growing influence of buoyancy-driven flow. The upward migration of isotherms at higher Ri indicates the formation of a thermal plume above the inner cylinder, characteristic of buoyancy-dominated convection. The left-right asymmetry intensifies with Ri in-

crease, showing that buoyancy breaks rotational symmetry and creates preferential heat pathways along the upward direction.

For $n = 0.6$, the higher mobility produces thinner thermal boundary layers and more dispersed temperature fields. In contrast, for $n = 1.6$, reduced motion thickens the thermal layers, limiting heat diffusion and causing localised heat build-up near the inner cylinder. This inverse relationship between viscosity

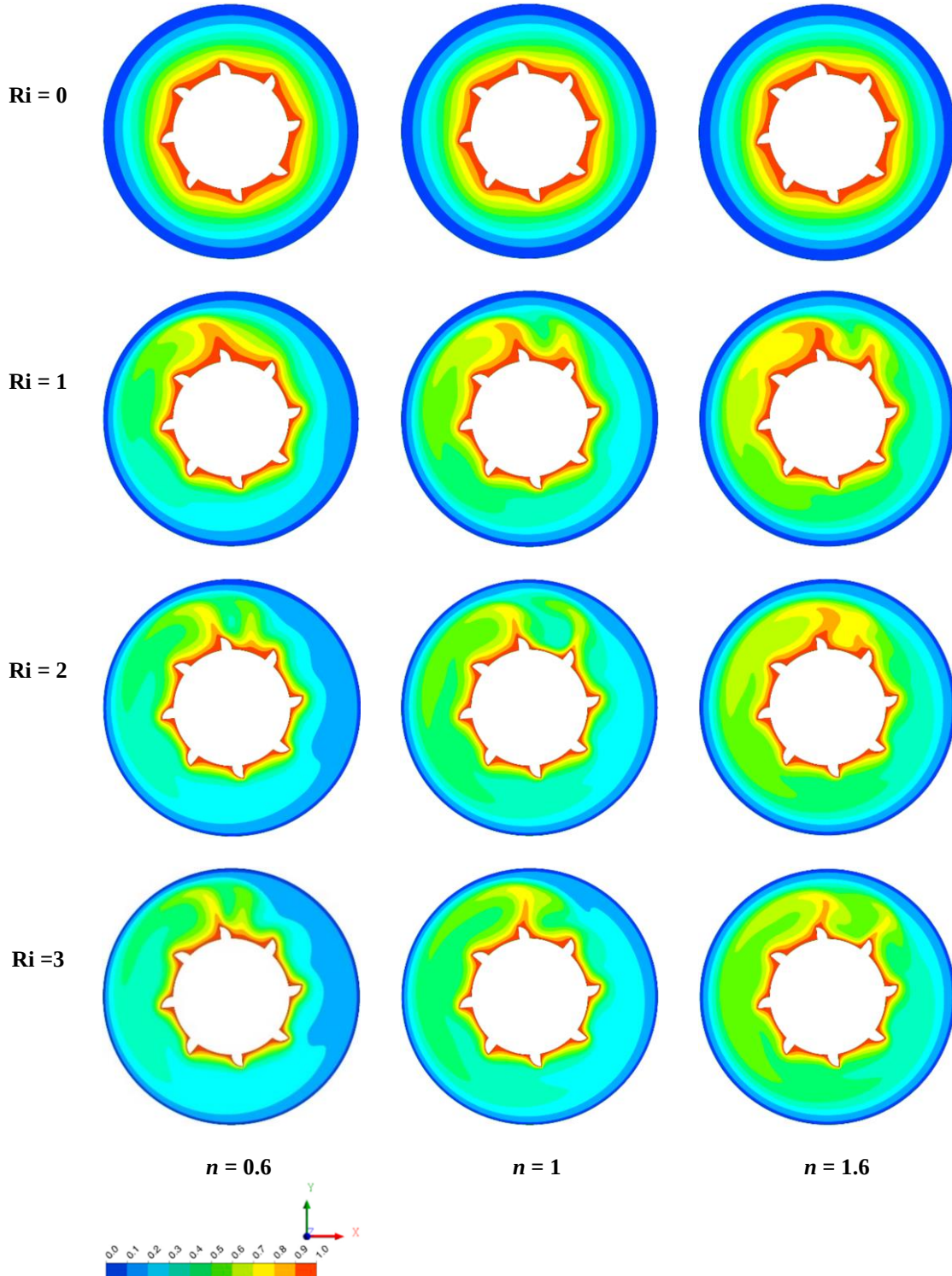


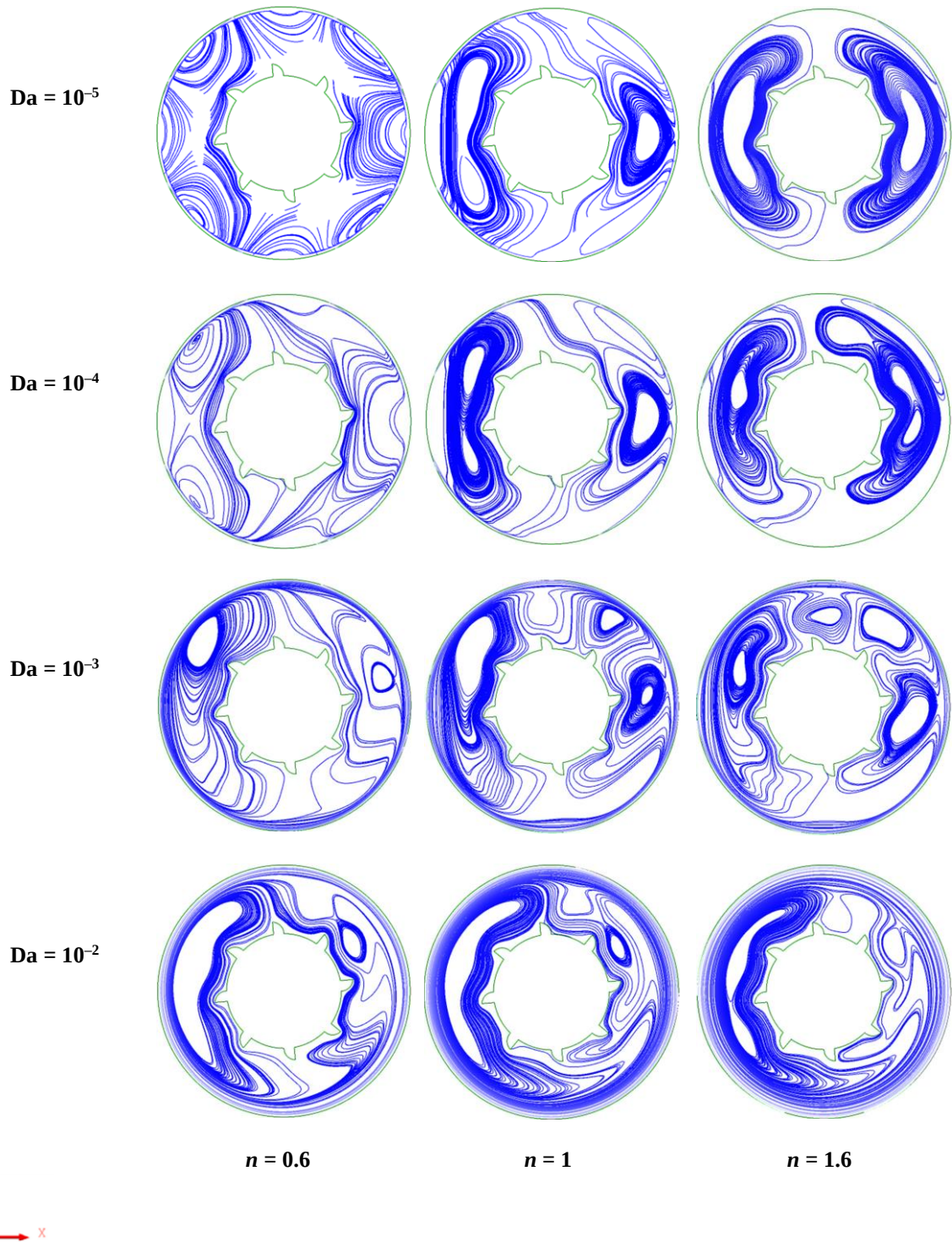
Fig. 7. Isotherms for various values of Ri and n at $Da = 10^{-2}$ and $Re = 25$.

and boundary-layer thickness confirms that rheological properties govern not only momentum diffusion, but also the thermal resistance adjacent to the heated surfaces.

Figure 8 shows the streamline patterns for various Darcy numbers and power-law indices, at fixed $Re = 25$ and $Ri = 3$. At low Da , the porous matrix restricts fluid motion, resulting in weak, localised recirculation around the finned cylinder. As Da increases, permeability improves, allowing stronger motion and

larger vortices to form. At intermediate Da values (10^{-4} – 10^{-3}), the competition between viscous drag and porous resistance yields a partially developed circulation where flow leakage through the porous matrix modifies vortex symmetry. This interplay introduces a mixed conduction–convection regime that is highly sensitive to n .

For $n = 0.6$, shear-thinning behaviour intensifies recirculation, especially at higher Da , while for $n = 1.6$, higher viscosity inhibits

Fig. 8. Streamlines for various values of Da and n at $Re = 25$ and $Ri = 3$.

vortex development, producing smaller and weaker zones. This behaviour underscores that the porous structure modulates the influence of non-Newtonian viscosity; permeability enhancement magnifies rheological effects, whereas low Da dampens them.

Figure 9 displays the associated isotherms for the same Da , Re and Ri conditions. At low Da , limited flow mobility yields nearly concentric isotherms, reflecting conduction-dominated heat transfer. As Da increases, enhanced convection stretches and distorts the isotherms outward, especially for $n = 0.6$, which

exhibits thinner boundary layers and better heat dispersion. The deformation of isotherms indicates a transition from localised conduction to bulk convection. At high Da , the temperature contours align with the dominant flow direction, confirming that fluid advection becomes the main heat transport mechanism.

For $n = 1.6$, high viscosity reduces fluid motion, producing thicker isotherms and localised thermal accumulation near the heated surface. These results emphasise the complex interaction between fluid rheology and porous-medium permeability in

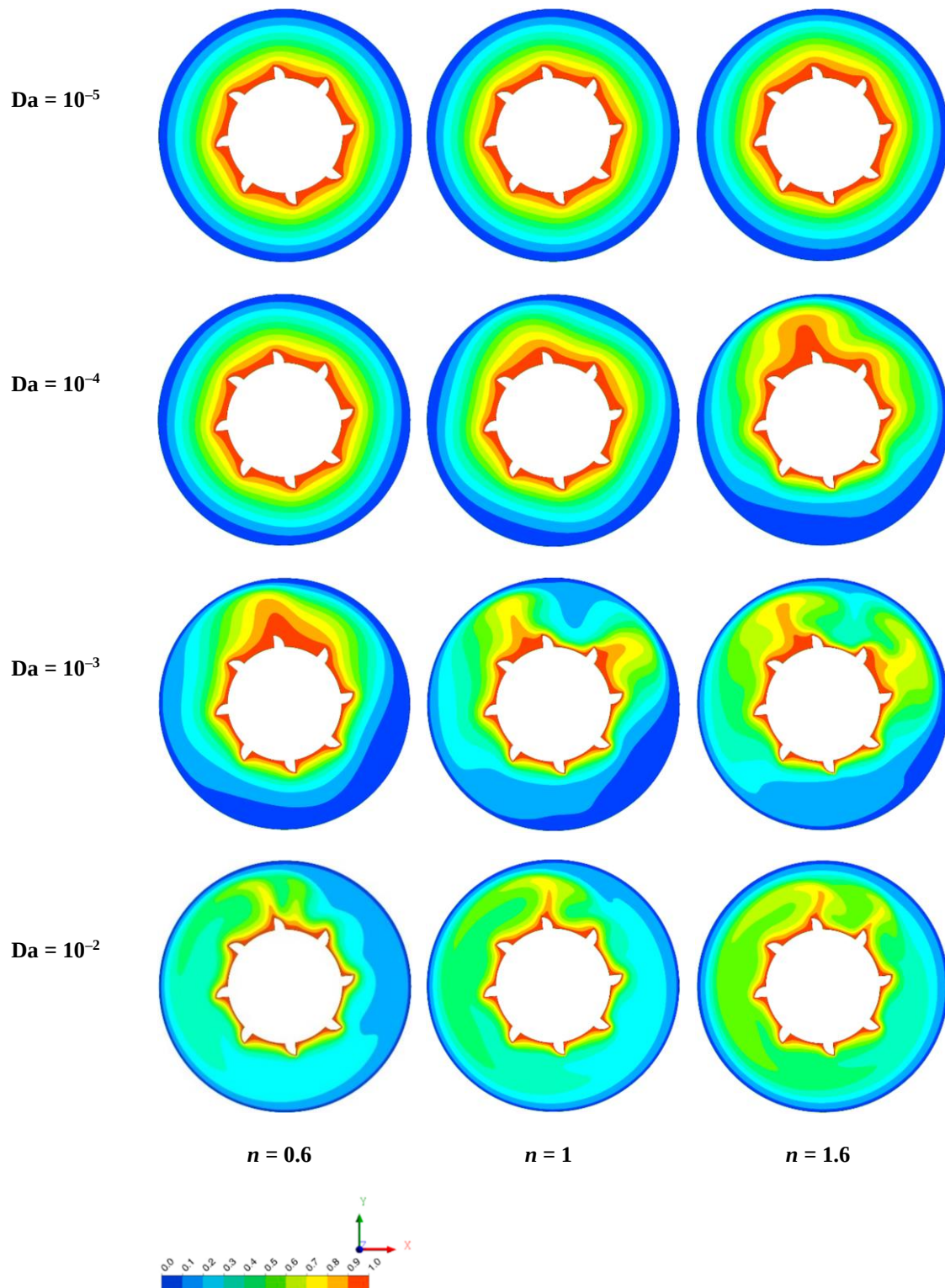


Fig. 9. Isotherms for various values of Da and n at $Re = 25$ and $Ri = 3$.

governing thermal transport. From a design standpoint, increasing Da enhances heat transfer up to a threshold, beyond which excessive permeability may shorten fluid residence time and slightly diminish local heat exchange.

Figure 10 illustrates how the average Nusselt number varies with Richardson number, power-law index and Reynolds number, at a fixed Darcy number ($Da = 10^{-2}$). At $Re = 5$, Nu remains nearly constant across Ri values, indicating that viscous effects dominate

and suppress convective mixing. In this low- Re regime, heat transfer occurs mainly through conduction-like mechanisms, with negligible contributions from buoyancy forces.

As Re increases to 15 and 25, Nu rises more noticeably with Ri , reflecting the strengthening interaction between forced and natural convection. In these regimes, forced convection provides the baseline flow, while buoyancy-driven motion enhances mixing and promotes more effective thermal transport. The highest

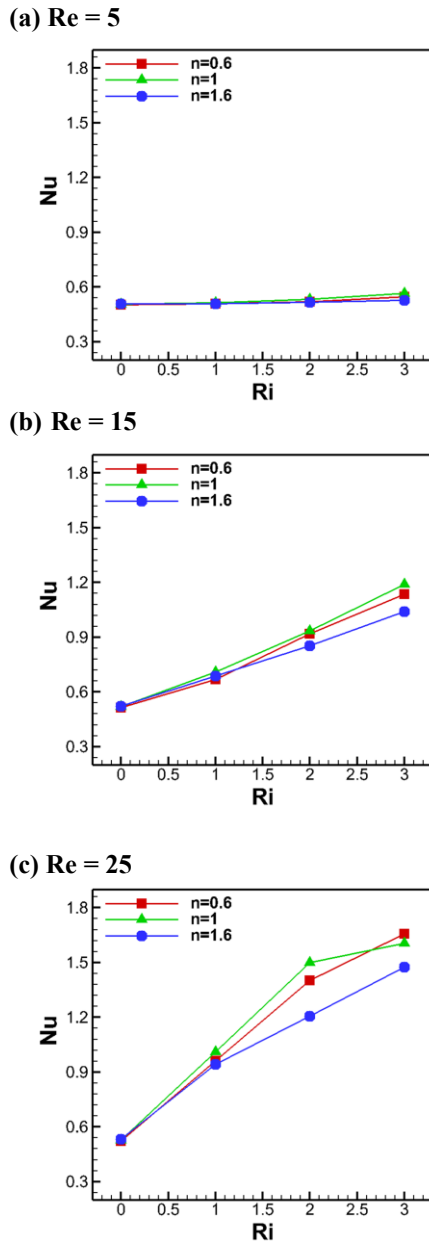


Fig. 10. Values of Nu vs. Ri and n at $Da = 10^{-2}$ for: (a) $Re = 5$, (b) $Re = 15$ and (c) $Re = 25$.

Nu values are typically observed for $n = 1$ (Newtonian fluid), which balances fluid mobility and thermal diffusion most effectively.

For $n = 0.6$ (shear-thinning fluid), Nu values are generally lower than those for Newtonian fluids under most conditions, particularly at lower Re or Ri. Although reduced viscosity promotes flow mobility, it may reduce thermal gradients and limit effective mixing. However, at higher Re and Ri (where inertial and buoyancy effects dominate) this enhanced mobility can improve thermal performance. For example, at $Re = 25$ and $Ri = 3$, Nu for $n = 0.6$ surpasses that of $n = 1$. On the other hand, for $n = 1.6$ (shear-thickening fluid), increased viscosity restricts motion and leads to the lowest Nu values across all conditions examined.

Figure 11 presents the variation of Nu with Ri, n and Da, at fixed $Re = 25$. The influence of Da is strongly dependent on fluid

rheology, indicating nonlinear interactions between permeability, viscosity behaviour and thermal gradients. At $Da = 10^{-5}$, the porous medium resists flow significantly, resulting in conduction-dominated heat transfer, where Nu remains low and nearly constant across all values of n and Ri. As Da increases to 10^{-4}

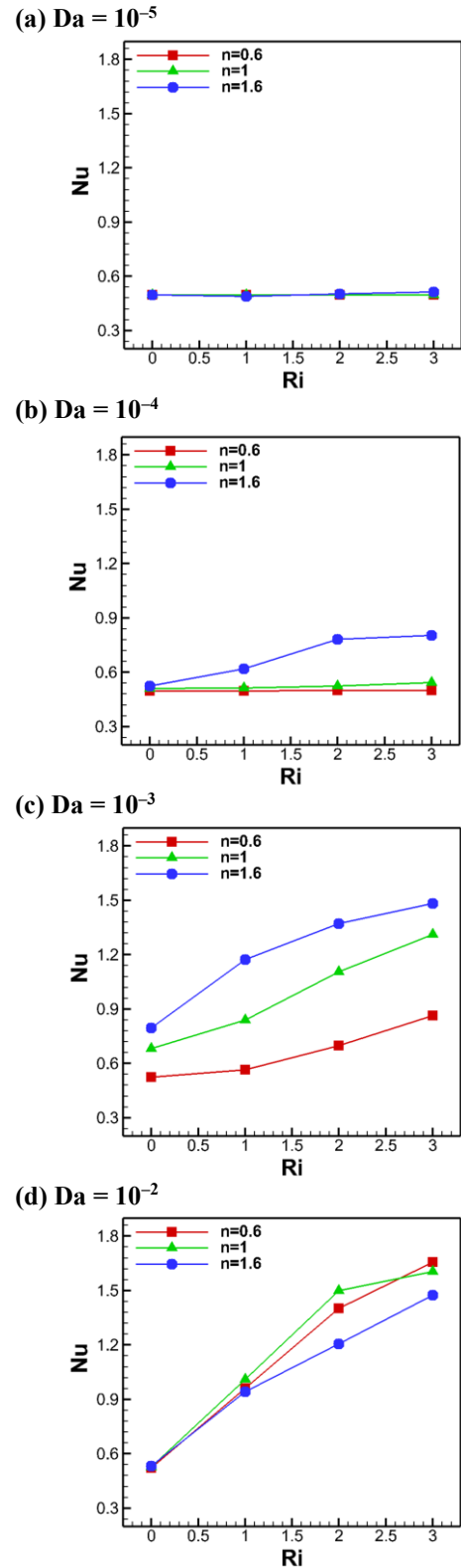


Fig. 11. Values of Nu vs. Ri and n at $Re = 25$ for: (a) $Da = 10^{-5}$, (b) $Da = 10^{-4}$, (c) $Da = 10^{-3}$ and (d) $Da = 10^{-2}$.

and 10^{-3} , permeability improves, allowing more vigorous convection and higher Nu values. Interestingly, the highest Nu values at these intermediate Da levels are observed for $n = 1.6$ (shear-thickening fluid). This suggests that, under moderate resistance, the increased shear-dependent viscosity promotes stronger temperature gradients and localised heat transfer enhancement.

At $Da = 10^{-2}$, the porous medium offers minimal resistance to flow, leading to robust convective heat transfer. Under these conditions, Newtonian fluids ($n = 1$) exhibit the highest Nu, benefiting from an optimal trade-off between flow and heat dispersion. Shear-thickening fluids ($n = 1.6$) underperform in this regime due to their higher viscosity, which limits motion and thermal mixing. Although shear-thinning fluids ($n = 0.6$) benefit from enhanced flow mobility, they do not achieve the highest Nu under any condition shown. This indicates that, despite faster flow, their reduced viscosity may hinder the development of strong thermal gradients, which are essential for efficient heat transfer.

A particularly noteworthy observation in Fig. 11 (for $Da = 10^{-2}$) is the critical Richardson number around $Ri = 2.6$, where the Nu values for the shear-thinning fluid ($n = 0.6$) and the Newtonian fluid ($n = 1$) intersect. This intersection marks a shift in the dominant heat transfer mechanism. For $Ri < 2.6$, Newtonian fluids outperform due to their balanced viscosity, which supports both momentum and thermal diffusion in mixed convection. As Ri increases and buoyancy dominates, the low viscosity of shear-thinning fluid allows for stronger buoyancy-driven flows and more effective convective mixing. Beyond $Ri = 2.6$, Nu for $n = 0.6$ exceeds that for $n = 1$, indicating better thermal performance in natural convection regimes. This finding highlights the nonlinear interaction between rheological properties and buoyancy forces, with important implications for optimising thermal systems involving non-Newtonian fluids in porous enclosures.

5. Conclusions

This study presented a numerical investigation of mixed convection heat transfer in a porous annulus formed by a stationary finned inner cylinder and a rotating outer cylinder, filled with a non-Newtonian power-law fluid. The governing equations were solved using the finite-volume method implemented in Ansys-CFX, with the fluid's rheology represented by the Ostwald's law model. The analysis systematically examined the combined influence of four key dimensionless parameters – the Reynolds number (Re), the Richardson number (Ri), the power-law index (n) and the Darcy number (Da) – on flow behaviour and heat transfer performance. The main conclusions are summarised as follows:

- 1) Increasing the Reynolds number from 5 to 25 enhances the flow circulation and convective strength, leading to a nearly 300% rise in the average Nusselt number at $Ri = 3$.
- 2) Shear-thinning fluids promote more vigorous recirculation and outperform Newtonian fluids ($n = 1.0$) under buoyancy-dominated conditions ($Ri \geq 2$), while shear-thickening fluids ($n = 1.6$) suppress motion and reduce thermal transport due to their elevated apparent viscosity.

- 3) A transitional Richardson number of about $Ri = 2.6$ (for $Da = 10^{-2}$) marks the point where the Nusselt number becomes nearly independent of the power-law index. Beyond this value, shear-thinning fluids achieve 15–20% higher heat transfer than Newtonian fluids due to stronger buoyancy-driven circulation.
- 4) Porous permeability (Da) strongly affects the convection regime. Low Da values produce conduction-dominated flow, while higher Da ($\geq 10^{-3}$) enhances convective mixing and results in substantially larger Nusselt numbers through the combined effect of rotation and buoyancy.
- 5) Flow and thermal structures demonstrate nonlinear coupling among rheology, porous resistance and rotation, which controls the balance between forced and natural convection and defines the system's overall heat transfer efficiency.

These findings provide practical design insights for engineers developing compact thermal systems, including porous heat exchangers, geothermal units and rotating machinery cooling devices. The results enable the optimised selection of working fluids and operating parameters to achieve efficient performance under different flow and thermal conditions. Moreover, the integration of machine learning and optimisation techniques offers promising opportunities for the next generation of energy and cooling systems, particularly in applications where the interaction between non-Newtonian fluids and porous media plays a crucial role in determining system efficiency and reliability.

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A method for determining the current-carrying capacity and durability of power cables based on variable ambient temperature and solar radiation

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Abstract

Modern power networks contain a large number of distributed energy sources, particularly photovoltaics. In the case of photovoltaic sources, the associated electrical installation components, including cables installed in free air, are exposed to solar radiation and significant ambient temperature variability. In the classical approach to calculating the current-carrying capacity of power cables in free air, a constant ambient temperature is assumed and the direct influence of solar radiation is usually not considered. Such an approach, under certain conditions for a given country, may lead to an underestimation of this capacity (a cold region of the country) or an overestimation (an exceptionally warm region of the country). This paper proposes an alternative approach for determining the current-carrying capacity of power cables. The concept takes into account daily and annual fluctuations in ambient temperature. This is based on actual data obtained from measurements with a measurement step of one hour. As a case study, the results of calculations for sample weather scenarios for northern Poland are shown. The analysis indicates that under certain real-world conditions, cables may be overheated, which reduces the durability of their insulation. For the case of non-rated ageing of cable insulation due to high temperature, a current-carrying capacity correction factor was determined. This approach, different from the normative scheme, allows for determining the thermal exposure of cables that is close to the real one. The analysis results can be useful in designing electrical installations, particularly when components are located on the building facade or roof.

Keywords: Electrical installations; Heat transfer; Power cables; Solar radiation

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1. Introduction

Determining the thermal parameters of electrical installations, including cable ratings, aroused the interest of researchers already at the end of the 19th century [1]. Currently, the main attention is paid to the continuous current-carrying capacity of power cables/wires, which depends on many factors, as stated in [2]. A general overview of the basic heat transfer concepts for cable systems and associated current-carrying capacity is given

in [3]. There are different methods for determining this capacity. As presented by the authors of the paper [4], these include mainly the finite element method (FEM), which is extensively analysed in [5] with respect to buried power cables in contact with native soil or thermal backfill, as well as the IEC 60287 method [6,7], but the latter is usually used in practice. Standard IEC 60287-1-1 [6], along with the second part IEC 60287-2-1 [7], provides relationships that precisely indicate what the current-carrying capacity depends on. In the con-

Nomenclature

I_{dd}	– current-carrying capacity of the cable, A
I_{dd-cor}	– current-carrying capacity of the cable with the effect of solar radiation, A
L_n	– rated insulation durability at temperature τ_{dd} , year
L_x	– insulation durability at temperature τ_x , year
L_{xi}	– insulation durability corresponding to a given temperature in a time interval t_i , year
L_{tot}	– resultant durability of insulation, year
R_e	– electrical resistance of the core of a power cable, Ω
R_c	– thermal resistance related to the heat emitted by the cable, K/W
t_i	– relative partial interval of insulation operating time at a specific temperature

Greek symbols

$\Delta\Theta_{dd}$ – highest permissible temperature rise of the power cable, K

$\Delta\Theta_{dd-cor}$	– corrected permissible temperature rise of the power cable, K
$\Delta\Theta_s$	– temp. rise of the power cable caused by solar radiation, K
$\Delta\tau$	– change in temperature which causes a twofold change in the durability of the insulation, K
τ_{dd}	– highest permissible long-term temperature of the power cable, $^{\circ}\text{C}$
τ_o	– ambient temperature, $^{\circ}\text{C}$
τ_x	– temperature other than the highest permissible long-term temperature, $^{\circ}\text{C}$

Abbreviations and Acronyms

FEM	– finite element method
IMGW-PIB	– Institute of Meteorology and Water Management – National Research Institute
PV	– photovoltaic
PVC	– polyvinyl chloride

text of the relationships indicated there, many publications analyse the detailed impact of various factors on cables.

Paper [8] indicates that underground cables are affected by the temperature distribution around the cables, the burial depth of the cables, distances between the cables, the thermal resistivity of the soil, as well as the material for cable backfilling. The influence of soil parameters and soil structure on cable current-carrying capacity is also analysed in [9,10]. These papers emphasise the accuracy of the capacity calculations when cables are laid in multi-layered soils or within stabilised/thermal backfill. Good modelling results for such systems have been achieved in these papers. Paper [11] considers the temperature distribution of cables arranged in layers in cable channels. The layered arrangement affects the uneven temperature distribution of the cable layers, which can lead to local overheating of the cable's insulation. As shown in [12], for high-voltage cables, the bonding method of the metal sheaths is also crucial. Currents circulating in the metal sheaths may significantly heat the cables, reducing their current-carrying capacity. The authors of the paper [13] draw attention to the specific heat transfer characteristics when cables are placed in ducts. Previous standard-compliant approaches may lead to unacceptable inaccuracies, so their paper proposes a more precise approach that is simple enough to implement in computer calculations. The permissible load of underground cables is also associated with the phenomenon of local drying out of the soil (due to heat emission from the cables or the influence of local heat sources). This issue is discussed in detail in [14]. In [15], the phenomenon of drying out of the soil is also considered, but in a different scheme – it is assumed that the top layer of soil near the ground surface dries out, while the layer of soil under the cables does not dry out. This is related to some extent to the effect of solar radiation on the ground surface. An important factor determining the current-carrying capacity is the ambient temperature [16] – as the authors note, it has been changing around the world in recent years and even in one country it may result in different permissible loads for the same cable systems. Very high air temperatures in summer may affect the drying out of the soil around the cables and, consequently, the deterioration of the current-carrying capacity of the cables, as discussed in [17]. The authors point out

that in some regions the temperatures may exceed 50°C . High summer temperatures and solar radiation may result (following strong heating of the ground surface) in a significant increase in the temperature of the soil surrounding the cable – it affects the permissible current of this cable, which is also raised in [18]. In some cases, a forced cooling system is used to improve the thermal conditions of buried power cables [19]. In advanced analysis using the FEM-based Arrhenius thermal aging model included in [20], the authors draw attention to the unfavourable thermal conditions of the cable environment as well. They point out possible solar irradiance and high ambient temperature levels, which should be taken into account when determining the current-carrying capacity of cables. In [21], the FEM-based Arrhenius approach is applied again; here, to assess the durability of cable lines exposed to thermal effects of a heating pipeline and several different types of pavements (for different levels of thermal conductivity, emissivity and solar absorptivity). Power cables may be laid in free air for some sections and exposed to direct solar radiation. This radiation is the source of the temperature increase of the cable insulation [22], even if cables are laid in protective pipes [23]. Particularly strong exposure of electrical installation elements, including cables, to high temperatures occurs in the case of photovoltaic (PV) systems, which are numerically demonstrated in [24] and measured in [25]. Paper [26] notes that PV cable systems are exposed to harsh operating conditions, among others, ultraviolet radiation and thermal stress. As a result, the cable insulation degrades, both in terms of electrical and mechanical properties. To decrease the temperature of the PV modules, cooling systems are applied [27,28] – this may beneficially reduce the temperature of the cable connectors in these modules.

A review of the literature and international standards allows us to draw the following conclusions:

- analyses of the current-carrying capacity of power cables usually concern the case when these cables are laid in the ground;
- previous studies have considered the direct impact of solar radiation on outdoor cables to a limited extent, but with the increasingly widespread use of PV systems, this issue has become highly relevant and large-scale;

- ambient temperature of cables (initial temperature before current loading) for a given region of the world is usually assumed to be constant throughout the year, as specified in the standard [29], however, in real conditions for electrical devices placed in the air, the ambient temperature can vary significantly throughout the day and throughout the year.

In the context of the above-mentioned observations, a research gap can be identified regarding a more detailed analysis of the thermal conditions of cables laid in the air – it is mainly about the direct impact of solar radiation and the daily-annual variability of ambient temperature. The exposure of cables to direct solar radiation, as well as the actual ambient temperature, is strongly related to the durability of cable insulation. It should be remembered that unexpected rapid ageing of this insulation reduces its insulating properties and may be the cause of fire and electric shock.

Therefore, this paper, being a modified and extended version of the conference paper [30], presents the results of measurements of daily-annual variability of ambient temperature for a selected region of Poland and the results of numerical calculations of the solar radiation effect. Based on this, sample scenarios of cable temperature changes throughout the day for different seasons are presented, enabling the assessment of the actual thermal exposure of cables in sunny locations. Having the expected temperatures of cable insulation throughout the year, their thermal durability is calculated. A correction factor for the current-carrying capacity of power cables is also determined to achieve insulation ageing at the rated level.

The concept proposed by the authors, based on numerical modelling of solar radiation and taking into account daily and annual ambient temperature variations, is an alternative approach to the design of cable systems. This concept can be used when designing power systems located outdoors and exposed to solar radiation.

2. Current-carrying capacity of cables and numerical calculations of the solar radiation effect

2.1. Current-carrying capacity of power cables according to the standards

The current-carrying capacity I_{dd} of power cables/wires for low-voltage systems can be determined based on the standards [6,7] and presented in the following simplified form [30]:

$$I_{dd} = \sqrt{\frac{\Delta\theta_{dd}}{R_e R_c}} = \sqrt{\frac{\tau_{dd} - \tau_o}{R_e R_c}}, \quad (1)$$

where: $\Delta\theta_{dd}$ – the highest permissible temperature rise of the power cable, τ_{dd} – the highest permissible long-term temperature of the power cable, τ_o – ambient temperature, R_e – electrical resistance of the core of a power cable, R_c – thermal resistance related to the heat emitted by the cable to the environment.

For further considerations, the key parameters in Eq. (1) are temperatures τ_{dd} and τ_o . The temperature τ_{dd} is related to the insulation material, and for polyvinyl chloride (PVC) insulated cables commonly used in electrical installations, it is $\tau_{dd} = 70^\circ\text{C}$.

For design purposes, the ambient temperature in Poland is assumed to be $\tau_o = 25^\circ\text{C}$ [29], and it is stated that the analysed element of the electrical installation is not exposed to solar radiation. Under such conditions, the maximum permissible temperature rise is $\Delta\theta_{dd} = 45\text{ K}$ ($70^\circ\text{C} - 25^\circ\text{C}$). In the case of solar radiation, the current-carrying capacity I_{dd} should be corrected, taking into account the temperature rise of the conductor caused by this radiation. This is presented in the following relation:

$$I_{dd-cor} = \sqrt{\frac{\Delta\theta_{dd-cor}}{R_e R_c}} = \sqrt{\frac{(\tau_{dd} - \tau_o) - \Delta\theta_s}{R_e R_c}}, \quad (2)$$

where: I_{dd-cor} – current-carrying capacity of the cable with the effect of solar radiation, $\Delta\theta_{dd-cor}$ – corrected permissible temperature rise of the power cable, $\Delta\theta_s$ – temperature rise of the power cable caused by solar radiation.

The main difficulty in determining the current-carrying capacity according to Eq. (2) is calculating the temperature rise caused by solar radiation. This rise depends, among other things, on solar irradiance, which in turn depends on latitude, season and time of the day. The temperature rise of the conductors caused by solar radiation has been estimated based on the developed numerical model.

2.2. Numerical calculations of the effect of solar radiation on cables

Numerical calculations were performed for a sample single-core low-voltage cable having a cross-sectional area of the core equal to 35 mm^2 (a cable with copper conductors, insulation and a PVC sheath, 2 m long), which is buried for half its length and arranged in free air for the remaining half (Fig. 1).

Since the ground surface is not perfectly smooth, a small obstacle (0.1 m high) is modelled in front of the cable. Computer simulations are intended to show, among other things, the cable's most vulnerable areas to overheating. The calculations

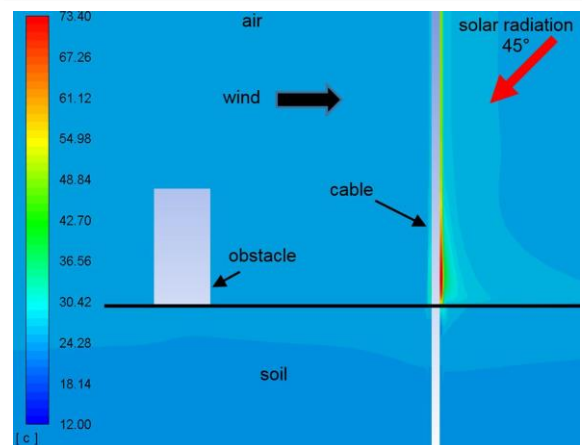


Fig. 1. Temperature distribution around a vertically placed cable exposed to solar radiation. The cable is not loaded with current. Maximum temperature: 73.40°C [30].

were performed for atmospheric conditions that often occur during summer in Poland or at similar latitudes. The air temperature was assumed to be 25°C, and the soil temperature at a depth of 1.2 m was taken as 12°C (consistent with geological data [31]). The calculations were performed for a wind speed of 1 m/s. The thermal conductivity of the soil was assumed to be 0.4 W/(m·K), which corresponds to a thermal resistivity of 2.5 (m·K)/W. It is considered to be quite unfavourable from the point of view of determining the continuous current-carrying capacity of power cables/wires [32,33]. The numerical model assumed solar radiation with an intensity of 1000 W/m² incident at an angle of 45° on the vertical cable – this angle reflects average summer conditions. The absorption coefficient of solar radiation for the cable insulation surface was set at 0.9. It was also assumed that the cable is not loaded with current. In this way, the effect of solar radiation alone on the cable temperature rise can be determined.

Stationary calculations were performed in Ansys Fluent, taking into account the effect of gravity. The k - ϵ standard turbulence model was used with the ‘enhanced wall treatment’ function activated, and ‘full buoyancy effects’ were enabled in the additional options to correctly model the effect of buoyancy during free convection modelling. The energy equation was activated to model thermal phenomena and obtain the temperature distribution in the computational domain. Air density varied with temperature according to the ideal gas equation of state. The airflow velocity was specified at the inlet, and the ‘pressure outlet’ boundary condition was applied at the outlet.

The temperature of the warmest points of the cable is as high as 73.40°C (Fig. 1), even though the cable is not loaded with current. The highest temperature area occurs just above the ground (the area above the ground is shielded from the wind by the aforementioned 0.1 m obstacle) on the side facing the incident sun.

Analysing the cable cross-section and the surrounding area at a height of 0.5 m above the ground, the simulation data shows that it is slightly over 51°C, which is significantly lower than for the section just above the ground, because air movement caused by the wind (cable cooling) is important. The exact temperature distribution in the cross-section at a height of 0.5 m is shown in Fig. 2.

Numerical analyses indicate that solar radiation can cause a cable temperature rise of:

- almost 50 K (73.40°C – 25°C) under very unfavourable conditions (intense sunlight, cable surrounded by still air, e.g. in a roof recess),
- 26 K (51°C – 25°C) under more favourable conditions (intense sunlight, but cable is cooled by wind).

The results of numerical analyses of the temperature increase of cable insulation caused by solar radiation were used to calculate the expected temperatures of this insulation for a selected region of Poland and assumed weather scenarios. The expected temperatures of the cables at specific times of day and year were used to determine the expected durability of the insulation.

3. Annual ambient temperature variation and cable thermal rating

Design calculations for cable current-carrying capacity and insulation durability usually refer to standard temperature conditions, which are generally considered to be the highest possible (most

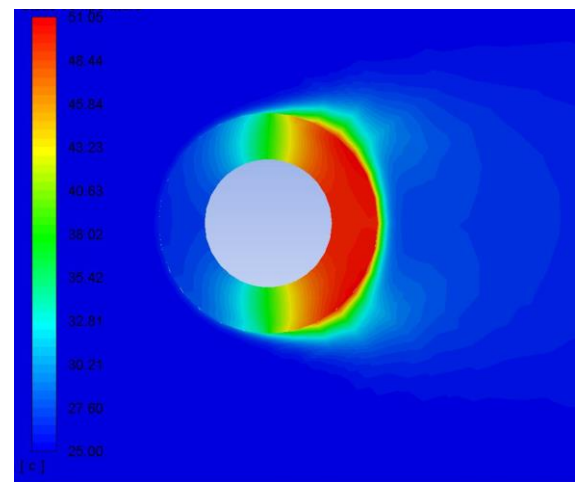


Fig. 2. Temperature fields in the cable cross-section at a height of 0.5 m above the ground. The arrangement is as in Fig. 1. The cable is not subjected to any current. The maximum temperature in the analysed cross-section is 51.05°C.

unfavourable) during the considered period of operation of the electrical installation. Design assumptions are generally known and available, but actual temperature conditions can vary significantly depending on the time of day and season. To provide a realistic estimate of the assumed ambient temperature variability for power cables installed outdoors, the authors of the study compiled daily temperatures for 2024, considering three weather scenarios:

- Scenario 1 (the most restrictive) – the highest temperature was assumed for each day considered;
- Scenario 2 – the average temperature was assumed for each day considered;
- Scenario 3 – the lowest temperature was assumed for each day considered.

The data regarding ambient temperature comes from the Gdańsk Świbno weather station, in northern Poland, and was provided by the Institute of Meteorology and Water Management – National Research Institute (Instytut Meteorologii i Gospodarki Wodnej – Państwowy Instytut Badawczy, IMGW-PIB).

As a result of the analysis of the distribution of the highest temperatures throughout the year (Scenario 1 – the most restrictive of those considered, Fig. 3), one warmest day was identified in each quarter of the year. The selected days and their temperature distributions are presented in Fig. 4 (bars ‘A’). It was then assumed that electric current flowing through the power cables is equal to the current-carrying capacity, resulting in a 45 K increase in the cable insulation temperature (bars ‘B’ in Fig. 4).

Furthermore, it was assumed that for half of each quarter, solar radiation affects the power cables and causes a specific increase in insulation temperature – elevated ‘C’ columns relative to ‘B’ columns at selected times of the day. Table 1 provides, for example, a detailed summary of temperatures on a given/selected warmest day in the third quarter of 2024 – this is a numerical representation of the data from Fig. 4c. Column 2 in Table 1 contains a summary of the maximum hourly ambient temperatures based on meteorological data. Column 3 contains the cable insulation temperatures, assuming that the current flowing through the cable is equal to the current-carrying capacity – then the cable insulation

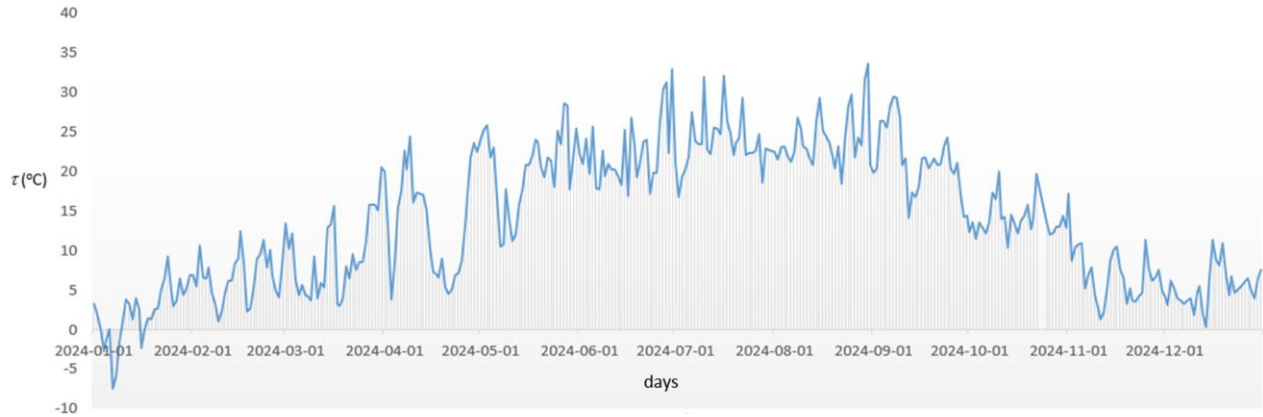


Fig. 3. The highest temperature for each day in 2024. Data from the IMGW-PIB Gdańsk Świbno station. Own elaboration based on data from IMGW-PIB [30].

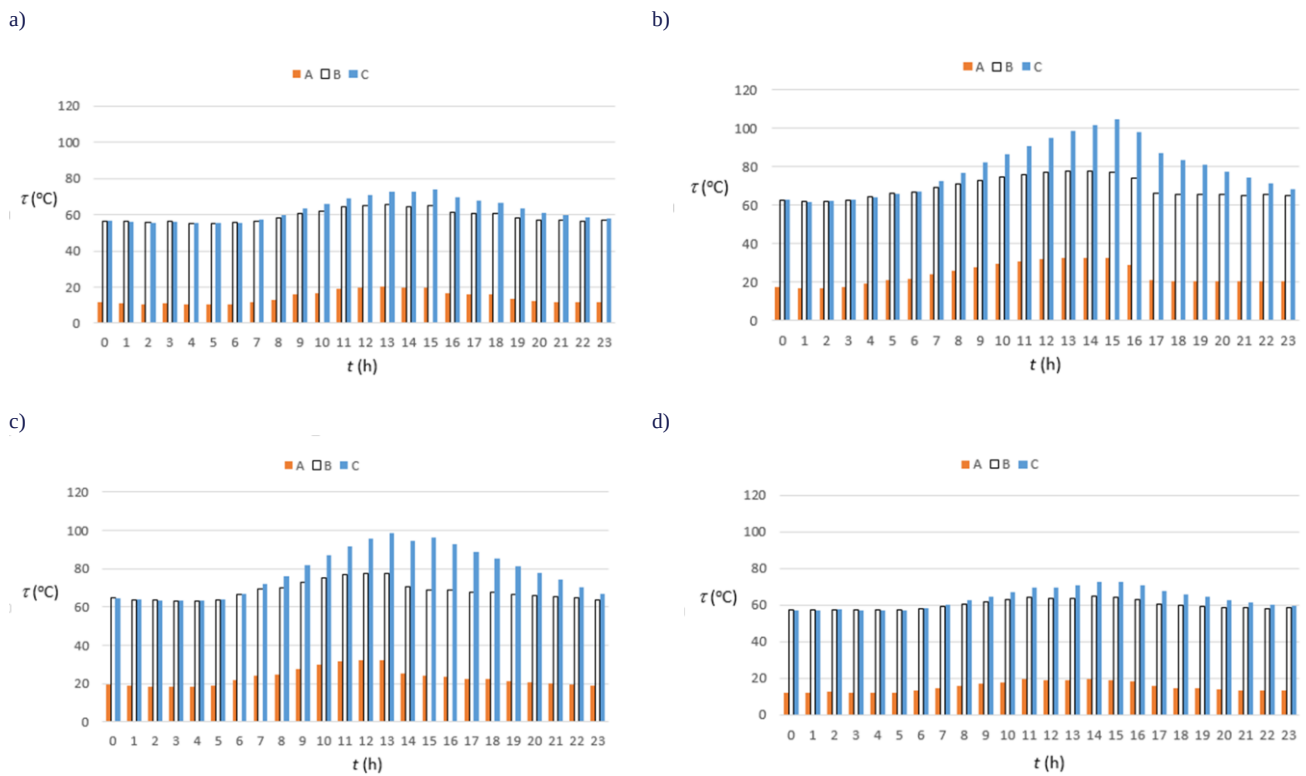


Fig. 4. Daily variation of air and cable insulation temperatures for selected days of 2024 in each quarter (Scenario 1): a) March 31 b) June 30, c) August 30, d) October 10. Bars: A – air temperature (IMGW-PIB data from the Gdańsk Świbno measuring station); B – calculated temperature of cable insulation in places shielded from direct solar radiation, cable loaded with current (I_{dd}); C – calculated temperature of cable insulation in places exposed to direct solar radiation, cable loaded with current (I_{dd}) [30].

temperature increase is 45 K. This corresponds to conditions without solar radiation lasting half of the quarter (assumed to be 45 days). In the case of solar radiation (the remainder of the quarter – also assumed to be 45 days), a further increase in insulation temperature is assumed at individual hours (column 4 in Table 1; Note: there is no column 4 for days without solar radiation). The resulting temperature values are shown in column 5. Similar calculations as in Table 1 were made for the remaining quarters of 2024.

The rated durability of the insulation (L_n) is assumed to be 25 years. The long-term permissible temperature τ_{dd} for PVC insulation is 70°C. The temperature τ_x was determined for each

hour of the day (Fig. 4 and sample detailed data in Table 1), with 45 days without solar radiation and 45 days with radiation at specific times assumed for each quarter.

Based on the assumptions made above, the durability of the insulation at temperature τ_x was calculated according to the following relationship:

$$L_x = L_n 2^{\frac{\tau_{dd} - \tau_x}{\Delta\tau}}, \quad (3)$$

where L_n is the rated insulation durability at temperature τ_{dd} , τ_{dd} – the highest permissible long-term temperature of the power cable, τ_x – temperature other than the highest permissible long-

Table 1. Hourly temperature distribution (°C) for the warmest day in the third quarter of 2024 (Scenario 1) – representative day 2024-08-30 (data from IMGW-PIB). Bars 'A', 'B', and 'C' from Fig. 4.

Time of the day, h	Ambient temp. (bars 'A'), °C	Temperature of the cable loaded with current (I_{dd}), in places not exposed to solar radiation (bars 'B'), °C	Assumed increase in cable temperature due to solar radiation, °C	Temperature of the cable loaded with current (I_{dd}), in places exposed to solar radiation (bars 'C'), °C
0:00	19.5	64.5	0	64.5
1:00	18.9	63.9	0	63.9
2:00	18.4	63.4	0	63.4
3:00	18.3	63.3	0	63.3
4:00	18.2	63.2	0	63.2
5:00	18.8	63.8	0	63.8
6:00	21.7	66.7	0	66.7
7:00	24.1	69.1	3	72.1
8:00	25.0	70.0	6	76.0
9:00	27.7	72.7	9	81.7
10:00	29.9	74.9	12	86.9
11:00	31.8	76.8	15	91.8
12:00	32.5	77.5	18	95.5
13:00	32.2	77.2	21	98.2
14:00	25.3	70.3	24	94.3
15:00	24.0	69.0	27	96.0
16:00	23.8	68.8	24	92.8
17:00	22.7	67.7	21	88.7
18:00	22.4	67.4	18	85.4
19:00	21.4	66.4	15	81.4
20:00	20.7	65.7	12	77.7
21:00	20.2	65.2	9	74.2
22:00	19.5	64.5	6	70.5
23:00	18.9	63.9	3	66.9

term temperature, $\Delta\tau$ – a change in temperature which causes a twofold change in the durability of the insulation (assumed 8 K). Sample calculation results based on Eq. (3) are included in Table 2.

Due to the estimated variation of the cable insulation temperature over time, it is necessary to use the relationship for the resultant insulation durability:

$$L_{tot} = \frac{1}{\sum_{i=1}^N \left(\frac{t_i}{L_{xi}} \right)}, \quad (4)$$

where L_{xi} – insulation durability corresponding to a given temperature in the considered partial time interval t_i , t_i – relative par-

Table 2. Example calculation results based on Eq. (3); Scenario 1.

Parameter	Value	L_x , year	Comments
τ_o , °C	29.9	–	–
τ_{dd} , °C	70	–	–
$\Delta\tau$, K	8	–	–
$\Delta\tau_{dd}$, K	45	–	–
τ_{x1} , °C	74.9	16.35	places not exposed to solar radiation
$\Delta\theta$, K	12	–	–
τ_{x2} , °C	86.9	5.78	places exposed to solar radiation

tial interval of insulation operating time at a specific temperature.

By applying Eq. (4), using the data presented in Fig. 4 and other assumptions, it was calculated that the insulation durability for the analysed conditions is 17.1 years. The calculations show that the expected actual insulation durability is significantly lower than the rated durability (25 years). Therefore, to maintain the rated insulation durability, the load should be reduced by 5.3% compared to the standard current-carrying capacity of power cables in the analysed case. Table 3 presents the results of the current-carrying capacity correction, for example, PVC-insulated cables with data included in the standard [33]. This correction allows for reducing the temperature rise of the cables caused by the flowing current from 45 K to approximately 40 K, which is sufficient to achieve an insulation durability of 25 years.

Table 3. Correction of the current-carrying capacity of PVC-insulated cables to achieve the rated insulation durability (25 years). Cables installed in locations exposed to solar radiation. Scenario 1.

Cross-section of conductor, mm ²	2.5	2.5	4	4	6	6
Number of loaded conductors	2	3	2	3	2	3
Current-carrying capacity without correction, A (according to [33])	23	19.5	31	26	39	33
Correction factor	1×0.947					
Current-carrying capacity after correction, A	21.8	18.5	29.4	24.6	36.9	31.3

For a different weather scenario, the correction factor will have a different value and can be estimated based on temperature data for the analysed region of the country/world using the methodology presented in this study.

In order to better illustrate the usefulness of the indicated calculation method for determining the correction factor, two further scenarios were used, based on the actual average temperatures (Scenario 2) and lowest temperatures (Scenario 3) from the Gdańsk Świbno station, and based on operating time at a specific temperature in the year 2024.

Scenario 2 focuses on the use of average temperatures throughout each analysed day. The annual distribution of average (median) temperatures throughout the year is shown in Fig. 5. Similarly to Scenario 1, in Scenario 2, each representative day in each quarter was selected. The selection was based on the temperature in each quarter, as shown in Fig. 6.

Bars 'A', 'B' and 'C' presented in Fig. 6 were determined similarly to Scenario 1. Based on the assumptions made in Scenario 1, the insulation durability at a given temperature was calculated. Example results are provided in Table 4. By applying Eq. (4) and the assumptions previously described, the resulting insulation durability is 35.31 years. The calculated durability is therefore significantly higher than the assumed rated durability of 25 years of the insulation.

To fully utilise the technically assumed durability of cables, it is possible to increase the current-carrying capacity of power cables. For the assumed weather Scenario 2, the current-carrying

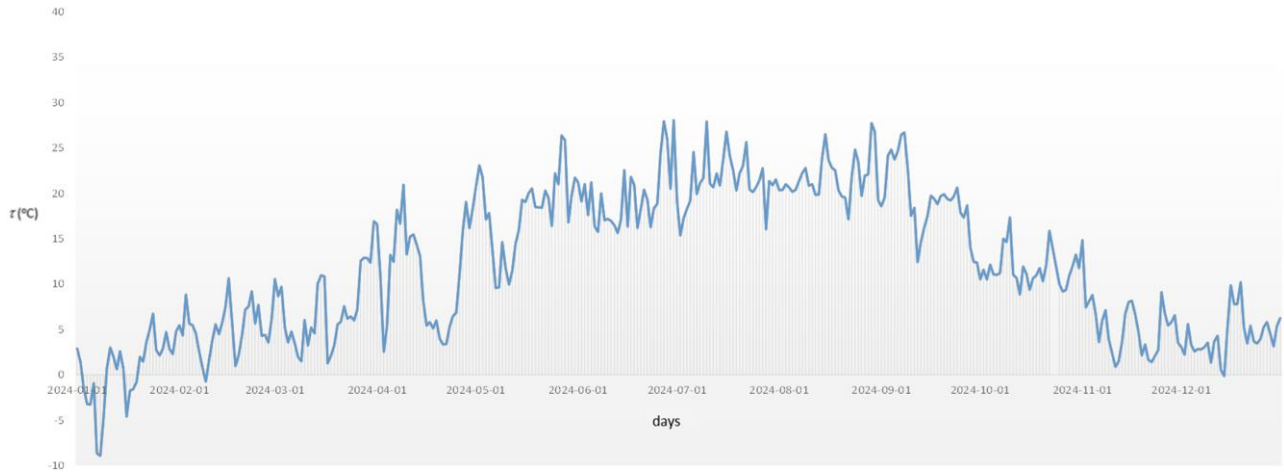


Fig. 5. The average temperature for each day in 2024. Data from the IMGW-PIB Gdańsk Świbno station. Own elaboration based on data from IMGW-PIB.

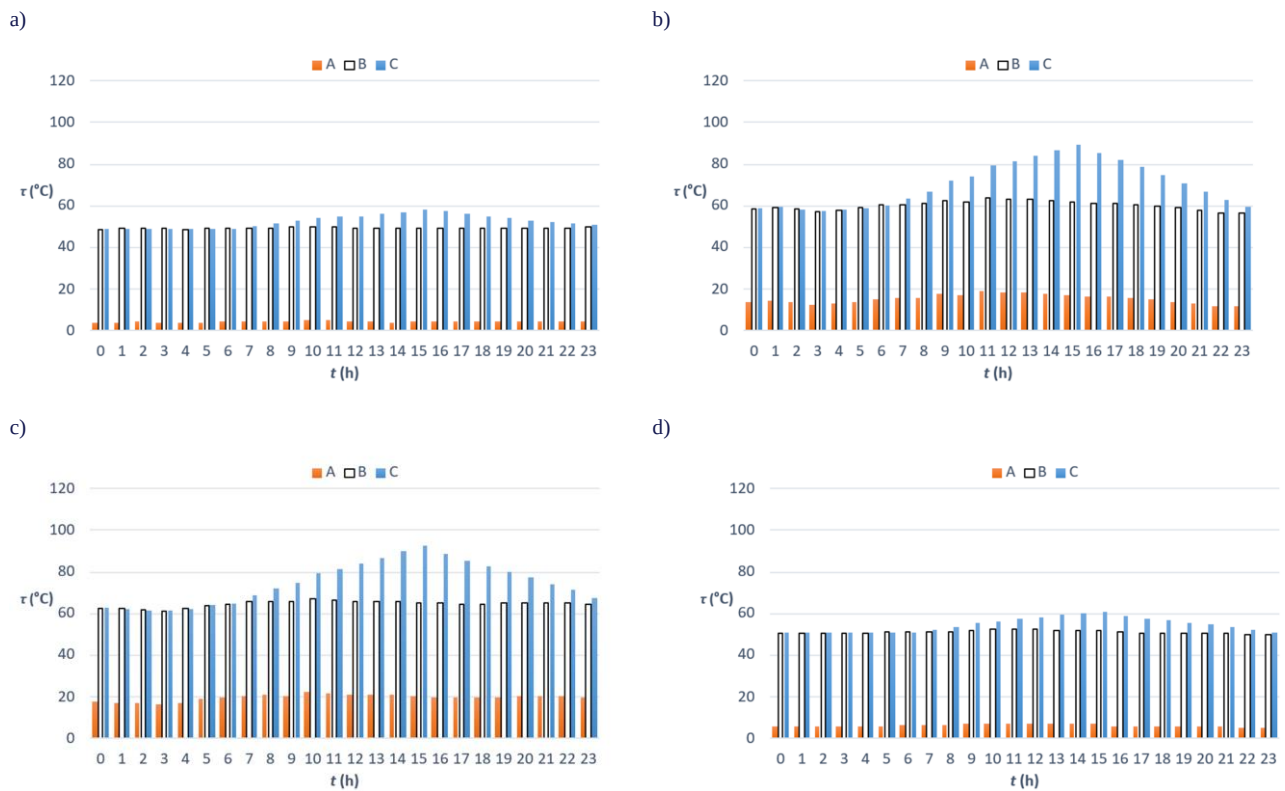


Fig. 6. Daily variation of air and cable insulation temperatures for selected days of 2024 in each quarter (Scenario 2):
a) February 20, b) June 9, c) July 12, d) November 17. Bars: A – air temperature (IMGW-PIB data from the Gdańsk Świbno measuring station); B – calculated temperature of cable insulation in places shielded from direct solar radiation, cable loaded with current I_{dd} ; C – calculated temperature of cable insulation in places exposed to direct solar radiation, cable loaded with current I_{dd} .

Table 4. Example calculation results based on Eq. (3); Scenario 2.

Parameter	Value	L_x (y)	Comments
τ_o , °C	22.1	–	–
τ_{dd} , °C	70	–	–
$\Delta\tau$, K	8	–	–
$\Delta\theta_{dd}$, K	45	–	–
τ_{x1} , °C	67.1	32.14	places not exposed to solar radiation
$\Delta\theta$, K	12	–	–
τ_{x2} , °C	79.1	11.36	places exposed to solar radiation

capacity of power cables can be adjusted/increased by 4.1%. Example values of the current-carrying capacity of power cables before and after applying the correction factor are given in Table 5.

The next scenario, i.e. Scenario 3, is based on the use of the lowest temperatures throughout each considered day. It is the most optimistic and somewhat risky scenario. The annual distribution of the lowest temperatures throughout the year is shown in Fig. 7. Similar to the previous two scenarios, each representative day in each quarter was selected.

Table 5. Correction of the current-carrying capacity of PVC-insulated cables to achieve the rated insulation durability (25 years). Cables installed in locations exposed to solar radiation. Scenario 2.

Cross-section of conductor, mm ²	2.5	2.5	4	4	6	6
Number of loaded conductors	2	3	2	3	2	3
Current-carrying capacity without correction, A (according to [33])	23	19.5	31	26	39	33
Correction factor	1×1.041					
Current-carrying capacity after correction, A	23.9	20.3	32.3	27.1	40.6	34.4

The selection was based on the coldest day in each quarter, as shown in Fig. 8. Bars 'A', 'B' and 'C' presented in Fig. 8 were determined similarly to Scenario 1 and Scenario 2. Based on the assumptions made in the example presented, the insulation durability at a given temperature was calculated. The results of a sample calculation are provided in Table 6. The resultant durability of insulation was obtained as 81.23 years. Similarly to Scenario 2, the resulting durability of insulation is higher than the rated durability of 25 years. Hence, it is possible to increase the current-carrying capacity of power cables by as much as 12.4%. Further, theoretically, it is possible to introduce a correction (in this case a significant increase) of the current-carrying capacity (Table 7).

It is obvious that for scenarios 2 and 3 there is no need to correct/increase the current-carrying capacity – it can remain at the normative level (to maintain a high margin of safety).

The presented results should be interpreted with consideration of the uncertainty associated with the adopted input data and modelling assumptions. First, the numerical analysis was performed for a selected set of environmental conditions, including air temperature of 25°C, soil temperature of 12°C, wind speed of 1 m/s, solar irradiance of 1000 W/m², solar incidence angle of 45°, soil thermal conductivity of 0.4 W/(m·K), and solar absorptivity of the cable surface equal to 0.9. Therefore, the obtained temperature rises should be treated as representative for the assumed scenario and may vary under different local operating conditions.

Table 6. Example calculation results based on Eq. (3); Scenario 3.

Parameter	Value	L _x , year	Comments
τ_{01} , °C	9.9	–	–
τ_{dd} , °C	70	–	–
$\Delta\tau$, K	8	–	–
$\Delta\Theta_{dd}$, K	45	–	–
τ_{x1} , °C	54.9	16.35	places not exposed to solar radiation
$\Delta\Theta$, K	6	–	–
τ_{x2} , °C	60.9	54.99	places exposed to solar radiation

An additional source of uncertainty results from the simplifications adopted in the numerical model. The calculations were carried out as steady-state simulations using the standard k - ϵ turbulence model with enhanced wall treatment and full buoyancy effects. In real outdoor conditions, however, airflow and solar exposure reveal transient behaviour and may change over short periods. Consequently, the model should be regarded as an engineering approximation of the thermal effect rather than reproducing every instantaneous operating condition exactly. The uncertainty of the cable's durability calculations is also related to the adopted

Table 7. Correction of the current-carrying capacity of PVC-insulated cables to achieve the rated insulation durability (25 years). Cables installed in locations exposed to solar radiation. Scenario 3.

Cross section of conductor, mm ²	2.5	2.5	4	4	6	6
Number of loaded conductors	2	3	2	3	2	3
Current-carrying capacity without correction, A (according to [33])	23	19.5	31	26	39	33
Correction factor	1×1.124					
Current-carrying capacity after correction, A	25.9	21.9	34.8	29.2	43.8	37.1

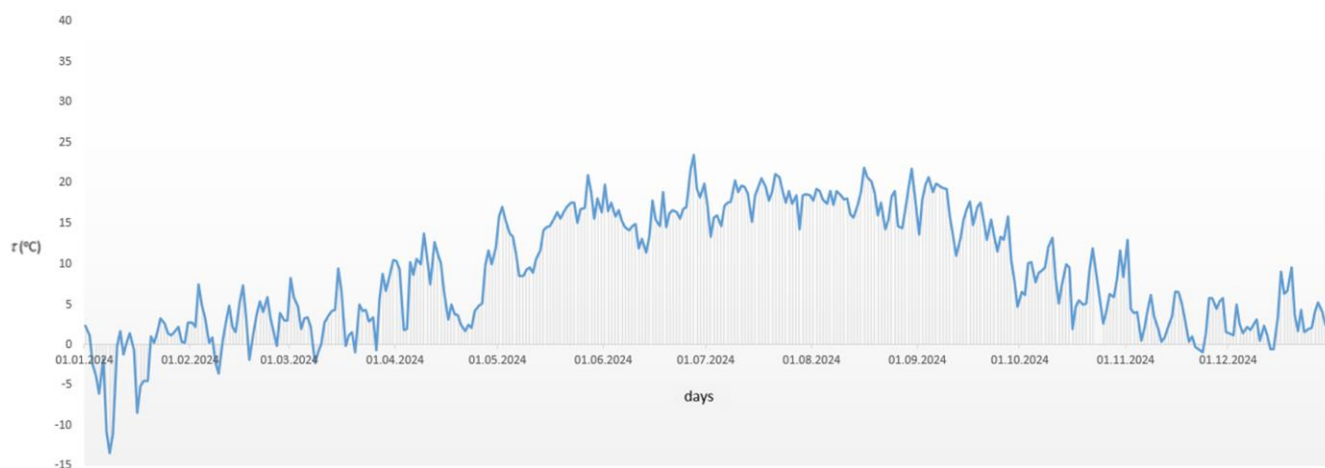


Fig. 7. The lowest temperature for each day in 2024. Data from the IMGW-PIB Gdańsk Świbno station. Own elaboration based on data from IMGW-PIB.

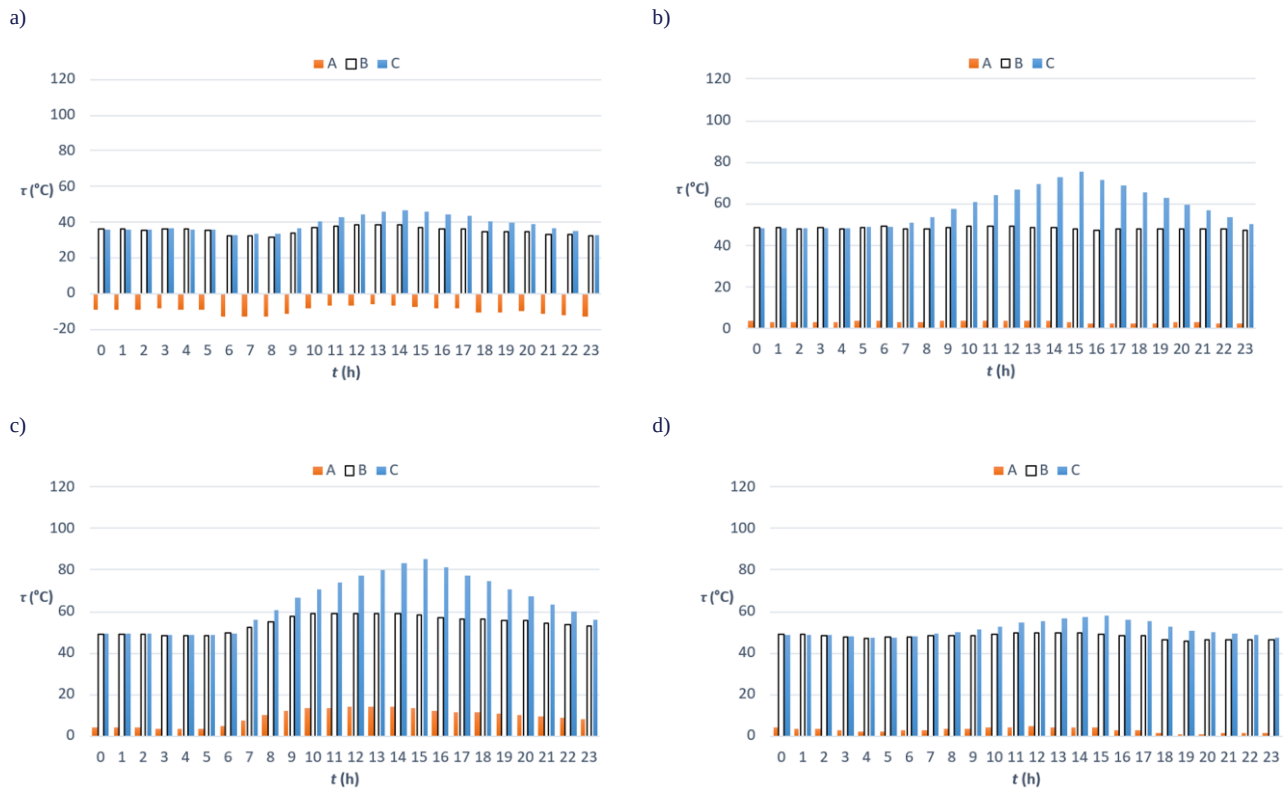


Fig. 8. Daily variation of air and cable insulation temperatures for selected days of 2024 in each quarter (Scenario 3): a) January 8, b) April 21, c) September 30, d) November 23. Bars: A – air temperature (IMGW-PIB data from the Gdańsk Świbno measuring station); B – calculated temperature of cable insulation in places shielded from direct solar radiation, cable loaded with current I_{dd} ; C – calculated temperature of cable insulation in places exposed to direct solar radiation, cable loaded with current I_{dd} .

thermal-ageing model. In particular, the insulation lifetime was estimated using the assumed temperature interval causing a two-fold change in durability, equal to 8 K, and by applying representative daily temperature profiles for selected quarters of the year. Thus, the calculated insulation lifetime/durability should be interpreted as an estimate of expected long-term thermal exposure under the assumed scenarios, rather than as an exact prediction for every installation site. Despite these limitations, the proposed methodology remains useful for engineering analysis because it captures the dominant factors affecting free-air cable thermal loading, namely solar radiation and time-varying ambient temperature. The results are therefore suitable for comparative assessments, conservative design studies, and the determination of practical correction factors for cable current-carrying capacity.

4. Conclusions

The standard and practical determination of the current-carrying capacity of power cables assumes a constant design ambient temperature throughout the year. The authors of this paper, however, have proposed an alternative approach based on daily-annual variability of ambient temperature with a step of one hour, and accounting for the direct solar radiation.

The presented method for determining the expected cable temperature and associated insulation durability can be applied to electrical installations located in selected regions of the country or world, knowing, among other things, the local daily and

annual ambient temperature variability. Taking into account local weather conditions, including expected solar radiation, allows for an estimate of cable insulation durability that is close to this realistic one. The approach proposed is particularly suitable for electrical installations placed on the facade or roof of a building.

The results obtained clearly demonstrate that solar radiation may significantly affect the thermal conditions of power cables installed in free air, which is consistent with previous findings reported in the literature. According to the obtained numerical modelling results, the authors' study indicates that under unfavourable local conditions (e.g. limited airflow), the temperature rise may reach values of up to 50 K above the ambient, which is comparable to or even exceeds typical design assumptions used in standard approaches.

In comparison with studies focusing on underground cables, where the thermal behaviour is strongly governed by soil properties and relatively stable boundary conditions, cables installed in free air are subject to much higher variability of environmental parameters. This variability is particularly related to insolation level, as well as daily and seasonal changes in ambient temperature, which are usually not taken into account in standard-based calculations (e.g., IEC 60287), where constant ambient temperature is assumed.

The results obtained in this paper are also consistent with other studies analysing the effect of ambient temperature profiles on cable current-carrying capacity, which indicate that the

use of constant design temperature may lead to inaccurate estimation of permissible current. However, the present study extends this approach by combining real hourly temperature data with the effect of solar radiation and linking these thermal conditions directly with insulation ageing.

The contribution of this work is the analytical evaluation of insulation durability under realistic operating conditions. While previous studies have mainly focused on temperature distribution or current-carrying capacity calculations, the present approach demonstrates that neglecting natural environmental variability may lead to a substantial reduction of insulation lifetime/durability (e.g. from 25 years to 17.1 years in the most restrictive scenario). This finding highlights the importance of considering both solar radiation and time-dependent ambient temperature in the design of outdoor electrical installations.

Furthermore, the proposed methodology allows for the determination of current-carrying capacity correction factor levels based on actual climatic data. This represents an advancement over standard approaches, which typically do not provide tools for such adjustments under variable environmental conditions. The obtained correction values (from approximately -5.3% to $+12.4\%$, depending on the scenario) indicate that both conservative and optimised design strategies can be developed using the presented method.

Among the presented scenarios/assumptions for calculating the resultant durability of insulation of power cables, the measurement data leading to the determination of the lowest durability should be selected. Regarding the data analysed by the authors, it is suggested to adopt Scenario 1 (the most restrictive, but the safest) as a representative scenario for determining the current-carrying capacity of cables. If the cable system is critical, one may consider analysing historical temperature data from the last few years and taking the least favourable year.

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A novel passive solar energy space heating design of an emergency tent

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Abstract

The adjustable tent is utilised in construction sites, military camps, tourist locations, disaster relief efforts, rescue operations, nomadic living, and other situations. Current methods offer thermal comfort in tents but may elevate weight, expense and complexity. This study introduces a design for a passive solar heating tent that controls internal temperatures using renewable solar energy. This tent was designed using SolidWorks, featuring a Nylon 6.6 cover and a metal frame. The study's results indicate that the aperture area of the sun's trajectory is essential for the tent's energy harvesting capability. The design's principal characteristic is the rectangular prism-shaped air containers, comprising three passive solar containers (PSAC) and two insulation containers (ISC). These containers (PSAC and ISC) can be utilised during the winter and stored during the summer. To determine the minimum necessary aperture area, we computed the tent's maximum thermal losses, representing the required space heating load of 1000 W. We formulated an equation to determine the aperture area of the three-sided tent on the sun-exposed sides based on the azimuth and elevation angles that affect the solar collectors. The integration of the five containers (PSAC and ISC) with the tent enhances thermal insulation by reducing the heat transfer coefficient from $6.5 \text{ W}/(\text{m}^2 \cdot \text{K})$ to $0.168 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Keywords: Passive solar heating; Rectangular prism-shaped air containers; Air containers; Emergency tent; SolidWorks.

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1. Introduction

Since the dawn of civilisation, humans have created and employed tents constructed from natural materials. These tents function as a critical form of emergency shelter [1]. The International Federation of Red Cross and Red Crescent Societies (IFRC) continuously invests in upgrading tents [2], pointing out the significance of tents as essential tools in the initial phases of humanitarian assistance. Temporary shelters must be used when calamities strike, and there are not enough facilities to house the displaced. Shelter is a human right and a necessity for the population's physical and mental well-being [3].

The unstable global situation, changing weather and climate conditions, medical risks, and natural disasters have necessitated the need for temporary housing alternatives for the affected population. In addition, well over 42 million individuals have been forcibly displaced due to conflicts over the past decade, while 2.9 billion people have been affected by natural disasters in the first twelve years of this century [4–6]. Relief organisations, as international aid is estimated in 2019, offer emergency shelter to numerous affected individuals [7,8].

Thermal comfort refers to being content with ambient temperatures. The absence of adverse sensations associated with heat impacts is also assessed. Thermal comfort necessitates a ba-

Nomenclature

A	– area, m^2
EoT	– equation of time
F	– factor
h	– convection heat transfer coefficient, $W/(m^2 \cdot K)$
H	– heat gain, W
I	– rate of solar radiation, W/m^2
K	– thermal conductivity, $W/(m \cdot K)$
l	– window length, m
L	– tent length, m
Lat	– latitude, $^\circ$
Lon	– longitude, $^\circ$
LT	– local time, s
n	– number of the day within the year
N	– number of persons
Q	– power, W
$\dot{Q}$	– heat flow rate, W
R	– thermal resistance, $(m^2 \cdot K)/W$
ST	– solar time
STM	– standard time meridian, $^\circ$
th	– time hour angle, $^\circ$
T	– temperature, $^\circ C$
U	– overall heat transfer coefficient, $W/(m^2 \cdot K)$
$\dot{V}$	– volumetric air flow rate, m^3/s
w	– window height, m
W	– tent height, m
Z	– azimuth angle, $^\circ$

Greek symbols

α	– elevation angle, $^\circ$
δ	– declination, $^\circ$

η – conversion efficiency of the black paint

Subscripts and Superscripts

a	– aperture
am	– ambient
c	– cross section
e	– equipment
ex	– external
$gain$	– gain
h	– harvested energy
hg	– heat gain
i	– internal static air
in	– internal
inf	– infiltration
l	– latent
$loss$	– heat losses
o	– outer moving air
P	– persons
r	– required power
S	– sensible
sp	– space
t	– total
u	– usage

Abbreviations and Acronyms

CLF	– cooling load factor
CPVC	– chlorinated polyvinyl chloride
IAC	– insulation air containers
PMV	– predicted mean vote
PPD	– predicted percentage of dissatisfied
PSAC	– passive solar air containers

lance between body heat production and loss [9–11]. The predicted mean vote (PMV) and predicted percentage of dissatisfied (PPD) indices are frequently employed to assess thermal comfort; however, some studies [12,13] suggest that the PPD index may not adequately represent individual perceptions in hot and humid environments, and PMV-PPD models exhibit certain limitations. Yang et al. [14] performed a supplementary study in Tianjin to corroborate this assertion. Environmental elements and tent attributes, such as materials, heating systems, location, and ventilation options, affect thermal comfort [15].

Conventional solar and non-solar air-conditioning tents combine renewable energy with temporary shelter, enhancing outdoor comfort. Air conditioning units in these tents maintain a stable temperature. Efficient air circulation, which means air-flow that quickly distributes cooled or heated air, is key to energy savings and user comfort. High-quality thermal insulation materials that slow heat loss or gain, and advanced solar panel technology that uses solar cells to convert sunlight into usable electricity, are essential. Furthermore, materials that retain heat (store warmth for later use), resist fire (are non-flammable), and provide waterproofing (prevent water entry) further ensure safety and comfort [19]. Solar panels exchange sunlight for electrical energy, which powers air conditioning. Other tents use air-supply pipes (ducts that guide air) and ventilation fans (devices that move air) to boost airflow [20].

Although these approaches promise the thermal comfort of conventional solar and non-solar air-conditioning tents, they may also increase the tent's weight, cost, electrical consumption and complexity. This study presents a novel theoretical approach to design a passive solar heating tent that utilises free and renewable solar energy to maintain interior temperatures effectively. Using SolidWorks, we were able to design and simulate this tent, which has a metal base and a Nylon 6.6 cover. Additionally, the tent employs a sustainable passive solar system, making it environmentally friendly. Furthermore, this study highlights the vital effect of the varying aperture area resulting from the sun's trajectory on the energy harvested by the tent.

2. Materials and methods

We designed a four-person tent featuring a passive solar heating system utilising the SolidWorks software. The structure consists of a metal frame; it also features an outer layer made of nylon 6.6 fabric, known for its distinctive properties [21] and commonly utilised in parachutes because of its excellent air insulation. The design comprises a rectangular prism consisting of five containers – passive solar air containers and insulation air containers (PSAC and IAC) that can be assembled in the winter and disassembled in the summer: three coated passive solar air canisters (PSAC) for the walls and two insulating air containers (IAC) for the doorway and roof. PSACs are covered with black

nitrocellulose paint on one external side of each PSAC to improve solar radiation absorption efficiency.

The uncoated sides of these PSACs, on the other hand, feature openings at the top and bottom. The openings at the top and bottom of the tent match with the openings on either side of the containers. All containers are secured to the tent with adhesive tapes bonded to both the tent and the containers. When installed within the tent, each painted side of the containers is oriented towards the east, south and west, respectively, to optimise solar energy collection on their surfaces. Figure 1 (a and b) displays the setting.

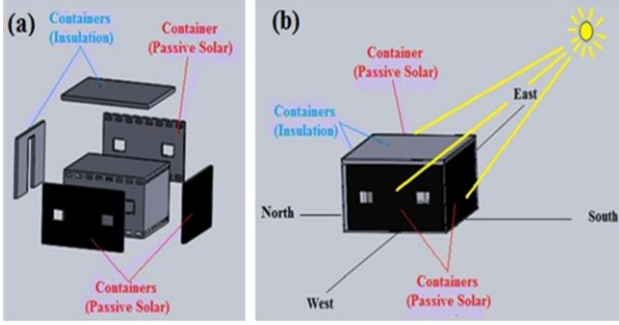


Fig. 1. (a) PSAC and IAC elements, and (b) PSAC and IAC assembled in the tent.

The tent's entrance is located on the northern side, which is the fourth side. The variation in density will result in the energy absorbed from solar radiation heating the air within the PSAC, causing it to ascend to the top, where it will enter the tent through the upper openings, while cooler air will enter the container through the lower openings. The air circulation system elevates the tent's temperature during the day by employing the thermosiphon principle. This study aims to propose a theoretical design for an emergency tent with a novel solar heating system. This design is ready to be installed at any time, and all necessary mathematical calculations to validate it will be presented.

2.1. Space heating load and harvested energy

The required heating load is calculated by subtracting the heat obtained from occupants and objects within the tent while considering the heat losses through the tent's walls, ceiling and floor, with an internal temperature set at 22°C and an outside temperature of 0°C. The heat losses depend on the air properties, and the thermal resistances (R_t) are displayed in Fig. 2.

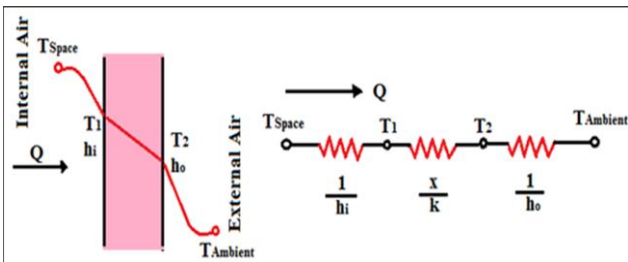


Fig. 2. Thermal resistances across the wall [23].

While the thermal conductivity (k) of the concrete floor equals 1.35 W/(m·K) [22], simultaneously, presuming the soil temperature to be between 10°C and 14°C. This study highlights two layers in the ground: the concrete layer (first) and the underlying soil layer (second). It is worth mentioning that this tent can also be set up directly on the ground without the use of a concrete layer.

2.1.1. Total amount of required power (Q_r)

The variation between heat losses (Q_{loss}) and heat gains (Q_{gain}) specifies the quantity of heat that must be supplied to the tent for the heating load (Q_r), which is:

$$Q_r = Q_{loss} - Q_{gain}, \quad (1)$$

where Q_r is the total amount of the required power in W.

• Energy losses (Q_{loss})

Load estimation is predicated on the assumption that the energy losses (Q_{loss}) of a tent are directly proportional to the temperature variation between its interior and exterior. In addition, the tent's wall cross sections that face the sun are involved in the calculations; despite the heat gain from the three PSAC, the calculations were based on the maximum thermal losses from the tent's walls, ceiling and floor:

$$Q_{loss} = UA_c(T_{sp} - T_{am}), \quad (2)$$

where A_c denotes the cross-sectional area of each layer. The overall heat transfer coefficient (U) for each element of the outer casing is obtained by the thermal resistance method, which accounts for the resistance of the convective heat transfer on the inner and outer surfaces and the thermal conductivity resistance of all the layers of the material.

The overall heat transfer coefficient is given by [23]:

$$U = \frac{1}{R_t} = \frac{1}{\frac{1}{h_i} + \sum \frac{x}{k} + \frac{1}{h_o}}, \quad (3)$$

where X is the nylon 6.6 fabric thickness (0.8 mm); the thermal conductivity (k) of nylon used in the walls and roof is 0.25 W/(m·K), whereas the thermal conductivity of air is 0.0257 W/(m·K), the containers' thickness is 15 cm. The floor thickness is specified as 10 cm for concrete and 1 m for soil.

The convection heat transfer coefficient for internal static air is 8.29 W/(m²·K), while the coefficient for moving air in winter is 34.1 W/(m²·K) [22].

The total losses can be calculated as:

$$Q_{loss} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_{inf}, \quad (4)$$

Taking into account thermal losses: through the tent walls (Q_1, Q_2, Q_3), windows (Q_4), door (Q_5), ceiling (Q_6), floor (Q_7), and through infiltration (Q_{inf}). The notations (1–7) correspond to the values of A_c listed in Table 1.

• Energy gain (Q_{gain})

Persons residing in air-conditioned environments experience a loss of sensible heat that fluctuates according to their condi-

tion. This occurs because their bodies release heat when exposed to the tent's inner air, which is at 37°C. Furthermore, latent heat is generated from sweat on the body's surface and the evaporation of water in the lungs, which contributes thermal energy and decreases the heating load, along with the heat gain from electrical equipment:

$$Q_{gain} = Q_p + Q_e. \quad (5)$$

The sensible heat gained by individuals (Q_s) can be calculated using the equation [22]:

$$Q_s = N_p H_s CLF. \quad (6)$$

In refrigeration calculations, CLF denotes the cooling load factor, but in this context, it refers to the heat gain load factor per person. The value of H_s is dependent upon the person's activities within the tent and its magnitude. The latent heat gain from occupants (Q_l) in a space is expressed by the subsequent equation:

$$Q_l = N_p H_l. \quad (7)$$

H_l denotes the latent heat rate for an individual within the space, with its value dependent upon the nature of the person's activity.

The total heat gain due to the presence of people inside the tent (Q_p) was calculated from the following equation:

$$Q_p = Q_s + Q_l, \quad (8)$$

When calculating the heat gain load from equipment in the tent, references equate this gain to the power consumption of the devices, taking into account usage factors, and calculate the heat gains as follows [22]:

$$Q_e = P_e F_u F_{hg}, \quad (9)$$

where Q_e is the heat gain from equipment (W), P_e is the equipment power (W), F_u is the usage factor

$$F_u = \frac{\text{Actual operating time}}{\text{Design period}}; \quad (10)$$

F_{hg} is the heat gain factor, and the design period is 10 hours.

2.2. The main components of the tent and solar heating system

2.2.1. The main metal structure assembly

Figure 3 illustrates a primary metal framework of the tent (of size 4 m × 4 m × 2 m) designed for fast assembly and disassembly, which utilises metal assemblies secured at each of its eight corners with screws and nuts. The subsequent assemblies are as follows: for each tent, two symmetrical side metal components, fabricated from 40 mm × 40 mm × 1.25 mm square hollow section tubes, are employed. The symmetrical complexes on the right and left are interconnected at the top by ribs made of a square hollow section, with bond plates at the ends that attach the components affixed to the aforementioned complexes.

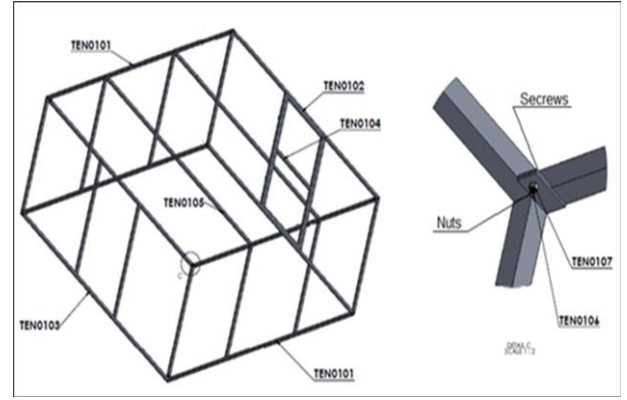


Fig. 3. The tent metal assemblies and their connections.

The next phase includes affixing the door frame to the primary construction of the tent utilising a door hinge. Subsequently, a body constructed from nylon 6.6 fabric is sewn and positioned over the framework in accordance with Fig. 4.

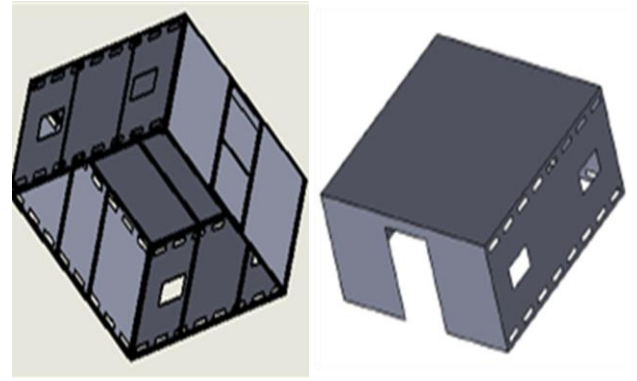


Fig. 4. The fabric framework of the tent.

Finally, the door fabric is attached to the door structure and metal frame using connecting belts and a fabric label. This connection allows air to pass between the frame and the fabric, effectively eliminating the effect of heat conduction between them.

2.3. PSAC and IAC

During this step, we have connected PSAC, specifically the left, right and rear, along with IAC for the door and roof. This procedure is illustrated in Fig. 5, which depicts the connections using belts and fabric sticker labels around the windows and their edges. PSAC and IAC are equipped with valves for the primary purpose of air filling. In order to take the form of a parallel rectangular shape, it employs an assembly of plastic tubes (2" C.P.V.C SCH 80) as illustrated in Fig. 6 to maintain their shapes by linking them at the outer corners and ends of the containers (PSAC and IAC), as depicted in Fig. 5.

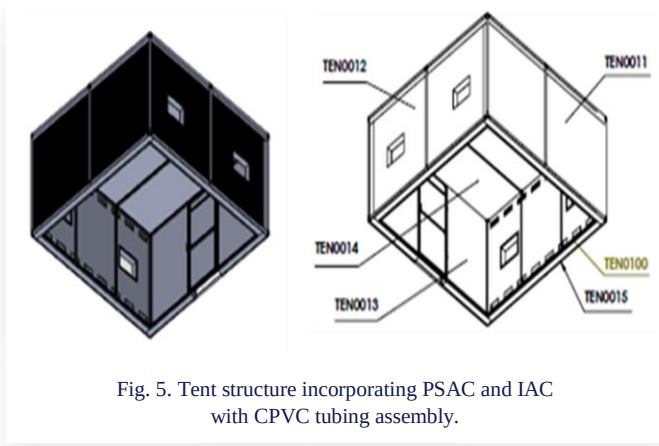


Fig. 5. Tent structure incorporating PSAC and IAC with CPVC tubing assembly.

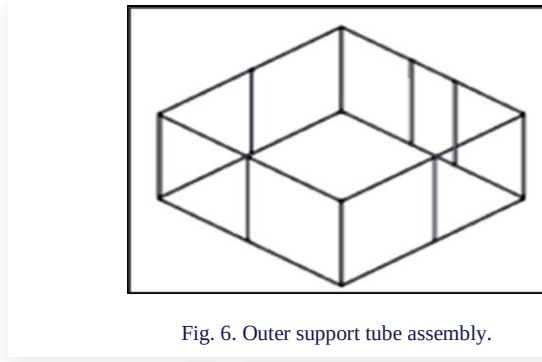


Fig. 6. Outer support tube assembly.

2.4. Sun trajectory calculations

Static solar containers (PSACs) capture solar radiation through their variable aperture area, defined as the section of PSAC oriented toward the sun that receives incident solar radiation. This mechanism controls the amount of solar energy harvested. The aperture area of PSAC varies from sunrise to sunset as the solar angles (azimuth and elevation) change. To compute the changing aperture of three PSACs facing the sun as a function of elevation and azimuth solar angles, the following mathematical relation was developed, Eq. (11):

$$A_a(z, \alpha) = L \cos(90 + Z) 2 \cos(90 - \alpha) + W \cos(Z) 2 \cos(\alpha). \quad (11a)$$

When subtracting the area of the tent windows, the equation becomes as follows:

$$A_a(z, \alpha) = L \cos(90 + Z) 2 \cos(90 - \alpha) + -2l \cos(90 + Z) w \cos(90 - \alpha) + W \cos(Z) 2 \cos(\alpha). \quad (11b)$$

where Z denotes the azimuth angle, α represents the elevation angle. L and W indicate the length and height of PSAC, while l and w identify the length and height of the windows. The elevation angle (α) is determined according to [24]:

$$\alpha = \sin^{-1}(\cos(Lat) \cos(\delta) \cos(th) + \sin(Lat) \sin(\delta)), \quad (12)$$

where th is the time hour angle, Lat is the position's latitude (33.27°), and δ is the declination. These parameters can be determined as [24–27]:

$$\delta = 23.45 \sin\left(\frac{360}{365}(284 + n)\right), \quad (13)$$

where n represents the number of the day within the year, and

$$th = 15(ST - 12). \quad (14)$$

The solar time (ST) can be determined from [19]:

$$ST = LT + 4(Lon - STM) + EoT, \quad (15)$$

where LT is the local time, Lon is the local longitude ($44.4^\circ E$), STM is the standard time meridian ($45^\circ E$), EoT is the equation of time, which reads as [28]:

$$EoT = 9.87 \sin(2\beta) - 7.53 \cos(\beta) - 1.5 \sin(\beta), \quad (16)$$

$$\beta = \frac{360}{364}(n - 81), \quad (17)$$

$$Z = \arcsin(\sec(\alpha) \cos(\delta) \sin(th)). \quad (18)$$

3. Results and discussion

3.1. Energy losses and energy gain

• Energy losses (Q_{loss})

First, we have to set the values for the overall heat transfer coefficients using Eq. (3): $U_1 = 0.168 \text{ W}/(\text{m}^2 \cdot \text{K})$ for the right, left and rear walls and also for the door wall and ceiling, $U_2 = 6.5 \text{ W}/(\text{m}^2 \cdot \text{K})$ for the door, $U_3 = 6.27 \text{ W}/(\text{m}^2 \cdot \text{K})$ for the windows, $U_4 = 5.14 \text{ W}/(\text{m}^2 \cdot \text{K})$ for the floor.

Using Eq. (2) to calculate the heat losses yields the results presented in Table 1 for the floor concrete thickness of 10 cm and for dry soil of thickness 1 m (effective depth).

Table 1. Details of heat losses calculations.

Heat losses	Cross section area, m^2	Values, W	Details
Q_1	7.52	55.58	right and left tent walls
Q_2	8	29.6	rear wall
Q_3	6.2	22.9	door wall
Q_4	0.96	132.4	windows
Q_5	1.8	257.4	door
Q_6	16	59.1	ceiling
Q_7	16	822.4	floor (concrete)
	16	80	floor (dry soil)

Note that the air infiltration ($\dot{Q}_{inf}$) is a contributing factor to heat loss and can be calculated using the method [22]:

$$\dot{Q}_{inf} = 1.22 \dot{V}(T_{sp} - T_{am}), \quad (19)$$

where $\dot{V}$ represents the volumetric rate of air exchange, determined as:

$$\dot{V} = \frac{V n}{3600}. \quad (20)$$

The value n indicates the quantity of air changes. Assuming one-time air change (1), V represents the volume of a tent covering 32 m³. Applying the preceding formula, we derive:

$$\dot{V} = 0.00889 \text{ m}^3/\text{s}.$$

By substituting values of $T_{sp} = 22^\circ\text{C}$ and $T_{am} = 0^\circ\text{C}$ into Eq. (19), we obtain:

$$\dot{Q}_{inf} = 238.8 \text{ W}.$$

Placing the data from Table 1 and $\dot{Q}_{inf}$ into Eq. (4) gives:

$$Q_{loss} = 1618 \text{ W}.$$

• Energy gain (Q_{gain})

In the context of minor activities such as sitting or light work, the CLF value is 0.65, while the CLF value varies according to the duration of an individual's presence, with a range of 0.5–0.79 for a 4-hour presence [22]. Inputting the values of H_s , CLF and the number of individuals, NP , into the aforementioned Eq. (6) yields:

$$Q_s = 182 \text{ W}.$$

In addition, for sitting or light work, H_l is measured at 55 W [22]. Substituting values of H_l and the number of individuals into Eq. (7) yields:

$$Q_l = 220 \text{ W}.$$

Finally, substituting the values of Q_s and Q_l into Eq. (8) results in:

$$Q_p = 402 \text{ W}.$$

Assuming that the devices operate at completely full power, the heat gain factor is equal to 1, and the usage factor is based on actual operating time of the equipment. The calculations of the heat gain by the equipment using Eq. 9 are detailed in Table 2. The total heat gain by the equipment is 216 W.

Table 2. Details of the heat gain load for the equipment.

Equipment	Device number	Total Power, W	Actual operating time, h	Usage factor	Heat gain, W
Laptop and charger	2	130	4	0.4	52
Mobile charger	4	40	0.5	0.05	2
Kettle	1	2000	0.5	0.05	100
Monitor LED	1	70	6	0.6	42
Lighting LED	2	20	10	1	20

By substituting values of $Q_p = 402 \text{ W}$ and $Q_d = 216 \text{ W}$ in Eq. (5), we obtained the following:

$$Q_{gain} = 618 \text{ W}.$$

3.2. Total amount of required power (Q_r)

From Eq. (1): $Q_r = 1000 \text{ W}$. Now the total energy (Q_t) required to heat the tent for a duration of 10 hours is as follows:

$$Q_t = Q_r \times \text{used hours of day}, \quad (21)$$

$$Q_t = 1000 \times 10 \text{ hours} \times 3600 = 36000 \text{ kJ} = 10 \text{ kWh}.$$

3.3. Harvested energy and optical calculations

The energy collected from the sun via the PSAC is called harvested energy (Q_h). This energy can be expressed as:

$$Q_h = I A_a \eta. \quad (22)$$

To find the minimum aperture area (A_a) of the passive solar energy collector that must be exposed to solar radiation to meet the required load (as much as possible), we must assume that the calculated total energy (36000 kJ) is equal to the harvested energy (Q_h). Now:

$$A_a = \frac{Q_t}{I \eta}. \quad (23)$$

Between October and March, the average solar radiation for the winter, over a six-month duration of 10 hours daily, is 3.868 kWh/m² [29]. The absorptance of the black paint (cellulose nitrate) is 0.78, representing η [30]. Consequently, the required aperture area for the passive solar collector is:

$$A_a = \frac{10 \text{ kWh}}{3.868 \text{ kWh/m}^2 \times 0.78} = 3.31 \text{ m}^2.$$

Applying a cumulative method with spatial adjustments based on the sun's trajectory (azimuth and elevation) as detailed in Eqs. (11)–(18), Table 3 presents the amount of harvested energy obtainable over a nine-hour period. Measurements were recorded every ten minutes (see the appendix); however, for simplicity, the data presented in Table 3 represents hourly averages throughout the day. The coordinates for Baghdad, Iraq (33.27°N, 44.38°E) were employed in the calculations.

Table 3. The detailed data of harvested energy and actual aperture area with corresponding azimuth and elevation angles for every hour.

Time, hh:mm	Azimuth, deg.	Elevation, deg.	Aperture area, m ²	Solar radiation, W/m ²	Energy, kJ
07:30 a.m.	−58.16	4.42	4.71	19	−
08:30 a.m.	−48.64	14.49	6.54	169	1508.3
09:30 a.m.	−37.24	23.05	7.65	300	4727.8
10:30 a.m.	−23.62	29.42	7.87	400	7832.4
11:30 a.m.	−8.04	32.85	7.23	454	9167.5
12:30 p.m.	8.35	32.82	7.23	454	9015.0
01:30 p.m.	23.90	29.32	7.87	403	9239.6
02:30 p.m.	37.48	22.90	7.65	311	7949.3
03:30 p.m.	48.84	14.31	6.54	174	4981.7
04:30 p.m.	58.33	4.22	4.71	21	1497.2

Since the outside air temperature was assumed to be 0°C and the inside tent temperature was 22°C , we could calculate the required heating load by calculating the tent's thermal losses. According to Eq. (21), the calculations produced a heating energy load of 36 000 kJ. If the indoor temperature rises above 22°C , the windows can be opened to maintain the desired value.

On December 21, the shortest day of the year, which produces the least amount of energy during the winter, the accumulation approach can be used to measure the outcome at 10-minute intervals [31].

Figure 7 depicts the variation in the aperture area (Col. Area) as it correlates with changes in azimuth (Azim.) and elevation (Elev.) throughout the day.

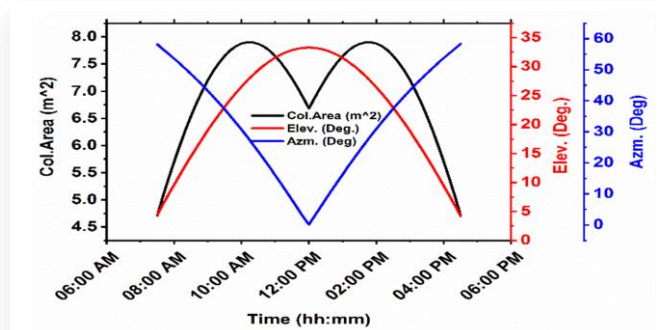


Fig. 7. The correlation involving aperture area, elevation and azimuth.

Figure 7 demonstrates that while elevation and azimuth varies throughout the day, the actual aperture area consistently exceeds its minimum calculated value of 3.31 m^2 , indicating that the energy collected would be greater than the requirements for space heating.

Figure 8 illustrates the variation in aperture area in connection with variations in azimuth and elevation over a half-day period (from 7:30 a.m. to 12:00 p.m.). The identical occurrence is expected to take place throughout the forthcoming half-day (from 12:00 p.m. to 4:30 p.m.). The changing aperture area and the intensity of solar radiation will influence the efficiency of energy harvested from PSAC. The section of the aperture area curve depicted in Fig. 8 aligns with the Gaussian model, as illustrated within the figure.

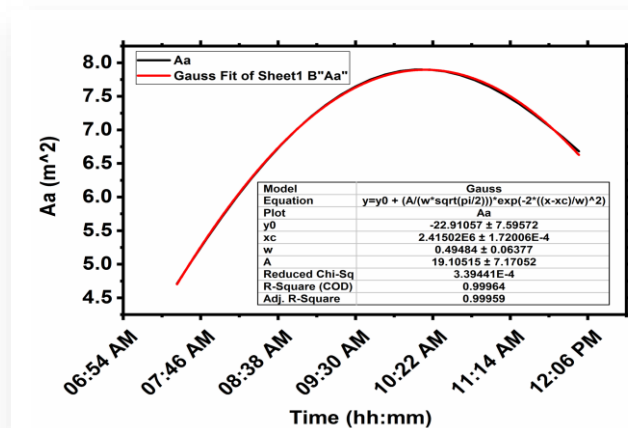


Fig. 8. Behaviour of aperture area variation with Gauss model fitting.

By employing Eq. (22), we can compute how much energy the PSAC captured from the sun's rays for the entire shortest winter day, as illustrated in Fig. 9. The result was 55 919.04 kJ. The solar radiation data for December 21 demonstrate the effect of changes in the aperture area (A_a) and solar radiation (S. Rad.) values on the quantity of harvested energy produced, as shown in Fig. 9.

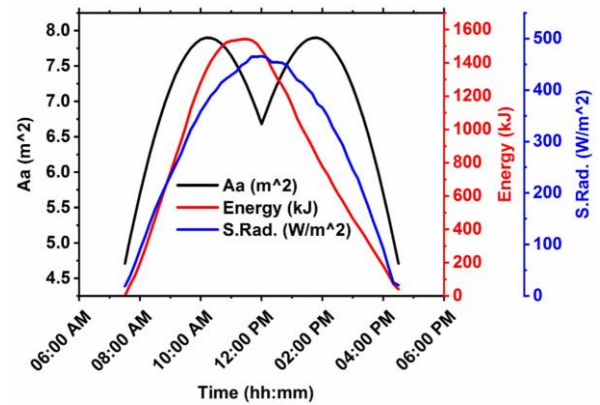


Fig. 9. The influence of aperture area and solar radiation on the energy harvested.

At 10:10 a.m., the maximum aperture area was measured as 7.898 m^2 , with solar radiation recorded at 374 W/m^2 , resulting in an energy gain of 1352.7 kJ. The highest recorded energy was 1542.44 kJ at 11:30 a.m. The aperture area measured 7.23 m^2 under solar radiation of 454 W/m^2 . This variation arises from the simultaneous influence of aperture areas and solar radiation values on the energy obtained.

6. Conclusions

The project aimed to develop an innovative technique for designing an emergency passive solar heating tent that utilises renewable and cost-free solar energy to maintain appropriate internal temperatures.

The research demonstrates that a tent measuring $4\text{ m} \times 4\text{ m} \times 2\text{ m}$ can accommodate up to four people while providing the necessary space-heating load and usable solar energy. Another advantage of the tent is the simplicity of assembly and disassembly. The aperture area of PSAC, which varies throughout the day as the sun traverses the sky, determines the amount of energy that may be harvested.

This design simulation focuses only on the volumetric rate of air exchange and assumes one-time air change for simplicity.

Additional practical parameters, such as wind speed, pressure differences, air temperature differences and minimum ventilation, could influence the calculation of the heat losses. All these parameters will be taken into account in our future work.

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Appendix

Table 4 presents the results of calculations of the harvested energy every ten minutes, along with the effects of azimuth and elevation on the aperture area and amount of energy harvested.

Table 4. The detailed data of harvested energy and actual aperture area every ten minutes with corresponding azimuth and elevation angles.

Time, hh:mm	Azimuth, deg.	Elevation, deg.	Aperture area, m ²	Solar radiation, W/m ²	Energy, kJ
07:30 AM	-58.16	4.42	4.71	19	-
07:40 AM	-56.68	6.19	5.05	38	65.11
07:50 AM	-55.17	7.91	5.38	63	123.34
08:00 AM	-53.60	9.61	5.70	90	198.38
08:10 AM	-51.10	11.28	5.10	116	281.88
08:20 AM	-50.35	12.90	6.27	143	371.90
08:30 AM	-48.64	14.49	6.54	169	467.74
08:40 AM	-46.88	16.04	6.78	192	562.44
08:50 AM	-45.08	17.54	6.10	213	652.80
09:00 AM	-43.20	18.10	7.19	234	742.30
09:10 AM	-41.28	20.40	7.37	254	831.67
09:20 AM	-39.29	21.75	7.52	274	919.93
09:30 AM	-37.24	23.05	7.65	300	1018.70
09:40 AM	-35.12	24.28	7.75	323	1122.28
09:50 AM	-32.96	25.45	7.82	340	1208.05
10:00 AM	-30.71	26.55	7.87	359	1283.91
10:10 AM	-28.41	27.58	7.898	374	1352.70
10:20 AM	-26.05	28.54	7.896	389	1410.05
10:30 AM	-23.62	29.42	7.87	400	1455.43
10:40 AM	-21.14	30.21	7.82	414	1494.01
10:50 AM	-18.62	30.92	7.74	422	1521.93
11:00 AM	-16.03	31.55	7.64	429	1531.93
11:10 AM	-13.40	32.08	7.52	436	1535.09
11:20 AM	-10.74	32.51	7.38	448	1542.14
11:30 AM	-8.04	32.85	7.23	454	1542.44
11:40 AM	-5.31	33.10	7.06	465	1536.25
11:50 AM	-2.59	33.24	6.87	465	1516.05
12:00 PM	0.16	33.29	6.68	466	1476.56
12:10 PM	2.90	33.23	6.875	463	1473.39
12:20 PM	5.63	33.08	7.06	454	1494.86
12:30 PM	8.35	32.82	7.23	454	1517.87
12:40 PM	11.05	32.47	7.38	453	1550.10
12:50 PM	13.70	32.02	7.52	448	1571.80
01:00 PM	16.33	31.48	7.64	433	1563.48
01:10 PM	18.90	30.85	7.74	420	1535.53
01:20 PM	21.43	30.13	7.82	414	1518.29
01:30 PM	23.90	29.32	7.87	403	1499.52
01:40 PM	26.33	28.43	7.90	393	1468.34
01:50 PM	28.67	27.47	7.90	373	1415.60
02:00 PM	30.97	26.43	7.87	366	1363.77
02:10 PM	33.20	25.32	7.82	350	1315.13
02:20 PM	35.37	24.15	7.75	328	1235.38
02:30 PM	37.48	22.90	7.65	311	1151.10
02:40 PM	39.52	21.60	7.52	293	1071.94
02:50 PM	41.50	20.25	7.37	268	977.43
03:00 PM	43.42	18.83	7.19	245	874.27
03:10 PM	45.28	17.37	6.10	228	785.47
03:20 PM	47.08	15.87	6.78	200	689.87
03:30 PM	48.84	14.31	6.54	174	582.69
03:40 PM	50.54	12.72	6.30	146	479.73
03:50 PM	52.18	11.09	5.10	119	380.51
04:00 PM	53.78	9.42	5.70	92	288.72
04:10 PM	55.34	7.72	5.38	57	193.19
04:20 PM	56.85	5.99	5.05	26	101.36
04:30 PM	58.33	4.22	4.71	21	53.69

Dual-use PV canopies for public gardens: Performance, techno-economic feasibility and CO₂ mitigation of an 84.5 kWp grid-tied system in Baghdad, Iraq

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Abstract

This study presents a comprehensive energy, techno-economic and environmental assessment of a dual-use grid-connected photovoltaic canopy system implemented in a public urban garden in Baghdad, Iraq, under hot-arid climatic conditions. The photovoltaic system, designed as shading canopies integrated within recreational areas, has a total installed capacity of 84.5 kWp and operates under a net-metering configuration without battery storage. The system performance was evaluated using PVsyst simulation and independently cross-checked through a simplified analytical model developed in a spreadsheet-based framework using monthly solar-resource inputs and aggregate loss assumptions. The analytical model reproduced the same seasonal behaviour as the PVsyst simulation, with an annual deviation of only 1.51%. The results indicate an annual electricity generation of 133 189 kWh, corresponding to a specific yield of 1 577 kWh/kWp per year and a performance ratio of 79.6%, demonstrating reliable operation despite high ambient temperatures, dust exposure and other site-related losses. The irrigation demand of the public garden is supplied by a 4-hp electric pump requiring approximately 2 200 kWh per year, which represents only about 1.6% of the total annual photovoltaic production. Consequently, more than 130 MWh per year of surplus electricity is available for export to the national grid. The economic analysis revealed a levelised cost of electricity of approximately 44 IQD/kWh (≈ 0.034 USD/kWh), substantially lower than the prevailing commercial electricity tariff of about 120 IQD/kWh (≈ 0.092 USD/kWh), with a simple payback period of about 6 years and an internal rate of return of approximately 18%. Environmentally, replacing the previous diesel-based irrigation supply with the photovoltaic-supported electric system leads to an estimated annual reduction of approximately 113 tonnes of CO₂ emissions. Overall, the findings demonstrate that photovoltaic canopy systems provide a technically reliable, economically attractive and environmentally effective solution for urban irrigation while simultaneously delivering shading benefits, supporting grid interaction and improving the sustainability of public green spaces in hot-arid regions.

Keywords: Photovoltaic canopy; Urban solar irrigation; Grid-connected photovoltaic system; Techno-economic assessment; CO₂ mitigation; Hot-arid climate

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1. Introduction

Urban green spaces such as public gardens and parks play a critical role in improving environmental quality, mitigating urban heat island effects and enhancing social well-being, particularly in densely populated cities located in hot-arid regions.

However, maintaining these green infrastructures requires reliable and continuous irrigation, which in many developing countries still depends heavily on diesel-powered pumping systems. These systems are associated with high operational costs, fuel supply vulnerability, noise and significant greenhouse gas emissions [1–3]. As cities expand and climate conditions deteriorate,

Nomenclature

$E_{analytical}$	– energy estimated by the simplified analytical model, kWh
E_m	– monthly electrical energy output, kWh
EPV_{syst}	– monthly or annual energy yield predicted by PVsyst, kWh
H_m	– monthly in-plane solar irradiation, h
P_r	– rated PV system capacity, kWp
PR_m	– monthly effective performance ratio

Abbreviations and Acronyms

AC	– alternating current
DC	– direct current
IQD	– Iraqi dinar
IRR	– internal rate of return
LCOE	– levelised cost of electricity
PV	– photovoltaic
STC	– standard test conditions

the energy–water nexus in urban irrigation has become an increasingly important challenge.

Photovoltaic (PV) energy has emerged as one of the most promising solutions for sustainable irrigation due to its modularity, declining costs, and suitability for regions with high solar irradiance [4–6]. In arid and semi-arid climates, solar energy availability often coincides with peak irrigation demand, making PV-powered irrigation systems particularly attractive. Nevertheless, most existing studies have focused on rural or agricultural irrigation schemes, frequently relying on off-grid configurations with battery storage, which increases system complexity, cost, and long-term maintenance requirements [7,8]. In contrast, grid-connected PV systems without storage, operating under net-metering schemes, remain underexplored for urban irrigation applications.

Recent research has emphasised the importance of system performance, degradation behaviour, and thermal effects on PV installations operating in harsh climates. Long-term exposure to high ambient temperatures, dust accumulation and thermal cycling can significantly affect PV efficiency and reliability [9–11]. Detailed performance assessment and loss analysis are therefore essential to ensure realistic energy yield predictions. Previous investigations have demonstrated that careful system design and monitoring can maintain high performance ratios even under challenging environmental conditions [12]. These aspects are particularly relevant for urban installations, where shading, architectural constraints and multifunctional land use must be considered.

An emerging concept in urban solar integration is the use of dual-use PV canopies, where PV modules are installed as shading structures over public spaces. Such systems offer multiple benefits: electricity generation, thermal comfort for users, reduced surface temperatures and improved land-use efficiency [13–15]. When applied to public gardens, PV canopies can simultaneously support irrigation energy demand and provide shaded recreational areas, contributing to both environmental sustainability and social value. Despite these advantages, quantitative assessments of PV canopy systems for urban irrigation, especially in hot-arid cities, remain limited in the literature.

In parallel, techno-economic analyses have become a cornerstone of evaluating renewable energy projects, particularly in regions where fossil-fuel-based solutions are still economically dominant. Studies conducted under Iraqi climatic conditions have demonstrated the critical role of thermal behaviour, material performance and system aging in determining the long-term viability of solar energy systems [16–18]. Moreover, replacing diesel-driven systems with electrically powered alternatives supplied by PV energy can yield substantial reductions in operating costs and carbon emissions, provided that system sizing

and grid interaction are properly optimised [19].

Against this background, the present study addresses a clear research gap by providing a comprehensive energy, techno-economic and environmental assessment of a grid-connected PV canopy system used for urban irrigation in a hot-arid climate. The study is based on a real public garden located in Baghdad, Iraq, where a dual-use PV canopy system with an installed capacity of 84.5 kWp supplies a relatively small irrigation load while exporting most of the generated electricity to the national grid. Using PVsyst-based simulation, an independent simplified analytical model, realistic load profiles and detailed economic and emissions analysis, this work aims to:

- quantify the energy performance of the PV canopy system under local climatic conditions,
- evaluate its economic feasibility in comparison with conventional diesel-based irrigation,
- assess its contribution to CO₂ emission mitigation.

In addition to detailed software-based simulation, the study introduces an independent engineering cross-check model to enhance methodological transparency and improve the reproducibility of PV canopy performance estimation under hot-arid operating conditions. The analytical cross-check showed an annual deviation of only 1.51% relative to the PVsyst prediction, confirming acceptable agreement between the two approaches. The results provide practical insights for municipalities and decision-makers seeking scalable, multifunctional solar solutions for sustainable urban green infrastructure.

2. System description and methodology

2.1. Site description and climatic conditions

The investigated system is installed in a public urban garden located in Al-Ghazālīyah district, Baghdad, Iraq, at geographic coordinates of 33.33°N latitude and 44.27°E longitude, with an average altitude of approximately 28 m above sea level. The site lies within a hot-arid climatic zone, characterised by long, dry summers, high solar irradiance and elevated ambient temperatures during the irrigation season.

Baghdad receives high annual solar potential, with average global horizontal irradiation exceeding 1 800 kWh/m² per year, making it highly suitable for photovoltaic energy deployment. However, the same climatic conditions impose challenges related to thermal losses, dust accumulation and long-term system performance, which necessitate realistic modelling and conservative performance assumptions [9,11]. The geographical location of the public garden, the spatial arrangement of the PV canopy system, and the near-shading environment are illustrated in Fig. 1.

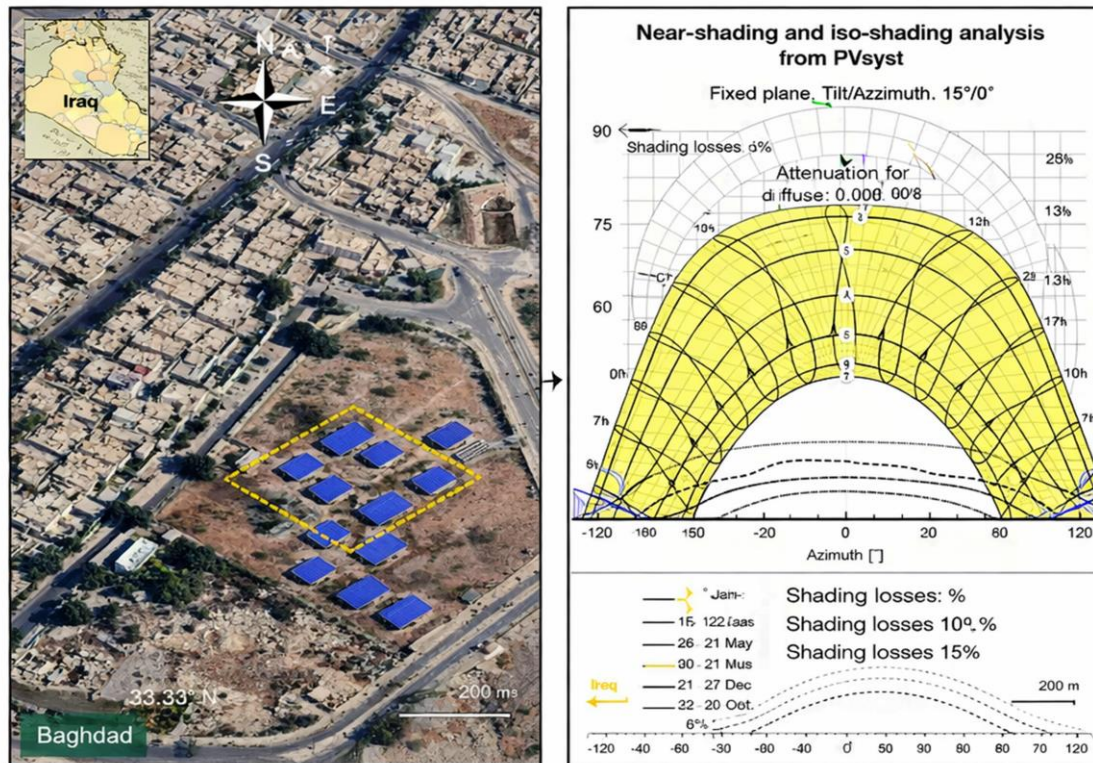


Fig. 1. Geographical location of the investigated public garden in Al-Ghazaliyah district, Baghdad, Iraq, showing the spatial layout of the PV canopy system and the corresponding near-shading and iso-shading analysis obtained from PVsyst.

2.2. Description of the PV canopy system

The photovoltaic installation is designed as a dual-use PV canopy system, where PV modules are mounted on shading structures integrated within the garden area. These canopies simultaneously provide electrical energy generation and shaded spaces for park users, thereby improving land-use efficiency and thermal comfort.

The total installed PV capacity of the system is 84.5 kWp, consisting of 128 monocrystalline PV modules, each rated at 660 Wp under standard test conditions (STC). The modules are arranged in eight strings of sixteen modules, mounted at a fixed tilt angle of 15° and an azimuth of 0°, optimised for annual energy production under local conditions. The total active module area is approximately 398 m². The structural configuration of the dual-use photovoltaic canopy system and its integration within the public garden are illustrated in Fig. 2.

Power conversion is achieved using two grid-connected inverters, each rated at 40 kWac, resulting in a direct current to alternating current (DC-to-AC) power ratio of approximately 1.06, which is consistent with best practices for minimising inverter clipping while enhancing energy yield. The system operates without battery storage and is directly connected to the national grid under a net-metering configuration.

The site characteristics and main technical specifications of the investigated grid-connected PV canopy system, including installed capacity, array configuration and inverter characteristics, are summarised in Table 1.

2.3. Irrigation load characterisation

The irrigation demand of the public garden is supplied by a single electric water pump rated at 4 horsepower (≈ 2.98 kW). Field inspection and operational assessment indicated that the garden requires approximately 10 000 litres of water per day, which can be delivered by operating the pump for two hours daily.

Under these operating conditions, the annual electricity consumption of the irrigation pump is approximately 2 200 kWh per year. Compared with the annual PV electricity



Fig. 2. Schematic configuration of the dual-use photovoltaic canopy system integrated within the public garden, showing the PV module arrangement, supporting structures, and the associated electrical components supplying irrigation and exporting surplus electricity to the grid.

Table 1. Site characteristics and technical specifications of the investigated grid-connected PV canopy system.

Parameter	Value	Unit
Location	Al-Ghazālīyah, Baghdad, Iraq	—
Latitude / Longitude	33.33° N / 44.27° E	—
Altitude	28	m
Climate zone	Hot-arid	—
PV system type	Grid-connected PV canopy (no storage)	—
Total installed capacity	84.5	kWp
Number of PV modules	128	units
Nominal power per module	660	Wp
PV module technology	Monocrystalline silicon	—
PV array configuration	8 strings × 16 modules	—
Tilt angle / azimuth	15° / 0°	—
Total module area	398	m ²
Number of inverters	2	units
Inverter rated power	40	kWac
Total inverter capacity	80	kWac
DC/AC ratio	1.06	—
Grid interaction	Net metering	—

production of the system, the irrigation load represents only about 1.6% of the total generated energy, allowing the majority of the PV output to be exported to the grid. This low load-to-generation ratio highlights the suitability of grid-connected operation and the economic importance of surplus energy export.

2.4. Performance modelling and simulation approach

The energy performance of the PV canopy system was evaluated using PVsyst (version 7.4.8), a widely validated simulation tool for photovoltaic system analysis. The model incorporates site-specific meteorological data derived from the Meteonorm database, including solar irradiance, ambient temperature and seasonal variability. The simulation accounts for the principal system losses, including soiling losses, thermal losses due to elevated module operating temperatures, light-induced degradation, module mismatch losses, ohmic losses in DC wiring, inverter conversion losses, and system unavailability. Near-shading effects caused by the canopy configuration were modelled using the linear shading approach implemented in PVsyst. The overall system performance was quantified using standard indicators, including annual energy yield, specific yield and performance ratio (PR).

2.5. Simplified analytical model and cross-validation approach

To complement the PVsyst-based simulation and improve methodological transparency, a simplified analytical model was developed to independently estimate the annual and monthly energy output of the PV canopy system. The analytical approach was based on the installed PV capacity, monthly solar resource and aggregate performance losses under local operating conditions.

The monthly electrical energy yield of the PV canopy system was estimated using the following simplified relationship:

$$E_m = P_r \times H_m \times PR_m, \quad (1)$$

where E_m is the monthly electrical energy output (kWh), P_r is the rated PV system capacity (kWp), H_m is the monthly in-plane solar irradiation expressed in equivalent peak-sun-hours, and PR_m is the monthly effective performance ratio. The term PR_m incorporates the main loss mechanisms affecting system operation, including thermal losses, soiling losses, mismatch losses, DC wiring losses, inverter losses and minor near-shading effects.

The simplified analytical model was implemented in a spreadsheet-based framework to provide an independent engineering cross-check of the simulation outputs. The purpose of this model is not to replace detailed photovoltaic simulation software, but rather to provide a transparent and reproducible validation tool for assessing the physical consistency of the predicted energy yield. The outputs of the analytical model were therefore compared with the PVsyst results at both monthly and annual scales in order to evaluate the agreement between the two approaches and strengthen confidence in the reported performance indicators.

To quantify the agreement between the analytical model and the PVsyst simulation, the relative deviation was calculated as follows:

$$\begin{aligned} \text{Relative deviation (\%)} &= \\ &= |E_{PVsyst} - E_{analytical}| / E_{PVsyst} \times 100, \end{aligned} \quad (2)$$

where E_{PVsyst} is the monthly or annual energy yield predicted by PVsyst and $E_{analytical}$ is the corresponding value estimated by the simplified analytical model. This comparison was used to evaluate the consistency of the two approaches at both monthly and annual scales.

At the annual scale, the simplified analytical model showed a deviation of 1.51% relative to the PVsyst prediction, which indicates acceptable agreement for validation purposes under the adopted assumptions.

2.6. Techno-economic and environmental assessment methodology

The techno-economic and environmental evaluation was conducted after confirming the consistency of the PV energy output using both PVsyst simulation and the independent simplified analytical model. The economic assessment was performed using a life-cycle cost approach, considering capital investment, operation and maintenance costs, energy savings from diesel displacement and revenue from surplus electricity exported to the grid. The primary economic indicators used in this study include the levelised cost of electricity (LCOE), simple payback period, and internal rate of return (IRR). The environmental assessment focused on the reduction of greenhouse gas emissions achieved by replacing the diesel-powered irrigation system with an electrically driven pump supplied by photovoltaic energy. Carbon dioxide emission reductions were estimated using standard

emission factors associated with diesel fuel consumption and grid electricity generation [9].

2.7. Data reliability and assumptions

All system parameters, load profiles, and economic inputs were selected to reflect realistic operating conditions based on field observations, local climatic conditions and representative cost structures. Conservative assumptions were adopted where uncertainty existed in order to avoid overestimation of system performance or financial benefits. In addition, the simplified analytical model was intentionally formulated as an independent cross-check tool rather than a substitute for detailed PV simulation. This approach improves methodological transparency while maintaining consistency with the technical assumptions used in the PVsyst-based analysis.

3. Energy performance results and loss analysis

3.1. Annual energy yield and system productivity

The PV canopy system exhibits a high annual energy yield despite operating under hot-arid climatic conditions. The simulated annual electricity production reaches 133 189 kWh, corresponding to a specific yield of 1 577 kWh/kWp per year. This value is notably high for an urban installation and reflects the favourable solar resource of Baghdad combined with appropriate system sizing and orientation. The calculated performance ratio (PR) of 79.6% indicates efficient conversion of available solar irradiation into usable electrical energy, even when accounting for thermal losses, soiling and inverter inefficiencies. Such a PR value is comparable to, and in some cases higher than, those reported for PV systems operating in temperate climates, confirming the technical viability of PV installations in hot-arid urban environments when proper design practices are applied.

3.2. Monthly energy production profile

Monthly energy generation follows the expected seasonal solar irradiation pattern. Peak electricity production occurs between May and August, coinciding with maximum global horizontal irradiation and extended daylight hours. During this period, monthly energy output exceeds 13 MWh, with the highest contribution recorded in June and July. Lower production values are observed during winter months, particularly in December and January, due to reduced solar irradiance and shorter daylight duration. Nevertheless, even during these months, the system maintains stable operation and sufficient energy availability to fully cover the irrigation load. The consistency of the monthly production profile demonstrates the robustness of the canopy-mounted configuration against seasonal variability. A detailed month-by-month and annual energy balance of the investigated PV canopy system, including irrigation demand and surplus electricity exported to the grid, is presented in Table 2. A comparison between the monthly photovoltaic electricity generation and the irrigation energy demand is presented in Fig. 3, clearly showing that the irrigation load represents only a small fraction of the total PV production throughout the year.

Table 2. Monthly and annual energy balance of the investigated PV canopy system, irrigation demand, and surplus electricity exported to the grid.

Month	PV energy production, kWh	Irrigation energy demand, kWh	Energy exported to grid, kWh
January	9 139	187	8 952
February	9 209	169	9 040
March	11 725	187	11 538
April	13 224	181	13 043
May	13 831	187	13 644
June	14 648	181	14 467
July	14 322	187	14 135
August	13 246	187	13 059
September	12 522	181	12 341
October	10 324	187	10 137
November	8 772	181	8 591
December	8 138	187	7 951
Total (annual)	133 189	2 200	130 989

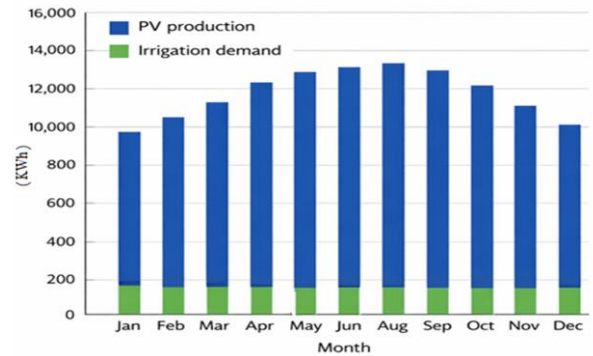


Fig. 3. Monthly photovoltaic electricity generation and irrigation energy demand of the investigated public garden, illustrating the large surplus electricity available for grid export throughout the year.

3.3. Load coverage and surplus energy export

The annual electricity demand of the irrigation system, estimated at 2 200 kWh, represents only approximately 1.6% of the total PV production. As a result, the system operates predominantly as an electricity-exporting installation, with more than 130 MWh per year of surplus energy injected into the national grid.

This pronounced mismatch between generation capacity and irrigation demand is intentional and reflects the dual-function design philosophy of the system. Rather than sizing the PV system solely to match the irrigation load, the canopy installation is optimised to maximise land-use efficiency and grid support while still fully satisfying local energy needs. From an operational standpoint, the irrigation load is entirely supplied by solar-generated electricity during daytime hours, while grid interaction ensures uninterrupted operation under all conditions.

3.4. Loss analysis and performance breakdown

A detailed loss analysis reveals the dominant mechanisms affecting system performance. Thermal losses constitute the large-

est contribution, accounting for approximately 8.4% of the total energy reduction, which is consistent with the high ambient temperatures characteristic of Baghdad's climate. Soiling losses are estimated at 3%, reflecting conservative assumptions for dust accumulation under regular maintenance conditions.

Additional losses include light-induced degradation (approximately 2%), module mismatch losses (2%) and ohmic losses in DC cabling (approximately 1%). Inverter-related losses remain limited to around 2%, highlighting the suitability of the selected inverter configuration and DC-to-AC sizing ratio. Near-shading losses associated with the canopy geometry are negligible, contributing less than 0.3% to the overall energy reduction.

The cumulative impact of these losses results in an effective energy output that remains high relative to the available solar resource, confirming that no single loss mechanism critically undermines system performance. The key energy performance indicators and principal loss contributions affecting the investigated PV canopy system are summarised in Table 3.

Table 3. Key energy performance indicators and principal loss contributions of the investigated PV canopy system.

Indicator	Value	Unit
Annual energy production	133 189	kWh per year
Specific yield	1 577	kWh/kWp per year
Performance ratio (PR)	79.6	%
Global horizontal irradiation	1 818	kWh/m ² per year
Effective irradiation on collectors	1 876	kWh/m ² per year
Near-shading losses	0.26	%
Soiling losses	3.0	%
Thermal losses	8.36	%
Inverter losses	2.01	%
System availability	97.8	%

The contribution of the main loss mechanisms affecting the annual energy yield of the PV canopy system is summarised in the loss diagram presented in Fig. 4.

3.5. Cross-validation of PVsyst results using the simplified analytical model

To assess the robustness of the simulation results, the annual and monthly PV energy outputs predicted by PVsyst were independently cross-checked using the simplified analytical model described in Section 2.5. The analytical model reproduced the same seasonal production trend observed in the software simulation, with peak generation during late spring and summer and lower output during winter months.

At the annual scale, the analytical estimation showed acceptable agreement with the PVsyst result, with a total annual deviation of 1.51%. This indicates that the predicted energy yield is physically reasonable for the installed capacity, local solar resource and assumed loss conditions of the Baghdad site. Minor monthly deviations between the two approaches are ex-

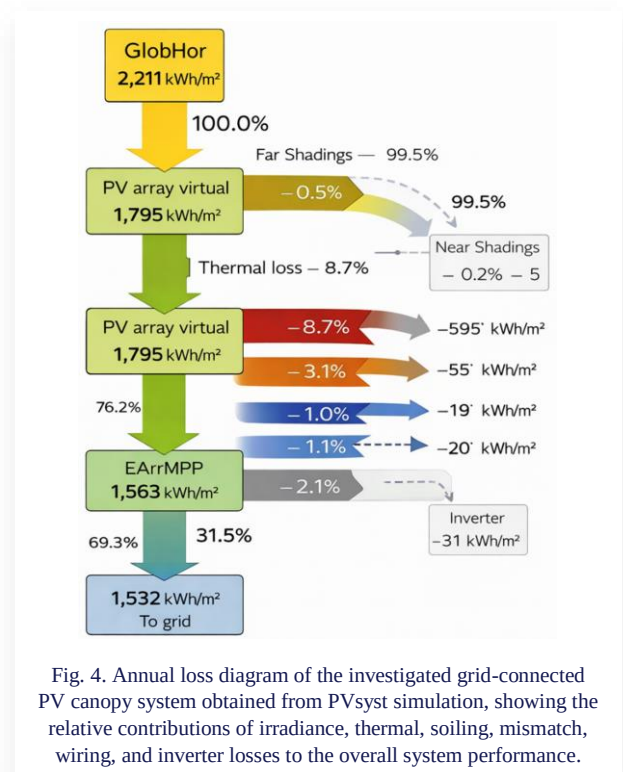


Fig. 4. Annual loss diagram of the investigated grid-connected PV canopy system obtained from PVsyst simulation, showing the relative contributions of irradiance, thermal, soiling, mismatch, wiring, and inverter losses to the overall system performance.

pected because PVsyst incorporates more detailed sub-models related to thermal behaviour, incidence-angle effects and inverter operation, whereas the simplified analytical model applies aggregate monthly correction factors.

Overall, the comparison demonstrates that the reported performance indicators are not solely dependent on commercial software output, but are also supported by an independent engineering calculation framework. The simplified analytical model reproduced the same overall seasonal behaviour and annual performance trend obtained from PVsyst, and the annual deviation between the two approaches was limited to 1.51%, confirming the physical consistency of the predicted system output under the adopted operating assumptions. A detailed month-by-month comparison between the PVsyst predictions and the simplified analytical model estimates is presented in Table 4.

Note: The simplified analytical estimates were derived using monthly solar-resource inputs and aggregate loss assumptions to provide an independent engineering cross-check of the PVsyst results. The analytical model is intended for validation and transparency purposes rather than as an exact replacement for the detailed PVsyst simulation.

3.6. Reliability under hot-arid operating conditions

Operating in a hot-arid urban environment imposes challenges related to elevated module temperatures, dust exposure and prolonged summer operation. Despite these constraints, the system demonstrates stable performance indicators across the annual cycle. The acceptable agreement between PVsyst and the simplified analytical model, with an annual deviation of only 1.51%, further strengthens confidence in the reliability of the reported performance indicators under hot-arid operating conditions. The absence of battery storage also enhances system reliability by eliminating one of the most failure-prone components

in photovoltaic systems. The results indicate that grid-connected PV canopy systems can achieve high energy productivity and operational stability in extreme climates, provided that appropriate assumptions regarding losses and maintenance are incorporated into the design stage.

Table 4. Month-by-month comparison between PVsyst predictions and simplified analytical model estimates for the investigated PV canopy system.

Month	PVsyst energy, kWh	Analytical model energy, kWh	Absolute deviation, kWh	Relative deviation, %
January	9 139	8 152	987	10.80
February	9 209	8 643	566	6.14
March	11 725	10 883	842	7.18
April	13 224	12 524	700	5.29
May	13 831	13 372	459	3.32
June	14 648	13 614	1 034	7.06
July	14 322	13 654	668	4.66
August	13 246	13 304	58	0.44
September	12 522	12 417	105	0.84
October	10 324	9 639	685	6.64
November	8 772	8 024	748	8.52
December	8 138	6 948	1 190	14.62
Annual total	133 189	131 174	2 015	1.51

3.7. Summary of performance findings

Overall, the combined PVsyst simulation and independent analytical cross-validation confirm that the investigated PV canopy system not only satisfies the local irrigation demand with a large safety margin, but also functions as a significant distributed generation asset. The analytical model reproduced the seasonal and annual performance trends obtained from PVsyst, with an annual deviation of only 1.51%, supporting the physical consistency of the reported energy yield. The combination of high specific yield, favourable performance ratio and substantial surplus electricity export establishes the system as an effective model for integrating renewable energy into urban green infrastructure under hot-arid climatic conditions.

4. Techno-economic assessment

4.1. Economic framework and assumptions

The techno-economic evaluation of the proposed PV canopy system was conducted using a life-cycle cost approach that reflects realistic operating conditions and local market parameters. Capital expenditure includes PV modules, inverters, mounting structures, electrical accessories, and installation costs, while operational expenditure accounts primarily for routine maintenance and system supervision. In line with common practice for grid-connected PV systems without storage, annual operation and maintenance costs were assumed to be modest due to the absence of batteries and rotating mechanical components. All

economic calculations were performed using conservative assumptions to avoid overestimation of financial benefits. Electricity tariffs, fuel prices and emission factors were selected based on prevailing local values and recent studies conducted under Iraqi climatic and economic conditions [9,11,16]. For clarity and international readability, selected monetary values originally expressed in Iraqi dinars (IQD) are also reported in equivalent U.S. dollars (USD) using a consistent representative exchange rate. An exchange rate of 1 USD = 1300 IQD was adopted for indicative currency conversion throughout the analysis.

4.2. Cost of electricity and competitiveness

The levelised cost of electricity (LCOE) produced by the PV canopy system was estimated at approximately 44 IQD/kWh (≈ 0.034 USD/kWh), considering the total life-cycle cost over an assumed system lifetime of 25 years. This value is significantly lower than the prevailing commercial electricity tariff of approximately 120 IQD/kWh (≈ 0.092 USD/kWh), indicating a cost reduction of nearly 63% per unit of delivered electricity.

This substantial difference highlights the economic advantage of photovoltaic energy in high-irradiance regions, particularly when systems are designed to operate without storage and are allowed to export surplus electricity to the grid. From a municipal perspective, such a reduction directly translates into long-term budgetary relief and improved energy cost predictability.

4.3. Diesel displacement and operating cost savings

Prior to PV system implementation, irrigation was supplied by a diesel-powered pump, characterised by high fuel consumption, frequent maintenance requirements, and sensitivity to fuel price fluctuations. Replacing this system with an electrically driven pump powered by solar-generated electricity results in immediate and sustained operating cost savings.

For the investigated case, the annual irrigation energy demand of 2 200 kWh is fully covered by PV generation, effectively eliminating diesel fuel consumption for irrigation purposes. The avoided cost of diesel fuel constitutes a significant portion of the annual savings, particularly when accounting for indirect costs associated with fuel transport, storage, and engine maintenance. These findings are consistent with previous studies demonstrating the economic vulnerability of diesel-based irrigation systems in comparison with solar-powered alternatives [6,7].

4.4. Revenue from surplus electricity export

A defining feature of the investigated system is the large surplus of electricity available for export to the national grid. Because the irrigation demand accounts for only a small fraction of the annual PV output, more than 130 MWh per year of electricity can be delivered to the grid under the adopted grid-connected operating scheme. This exported energy represents an important economic advantage of the system, as it improves overall electricity utilisation and enhances the financial value of the installation beyond its direct irrigation function. Unlike stand-alone systems, where excess generation may remain underutilised, the

grid-connected configuration enables productive use of the surplus electricity and strengthens the economic case for dual-use PV canopy deployment in urban public spaces.

4.5. Payback period and investment attractiveness

Based on the combined effects of reduced diesel dependence, lower electricity costs, and the economic value of surplus electricity export, the investigated PV canopy system achieves a simple payback period of approximately 6 years. This value falls within a practically acceptable range for municipal renewable energy investments and leaves a substantial portion of the system lifetime available for net economic benefit. The calculated internal rate of return (IRR) of approximately 18% further confirms the financial attractiveness of the project. These indicators show that the proposed system is not only environmentally beneficial, but also economically competitive as a long-term urban infrastructure investment.

4.6. Economic robustness and long-term perspective

One of the principal strengths of the investigated PV canopy system is its resilience to future economic uncertainty. In contrast to diesel-based irrigation systems, which remain highly sensitive to fuel price volatility and recurrent operating costs, the photovoltaic system is characterised by comparatively stable long-term operating expenses. This economic robustness becomes more pronounced when surplus electricity can be productively exported to the grid under a grid-connected operating scheme. In addition, the dual-use canopy configuration provides non-energy benefits, including shading and improved usability of public space, which further strengthen the overall value proposition of the system from a municipal planning perspective.

4.7. Summary of techno-economic findings

The techno-economic assessment confirms that the investigated PV canopy system represents a financially viable solution for urban irrigation in hot-arid regions. The combination of low electricity cost, productive utilisation of surplus generation through grid export, acceptable payback period, and favourable internal rate of return demonstrates that the system can deliver long-term economic value in addition to its environmental and functional benefits. The main techno-economic and environmental indicators of the investigated PV canopy system are summarised in Table 5.

5. Environmental impact and CO₂ mitigation

5.1. Emission reduction through diesel replacement

The environmental benefits of the PV canopy system are primarily associated with replacing the previous diesel-powered irrigation supply with an electrically driven pump supported by photovoltaic energy. Diesel-based irrigation systems are a notable source of greenhouse gas emissions due to continuous fuel combustion and relatively low overall efficiency. In the investigated case, the transition to photovoltaic energy effectively eliminates diesel use for irrigation purposes.

Table 5. Main techno-economic and environmental indicators of the investigated PV canopy system.

Parameter	Value	Unit
System lifetime	25	years
Levelised cost of electricity (LCOE)	44 (≈ 0.034)	IQD/kWh (USD/kWh)
Commercial electricity tariff	120 (≈ 0.092)	IQD/kWh (USD/kWh)
Cost reduction compared to grid	63	%
Simple payback period	≈ 6	years
Internal rate of return (IRR)	≈ 18	%
Annual diesel fuel displacement	Fully eliminated	—
Annual CO ₂ emission reduction	113	tonnes
Annual surplus energy exported	≈ 131	MWh

Based on the adopted diesel displacement and grid-related emission assumptions, the avoided diesel fuel consumption corresponds to an estimated annual reduction of approximately 113 tonnes of CO₂. This reduction is achieved without compromising irrigation reliability or water delivery, highlighting the effectiveness of photovoltaic systems as a direct decarbonisation measure for urban service infrastructure. The environmental benefit of replacing diesel-powered irrigation with photovoltaic-powered irrigation is illustrated in Fig. 5, showing a substantial reduction in annual CO₂ emissions.

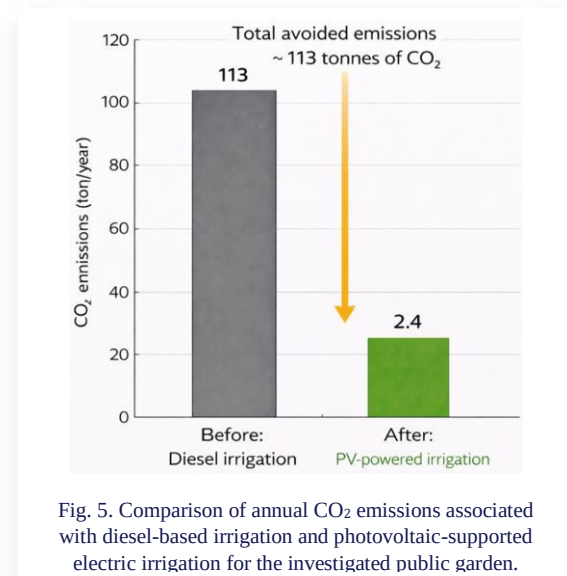


Fig. 5. Comparison of annual CO₂ emissions associated with diesel-based irrigation and photovoltaic-supported electric irrigation for the investigated public garden.

5.2. Contribution to grid decarbonisation

In addition to reducing local emissions through diesel displacement, the investigated PV canopy system contributes to grid decarbonisation by exporting more than 130 MWh per year of surplus solar electricity under the adopted grid-connected operating scheme. This exported electricity can offset part of the fossil-fuel-based generation that would otherwise be required to meet daytime demand. Although the carbon intensity of the national

grid may vary over time, the cumulative contribution of distributed photovoltaic generation remains environmentally significant, particularly in high-irradiance regions where daytime solar production is substantial. Accordingly, the environmental value of the system extends beyond the boundaries of the public garden itself and supports broader urban decarbonisation efforts.

5.3. Water–energy–climate co-benefits

The PV canopy configuration provides additional co-benefits that extend beyond direct electricity generation and carbon emission reduction. By functioning as shading structures within the public garden, the canopies can help reduce surface temperatures and limit direct solar exposure of soil and vegetation during peak summer conditions. This shading function may contribute to reduced evapotranspiration and improved local water-use efficiency, although these effects were not quantified in the present study. In addition, the integration of renewable energy generation into urban green spaces can support climate adaptation strategies by improving thermal comfort for users and enhancing the multifunctional value of public infrastructure.

5.4. Environmental significance at an urban scale

Although the present analysis focuses on a single public garden, the broader environmental implications become more significant when similar PV canopy systems are deployed across multiple urban parks and green spaces. Replication of this concept at the city scale could lead to cumulative reductions in diesel use, lower dependence on fossil-fuel-based electricity, and measurable decreases in urban carbon emissions. In addition to these direct energy-related benefits, wider deployment of PV canopies could support more climate-responsive urban infrastructure by combining renewable electricity generation with shading and improved public-space functionality. These considerations indicate that the environmental significance of the proposed system extends beyond the individual case study and may inform sustainable urban planning strategies in hot-arid cities.

6. Discussion

6.1. Comparison with existing studies

The performance indicators obtained in this study, particularly the specific yield of 1 577 kWh/kWp per year and the performance ratio of 79.6%, are consistent with, and in some cases superior to, values reported for PV systems operating in comparable hot-arid regions. Unlike many previous studies that focused mainly on ground-mounted or rooftop PV systems, the present work demonstrates that canopy-mounted configurations integrated into public urban spaces can achieve high productivity despite structural and near-shading constraints. In addition, the present study does not rely solely on commercial software output, as the PVsyst results were independently cross-checked using a simplified analytical model. This additional validation step improves the methodological transparency of the analysis and strengthens confidence in the reported energy performance trends. In particular, the annual deviation between the PVsyst output and the simplified analytical model was limited to 1.51%,

indicating acceptable agreement between the two approaches for engineering validation purposes. From an economic standpoint, the reported LCOE of 44 IQD/kWh (≈ 0.034 USD/kWh) and payback period of approximately 6 years compare favourably with other solar irrigation solutions, particularly off-grid systems that rely on battery storage. The absence of storage not only reduces costs but also improves long-term reliability, which is a critical consideration for municipal infrastructure.

6.2. Design philosophy and system sizing

A key design insight emerging from this study is the deliberate oversizing of the PV canopy system relative to the direct irrigation load. Rather than minimising the installed PV capacity to match only the local pumping demand, the system was configured to maximise the functional value of the available urban space through simultaneous electricity generation, shading provision and grid interaction. This design philosophy differs from conventional solar irrigation configurations, which are typically sized to satisfy a specific agricultural or pumping load with limited consideration of broader urban functionality. In the investigated case, the relatively small irrigation demand enables most of the generated electricity to be exported to the grid, thereby improving the economic value of the installation while preserving its primary service role within the public garden. This multi-objective sizing strategy demonstrates that PV canopy systems can be planned not only as energy supply units, but also as integrated urban infrastructure assets.

6.3. Practical implications for municipalities

For municipalities in hot-arid regions, the results highlight a practical pathway for reducing dependence on diesel-based irrigation without introducing significant operational complexity. Grid-connected PV canopy systems offer predictable energy costs, minimal maintenance requirements and measurable environmental benefits while simultaneously improving the functional quality of public spaces through shading. In addition to supplying local irrigation demand, such systems can contribute surplus electricity to the grid, thereby enhancing the economic value of municipal solar investments. Importantly, the dual-use nature of the canopy installation may improve public acceptance by combining visible social benefits with clean energy generation. These features make PV canopy systems a promising option for sustainable urban green infrastructure planning in cities with high solar potential.

6.4. Limitations and future research

While the present study provides a comprehensive assessment based on detailed simulation and realistic operational assumptions, future work could extend the analysis through long-term monitoring of actual system performance under field conditions. Further research may also quantify microclimatic effects, water-saving potential and user comfort improvements associated with canopy shading, thereby strengthening the interdisciplinary value of such systems. Another limitation is that the independent analytical model was intentionally formulated as a simplified cross-check tool for validation purposes rather than as a repla-

cement for detailed dynamic PV simulation software. Nevertheless, its agreement with the PVsyst results, reflected by an annual deviation of only 1.51%, improves confidence in the physical consistency of the reported energy performance indicators.

7. Conclusions

This study evaluated the energy performance, techno-economic feasibility, and environmental impact of a grid-connected photovoltaic canopy system implemented for urban irrigation in a hot-arid climate using both detailed PVsyst simulation and an independent simplified analytical cross-check model. Based on a real public garden case in Baghdad, the system demonstrates high annual energy productivity, with 133 189 kWh per year of electricity generation, a specific yield of 1 577 kWh/kWp per year, and a performance ratio of 79.6%.

The irrigation load, accounting for only 1.6% of total PV production, is fully supplied by solar electricity, while the majority of generated energy is exported to the national grid. Economic analysis confirms the competitiveness of the system, with a levelised cost of electricity of 44 IQD/kWh (≈ 0.034 USD/kWh), a payback period of approximately 6 years, and an internal rate of return of about 18%. Environmentally, the system achieves an estimated annual reduction of approximately 113 tonnes of CO₂, primarily through diesel fuel displacement and grid support.

The close agreement between the software-based simulation and the independent analytical estimation enhances confidence in the reported results and improves the methodological transparency of the study. In particular, the annual deviation between the PVsyst prediction and the simplified analytical model was limited to 1.51%, confirming acceptable agreement between the two approaches for validation purposes. Overall, the findings demonstrate that PV canopy systems represent a technically reliable, economically viable, and environmentally effective solution for integrating renewable energy into urban green infrastructure. Beyond irrigation, such systems provide a scalable model for multifunctional solar deployment in hot-arid cities, supporting sustainable urban development and long-term decarbonisation strategies.

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Numerical investigation of heat transfer in a partially porous-filled channel with core and wall configurations

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Abstract

The numerical simulations are carried out for analysis of heat transfer enhancement in the partially filled channel with the horizontal porous layer, considering two different configurations: at the core and at the walls of the channel. The Darcy-Brinkman-Forchheimer equation is used to model flow through the porous medium, and the governing equations are solved using a finite volume method. The study investigates the effect of the porous layer thickness, thermal conductivity and Darcy number on heat transfer performance and pressure drop. The detailed parametric analysis quantifies variations in the Nusselt number and pressure drop for both configurations. The study reveals that the pressure drop is consistently lower for porous layers located at the channel walls compared to the core for all Darcy numbers. The Nusselt number increases by up to 88% as the porous layer thickness ratio rises to 0.8, then decreases by approximately 19.5% at a lower thermal conductivity ratio of 2.5, but continues to rise at a higher ratio of 25. Increasing the thermal conductivity ratio to 25 improves the Nusselt number by up to 80.5% compared to a ratio of 2.5. Additionally, for lower Darcy numbers (up to 10^{-4}), the higher Nusselt number is achieved when the porous layer is located at the core. However, for higher Darcy numbers (above 10^{-3}), the Nusselt number is higher when the porous layer is at the walls. The results reveal that the position of the porous layer within the channel significantly affects its thermal and hydrodynamic performance.

Keywords: Channel flow; Darcy number; Heat transfer; Porous media; Pressure drop

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1. Introduction

Heat management in electronic components has become challenging due to technological advancement and miniaturisation. As conventional cooling methods are becoming obsolete, researchers are exploring alternative cooling technologies. The integration of porous materials into channels has shown great potential for enhancing heat transfer rates, with significant applications in electronic thermal management. The insertion of porous media in fluid channels modify the flow dynamics, resulting in reduced boundary layer thickness and enhanced heat transfer. Therefore, a detailed investigation of the heat transfer

analysis is required for the optimal design of a channel containing porous inserts.

Heat transfer enhancement is crucial for improving the thermal performance of various industrial systems, including heat exchangers, cooling systems and electronic devices. The heat enhancement techniques are vital for sustainability and are categorised as either passive technique (e.g. porous inserts, nanofluids, turbulators), which require no extra energy, or active technique (e.g. magnetic fields, external pumps). Passive techniques are typically preferred for their low energy demands, offering a more sustainable alternative for improving heat transfer and flow without consuming large amounts of energy, which is increasingly important for energy-conscious industries.

Nomenclature

C – inertial coefficient
 C_p – specific heat, J/(kg K)
 Da – Darcy number
 F – Forchheimer coefficient
 g – gravitational acceleration, m/s²
 H – channel height, m
 k – thermal conductivity, W/(m K)
 k_r – thermal conductivity ratio
 K – permeability, m²
 Nu – Nusselt number
 p – pressure, Pa
 Pr – Prandtl number
 q'' – heat flux, W/m²
 r – porous layer thickness, m
 r_p – porous layer thickness ratio
 Re – Reynolds number

T – temperature, K
 u – velocity in x -direction, m/s
 $\vec{v}$ – velocity vector, m/s
 x, y – Cartesian coordinates, m

Greek symbols

ε – porosity
 θ – non-dimensional temperature
 μ – dynamic viscosity, Pa·s
 ρ – density, kg/m³

Subscripts and Superscripts

c – core
 e – effective
 in – inlet
 m – bulk mean
 w – wall

The use of nanofluids, turbulators and porous inserts has been extensively studied in the literature as effective methods for enhancing heat transfer in various cooling systems. The details of these techniques, along with their performance metrics, working conditions and applications, are summarised in Table 1. Said et al. [1] presented a comprehensive review of passive heat transfer enhancement techniques employing nanofluids and turbulators. Alenezi et al. [2] experimentally investigated the effect of ultrasonic waves on heat transfer enhancement using the propylene glycol/water nanofluid in the minichannel. Their results showed that ultrasound at 27.8 kHz improved heat transfer, achieving a maximum Nusselt number increase of 11.3% for de-ionised water at the lowest flow rate (0.2 l/min). Although the study demonstrates the potential of active methods, the energy demand limits their applicability in low-power systems. The use of porous media is a well-established method for heat transfer enhancement, primarily due to its high surface area-to-volume ratio and ability to promote flow mixing. Hadim and Bethancourt [3] identified that full porous fills lead to prohibitive pressure drops, suggesting partial filling as a solution. Subsequent studies, such as Baig et al. [4], have continued this approach, often combining porous inserts with other methods like nanofluids. While the nanofluids and turbulators have been widely investigated, porous inserts have also shown more effective heat transfer enhancement while maintaining relatively low pressure drops and requiring no additional energy input. Owing to these advantages, porous materials represent an efficient and practical passive technique, motivating the present study to focus specifically on porous inserts for further investigation.

The study of heat transfer in porous channels or pipes, particularly under conditions such as constant wall temperature, constant heat flux and discrete heat sources, has received significant attention in recent research in the literature [3–9]. Several investigations have focused on understanding how these thermal boundary conditions influence flow behaviour and heat transfer in porous media. Hadim and Bethancourt [3] also investigated the parametric variation of heat source width in a channel and

revealed that when the width of the heat source decreases, a significant increase in the heat transfer occurs and reduces the pressure drop. Hadim et al. [9] investigated the effect of discrete heat sources on the bottom wall of the porous channel using two different cases: full and partial porous channels. The results show the same heat transfer in both cases at low Darcy number and higher pressure drop in fully porous channels. Jen and Yan [10] proposed a 3D model to investigate the flow patterns in partially filled porous media. They revealed that strong counter rotating vortices are present in the channel because of the blowing effect of the porous layer. Hadim and North [11] investigated the effect of multiple inlet and outlet in a porous channel with a constant heat flux condition at the bottom wall. They observed that the pressure drop is minimised when the channel height equals the inlet width, while maintaining the same level of heat transfer enhancement. They also proposed the length-averaged correlation for the Nusselt number.

The use of porous materials in channels has been widely analysed with respect to its effects on flow behaviour, temperature change and pressure drop. Rachedi and Chikh [12] examined mixed convection heat transfer between a channel and an electronic component, inserting a porous material between the two. They demonstrated that at lower thermal conductivity ratios (up to 2), the effect on temperature drop is minimal. However, at higher thermal conductivity ratios (from 2 to 10), a temperature drop of 15% can be achieved. Yucel and Guven [13] studied forced convection heat transfer in parallel plates with multiple heat sources. They observed a significant rise in pressure drop with increasing Reynolds numbers in the presence of a porous layer. Huang et al. [14] investigated the effect of a porous cover on heat transfer enhancement in a channel using multiple heat sources. They revealed that the presence of a porous cover significantly enhances heat transfer compared to a setup without it. Da Silva Miranda and Anand [15] explored the role of porous baffles in enhancing convective heat transfer within channels. They found that a smaller spacing between the baffles results in better heat transfer performance, and that an increase in the

Table 1: Summary of important studies in heat transfer enhancement for channel/internal flows.

Authors	Geometry	Working fluid	Applications	Key findings
Alenezi et al. [2]	Minichannel	Propylene glycol/water mixtures	Minichannel heat sinks, electronics cooling	1. Ultrasound (27.8 kHz) enhanced heat transfer, with a maximum Nusselt number increase of 11.3% for deionised water at the lowest flow rate (0.2 l/min).
Hadim and Bethancourt [3]	Porous channel	Dielectric fluid	Electronic cooling	1. A partially porous channel (porous layers only above heaters) achieves comparable heat transfer to a fully porous channel but with a much lower pressure drop. 2. Decreasing heat source width moderately improves heat transfer while significantly reducing pressure drop.
Baig et al. [4]	Microchannel	Al ₂ O ₃ -water nanofluid	Microchannel cooling, electronics thermal management	1. Finned cooking vessels significantly improve thermal performance and cooking rate compared to an unfinned vessel. 2. Hexagonal fins performed best, reducing boiling time by 13.63% and increasing maximum energy efficiency to 64.46%. 3. An economic analysis showed the hexagonal fin-based cooker could save up to 1495 MJ of fuel equivalent per year.
Promvonge [5]	Square duct with combined 30° V-fins and quadruple counter-twisted tapes	Air	Heat exchangers	1. The combined V-finned twisted tapes provide higher heat transfer and thermal performance than twisted tapes alone. 2. Heat transfer and friction factor increase with relative fin height but decrease with relative fin pitch.
Kalidasan et al. [6]	Solar box cooker with different fin configurations (pin, cross, hexagonal) on the cooking vessel	Water	Solar cooking	1. The fully wavy wall microchannel significantly enhances heat transfer, with a Nusselt number up to 3.87 times higher and thermal resistance only 0.41 times that of a smooth microchannel. 2. The flow becomes chaotic, intensifying fluid mixing and heat transfer with a relatively small pressure drop penalty.
Zhong et al. [7]	Square cross-section microchannel with fully wavy walls (all four sides)	Deionised water	Microchannel heat sinks for electronic cooling	1. The fully wavy wall microchannel significantly enhances heat transfer, with a Nusselt number up to 3.87 times higher and thermal resistance only 0.41 times that of a smooth microchannel. 2. The flow becomes chaotic, intensifying fluid mixing and heat transfer with a relatively small pressure drop penalty.
Mohammed et al. [8]	2D rib-groove channel (9 combinations of rectangular, triangular, trapezoidal ribs and grooves)	Nanofluids (Al ₂ O ₃ , CuO, SiO ₂ , ZnO with water, glycerin, engine oil)	Heat exchangers, turbine cooling, solar heaters	1. Rectangular rib-triangular groove provided the highest Nusselt number and best thermal enhancement factor. 2. SiO ₂ nanofluid showed the highest heat transfer performance, despite its moderate conductivity, due to favourable thermophysical properties.

thermal conductivity ratio enhances heat transfer. Ko and Anand [16] also examined the effect of porous baffles on convective heat transfer in a channel, with the baffles installed alternately at the top and bottom walls. They revealed a 300% enhancement in heat transfer with baffles compared to the channel without them. Additionally, they developed a correlation between the ratio of heat transfer enhancement and the Reynolds number. Kim et al. [17] investigated the effect of porous material in a channel. They developed a correlation for the Nusselt number, specifically for Reynolds numbers in the range of 1000 to 3000.

The influence of temperature-dependent thermophysical properties on heat transfer enhancement in fully or partially porous channels has been examined in several studies. Dehghan et al. [18,19] examined the effect of thermal conductivity in porous channels. They proposed a correlation for the Nusselt number based on thermal conductivity and the shape of the channel. Mehata et al. [20] examined forced convection heat transfer in a

wavy solar plant using porous blocks. They found that the average Nusselt number increased by 83.95% with the use of porous blocks compared to without them. Kaur et al. [21] investigated the effect of circular and semicircular heat sources in porous media and found that the lowest heat transfer occurred in the circular channel. Li et al. [22] examined the effect of multilayer porous inserts in a channel under non-equilibrium thermal conditions. The results showed that the pressure drop is directly proportional to the number of porous layers.

Due to their ability to enhance heat dissipation while maintaining a compact system design, channels partially filled with porous media have attracted considerable research attention. The temperature profile and Nusselt number used to quantify the heat transfer characteristics in such microchannels have been presented in previous studies [19,23]. Mahmoudi [24] developed analytical expressions for both the temperature field and Nusselt number in a microchannel. Dehghan [25] performed an

analytical study on the thermal behaviour of a channel partially filled with porous media. The addition of a porous material to a fluid undergoing convection results in a more uniform temperature distribution, which reduces thermal resistance and enhances heat transfer. The study also found that as the thickness of the porous material increases, the proportion of heat transferred by the porous medium relative to that transferred by the clear fluid flow increases. Dehghan et al. [26] developed analytical expressions for the local Nusselt number and the dimensionless temperature difference between the solid and fluid phases in a porous material placed inside a channel within the thermally developing region. Bibin et al. [27] numerically examined heat transfer in a partially filled porous channel using OpenFOAM. Their study reported that introducing a porous medium can enhance the heat transfer rate by 10.4 times compared to a channel without porous filling. They also found that this enhancement is most pronounced when the porous layer thickness ratio lies between 0.7 and 0.9. Kumar et al. [28] analysed heat transfer in sinusoidal, triangular, and trapezoidal channels containing porous media at their core. Their findings indicate that the trapezoidal channel achieves the highest thermal performance, with a maximum enhancement of 90%. Alkhamis [29] investigated heat transfer in a channel partially filled with porous material and equipped with solid turbulators. Their results indicate that the use of metal foam along with turbulator enhances both heat transfer and the friction factor. The combined effect of the metal foam and turbulators led to a fivefold improvement in thermal performance. Duan et al. [30] performed a numerical investigation of heat transfer in a wavy metal-foam sandwich configuration. Their analysis demonstrated that, under laminar flow conditions, the Nusselt number can increase by up to 118%.

Although many investigations have been conducted on heat transfer enhancement using porous layers in channels, a critical research gap remains. While the beneficial effects of partial porous layers are well recognised, a systematic numerical investigation into the influence of porous layer location and thickness ratio within a channel remains largely unexplored, particularly under constant heat flux boundary conditions. Addressing this gap, the present study focuses on analysing the effects of location of porous layer and thickness ratio on thermal and hydrodynamic performance to identify configurations that maximise heat transfer while minimising pressure losses. This study investigates convection heat transfer in a channel containing the porous layer with two different configurations: one near the walls and the other at the core of the channel. Partially filling the channel with the porous layer offers two primary benefits: it improves heat transfer performance and results in less pressure drop compared to a fully porous-filled channel. This work can be beneficial in industrial applications, such as heat exchangers, solar energy storage and electronic cooling. A uniform heat flux condition is applied at the channel wall. The effects of the thickness of the porous insert, thermal conductivity ratio and Darcy number on heat transfer enhancement are examined. Additionally, the velocity profile and pressure drop at different Darcy numbers are analysed to determine the optimal design of the channel.

2. Materials and methods

2.1 Governing equations

The governing equation for mass and momentum conservation for the fluid and porous region is explained in this section. The problem is simplified by assuming that the flow is 2D, steady and laminar. The Darcy-Brinkman-Forchheimer model [31] is employed to address the inertial effects in porous material.

The governing equations for mass and momentum conservation in the fluid region are as follows:

$$\nabla \cdot \vec{v} = 0, \quad (1)$$

$$\rho(\vec{v} \cdot \nabla)\vec{v} = -\nabla p + \mu \nabla^2 \vec{v}, \quad (2)$$

where $\vec{v}$ represents the velocity vector, μ denotes the dynamic viscosity, ρ is the density and p is the pressure.

The momentum equation, which incorporates the resistance of the porous medium using the Darcy-Forchheimer model [33], is given by [32]:

$$\frac{\rho}{\varepsilon}(\vec{v} \cdot \nabla)\vec{v} = -\nabla p + \frac{\mu}{\varepsilon} \nabla^2 \vec{v} - \frac{\mu}{K} \vec{v} - \frac{\rho F \varepsilon}{\sqrt{K}} |\vec{v}| \vec{v}, \quad (3)$$

where K represents the permeability, ε denotes the porosity, and F is the Forchheimer coefficient, which is a function of the porous geometry. Local thermal equilibrium is considered to be valid at the interface of the fluid and solid within the porous region.

The energy equation for fluid and porous region are written as [32]:

fluid region:

$$(\vec{v} \cdot \nabla)T = \frac{k}{\rho c_p} \nabla^2 T, \quad (4)$$

porous region:

$$(\vec{v} \cdot \nabla)T = \frac{k_e}{\rho c_p} \nabla^2 T, \quad (5)$$

where k is the thermal conductivity, c_p is the specific heat and T is temperature of the fluid, and k_e represents the effective thermal conductivity, which is influenced by the thermal conductivity of the solid phase.

To derive the non-dimensional form of the governing equations, scaling is applied to the velocity components, temperature and length, using the characteristic reference quantities: inlet velocity, heat flux and channel height, respectively. The dimensionless parameters are defined as follows:

$$\begin{aligned} X &= \frac{x}{H}, & Y &= \frac{y}{H}, & \vec{V} &= \frac{\vec{v}}{u_{in}}, & P &= \frac{p}{\rho u_{in}^2}, \\ \text{Re} &= \frac{\rho u_0 H}{\mu}, & \text{Da} &= \frac{K}{H^2}, & \text{Pr} &= \frac{\mu c_p}{k}, & \theta &= \frac{T - T_{in}}{q''(H/k)}, \\ C &= F\varepsilon, & r_p &= \frac{r}{H}, & k_r &= \frac{k_e}{k}, \end{aligned}$$

where H is the channel height, u_{in} is the inlet velocity in x-direction, u_0 is the average velocity in x-direction, q'' is the heat flux at the wall, θ is the non-dimensional temperature, r is the porous layer thickness, r_p is the porous layer thickness ratio, k_r is the thermal conductivity ratio, Pr is the Prandtl number and Re is the Reynolds number.

The non-dimensional form of the governing equations for mass and momentum conservation is written as:

$$\nabla \cdot \vec{V} = 0, \quad (6)$$

$$\frac{1}{\varepsilon} (\vec{V} \cdot \nabla) \vec{V} = -\nabla P + \frac{1}{\varepsilon \text{Re}} \nabla^2 \vec{V} - \frac{1}{\text{Da Re}} \vec{V} - \frac{c}{\sqrt{\text{Da}}} |\vec{V}| \vec{V}. \quad (7)$$

The energy equation in non-dimensional form is written as

$$(\vec{V} \cdot \nabla) \theta = \frac{k_r}{\text{Re Pr}} \nabla^2 \theta. \quad (8)$$

For flow in a channel, the Nusselt number for constant heat flux is calculated as [3]

$$\text{Nu} = \frac{q'' H}{k (T_w - T_m)} = \frac{1}{\theta_w - \theta_m}. \quad (9)$$

2.2 Problem statement and validation

Figure 1 illustrates the schematic of the computational domain. In one case, the porous layer is placed on the channel walls, and in the other case, the same porous material is located at the core of the channel. Geometric configurations partially or fully filled

with a porous material are subjected to a constant and uniform heat flux. The flow enters the domain at a constant velocity and temperature. The thermophysical properties of the air are assumed to remain constant throughout the simulations. The model is based on assumptions that the porous region is uniform, isotropic, and filled with a single-phase fluid. The analysis spans a range of Darcy numbers (Da) from 10^{-6} to 1 and porous layer thickness ratio (r_p) from 0.2 to 1.

To mitigate the influence of grid-related effects on the simulation results, a grid independence study is conducted. The study examines the dimensionless axial velocity (u/u_{in}) profile in a partially filled channel with porous media. Three different mesh resolutions are tested, corresponding to 7500, 30 000 and 46 875 elements. The resulting axial velocity profiles are presented in Fig. 2. The variation in axial velocity for the 30 000 and 46 875 element cases is minimal, indicating grid independence. Based on these findings, a grid resolution of 30 000 elements is selected for further analysis to reduce computational cost.

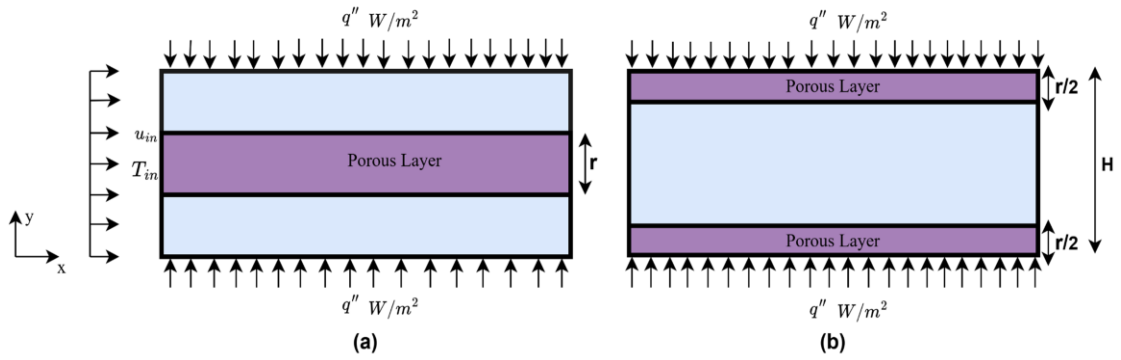


Fig. 1. The schematic representation of a partially filled channel used in the present study with two different configurations: (a) porous layer in the core of the channel, (b) porous layer at the walls of the channel.

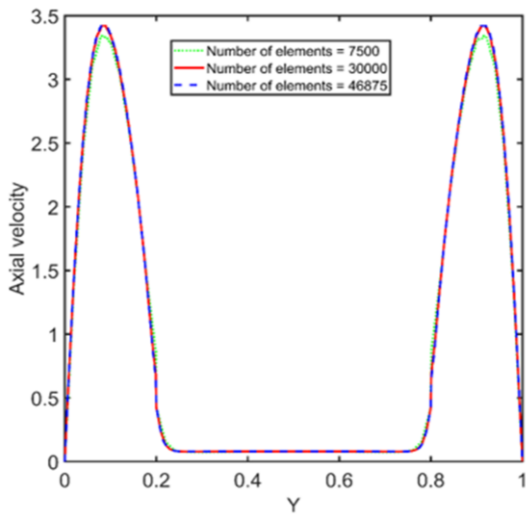


Fig. 2. Non-dimensional axial velocity variation for different grid resolutions at $r_p = 0.6$ and $\text{Da} = 10^{-4}$.

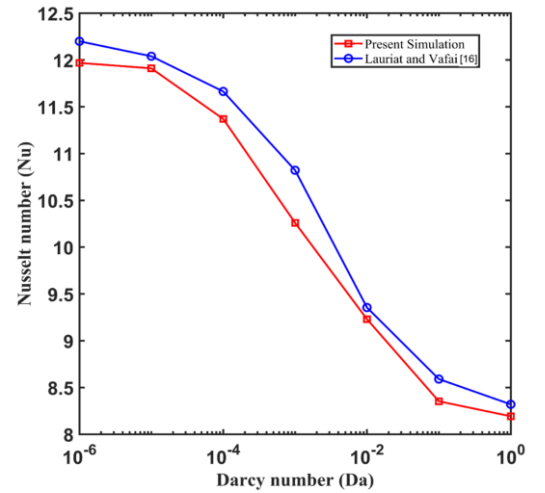


Fig. 3. Validation of the numerical model by comparing the Nusselt number variation with the Darcy number against benchmark results [17] for a fully porous channel.

To validate the numerical model, its results were compared with data reported in the literature. Specifically, the Nusselt number for flow through a fully porous channel with constant heat flux applied to the walls was compared to the findings of Lauriat and Vafai [34]. This comparison is presented in Fig. 3, where the agreement between the two sets of results demonstrates the accuracy of the developed numerical model in capturing the thermal performance of the porous medium.

To further establish the accuracy and reliability of the present numerical model, an additional validation is performed for a channel partially filled with porous media. The channel is partially porous, with porous layers placed directly above the heat sources, while the remaining regions of the channel are nonporous. The schematic representation of the validation geometry is shown in Fig. 4. The porous medium is modelled as homogeneous and isotropic and is saturated with a single-phase fluid that remains in thermal equilibrium with the solid matrix. The porosity and permeability are assumed to remain essentially uniform throughout the entire domain, including regions close to the channel walls.

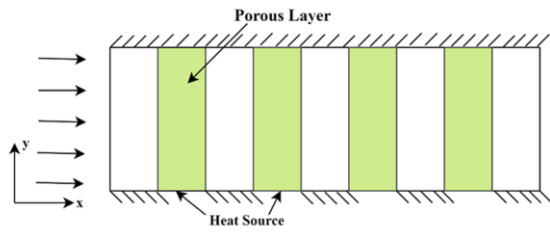


Fig. 4. Schematic diagram of a partially filled porous channel.

Additionally, the length-averaged Nusselt number is calculated at different Reynolds numbers and Darcy numbers for a partially filled porous channel. The obtained results (Fig. 5) exhibit good agreement with those reported in the previous study [9], further validating the numerical model used in the present work.

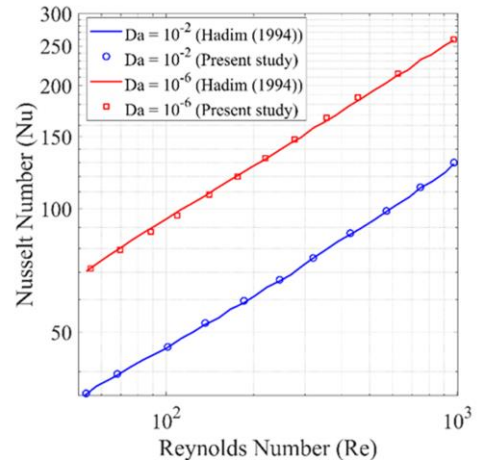


Fig. 5. Variation of the Nusselt number with the Reynolds number in a partially filled porous channel.

3. Results and discussion

The present study investigates the influence of a porous layer on heat transfer in a channel with two different configurations: one near the walls and the other in the core of the channel. The effects of the thickness ratio of the porous layer, the thermal conductivity ratio (k_r) and the Darcy number on heat transfer enhancement are examined. Additionally, the study examines the dimensionless axial velocity and pressure drop to optimise the balance between heat transfer enhancement and pumping power requirements.

3.1. Hydrodynamic characteristics

Figure 6 presents the axial velocity profiles for different values of porous layer thickness ratio (r_p). As the thickness ratio r_p increases, the fluid exhibits a channelling phenomenon, in which the flow is preferentially directed through the clear-fluid region between the porous layer and the channel wall. This results in

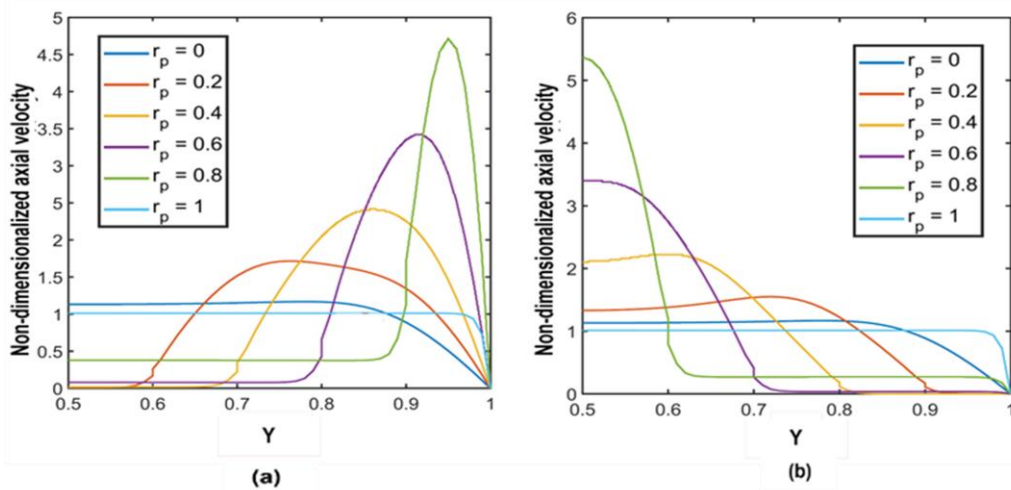


Fig. 6. The non-dimensional axial velocity profile for different porous layer thickness ratio (r_p) for both configurations: (a) porous layer embedded within the core of the channel, (b) porous layer at the walls of the channel.

a non-uniform distribution of flow, with fluid concentration occurring in localised regions around the porous layer, rather than maintaining a uniform flow across the entire cross-sectional area. For the case with a porous layer at the core (Fig. 6a), the maximum velocity of the fluid within the channel tends to increase and shift closer to the pipe surface as the thickness ratio increases, reaching a peak at $r_p = 0.8$. This increase in velocity is primarily attributed to the decrease in the cross-sectional area available for flow, which forces the fluid to accelerate to maintain the same flow rate. The peak velocity at $r_p = 0.8$ indicates an optimal configuration where the balance between the porous material's presence and the flow area is most favourable for fluid motion. For the configuration with the porous layer near the wall (Fig. 6b), the maximum fluid velocity increases and shifts toward the centre of the channel as the thickness ratio increases, reaching a peak at $r_p = 0.8$. This behaviour highlights the significant influence of the porous layer configuration on fluid dynamics, suggesting that careful design of these configurations can enhance thermal performance in applications, such as heat exchangers.

3.2. Pressure drop

The pressure drop in a channel is related to energy consumption. Reduced pressure losses allow for efficient fluid flow and enhance thermal performance. The pressure drop variation with different porous layer thickness ratios for the geometric configuration at different Darcy numbers is shown in Fig. 7. In this figure, subscript *c* represents the configuration when the porous layer is embedded within the core of the channel, and *w* represents the configuration when the porous layer is present at the wall of the channel. As the thickness of the porous layer increases, the resistance to fluid flow also rises, leading to higher pressure losses in both configurations of the channel, as depicted in Fig. 6. Similar pressure drop behaviour has been reported in previous studies [35,36]. This is primarily due to the increased surface area for frictional interactions and the longer path the fluid must traverse, which amplifies the energy required to maintain flow. Conversely, Darcy number is a measure of the porous medium's permeability relative to the fluid's viscosity,

plays a crucial role in determining how easily fluid can flow through the material.

A higher Darcy number indicates greater permeability, which typically results in lower pressure losses for a given inlet velocity. It is interesting to note that there is a lower pressure drop when the porous layer is located near the wall than at the core of the channel. This trend becomes more pronounced as the thickness of the porous insert increases. This is because the porous layer on the walls allows for a more streamlined flow path, reducing turbulence and frictional losses. In contrast, a porous insert in the core can disrupt the flow more significantly, leading to increased resistance and higher pressure drop. For a fully porous channel, the pressure loss values are the same for both configurations.

3.3. Heat transfer characteristics

Figure 8 depicts the relationship between the Nusselt number (Nu) and the porous layer thickness ratio for a porous layer present in the channel core, under different Darcy numbers, at a lower thermal conductivity ratio. For a Darcy number of 10^{-2} , the thermal performance improves with increasing porous layer thickness, peaking at a porous layer thickness ratio of 0.6. However, as the porous layer thickness ratio is further increased to 0.8 and then to 1.0, the Nusselt number begins to decrease. In contrast, when the Darcy number is reduced to a value ranging between 10^{-3} and 10^{-6} , the trend changes. In these cases, the heat transfer performance continues to improve with increasing r_p up to 0.8, which results in significant enhancement of thermal performance compared to other cases. Similar to the previous case, a further increase in r_p from 0.8 to 1.0 results in a decrease in thermal performance. This is due to the channelling effect occurring for porous layer thickness ratio, $r_p > 0.8$. As the porous layer thickness increases beyond this point, the clear fluid gap becomes smaller, resulting in increased flow resistance. Consequently, the flow loses a distinct preferential path, leading to

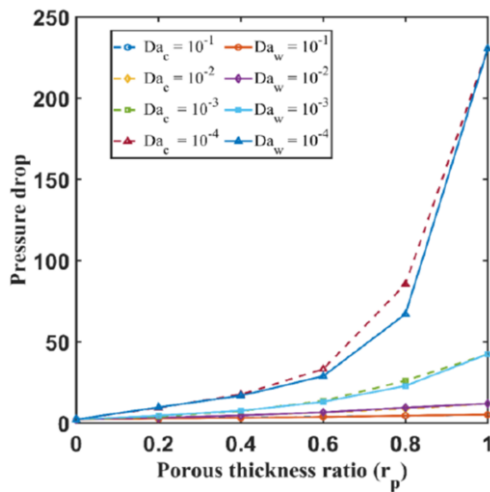


Fig. 7. Pressure drop with porous layer thickness ratio (r_p) at different Darcy numbers for both geometric configurations.

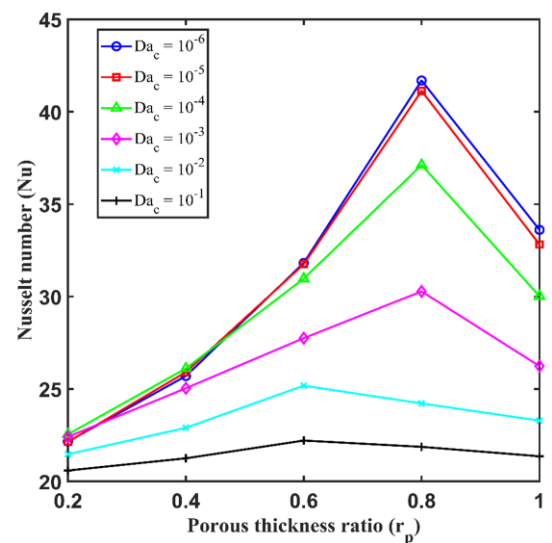


Fig. 8. Variation of the Nusselt number with the porous layer thickness ratio (r_p) at a thermal conductivity ratio (k_r) of 2.5 and different Darcy number (Da) values, when a porous layer is at the channel core.

a reduction in heat transfer performance, and resulting in a decrease in the Nusselt number. A similar variation in the Nusselt number has been reported in previous studies [36].

Figure 9 demonstrates the relationship between the Nusselt number (Nu), the porous layer thickness ratio (r_p) and the Darcy number (Da) for a higher thermal conductivity ratio. The results indicate that an increase in the thermal conductivity of the solid matrix improves the heat transfer rate. As the thermal conductivity ratio (k_r) increases, the porous material's ability to conduct heat also improves, resulting in higher Nusselt numbers. Notably, the impact of increasing thermal conductivity ratio becomes significant at higher porous layer thickness ratios. This is because as the porous layer thickness ratio (r_p) increases, the flow through the porous medium becomes less efficient, making the thermal conductivity of the solid matrix more important for heat transfer. Furthermore, when the porous material completely fills the channel ($r_p = 1.0$), the increase in thermal conductivity of the solid matrix results in the most substantial enhancement in heat transfer rates. In this fully filled configuration, the solid matrix can effectively conduct heat, maximising the thermal performance of the system. These findings emphasise that both the porous layer thickness ratio and the thermal conductivity of the solid matrix play a crucial role in heat transfer rates, with the extent of their impact being influenced by Darcy numbers.

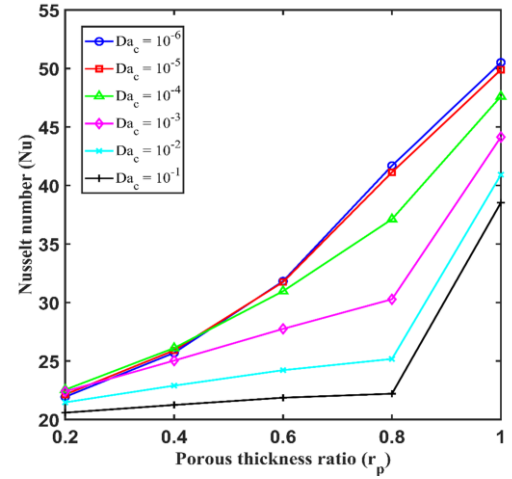


Fig. 9. Variation of the Nusselt number with the porous layer thickness ratio (r_p) at a thermal conductivity ratio (k_r) of 25 and for different Darcy number (Da) values, when a porous layer is at the channel core.

Figure 10 further demonstrates the influence of a higher thermal conductivity ratio on thermal performance across various configurations, comparing the porous layer placed at the core versus at the walls, for different Darcy numbers. Figures 10a and

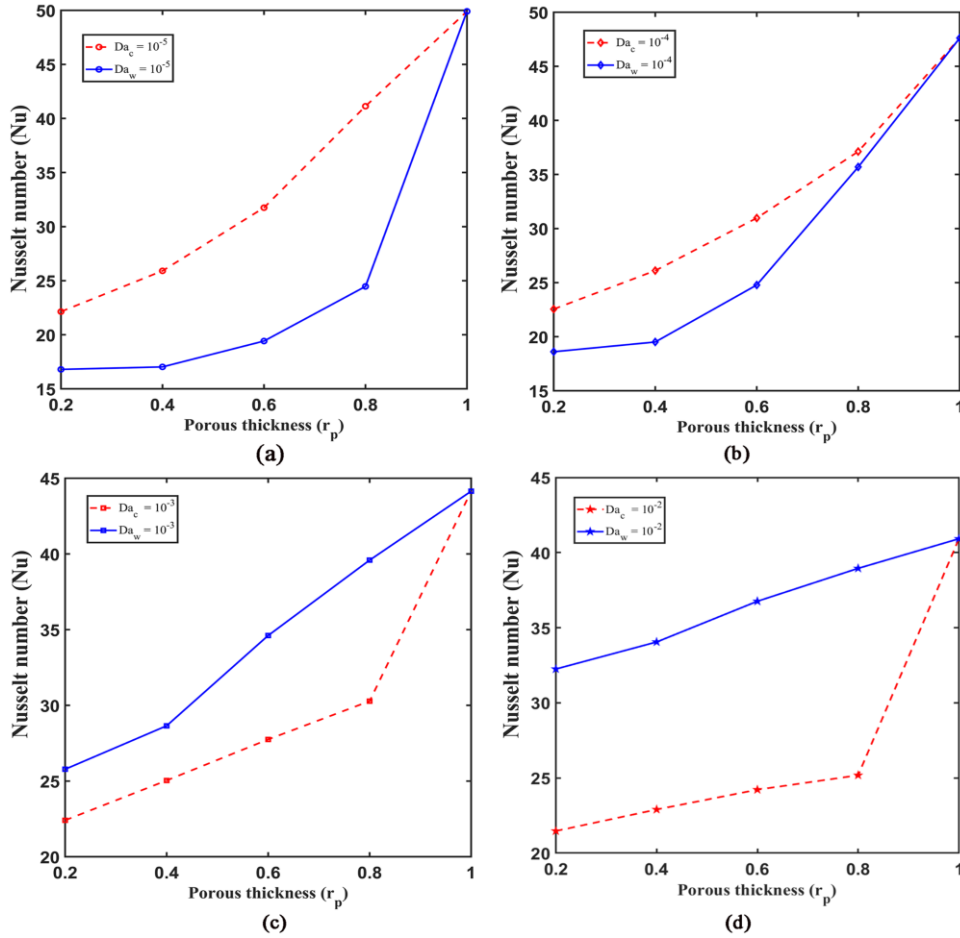


Fig. 10. Variation of the Nusselt number with respect to the porous layer thickness ratio (r_p) for two configurations: a porous layer in the core and a porous layer at the walls, for different Darcy numbers.

10b, corresponding to Darcy numbers of 10^{-5} and 10^{-4} , respectively, show that for less permeable porous media, positioning the porous layer in the channel core results in higher Nusselt numbers. However, this configuration also leads to higher pressure losses compared to the configuration with the porous layer adjacent to the walls, especially at lower Darcy numbers. In contrast, Figs. 10c and 10d, corresponding to Darcy numbers of 10^{-3} and 10^{-2} , respectively, reveal that higher permeability yields better thermal performance when the porous layer is located near the walls. This configuration achieves higher Nusselt numbers with lower pressure drops. The improved thermal performance in this case is due to higher permeability, which enhances fluid-wall interaction when the porous medium is adjacent to the walls, allowing the fluid to flow more freely and improving contact with the wall surface. This, in turn, increases convective heat transfer, resulting in a higher Nusselt number. Nusselt number as a function of the Darcy number for both cases for a porous layer thickness ratio of 0.8 is depicted in Fig. 11. The same value of the Nusselt number is observed for both configurations when the Darcy number is between 10^{-4} to 10^{-3} . The same transition of the Nusselt number with the Darcy number is seen for all porous layer thickness in this study. The thickness ratio of $r_p = 0.8$ is selected as it represents the optimal configuration that yields the maximum heat transfer enhancement among all thickness ratios investigated in the present study.

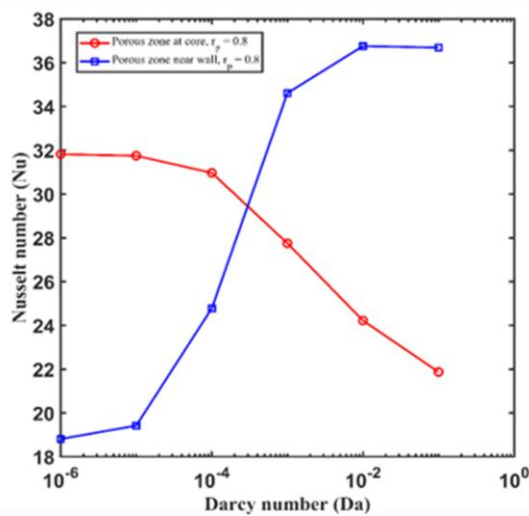


Fig. 11. Variation of the Nusselt number with the Darcy number at $r_p = 0.8$ for both configurations.

6. Conclusions

The present study numerically investigates the heat transfer enhancement in the channel using two different configurations: a porous layer in the channel core and a porous layer located near the channel wall. The effects of Darcy number ($0.000001 < Da < 0.01$), the ratio of porous layer thickness ($0.2 < r_p < 1$) and the thermal conductivity ratio are studied to understand how porous medium properties govern flow resistance and thermal performance. The main outcome of the study is summarised below:

1. The highest fluid velocity is observed when the porous la-

yer thickness ratio is 0.8 in both geometric configurations. This indicates the optimal configuration for enhancing convective heat transfer between the fluid and the channel wall.

2. Pressure-drop analysis indicates that positioning the porous layer at the walls consistently results in lower pressure losses for all Darcy numbers, whereas placing the porous layer at the core introduces a stronger flow obstruction and therefore higher-pressure penalties. The pressure drop increases as the porous layer thickness ratio increases.
3. The results show that the porous layer thickness ratio plays a dominant role in flow-thermal behaviour. The maximum improvement in heat transfer is achieved at a thickness ratio of 0.8, where the Nusselt number increases by about 88% compared to the baseline. For the low thermal conductivity ratio of 2.5, increasing thickness beyond this point leads to a 19.5% reduction in the Nusselt number due to increased thermal resistance. In contrast, for a high conductivity ratio of 25, the heat transfer rate continues to rise with thickness, yielding as much as 80.5% improvement relative to the low-conductivity case.
4. At lower Darcy numbers, the Nusselt number is higher when the porous layer is located at the core of the channel, irrespective of the porous layer thickness ratio. In contrast, at higher Darcy numbers, the Nusselt number is higher when the porous layer is positioned near the walls, especially for high thermal conductivity ratios and for all porous layer thickness ratios.

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Mixed convective chemically reacted hybrid nanofluid flowing over a non-isothermal/non-isosolutal cone/wedge with heat generation/absorption: A numerical study

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Abstract

The current study explores a numerical investigation of steady-state, two-dimensional, incompressible, laminar mixed convective flow of a chemically reactive hybrid nanofluid composed of $\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ over a vertical non-isothermal/isosolutal wedge and cone configurations, taking into account the impact of a heat absorption or generation parameter. Utilising similarity transformations, the set of governing equations was reduced to a set of ordinary differential equations. This model incorporates the conservation equations for energy, mass and momentum, which are solved numerically by using the BVP-5C method and the Runge-Kutta shooting technique in MATLAB. The influences of key factors on the flow fields, as well as on the mass and heat transmission rates, are analysed and presented through graphical and tabular formats. The outcome reveals that both solutal and thermal Grashof numbers display a comparable influence on the flow behaviour. As these parameters increase, an enhancement in the velocity profile is observed, whereas concentration and temperature distributions tend to decline. Additionally, an increase in the heat generation and thermal radiation parameters results in elevated temperature distributions for both wedge and cone geometries. In contrast, the chemical reaction factor contributes to a decrease in concentration distribution. The influence is more prominent in the wedge configuration than in the cone, where the profile exhibits an increasing behaviour, while the cone shows a stronger response in scenarios where the profile decreases. The mass and heat transmission rates corresponding to the key parameters are presented in tabulated form. The proposed model has been rigorously verified against previously reported data and demonstrates excellent agreement.

Keywords: Wedge/cone; Thermal/mass Grashof numbers; Heat generation/absorption parameter; Chemical reaction

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1. Introduction

Hybrid nanofluids are engineered by suspending a combination of two or more different nanoparticles within a conventional heat transmission medium such as water (H_2O), motor oil, or-

ganic solvents or biological fluids. When these nanoparticles are uniformly dispersed, the resulting fluid exhibits significantly improved thermal conductivity and superior heat transfer features. As an innovative class of working liquids, hybrid nanofluids have shown great promise in various engineering and scien-

Nomenclature

a, b – constants
 Bi – Biot number
 C – concentration, kg/m^3
 C_f – skin friction coefficient
 C_w – concentration at wall, kg/m^3
 C_∞ – ambient concentration (kg/m^3)
 D_{hnf} – diffusion coefficient, m^2/s
 g – acceleration due to gravity, m/s^2
 Gc – mass Grashof number
 Gr – thermal Grashof number
 h_f – convective heat transfer coefficient, $\text{W}/(\text{m}^2 \text{ K})$
 k_{hnf} – thermal conductivity of hybrid nanofluid, $\text{W}/(\text{m K})$
 k^* – mean absorption coefficient, m^{-1}
 l – characteristic length of cone/wedge, m
 n – power index
 Nu_x – Nusselt number
 Pr – Prandtl number
 Q – heat generation/absorption parameter
 q_r – heat flux, W/m^2
 r – radius of the cone, m
 Rc – chemical reaction parameter
 Rd – thermal radiation
 Re_x – local Reynolds number
 s_1 – wall temperature parameter
 s_2 – wall concentration parameter
 S – suction parameter

Sc – Schmidt number
 Sh_x – Sherwood number
 T – temperature, K
 T_w – temperature at wall, K
 T_∞ – ambient temperature, K
 u, v – velocity components along the x - and y -axis, m/s
 u_w – stretching velocity, m/s

Greek symbols

α – half-angle
 β_c – solutal expansion coefficient, K^{-1}
 β_T – thermal expansion coefficient, K^{-1}
 μ_{hnf} – dynamic viscosity of hybrid nanofluid, $\text{kg}/(\text{m s})$
 ρ_{hnf} – density of hybrid nanofluid, kg/m^3
 $(\rho C_p)_{hnf}$ – heat capacity of hybrid nanofluids, $\text{J}/(\text{kg K})$
 σ^* – Stefan-Boltzmann constant, $\text{W}/(\text{m}^2 \text{ K}^4)$
 ϕ_1, ϕ_2 – Cu and Al_2O_3 nano particles volume fraction

Subscripts and Superscripts

∞ – free stream
 f – fluid
 hnf – hybrid nanofluids
 w – at the wall/surface

Abbreviations and Acronyms

EG – ethylene glycol
MHD – magnetohydrodynamics
ODEs – ordinary differential equations

tific domains, including microfluidic systems, transportation technologies, biomedical applications, defence mechanisms, naval architecture and acoustic devices. Due to its extensive spectrum of applications in engineering and science, the investigation of fluid flow over different geometrical shapes such as vertical cones and wedges has attracted considerable interest from researchers. These flow configurations are crucial in various industrial and technological operations, including fibre production, advanced thermal aerodynamics, cooling of nuclear systems, surface modification, spray deposition techniques, and polymer fabrication. Consequently, a substantial amount of analytical and computational investigations have led to explore the flow behaviour of Newtonian as well as non-Newtonian fluids across such geometrical domains.

Dawar et al. [1] analysed the convective transport of Williamson's nanoliquid through wedge and cone geometries by employing convective boundary conditions. The flow was studied under varying thermal and solutal conditions that were not uniform across the surface. Kumar et al. [2] explored the laminar flow of a Newtonian composite nanofluid containing various nanoparticles embedded in a basic fluid, considering both temperature-dependent and exponentially changing spatial heat generation over an isothermal wedge. Yahaya et al. [3] examined steady-state hybrid nanoliquid flowing over a penetrable, non-isothermal wedge and cone, incorporating the influence of radiation and convective boundary conditions in their heat transmission analysis. Sushila et al. [4] analysed the heat transmission behaviour of TiO_2 – CuO /water hybrid nanoliquid flowing over

both static and moving wedges, taking into account the effects of thermal radiation and magnetic field within a porous medium.

Mixed convection represents fluid flow in which both forced convection, caused by external pressure gradients, and free convection, resulting from buoyancy forces, act simultaneously. This type of flow plays a substantial role in an extensive array of applications, like low-speed heat exchangers, cooling of electronic components, crystal formation, underground cable networks, chemical manufacturing, lubrication systems, boilers, emergency cooling in nuclear reactors, solar thermal systems, defrosting mechanisms, atmospheric circulation, vehicle windshield demisters, and ocean currents. Ramzan et al. [5] inspected mixed convection flow of a ternary hybrid nanoliquid over wedge and cone geometries characterised by non-isothermal and non-isosolutal distributions. Reddy et al. [6] investigated mixed convective magnetohydrodynamics (MHD) radiative flow of a Williamson's nanoliquid over a wedge, including the influence of inner heat generation and binary chemical reactions. The mixed convective heat and mass transfer behaviour of a magneto-nanofluid over a moving vertical plate, including thermal radiation, was examined by Chandra Reddy et al. [7]. The influence of time-dependent MHD mixed convection on hybrid nanofluid flow past a nonlinearly stretching surface in a porous medium was investigated by Mathews and Hymavathi [8].

The study of fluid flow involving chemical reactions is vital due to its substantial influence on both environmental and industrial operations. Such reactions play a crucial role in a wide-spread range of applications, such as nuclear reactor systems,

atomic energy generation, plastic fabrication, cable insulation processes, and many other industrial sectors. Amit and Shalini [9] conducted a numerical study on the influence of a nonlinear chemically reacting MHD convective flow of Maxwell liquid over penetrable surfaces, including a vertical cone/wedge/plate. AL Garalleh [10] investigated the mixed convection flow of a ternary hybrid nanoliquid composed of TiO_2 , Al_2O_3 and SiO_2 over a stretched/shrunk wedge, considering the combined influences of chemical reaction and activation energy. Arabi et al. [11] explored the convection behaviour of a Williamson hybrid nanoliquid flowing over wedge and cone geometries within a porous surface, considering various nanoparticle shapes and the effect of a chemical response.

The study of temperature distribution influenced by heat generation (absorption) in moving fluids plays a crucial role in numerous physical contexts, including chemical reaction processes and the behaviour of dissociating fluids. Numerous investigations in the literature focus on heat transmission in the laminar flow of viscous liquids. In some works, heat generation is modelled either as a constant or as a function of spatial coordinates, while others have incorporated the effects of frictional heating and fluid expansion. Vajravelu and Nayfeh [12] investigated MHD convection along cone and wedge surfaces, considering variable surface temperatures and the presence of inner heat absorption/generation. Rekha et al. [13] studied the influence of a uniform heat sink/source on the flow of an incompressible nanofluid over cone and wedge geometries within a permeable medium. The influence of heat generation and thermal radiation under exponentially varying temperature and concentration conditions was investigated by Chandra Reddy et al. [14]. The same authors, in [15], carried out a numerical investigation of free convection heat and mass transfer characteristics in a heat-producing magneto-nanofluid past a vertically moving plate. Mathews and Hymavathi [16] explored the effects of a magnetic field, viscous dissipation, thermal radiation and internal heat generation/absorption on flows involving metallic and oxide nanoparticles over a permeable stretching sheet.

By applying the MATLAB BVP4C boundary-value problem solver, Ali et al. [17,18] numerically explored the influence of various nanoparticle shape factors, including blade, brick, cylindrical, platelet, and spherical forms, on nanofluid flow characteristics, considering unsteady $\text{SiO}_2 - \text{H}_2\text{O}$ motion along a stretchable cylinder and steady $\text{EG} - \text{Cu}$ flow over a fixed stretching surface under Joule heating effects. Shahzad et al. [19] examined the characteristics of axisymmetric liquid film flow and the associated heat transfer over a time-dependent radially stretching surface, accounting for the influence of an applied transverse magnetic field. Zaman et al. [20] explored the thermal behaviour of Williamson nanofluids incorporating upward-moving motile thermophilic microorganisms along wedge and cone surfaces and further enhanced their study by employing an artificial neural network to support the numerical findings. Ahmed et al. [21] investigated the effects of thermal radiation on chemically reactive MHD hybrid nanofluid flow over elastically deformable wedge, plate, and cone configurations, while also accounting for surface tension gradients, Joule heating, and permeability of the medium. Sadiya et al. [22] analysed the simultaneous effects of these parameters on the flow behav-

iour of a ternary nanofluid moving through a porous medium over a rotating and stretching surface. Qureshi et al. [23] conducted a machine-learning-based analysis of MHD radiative ternary hybrid nanofluid flow, comprising multi-walled carbon nanotubes (MWCNTs) and Au/Ag nanoparticles suspended in blood, over a porous stretching sheet.

A thorough examination of existing literature demonstrates that the mixed convection flow of a chemically reactive $\text{Al}_2\text{O}_3 - \text{Cu/H}_2\text{O}$ hybrid nanofluid over wedge and cone geometries, considering the simultaneous influence of heat generation (absorption) and chemical reaction effects, remains uninvestigated. Aiming to address this gap, the present analysis explores the governing factors of heat and mass transfer in the flow system and builds upon the study of Yahaya et al. [3] by introducing heat generation within the boundary layer. The novelty of this study lies in its numerical assessment of how thermal and solutal Grashof numbers, together with heat generation (absorption) and chemical reaction factors, affect the behaviour of the hybrid nanoliquid over non-isothermal and non-isosolutal wedge and cone surfaces. A comparative evaluation between the two geometries is carried out to highlight the influence of the selected physical elements. This problem is tackled using the BVP5C numerical solver. Graphical representations are used to illustrate the influence of key variables, and tables are provided to report skin friction, mass and heat transmission rates for chosen parameter values. The results show strong consistency and offer valuable insights into the transport phenomena involved.

2. Mathematical modelling

This research presents a numerical exploration of steady-state, laminar, 2D incompressible mixed convective flow of a chemically reactive hybrid nanofluid composed of $\text{Al}_2\text{O}_3 - \text{Cu/H}_2\text{O}$ over vertical non-isothermal and non-isosolutal cone and wedge configurations, accounting for the impact of heat generation or absorption. Initially, a nanofluid is prepared by scattering copper (Cu) nanoparticles into water (H_2O), the basic fluid, with the volume fraction represented by ϕ_1 . This $\text{Cu} - \text{H}_2\text{O}$ nanofluid is then further modified into a hybrid nanofluid, $\text{Al}_2\text{O}_3 - \text{Cu/H}_2\text{O}$, by incorporating aluminium oxide (Al_2O_3) nanoparticles, known for their superior thermal conductivity at a designated volume fraction ϕ_2 . This study assumes that the nanoparticles are uniformly shaped and sized and that thermal equilibrium exists between the basic liquids and the suspended nanoparticles.

Figure 1 demonstrates the configuration and co-ordinate system for the 2D flow scenario, here the x -axis runs along the surface area of the cone or wedge and the y -axis is oriented perpendicular to it. The stretched velocity of the surface is defined as $u_w = v_f x/l^2$, and the liquid occupies the area $y \geq 0$. The surface temperature distribution is modelled as $T_w(x) = T_\infty + ax^{s_1}$, while the nanoparticle concentration at the surface is given by $C_w(x) = C_\infty + bx^{s_2}$, with the exponents s_1 and s_2 governing the spatial variation of temperature and concentration, respectively. A value of $s_1 = 0$ represents a surface with constant temperature (isothermal), and $s_2 = 0$ indicates a surface with uniform nanoparticle concentration (isosolutal). The cone or wedge is characterised by a half-angle α , with its radius defined as $r = x \sin \alpha$.

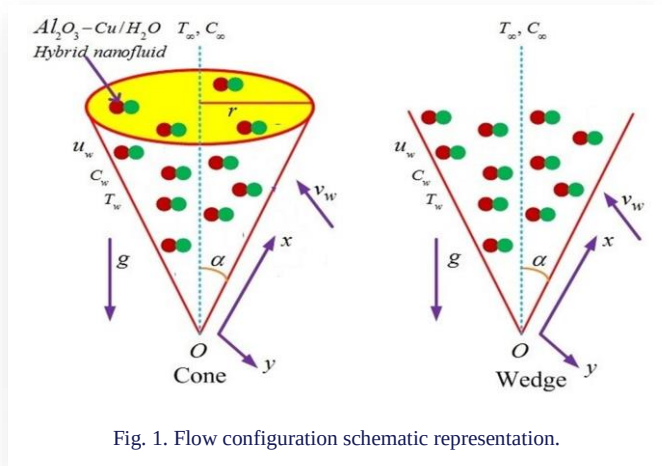


Fig. 1. Flow configuration schematic representation.

The two geometric configurations, cone and wedge, are distinguished using the following criteria:

- for $\alpha \neq 0$ and $n = 0$, the structure corresponds to a wedge,
- for $\alpha \neq 0$ and $n = 1$, the configuration represents a cone.

Taking into account the aforementioned suppositions, the equations that govern this flow problem are established as follows [1,3,5,24]:

$$\frac{\partial}{\partial x}(r^n u) + \frac{\partial}{\partial y}(r^n v) = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} + g [\beta_T(T - T_\infty) + \beta_c(C - C_\infty)] \cos \alpha, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{hnf}} \frac{\partial q_r}{\partial y} + \frac{Q^*}{(\rho c_p)_{hnf}} (T - T_\infty), \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_{hnf} \frac{\partial^2 C}{\partial y^2} - K^*(C - C_\infty). \quad (4)$$

The corresponding boundary constraints [3,5] are:

at $y = 0$:

$$u = u_w, \quad v = v_w, \quad -k_{hnf} \frac{\partial T}{\partial y} = h_f(T_w - T), \quad C = C_w, \quad (5a)$$

as $y \rightarrow \infty$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty. \quad (5b)$$

To simplify the analysis, the following similarity transformations are introduced [1,3]:

$$\eta = \frac{y}{l}, \quad u = \left(\frac{\nu_f x}{l^2} \right) f'(\eta),$$

$$v = - \left(\frac{\nu_f}{l} \right) (1 + n) f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (6)$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}.$$

Upon applying the similarity parameters defined by Eq. (6) to the governing Eqs. (1) to (4), it is observed that Eq. (1) is

naturally satisfied, while Eqs. (2)–(4) are simplified to the non-linear ODEs shown below:

$$R_1 f''' + R_2(1 + n)ff'' - R_2 f'^2 + R_2(Gr\theta + Gc\phi) \cos \alpha = 0, \quad (7)$$

$$\left(R_4 + \frac{4}{3} R_d \right) \theta'' + Pr Q \theta + Pr R_3(1 + n)f\theta' - s_1 Pr R_3 f'\theta = 0, \quad (8)$$

$$\phi'' + \frac{Sc}{R_5} [(1 + n)f\phi' - s_2 f'\phi - Rc\phi] = 0. \quad (9)$$

Here:

$$R_1 = \frac{\mu_{hnf}}{\mu_f}, \quad R_2 = \frac{\rho_{hnf}}{\rho_f}, \quad R_3 = \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}, \quad R_4 = \frac{k_{hnf}}{k_f}, \quad R_5 = \frac{D_{hnf}}{D_f}.$$

The corresponding boundary constraints are transformed to:

$$f'(0) = \phi(0) = 1, \quad f(0) = S \frac{1}{1+n}, \quad \theta'(0) = -Bi \frac{(1-\theta(0))}{R_4},$$

$$f'(\infty) = \phi(\infty) = \theta(\infty) = 0. \quad (10)$$

Dimensionless quantities utilised in the governing equations are defined as follows [1,3,5,25]:

- thermal Grashof number: $Gr = \frac{l^2 g \beta_T (T_w - T_\infty)}{\nu_f u_w}$,
- mass Grashof number: $Gc = \frac{l^2 g \beta_c (C_w - C_\infty)}{\nu_f u_w}$,
- heat generation/absorption: $S = -\frac{lv_w}{v_f}$ ($S > 0$ indicating suction and $S < 0$ indicating injection),
- Biot number: $Bi = \frac{h_f l}{k_f}$,
- Schmidt number: $Sc = \frac{\nu_f}{D_f}$,
- chemical reaction parameter: $Rc = \frac{K^* l^2}{\nu_f}$.

As per Rosseland's [26] diffusion estimate, the radiative heat flux q_r is formulated in the following manner, as outlined in references [27–29]:

$$q_r = -\frac{4}{3} \frac{\sigma^*}{k^*} \frac{\partial T^4}{\partial y}.$$

The factors k^* and σ^* designate the mean absorption coefficient and Stefan–Boltzmann constant respectively. By applying a Taylor's series extension to T^4 around the ambient temperature T_∞ and avoiding higher order terms, the expression streamlines to: $T^4 \approx 4 T_\infty^3 T - 3 T_\infty^4$.

Physical and thermal characterisations of the basic liquid H_2O and nanoparticles Al_2O_3 , Cu , along with those of the hybrid nanofluid $Al_2O_3 - Cu/H_2O$, are presented in Tables 1 and 2 given below.

 Table 1. Al_2O_3 , Cu and H_2O thermo-physical properties [33,34].

Properties	Cu	Al_2O_3	H_2O
C_p , J/(kg K)	385	765	4179
ρ , kg/m ³	8933	3970	997.1
k , W/(m K)	400	40	0.613
Pr			6.2

Table 2. Al₂O₃ – Cu/H₂O hybrid nanoliquid thermo-physical properties [1,5,30–32].

Properties	Al ₂ O ₃ – Cu/H ₂ O hybrid nanoliquid
Viscosity (μ)	$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_1 - \phi_2)^{2.5}}$
Density (ρ)	$\rho_{hnf} = \phi_2 \rho_2 + \phi_1 \rho_1 + (1 - \phi_1 - \phi_2) \rho_f$
Heat capacity (ρC_p)	$(\rho C_p)_{hnf} = \phi_2 (\rho C_p)_2 + \phi_1 (\rho C_p)_1 + (1 - \phi_1 - \phi_2) (\rho C_p)_f$
Thermal conductivity (k)	$\frac{k_{hnf}}{k_f} = \frac{\frac{\phi_2 k_2 + \phi_1 k_1}{\phi_2 + \phi_1} + 2k_f + 2(\phi_2 k_2 + \phi_1 k_1) - 2(\phi_2 + \phi_1)k_f}{\frac{\phi_2 k_2 + \phi_1 k_1}{\phi_2 + \phi_1} + 2k_f - (\phi_2 k_2 + \phi_1 k_1) + (\phi_2 + \phi_1)k_f}$
Diffusion coefficient (D)	$D_{hnf} = (1 - \phi_1 - \phi_2) D_f$

The key physical quantities appropriate to the current problem are discussed as follows:

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad (11a)$$

$$Nu_x = \frac{-x}{k_f (T_w - T_\infty)} \left[\left(k_{hnf} \frac{\partial T}{\partial y} \right)_{y=0} - (q_r)_{y=0} \right], \quad (11b)$$

$$Sh_x = \frac{-x}{D_B (C_w - C_\infty)} \left(\frac{\partial C}{\partial y} \right)_{y=0}. \quad (11c)$$

Substituting the similarity variables defined in Eq. (6) into Eq. (11) yields the non-dimensional forms of the physical quantities, as presented below:

$$Re_x^{1/2} C_f = R_1 f''(0), \quad (12a)$$

$$Re_x^{-1/2} Nu_x = - \left(R_4 + \frac{4}{3} Rd \right) \theta'(0), \quad (12b)$$

$$Re_x^{-1/2} Sh_x = -\phi'(0). \quad (12c)$$

3. Method of solution

The system of Eqs. (7) to (9), along with the corresponding boundary constraints, is solved numerically by utilising the BVP5C solver available in MATLAB. To implement this method, all higher-order differential equations (DEs) are first reduced to a set of first-order equations. The formulation of the problem and procedural flow of the BVP5C technique are illustrated in Fig. 2.

Let:

$$f = y_1, \quad \frac{df}{d\eta} = \frac{dy_1}{d\eta} = y_2, \quad \frac{d^2 f}{d\eta^2} = \frac{dy_2}{d\eta} = y_3, \quad \frac{d^3 f}{d\eta^3} = \frac{dy_3}{d\eta}, \quad (13)$$

$$\theta = y_4, \quad \frac{d\theta}{d\eta} = \frac{dy_4}{d\eta} = y_5, \quad \frac{d^2 \theta}{d\eta^2} = \frac{dy_5}{d\eta}, \quad (14)$$

$$\phi = y_6, \quad \frac{d\phi}{d\eta} = \frac{dy_6}{d\eta} = y_7, \quad \frac{d^2 \phi}{d\eta^2} = \frac{dy_7}{d\eta}. \quad (15)$$

Substitution of Eqs. (13)–(15) into Eqs. (7)–(9) yields the transformed equations given below:

$$R_1 \frac{dy_3}{d\eta} + R_2 (1+n) y_1 y_3 - R_2 y_2^2 + R_2 (Gr y_4 + Gc y_6) \cos \alpha = 0, \quad (16)$$

$$\left(R_4 + \frac{4}{3} Rd \right) \frac{dy_5}{d\eta} + \text{Pr} (Q y_4 + R_3 (1+n) y_1 y_5 - s_1 R_3 y_2 y_4) = 0, \quad (17)$$

$$\frac{dy_7}{d\eta} + \frac{Sc}{R_5} [(1+n) y_1 y_7 - s_2 y_2 y_6 - Rc y_6] = 0, \quad (18)$$

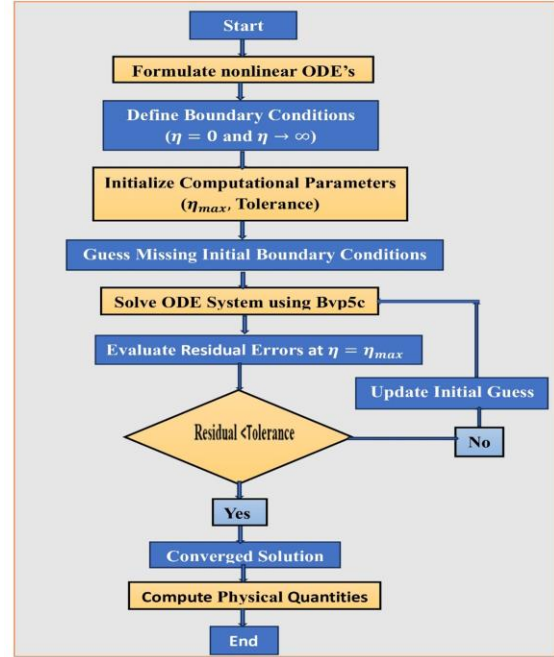


Fig. 2. Iterative BVP5C solution flowchart.

along with the boundary constraints:

$$y_1(0) = \frac{S}{1+n}, \quad y_2(0) = y_6(0) = 1,$$

$$y_5(0) = -\frac{Bi(1-y_4(0))}{R_4}, \quad (19)$$

$$y_2(\infty) = y_4(\infty) = y_6(\infty) = 0, \quad (20)$$

$$\frac{dy_3}{d\eta} = \frac{-R_2(1+n)y_1 y_3 + R_2 y_2^2 - R_2 (Gr y_4 + Gc y_6) \cos \alpha}{R_1}, \quad (21)$$

$$\frac{dy_5}{d\eta} = \frac{-\text{Pr} (Q y_4 + R_3 (1+n) y_1 y_5 - s_1 R_3 y_2 y_4)}{\left(R_4 + \frac{4}{3} Rd \right)}, \quad (22)$$

$$\frac{dy_7}{d\eta} = -\frac{Sc}{R_5} [(1+n) y_1 y_7 - s_2 y_2 y_6 - Rc y_6]. \quad (23)$$

The BVP5C solver generates a solution that is continuous and possesses a continuous first-order derivative over the interval $[0, \infty]$. Equations (19)–(23) are programmed in MATLAB, and their solutions are obtained using the BVP5C routine. This approach ensures that the convergence criteria are adequately met 10^{-6} .

4. Results and discussion

This section presents graphical illustrations of the temperature $\theta(\eta)$, velocity $f'(\eta)$ and concentration $\phi(\eta)$ distributions for the

hybrid nanofluid $\text{Al}_2\text{O}_3 - \text{Cu}/\text{H}_2\text{O}$ flowing over a vertical, non-isothermal and non-isosolutal, penetrable wedge and cone. A comparative assessment of the cone and wedge geometries is carried out to evaluate how the leading governing factors impact the flow, temperature and concentration characteristics. The influence of various factors, such as thermal Grashof number Gr , mass Grashof number Gc , heat absorption or generation parameter Q , radiation parameter Rd , Schmidt number Sc , chemical reaction parameter Rc , Biot number Bi , and suction parameter S , on these distributions is analysed and discussed. To explore the physical performance of the structure, the boundary value problem stated in Eqs. (16)–(20) is solved utilising MATLAB's BVP5C solver. The results are provided in both graphical and tabular representations to provide a comprehensive understanding of the underlying physical mechanism. A detailed investigation has been conducted on the skin friction coefficient, Sherwood number and Nusselt number with respect to the major influencing parameters.

The effect of thermal Grashof number, ranging between $1 \leq Gr \leq 8$, on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ distributions of $\text{Al}_2\text{O}_3 - \text{Cu}/\text{H}_2\text{O}$ hybrid nanoliquid flowing over a wedge and cone is presented in Figs. 3 to 5.

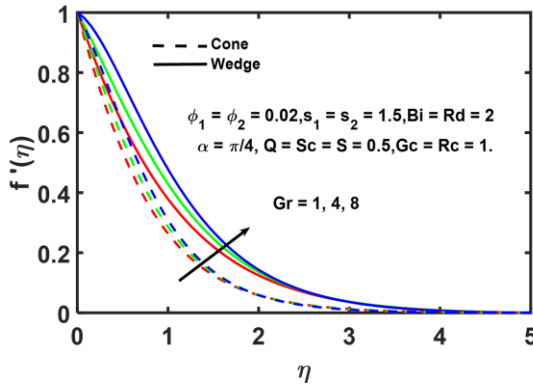


Fig. 3. Thermal Grashof number's effect on velocity profile.

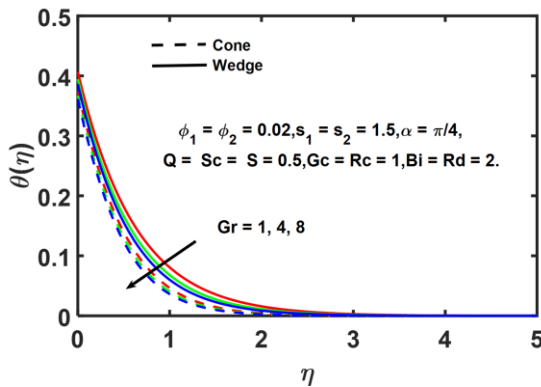


Fig. 4. Thermal Grashof number's effect on temperature profile.

As the values of Gr increase, the velocity distribution rises, while the temperature and concentration distributions exhibit a decreasing tendency. An increase in Gr reduces the boundary layer thickness due to stronger buoyancy forces generated by temperature transformations. This enhances the liquid motion and raises the velocity distribution. This increased acceleration

promotes more effective mixing and advection, resulting in a dilution phenomenon that reduces solute concentration. As a result, the concentration boundary layer becomes thinner, further diminishing the concentration distribution. In addition, larger temperature gradients produce stronger buoyant forces, which enhance convective heat transfer. This intensified convection helps dissipate heat more efficiently, thereby reducing the liquid's temperature. The impact is more pronounced for the wedge compared to the cone.

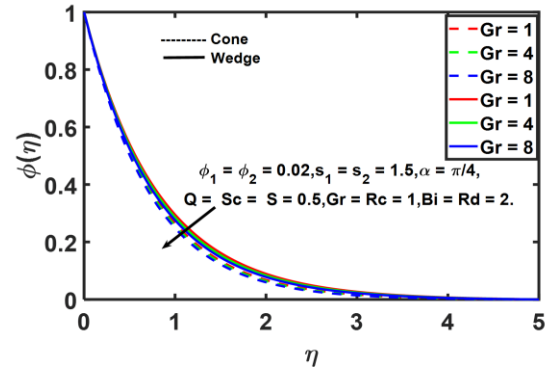


Fig. 5. Thermal Grashof number's effect on concentration profile.

Figures 6 to 8 depict the influence of the mass Grashof number, ranging between $0.1 \leq Gc \leq 0.9$, on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ profiles for both wedge and cone configurations. In a manner similar to Gr , increasing Gc enhances the $f'(\eta)$ profile, whereas $\theta(\eta)$ and $\phi(\eta)$ profiles show a decreasing trend.

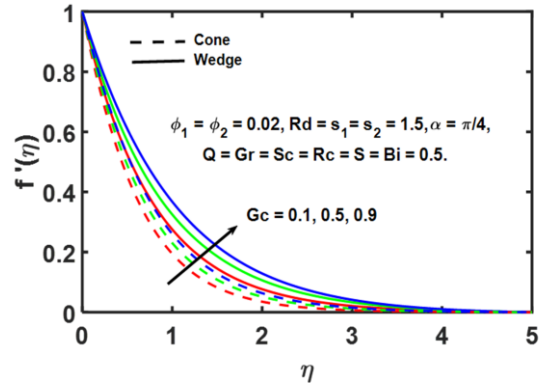


Fig. 6. Mass Grashof number's effect on velocity profile.

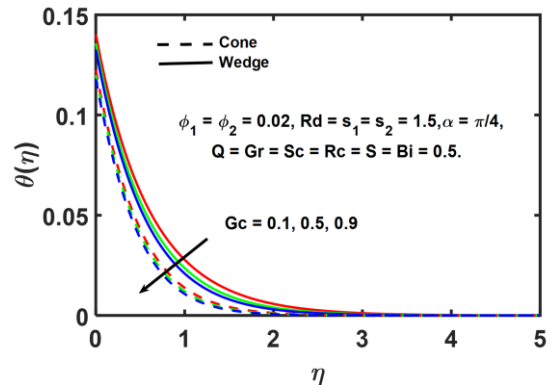


Fig. 7. Mass Grashof number's effect on temperature profile.

This phenomenon arises because the mass Grashof number is directly associated with buoyancy forces; as Gc rises, buoyancy effects become more significant, thereby accelerating the fluid flow. It is also witnessed that the impact of Gc is more pronounced for the wedge compared to the cone in all three profiles.

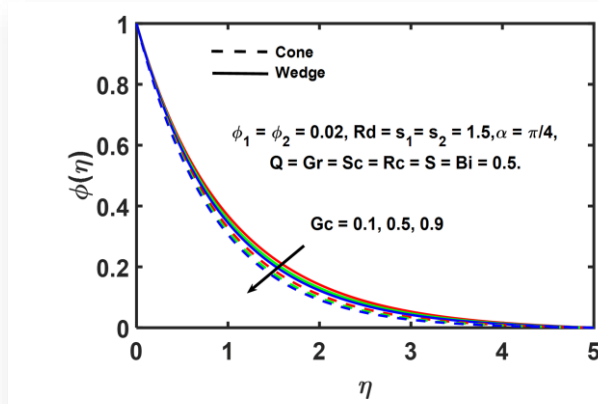


Fig. 8. Mass Grashof number's effect on concentration profile.

Figure 9 illustrates the variation in the $\theta(\eta)$ profile with cumulative values of the radiation parameter, within the range $1 \leq Rd \leq 2$, for both the wedge and cone. It is noted that as Rd increases, the temperature distribution in the $Al_2O_3 - Cu/H_2O$ hybrid nanofluid rises for both geometries. This occurs because a higher Rd indicates greater thermal radiation, which contributes additional energy to the fluid. Accordingly, more heat is absorbed by the hybrid nanofluid particles, leading to a thicker thermal boundary layer and an elevated temperature profile.

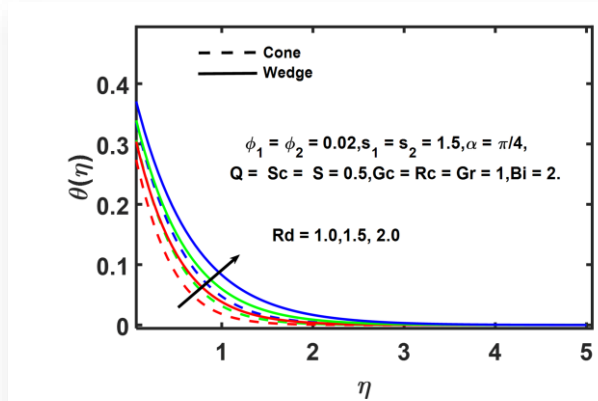


Fig. 9. Thermal radiation parameter's influence on temperature profile.

Figure 10 illustrates the effect of increasing the heat generation parameter, within the range $1.1 \leq Q \leq 1.3$, on the $\theta(\eta)$ profile for both the wedge and cone geometries. As Q increases, more thermal energy is transferred to the hybrid nanofluid, leading to a rise in stored internal energy within the liquid particles. This results in a thicker boundary layer and a corresponding rise in the temperature profile.

Figure 11 presents the variation of the temperature profile $\theta(\eta)$ of the hybrid nanofluid with respect to changes in the wall temperature parameter $1 \leq s_1 \leq 2$, for both wedge and cone geometries. It is evident that $\theta(\eta)$ reduces with the increasing s_1 value. This reduction is due to the strong influence of higher s_1 values on the thermal boundary layer, which becomes thinner,

resulting in a lower temperature distribution. Among these three parameters, their effects on $\theta(\eta)$ are found to be more important in the case of the wedge compared to that of the cone.

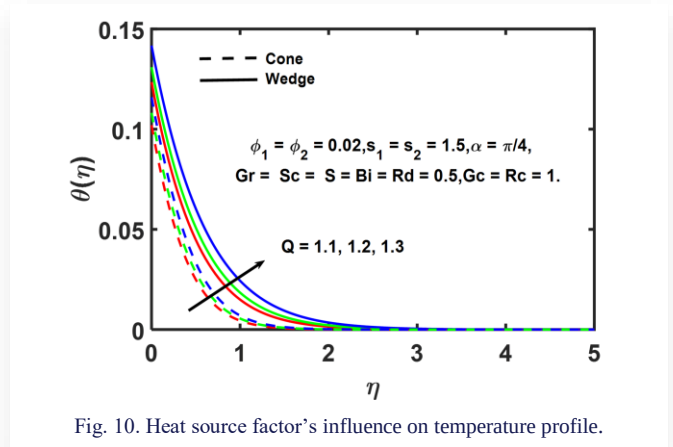


Fig. 10. Heat source factor's influence on temperature profile.

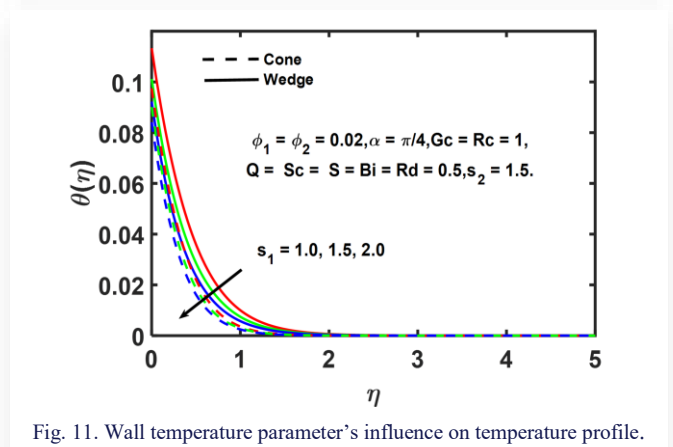


Fig. 11. Wall temperature parameter's influence on temperature profile.

Figures 12 to 14 demonstrate the influence of the chemical reaction factor ($1 \leq c \leq 3$), Schmidt number ($2 \leq Sc \leq 4$), and wall concentration parameter ($1 \leq s_2 \leq 3$) on the $\phi(\eta)$ profile for both the wedge and cone. The plots reveal a consistent decline in $\phi(\eta)$ with the increasing values of Rc , Sc and s_2 . This behaviour is attributed to the inverse relationship between viscosity and the chemical reaction rate; an increase in Rc results in a reduction of both viscosity and concentration distribution. Similarly, a rise in Sc implies a decrease in the molecular diffusivity of the liquid, which in turn diminishes the concentration levels within the boundary layer. Regarding the wall concentration factor, as s_2 upsurges, the boundary layer becomes thinner, leading to a decrease in the concentration profile. In all these three cases, the decrease in $\phi(\eta)$ is more pronounced for the cone configuration than for the wedge.

Figures 15 to 17 present the impact of the suction factor within the range $1 \leq S \leq 2$ on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ profiles for both cone and wedge. As S increases, all three profiles exhibit a downward trend. This behaviour is attributed to the stronger suction drawing liquid toward the surface, which modifies the boundary layer and reduces the fluid velocity. The inward pull of the fluid lessens surface friction, thereby increasing the rate of heat transfer and resulting in a decreased $\theta(\eta)$ distribution. Suction also eliminates low-energy fluid near the surface, effect-

tively drawing the hybrid nanoliquid closer to the wedge or cone surfaces. The decline in $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ is more significant in the case of the cone than in the wedge.

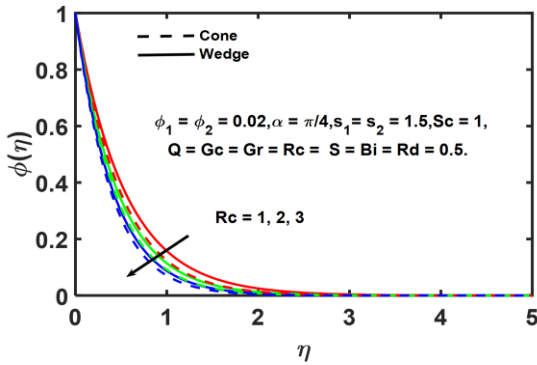


Fig. 12. Chemical reaction parameter's effect on concentration profile.

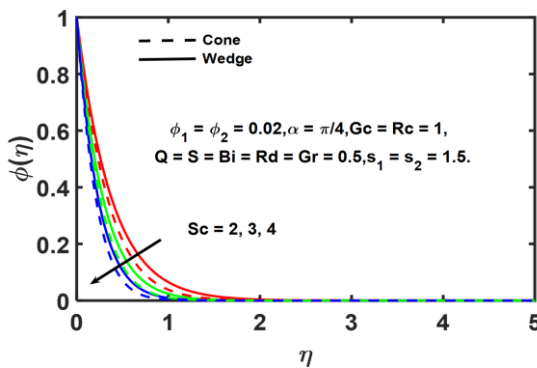


Fig. 13. Schmidt quantity's effect on concentration profile.

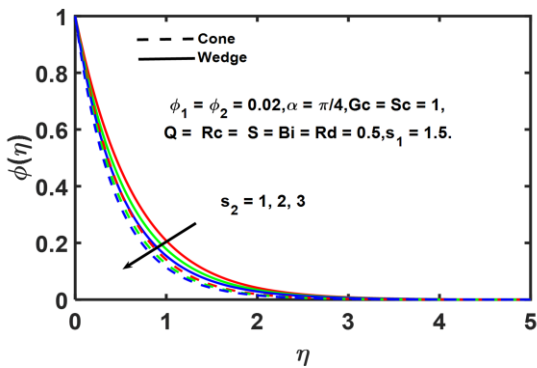


Fig. 14. Wall concentration parameter's effect on concentration profile.

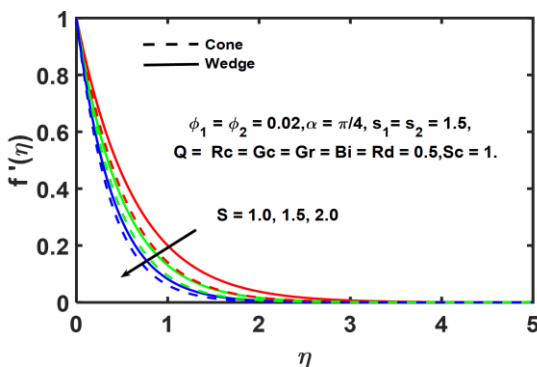


Fig. 15. Suction factor's influence on velocity profile.

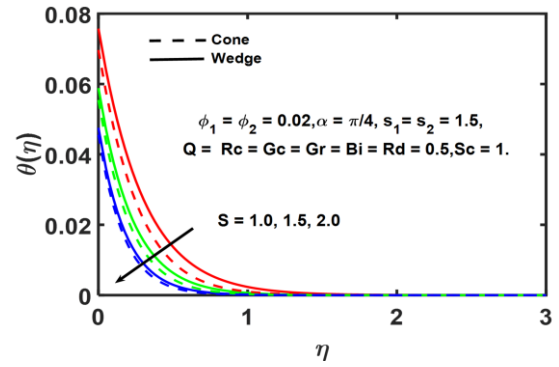


Fig. 16. Suction factor's influence on temperature profile.

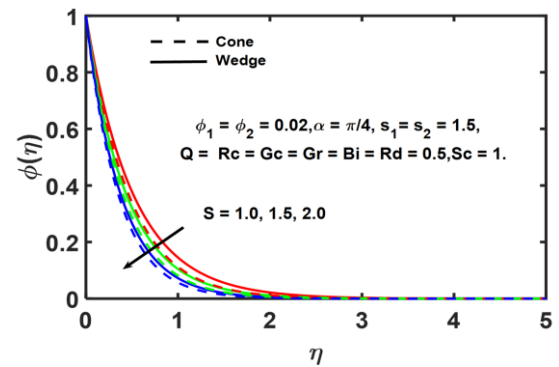


Fig. 17. Suction factor's influence on concentration profile.

Figure 18 illustrates the effect of Biot number, ranging between $1 \leq Bi \leq 2$, on the $\theta(\eta)$ profile for both the cone and wedge. Bi represents the ratio of convective heat transfer at the surface to conductive heat transfer within the surface. An increase in Bi indicates stronger convective heat exchange between the cone and wedge surface and the neighbouring hybrid nanofluid. As a result, $\theta(\eta)$ increases, leading to a thicker thermal boundary layer. This influence is more important in the wedge configuration than in the case of the cone.

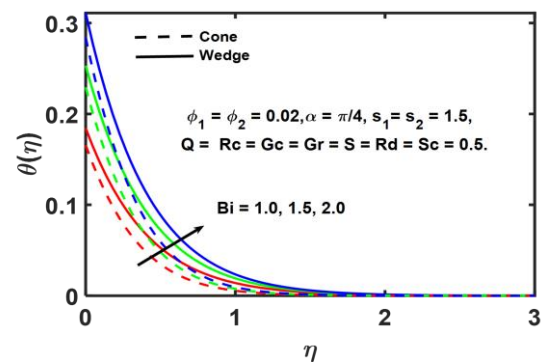


Fig. 18. Biot number's influence on the temperature profile.

Tables 3 and 4 demonstrate the effects of Gr, Gc, Rd, Q, Sc and Rc on the skin friction coefficient C_f , Nusselt number Nu_x and Sherwood number Sh_x for the hybrid nanofluid $Al_2O_3 - Cu/H_2O$ over both cone and wedge geometries. The computations are carried out with fixed values: $\phi_1 = \phi_2 = 0.02$, $\alpha = \pi/4$, $s_1 = s_2 = 1$, $Sc = 0.2$, $Rd = Q = Gr = Gc = Rc = S =$

Table 3. Influences of Gr , Gc , Rd , Q , Sc and Rc on C_f and Nu_x for both the cone and wedge.

Gr	Gc	Rd	Q	Sc	Rc	C_f		Nu_x	
						Cone	Wedge	Cone	Wedge
0.1						-1.411734	-1.176412	0.616064	0.605640
0.2						-1.409805	-1.173662	0.616071	0.605653
0.3						-1.407876	-1.170915	0.616077	0.605667
0.4						-1.405948	-1.168170	0.616083	0.605681
	0.1					-1.642275	-1.420108	0.614557	0.603147
	0.4					-1.460673	-1.226097	0.615756	0.605161
	0.7					-1.294884	-1.048459	0.616690	0.606633
	1.0					-1.139337	-0.881631	0.617470	0.607814
		1.0				-1.400719	-1.160468	0.765981	0.749786
		1.5				-1.397266	-1.155199	0.911128	0.887971
		2.0				-1.393672	-1.149638	1.051824	1.020541
		2.5				-1.389945	-1.143819	1.188288	1.147751
			0.1			-1.405233	-1.167917	0.621241	0.614034
			0.2			-1.404964	-1.167401	0.620081	0.612257
			0.3			-1.404675	-1.166823	0.618843	0.610299
			0.4			-1.404361	-1.166172	0.617516	0.608127
				0.2		-1.404021	-1.165427	0.616090	0.605693
				0.3		-1.441769	-1.199960	0.615791	0.605267
				0.4		-1.470497	-1.227122	0.615558	0.604919
				0.5		-1.492936	-1.249013	0.615375	0.604632
					0.5	-1.404021	-1.165427	0.616090	0.605693
					0.6	-1.408361	-1.170228	0.616056	0.605635
					0.7	-1.412498	-1.174799	0.616024	0.605580
					0.8	-1.416448	-1.179155	0.615993	0.605527

Table 4. Influences of Gr , Gc , Rd , Q , Sc and Rc on Sh_x for both the cone and wedge.

Gr	Gc	Rd	Q	Sc	Rc	Sh_x	
						Cone	Wedge
0.1						0.606284	0.571215
0.2						0.606336	0.571292
0.3						0.606387	0.571368
0.4						0.606439	0.571445
	0.1					0.582707	0.550370
	0.4					0.601217	0.566799
	0.7					0.616125	0.580182
	1.0					0.628864	0.591689
		1.0				0.606641	0.571756
		1.5				0.606824	0.572047
		2.0				0.607042	0.572397
		2.5				0.607293	0.572802
			0.1			0.606447	0.571426
			0.2			0.606456	0.571445
			0.3			0.606467	0.571467
			0.4			0.606478	0.571492
				0.2		0.606491	0.571521
				0.3		0.773045	0.721728
				0.4		0.925505	0.858809
				0.5		1.067523	0.986330
					0.5	0.606491	0.571521
					0.6	0.625225	0.590917
					0.7	0.643397	0.609699
					0.8	0.661046	0.627916

$Bi = 0.5$. Note that when evaluating the impact of a specific parameter, all other parameters are kept constant to isolate its effect on C_f , Nu_x and Sh_x .

Tables 3 and 4 indicate that an increase in Gr from 0.1 to 0.4 leads to a slight rise in C_f , Nu_x and Sh_x for both cone and wedge configurations. For the cone, these values are increased by approximately 0.4%, 0.003% and 0.03%, respectively, while for the wedge, the corresponding increases are about 0.7%, 0.006% and 0.04%. When Gc is increased from 0.1 to 1.0, noticeable enhancements are observed. For the cone, C_f , Nu_x and Sh_x increase by approximately 31%, 0.5% and 8%, respectively, while for the wedge, the corresponding increases are about 38%, 0.8% and 8%. As Rd increases from 1.0 to 2.5, the cone configuration shows increases of approximately 0.8% in C_f , 55% in Nu_x and 0.1% in Sh_x . For the wedge, the corresponding increases are about 1.4%, 53% and 0.2%, respectively. As Q increases from 0.1 to 0.4, both C_f and Sh_x exhibit slight increases by 0.06% and 0.01%, respectively, for the cone and by 0.15% and 0.01% for the wedge. In contrast, Nu_x decreases by 0.6% for the cone and 1% for the wedge. As $0.2 \leq Sc \leq 0.5$, both C_f and Nu_x decrease by 6% and 0.1% for the cone and by 7% and 0.2% for the wedge, respectively. In contrast, Sh_x shows a substantial increase of 76% for the cone and 73% for the wedge. Lastly, increasing Rc from 0.5 to 0.8 results in slight reductions in C_f and Nu_x by 0.9% and 0.02% for the cone, and by 1.18% and 0.03% for the wedge. Meanwhile, Sh_x increases by 9% for the cone and 10% for the wedge.

To ensure the validity of the present model, Tables 5 and 6, along with the bar diagrams shown in Figs. 19 and 20, present the computed values of $-f''(0)$ and $-\theta'(0)$ for the specific case when $\phi_1 = 0.00001$ and all other parameters ϕ_2 , Rd , s_2 , Q , Gr , Gc , Sc and Rc are set to zero, with $Bi = 10^6$. The outcomes demonstrate strong agreement with the existing results reported

in [3,35,36] for the cone configuration, thereby supporting the accuracy of the present formulation.

Table 5. Validation of $-f''(0)$ against existing results for a wedge.

Pr	α	s_1	S	$-f''(0)$		
				Hassanien et al. [35]	Yahaya et al. [3]	Present results
6.2	1	1	1	1.618034	1.61803	1.618281
			0.5	1.280777	1.28078	1.281515
			-0.5	0.780781	0.78078	0.782412
			0	1.00000	1.00001	1.001396

Table 6. Validation of $-\theta'(0)$ against existing results for a wedge.

Pr	α	s_1	S	$-\theta'(0)$		
				Waini et al. [36]	Yahaya et al. [3]	Present results
1	0	0	0	0.5820	0.58201	0.582013
			1	1.0000	1.00001	1.000011
			2	1.3333	1.33333	1.333333
0.72		1		0.8086	0.80864	0.808645

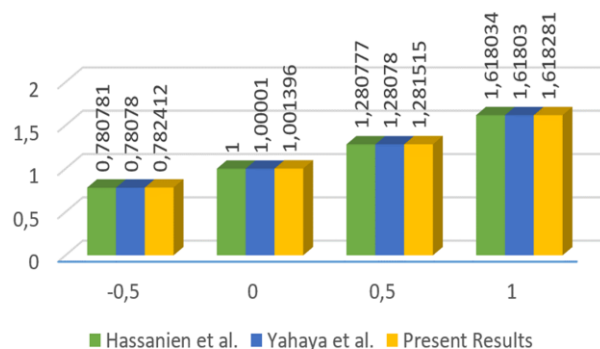


Fig. 19. Comparison of $-f''(0)$ for different values of S.



Fig. 20. Comparison of $-\theta'(0)$ for different values of s_1 .

5. Conclusions

This work presents a mathematical model for analysing the steady, laminar, two-dimensional mixed convective flow

of a chemically reactive hybrid nanoliquid consisting of $Al_2O_3 - Cu/H_2O$ over vertical non-isothermal/non-isosolutal wedge and cone surfaces, incorporating the influence of the heat generation factor. The nonlinear ODE system is numerically solved using MATLAB's BVP-5C solver. The key findings derived from this analysis are outlined below:

- As the values of Gr increase, an enhancement in the velocity profile is observed, whereas both concentration and temperature profiles tend to decline. A similar performance is noted for Gc as well.
- Rising values of Rd, Q and Bi leads to a rise in the temperature distribution of the hybrid nanoliquid for both geometries, whereas an opposite behaviour is noticed with the rise in s_1 .
- A consistent decline in $\phi(\eta)$ is observed with increasing values of Rc, Sc and s_2 . With an increase in S, all three profiles $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ show a decreasing trend.
- An increase in Gr, Gc, Rd and Q leads to a rise in the skin friction coefficient, whereas Rc produces a contrasting effect. The rate of heat transfer, represented by the Nusselt number, improves with higher Gr, Gc and Rd but declines as Q and Rc increase. Moreover, the Sherwood number is observed to increase with growing values of Gr, Gc, Rd, Q and Rc for cone and wedge geometries.

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