

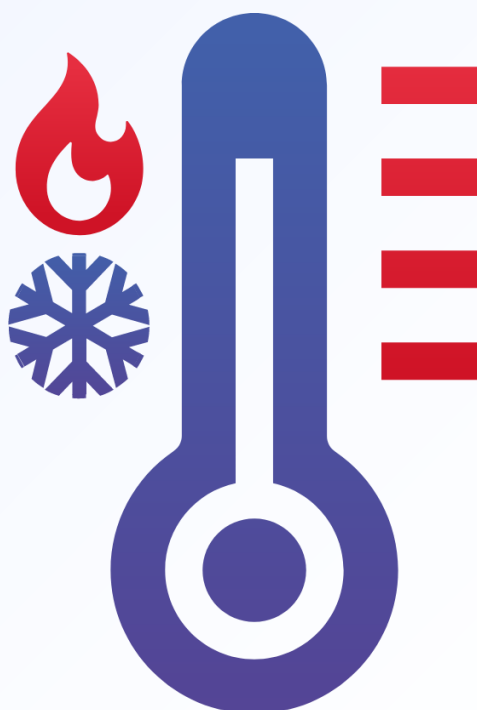
Vol. 46 No. 4 (2025)



ARCHIVES OF THERMODYNAMICS

ISSN: 1231-0956

ISSN: 2083-6023 (Online)



Co-published by



Institute of Fluid Flow Machinery
Polish Academy of Sciences



**Committee on Thermodynamics
and Combustion**
Polish Academy of Sciences



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The aim of the quarterly journal Archives of Thermodynamics (AoT) is to disseminate knowledge between scientists and engineers worldwide and to provide a forum for original research conducted in the field of thermodynamics, heat transfer, fluid flow, combustion and energy conversion in various aspects of thermal sciences, mechanical and power engineering. Besides original research papers, review articles are also welcome.

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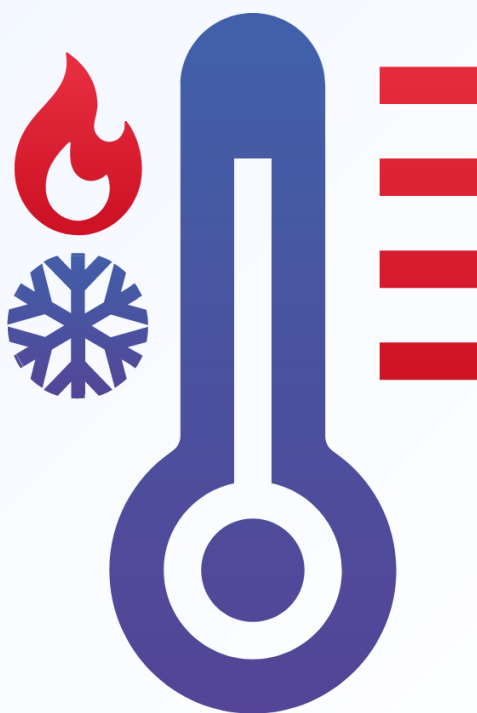
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ISSN 1231-0956

ISSN 2083-6023 (Online)

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Publication funding of this journal is provided by resources of the Polish Academy of Sciences and the Institute of Fluid Flow Machinery

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Thermodynamics Frontier Award 2024

The Thermodynamics Frontier Award (TheFA) is a yearly distinction for the best paper published in Archives of Thermodynamics (AoT) in the preceding calendar year, recognising work that pushes the frontiers of thermodynamics. The assessment criteria are originality, scientific significance, methodological quality, and impact on the field. As a rule, the AoT Editors nominate 8–10 papers, and the Award Donor, in cooperation with the Programme Council (advisory vote), selects the winner. In the 2024 edition, 12 papers were nominated:

1. Singh, J., & Lanjewar, A. (2024). Experimental investigation and performance prediction of SAH using different arc rib roughness geometries – A comparative study. *Archives of Thermodynamics*, 45(1), 33–44. doi: 10.24425/ather.2024.150436
2. Wajs, J., & Lukasik, J. (2024). Assessment of the impact of jet impingement technique on the energy efficiency of air-cooled BIPV/T roof tile. *Archives of Thermodynamics*, 45(2), 5–18. doi: 10.24425/ather.2024.150847
3. Sikora, M., Bohdal, T., Witczak, S., & Ligus, G. (2024). Flow maps in multiphase flows. *Archives of Thermodynamics*, 45(2), 19–28. doi: 10.24425/ather.2024.150848
4. Gurgul, S., & Fornalik-Wajs, E. (2024). Understanding of RANS-modelled impinging jet heat transfer through turbulence kinetic energy, momentum and energy budgets. *Archives of Thermodynamics*, 45(3), 13–30. doi: 10.24425/ather.2024.150451
5. Kaniowski, R.M. (2024). Pool boiling for water on surfaces with inclined microchannel. *Archives of Thermodynamics*, 45(2), 41–49. doi: 10.24425/ather.2024.150850
6. Rafałko, G., Mosdorf, R., Grzybowski, H., Dzienis, P., & Górski, G. (2024). Multiscale entropy applications for complexity analysis of two-phase flow. *Archives of Thermodynamics*, 45(2), 83–90. doi: 10.24425/ather.2024.150854
7. Bobrowski, J., & Gutkowski, A. (2024). Influence of wall temperature on condensation rate in duct flow of humid air: a comprehensive computational study. *Archives of Thermodynamics*, 45(3), 127–133. doi: 10.24425/ather.2024.151229
8. Sutjahja, I.M., Yusuf, A., Anggraini, Y., Ulhaq, S.D., Kurnia, D., & Wonorahardjo, S. (2024). Asymmetrical melting and solidification processes of phase change material and the challenges for thermal energy storage systems. *Archives of Thermodynamics*, 45(3), 135–147. doi: 10.24425/ather.2024.151224
9. Wang, Y., Ma, Z., Gu, Y., & Guo, Q. (2017). Multi-optimization of thermodynamic performance of an HT-PEM fuel cell based on MOPSO algorithm. *Archives of Thermodynamics*, 45(3), 197–208. doi: 10.24425/ather.2024.151231
10. Mamache, S., Mendil, F., & Bouda, F.N. (2024). Thermal instability of three-dimensional boundary layer stagnation point flow towards a rotating disc. *Archives of Thermodynamics*, 45(4), 61–72. doi: 10.24425/ather.2024.151997
11. Kuś, T., & Madejski, P. (2024). Numerical investigation of thermal-flow processes in the ejector-condenser for selected geometrical parameters. *Archives of Thermodynamics*, 45(4), 73–83. doi: 10.24425/ather.2024.151998
12. Kalina, J. (2024). Sizing large-scale industrial heat pump for heat recovery from treated municipal sewage in coal-fired district heating system. *Archives of Thermodynamics*, 45(4), 107–124. doi: 10.24425/ather.2024.152001

On 24 June 2025, a meeting of the Programme Council of AoT was held to select the laureate of TheFA for the year 2024. The meeting was attended by:

- Prof. Janusz Badur (Donor of TheFA 2024),
- Prof. Tadeusz Chmielniak,
- Prof. Piotr Furmański,
- Prof. Dariusz Kardaś (Chairman of the Council),
- Prof. Robert Kobyłecki.

Council members presented their views on 12 articles nominated by AoT Editors. The discussion covered both the scientific merit of each paper and their links to fundamental issues in thermodynamics. Finally, the Donor of the Award 2024, a renowned thermodynamicist – Professor Janusz Badur, presented his assessment of the most noteworthy articles. From among the nominated works, he decided to bestow his award on the paper:

Gurgul, S., & Fornalik-Wajs, E. (2024). Understanding of RANS-modelled impinging jet heat transfer through turbulence kinetic energy, momentum and energy budgets. *Archives of Thermodynamics*, 45(3), 13–30. doi: 10.24425/ather.2024.150451

The Donor underlined that the awarded paper is deeply rooted in classical thermodynamics and that its subject—modelling turbulent heat transfer—rests on analysing temperature and velocity fluctuations. The work offers an alternative, novel approach to heat transfer in impinging-jet flows, and the selected application example helped to highlight key features of turbulent phenomena.

Short scientific biography of the Donor of TheFA 2024 – Professor Janusz Badur

Professor Janusz Badur (born 1952 in Iława, Warmia and Mazury Voivodeship, Poland) is a thermodynamicist who has been specialising for years in numerical modelling of combustion and phase transitions in the flow of working fluids through technical devices such as reactors, combustion chambers or turbines. Professionally, he is associated with the Institute of Fluid-Flow Machinery of the Polish Academy of Sciences in Gdańsk. He was also associated with such excellent technical universities as: Ruhr-Universität Bochum (1980–1993), Université de Poitiers (1994), Cape Town University (1996), Manchester University (1995), Technical University of Gdańsk (1996–2000, 2009–2015), Technical University of Koszalin (1999–2015), University of Warmia and Mazury (1998–2002), Kazimierz Wielki University of Bydgoszcz (2016–2022).

Prof. Badur's deepest professional interest and practical skills stem from his long and technically-oriented education – he learned the trade of a bricklayer at a vocational school in Prabuty (1965–1968) and the trade of a technician in Olsztyn (1968–1971).

He studied at Gdańsk Technical University (1971–1974) and Cracow Technical University (1974–1976). He worked part-time at numerous companies and offices, his longest stint being at the Gdańsk Industrial Construction Design Office (1976–1978).

His scientific ties lie primarily with the Institute of Fluid-Flow Machinery PAS, where he earned his doctorate in 1985 and habilitation in 1993. In 1993, as part of the Institute's transformation, Professor Zbigniew Bilicki entrusted him with the challenging task of building a department dedicated to modern methods of computational mechanics (CFD, CSD, FSI).

Thanks to the focused attention and talent of his staff and doctoral students, the Energy Conversion Department has developed computational programs based on mathematical models combining thermodynamics, fluid flow and solid deformation. These tools are continually developed and supplemented with elements resulting from practical needs.

In 1993–2013, the research team of the Energy Conversion Department, led by Professor Badur, participated in Alstom Power's large-scale program to modernise the Polish energy sector, which aimed at emission-free, highly efficient and cheap electric energy production. This was a major modernisation program for Polish power plants, which posed numerous challenges and scientific problems. Among them were the problems of numerical simulation of real, full-scale and unrecognised phenomena occurring in turbines and reactors. Many of these problems became the subjects of 54 doctoral theses supervised by Professor Badur.

Between 2013 and 2025, the research efforts of Professor Badur and his Energy Conversion Department focused on direct collaboration with the largest domestic power plants. A series of projects were carried out for power plants in: Bełchatów, Ostrołęka, Opole, Turów and Gorzów Wielkopolski. These projects yielded tangible financial benefits and improved the country's energy security.

Recently, Prof. Badur's main scientific specialisation, presented in over 90 publications, is the development of computational tools for the science called: fluid-solid interactions (FSI), where new models of boundary conditions for fluid-solid contact are necessary, since the basic phenomena are concentrated there.



In particular, the FSI formalism requires that many phenomena that either originate at the surface or are induced by the interface between two bodies, be reformulated or expressed completely differently. This applies especially to the surface boiling, cavitation, spontaneous condensation, catalytic combustion, wall turbulence, surface cracking, scaling, wear and friction, surface plasticity, and surface heat and mass transport. Due to this large accumulation of surface phenomena, numerical FSI methods must be of the "preserving structure" type, which is another major challenge in computational thermodynamics. Therefore, Professor Badur is developing the mathematical foundations of FSI within an approach called: "post-Hamiltonian thermodynamics."

Prof. Badur is also the author of a number of scientific monographs, among which the most famous is the 1160-pages work "The Evolution of the Concept of Energy" (2009). He reevaluated and modernised the Carnot theory of heat exchange and the laws of thermodynamics established by Lazar Carnot and Sadi Carnot. In his monograph "Eternal Energy Conversion" (2017), he reevaluated Aristotle's thermodynamics and examined its four renaissances. He showed that Aristotle's concept of teleology and his principle of least action could be a competing paradigm of thought compared to that of Clausius.

Similarly, in his monograph "Eternal Symmetry of Noether" (2019), drawing on historical research, he demonstrated the superiority of thermodynamic principles over the symmetries of gauge field theory.

Also, having a thermodynamic point of view on the problems of interaction of matter and fields, in the monograph "Eternal Relativity of Whitehead" (2022) he showed that excessive geometrisation of physics leads to models that violate the first law of thermodynamics, while algebraisation of physics, on the contrary, allows the elevation of the first and second laws of thermodynamics to the role of superior laws.

Janusz Badur, like Socrates, loves to talk and debate with young scientists. This is because he was fortunate enough to meet sages such as Anna Stefaniak, Janusz Chojnacki, Andrzej Walicki, Zbigniew Bilicki, Stanisław Sieniutycz, and others. From these science-minded individuals, he received much kindness and financial support. Therefore, the Thermodynamics Frontier Award (TheFA) is the best possible continuation of this approach.

AoT Editorial Board

Numerical Investigation of a Two-Stage Ejector Based on a Constant Rate of Momentum Change: Performance Analysis and Optimisation

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Received: 01.04.2025; revised: 17.07.2025; accepted: 19.07.2025

Abstract

A two-stage ejector based on a constant rate of momentum change is a geometrical arrangement of a single-stage ejector to further improve its performance, utilising water vapour as a working fluid for both primary and secondary flows. Creating an additional secondary inlet in the single-stage ejector helps to further entrain the secondary mass flow, resulting in a better entrainment ratio. The present study utilises the constant rate of momentum change, and a 1D gas dynamic approach to compute the geometrical profile and flow parameters using MATLAB. The numerical software ANSYS Fluent 18.0 is utilised to analyse the two-stage ejector geometry. The global performance ejector entrainment was computed at on and off design conditions. The results show that the two-stage ejector entrainment ratio is significantly higher than that of the previously studied single-stage ejector. The entrainment ratio of the two-stage ejector increases to reach the on-design value of operating conditions, and then starts decreasing.

Keywords: Constant rate of momentum change; Ejector; Two-stage; CFD.

Vol. 46(2025), No. 4, 7–17; doi: 10.24425/ather.2025.156597

Cite this manuscript as: Kumar, A., Yadav, S. K., & Kumar, V. (2025). Numerical Investigation of a Two-Stage Ejector Based on a Constant Rate of Momentum Change: Performance Analysis and Optimisation. *Archives of Thermodynamics*, 46(4), 7–17.

1. Introduction

An ejector is a device that utilises the Venturi effect to create a vacuum and entrain gas or vapour from a desired vessel or system. In this study, the two-stage ejector (TSE) operates with water vapour as a working fluid for both primary and secondary flows, leveraging the constant rate of momentum change (CRMC) approach to enhance the ejector performance. It works similarly to a vacuum pump but without any moving parts. In the early 19th century (1942), the ejector model was developed by Keenan J. H. and Neumann E. P. These ejectors are built on constant area and pressure mixing (CAM and CPM) approaches. Several efforts have been made to optimise the ejector parameters, viz. nozzle parameters [1], mixing section geometrical pa-

rameters [2], diffuser geometrical parameters [3] and ejector operating parameters [4]. Even after all efforts made to improve both CAM and CPM single stage ejectors, their performance is limited. It is found that the CPM ejector exhibits a better performance than that of the CAM ejectors [5]. A novel method for a comprehensive analysis of geometric properties that can be used to alter the mass flow ratio and the hydrogen mass flow was introduced. Additionally, the dynamic behaviour of the fuel cell system's ejector was investigated in [6]. According to energy, exergy, and economic considerations, the goal of the recent study [7] was to thoroughly examine the performance of a solar ejector cooling system. It was meant to provide a complete view of the solar cooling system performance under typical operational circumstances.

Nomenclature

L – length, m
 M – Mach number
 $\dot{m}$ – mass flow rate, kg/s
 T – temperature, K
 P_d – discharge pressure, Pa
 P_s – suction pressure, Pa
 u – velocity, m/s

Greek symbols

ρ – density, kg/m³
 ϕ – entrainment ratio

Subscripts

e – exit flow
 I, J – space component

m – mixing
 n – nozzle
 o – stagnation condition
 p – primary fluid flow
 s – secondary fluid flow
 $s1$ – secondary fluid flow (Stage I)
 $s2$ – secondary fluid flow (Stage II)

Abbreviations and Acronyms

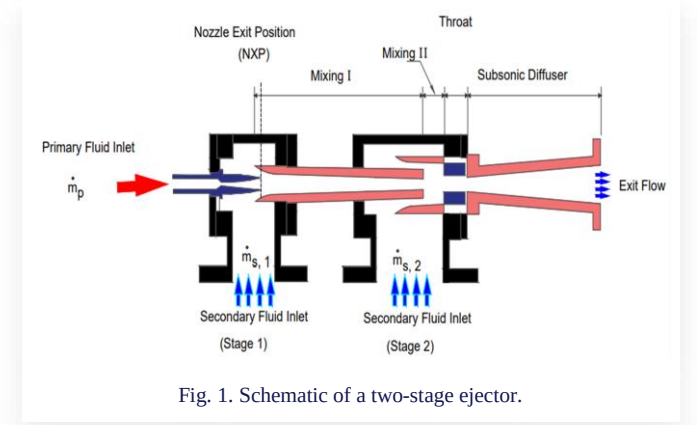
CAM – constant area mixing
 CFD – computational fluid dynamics
 CPM – constant pressure mixing
 CRMC – constant rate of momentum change
 RANS – Reynolds-averaged Navier-Stokes
 SSE – single-stage ejector
 TEWI – total equivalent warming impact
 TSE – two-stage ejector

Kanbur et al. (2025) [8] used a coupled CFD and thermodynamic model to define outlet boundary conditions through multiphase CFD simulations. The study claims that this framework reduces reliance on experimental data. A comprehensive assessment of energy, exergy, and economic factors conducted by Ogaili et al. [9] highlights an integrated solar-thermal power system, which leads to more sustainable and cost-effective energy solutions. The single-compression multi-temperature ejector refrigeration sequence architecture is proposed by Fabris et al. [10] for last-mile deliveries of multi-temperature goods in urban environments. The design of ejectors helps effectively to reduce the overall carbon footprint associated with road transport refrigeration systems. These results have significant implications for the design and deployment of hybrid refrigeration systems that seek to reduce their ecological influence while upholding operational flexibility [11].

The current investigation involved an experimental analysis of a gas ejector with a spindle calculated for use with propane (R290) as a working fluid. The performance of the ejector was assessed using conventional literature notation, the critical temperature and entrainment ratio, which are the supreme important factors for adjusting cooling capability in ejector-based refrigeration methods [12]. In order to optimise the ejector performance, the surrogate model developed in [13] uses a multi-objective evolutionary approach to increase entrainment and compression ratios while reducing entropy output. The stagnation temperature ratio is found to be one of the main factors that enhances mixing layer growth and ejector performance. The optimised ejector shows a roughly 25% boost in efficiency over a non-optimised one. The study [14] looks at how well four cutting-edge cascaded refrigeration systems work to cool at lower temperatures while consuming less energy. In a flash binary geothermal cycle, the study [15] offers a new method to increase the efficiency of a geothermal-based power plant by substituting an ejector for conventional expansion valves.

A two-stage ejector (TSE) is a type of ejector system that involves two stages of compression or fluid entrainment to achieve the desired pressure or flow rate (refer to Fig. 1). Ejectors, also known as jet pumps, are devices that use the principle of fluid dynamics to transfer or compress fluids. They operate

by entraining a secondary fluid or gas using a high-speed primary fluid, typically at a nozzle or diffuser, creating a low-pressure section that draws in the secondary fluid.



1.1. Performance evaluation

Performance evaluation of a TSE involves assessing the efficiency and effectiveness of a system that uses two ejectors in series to entrain and compress the secondary fluid (suction fluid). This configuration allows for higher compression ratios and is often used in applications requiring higher pressures, such as in studies involving entrainment ratio, discharge pressure, suction pressure, critical pressure ratio, and experimental testing.

1.2. Entrainment ratio

The entrainment ratio of a two-stage ejector denotes the proportion of mass flow rate attributed to the secondary fluid (often referred to as entrained or suction fluid) with respect to the flow rate of primary fluid (typically termed motive fluid) within each stage of the ejector. The equation which can be used to calculate the entrainment ratio is given as follows:

$$ER = \frac{\dot{m}_{s,1} + \dot{m}_{s,2}}{\dot{m}_p} \quad (1)$$

Here, $\dot{m}_{s,1}$ is the mass flow rate of the secondary fluid entrained at the first stage, where the primary fluid (motive fluid)

interacts with the secondary fluid. The mixed fluid from the first stage then acts as the motive fluid for the second stage, entraining an additional secondary mass flow rate $\dot{m}_{s2}$ at the second inlet. This distinction ensures that the contributions of both stages to the overall entrainment are clearly captured.

1.3. Compression ratio/lift pressure ratio

The compression ratio is given as follows:

$$CR = \frac{P_{dis}}{P_{suc}}. \quad (2)$$

Here, P_{dis} denotes the pressure at the discharge boundary, and P_{suc} signifies the pressure at the suction boundary.

Although single-stage ejectors are frequently used to create vacuum, they have some disadvantages that can affect their performance in some applications, including limited vacuum generation, decreased efficiency, reduced performance at low pressures, and a larger size. However, the addition of a two-stage ejector contributes to getting over these restrictions as explained by various researchers.

In the study of a two-stage ejector [16], the numerical technique utilising Navier-Stokes equations and the theoretical assessment using a 1-dimensional model have been used to critically evaluate the flow phenomena within ejector systems. Existing experimental data were used to validate both numerical and theoretical results. In [17], to determine the ideal design parameters, numerical studies utilising computational methods were used to estimate the performance phenomena of the two-stage ejector system. It has been suggested that a TSE system is a valuable different configuration for making use of the fired thrust of the discharged flow. As compared to the traditional single-stage ejector system, the performance can be greatly enhanced. It is shown in [18] that the two-stage ejector-diffuser system performs approximately four times better than the single ejector system. A trial exploration of a two-stage ejector refrigeration system, encompassing activities such as design of the system, operation, and empirical assessment was made in [19]. It was examined using independently varying temperatures for evaporation and condensation to examine the likelihood for a system performance change. The blended performances of both the ejectors used in the two-stage ejector-based multi-evaporator refrigeration system (TEMERS) were examined in [20] utilising a two-dimensional CFD modelling method. First, the cooling capacity requirements of the refrigeration and air-conditioning chamber define the main nozzle diameters. Using the verified Reynolds-averaged Navier-Stokes (RANS) simulation model, a shape optimisation was performed to maximise the entrainment performance of the R134a two-phase ejector [21]. The suggested review updates the body of prior research on the topic by talking about the pertinent and latest material. Expressive and latest improvements in refrigeration cycles, two-phase ejector modelling and other potential applications are shown and addressed in their respective contexts [22]. The concept of a two-stage ejector design was proposed, incorporating a second inlet. In the replication, the steam ejector refrigeration system's varied operating parameters were used to evaluate the TSE performance, which was then compared to the SSE performance [23].

The current two-stage ejector, which is CRMC-based, can highlight recent and important developments in the area based on supersonic design. ANSYS Fluent 14.0 was used to examine the calculated model numerically for the specified design input parameters and was validated by numerical data [24]. Using thermodynamic analytical techniques, the two-stage cascade refrigeration cycle (CARC) with an enhanced ejector was compared in [25] with the conventional CARC for effectiveness. It has been suggested that the TSE should replace the single-stage ejector due to its superior geometrical configuration, which allows better utilisation of the discharge flow's terminated momentum to generate secondary flow [26]. To upgrade the energy effectiveness of the refrigeration system on a fishing craft, the study [27] proposed a TSE based subzero refrigeration system in order to construct and achieve the best entrainment performance, where each stage's specified parameter is developed with the consideration of the area ratio. A pattern of established and newly developed exergy approaches is utilised in [28] to assess the exergy destruction properties inside the system. Another investigation provides the optimum design and process control methodologies for the hydrogen passage system with a dual ejector to support all aspects of PEMFC system processes [29]. To enhance CO₂ two-phase ejectors' off-design performance, a new swirl-bypass nozzle ejector-based design is put forth in [30]. A 3D multiphase CFD is used to investigate the design of such a device. This study looks at the impact of adding an ejector to two-phase cascades for refrigeration at very low temperatures. The impacts of various ejector configurations on thermodynamics, the environment, and the economy are compared. The main characteristics examined are the volumetric chilling capacity, best cascade temperature, economic analysis, and total equivalent warming impact (TEWI). The impact of various refrigerants on the cycle of low-temperature of the two-stage cascade is also examined [31]. The report [32] presents the findings of a computer-based fluid dynamic study on the TSE execution in a system of refrigeration using a steam ejector, along with a comparison to an SSE without altering the ejector's area ratio. There is a discussion of the elevated entrainment ratio, coefficient of performance (COP), and the additional adaptable TSE functioning.

Environmentally friendly refrigerants are replacing those whose use is either completely prohibited or highly restricted by the Kyoto Protocol. One such refrigerant, R134a, has had its use restricted and will eventually be completely outlawed. R134a has several substitutes, and various studies have been performed to determine possible replacements based on performance criteria. In the study [33], the effects of using R1234zeE, R1234yf, or R600a as an alternative for R134a in the ejector-based bus air conditioning for single and dual ejectors were investigated. Given the increasing demand for heating, the implementation of high-efficacy heating within combined heat and power systems is regarded as a pragmatic method toward conserving energy and reducing emissions [34]. A novel two-stage ejector featuring a control switching technique is presented in [35]. The main purpose is to ensure consistent and stable performance of the TSE across a wide range of primary flow pressures. To achieve this, an optimal switching approach has been devised. Accord-

ing to the numerical findings, when 60% of the pressure reaches the specified primary flow pressure, the TSE demonstrates an impressive 79.4% increase in entrainment ratio as compared to the traditional single-stage ejector.

Recently, the multi-stage ejector is considered as a high-performance device as compared to a single stage ejector. In the present study, the 1-D gas dynamic model of a single stage ejector is modified by creating a second opening at the exit of the mixing section, where the secondary flow can again be dragged through the momentum of the mixed fluid from a single stage. ANSYS Fluent 18.0 is used to analyse the two-stage ejector geometry for on and off design operating conditions.

2. Mathematical computation of the two-stage ejector geometry

The analysis is limited to the axi-symmetric ejector. This section explains the compressible flow theory in one dimension, utilising the CRMC method with frictional effects for real fluids. The design approach is based on the equations of steady flow in an adiabatic steady-state system.

2.1. Computation of flow properties at the exit from the mixing region for a two-stage ejector.

In addition, SSE [36] was modified to become a TSE. Utilising superfluous momentum from the initial stage to further entrain the secondary flow is made possible by the TSE's additional en-

trainment passageway at the mixing section's outlet. The geometry calculation procedure for a TSE is nearly identical to that of a single-stage supersonic ejector, except for the calculation of the ejector's second stage. To determine the equilibrium properties at the ejector section J-J', which is necessary for computing the TSE diffuser region, certain alterations were completed to the creation of section J-J', which is shown in Fig. 2. The regions of nozzle and mixing area of the computed two-stage ejector remain the same as those of the single-stage ejector (SSE), but the diffuser section of TSE was computed separately. The geometry of TSE is computed using the CRMC approach with the key parameters including the nozzle diameter ($d_n = 99.5$ mm), mixing section length ($L_m = 95$ mm), and diffuser length ($L_d = 250$ mm), as illustrated in Fig. 3. These parameters are derived from the 1D gas dynamic model to ensure optimal entrainment and pressure recovery.

$$C_{n,e} = C_{m,e}, \quad (3)$$

$$\varphi \rightarrow \varphi = \frac{\dot{m}_{s,2}}{\dot{m}_{m,e}}, \quad (4)$$

$$\dot{m}_{m,e} = \dot{m}_p + \dot{m}_{s1}, \quad (5)$$

$$T_{0,p} \rightarrow T_{0,m}, \quad (6)$$

$$T_{0,s1} \rightarrow T_{0,s2}, \quad (7)$$

$$T_{0,s1} \rightarrow T_{0,s2}, \quad (8)$$

$$\rho_{0,s1} \rightarrow \rho_{0,s2}. \quad (9)$$

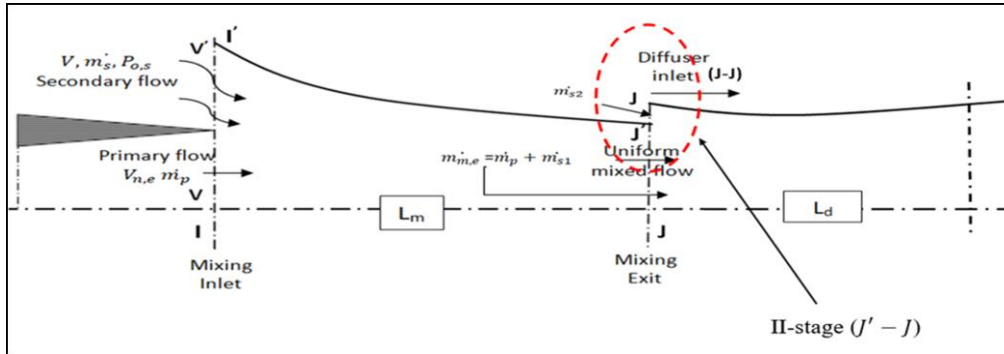


Fig. 2. Discrete modelling of a two-stage ejector.

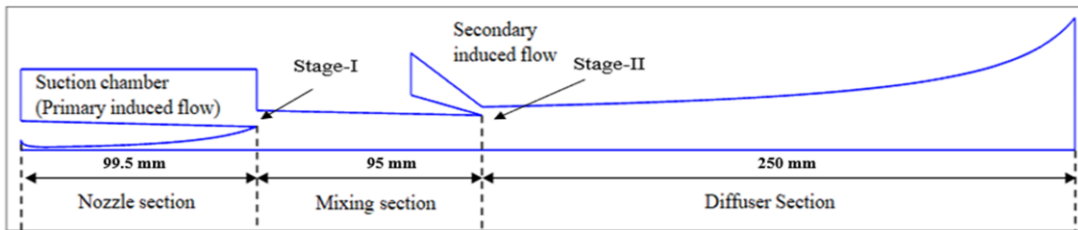


Fig. 3. CRMC two-stage ejector system.

2.3 Selection of design conditions

The working fluid for both primary and secondary flows is water vapour (steam). Steam superheating is critical for ejector performance, as it ensures that the primary flow remains gaseous dur-

ing expansion in the nozzle, preventing condensation that could disrupt supersonic flow and reduce entrainment efficiency. The degree of superheating, defined as the temperature difference above the saturation point at a given pressure, enhances the thermal energy available for driving the entrainment process. For

the primary flow, a superheating degree is approximately 30 K above the saturation temperature, which ensures stable operation and maximises momentum transfer to the secondary flow.

The primary flow is superheated steam at a pressure of 2.2 bar and temperature of 423 K, with a superheating degree of approximately 30 K above the saturation temperature. The secondary flow is at 0.014 bar and 300 K, as specified in Table 1. To conduct the analytical and numerical examination, the design data points specified in Table 1 were taken into consideration.

Table 1. Design data points for two-stage ejector.

Parameters	Primary flow	Secondary flow
Pressure (Pa)	2.2×10 ⁵	1×10 ³
Temperature(K)	423	300
Degree of superheating (K)	30	0 (saturated)
Mass flow rate(kg/s)	0.01	—
Nozzle exit position = 0	—	—

3. Computational analysis

The chapter describes the use of a 1-D gas dynamic approach and computational fluid dynamics (CFD) for designing and optimisation of ejector systems. The 1-D gas dynamic approach considers axi-symmetric, steady, turbulent, compressible flow that is equivalent to area-averaged axis-symmetric flow. This approach is used to generate the ejector geometry, which is required for CFD analysis. The ANSYS Fluent 18.0 commercial software is used for CFD analysis, and it is specifically designed for simulating turbulent supersonic flow. This software is efficient in optimising and simulating flow properties, which can help in improving the ejector system's performance. It is important to note that CFD analysis can only be performed when the geometry is known. Therefore, the 1-D gas dynamic CRMC approach is a necessary step in the ejector system design process. The ejector geometries are quantified through a comprehensive study using ANSYS Fluent 18.0. Overall, the combination of the 1-D gas dynamic approach and CFD analysis can provide a powerful tool for designing and optimising ejector systems.

The evaluation of flow properties such as the Mach number, pressure, and temperature has been carried out along ejector's axial direction. The computation of the 'entrainment ratio,' a universal performance parameter, has been supported by the outcomes predicted by the 1-D gas dynamic model under on-design conditions. Additionally, the ejector's off-design features were examined to ascertain its adaptability to varying operating conditions and working fluids in diverse fields of application. In the off-design investigation, parameters were varied one at a time, while the remaining parameters were kept constant.

The Navier-Stokes equation was numerically solved for the 2-dimensional axi-symmetric field using a steady-state turbulence model. Listed below are the governing equations in the compact Cartesian form [36]:

- mass equation:

$$\frac{\partial}{\partial x_i} (\rho u_j) = 0, \quad (10)$$

- momentum equation:

$$\frac{\partial}{\partial x_i} (\rho u_i u_j) = - \frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j}, \quad (11)$$

where:

$$\tau_{ij} = \mu_{eff} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu_{eff} \frac{\partial u_k}{\partial x_k} \beta_{ij}, \quad (12)$$

- energy equation:

$$\frac{\partial}{\partial x_i} (u_i (\rho E + P)) = \nabla \cdot \left(\epsilon_{eff} \frac{\partial T}{\partial x_i} + u_j \tau_{ji} \right). \quad (13)$$

Here, ρ is the mixture density, u is the velocity vector, E – total energy, P – pressure, ϵ_{eff} – effective thermal conductivity, μ_{eff} – effective dynamic viscosity and β_{ij} is the Kronecker function.

The shear stress transport (SST) k - ω turbulence model was selected for numerical simulation of the supersonic ejector configuration due to its superior ability to capture mixing layers and simulate free shear flows in round and radial jets, which are critical for accurately modelling the complex flow interactions in TSE. Unlike the k - ϵ model, which is less effective in resolving near-wall effects and separated flows, the k - ω SST model combines the robustness of k - ω near walls with the accuracy of k - ϵ in free-stream regions, making it ideal for this study (Ariaifar et al. [37]). Below are the governing equations of this model:

- turbulence kinetic energy:

$$\frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - Y_k + S_k, \quad (14)$$

- specific dissipation rate:

$$\frac{\partial}{\partial x_i} (\rho \omega u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] + G_\omega - Y_\omega + D_\omega + S_\omega. \quad (15)$$

Here:

$$G_k = - \overline{\rho u'_i u'_j} \frac{\partial u_j}{\partial x_i}, \quad Y_k = \rho \beta^* k \omega, \quad Y_\omega = \rho \beta \omega^2,$$

$$D_\omega = 2(1 - F_1) \rho \sigma_{\omega,2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_j},$$

where G_k , Y_k and S_k are the generation of turbulent kinetic energy owing to mean velocity gradients, the generation of turbulence kinetic energy owing to buoyancy and the influence of the variable dilatation in compressible turbulence on the overall dissipation rate, respectively. These terms are expressed as in ANSYS Fluent 18.0 (user manual). G_ω and Y_ω represent generation and dissipation of omega, respectively.

ANSYS Fluent 18.0 was used to employ control volume-based discretisation techniques in solving the governing equations of the problem. The equation's conventional terms were discretised using a second-order upwind approach in order to attain higher-order precision. After that, an implicit method was used to resolve the subsequent arithmetic equations using a density-based joint solver. The algorithm stability was maintained by setting a CFL criterion of 5. At the primary and secondary flow exits and inlets of both ejectors, the pressure boundary condition was applied. In order to achieve greater orthogonality and

better control over mesh quality, the structured quadrilateral mesh was employed in the computational zone of TSE geometries (Fig. 4). There were comparatively more cells used in the area with the higher velocity gradient.

The boundaries are assumed to have zero normal gradients (refer to Fig. 5), resulting in no-slip conditions. In all simulation scenarios, the convergence criterion for the continuity equation and various equations, including X and Y velocities, energy, turbulence kinetic energy, and turbulence dissipation rate, is set to 10^{-5} . For the TSE geometry, the residual convergence criterion for the continuity equation is 10^{-9} . The supersonic flow computational model is solved using a second-order upwind scheme with an under-relaxation factor of 0.3, and standard wall treatment is used near the wall.

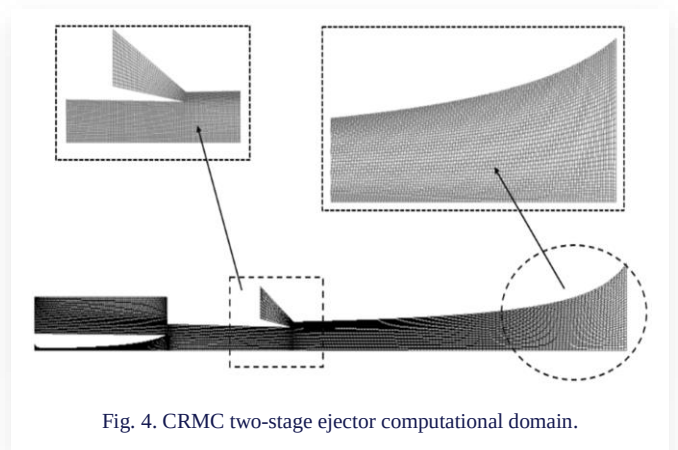


Fig. 4. CRMC two-stage ejector computational domain.

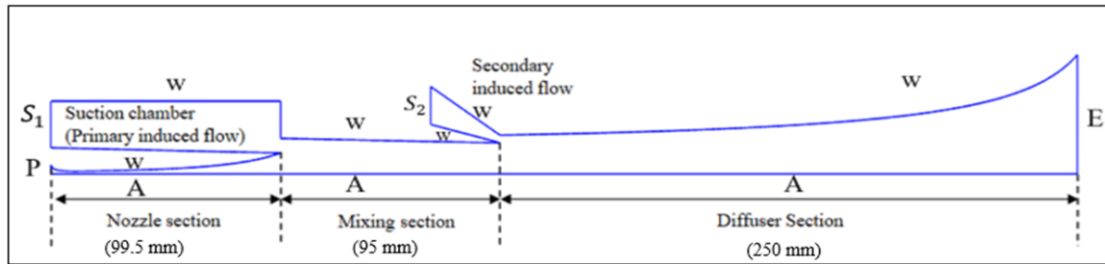


Fig. 5. Boundary conditions employed in TSE ejector computational domain.

3.1. Grid Independence study

The $k-\omega$ SST model was used to conduct a grid independence study on both computational domains. Initially, 60 432 elements were created. The grid-independence study was conducted to assess the performance parameter known as the "entrainment ratio". This study involved increasing the number of grid elements from 60 432 to 110 246. The results indicated that the variation was less than 5%. Based on this assessment, the number of elements was established at 110 246. However, a higher number of grid elements was defined in the ejector's mixing region to accurately capture the mixing phenomenon. The mesh quality was assessed based on the mesh orthogonality and aspect ratio. The results (refer to Table 2) indicate that changes of the entrainment ratio were insignificant beyond ~ 110 246 elements, thus further study was conducted using the number of cells around this value.

Table 2. Detailed mesh used for TSE computational domain.

Model No.	No. of cell	Entrainment ratio (ϕ)
TM-1	60432	0.624
TM-2	80642	0.649
TM-3	110246	0.657
TM-4	130324	0.658
TM-5	150132	0.656

4. Validation of the CRMC approach for the two-stage ejector diffuser section

The validation of the 1D ejector model incorporating the CRMC approach with CFD was carried out. The purpose of this validation was to verify the 1D ejector model before advancing to the

study of operating parameters. On-design data (see Table 1) were used for this analysis. The results obtained from the CFD study, particularly the static pressure, were compared with those calculated from the 1D ejector model under design conditions. In the CFD results, the combined flow of the single-stage ejector and the secondary auxiliary fluid enters and mixes at the second stage, exhibiting oscillations near the diffuser inlet. However, this behaviour is not captured by the 1D ejector model. The static pressure variation along the diffuser region of the 1D ejector model and CFD centerline results are shown in Fig. 6. Mixed fluid of SSE and secondary auxiliary fluid exchange their kinetic energy and momentum during mixing and grasp up to equilibrium with uniform flow properties. The flow downstream of the diffuser shows a diminishing deviation.

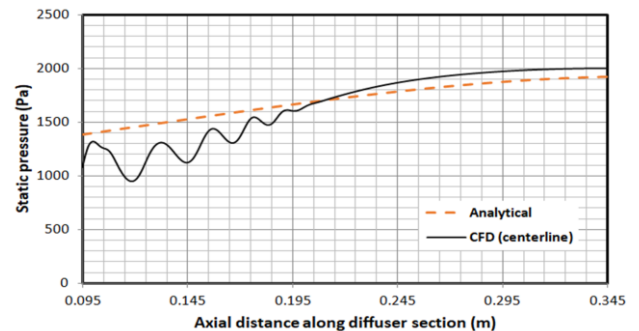


Fig. 6. Variation of static pressure along the diffuser section under on-design conditions ($P_{op} = 2.2$ bar, $P_{os} = 0.014$ bar, $P_e = 0.01925$ bar).

5. Results and discussion

The computational study has been performed on the two-stage ejector model, which was considered for mathematical study. The effect of operating parameters on the performance of the ejector model is presented here. Mach number contours are employed to analyse the operating effects on the performance.

Figure 7 shows the contours of Mach number. The flow characteristics within the ejector can be perceived along with first

and second series of oblique shock waves, diamond wave and shear layer. The primary supersonic flow and secondary subsonic flow start interacting after entering the mixing section, and move towards the second stage with uniform flow properties. The effective area plays an important role for entering secondary flow. The uniformly mixed fluid from SSE acts as the primary fluid, enabling further entrainment of secondary fluid in TSE.

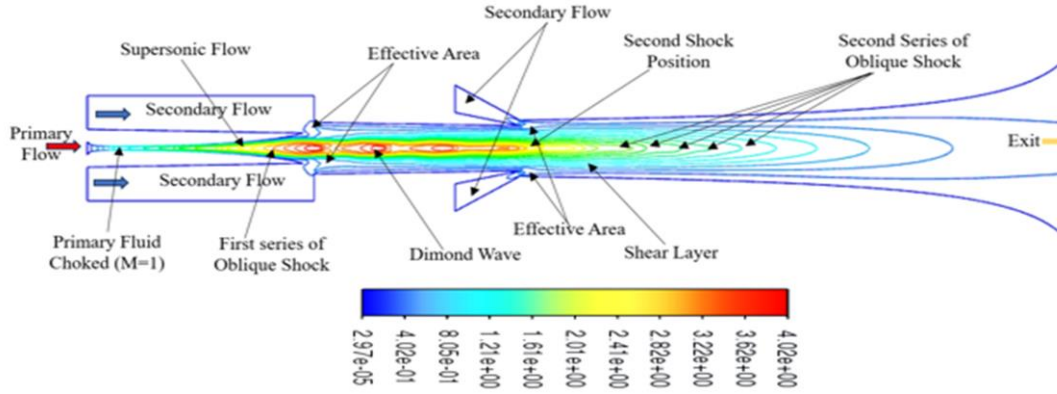


Fig. 7. Mach number contours of the two-stage ejector under on-design conditions ($P_{op} = 2.2$ bar, $P_{oss}=0.014$ bar, $P_e = 0.01925$ bar).

5.1. On-design study

A CFD examination was conducted to analyse the design of two-stage ejectors. Figure 8 displays the pressure fluctuations during the regions of mixing and diffuser. At stage I, the primary and secondary fluid flows strongly interact, causing pressure pulsation in the mixing section. Furthermore, pressure oscillation was noted in the diffuser section as a result of the contact of secondary and mixed flow at stage II.

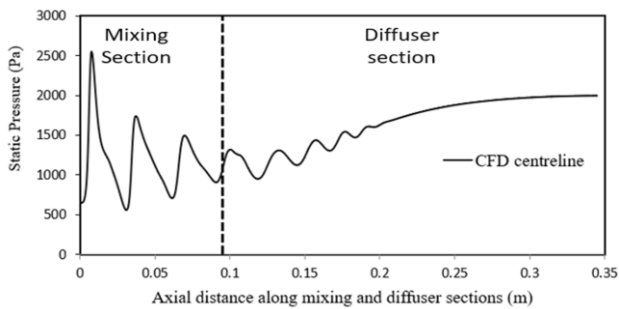


Fig. 8. Static pressure variation along the two-stage ejector under on-design conditions ($P_{op} = 2.2$ bar, $P_{oss}=0.014$ bar, $P_e = 0.01925$ bar).

The pressure gradually increased in the diffuser region after the mixing of the motive and secondary fluids at both stages. The static pressure at the diffuser section exit was predicted to be 0.01925 bar through numerical analysis.

Figure 9 depicts the Mach number fluctuation throughout the two-stage ejector. The anticipated Mach number at the centreline calculated numerically is 4.3, while the design Mach number determined analytically is 3.03. The average Mach number was taken from the analytical design, whereas the centreline Mach number was obtained from the CFD investigation.

Figure 10 displays the overall pressure fluctuation occurring alongside the mixing and diffuser region of TSE. The mixing section exhibits a discernible loss of total pressure (as depicted in Fig. 10) owing to the strong interaction between primary and

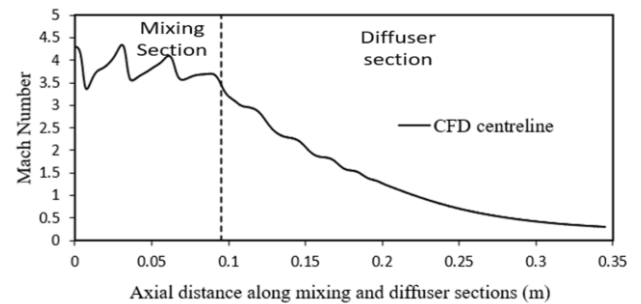


Fig. 9. Mach number variation along the two-stage ejector under on-design conditions ($P_{op} = 2.2$ bar, $P_{oss}=0.014$ bar, $P_e = 0.01925$ bar).

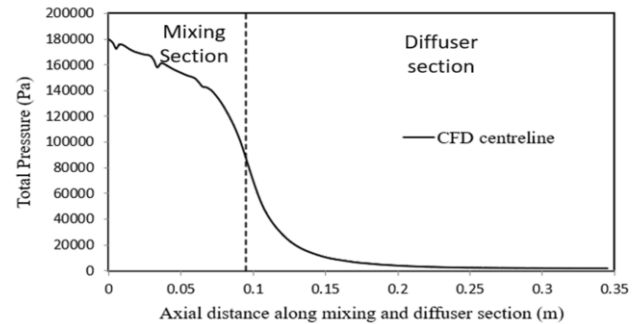


Fig. 10. Total pressure variation along the two-stage ejector under on-design conditions ($P_{op} = 2.2$ bar, $P_{oss}=0.014$ bar, $P_e = 0.01925$ bar).

secondary fluid at the section inlet. The shift in the total pressure occurs gradually and uniformly, thanks to the CRMC method that prevents any thermodynamic shock occurring at the diffuser section inlet, unlike for the conventional ejector system.

Figure 11 illustrates the changes in static temperature as it traverses the mixing and diffuser sections of a two-stage ejector system. The static temperature experiences a decrease initially in stage I because of the interplay between the primary and secondary fluid flows. However, it subsequently gradually increases along the diffuser section and ultimately reaches 358 K at the exit of the diffuser section.

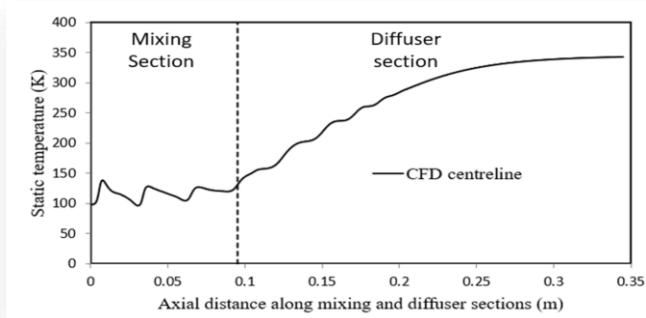


Fig. 11. Static temperature variation along the two-stage ejector under on-design conditions ($P_{op} = 2.2$ bar, $P_{os} = 0.014$ bar, $P_e = 0.01925$ bar).

5.2. Off-design optimisation

Numerical analysis has been conducted to examine the performance of the TSE at different operating pressures. One operating pressure was altered at a time while the other remained constant as per the design. The numerical analysis optimised the TSE performance by evaluating the entrainment ratio (ϕ) under varying primary flow pressure, exit pressure, and nozzle exit position (NXP). The results (Fig. 12) show that the entrainment ratio peaks at a primary flow pressure of 2.2 bar under on-design conditions.

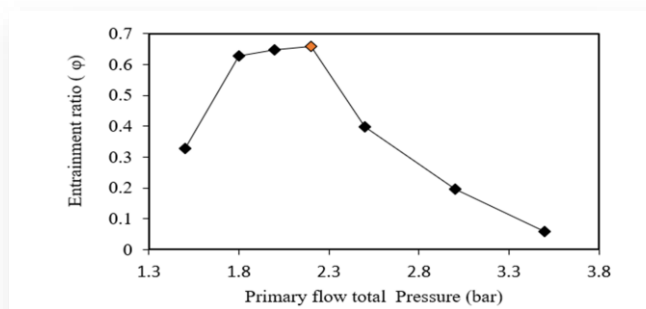


Fig. 12. Effect of entrainment ratio (ϕ) on primary flow total pressure.

Figure 12 depicts the correlation between the entrainment ratio and the total pressure of the primary flow. The mass inflow and momentum during the nozzle's flow at exit increase when the primary flow's total pressure is raised. In the area of the mixing section, this increase in momentum speeds up the entrainment process. Nevertheless, the secondary mass flow rate that must be entrained in Stages I and II increases more slowly than the primary mass flow rate. Consequently, the entrainment ratio shows an initial rise in stages I and II as the primary flow total

pressure increases (below the on-design primary flow total pressure of approximately 2.2 bar). After this point, the entrainment ratio begins to decrease.

Figure 13 displays the Mach number contours of a two-stage ejector, which can help explain the physical phenomena related to this pattern. When the primary flow total pressure increases, the secondary shock location moves downstream, causing the pressure within the suction section to increase for the same exit pressure ($P_e = 0.01925$ bar). However, above the on-design point, an increase in primary flow pressure decreases the pressure difference inside the suction chamber, reducing the trend to entrain secondary flow. Moreover, due to a decrease in the effective area at stages I and II, the ejector can operate at higher critical pressure levels. Consequently, the overall entrainment ratio decreases.

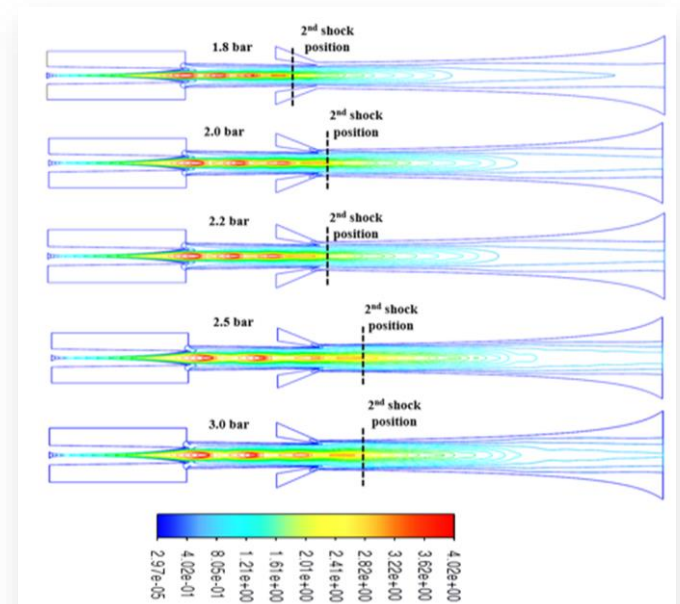


Fig. 13. Primary flow pressure impact on secondary shock position.

Figure 14 depicts how the entrainment ratio changes with the exit pressure. This numerical investigation was conducted when NXP was equal to zero. The ejector's performance curvature is classified into three types: choked, unchoked and reverse flow. As the exit pressure rises, the oblique shock becomes more powerful, and shock waves travel upstream. As a result, both of the flows, i.e. primary and secondary flows, are disrupted at stages I and II.

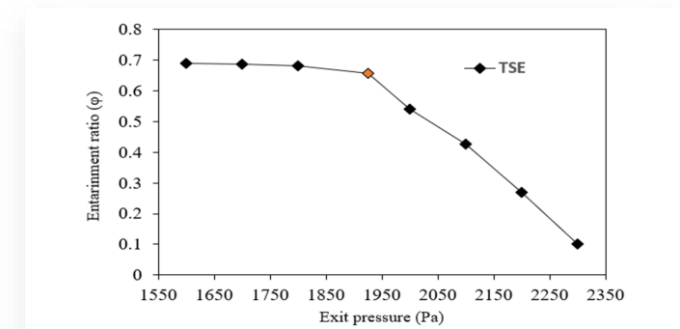


Fig. 14 Effect of exit pressure on entrainment ratio (ϕ).

When the exit pressure is further increased, the shock waves become stronger and impede the secondary flow from entraining at stages I and II, leading to a reduction in the entrainment ratio.

In this investigation, the ejector was found to operate under double fluid choking at pressures up to 0.01925 bar, with minimal variation in the entrainment ratio. However, when the primary fluid alone was choked (between 0.01925 bar and 0.02300 bar), there was a dramatic decrease in the system's entrainment ratio. The system reached its breakdown point at 0.02350 bar.

To avoid choking of both primary and secondary flows during reverse flow, the primary flow is directed into the suction chamber. This study shows that the ejector must operate in a choked flow region, with a critical pressure separation, to achieve the highest entrainment ratio.

The Mach number contours in Fig. 15 illustrate how the shock position of TSE is influenced by the exit pressure. The second series oblique shock wave position's distance decreases with the increasing exit pressure, which affects the mixing phenomenon in the stage I and II entrainment zone. Still, the entrainment ratio stays constant up to the on-design departure pressure ($P_e = 0.01925$ bar). Beyond this threshold, the entrainment ratio decreases as exit pressure rises. This study, which relies solely on the CRMC theory, identified the critical exit pressure as the on-design condition for the two-stage ejector through analytical analysis. Numerical analysis confirms that the ejector performs optimally under double choked conditions (primary and secondary flow) at the on-design exit pressure.

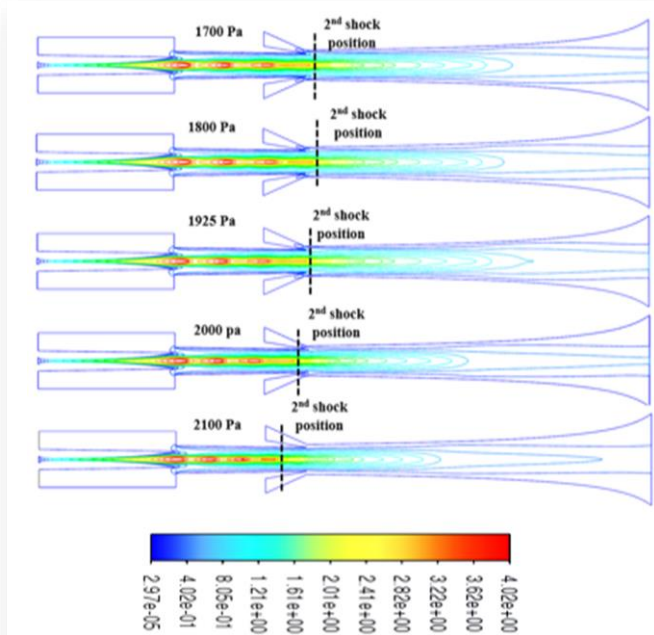


Fig. 15. Impact of exit pressure on secondary shock position.

The placement of the nozzle exit is a critical geometric factor for two-stage ejectors as it dictates the expansion angles of the jet and the converging passageway that induces flow in the entrainment region. In the two-stage ejector system, the entrainment area is not explicitly calculated through analytical modeling. To assess the impact of the nozzle exit position, the varia-

tion was tested comparative to the on-design situation ($NXP = 0$) in both the upstream (-4 mm) and downstream ($+3$ mm) directions of the flow at 1 mm intervals (refer to Fig. 16). The numerical results demonstrate that the entrainment ratio increases up to -2 mm when moving away from $NXP = 0$ in the upstream direction, followed by a decrease. Conversely, moving from $NXP = 0$ into the mixing area when going downstream decreases the entrainment. As a whole, it is clear that adjusting the nozzle exit location away or towards the mixing area from $NXP=0$ results in a significant alteration of the entrainment ratio.

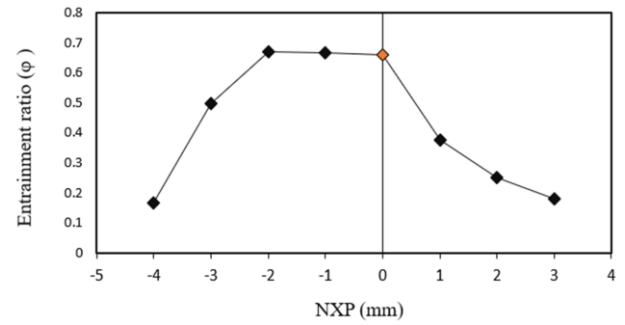


Fig. 16. Effect of entrainment ratio (ϕ) on nozzle exit position (NXP).

By altering the expansion of the jet and induced flow pathways, entrainment varies with NXP. In the mixing portion, moving downstream lowers the induced flow passage and expansion angle, which lessens entrainment. Furthermore, the momentum of motive flow responsible for induced secondary flow decreases with the increasing distance from the suction port because of positive NXPs. On the other hand, ejector entrainment performance is enhanced when positioned upstream (up to -2 mm from the mixing plane inlet), which increases the expansion angle and induced flow passage. The findings of the CFD and 1-D analyses are compared in Fig. 17.

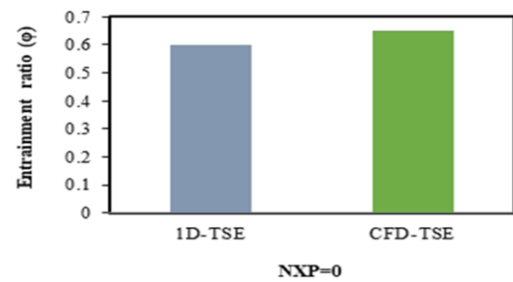


Fig. 17. Comparison of 1-D and CFD entrainment ratios (on-design).

6. Conclusions

The physics based two-stage ejector was designed and validated using on-design CFD results. Operational characteristics like the main fluid pressure, exit fluid back pressure, and nozzle exit location were also subjected to numerical analysis. It has been examined how operating parameters affect the entrainment ratio. The following is a list of the study's findings:

- The flow characteristic inside the two-stage ejector plays a very important role in identifying the oblique shock positions, diamond type wave, effective area and shear layer, and in developing a high performance ejector.
- Mach number contours are taken into consideration in order to forecast the entrainment effect and the flow behaviour inside the ejector.
- The CFD results of static pressure variation closely match the 1D on design ejector obtained results.
- The entrainment ratio increases with the increase in primary pressure up to the on-design pressure. Beyond this value, the entrainment ratio starts to fall if the exit pressure increases.

References

- [1] Ma, X., Sun, W., Ma, S., Zhang, H., Zhang, Y., Liu, C., Jia, L., & Xue, H. (2023). A streamlining method for ejector nozzle profile optimization based on polynomial function. *Applied Thermal Engineering*, 219, A, 119380. doi: 10.1016/j.applthermaleng.2022.119380
- [2] Muzaber, A., Bassmaji, N., & Kaddah, A. (2024). A new mixing chamber geometry design for supersonic ejector performance optimization using computational fluid dynamics. *International Journal of Thermofluids*, 101047. doi: 10.1016/j.ijft.2024.101047
- [3] Zheng, L., Wang, W., Zhang, Y., Wang, L., & Lu, W. (2022). Optimization Analysis of the Mixing Chamber and Diffuser of Ejector Based on Fano Flow Model. *Computer Modelling in Engineering & Sciences*, 133(1), 153–170. doi: 10.32604/cmescs.2022.021235.
- [4] Yan, J., Li, Z., & Zhang, H. (2023). Investigation on key geometries optimization and effect of variable operating conditions of a transcritical R744 two-phase ejector. *Applied Thermal Engineering*, 230, 120733. doi: 10.1016/j.applthermaleng.2023.120733
- [5] Kumar, V., Singhal, G., & Subbarao, P.M.V. (2013). Study of supersonic flow in a constant rate of momentum change (CRMC) ejector with frictional effects. *Applied Thermal Engineering*, 60(1–2), 61–71. doi: 10.1016/j.applthermaleng.2013.06.045
- [6] Seth, B., Knecht, S., Szalai, M., & Haußmann, J. (2025). Design of a variable passive ejector for hydrogen recirculation of a PEM fuel cell system. *International Journal of Hydrogen Energy*, 99, 956–965. doi: 10.1016/j.ijhydene.2024.12.091
- [7] Khennich, M., Galanis, N., Sorin, M., Orfi, J., Sarrafi, A., & Dupuis, S. (2024). Thermodynamic analysis of a solar powered ejector cooling system. *International Journal of Thermofluids*, 24, 100960. doi: 10.1016/j.ijft.2024.100960
- [8] Kanbur, B.B., Kriezi, E.E., Markussen, W.B., Kærn, M.R., Busch, A., & Kristófersson, J. (2025). Framework for prediction of two-phase R-744 ejector performance based on integration of thermodynamic models with multiphase mixture CFD simulations. *Applied Thermal Engineering*, 258, C, 124888. doi: 10.1016/j.applthermaleng.2024.124888
- [9] Ogaili, H.H., Khalilarya, S., Chitsaz, A., & Mojaver, P. (2025). Energy, exergy, and economic performance analysis of integrated parabolic trough collector with organic Rankine cycle and ejector refrigeration cycle. *Energy Conversion and Management: X*, 25, 100843. doi: 10.1016/j.ecmx.2024.100843
- [10] Fabris, F., Bodys, J., Marinetti, S., Minetto, S., Smolka, J., & Rossetti, A. (2024). Numerical modelling of a single-compression multi-temperature ejector-supported R744 refrigeration unit for last mile delivery. *International Journal of Refrigeration*, 160, 65–75. doi: 10.1016/j.ijrefrig.2024.01.014
- [11] Gagan, J., Śmierciew, K., Pawluczuk, A., Łukaszuk, M., Dudar, A., & Butrymowicz, D. (2025). The First experimental tests of novel bivalent hybrid ejector-compressor refrigeration system. *Energy*, 317, 134646. doi: 10.1016/j.energy.2025.134646
- [12] Fingas, R., Haida, M., Smolka, J., Croci, L., Bodys, J., Palacz, M., & Besagni, G. (2025). Experimental analysis of the R290 variable geometry ejector with a spindle. *Applied Thermal Engineering*, 258, C, 124632. doi: 10.1016/j.applthermaleng.2024.124632
- [13] Paul, S., Srikar Reddy, P., Rao, S.M., & Kumar, P. (2025). Surrogate model based multi-objective optimisation of supercritical CO₂ ejectors. *The Journal of Supercritical Fluids*, 218, 106493. doi: 10.1016/j.supflu.2024.106493
- [14] Siddique, S.M., Uzzaman, M.M., Ehsan, M.M., & Khan, Y. (2025). Thermal analysis of a novel configuration of double effect absorption system cascaded with ejector and injection enhanced compression refrigeration cycle. *Energy Conversion and Management*, 326, 119435. doi: 10.1016/j.enconman.2024.119435
- [15] Li, M., Xu, L., & Gong, H. (2025). Examination of an Ejector-Enhanced Geothermal Plant for Power Generation Improvement. *Renewable Energy*, 242, 122400. doi: 10.1016/j.renene.2025.122400
- [16] Kong, F., & Kim, H.D. (2015). Analytical and computational studies on the performance of a two-stage ejector–diffuser system. *International Journal of Heat and Mass Transfer*, 85, 71–87. doi: 10.1016/j.ijheatmasstransfer.2015.01.117
- [17] Ding, Z., Wang, L., Zhao, H., Zhang, H., & Wang, C. (2016). Numerical study and design of a two-stage ejector for subzero refrigeration. *Applied Thermal Engineering*, 108, 436–448. doi: 10.1016/j.applthermaleng.2016.07.104
- [18] Kong, F., & Kim, H.D. (2016). Optimization study of a two-stage ejector–diffuser system. *International Journal of Heat and Mass Transfer*, 101, 1151–1162. doi: 10.1016/j.ijheatmasstransfer.2016.05.129
- [19] Chen, G., Zhang, R., Zhu, D., Chen, S., Fang, L., & Hao, X. (2017). Experimental study on two-stage ejector refrigeration system driven by two heat sources. *International Journal of Refrigeration*, 74, 295–303. doi: 10.1016/j.ijrefrig.2016.10.010
- [20] Yan, J., Wen, N., Wang, L., Li, X., Liu, Z., & Li, S. (2018). Optimization on ejector key geometries of a two-stage ejector-based multi-evaporator refrigeration system. *Energy Conversion and Management*, 175, 142–150. doi: 10.1016/j.enconman.2018.08.110
- [21] Baek, S., & Song, S. (2018). Numerical study for the design optimization of a two-phase ejector with R134a refrigerant. *Journal of Mechanical Science and Technology*, 32, 4231–4236.
- [22] Aidoun, Z., Ameer, K., Falsafioon, M., & Badache, M. (2019). Current advances in ejector modeling, experimentation and applications for refrigeration and heat pumps. Part 2: Two-phase ejectors. *Inventions*, 4(1), 16. doi: 10.3390/inventions4010016
- [23] Suvarnakuta, N., Pianthong, K., Sriveerakul, T., & Seehanam, W. (2020). Performance analysis of a two-stage ejector in an ejector refrigeration system using computational fluid dynamics. *Engineering Applications of Computational Fluid Mechanics*, 14(1), 669–682. doi: 10.1080/19942060.2020.1756913
- [24] Yadav, S.K., Pandey, K.M., Kumar, V., & Gupta, R. (2021). Computational analysis of a supersonic two-stage ejector. *Materials Today: Proceedings*, 38, 2700–2705. doi: 10.1016/j.matpr.2020.08.483
- [25] Bai, T., Lu, Y., Yan, G., & Yu, J. (2021). Performance analysis of an ejector enhanced two-stage auto-cascade refrigeration cycle

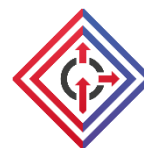
- for low temperature freezer. *Journal of Thermal Science*, 30, 2015–2026. doi: 10.1007/s11630-020-1290-6
- [26] Kumar, A., Kumar, V., Subbarao, P.M.V., Yadav, S.K., & Singhal, G. (2022). Numerical assessment on the performance of variable area single-and two-stage ejectors: A comparative study. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, 236, 1, 114–125. doi: 10.1177/09544089211033129
- [27] Wang, X., Wang, L., Song, Y., Deng, J., & Zhan, Y. (2021). Optimal design of two-stage ejector for subzero refrigeration system on fishing vessel. *Applied Thermal Engineering*, 187, 116565. doi: 10.1016/j.applthermaleng.2021.116565
- [28] Yang, D., Li, Y., Xie, J., & Wang, J. (2022). Exergy destruction characteristics of a transcritical carbon dioxide two-stage compression/ejector refrigeration system for low-temperature cold storage. *Energy Reports*, 8, 8546–8562. doi: 10.1016/j.egyr.2022.06.066
- [29] Chen, L., Xu, K., Yang, Z., Yan, Z., & Dong, Z. (2022). Optimal design and operation of dual-ejector PEMFC hydrogen supply and circulation system. *Energies*, 15(15), 5427. doi: 10.3390/en15155427
- [30] Ringstad, K.E., Banasiak, K., Ervik, Å., & Hafner, A. (2022). Swirl-Bypass Nozzle for CO₂ Two-Phase Ejectors: Numerical Design Exploration. *Energies*, 15(18), 6765. doi: 10.3390/en15186765
- [31] Udroui, C.M., Mota-Babiloni, A., Giménez-Prades, P., Barragán-Cervera, Á., & Navarro-Esbrí, J. (2023). Two-stage cascade configurations based on ejectors for ultra-low temperature refrigeration with natural refrigerants. *International Journal of Thermofluids*, 17, 100287. doi: 10.1016/j.ijft.2023.100287
- [32] Suvarnakuta, N., Pianthong, K., Sriveerakul, T., & Seehanam, W. (2020). Performance analysis of a two-stage ejector in an ejector refrigeration system using computational fluid dynamics. *Engineering Applications of Computational Fluid Mechanics*, 14(1), 669–682. doi: 10.1080/19942060.2020.1756913
- [33] Ünal, Ş., Erdiñç, M.T., Akgün, H., & Bilgili, M. (2023). Effects of alternative refrigerants on the ejector dimensions for single and dual ejectors enhanced bus air conditioning system. *International Communications in Heat and Mass Transfer*, 143, 106685. doi: 10.1016/j.icheatmasstransfer.2023.106685
- [34] Pang, J., Ge, Z., Zhang, Y., & Du, X. (2023). Comparative designs and optimizations of the steam ejector for a CHP system. *Applied Thermal Engineering*, 226, 120345. doi: 10.1016/j.applthermaleng.2023.120345
- [35] Han, Q., Liu, C., Xue, H., Zhang, H., Sun, W., Sun, W., & Jiaa, L. (2023). Working condition expansion and performance optimization of two-stage ejector based on optimal switching strategy. *Energy*, 282, 128376. doi: 10.1016/j.energy.2023.128376
- [36] Yadav, S.K., Pandey, K.M., Gupta, R., & Kumar, V. (2021). Numerical study for the influences of nozzle exit position, mixing, and diffuser section lengths on performance of CRMC ejector. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 43(496), 1–14. doi: 10.1007/s40430-021-03207-x
- [37] Ariaifar, K., Buttsworth, D., Al-Doori, G., & Malpress, R. (2015). Effect of mixing on the performance of wet steam ejectors. *Energy*, 93, 2030–2041. doi: 10.1016/j.energy.2015.10.082



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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Hybrid nanofluid past time-dependent radially stretched sheet with Dufour and Soret effects

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Received: 11.12.2024; revised: 01.08.2025; accepted: 03.08.2025

Abstract

The steady laminar flow past a time-dependent radially stretching sheet with Soret and Dufour effects within a hybrid nanofluid is studied. The governing equations are transformed into ordinary differential equations utilising the similarity transformations. Successive linearization is employed to linearise the nonlinear system of equations. The resultant system of equations is solved using the Chebyshev collocation method. Plots of the velocity, temperature, and concentration for chosen parameters are displayed in conjunction with the Sherwood number, Nusselt number, and coefficient of skin friction. As the volume fraction of copper (Cu) nanoparticles increases, the important values of these variables decrease, while increasing the amount of alumina (Al_2O_3) nanoparticles causes them to rise. The hybrid nanofluid demonstrates a faster heat transfer rate than the nanofluid on the radially stretched surface. Additionally, it has been discovered that increasing the volume fractions of copper (Cu) nanoparticles minimizes the coefficient of skin friction, the Sherwood number, and the Nusselt number for the stretching surface, while increasing the volume fractions of alumina (Al_2O_3) nanoparticles boosts the skin friction coefficient, the Sherwood number, and lowers the Nusselt number. Furthermore, increasing the Dufour number maintains the Sherwood number at a constant level while decreasing the Nusselt number; in contrast, enhancing the Soret number decreases the Sherwood number and increases the Nusselt number.

Keywords: Hybrid nanofluid; Radially stretching sheet; Successive linearisation method; Dufour and Soret effects

Vol. 46(2025), No. 4, 19–30; doi: 10.24425/ather.2025.156595

Cite this manuscript as: Srinivasacharya, D., & Sreekantha Reddy, M.V. (2025). Hybrid nanofluid past time-dependent radially stretched sheet with Dufour and Soret effects. *Archives of Thermodynamics*, 46(4), 19–30.

1. Introduction

Fluid flow across a sheet that is stretched radially is a significant subject in chemical, metallurgical, and biological engineering applications. It also plays a key role in wire drawing, hot rolling, polymer extrusion, metal spinning, and liquid metal processing. Numerous researchers have examined the features of mass and heat transmission in Newtonian and non-Newtonian fluid flow. Khan et al. [1] addressed the heat transfer and axisymmetric

flow of a cross fluid on a radially stretched sheet. Ahmed et al. [2] investigated the effects of radiation and an angled magnetic field on the Sisko fluid across a sheet that is stretched radially. Utilising the homotopy analysis approach, Sreelakshmi et al. [3] explored the time-dependent Jeffrey nanofluid flow across a surface that is expanding radially by employing convective boundary conditions. Khan et al. [4] investigated the flow across a stretched sheet involving several slips, radiation, chemical reactions, and magnetic impacts. Nayak et al. [5] evaluated the

Nomenclature

A – unsteadiness parameter, (c/a)
 $a, b, c, d (\geq 0)$ – constants
 $C(r, z, t)$ – concentration, mol
 C_∞ – concentration of the surrounding medium, mol
 C_f – coefficient of skin friction
 C_p – specific heat at constant pressure, $J/(kg\ K)$
 C_s – concentration susceptibility of the hybrid nanofluid
 $C_w (C_w > C_\infty)$ – concentration at the stretched surface, mol
 Df – Dufour number $\left(\frac{D_m K_t (C_w - C_\infty)}{C_s C_p \nu (T_w - T_\infty)} \right)$
 D_m – mass diffusivity, (m^2/s)
 $f'(\eta)$ – dimensionless velocity
 K_T – thermal diffusion ratio, $1/K$
 k_{hnf} – thermal conductivity of the hybrid nanofluid, $W/(m\ K)$
 Nu – Nusselt number
 Pr – Prandtl number, (ν/α_f)
 r, z – radial and axial coordinate axes, m
 Re – local Reynolds number, (rU_w/ν)
 Sh – Sherwood number
 Sc – Schmidt number (ν/D_m)
 Sr – Soret number $\left(\frac{D_m K_t (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)} \right)$
 $T_\infty (T_w > T_\infty)$ – temperature of the surrounding medium, K

$T(r, z, t)$ – temperature, K
 T_m – mean fluid temperature, K
 T_w – temperature at the stretched surface, K
 $u(r, z, t), w(r, z, t)$ – velocity components in r and z direction, m/s
 U_w – time-dependent stretching velocity, m/s
 W_0 – suction/injection velocity

Greek symbols

α – thermal diffusivity, m^2/s
 μ – dynamic viscosity, Pa-s
 ν – kinematic viscosity, m^2/s
 ρ – density, kg/m^3
 $\psi(r, z)$ – stream function

Subscripts and Superscripts

$n1$ – for the Al_2O_3 nanoparticles
 $n2$ – for the Cu nanoparticles
 f – for the fluid
 hnf – for hybrid nanofluid

Abbreviations and Acronyms

CNT – carbon nanotube
 MHD – magnetohydrodynamics
 SLM – successive linearisation method

influence of a chemical reaction on mass transfer flow across a radially elongated sheet. The majority of research on the radially expanded surface focuses on steady-state conditions. On the other hand, unsteadiness becomes a crucial subject of analysis in many engineering processes. Shahzad et al. [6,7] analysed the effect of axially symmetrical flow and heat exchange on a time dependent radially stretching surface. Srinivasacharya and Shravan Kumar [8] used artificial neural networks to investigate the movement of Casson fluid across an unsteady sheet that is expanding radially under Soret and Dufour's influence.

Further, simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications. Chamkha and Khaled [9] considered simultaneous heat and mass transfer of an electrically-conducting fluid by mixed convection in a stagnation flow over a flat plate embedded in a porous medium in the presence of wall blowing or suction, magnetic field effects, and temperature-dependent heat generation or absorption effects. Krishna et al. [10] studied the effects of heat and mass transfer on free convective flow of micropolar fluid over an infinite vertical porous plate in the presence of an inclined magnetic field with a constant suction velocity and taking Hall current into account. Krishna et al. [11] explored the Hall and ion slip impacts on an unsteady laminar magnetohydrodynamics (MHD) convective rotating flow of heat generating or absorbing second grade fluid over a semi-infinite vertical moving permeable surface.

The topic of heat transfer improvement has gained lots of attention in the last several years. Since nanoparticles have a higher thermal conductivity than base fluid, thermal scientists have proposed including metallic or non-metallic nanoparticles in the base fluid to boost its thermal conductivity. A nanofluid is the resultant blend with enhanced chemical and physical characteristics. In 1995, Choi and Eastman [12] coined the term "nanofluid." However, to accelerate heat transmission, research-

ers are looking into hybrid nanofluids, a new kind of nanofluids. Ordinary nanofluid contains only one nanoparticle in the base fluid, whereas hybrid nanofluid is considered a superior nanofluid, consisting of two distinct nanoparticles. Hybrid nanofluid provides better thermophysical qualities, thereby enhancing heat transfer efficiency. Other than the atypically high effective thermal conductivity, hybrid nanofluid can provide great benefits when the nanosized particles are adequately dispersed. Hybrid nanofluid has been introduced to investigate the properties of heat transfer and make it even better. Heat transfer in the fields of microelectronics, microfluidics, manufacturing, and medicine are a few of the applications for hybrid nanofluids. Several researchers have analysed the heat transfer and boundary layer flow of hybrid nanofluids. Devi and Devi [13,14] examined Cu nanoparticles regardless of magnetic effects when studying the motion of a hybrid nanofluid across a stretched surface. They discovered that, compared to ordinary nanofluid, hybrid nanofluid increases the heat transmission rate. Recently, many researchers, including Hayat and Nadeem [15], Yousefi et al. [16], Subhani and Nadeem [17], Ghadikolaei et al. [18], Usman et al. [19], Rostami et al. [20], and Waini et al. [21,22] have analysed the heat and mass transfer properties in a hybrid nanofluid flow.

Gangadhar et al. [23] studied the impact of convective heating boundary on a second-grade nanofluid flow alongside a Riga pattern using a Grinberg term and a Lorentz force applied parallel to the wall of the Riga plate. Ontela et al. [24] studied the behaviour of a conducting hybrid nanofluid comprised of CuO and Cu nanoparticles over a nonlinear stretching surface filled with a porous matrix. Panda et al. [25] investigated the optimisation of heat transportation rate in a nonlinear radiative buoyancy-driven hydromagnetic carbon nanotube (CNT) hybrid nanofluid flow.

Pattnaik et al. [26] analysed the convective transport of radiative blood flow properties in the context of a heated stretch-

ing/shrinking conducting Riga sensor plate. Panda et al. [27] considered the influence of radiating heat on the flow of Powell-Eyring fluid over a Riga plate subjected to thermal stratification, utilising the Cattaneo-Christov heat flux model.

The Dufour and Soret effects in hybrid nanofluids have been the subject of extensive research in recent years. Mass and heat transmission in a fluid flow are referred to as Dufour and Soret effects. In addition to mass transfer (the Soret effect), the temperature gradient produces variations in concentration. These two effects, Dufour and Soret, are simultaneous coupling processes. Nisar et al. [28] studied the effects of Dufour and Soret in the flow of hybrid nanofluids using numerical methods. Abad et al. [29] have documented their exploration of the implications of the stagnation point on a hybrid nanofluid based on water, in which alumina and Cu are the nanoparticles. Hayat et al. [30] examined the consequences of Soret and Dufour numbers on the magnetohydrodynamic movement of the Casson fluid across an extended surface. Kameswaran et al. [31] addressed the stagnation point flow of a Casson fluid along a stretched sheet in the presence of Dufour and Soret effects.

Many investigators have analysed the Soret and Dufour effects on the flow of hybrid nanofluid in various geometries. Isa et al. [32] researched the consequence of the Soret and Dufour effects on the flow of a hybrid nanofluid made of copper and alumina nanoparticles in water. Prathiba et al. [33] explored the Soret and Dufour effects over a rotating hybrid nanofluid. In the context of Dufour and Soret effects, Arif et al. [34] assessed numerically the simultaneous transmission of mass and heat in a Maxwell hybrid nanofluid. Sharma et al. [35] performed a comparative analysis of the Ag-CuO/H₂O hybrid nanofluid flow across an inclined stretched surface incorporating Dufour and Soret effects. Vijay et al. [36] evaluated magnetohydrodynamic hybrid nanofluid flow on a revolving, slowing-down disk using Dufour and Soret effects.

From the available literature, the flow of hybrid nanofluid over radially shrinking and stretching sheets is extremely rare, and none of the published articles consider separately the effect of Dufour and Soret parameters on such a fluid stream. Most of the investigations on the radially stretching sheet are restricted to steady-state. However, when the flow depends on time, unsteadiness becomes an important component of study in a variety of engineering processes, e.g. in solar energy, refrigeration, heating, ventilation, and air conditioning applications. Hence, to bridge the gap, this paper investigates the hybrid nanofluid flow behaviour and heat transmission across the time-dependent radially extending surface with Soret and Dufour effects utilising the Tiwari and Das nanofluid model. Hybrid nanofluid is generated by suspending alumina and Cu nanoparticles in water.

In the present study, the aluminium and copper nanoparticles are chosen because of their unique thermal properties. The combination of aluminium oxide and copper helps to maintain nanofluid consistency over time while also giving strong thermal characteristics. Cu – Al₂O₃: H₂O is one of the attractive choices for improved heat transfer in industries like electronics cooling, power generation, and chemical processing. As copper is one of the most thermally conductive metals, it enhances the thermal conductivity significantly, and Al₂O₃ adds stability and strength. The data of the current analysis is compared with the numerical results to verify the findings.

2. Problem formulation

The unsteady and laminar flow of a hybrid nanofluid over a radially stretching surface with Soret and Dufour effects is considered here. Figure 1 depicts the coordinate system and geometry. The flow is a consequence of the radial stretching of the surface, which has a time-dependent velocity: $U_w = ar/(1-ct)$. Let $C_w = C_\infty + dr/(1-ct)$ and $T_w = T_\infty + br/(1-ct)$ indicate the concentration and the temperature at the stretched surface, respectively. Here, C_∞ ($C_w > C_\infty$) and T_∞ ($T_w > T_\infty$) are the concentration and temperature of the surrounding medium, with $ct < 1$ and $a, b, c, d (\geq 0)$ are constants. ∞

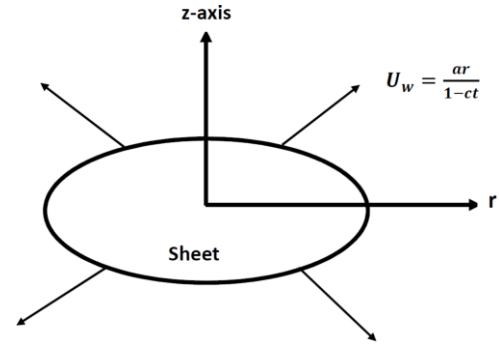


Fig. 1. Coordinate system and geometry.

The equations that govern the flow are [37–39]:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = \frac{\mu_{nnf}}{\rho_{nnf}} \frac{\partial^2 u}{\partial z^2}, \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{K_{nnf}}{(\rho C_p)_{nnf}} \frac{\partial^2 T}{\partial z^2} + \frac{D_{mK_T}}{C_{SCP}} \frac{\partial^2 C}{\partial z^2}, \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} = D_m \frac{\partial^2 C}{\partial z^2} + \frac{D_{mK_T}}{T_m} \frac{\partial^2 T}{\partial z^2}. \quad (4)$$

Following Devi and Devi [13,14], and Oztop and Abu-Nada [40], fluid and hybrid nanofluid thermophysical properties are assessed using the equations demonstrated in Table 1.

Table 1. Thermophysical characteristics of hybrid nanofluid and fluid.

Property	Fluid	Hybrid nanofluid [13,14,40]
Heat capacity	$(\rho C_p)_f$	$(\rho C_p)_{nnf} = (1 - \phi_2) \times [(1 - \phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{n1}] + \phi_2(\rho C_p)_{n2}$
Dynamic viscosity	μ_f	$\mu_{nnf} = \left[\frac{\mu_f}{((1 - \phi_1)(1 - \phi_2))^{2.5}} \right]$
Density	ρ_f	$\rho_{nnf} = \phi_2 \rho_{n2} + (1 - \phi_2)[(1 - \phi_1)\rho_f + \phi_1 \rho_{n1}]$
Thermal conductivity	k_f	$\frac{k_{nnf}}{k_{nf}} = \left[\frac{k_{n2} - 2\phi_2(k_{nf} - k_{n2} + 2k_{nf})}{k_{n2} + \phi_2(k_{nf} - k_{n2}) + 2k_{nf}} \right]$ where: $\frac{k_{nf}}{k_f} = \left[\frac{k_{n1} - 2\phi_1(k_f - k_{n1}) + 2k_f}{k_{n1} + \phi_1(k_f - k_{n1}) + 2k_f} \right]$

Table 2 gives the same information as Oztop and Abu-Nada [40] regarding the base fluid and nanoparticles. In this case, the subscripts $n1$, $n2$, f , and hnf stand for the Al_2O_3 , Cu nanoparticles solid components, fluid, and hybrid nanofluid.

Table 2. Fluid thermophysical characteristics and nanoparticles [40].

Characteristics	Cu	Al_2O_3	Water
$k, W/(m \ K)$	400	40	0.613
$C_p, J/(kg \ K)$	385	765	4179
$\rho, kg/m^3$	8933	3970	997.1

The boundary conditions on the surface are:

$$\begin{aligned} w &= W_0, \quad u = U_w = \frac{ra}{1-tc}, \quad T_w = T_\infty + \frac{rb}{1-tc}, \\ C_w &= C_\infty + \frac{rd}{1-tc}atz = 0, \\ u &\rightarrow 0, \quad C \rightarrow C_\infty, \quad T \rightarrow T_\infty asz \rightarrow \infty, \end{aligned} \quad (5)$$

where U_w is the velocity of stretching and $W_0 = -2S \left[\frac{\nu U_w}{r} \right]^{\frac{1}{2}}$. Here on the surface, $W_0 < 0$ ($S > 0$) signifies suction and $W_0 > 0$ ($S < 0$) signifies injection.

The similarity transformations for the present problem are given by:

$$\begin{aligned} \psi(r, z) &= -r^2 U_w Re^{-1/2} f(\eta), \quad \eta = \frac{z}{r} Re^{1/2}, \\ \theta &= \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \end{aligned} \quad (6)$$

where $\psi(r, z)$ represents the stream function defined by $u = -\frac{1}{r} \frac{\partial \psi}{\partial z}$ and $w = \frac{1}{r} \frac{\partial \psi}{\partial r}$. $Re = \frac{r U_w}{\nu}$ denotes the local Reynolds number.

Substituting Eq. (6) into Eqs. (1)–(4), leads to the dimensionless equations shown below:

$$\left[\frac{\mu_{hnf} \rho_f}{\rho_{hnf} \mu_f} \right] f''' + 2f f'' - f'^2 - A \left[\frac{1}{2} \eta f'' + f' \right] = 0, \quad (7)$$

$$\begin{aligned} \left[\frac{1}{Pr} \frac{K_{hnf}(\rho C_p)_f}{(\rho C_p)_{hnf} K_f} \right] \theta'' + Df \cdot \phi'' + 2f \theta' + \\ -f' \theta - A \left[\frac{1}{2} \eta \theta' + \theta \right] = 0, \end{aligned} \quad (8)$$

$$Sr \cdot \theta'' + \frac{1}{Sc} \phi'' + 2f \phi' - f' \phi - A \left[\phi + \frac{1}{2} \eta \phi' \right] = 0. \quad (9)$$

The accompanying boundary conditions are changed to the subsequent form:
at $\eta = 0$:

$$\theta(0) = 1, \quad f'(0) = 1, \quad f(0) = S, \quad \phi(0) = 1, \quad (10a)$$

as $\eta \rightarrow \infty$:

$$\theta(\eta) \rightarrow 0, \quad f'(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0. \quad (10b)$$

The mass transfer rate (Sherwood number Sh), the heat transfer rate (Nusselt number Nu), and the coefficient of skin friction C_f are defined by:

$$\begin{aligned} Sh &= -\frac{r}{(C_w - C_\infty)} \left(\frac{\partial C}{\partial z} \right)_{z=0}, \quad C_f = \frac{2}{\rho U_w^2} \mu \left(\frac{\partial u}{\partial z} \right)_{z=0}, \\ Nu &= -\frac{r}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial z} \right)_{z=0}. \end{aligned} \quad (11)$$

The non-dimensional form of Sherwood number Sh , skin friction coefficient C_f , and Nusselt number Nu are:

$$\begin{aligned} Re^{-\frac{1}{2}} Sh &= -\phi'(0), \quad \frac{1}{2} Re^{\frac{1}{2}} C_f = f''(0), \\ Re^{-\frac{1}{2}} Nu &= -\theta'(0). \end{aligned} \quad (12)$$

3. Method of solution

The basic principle of the successive linearisation method (SLM) is the assumption that the functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ can be interpreted as:

$$\begin{aligned} f(\eta) &= f_k(\eta) + \sum_{m=0}^{k-1} f_m(\eta), \\ \theta(\eta) &= \theta_k(\eta) + \sum_{m=0}^{k-1} \theta_m(\eta), \\ \phi(\eta) &= \phi_k(\eta) + \sum_{m=0}^{k-1} \phi_m(\eta). \end{aligned} \quad (13)$$

Here, f_k , θ_k and ϕ_k ($k = 1, 2, 3, \dots$) are unknown functions and θ_m , f_m , and ϕ_m ($m \geq 1$) are the approximations that can be acquired repeatedly by computing the solution of the linear portion of the equations that arises from putting Eq. (13) into Eqs. (7)–(9). The basic principle concerning SLM is that when k grows in size, f_k , θ_k and ϕ_k become progressively smaller and thus ignored, i.e.

$$\lim_{k \rightarrow \infty} f_k = \lim_{k \rightarrow \infty} \theta_k = \lim_{k \rightarrow \infty} \phi_k = 0. \quad (14)$$

The initial assumptions for $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$ are made to fulfil the boundary conditions. The differential equations for f_k , θ_k and ϕ_k ($k \geq 1$) are found by substituting Eq. (13) into Eqs. (7)–(9) and then taking only the linear terms.

$$a_1 f_k''' + a_2 f_k'' + a_3 f_k' + a_4 f_k = r_1, \quad (15)$$

$$b_1 f_k' + b_2 f_k + b_3 \theta_k'' + b_4 \theta_k' + b_5 \theta_k + Df \cdot \phi_k'' = r_2, \quad (16)$$

$$c_1 f_k' + c_2 f_k + Sr \cdot \theta_k'' + \frac{1}{Sc} \phi_k'' + c_3 \phi_k' + c_4 \phi_k = r_3. \quad (17)$$

The linearised boundary conditions are:

$$\begin{aligned} \phi_k(0) &= \phi_k(\infty) = 0, \\ f_k(0) &= f_k'(0) = f_k'(\infty) = 0, \\ \theta_k(0) &= \theta_k(\infty) = 0, \end{aligned} \quad (18)$$

where the coefficients a_q ($q = 1, 2, 3, 4$), b_z ($z = 1, 2, 3, 4, 5$), c_n ($n = 1, 2, 3, 4$), and r_p ($p = 1, 2, 3$) are:

$$a_1 = \frac{\mu_{hnf} \rho_f}{\rho_{hnf} \mu_f}, \quad a_2 = 2 \sum_{m=0}^{k-1} f_m' - \frac{A}{2} \eta,$$

$$a_3 = -[2 \sum_{m=0}^{k-1} f_m' + A], \quad a_4 = 2 \sum_{m=0}^{k-1} f_m'',$$

$$\begin{aligned} r_1 &= -a_1 \sum_{m=0}^{k-1} f_m''' + \left[\frac{A}{2} \eta - 2 \sum_{m=0}^{k-1} f_m' \right] \sum_{m=0}^{k-1} f_m'' + \\ &+ (\sum_{m=0}^{k-1} f_m')^2 + A \sum_{m=0}^{k-1} f_m', \end{aligned}$$

$$b_1 = -\sum_{m=0}^{k-1} \theta_m, \quad b_2 = 2 \sum_{m=0}^{k-1} \theta'_m, \quad b_3 = \frac{1}{\text{Pr}} \frac{K_{hnf}(\rho C_p)_f}{(\rho C_p)_{hnf} K_f},$$

$$b_4 = 2 \sum_{m=0}^{k-1} f_m - \frac{A}{2} \eta, \quad b_5 = -[A + \sum_{m=0}^{k-1} f'_m],$$

$$r_2 = -b_3 \sum_{m=0}^{k-1} \theta''_m + \left[\frac{A}{2} \eta - 2 \sum_{m=0}^{k-1} f_m \right] \sum_{m=0}^{k-1} \theta'_m + (\sum_{m=0}^{k-1} f'_m + A) \sum_{m=0}^{k-1} \theta_m - \text{Df} \sum_{m=0}^{k-1} \phi''_m,$$

$$c_1 = -\sum_{m=0}^{k-1} \phi_m, \quad c_2 = 2 \sum_{m=0}^{k-1} \phi'_m, \quad ,$$

$$c_3 = 2 \sum_{m=0}^{k-1} f_m - \frac{A}{2}, \quad c_4 = -[A + \sum_{m=0}^{k-1} f'_m],$$

$$r_3 = -\frac{1}{\text{Sc}} \sum_{m=0}^{k-1} \phi''_m + \left[\frac{A}{2} \eta - 2 \sum_{m=0}^{k-1} f_m \right] \sum_{m=0}^{k-1} \phi'_m + (\sum_{m=0}^{k-1} f'_m + A) \sum_{m=0}^{k-1} \phi_m - \text{Sr} \sum_{m=0}^{k-1} \theta''_m.$$

After calculating each solution for f_k , ϕ_k and θ_k ($k \geq 1$) by repeatedly solving Eqs. (15)–(18) the following are the approximate values for $f(\eta)$, $\phi(\eta)$ and $\theta(\eta)$:

$$\begin{aligned} f(\eta) &\approx \sum_{s=0}^S f_s(\eta), \\ \phi(\eta) &\approx \sum_{s=0}^S \phi_s(\eta), \\ \theta(\eta) &\approx \sum_{s=0}^S \theta_s(\eta), \end{aligned} \quad (19)$$

where S signifies the SLM approximation order.

The solution to the system of Eqs. (15)–(16) may be computed analytically (if possible) or numerically using any methods, most notably collocation methods, finite differences, shooting technique based on Runge-Kutta, and finite elements. This study uses the spectral collocation method using Chebyshev polynomials. This technique is based on using the Chebyshev interpolating polynomials to approximate the unknown functions in a way that allows them to collocate at the Gauss-Lobatto points:

$$\xi_k = \cos \frac{\pi k}{M}, \quad k = 0, 1, \dots, M, \quad (20)$$

where $M + 1$ collocation points are utilised. To put the method into practice, the domain truncation technique is used to change the physical domain into the domain $[0, L]$. The domain is changed to $[-1, 1]$ using the following mapping:

$$\frac{\eta}{L} = \frac{\xi + 1}{2}, \quad -1 \leq \xi \leq 1, \quad (21)$$

where L stands for the scaling value at infinity that activates the boundary condition. At the collocation points, the undefined functions f_k , θ_k and ϕ_k are approximated by the Chebyshev interpolating polynomials:

$$\begin{aligned} f_k(\xi) &\approx \sum_{n=0}^M f_k(\xi_n) T_n(\xi_k), \\ \theta_k(\xi) &\approx \sum_{n=0}^M \theta_k(\xi_n) T_n(\xi_k), \\ \phi_k(\xi) &\approx \sum_{n=0}^M \phi_k(\xi_n) T_n(\xi_k), \end{aligned} \quad (22)$$

where $k = 0, 1, 2, \dots, M$ and T_n is the n^{th} Chebyshev polynomial described as:

$$T_n(\xi) = \cos[n \cos^{-1}(\xi)]. \quad (23)$$

The unknown function derivatives are computed as:

$$\begin{aligned} \frac{d^a f_k}{d\eta^a} &= \sum_{n=0}^M D_{nj}^a f_k(\xi_n), \\ \frac{d^a \theta_k}{d\eta^a} &= \sum_{n=0}^M D_{nj}^a \theta_k(\xi_n), \\ \frac{d^a \phi_k}{d\eta^a} &= \sum_{n=0}^M D_{nj}^a \phi_k(\xi_n), \end{aligned} \quad (24)$$

where $j = 0, 1, 2, \dots, M$. Here $\mathbf{D} = 2D/L$, where D is the Chebyshev spectral differentiation matrix and a is the order of differentiation. The matrix equation that emerges from substituting Eqs. (21)–(24) in Eqs. (15)–(18) is as follows.

$$\mathbf{B}_{k-1} \mathbf{X}_k = \mathbf{Z}_{k-1}, \quad (25)$$

along with the conditions on the boundary:

$$\begin{aligned} f_k(\xi_M) &= \sum_{n=0}^M D_{0n} f_k(\xi_n) = \sum_{n=0}^M D_{Nn} f_k(\xi_n) = 0, \\ \theta_k(\xi_M) &= \theta_k(\xi_0) = \phi_k(\xi_M) = \phi_k(\xi_0) = 0, \end{aligned} \quad (26)$$

in which $\mathbf{B}_{k-1}$ is a square matrix of order $3(M + 1)$ and $\mathbf{X}_k$ $\mathbf{B}_{k-1}$ are column vectors $3(M + 1) \times 1$ described by:

$$\mathbf{B}_{k-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}, \quad \mathbf{X}_k = \begin{bmatrix} F_k \\ \theta_k \\ \phi_k \end{bmatrix}, \quad \mathbf{Z}_{k-1} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}, \quad (27)$$

where:

$$\begin{aligned} B_{11} &= a_1 D^3 + a_2 D^2 + a_3 D + a_4, \quad B_{12} = 0, \quad B_{13} = 0, \\ B_{21} &= b_1 D + b_2, \quad B_{22} = b_3 D^2 + b_4 D + b_5, \quad B_{23} = \text{Df} D^2, \\ B_{31} &= c_1 D + c_2, \quad B_{32} = \text{Sr} D^2, \quad B_{33} = \frac{1}{\text{Sc}} D^2 + c_3 D + c_4, \\ F_k &= [f_k(\xi_0), f_k(\xi_1), \dots, f_k(\xi_{M-1}), f_k(\xi_M)]^T, \\ \theta_k &= [\theta_k(\xi_0), \theta_k(\xi_1), \dots, \theta_k(\xi_{M-1}), \theta_k(\xi_M)]^T, \\ \phi_k &= [\phi_k(\xi_0), \phi_k(\xi_1), \dots, \phi_k(\xi_{M-1}), \phi_k(\xi_M)]^T, \\ z_1 &= [r_1(\xi_0), r_1(\xi_1), \dots, r_1(\xi_{M-1}), r_1(\xi_M)]^T, \\ z_2 &= [r_2(\xi_0), r_2(\xi_1), \dots, r_2(\xi_{M-1}), r_2(\xi_M)]^T, \\ z_3 &= [r_3(\xi_0), r_3(\xi_1), \dots, r_3(\xi_{M-1}), r_3(\xi_M)]^T, \\ a_f \quad (f = 2, 3, 4), \quad b_m \quad (m = 1, 2, 4, 5), \quad c_n \quad (n = 1, 2, 3, 4), \end{aligned}$$

defined above are diagonal matrices of dimension $(M + 1)$, and the transpose is indicated by the superscript T . On Eq. (25), the boundary constraints (26) are applied by altering the first and last rows of B_{mn} ($m = 1, 2, 3$; $n = 1, 2, 3$) and z_p ($p = 1, 2, 3$).

4. Findings and discussion

In the current investigation, the Sherwood number Sh , velocity, Nusselt number Nu , temperature, coefficient of skin friction C_f , and concentration are evaluated for distinct values of solid volume fraction of ϕ_1 (Al_2O_3), ϕ_2 (Cu), unsteadiness parameter A , Schmidt number Sc , suction/ injection parameter S , Soret number Sr , and Dufour number Df and are graphically represented. Unless otherwise stated, numerical computations are performed using $\phi_1 = 0.1$, $\phi_2 = 0.1$, $A = 0.5$, $S = 0.5$, $Df = 0.5$, $Sc = 0.2$ and $Sr = 0.2$ to assess the influence of all the parameters involved. Eqs. (7)–(9) are linearised using the successive linearisation procedure, and then the system is resolved by applying

the collocation technique. The resultant skin friction coefficient and Nusselt number are compared with the findings of previous investigations for $\phi_1 = \phi_2 = 0$. A well-justified comparison between our results and those of for $\theta'(0)$ and $f''(0)$ is demonstrated in Table 3.

Table 3. Comparison between $f''(0)$ and $\theta'(0)$ values at the surface by the current strategy with Shahzad et al. [6].

A	S	Pr	$f''(0)$		$\theta'(0)$	
			Current values	Shahzad et al. [6]	Current values	Shahzad et al. [6]
0.5	1	1	-2.6713727	-2.655999	-2.4191865	-2.655999
0.5	0.5	1	-1.9147295	-1.907999	-1.7474918	-1.907999
0.5	0	1	-1.3104137	-1.308999	-1.2096476	-1.308999
0.5	-0.5	1	-0.8867758	-0.887200	-0.8309926	-0.887200
0.5	-1	1	-0.6192663	-0.620400	-0.5899393	-0.620400
0	0.5	1	-1.8041331	-1.798999	-1.6397280	-1.798999
0.5	0.5	0.5	-1.9147295	-1.907999	-1.0313962	-1.119999
0.5	0.5	0.7	-1.9147295	-1.907999	-1.3312163	-1.450000
1	0.5	1	-2.0244687	-2.016999	-1.8528471	-2.016999

Figure 2 demonstrates the influence of unsteadiness variable A over the velocity, heat transfer rate $-\theta'(0)$, temperature, and mass transfer rate, $-\phi'(0)$, concentration, and coefficient of skin friction $f''(0)$. An enhancement in A decreases the $f''(0)$, as demonstrated in Fig. 2a. This is due to the development of the velocity boundary layer is caused solely on the stretching plate. Figure 2b and Fig. 2c demonstrate that $-\phi'(0)$ and $-\theta'(0)$ enhance when parameter A increases. Physically, it means that the temperature gradient at the surface increases as A increases, which implies an increase of the heat transfer rate at the surface. The rise in parameter A led to an initial decrease in velocity, but later to a slow increment in the velocity, depicted in Fig. 2d. It can be viewed from Fig. 2e and Fig. 2f that the rise in parameter A led to a decrease in concentration and temperature.

The impact of ϕ_1 on the heat transfer rate $-\theta'(0)$, concentration, coefficient of skin friction $f''(0)$, temperature, mass transfer rate $-\phi'(0)$, and velocity is described in Fig. 3. It has been noted that as ϕ_1 increases, the $f''(0)$ and $-\phi'(0)$ escalate, as demonstrated in Fig. 3a and Fig. 3c, and $-\theta'(0)$ diminishes, as depicted in Fig. 3b. It appears that the rise in parameter ϕ_1 the enhances temperature and velocity, as shown in Fig. 3d and Fig. 3e and decreases the concentration, as displayed in Fig. 3f.

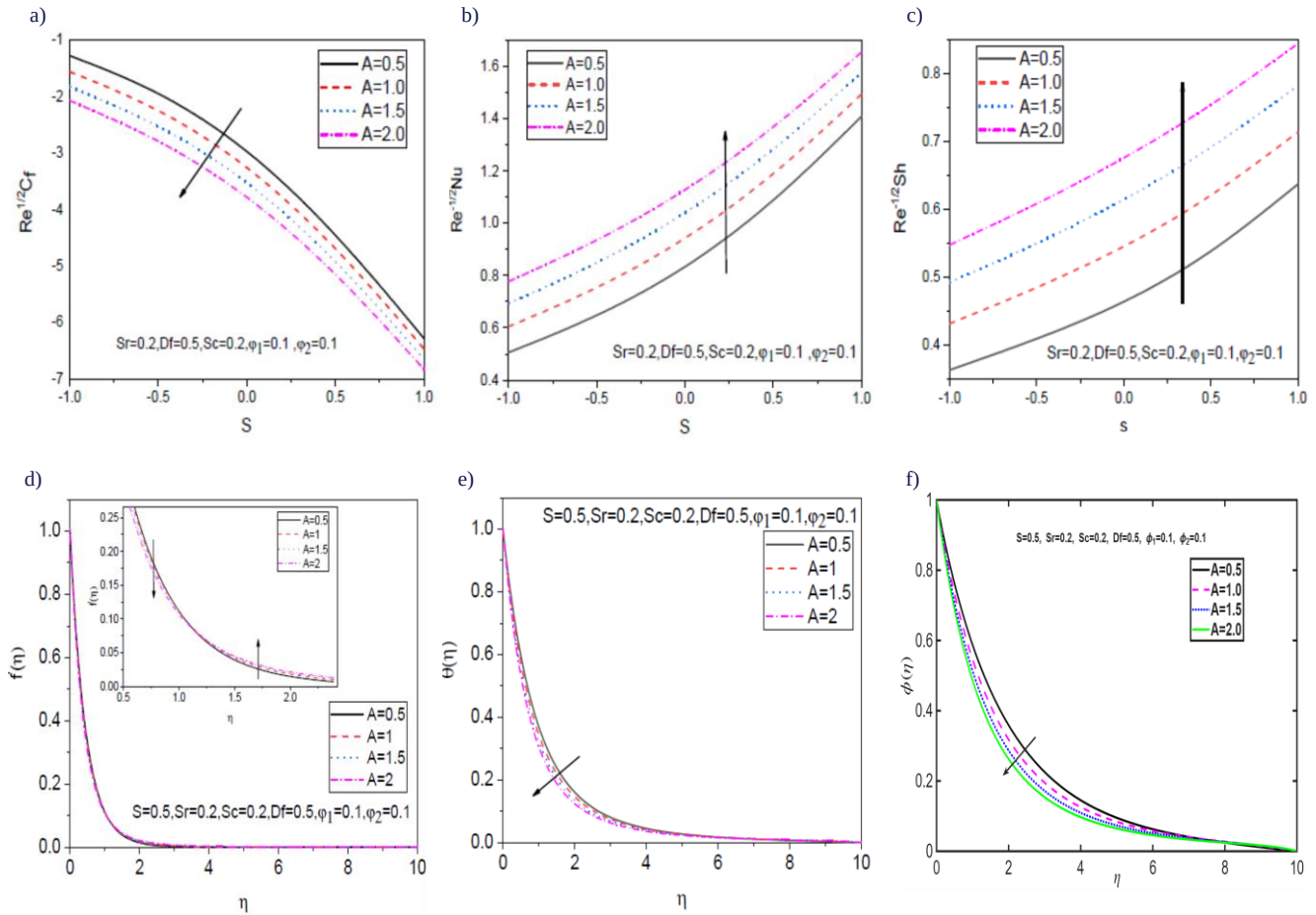
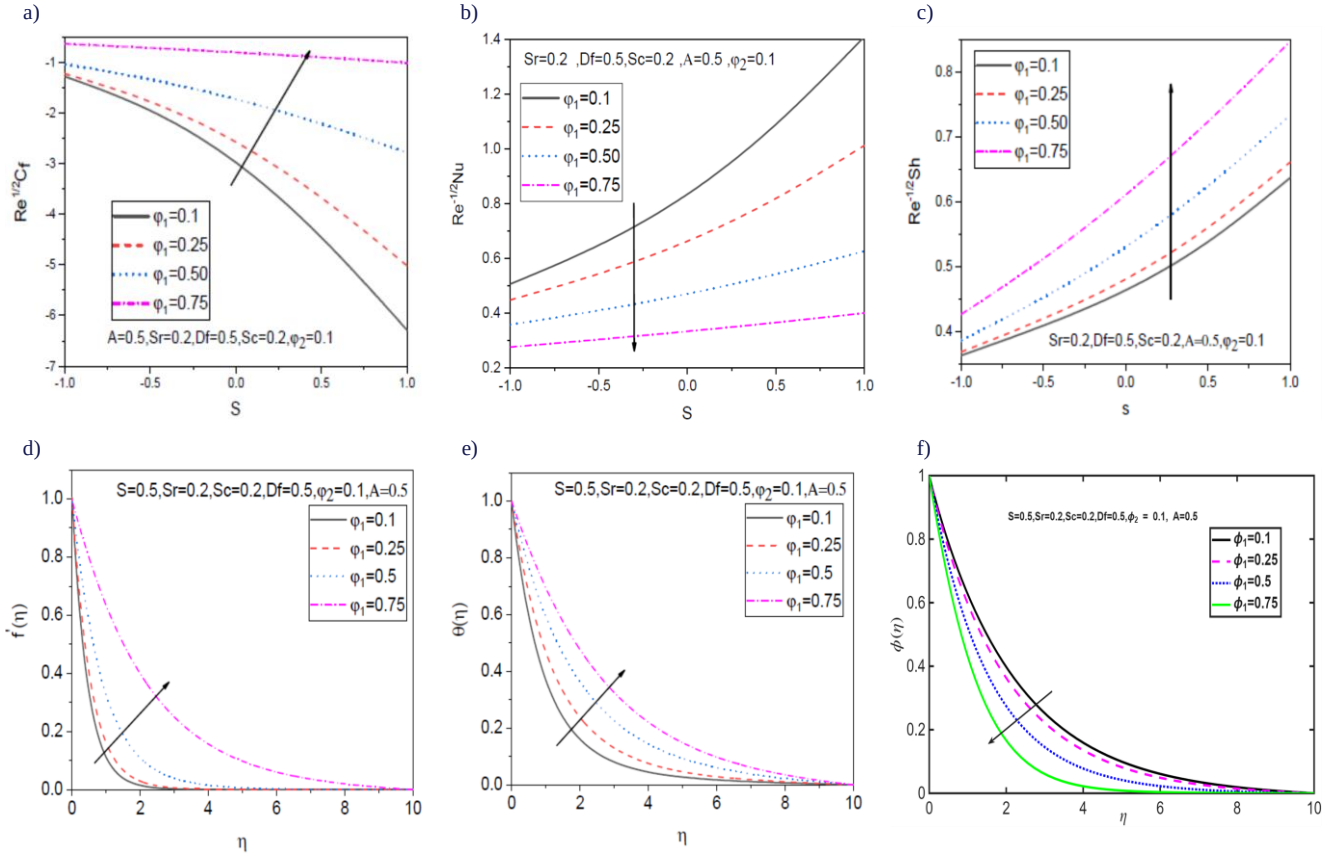
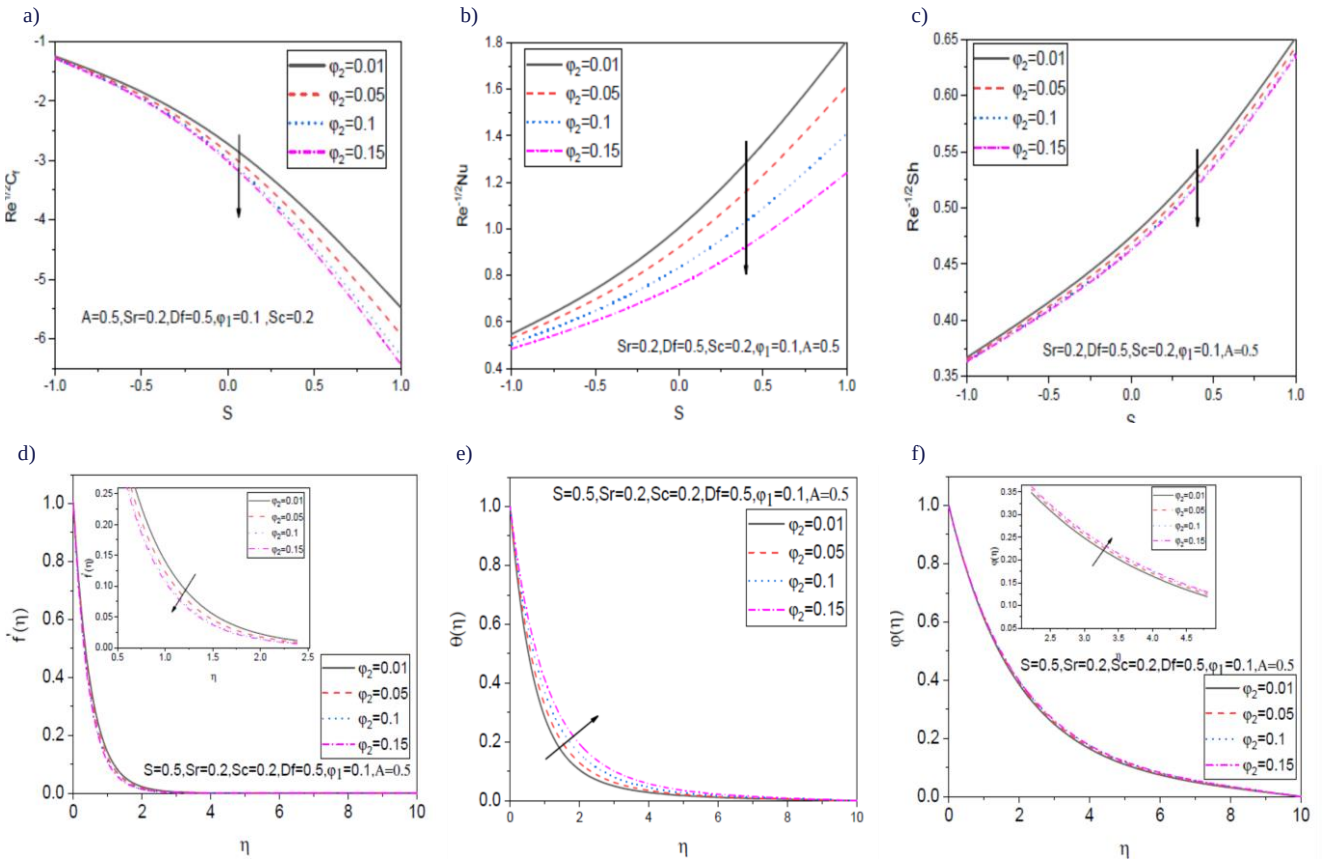


Fig. 2. Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with unsteadiness parameter A .

The influence of ϕ_2 on velocity, skin friction coefficient $f''(0)$, temperature, mass transfer rate $-\phi'(0)$, concentration, and heat transfer rate $-\theta'(0)$, are shown in Fig. 4. As ϕ_2 increases,

$f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ reduce, depicted in Fig. 4a, Fig. 4b, and Fig. 4c. It is noticed that the boost in parameter ϕ_2 lowers velocity, as shown in Fig. 4d, but increases temperature and concentration as illustrated in Fig. 4e and Fig. 4f.


 Fig. 3. Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with nanoparticle volume fraction ϕ_1 .

 Fig. 4. Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with nanoparticle volume fraction ϕ_2 .

The impact of Schmidt number Sc on the velocity, mass transfer rate $-\phi'(0)$, concentration, heat transfer rate $-\theta'(0)$, temperature, and skin friction coefficient $f''(0)$ is demonstrated in Fig. 5. It has been noted that $f''(0)$, is unaffected by an increase in parameter Sc , as depicted in Fig. 5a. Figures 5b and 5c

demonstrate that as Sc grows, $-\theta'(0)$ drops and $-\phi'(0)$ increases. It is noticed from Fig. 5d that the boost in parameter Sc does not affect velocity. Figure 5e demonstrates that as Sc enhances, the temperature initially increases and later diminishes. It is apparent from Fig. 5f that a spike in Sc reduces the concentration.

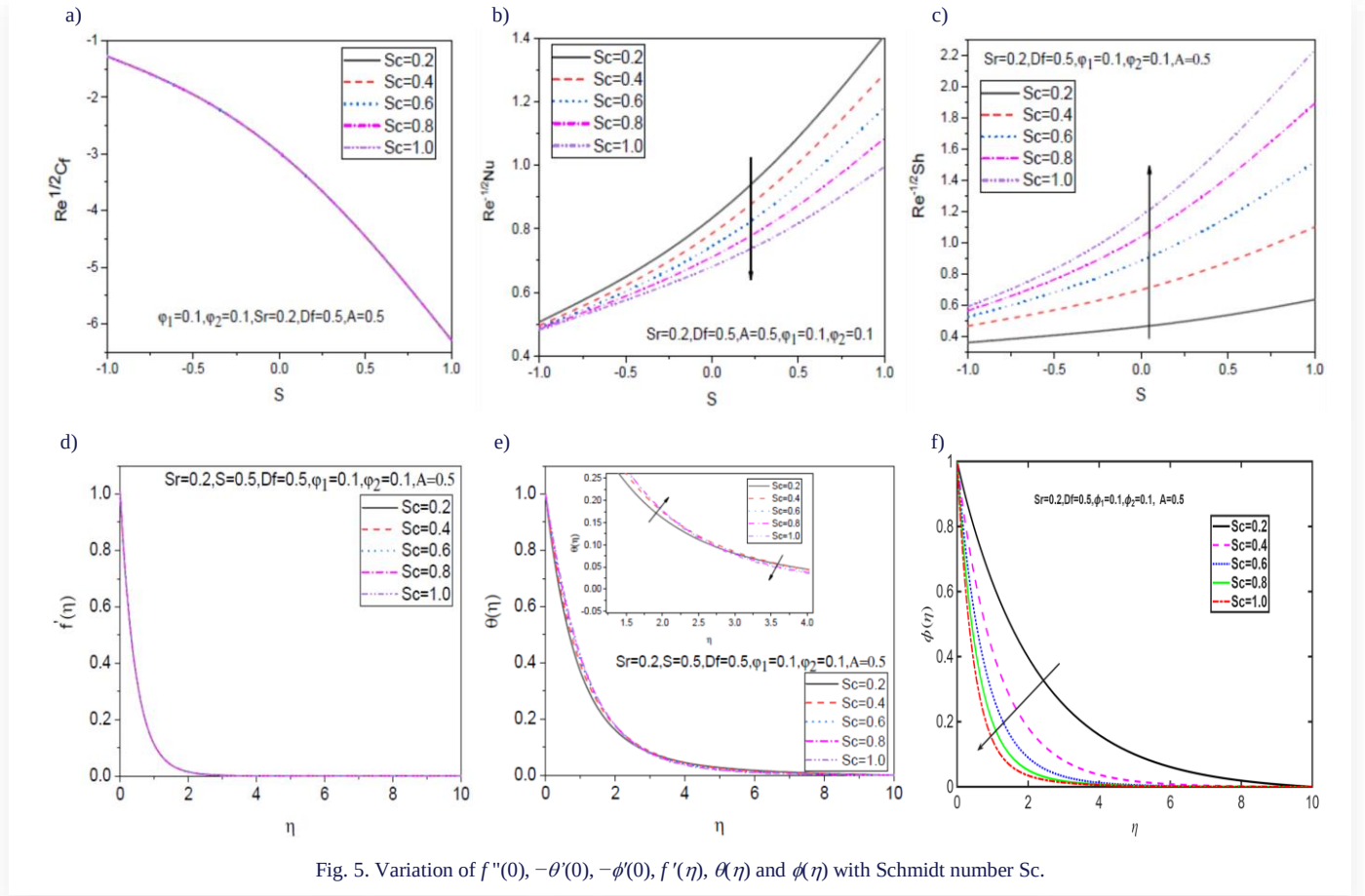


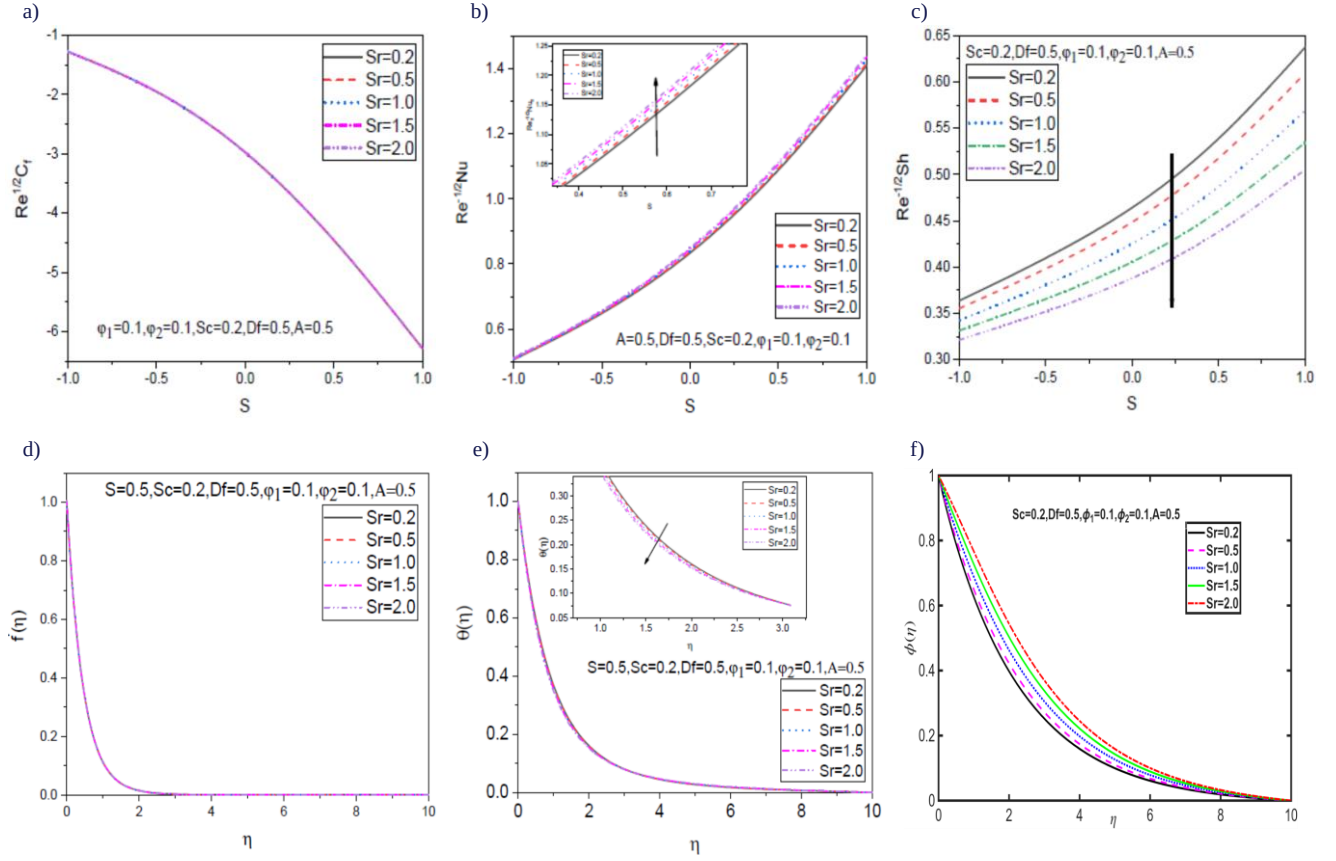
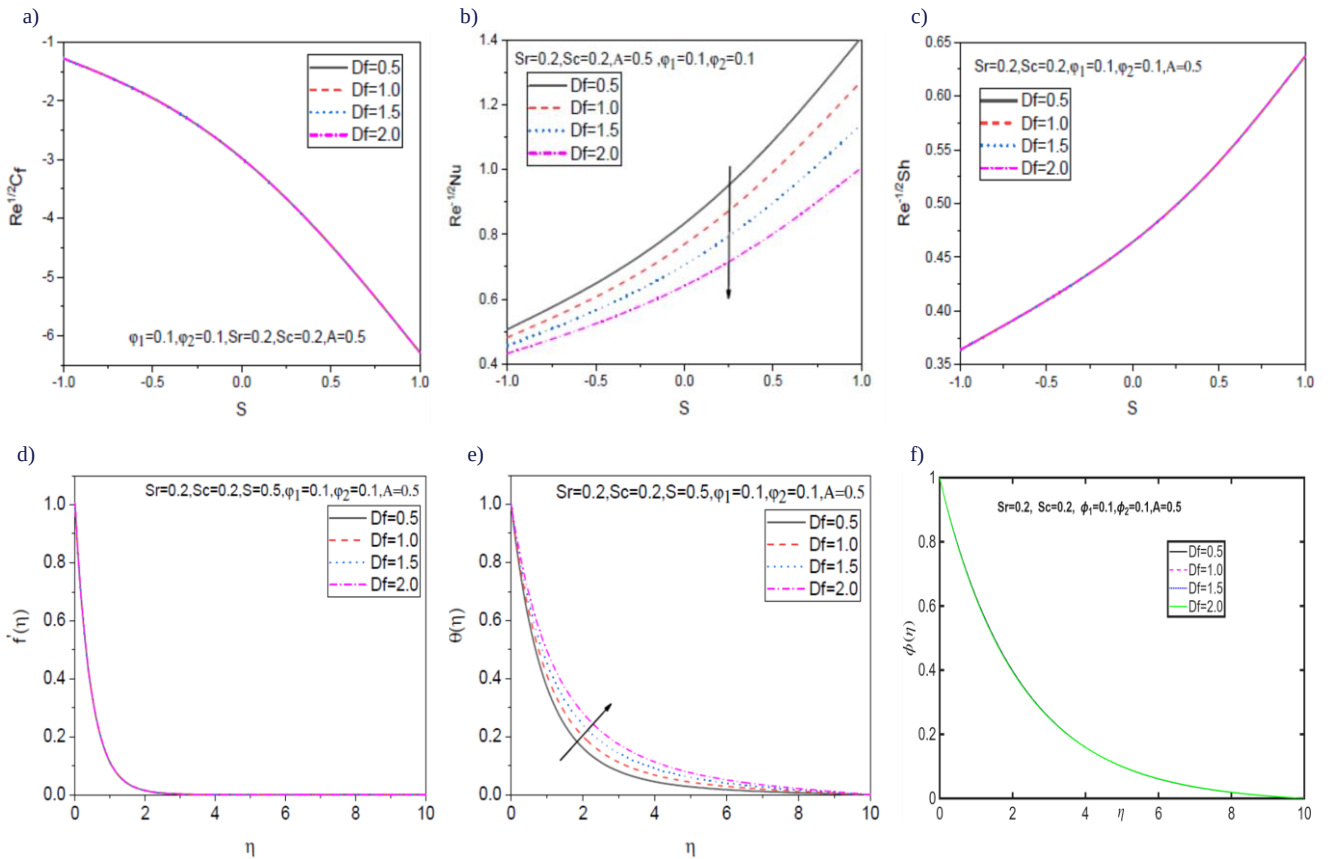
Fig. 5. Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with Schmidt number Sc .

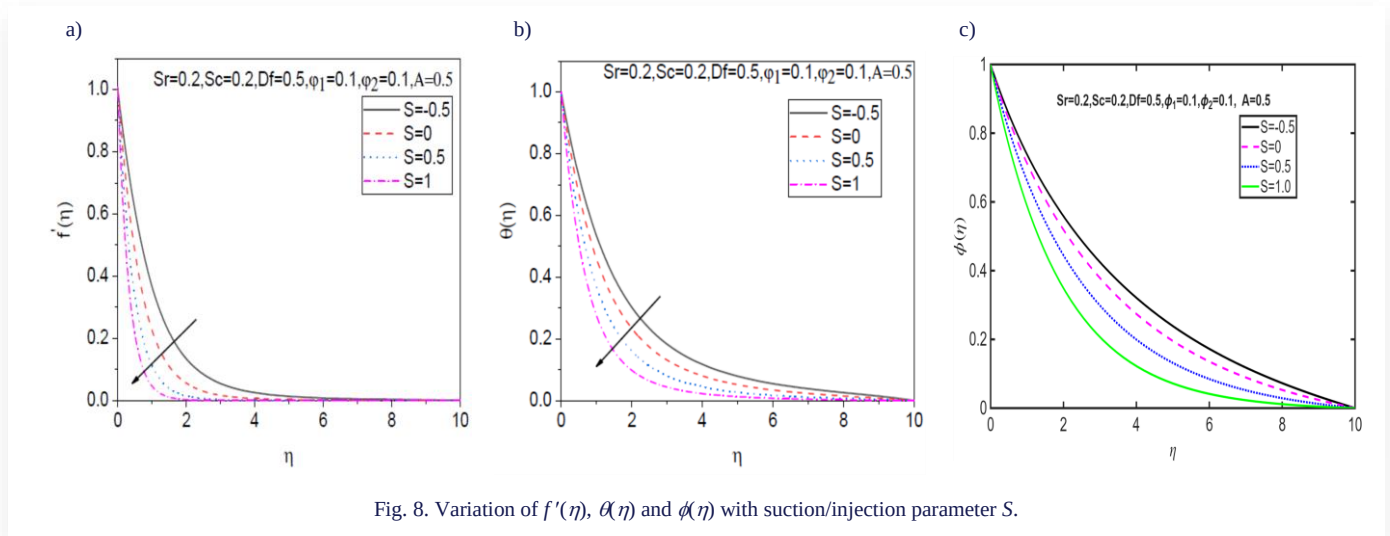
Figure 6 demonstrates the influence of the Soret number Sr over temperature and mass transfer rate $-\phi'(0)$, skin friction coefficient $f''(0)$, concentration, velocity, and heat transfer rate $-\theta'(0)$. The Soret number (Sr) describes the relative significance of mass diffusion (species diffusion) and thermal diffusion (heat conduction) in a fluid combination. When temperature gradients and concentration gradients combine, as they do in combustion, chemical processes, and in other reactive flow systems, the parameter Sr is especially important. It is noticed that as Sr increases, $f''(0)$ is constant and $-\theta'(0)$ escalates, as displayed in Fig. 6a and Fig. 6b. Figure 6c demonstrates that as Sr grows, $-\phi'(0)$ diminishes. Note that the rise in parameter Sr has no effect on velocity, but the temperature decreases, as shown in Fig. 6d and Fig. 6e. Figure 6f reveals that an increase in Sr increases concentration.

The enhancement in concentration is due to the Soret effect, where thermal gradients drive mass diffusion, enriching the species concentration near the surface. This enhanced diffusion also contributes to increased tangential motion. Conversely, the reduced temperature profile indicates that thermal energy is being redistributed to aid species transport, leading to a lower thermal boundary layer.

The influence of Dufour number Df on the heat transfer rate $-\theta'(0)$, coefficient of skin friction $f''(0)$, mass transfer rate $-\phi'(0)$, velocity, temperature, and concentration is presented in Fig. 7. The Dufour number Df describes how the diffusion coefficient in thermodynamics and transport phenomena changes with temperature $\theta(\eta)$. It reflects the temperature-dependent variation in the rate of diffusion and can be useful for understanding and simulating diffusion processes in a variety of systems, such as gases, liquids, and solids. Figure 7a depicts that the rise in the parameter Df does not influence the $f''(0)$. It is noticed from Fig. 7b and Fig. 7c that when the magnitude of the variable Df grows, $(-\theta')$ drops and $(-\phi')$ remains constant. The rise in parameter Df has no impact on velocity, but the increment in temperature is shown in Fig. 7d and Fig. 7e. This behaviour is attributed to the Dufour effect, where the energy flux induced by concentration gradients enhances thermal motion. It can be spotted from Fig. 7f that a spike in variable Df does not affect the concentration.

The outcome of the suction/injection parameter S on the concentration, velocity and temperature is apparent in Fig. 8. As the value of S rises, the temperature, velocity, and concentration diminish as presented in Figs. 8a, 8b and 8c.


 Fig. 6. Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with Soret number Sr .

 Fig. 7. Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with Dufour parameter Df .

Fig. 8. Variation of $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with suction/injection parameter S .

5. Conclusions

In this paper, the nonlinear differential equations arising from the hybrid nanofluid flow past a time-dependent sheet that is stretching radially with Dufour and Soret effects under the influence of the unsteadiness parameter A , suction or injection parameter, ϕ_1 and ϕ_2 at the surface, are solved employing the SLM approach. The effect of variables Sr , A , S , Sc , ϕ_1 , ϕ_2 and Df over the pertinent physical quantities is scrutinised. The key findings are outlined below:

- The skin friction falls when the unsteadiness parameter ϕ_2 rises, and as ϕ_1 values rise, the skin friction enhances. As the Soret, Schmidt, and Dufour numbers rise, the skin friction doesn't alter.
- As the unsteadiness parameter increases, the Nusselt number rises, and decreases when ϕ_1 and ϕ_2 are increased. The Nusselt number diminishes as the Schmidt and Dufour numbers escalate but increases as the Soret number boosts.
- As ϕ_2 gets higher, the Sherwood number goes down and rises when the unsteadiness parameter ϕ_1 increases. The Sherwood number advances with the Schmidt number, but it stays constant while the Dufour number rises and falls with the Soret number.
- The velocity initially reduces and then increases when the unsteadiness parameter grows. The velocity decreases as the suction/injection parameter ϕ_2 increases, but the velocity escalates when ϕ_1 increases. As the Dufour, Schmidt, and Soret numbers grow, the velocity remains constant.
- The temperature decreases as the suction/injection parameter, Soret number, and unsteadiness parameter increase, and rises when ϕ_1 , ϕ_2 , and Dufour number are increased. The temperature increases initially and later diminishes while the Schmidt number grows.
- The concentration drops while the Schmidt number, unsteadiness parameter ϕ_1 and suction/injection parameter enhance, and rises when ϕ_2 and Soret number are increased. The rise in the Dufour number does not affect the concentration.

References

- [1] Khan, M., Manzur, M., & ur Rahman, M. (2017). On axisymmetric flow and heat transfer of Cross fluid over a radially stretching sheet. *Results in Physics*, 7, 3767–3772. doi: 10.1016/j.rinp.2017.08.039
- [2] Ahmed, J., Shahzad, A., Begum, A., Ali, R., & Siddiqui, N. (2017). Effects of inclined Lorentz forces on boundary layer flow of Sisko fluid over a radially stretching sheet with radiative heat transfer. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 39, 3039–3050. doi: 10.1007/s40430-017-0759-z
- [3] Sreelakshmi, K., Sarojamma, G., & Josyula, R.M. (2018). Homotopy analysis of an unsteady flow heat transfer of a Jeffrey nanofluid over a radially stretching convective surface. *Journal of Nanofluids*, 7(1), 62–71. doi 10.1166/jon.2018.1432
- [4] Khan, S.A., Nie, Y., & Ali, B. (2019). Multiple slip effects on magnetohydrodynamic axisymmetric buoyant nanofluid flow above a stretching sheet with radiation and chemical reaction. *Symmetry*, 11(9), 1171. doi: 10.3390/sym11091171
- [5] Nayak, B., Mishra, S.R., & Krishna, G.G. (2019). Chemical reaction effect of an axisymmetric flow over radially stretched sheet. *Propulsion and Power Research*, 8(1), 79–84. doi: 10.1016/j.jprr.2019.01.002
- [6] Shahzad, A., Ali, R., Hussain, M., & Kamran, M. (2017). Unsteady axisymmetric flow and heat transfer over time-dependent radially stretching sheet. *Alexandria Engineering Journal*, 56(1), 35–41. doi: 10.1016/j.aej.2016.08.030
- [7] Shahzad, A., Gulistan, U., Ali, R., Iqbal, A., Benim, A.C., Kamran, M., Khan, S.U-D, Ud-Din Khan, S., & Farooq, A. (2020). Numerical study of axisymmetric flow and heat transfer in a liquid film over an unsteady radially stretching surface. *Mathematical Problems in Engineering*, 2020, 1–9. doi: 10.1155/2020/6737243
- [8] Srinivasacharya, D., & Kumar, R.S. (2022) Artificial neural network modeling of the Casson fluid flow over unsteady radially stretching sheet with Soret and Dufour effects. *Journal of Thermal Analysis and Calorimetry*, 147(24), 14891–14903. doi: 10.1007/s10973-022-11694-w
- [9] Chamkha, A.J., & Khaled, A.R.A. (2000). Similarity solution for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media. *International Journal of Numerical Methods for Heat & Fluid Flow*, 10(1), 94–115. doi: 10.1108/09615530010306939

- [10] Krishna, M.V., Anand, P.V.S., & Chamkha, A.J. (2019). Heat and mass transfer on free convective flow of a micropolar fluid through a porous surface with inclined magnetic field and Hall effects. *Special Topics & Reviews in Porous Media: An International Journal*, 10(3), 203–223. doi: 10.1615/SpecialTopicsRevPorousMedia.2018026943
- [11] Krishna, M.V., Ahamad, A.N., & Chamkha, A.J. (2021). Hall and ion slip impacts on unsteady MHD convective rotating flow of heat generating/absorbing second grade fluid. *Alexandria Engineering Journal*, 60(1), 845–858. doi: 10.1016/j.aej.2020.10.013
- [12] Choi, S.U.S., & Eastman, J.A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. No. ANL/MSD/CP-84938; CONF-951135-29, ASME International Engineering Congre & Expositon, November 12–17, San Francisco, USA.
- [13] Devi, S.A., & Devi, S.S.U. (2016). Numerical investigation of hydromagnetic hybrid Cu-Al₂O₃/water nanofluid flow over a permeable stretching sheet with suction. *International Journal of Nonlinear Sciences and Numerical Simulation*, 17(5), 249–257. doi: 10.1515/ijnsns-2016-0037
- [14] Devi, S.U., & Devi, S.A. (2017). Heat transfer enhancement of Cu-Al₂O₃/water hybrid nanofluid flow over a stretching sheet. *Journal of the Nigerian Mathematical Society*, 36(2), 419–433.
- [15] Hayat, T., & Nadeem, S. (2017). Heat transfer enhancement with Ag-CuO/water hybrid nanofluid. *Results in Physics*, 7, 2317–2324. doi: 10.1016/j.rinp.2017.06.034
- [16] Yousefi, M., Dinarvand, S., Yazdi, M.E., & Pop, I. (2018). Stagnation-point flow of an aqueous titania-copper hybrid nanofluid toward a wavy cylinder. *International Journal of Numerical Methods for Heat and Fluid Flow*, 28(7), 1716–1735. doi: 10.1108/HFF-01-2018-0009
- [17] Subhani, M., & Nadeem, S. (2019). Numerical analysis of micropolar hybrid nanofluid. *Applied Nanoscience*, 9(4), 447–459. doi: 10.1007/s13204-018-0926-2
- [18] Ghadikolaei, S.S., Yassari, M., Sadeghi, H., Hosseinzadeh, K., & Ganji, D.D. (2017). Investigation on thermophysical properties of TiO₂-Cu/H₂O hybrid nanofluid transport dependent on shape factor in MHD stagnation point flow. *Powder Technology*, 322, 428–438. doi: 10.1016/j.powtec.2017.09.006
- [19] Usman, M., Hamid, M., Zubair, T., Ul Haq, R., & Wang, W. (2018). Cu-Al₂O₃/water hybrid nanofluid through a permeable surface in the presence of nonlinear radiation and variable thermal conductivity via LSM. *International Journal of Heat and Mass Transfer*, 126, 1347–1356. doi: 10.1016/j.ijheatmasstransfer.2018.06.005
- [20] Rostami, M.N., Dinarvand, S., & Pop, I. (2018). Dual solutions for mixed convective stagnation-point flow of an aqueous silica-alumina hybrid nanofluid. *Chinese Journal of Physics*, 56(5), 2465–2478. doi: 10.1016/j.cjph.2018.06.013
- [21] Waini, I., Ishak, A., & Pop, I. (2019). Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid. *International Journal of Heat and Mass Transfer*, 136, 288–297. doi: 10.1016/j.ijheatmasstransfer.2019.02.101
- [22] Waini, I., Ishak, A., & Pop, I. (2019). Hybrid nanofluid flow and heat transfer past a permeable stretching/shrinking surface with a convective boundary condition. In *Journal of Physics: Conference Series*, 1366(1), 012022. IOP Publishing. doi: 10.1088/1742-6596/1366/1/012022
- [23] Gangadhar, K., Kumari, M.A. & Chamkha, A.J. EMHD Flow of Radiative Second-Grade Nanofluid over a Riga Plate due to Convective Heating: Revised Buongiorno's Nanofluid Model. *Arabian Journal for Science and Engineering*, 47(3), 8093–8103 (2022). doi: 10.1007/s13369-021-06092-7
- [24] Ontela, S., Pattnaik, P.K., Panda, S., & Mishra, S.R. (2024). Optimizing heat transfer rate and sensitivity analysis of hybrid nanofluid flow over a radiating sheet: Applications in solar-powered charging stations. *Numerical Heat Transfer, Part B: Fundamentals*, 1–21. doi: 10.1080/10407790.2024.2341439
- [25] Panda, S., Ontela, S., Pattnaik, P.K., & Mishra, S.R. (2024). Optimizing heat transfer rate with sensitivity analysis on nonlinear radiative hydromagnetic hybrid nanofluid flow considering catalytic effects and slip condition: Hamilton–Crosser and Yamada–Ota modelling. *Journal of Applied Mathematics and Mechanics (Zeitschrift für Angewandte Mathematik und Mechanik)*, 104, e202301064. doi: 10.1002/zamm.202301064
- [26] Pattnaik, P.K., Mishra, S.R., Thumma, T., Panda, S., & Ontela, S. (2024). Numerical investigation of radiative blood-based aluminum alloys nanofluid over a convective Riga sensor plate with the impact of diverse particle shape. *Journal of Thermal Analysis and Calorimetry*, 149, 2317–2329. doi: 10.1007/s10973-023-12820-y
- [27] Panda, S., Ontela, S., Pattnaik, P.K., & Mishra, S.R., (2024). Radiating heat effect on Powell–Eyring blood-based hybrid nanofluid over a Riga plate with thermal stratification Cattaneo–Christov heat flux model, *Partial Differential Equations in Applied Mathematics*, 11, 100769. doi: 10.1016/j.padiff.2024.100769
- [28] Nisar, K.S., Khan, U., Zaib, A., Khan, I. & Baleanu, D., (2020). Exploration of aluminum and titanium alloys in the stream-wise and secondary flow directions comprising the significant impacts of magnetohydrodynamic and hybrid nanofluid. *Crystals*, 10(8), 679. doi: 10.3390/cryst10080679
- [29] Abad, J.M.N., Alizadeh, R., Fattahi, A., Doranehgard, M.H., Alhajri, E., & Karimi, N. (2020). Analysis of transport processes in a reacting flow of hybrid nanofluid around a bluff-body embedded in porous media using artificial neural network and particle swarm optimization. *Journal of Molecular Liquids*, 313, 113492. doi: 10.1016/j.molliq.2020.113492
- [30] Hayat, T., Shehzad, S.A., & Alsaedi, A. (2012). Soret and Dufour effects on magnetohydrodynamic (MHD) flow of Casson fluid. *Applied Mathematics and Mechanics*, 33(10), 1301–1312. doi: 10.1007/s10483-012-1623-6
- [31] Kameswaran, P.K., Shaw, S., & Sibanda, P. (2014). Dual solutions of Casson fluid flow over a stretching or shrinking sheet. *Sādhanā*, 39(6), 1573–1583. doi: 10.1007/s12046-014-0289-7
- [32] Isa, S.S.P.M., Parvin, S., Arifin, N.M., Ali, F.M. & Ahmad, K. (2023). Soret-Dufour effects on the waterbased hybrid nanofluid flow with nanoparticles of Alumina and Copper. *Malaysian Journal of Mathematical Sciences*, 17(3), 283–304. doi: 10.47836/mjms.17.3.04
- [33] Prathiba, A., Lakshmi, V.A., & Ramesh, V. (2023). A Computational Analysis of the Soret and Dufour Effects on a Rotating Hybrid Nanofluid. *Communications in Mathematics and Applications*, 14(2), 881. doi: 10.26713/cma.v14i2.2124
- [34] Arif, U., Nawaz, M., & Salmi, A. (2022). Numerical study of simultaneous transport of heat and mass transfer in Maxwell hybrid nanofluid in the presence of Soret and Dufour effects. *Physica Scripta*, 97(2), 025207. doi: 10.1088/1402-4896/ac4d46
- [35] Sharma, R.P., Gorai, D., & Das, K. (2022). Comparative study on hybrid nanofluid flow of Ag-CuO/H₂O over a curved stretching surface with Soret and Dufour effects. *Heat Transfer*, 51(7), 6365–6383. doi: 10.1002/hjt.22595
- [36] Vijay, N., & Sharma, K. (2023). Magnetohydrodynamic hybrid nanofluid flow over a decelerating rotating disk with Soret and Dufour effects. *Multidiscipline Modeling in Materials and Structures*, 19(2), 253–276. doi: 10.1108/MMMS-08-2022-0160
- [37] Das, S.K., Choi, S.U., Yu, W. & Pradeep, T. (2007). *Nanofluids: Science and Technology*. John Wiley & Sons.

- [38] Bachok, N., Ishak, A., & Pop, I. (2011). Flow and heat transfer over a rotating porous disk in a nanofluid, *Physica B: Condensed Matter*, 406(9), 1767–1772. doi: 10.1016/j.physb.2011.02.024
- [39] Tassaddiq, A., Khan, S., Bilal, M., Gul, T., Mukhtar, S., Shah, Z., & Bonyah, E. (2020). Heat and mass transfer together with hybrid nanofluid flow over a rotating disk. *AIP Advances*, 10(5), 055317. doi: 10.1063/5.0010181
- [40] Oztop, H.F., & Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, 29(5), 1326–1336. doi: 10.1016/j.ijheatfluidflow.2008.04.009



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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<http://www.imp.gda.pl/archives-of-thermodynamics/>



Thermal Performance of MHD Radiative Nanofluid Convection over an Exponentially Stretching Surface in a Porous Medium

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Received: 21.02.2025; revised: 10.07.2025; accepted: 11.07.2025

Abstract

This study investigates the hydromagnetic convection of a radiating nanofluid flowing over an exponentially stretched sheet embedded in a porous medium, with emphasis on the influences of thermal radiation, magnetic fields, Joule heating, viscous dissipation, porous permeability, and nanoparticle volume fraction on flow and heat transfer behaviour. The governing momentum and energy equations are formulated using boundary layer theory and reduced to nonlinear ordinary differential equations via similarity transformations. We numerically solve these equations using the shooting method in conjunction with the Runge-Kutta-Fehlberg integration scheme. The analysis looks at how important factors like the strength of the magnetic field, thermal radiation, Prandtl number, and how easily fluid can flow through a porous material impact the system, showing how magnetic and thermal effects play a big role in how fluids move. One of the main discoveries is that stronger magnetic fields and more thermal radiation together lower the Nusselt number, highlighting the balance between heat transfer through radiation and convection, which is important in high-temperature industrial processes. The study offers important insights for optimising a range of engineering applications, including nanomaterial-based thermal coatings, energy systems, and magnetically influenced flow control that ultimately aid advancements in energy efficiency, manufacturing processes, and environmental technologies.

Keywords: Nanofluid; Magnetic field; Thermal radiation; Porous medium; Heat transfer

Vol. 46(2025), No. 4, 31–39; doi: 10.24425/ather.2025.156596

Cite this manuscript as: Eegunjobi, A.S., & Makinde, O.D. (2025). Thermal Performance of MHD Radiative Nanofluid Convection over an Exponentially Stretching Surface in a Porous Medium. *Archives of Thermodynamics*, 46(4), 31–39.

1. Introduction

Research on the hydromagnetic convection of radiative nanofluids over exponentially stretching sheets within porous media is highly significant, as it effectively tackles complex multiphysics problems encountered in engineering and applied sciences. Nanofluids, known for their superior thermal conductivity, play a vital role in enhancing heat transfer performance. Their response to the combined effects of magnetic fields, thermal radiation, and porous environments helps optimise diverse industrial processes [1]. Incorporating a porous medium enables re-

alistic simulation of systems such as geothermal reservoirs, packed bed reactors and filtration units, while the exponentially stretching sheet models practical scenarios in manufacturing, including polymer extrusion, thin-film fabrication, and nanomaterial coating technologies [2]. The research also provides clarity regarding how magnetic fields affect fluid dynamics and energy transport, which is essential for applications such as electromagnetic pumps, magnetic damping systems, and magnetically actuated microfluidic devices. Additionally, the results can be used in high-efficiency thermal energy systems like solar collectors, heat exchangers, and thermal storage units, as well as in medical

Nomenclature

C_f – skin friction coefficient
 C_p – specific heat capacity, J/(kg·K)
 Da – Darcy number
 Ec – Eckert number
 k – thermal conductivity, W·m⁻¹·K⁻¹
 k^* – mean absorption coefficient, m⁻¹
 K – porous medium permeability, m²
 L – length, m
 M – magnetic field parameter
 Nu – Nusselt number
 Pr – Prandtl number
 q_r – radiative heat flux, W·m⁻²
 q_w – local heat flux, W·m⁻²
 Ra – thermal radiation parameter
 Re_x – local Reynolds number
 T, T_w, T_∞ – temperatures, K
 u, v – velocity components (in the x, y directions, respectively), m/s
 $U_w(x)$ – sheet exponential stretching velocity, m/s
 x, y – directions, m

Greek symbols

μ – dynamic viscosity, Pa·s
 ρ – density, kg/m³
 σ – electrical conductivity, S·m⁻¹
 σ^* – Stefan-Boltzmann constant, W·m⁻²·K⁻⁴
 τ – wall shear stress, N·m⁻²
 ϕ – volume fraction

Subscripts and Superscripts

f – fluid
 nf – nanofluid
 s – nanoparticles
 w – sheet surface
 ∞ – free stream

Abbreviations and Acronyms

LTE – local thermal equilibrium
 MHD – magnetohydrodynamic
 ODE – ordinary differential equations

technologies such as targeted drug delivery using magnetic nanofluids and hyperthermia treatment. Additionally, the research contributes to advancements in geothermal and environmental engineering, supports efficient geothermal energy extraction, manages pollutant transport in aquifers, and prompts the development of magnetohydrodynamics (MHD) systems, including MHD generators and nuclear reactor cooling technologies.

The investigation of fluid flow over stretching surfaces has garnered significant attention due to its industrial relevance. Seminal work [3] on flow past a stretching plate laid the groundwork for further developments in this domain. Building on this, researchers such as those in [4–6] explored heat transfer over stretching surfaces under various boundary conditions, including scenarios involving variable surface heat flux. The influence of fluid properties like variable viscosity on flow and thermal behaviour was analysed in [7], highlighting the complexity of real-world applications. The incorporation of exponentially stretching surfaces introduced new dimensions to this field, allowing for the analysis of more intricate flow behaviours. As emphasised in [1], the unique geometry of exponentially stretching sheets, where the stretching rate increases exponentially with distance, has a profound effect on the fluid dynamics. Further studies [8–9] examined how surface curvature interacts with exponential stretching, offering valuable insights for optimising surface configurations in engineering systems. Research has also extended to different fluid types, with analysing second-grade fluids over stretching sheets [10] and investigating flow and heat transfer over curved surfaces [11], enriching the understanding of these complex flow environments.

The emergence of nanofluids has significantly transformed the landscape of heat transfer research. Notably, studies like [12] have investigated nanofluid flow over exponentially stretching sheets, taking into account critical factors such as thermal radiation, magnetic fields, and internal heat generation. These studies highlighted the capacity of nanofluids to enhance heat transfer performance in systems involving such stretching surfaces. The presence of nanoparticles dispersed in base fluids, which enhance

properties like thermal conductivity, viscosity, and overall stability, is largely responsible for this improvement. A broad spectrum of research [13–16] has been dedicated to examining nanofluid dynamics under varied flow conditions. In particular, investigations in [17–20] have focused on flow and heat transfer over stretching surfaces while incorporating the influences of magnetic fields, heat sources, and radiation. Additionally, the influence of magnetic fields on nanofluid behaviour has been studied in detail [21–23], with findings emphasising the interaction between magnetic forces and fluid motion. Further studies have explored the impact of parameters such as Brownian motion, thermophoresis, heat generation or absorption, and the presence of porous media, offering valuable insights into optimising nanofluid-based heat transfer systems across a range of engineering applications.

Magnetohydrodynamics (MHD), which examines the interaction between electrically conducting fluids and magnetic fields, plays a pivotal role in various fields such as materials processing, astrophysics, and geophysics. Its principles underpin several advanced technologies, including MHD propulsion systems, power generation units, and electromagnetic pumps. Extensive research on MHD heat transfer in nanofluids has shed light on its unique thermal characteristics and flow behaviour. Studies [24–26] have specifically explored the influence of magnetic fields on flow dynamics, thermal radiation, and heat transfer, enhancing our comprehension of nanofluid performance under magnetic effects. Further investigations [27–29] into MHD nanofluid flow over stretching surfaces have examined how magnetic forces affect velocity fields, temperature profiles, and heat transfer rates. These works collectively highlight the intricate interplay between magnetic fields and buoyancy-driven convection, offering more profound insights into the complex thermal-fluid behaviour of nanofluids exposed to simultaneous magnetic and thermal influences. Several notable studies on hydromagnetic nanofluid convection over a stretching sheet can be found in Refs. [35–38].

In this study, we look at how several important factors, including magnetic field strength, the ability of the porous material to conduct magnetic fields, thermal radiation, heat loss due to fluid movement, the amount of nanoparticles, and Joule heating, affect the movement of a nanofluid flowing over a heated surface that stretches exponentially, especially regarding heat transfer. To the best of our knowledge, this kind of comprehensive analysis is scarcely addressed in the existing literature, aiming here to bridge that gap. The multifaceted approach provides helpful suggestions for enhancing heat transfer performance in complex systems where multiple physical effects interact simultaneously. Such investigations are essential for advancing thermal management strategies in critical applications, including electronic device cooling, metal extrusion, polymer manufacturing, nuclear reactor safety, and energy harvesting systems. By understanding how these parameters affect temperature profiles, flow dynamics, and heat transfer rates, engineers can develop more efficient, robust, and high-performance systems capable of functioning under extreme thermal and electromagnetic conditions. The subsequent sections detail the mathematical modeling, analytical framework, and numerical findings, accompanied by a thorough quantitative discussion.

2. Model description

We examine the laminar flow of a steady, two-dimensional, incompressible, viscous, electrically conducting, and radiative nanofluid over an exponentially stretching sheet embedded in a porous medium. The coordinate system is defined such that the x -axis lies along the surface of the sheet, while the y -axis is oriented normal to it. A transverse magnetic field of constant strength B_0 is applied perpendicular to the flow direction, as depicted in Fig. 1.

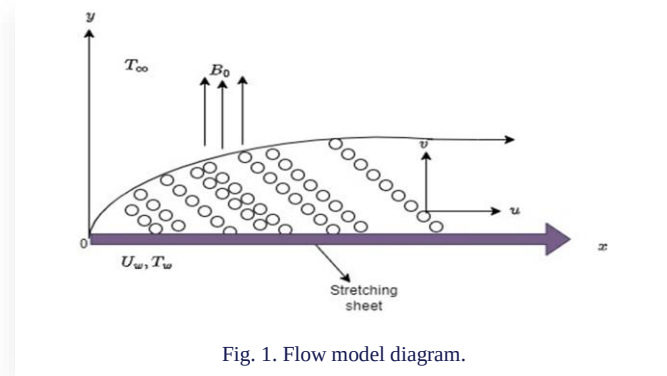


Fig. 1. Flow model diagram.

The induced magnetic field is considered negligible due to a low magnetic Reynolds number, and the Hall effect is disregarded, assuming a minimal overall current density. The porous medium, characterised by negligible inertia effects, and the nanofluid are assumed to be in local thermal equilibrium (LTE). Applying the standard boundary layer approximations, the governing continuity, momentum, and energy balance equations for the problem are formulated as follows [2,9,36,37]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} B_0^2 u}{\rho_{nf}} - \frac{\mu_{nf} u}{\rho_{nf} K}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B_0^2 u^2}{(\rho C_p)_{nf}} + \frac{\mu_{nf} u^2}{(\rho C_p)_{nf} K} - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial y}, \quad (3)$$

along with the corresponding boundary conditions:

- at $y = 0$:

$$u = U_w(x) = a e^{\frac{x}{L}}, v = 0, \quad T = T_w = T_\infty + T_0 e^{\frac{x}{2L}}, \quad (4)$$

- as $y \rightarrow \infty$:

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad (5)$$

where (u, v) denote the velocity components of the nanofluid in the x - and y -directions, respectively, $U_w(x)$ is the velocity of the exponentially stretching sheet, K denotes the porous medium permeability, T is the nanofluid temperature, T_∞ represents the free stream temperature, T_w is the temperature of the sheet surface of the hot fluid below the sheet, ρ_{nf} represents the nanofluid density, μ_{nf} is the nanofluid dynamic viscosity, k_{nf} denotes the nanofluid thermal conductivity, $(\rho C_p)_{nf}$ is the nanofluid heat capacitance, L is characteristic length and σ_{nf} is the nanofluid electrical conductivity. It is important to note from the boundary conditions in Eq. (4) that the surface of the exponentially stretching sheet is impermeable and maintained at a non-uniform temperature that increases along the sheet. Following [32], the thermophysical expressions relating ferrofluids to base fluid and nanoparticles are given as follows:

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad (6a)$$

$$(\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_s, \quad r = \frac{\sigma_s}{\sigma_f}, \quad (6b)$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \quad \frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(r - 1)\phi}{(r + 2) - (r - 1)\phi}, \quad (6c)$$

where ρ_f refers to the density of the base fluid, ρ_s is the density of the solid nanoparticle, k_f is the base fluid thermal conductivity, k_s is the thermal conductivity of the nanoparticles, σ_f is the electrical conductivity of the base fluid, σ_s is the electrical conductivity of the nanoparticles, ϕ represents the volume fraction of the nanoparticles, μ_f is the base fluid's dynamic viscosity C_{pf} is the specific heat capacity of the base fluid, and C_{ps} is the specific heat capacity of the nanoparticles. The physical properties of H_2O and Fe_3O_4 nanoparticles are listed in Table 1 below.

Table 1. Nanoparticles and base fluid thermophysical properties [33].

Physical properties	ρ , kg/m ³	C_p , J/(kg·K)	k , W/(m·K)	σ , S/m
H ₂ O	997.1	4179	0.613	5×10 ⁻⁵
Fe ₃ O ₄	5180	670	9.7	2.5×10 ⁶

Based on the Roseland approximation [34], the local radiative heat flux term for optically thick gray nanofluid can be written as:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \approx -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial T}{\partial y}, \quad (7)$$

where $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$ (by employing the Taylor series approximation), k^* denotes the mean absorption coefficient, and σ^* is the Stefan-Boltzmann constant.

Thus,

$$\frac{\partial q_r}{\partial y} \approx -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (8)$$

Inserting Eq. (8) into Eq. (3), we obtain the energy balance equation as:

$$\begin{aligned} \frac{(\rho c_p)_{nf}}{k_f} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= \left[\left(\frac{k_{nf}}{k_f} \right) + \frac{16\sigma^* T_\infty^3}{3k^* k_f} \right] \frac{\partial^2 T}{\partial y^2} + \\ &+ \frac{\mu_{nf}}{k_f} \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\mu_{nf}}{k_f} + \frac{\sigma_{nf} B_0^2}{k_f} \right) u^2 \end{aligned} \quad (9)$$

Similarity transformations defined below are considered to simplify the governing partial differential equations into a system of ordinary differential equations

$$\begin{aligned} \eta &= y e^{\frac{x}{2L}} \sqrt{\frac{a}{2\nu_f L}}, \quad v = -e^{\frac{x}{2L}} \sqrt{\frac{a\nu_f}{2L}} (\eta f'(\eta) + f(\eta)), \\ M &= \frac{\sigma_f B_0^2 L}{\rho_f \mu_w}, \quad A_5 = \frac{(\rho c_p)_{nf}}{(\rho c_p)_f}, \quad A_1 = \frac{\mu_{nf}}{\mu_f}, \quad A_2 = \frac{\rho_{nf}}{\rho_f}, \\ A_3 &= \frac{k_{nf}}{k_f}, \quad Ec = \frac{U_w^2}{c_{pf}(T_w - T_\infty)}, \quad u = a e^{\frac{x}{2L}} f'(\eta), \\ A_4 &= \frac{\sigma_{nf}}{\sigma_f}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_w = T_\infty + T_0 e^{\frac{x}{2L}}, \\ Da &= \frac{K U_w}{\nu_f L}, \quad Ra = \frac{4\sigma^* T_\infty^3}{k^* k_f}, \quad Pr = \frac{\mu_f c_{pf}}{k_f}. \end{aligned} \quad (10)$$

Substituting Eq. (10) into Eqs. (2)–(9), we obtain the following local similarity equations:

$$f''' + \frac{A_2}{A_1} [f f'' - 2(f')^2] - 2 \left(\frac{A_4}{A_1} M + \frac{1}{Da} \right) f' = 0, \quad (11)$$

$$\begin{aligned} \left(A_3 + \frac{4Ra}{3} \right) \theta'' + A_5 Pr [f \theta' - f' \theta] + A_1 Pr Ec (f'')^2 + \\ + Pr Ec \left(A_4 M + \frac{A_1}{Da} \right) (f')^2 = 0 \end{aligned} \quad (12)$$

with the associated boundary conditions given as:

- at $y = 0$: $f'(\eta) = 1, \quad f(\eta) = 0, \quad \theta(\eta) = 1$ (13a)

- as $y \rightarrow \infty$: $f'(\eta) = 0, \quad \theta(\eta) = 0,$ (13b)

where Pr (≈ 6.2) is the Prandtl number (for water-based nanofluid), Ra is the thermal radiation parameter, with M as the magnetic field parameter, Ec is the Eckert number, and Da is the Darcy number. Other related quantities of interest include the skin friction coefficients (C_f) and the Nusselt number (Nu), which are given as:

$$C_f Re_x^{1/2} = A_1 \frac{d^2 f}{d\eta^2}(0), \quad (14a)$$

$$Nu Re_x^{-1/2} = - \left(A_3 + \frac{4Ra}{3} \right) \frac{d\theta}{d\eta}(0), \quad (14b)$$

where:

$$C_f = \frac{\tau_w}{\rho_f U_w^2}, \quad Nu = \frac{x q_w}{k_f (T_f - T_\infty)}, \quad \tau_w = \mu_{nf} \frac{\partial u}{\partial y}, \quad (15a)$$

$$q_w = - \left(k_{nf} + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial T}{\partial y}, \quad Re_x = \frac{x U_w}{\nu_f}. \quad (15b)$$

3. Numerical procedure

The nonlinear boundary value problem defined by Eqs. (11) and (12), along with the boundary conditions specified in Eq. (13), is solved numerically using the shooting method combined with the Runge-Kutta-Fehlberg integration technique [30–31]. To facilitate this approach, the boundary value problem (BVP) is reformulated as an initial value problem (IVP), and numerical integration is carried out using the Runge-Kutta-Fehlberg method (RKF45). To implement the method, new variables are introduced to represent the derivatives, such that:

$$f_1 = f, f_2 = f', f_3 = f'', \theta_1 = \theta, \theta_2 = \theta'. \quad (16)$$

Implementing Eq. (16), transform Eqs. (11)–(13) to:

$$f_3' = \frac{A_2}{A_1} (2f_2^2 - f_3 f_1) + 2f_2 \left(\frac{A_4}{A_1} M + \frac{1}{Da} \right), \quad (17)$$

$$\theta_2' = \frac{A_5 Pr (f_2 \theta_1 - f_1 \theta_2) - A_1 Pr Ec f_3^2 - Pr Ec f_2^2 \left(A_4 M + \frac{A_1}{Da} \right)}{A_3 + \frac{4Ra}{3}}, \quad (18)$$

with initial conditions:

$$\begin{aligned} f_2(0) = 1, \quad f_1(0) = 0, \quad \theta_1(0) = 1, \\ f_3(0) = a_1, \quad \theta_2(0) = a_2. \end{aligned} \quad (19)$$

The unspecified, initial values of a_1 and a_2 that emerged in Eq. (19) are first assumed, and then the values are estimated accurately through a shooting methodology using an iterative method employing Newton-Raphson's method. We define the first-order system of ODEs for f and θ as:

$$\mathbf{y}' = \mathbf{F}(\eta, \mathbf{y}), \quad (20)$$

where $\mathbf{y} = [f_1, f_2, f_3, \theta_1, \theta_2]^T$ and the components of $\mathbf{F}(\eta, \mathbf{y})$ are derivatives of Eqs. (16)–(18), and thereafter, the Runge-Kutta-Fehlberg integration scheme is applied. To validate the accuracy of our numerical procedure and benchmark the present results, we compare a special case from this study with previously published values of the skin friction coefficient reported in Refs. [36,37] for the conditions $\phi = 0$ and $Da \rightarrow \infty$, as presented in Table 2. The comparison demonstrates excellent agreement both qualitatively and quantitatively.

Table 2. Comparative data of $f''(0)$ for $2M$ when $\phi = 0$, $Da \rightarrow \infty$.

2M	Ref. [36]	Ref. [37]	Present
0.0	1.0000	1.0000	1.00000
0.25	1.1180	1.1180	1.11803
1.0	1.4137	1.4142	1.41421
5.0	2.4495	2.4495	2.44949
10	3.3166	3.3166	3.31662

2. Results and discussion

The graphical results derived from Eqs. (11) to (13) vividly illustrate the influence of key thermophysical flow parameters on the behaviour of nanofluid motion within porous and magnetically influenced environments, with direct relevance to various industrial and engineering systems. Figure 2 emphasises the role of the porous medium's permeability, quantified by the Darcy number, in the velocity distribution. As the Darcy number increases, indicating a more permeable medium, the fluid experiences reduced resistance due to lower drag forces imposed by the solid matrix. This results in a broadened momentum boundary layer and enhanced nanofluid velocity, which is critical for optimising flow systems such as packed-bed reactors, geothermal reservoirs, and filtration devices, where controlled permeability is essential for performance.

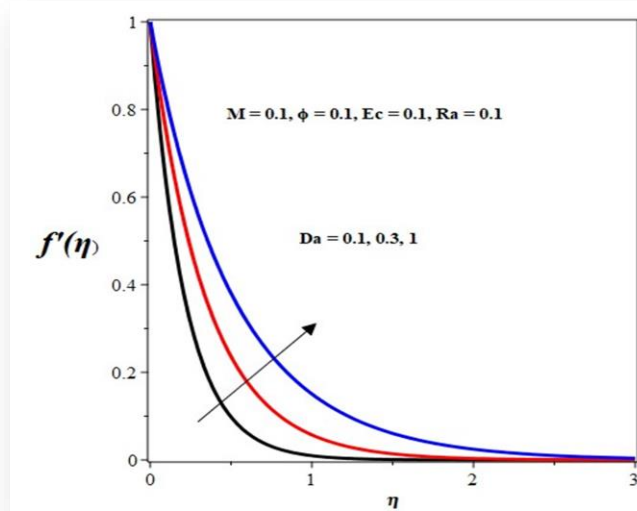


Fig. 2. Velocity profile with increasing Da .

Figure 3 shows how the strength of the magnetic field affects the fluid's speed, demonstrating that the Lorentz force works against the fluid's movement, which slows it down. At a nanoparticle concentration of 10%, the magnetic field's damping effect becomes more pronounced, especially in regions away from the boundary, eventually causing the velocity to approach zero in the free stream. Strong magnetic fields effectively draw the nanofluid closer to the heated surface, compressing the momentum boundary layer and thereby enhancing surface heat absorption.

This MHD behaviour is particularly advantageous in applications such as electromagnetic pumps, nuclear reactor cooling systems, and advanced cooling technologies in electronics and metallurgy, where precise control over flow and heat transfer is vital. Additionally, being able to control fluid flow and transfer heat by adjusting permeability and magnetic fields allows for the creation of advanced thermal systems and nanofluid heat exchangers in fields like aerospace, healthcare, and energy conversion.

Figures 4 to 8 clearly show how important thermophysical factors affect temperature distribution in the flow problem being studied, which is important for various engineering and industrial uses like thermal management systems, energy devices,

and material processing. Figure 4 shows that when the porous material allows more fluid to pass through, it improves heat transfer by convection rather than conduction because there is less resistance to the fluid's movement. In areas with low porosity, high friction creates a lot of internal heat, but as porosity increases, this heat is reduced, leading to lower temperatures. This is especially important in fields like geothermal engineering, filtration systems, and catalytic reactors where porous materials are essential.

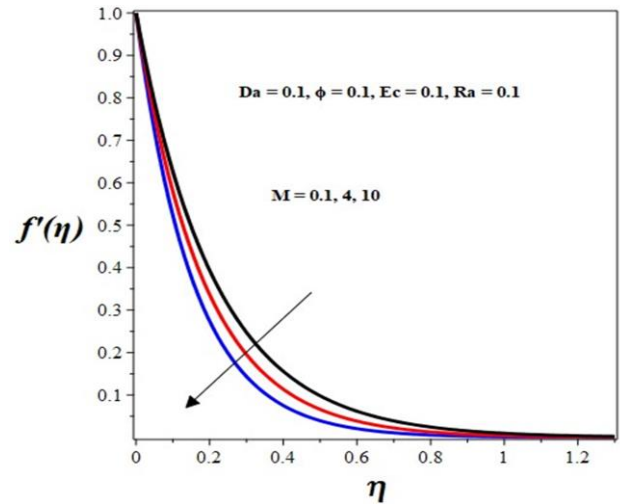


Fig. 3. Velocity profile with increasing M .

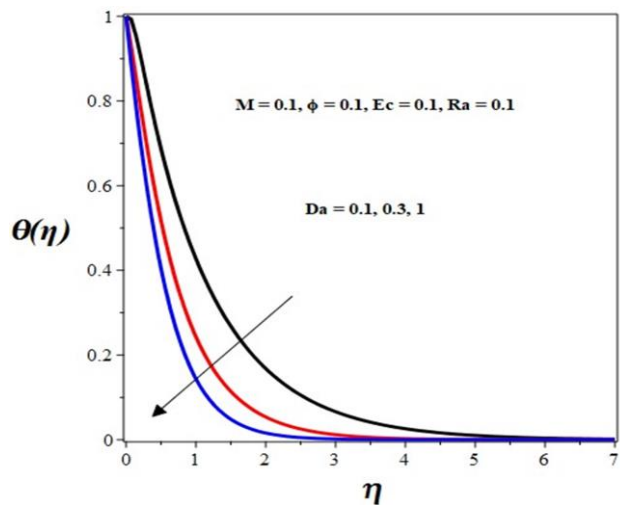


Fig. 4. Temperature profile with increasing Da .

Figure 5 shows that the introduction of the magnetic field imposes a Lorentz force that resists fluid flow and induces Joule heating by transforming kinetic energy into thermal energy. This results in a higher temperature within the boundary layer, especially near the surface, making magnetic control valuable in MHD power generators, plasma confinement, and metallurgical cooling processes. Figure 6 emphasises the role of thermal radiation, where increased radiative heat transfer augments the effective thermal conductivity of nanofluids. This mechanism, vital in high-temperature environments such as aerospace thermal shields and solar energy collectors, promotes enhanced thermal

dispersion and results in elevated fluid temperatures. In Fig. 7, the impact of the Eckert number, representing viscous dissipa-

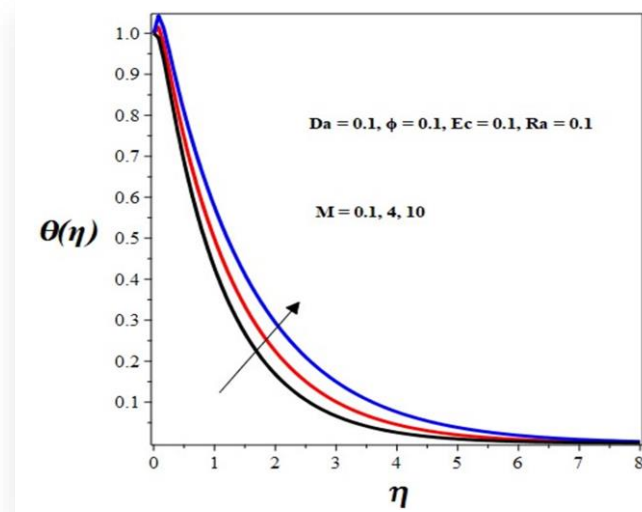


Fig. 5. Temperature profile with increasing M .

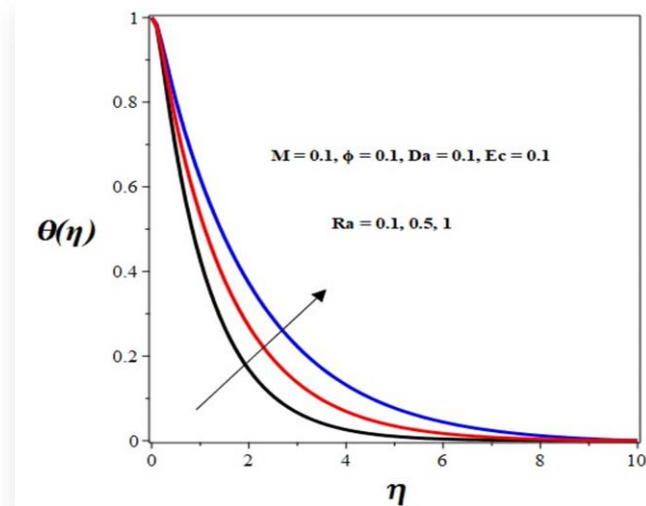


Fig. 6. Temperature profile with increasing Ra .

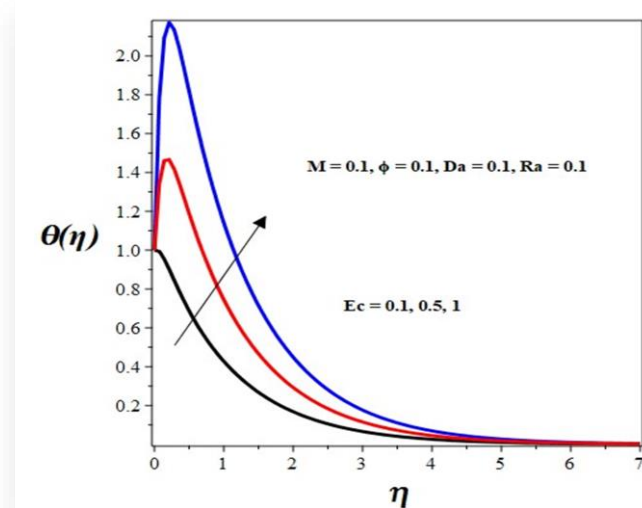


Fig. 7. Temperature profile with increasing Ec .

tion, becomes apparent: as this parameter rises, the conversion of kinetic energy into thermal energy intensifies, particularly near the heated plate surface, where velocity gradients are steep. This phenomenon is critical in high-speed fluid applications like turbine blade cooling and polymer extrusion processes.

Finally, Fig. 8 underscores the importance of nanoparticle volume fraction in nanofluid behaviour. An increase from 5% to 10% significantly thickens the thermal boundary layer and accelerates heat transfer, leading to a marked temperature rise. Such behaviour is highly advantageous in advanced heat exchangers, electronic cooling systems, and biomedical applications, where precise thermal control and efficient heat dispersal are paramount. Collectively, these findings underscore the pivotal role of thermophysical parameter tuning in optimising thermal performance across a wide range of engineering domains.

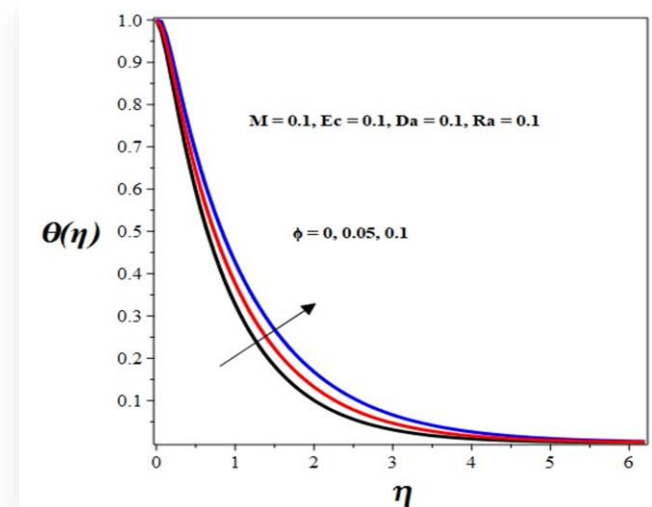


Fig. 8. Temperature profile with increasing ϕ .

Figures 9 to 11 show how the magnetic field parameter, Darcy number, nanoparticle volume fraction, and thermal radiation interact and affect the skin friction and Nusselt number, which are important in fluid dynamics and heat transfer. Figure 9 shows that when the magnetic field parameter is increased, skin friction goes up because the Lorentz force makes it harder for the fluid to move, pushing the flow closer to the surface and increasing resistance and possible wear on the surface. This effect is especially important in MHD applications like cooling nuclear reactors, using electromagnetic pumps, and in metal processing, where controlling fluid movement is essential. The amount of nanoparticles in a fluid is important; when there are more nanoparticles, it increases skin friction because the fluid becomes denser and thicker. This is especially important in areas like targeted drug delivery, solar energy systems, and managing heat in electronics. The Darcy number, which shows how easily fluid can flow through a porous material, is very important; low Darcy values mean less space for fluid to move, causing more friction, while higher values allow fluid to flow more easily and reduce friction, which is essential for creating efficient packed bed reactors, geothermal reservoirs, and filtration systems. Figure 10 shows that having too many particles in the mixture lowers the Nusselt number, meaning that too many

nanoparticles can act as an insulator and reduce heat flow. This information is important for optimising heat exchangers and thermal interface materials. Moreover, as the Darcy number increases, heat transfer at the surface improves due to enhanced convective flow, while stronger magnetic fields suppress the Nusselt number by impeding thermal boundary layer motion – key in applications involving MHD flows in aerospace engineering and advanced energy systems.

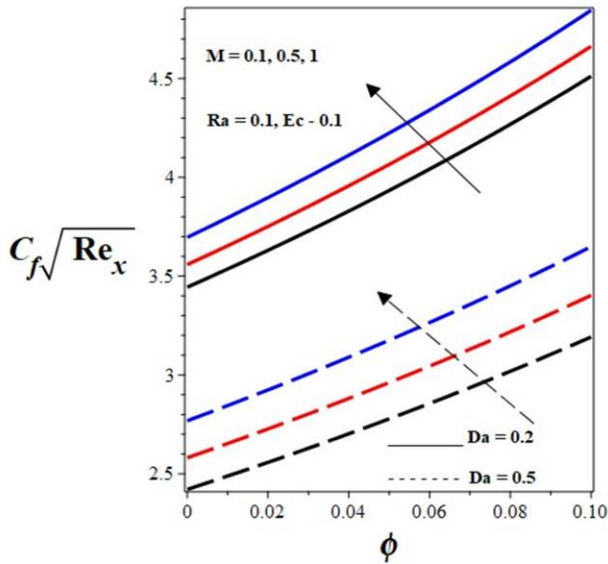


Fig. 9. Skin friction with increasing M and Da versus ϕ .

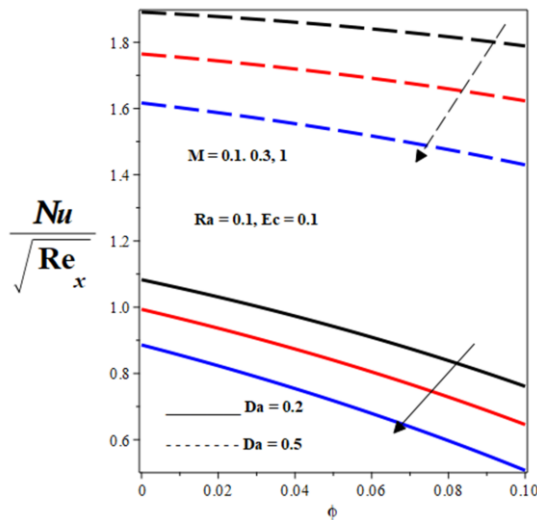


Fig. 10. Nusselt number with increasing M and Da versus ϕ .

Finally, Fig. 11 shows that both magnetic fields and thermal radiation significantly lower the Nusselt number, which means they reduce the ability to transfer heat through convection. This phenomenon has significant implications in high-temperature environments such as space vehicle thermal shields, radiative cooling systems, and solar thermal power plants, where understanding and mitigating heat transfer suppression due to electro-

magnetic and radiative forces is essential for maintaining performance and structural integrity.

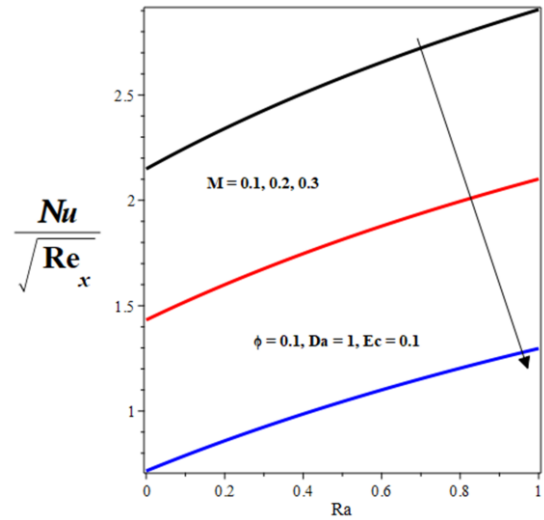


Fig. 11. Nusselt number with increasing Ec versus Ra .

5. Conclusions

This study examines the hydromagnetic convective flow of a radiating nanofluid past an exponentially stretching sheet embedded in a porous medium. The shooting numerical technique coupled with the Runge-Kutta-Fehlberg integration scheme is employed to tackle the nonlinear model problem. We present graphically and discuss pertinent results on the effects of various thermophysical parameters on the overall flow structure and thermal enhancement. The research elaborates its findings as follows:

- The Darcy number, which measures the permeability of the porous medium, increases fluid velocity by reducing flow resistance, impacting fields like filtration, petroleum engineering, and soil mechanics. In contrast, a magnetic field slows down fluid velocity due to the MHD effect, where the magnetic field interacts with charged particles in the fluid. This effect is important in industries like nuclear reactors, electrical power generation, and materials processing, where magnetic fields are used to control or stabilise fluid flows, especially in high-temperature environments.
- The study shows that the magnetic field and thermal radiation parameters increase the fluid's temperature profile, while the Eckert number raises the temperature by converting kinetic energy to thermal energy. Increased nanoparticle volume enhances thermal conductivity, boosting the temperature profile, while a higher Darcy number reduces the temperature by lowering the fluid's residence time in porous media. These factors are crucial in applications like cooling systems, aerospace, high-speed fluid flow, heat exchangers, and geothermal energy systems.
- The skin friction increases with a growing magnetic field, higher Darcy numbers, and larger nanoparticle volume fractions. The MHD effect raises viscous drag, while

higher Darcy numbers and nanoparticle concentrations enhance fluid viscosity, leading to more friction. These factors are critical in designing efficient fluid flow systems, porous media-based filtration, and catalysis, where minimising frictional losses is essential.

- The Nusselt number, representing convective heat transfer, decreases with higher magnetic field strength, as does the Darcy number, reducing heat transfer efficiency in systems like heat exchangers and reactors. A higher amount of nanoparticles makes it harder for heat to transfer in cooling systems because it thickens the layer of heat around them. Also, when stronger magnetic fields and more thermal radiation work together, they further lower the Nusselt number, showing the balance between radiation and convection, which is crucial in high-temperature industries like materials processing and aerospace.

The study's findings are relevant to several industrial applications involving fluid flow in porous media, heat transfer, and nanofluids. These include energy systems (geothermal, oil recovery, and heat exchangers), where managing Darcy numbers, magnetic fields, and nanoparticle contents enhances efficiency. Nanofluids are also used in cooling systems for electronics, automotive engines, and industrial heat exchangers, where controlling nanoparticle volume and magnetic fields optimises performance. In chemical engineering, these results benefit processes like catalysis and filtration by improving fluid flow and heat transfer. Additionally, the findings assist in managing heat dissipation in aerospace and high-temperature systems like combustion chambers and nuclear reactors. Overall, the study provides valuable insights for optimising heat transfer and flow in various engineering applications. In the future, this study will be broadened to look into how changes in flow, shrinking and slippery surfaces, the effects of porous materials, entropy, thermophoresis, and the movement of nanoparticles affect the results.

References

- [1] Abbas, N., Shatanawi, W., Hasan, F., & Shatnawi Taqi, A.M. (2025). Investigation of MHD radiative Casson micropolar hybrid nanofluid over an exponential curved stretching sheet. *Journal of Radiation Research and Applied Sciences*, 18(1), 101269. doi: 10.1016/j.jrras.2024.101269
- [2] Nayak, M.K., Pandey, V.S., Tripathi, D., Akbar, N.S., & Makinde, O.D. (2020). 3D MHD cross flow over an exponential stretching porous surface. *Heat Transfer*, 49(3), 1256–1280. doi: 10.1002/htj.21661
- [3] Crane, L.J. (1970). Flow past a stretching plate. *Journal of Applied Mathematics and Physics (Zeitschrift für Angewandte Mathematik und Physik)*, 21, 645–647. doi: 10.1007/BF01587695
- [4] Elbashbeshy, E.M.A. (1998). Heat transfer over a stretching surface with variable surface heat flux. *Journal of Physics. D: Applied Physics*, 31(16), 1951–1954.
- [5] Mahapatra, T.R., & Gupta, A.S. (2002). Heat transfer in stagnation-point flow towards a stretching sheet. *Heat and Mass Transfer*, 38, 517–521. doi: 10.1007/s002310100215
- [6] Chaudhary, S., Choudhary, M.K., & Sharma, R. (2015). Effects of thermal radiation on hydromagnetic flow over an unsteady stretching sheet embedded in a porous medium in the presence of a heat source or sink. *Meccanica*, 50, 1977–1987. doi: 10.1007/s11012-015-0137-9
- [7] Alkaoud, A., Khader, M.M., Eid, A. & Megahed, A.M. (2024). Numerical simulation of the flow of a tangent hyperbolic fluid over a stretching sheet within a porous medium, accounting for slip conditions. *Heliyon*, 10, e28683. doi: 10.1016/j.heliyon.2024.e28683
- [8] Nawaz, Y., Arif, M.S., Nazeer, A., Abbasi, J.N., & Abodayeh, K. (2024). A two-stage reliable computational scheme for stochastic unsteady mixed convection flow of Casson nanofluid. *International Journal for Numerical Methods in Fluids*, 96(5), 719–737. doi: 10.1002/flid.5264
- [9] Saleem, M., Hussain, M., & Inc, M. (2023). Significance of Darcy–Forchheimer law and magnetic field on the comparison of Williamson–Casson fluid subject to an exponential stretching sheet. *International Journal of Modern Physics B*, 37(27), 2350315. doi: 10.1142/S0217979223503150
- [10] Vajravelu, K., & Roper, T. (1999). Flow and heat transfer in a second-grade fluid over a stretching sheet. *International Journal of Non-linear Mechanics*, 34(6), 1031–1036. doi: 10.1016/S0020-7462(98)00073-0
- [11] Sajid, M., Ali, N., Javed, T., & Abbas, Z. (2010). Stretching a curved surface in a viscous fluid. *Chinese Physics Letters*, 27(2), 024703. doi: 10.1088/0256-307X/27/2/024703
- [12] Abbas, Z., Naveed, M., & Sajid, M. (2016). Hydromagnetic slip flow of nanofluid over a curved stretching surface with heat generation and thermal radiation. *Journal of Molecular Liquids*, 215, 756–762. doi: 10.1016/j.molliq.2016.01.012
- [13] Hussain, Z., Hayat, T., Alsaedi, A., & Anwar, M.S. (2021). Mixed convective flow of CNTs nanofluid subject to varying viscosity and reactions. *Scientific Reports*, 11, 22838. doi: 10.1038/s41598-021-02228-9
- [14] Othman, M.N., Jedi, A., & Bakar, N.A.A. (2021). MHD stagnation point on nanofluid flow and heat transfer of carbon nanotube over a shrinking surface with heat sink effect. *Molecules*, 26, 7441. doi: 10.3390/molecules26247441
- [15] Majeed, A., Zeeshan, A., & Alam, T. (2023). Mathematical analysis of MHD CNTs of rotating nanofluid flow over a permeable stretching surface. *Arab J. Sci. Eng*, 48, 727–737, doi: 10.1007/s13369-022-06871-w
- [16] Choudhary, P., Choudhary, S., Jat, K., Loganathan, K., & Eswaramoorthi, S. (2024). Impacts of unsteady MHD hybrid nanofluid over a non-linear stretchable porous sheet with thermal radiation and gyrotactic microorganisms. *International Journal of Thermofluids*, 23, 100788, doi: 10.1016/j.ijft.2024.100788
- [17] Sharma, R.P., Baag, S., Mishra, S.R., & Sahoo, P. (2023). On the radiative heat transport phenomena in MHD Williamson nanofluid flow past an expanding surface with an interaction of inclined magnetic field. *Journal of Thermal Analysis Calorimetry*, 148(14). doi: 10.1007/s10973-023-12206-0
- [18] Sakkaravarthi, K., & Reddy, B.A. (2023). Entropy generation on MHD Casson and Williamson hybrid nanofluid over a curved stretching sheet with the Cattaneo–Christov heat flux model: semi-analytical and numerical simulations. *International Journal of Ambient Energy*, 44(1). doi: 10.1080/01430750.2023.2190338
- [19] Makinde, O.D., & Eegunjobi, A.S. (2017). MHD couple stress nanofluid flow in a permeable wall channel with entropy generation and nonlinear radiative heat. *Journal of Thermal Science and Technology*, 12(2). doi: 10.1299/jtst.2017jtst0033
- [20] Srinivasulu, T., & Goud, B.S. (2021). Effect of inclined magnetic field on flow, heat, and mass transfer of Williamson nanofluid over a stretching sheet, *Case Studies in Thermal Engineering*, 23, 100819. doi: 10.1016/j.csite.2020.100819

- [21] Kiwan, S., & Ali, M.E. (2008). Near-slit effects on the flow and heat transfer from a stretching plate in a porous medium. *Numerical Heat Transfer, Part A: Applications*, 54(1), 93–108. doi: 10.1080/10407780802025275
- [22] Tamayol, A., Hooman, K., & Bahrami, M. (2010). Thermal analysis of flow in a porous medium over a permeable stretching wall. *Transport in Porous Media*, 85(3), 661–676. doi: 10.1007/s11242-010-9584-x
- [23] Pal, D., & Chatterjee, S. (2010). Heat and mass transfer in MHD non-Darcian flow of a micropolar fluid over a stretching sheet embedded in a porous medium with non-uniform heat source and thermal radiation. *Communications in Nonlinear Science and Numerical Simulation*, 15(7), 1843–1857. doi: 10.1016/j.cnsns.2009.07.024
- [24] Krishna, M.V., Ahammad, N.A., & Chamkha, A.J. (2021). Radiative MHD flow of Casson hybrid nanofluid over an infinite exponentially accelerated vertical porous surface. *Case Studies in Thermal Engineering*, 27, 101229. doi: 10.1016/j.csite.2021.101229
- [25] Eegunjobi, A.S., & Makinde, O.D. (2014). Second law analysis for MHD permeable channel flow with variable electrical conductivity and asymmetric Navier slips. *Open Physics*, 13(1), 100–110, doi: 10.1515/phys-2015-0014.
- [26] Chamkha, A. J, Issa, C., & Khanafer, K. (2002). Natural convection from an inclined plate embedded in a variable porosity porous medium due to solar radiation. *International Journal of Thermal Sciences*, 41(1), 73–81. doi: 10.1016/S1290-0729(01)01305-9
- [27] Makinde, O.D., & Sibanda, P. (2008). Magnetohydrodynamic mixed convective flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction. *Journal of Heat Transfer*, 130(11). doi: 10.1115/1.2955471
- [28] Hayat, T., Qasim, M., & Mesloub, S. (2011). MHD flow and heat transfer over permeable stretching sheet with slip conditions. *International Journal for Numerical Methods in Fluids*, 66, 963–975. doi: 10.1002/flid.2294
- [29] Khan, M.S., Karim, I., Islam, M.S., & Wahiduzzaman, M. (2014). MHD boundary layer radiative, heat-generating, and chemical reacting flow past a wedge moving in a nanofluid. *Nano Convergence*, 1(20). doi: 10.1186/s40580-014-0020-8
- [30] Cebeci, T., & Bradshaw, P. (1988). *Physical and computational aspects of convective heat transfer*. New York, USA, Springer.
- [31] Na, T.Y. (1979). *Computational methods in engineering boundary value problems*. Academic Press. New York.
- [32] Brinkman, H.C. (1952). The viscosity of concentrated suspensions and solutions. *The Journal of Chemical Physics*, 20(4), 571–571. doi: 10.1063/1.1700493
- [33] Kefene, Z.M., Makinde, O.D., & Enyadene, G.L. (2022). Magnetic Effects of Ferrofluid Flow in a Heated Micro-Channel with Permeable Walls. *Latin American Applied Research*, 52(3), 247–257. doi: 10.52292/j.laar.2022.666
- [34] Eegunjobi, A.S., & Makinde, O.D. (2017). Irreversibility analysis of hydromagnetic flow of couple stress fluid with radiative heat in a channel filled with a porous medium. *Results in Physics*, 7, 459–469. doi: 10.1016/j.rinp.2017.01.002
- [35] Putra, N., Roetzel, W. & Das, S.K., (2003). Natural convection of nanofluids. *Heat and Mass Transfer*, 39, 775–784. doi: 10.1007/s00231-002-0382-z
- [36] Waqas, H., Kafait, A., Muhammad, T., & Farooq, U. (2022). Numerical study for bio-convection flow of tangent hyperbolic nanofluid over a Riga plate with activation energy. *Alexandria Engineering Journal*, 61(2), 1803–1814. doi: 10.1016/j.aej.2021.06.068
- [37] Amjad, M., Khan, M.N., Ahmed, K., Ahmed, I., Akbar, T., & Eldin, S.M., (2023). Magnetohydrodynamics tangent hyperbolic nanofluid flow over an exponentially stretching sheet: Numerical investigation. *Case Studies in Thermal Engineering*, 45, 102900. doi: 10.1016/j.csite.2023.102900
- [38] Chamkha, A.J., & Pop, I. (2004). Effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium. *International Communications in Heat and Mass Transfer*, 31(3), 421–430. doi: 10.1016/j.icheatmasstransfer.2004.02.012

Optimisation of the control curves of load-changing nuclear power units taking into account thermal stress

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Received: 22.01.2025; revised: 22.10.2025; accepted: 21.12.2025

Abstract

Besides the increasing share of uncontrollable renewables in the energy mix, the consumption/production balance of the grid must be permanently assured. Additionally, the successive stopping of traditional, fossil-fuelled power plants necessitates the controllability also of nuclear power plants. The major drawback of the load-following operation of such units is the side effect of load changes. They are, namely, carried out via non-stationary transitions, during which the temperature changes result in thermal stresses. The most critical, practically unchangeable part of the plant is the reactor pressure vessel, which is a thick-walled component of complex geometry. Hence, determining the optimal curves of the unit's available control inputs is a crucial issue. For this, several tools are needed, such as a dynamical model of the entire power unit, a dynamical description of the thermal stress in the most critical points, a practical system of criteria of the optimisation condition, and numerical method for seeking the optimal pathway. The results demonstrate that optimised actuator trajectories can reduce thermally induced stress in the nozzle corner of the reactor pressure vessel during load-following operation.

Keywords: Thermal stress; Load change; Optimised control; Nuclear power plant; Reactor pressure vessel

Vol. 46(2025), No. 4, 41–48; doi: 10.24425/ather.2025.156835

Cite this manuscript as: Pudleiner, B., Szentannai, P., Désesquelles, L., Matter, C., & Fekete, T. (2025). Optimisation of the control curves of load-changing nuclear power units taking into account thermal stress. *Archives of Thermodynamics*, 46(4), 41–48.

1. Introduction

The major bottleneck of electrical energy is well known, as its storage is still not solved in an efficient way and in an appropriate amount. Consequently, a permanent balance between actual consumption and production is a must. Because the consumption side practically cannot be influenced and because some uncontrollable elements have also been appearing on the production side, the rest of this side must satisfy the entire burden of controllability. As highlighted by Marusic et al. [1], the growing

share of uncontrollable renewables, together with the decreasing amount of easy-to-control fossil-fuelled power plants, results in the need for the load-following operation of rather large power-generating units. The difficulty is that such units were traditionally handled as permanent-load ones.

Nuclear power plants (NPPs) are typical examples of this type, as discussed in the study by Raskovic et al. [2], and they are characterised by outstandingly high relevance of security, economic, and ecological issues. Just because of these reasons, a basic question is whether they are capable of load-following operation at all.

Nomenclature

c	– specific heat capacity, J/(kg K)
E	– total internal energy, J
E	– Young’s modulus, Pa
$\mathbf{F}$	– force vector, N
h	– specific enthalpy, J/kg
$\mathbf{I}$	– identity tensor
J	– dimensionless cost function
m	– mass, kg
$\dot{m}$	– mass flow rate, kg/s
p	– pressure, Pa
$\mathbf{Q}$	– weighting matrix for the control accuracy
$\dot{Q}$	– heat transfer rate into the system, W
$\dot{q}$	– volumetric heat transfer rate, W/m ³
$\mathbf{r}$	– reference signal
$\mathbf{R}$	– weighting matrix for the manipulated variable change
t	– time, s
T	– temperature, K
$\mathbf{u}$	– local fluid velocity vector, m/s
$\Delta\mathbf{u}$	– manipulated variable change (between two time steps)
$\mathbf{v}$	– velocity vector, m/s
V	– volume, m ³
$\dot{W}$	– work done by the system, W
y	– controlled variable (process output)

Greek symbols

$\boldsymbol{\varepsilon}$	– strain tensor
κ	– thermal conductivity, W/(m K)
ν	– Poisson’s ratio

ρ	– density, kg/m ³
σ	– stress, Pa
$\boldsymbol{\sigma}$	– stress tensor, Pa
$\boldsymbol{\tau}$	– shear stress tensor, Pa

Subscripts and Superscripts

1, 2, 3–	first, second, third
<i>ext</i>	– external
<i>in</i>	– incoming
<i>l</i>	– liquid
<i>ls</i>	– fluid and solid
<i>out</i>	– outgoing
<i>s</i>	– solid
<i>T</i>	– transpose of the vector
VM–	von Mises
<i>I</i>	– first scalar invariant

Abbreviations and Acronyms

FEA	– finite element analysis
normd.	– normalised: divided by the maximal value*
NPP	– nuclear power plant
PID	– proportional-integral-derivative
Pos.	– position
Rel.	– relative: deviation from the point of linearization*
RPV	– reactor pressure vessel
vlv	– valve

* Note that most numerical results are shown as ‘rel., normd.’ as deviations from the point of linearisation in a normalised (dimensionless), hence have an easy-to-compare form.

A high number of operational experiences definitely answers this question with a yes. Germany is a well-documented example, where several units operated in a load-following mode nearly continuously throughout the year, as confirmed by Ludwig et al. [3] and Grünwald and Caviezel [4]. This operational flexibility is often referred to as ‘manoeuvrability’, a term borrowed from aviation, where faster control typically comes at the cost of higher structural stress [5].

The most critical technical challenge of manoeuvring operation in NPPs is the thermal stress induced in thick-walled components, especially in the reactor pressure vessel (RPV). Among its regions, the nozzle corner is widely recognised as the most vulnerable due to geometric complexity and stress concentration effects [6–8].

While the control side of load-following has been extensively studied, the structural integrity side – particularly the dynamic modelling of thermal stress – has received less attention. Recent studies [5] have proposed integrated modelling approaches that combine fluid dynamics, heat transfer and solid mechanics to derive time-dependent stress functions suitable for control-oriented optimisation. Furthermore, the long-term impact of thermal stress is material ageing, which can be assessed using either cycle-based (Palmgren-Miner) or time-dependent (Lorenzo-Ray) damage models [9,10]. These models are increasingly being incorporated into control strategies to extend component lifetime while maintaining grid compliance.

For investigating the potential of NPPs to participate in the load control, it is necessary to determine their optimal behaviour. For this, these dynamical systems must be mathematically described, that is, modelled. If a model is ready and programmed, it can be easily turned into a linearised one. Because of some practical reasons, for simplicity, throughout this study, we consider the linearised model and the linear ramp of the load change as the expectation of the grid operation.

The main contribution of this study is to introduce a new procedure for determining the optimal route of the plant’s real inputs, i.e. the actuators, while also considering the side-effects, i.e. the caused thermal stresses. The resulting trajectories can then be used as reference pathways for traditional proportional-integral-derivative (PID) controllers or as benchmarks for assessing any controller performances.

2. Dynamical model of the power unit

The power unit itself has to be modelled in the first step, and, on this basis, a model for calculating thermal stress must be built. Both model parts are needed for the optimisation of the control curves of the entire system. The first part of the dynamical model is introduced in the current section (Sec. 2), while the second one is presented in the subsequent one (Sec. 3).

The inputs of the model of the power unit are the variables that also serve as inputs in reality, that is, the control rod positions and the main steam valve positions. Similarly, all those variables are modelled as outputs that may be used as measured

values for any control strategies, such as temperatures in both the primary and secondary circuits, flow rates, neutron fluxes, water levels, etc. Further, the process variables that can act as the inputs for thermal stress calculations are also handled as outputs in this model part.

A central item in any lumped dynamical model is the state variable. This variable defines the model's internal state at any given time and serves as the foundation for describing time-dependent processes. Accordingly, these models are often referred to as state space models, and their dynamical nature arises precisely from the evolution of state variables over time. At the heart of lumped models lie the fundamental conservation laws, which form the primary differential equations governing the system's behaviour. These include: conservation of mass, conservation of momentum and conservation of energy.

The general form of these conservation laws can be expressed as follows:

Conservation of mass:

$$\frac{dm}{dt} = \Sigma \dot{m}_{in} - \Sigma \dot{m}_{out}, \quad (1)$$

where m is the total mass within the control volume, $\dot{m}_{in}$ and $\dot{m}_{out}$ are the incoming and outgoing mass flow rates, respectively.

Conservation of momentum, which is typically omitted in lumped models unless forces or accelerations are of primary concern; if required, it takes the form:

$$\frac{d(\rho V \mathbf{v})}{dt} = \Sigma \mathbf{F}, \quad (2)$$

where ρ is the fluid density, V is the volume of the control region, $\mathbf{v}$ is the velocity vector and $\mathbf{F}$ is the force vector acting on the control volume. However, in the current model, momentum conservation is not explicitly included.

Conservation of energy:

$$\frac{dE}{dt} = \Sigma \dot{Q} - \Sigma \dot{W} + \Sigma \dot{m}h, \quad (3)$$

where E is the total internal energy of the control volume, $\dot{Q}$ is the heat transfer into the system, $\dot{W}$ is the work done by the system (e.g. shaft or flow work), $\dot{m}$ is the mass flow rate and h is the specific enthalpy of the inflowing or outflowing fluid.

A well-balanced mathematical model was set up by targeting accuracy and numerical effectiveness at the same time. It was achieved by our in-house model, which includes 12 state variables covering the above conservation laws as per Table 1.

Table 1. Conservation equations in the dynamical model of the power unit.

Type	No.	Applied for
Mass conservation	4	Neutrons, delayed neutron precursor atoms, steam (several locations)
Momentum conservation	0	–
Energy conservation	8	Thermal energy stored in the fuel rods, cladding, moderator (several locations), steam generator and pipe (several locations)

Besides the above conservation equations, their internal variables also have to be calculated. It can be done by algebraic equations (no differential equations) and their sources are rather diverse. Throughout setting up the lumped model, its parts can be taken from the basic physical rules, reliable approximations and phenomenological equations, and their selection and formulation are exciting aspects of model building. Textbooks can be beneficially used here (see, e.g. Walter et al. [11] for the most relevant systems of the current topic, and Corriou [12] for additional insights), along with tables or calculation methods for material properties and often also the latest publications and own measurements. It should be noted that this way of model development is referred to as theoretical or first-principle model building, in contrast to the empirical method. The latter relies on input-output data measured on the existing full-sized facility. A mixed use of these two basic directions is also possible, as demonstrated by Rossi et al. [13].

Accordingly, the first-principle modelling approach was applied in the actual work, and because control systems may sense and act on discrete points only, the proposed model class is that of the lumped models, as introduced in more detail by Szentannai et al. [14].

3. Dynamical calculation of thermal stress at the most critical point

Non-stationary states cause thermal stresses in the structural materials of nuclear power plants. Among these, the RPV is of particular concern due to its large wall thickness, complex geometry, and the fact that it is practically irreplaceable. Within the RPV, the nozzle region – especially the nozzle corner – has been consistently identified as the most critical location in terms of stress concentration and fatigue vulnerability. This is supported by multiple studies.

Li et al. [6] conducted an engineering critical assessment of RPV nozzle corner cracks under pressurised thermal shocks and highlighted the nozzle corner as the most vulnerable area due to geometric stress intensification. Liu et al. [7] and Cheng et al. [15] further confirmed that the nozzle-cylinder intersection is highly susceptible to crack initiation and propagation, particularly under transient thermal loads. Trampus [8] emphasised that the nozzle corner's geometry and material constraints make it the limiting factor in manoeuvring operations. Additionally, Rabazzi et al. [16] performed a detailed structural integrity assessment of the nozzle during thermal shock events, reinforcing its criticality.

Based on these findings, the nozzle corner was selected as the focus of our analysis. To quantify the thermal stresses in this region, we developed a dynamic model through an in-depth investigation.

A finite element analysis was carried out, which consisted of three main steps:

- fluid mechanical calculations,
- numerical modelling of heat transfer phenomena, and
- computation of stress-strain fields within the steel body.

The fluid flow in the analysed region was modelled using the Navier-Stokes equation and the continuity equation, both applied for incompressible fluids:

$$\rho_l \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \mathbf{F}_{ext}, \quad (4)$$

$$\nabla \cdot (\rho_l \mathbf{u}) = 0, \quad (5)$$

where $\mathbf{u}$ is the local fluid velocity vector, ρ is the fluid density, p is the pressure, $\boldsymbol{\tau}$ is the local shear stress tensor and $\mathbf{F}_{ext}$ summarises the external forces.

The fluid flow governs the heat exchange between the fluid and solid domains. The energy equation for the fluid phase is

$$\rho_l c_l \frac{\partial T_l}{\partial t} + \rho_l c_l (\mathbf{u} \cdot \nabla) T_l - \nabla \cdot (\kappa_l \nabla T_l) = \dot{q}_{ls}, \quad (6)$$

where T_l is the fluid temperature, c_l is the fluid specific heat capacity, κ_l is the fluid thermal conductivity and $\dot{q}_{ls}$ is the volumetric heat transfer rate between fluid and solid.

In the solid domain, where there is no convective motion, the energy equation simplifies to

$$\rho_s c_s \frac{\partial T_s}{\partial t} - \nabla \cdot (\kappa_s \nabla T_s) = -\dot{q}_{ls}, \quad (7)$$

where T_s is the solid temperature, ρ_s is the solid density, c_s is the solid specific heat capacity and κ_s is its thermal conductivity. In regions away from fluid contact $\dot{q}_{ls} = 0$.

The steel structure is considered to behave as an isotropic, linear thermoelastic material. The stress tensor $\boldsymbol{\sigma}$ is given by the following formulation [19]:

$$\boldsymbol{\sigma} = \frac{E}{1+\nu} \left[\boldsymbol{\varepsilon} + \frac{\nu}{1-2\nu} \varepsilon_I \mathbf{I} \right] - \frac{E}{1-2\nu} \alpha (T - T_{ref}) \mathbf{I}, \quad (8)$$

where E is Young's modulus, ν is Poisson's ratio, α is the thermal expansion coefficient, $\boldsymbol{\varepsilon}$ is the strain tensor, ε_I is the first scalar invariant of the strain tensor, T is the local temperature, T_{ref} is the reference temperature and $\mathbf{I}$ is the identity tensor.

The principal stresses σ_1 , σ_2 , and σ_3 are calculated as the eigenvalues of $\boldsymbol{\sigma}$. From these, the von Mises stress is determined as

$$\sigma_{VM} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}. \quad (9)$$

This scalar quantity characterises the effective stress at each point in the material. The regions of highest σ_{VM} values align with the expected stress concentrations. Due to symmetry, only one critical zone is considered for further calculations.

As a result of the above series of finite element calculations, the stress field around the critical point inside the steel body could be obtained, as shown in Fig. 1.

The time-dependent maximum of the thermal stress was then calculated as a consequence of a (theoretically applied) stepwise change in the input. This time function is the *step response function* [12] of this subsystem, which can already be integrated with the dynamical model of the power unit, as discussed in the previous subsection.

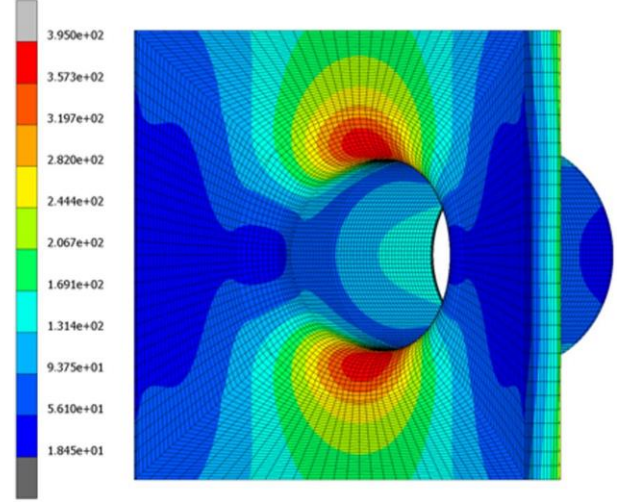


Fig. 1. Von Mises stress field around the critical point in the course of increasing the unit load; 300 s after stepwise temperature increase.

4. System of criteria for the optimisation

The consumer of the product that a power plant offers is the electrical grid, hence, the criteria formulated on this side are the first ones to be considered. In the synchronous area of Continental Europe, it is clearly formulated and summarised by a regulation of the European Commission [17]. According to this, limiting curves are given, between which the electrical power output must be kept throughout any load-changing transients. Although this regulation allows a certain freedom between the two limiting curves, throughout this study, we simply follow the more remote limiting line. This is because our preliminary investigations show that the optimum is in the very close proximity of this line (Fig. 2), hence, this simplifying decision does not result in any noticeable influences on the global optimisation result.

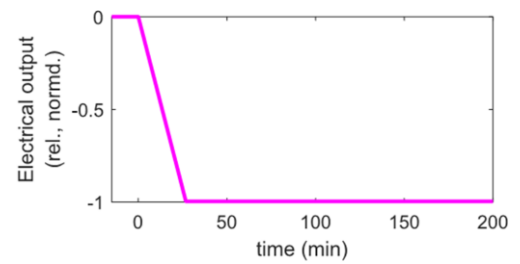


Fig. 2. The setpoint of the electrical power output vs. time as one of the criteria for transient load change formulated by the grid operator.

A first attempt at solving the control problem could be a proportional, purely linear moving of both actuators within the prescribed time interval from their original positions to the final ones, determined by the final desired operating point (Fig. 3). In the presence of a dynamical plant model (see Sec. 2), the plant outputs can be calculated as functions of time, as shown in Fig. 4. These plant outputs are the electrical output, the steam pressure

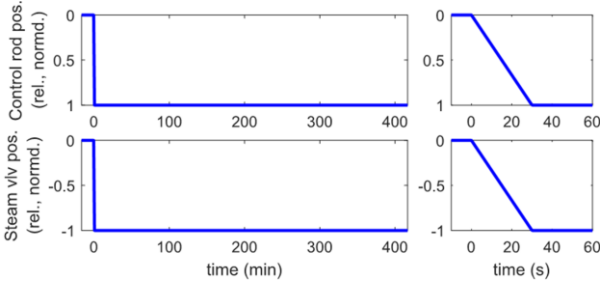


Fig. 3. Purely linear control of the actuators between their initial and final positions with filling the allowed time frame – to be used as the reference case throughout the present study.

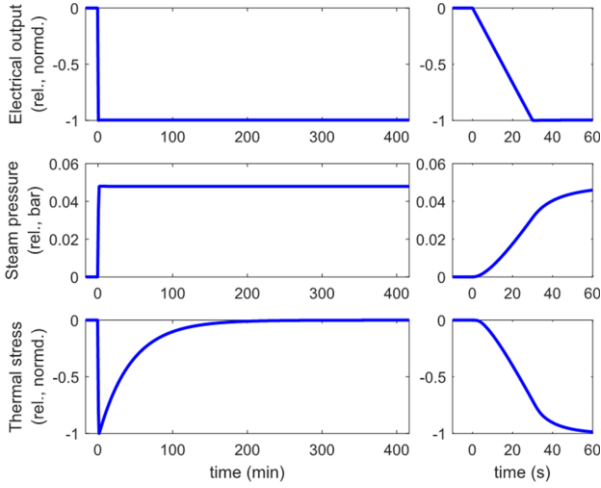


Fig. 4. Plant outputs as functions of time as consequences of the basic control strategy – to be used as the reference case throughout the present study.

in the secondary circuit and the von Mises stress at the most critical point. As visible in the top right high-time-resolution graph in this figure, this control concept fulfils the first criterion. Note that this scenario will be used as the reference one with the same normalising values throughout this study.

Further optimisation criteria come from the capabilities of the plant itself. As the first one, the speed of the control rods is limited to 0.2 m/s in the investigated plant. For formulating the next one, the degree of freedom of such plants must be considered. Based on these thoughts and based on the two known modes of traditional reactor control, either the live steam pressure in the secondary circuit or the average coolant temperature in the primary circuit can be freely chosen and kept constant. Throughout this study, we consequently apply the former one, as this seems to be the more often implemented one. Finally, all criteria together must be formulated in an integrated form. For this, we apply the quadratic cost function:

$$J = \int_{\text{time frame}} (\mathbf{r} - \mathbf{y})^T \mathbf{Q} (\mathbf{r} - \mathbf{y}) + \Delta \mathbf{u}^T \mathbf{R} \Delta \mathbf{u} dt. \quad (10)$$

This form is extensively applied in both theoretical and practical control applications due to its analytical tractability and effectiveness in balancing multiple objectives. The definitions of the vector variables and matrices are as follows: $\mathbf{r}(t)$ – the reference

signal (desired output trajectory), $\mathbf{y}(t)$ – the actual process output, $\mathbf{u}(t)$ – the control input (e.g. actuator positions), $\Delta \mathbf{u}(t)$ – the rate of change of the control input (i.e. actuator speed), $\mathbf{Q}$ – weighting matrix for the control accuracy, penalising output tracking error, $\mathbf{R}$ – weighting matrix for the manipulated variable change penalising control effort.

This quadratic optimisation problem can also be numerically solved while considering several constant or time-dependent constraints. The cost function defined by Eq. (10) is a widely used quadratic form in optimal control theory [18]. Its application in nuclear reactor control has also been demonstrated in recent studies, where it enables the simultaneous consideration of control accuracy, actuator effort and thermal stress.

Based on the above possibilities, also for considering thermal stress, the same optimum-seeking procedure can be applied in two ways: either by putting a constraint or by making the thermal stress more marked. These two methods are detailed and commented on below.

4.1. Weight on thermal stress

To follow established practices, the pressure should return to its initial value after the transitory state. In order to do so, a weight can be put on this output to be stricter on the correspondence between this value and its reference, which remains unchanged. A time-varying weight is applied to the pressure value to make it slowly but surely reach the right value. This weight on the pressure, increasing linearly over time, assures stabilising the pressure at its initial value at the end of the transient.

Of course, thermal stress must also be weighted. A very large weight was applied throughout the transient period, which can then be removed later. An appropriate set of these two time-dependent weights was applied, as shown in Fig. 5.

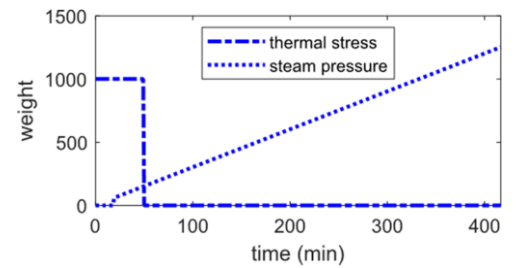


Fig. 5. Time-dependent weights applied in the cost function for fulfilling the system of criteria.

4.2. Constraint on thermal stress

Another possibility offered by the quadratic optimisation method is to apply constraints on thermal stress. This approach is useful in cases when material ageing is considered to be negligible by low thermal stress values, while it becomes harmful after overstepping a certain limit.

5. Numerical method for seeking the optimal pathway

Effective numerical optimum seeking procedures have been available for many decades. In the current study, its realisation

within the Matlab framework, as demonstrated by Bemporad et al. [18], was applied. According to its hints, we applied a simplification to achieve a numerically very cheap and effective solution. It is based on a step-by-step optimisation of Eq. (10) and constraints, in such a way that it always seeks the next optimal move of the process inputs, instead of doing the same for the entire set of future time instances.

Throughout the dynamic optimisation procedure, the σ_{VM} thermal stress was considered as one component in the y output vector of the process. This σ_{VM} was calculated by means of the *step response function*, which had to be achieved once as the final result of the finite element modelling, as described in Section 3.

6. Results and discussion

The optimisation procedure presented in this study aimed to determine actuator trajectories that minimise thermally induced stress in critical components of nuclear power plants during load-following operation. The focus was on the nozzle corner of the reactor pressure vessel, which is widely recognised as the most vulnerable region due to its geometric complexity and stress concentration effects. Two optimisation strategies were investigated: one treating thermal stress as a constrained variable and another, incorporating it as a weighted term in the cost function. In both cases, the optimisation was performed using a two-scale time framework: a fast scale (seconds) to satisfy grid-side ramping requirements, and a slow scale (minutes to hours) to capture the evolution of thermal stress.

6.1. Weight on thermal stress

The optimisation was carried out for two cases to demonstrate the effect of changing the weights. In the first one (dashed blue line in Figs. 6 and 7), there was a bigger weight on the steam pressure, while in the second one (thick, solid blue line), the thermal stress was more heavily weighted. As visible in Fig. 7, the latter one results in a fairly acceptable deviation in the main steam pressure, which allows a significantly lower absolute peak in the thermal stress.

The basis for the proposed weight functions for both pressure and thermal stress lies in the physical interpretation of the transient process and the optimisation strategy used in predictive control frameworks. As can be observed, in the case where there was a higher weight on the thermal stress (solid line), in exchange for a lower peak in the thermal stress, we had to allow

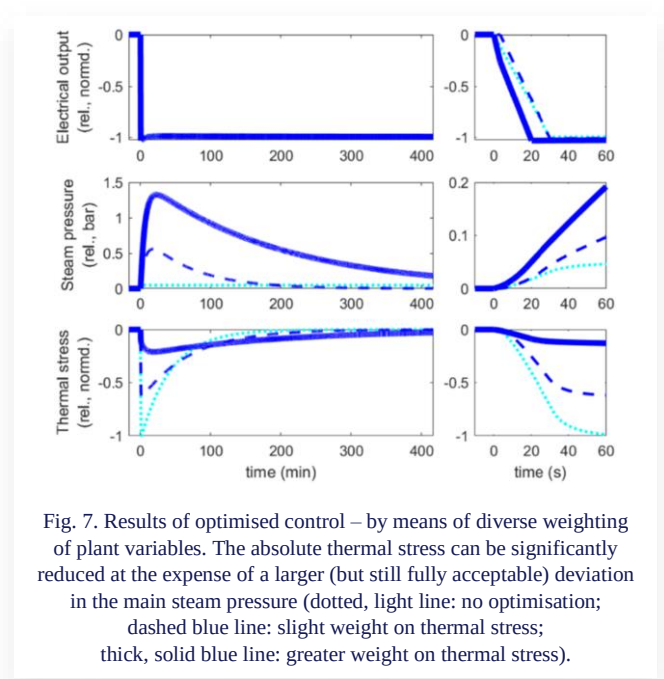


Fig. 7. Results of optimised control – by means of diverse weighting of plant variables. The absolute thermal stress can be significantly reduced at the expense of a larger (but still fully acceptable) deviation in the main steam pressure (dotted, light line: no optimisation; dashed blue line: slight weight on thermal stress; thick, solid blue line: greater weight on thermal stress).

a much larger deviation in the main steam pressure. While the peak thermal stress was reduced by more than 50%, the peak value of the main steam pressure increased by almost 1.5 times compared to the first case (dashed line). However, this is in our favour as the thermal stress is more significant for the ageing of the structural materials. Compared to the purely linear actuator control, the optimised trajectories achieved the same load-following performance while significantly reducing thermal stress.

6.2. Constraint on thermal stress

To demonstrate the effectiveness of constrained optimisation, two cases will also be shown with two different values as the constraints of thermal stress. In Figs. 8 and 9, these cases were marked consequently by a dashed blue line (less strict constraint) and a thick, solid blue line (more strict constraint). The figures show a similar trend: less absolute thermal stress can be achieved at the price of a higher transitional pressure peak.

6.3. Overall observations and causes

The results show that the optimised control curves significantly reduce the peak thermal stress compared to heuristic or linearly ramped control actions. In the constrained case, the stress was

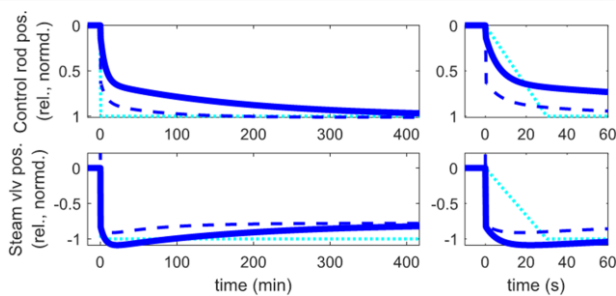


Fig. 6. Optimised control of the actuators (solid and dashed blue lines), together with the pure linear control (dotted, light line) – by means of diverse weighting of plant variables.

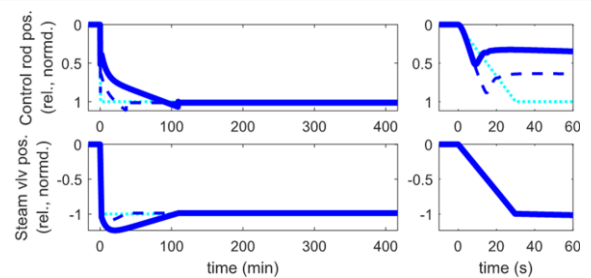


Fig. 8. Optimised control of the actuators (solid and dashed blue lines), together with the pure linear control (dotted, light line) – by means of constraining the resulted thermal stress.

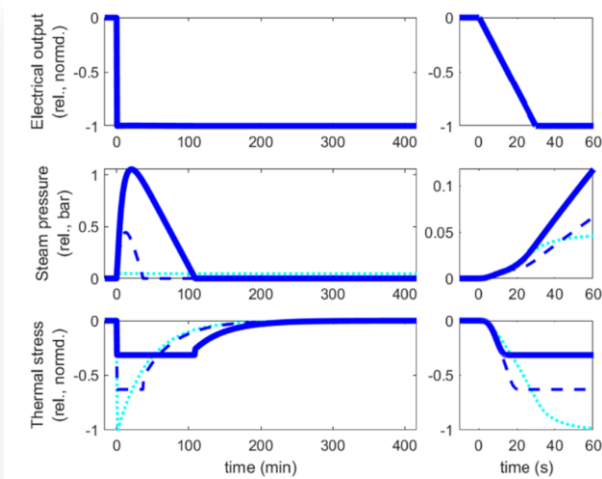


Fig. 9. Results of optimised control – by means of diverse constraining the resulted thermal stress. The absolute thermal stress can be significantly reduced on the price of larger (but still fully acceptable) deviation in the main steam pressure (dotted, light line – no optimisation; dashed blue line – slight constraint on thermal stress; thick, solid blue line – more strict constraint on thermal stress).

kept below the predefined limit throughout the transient. In the weighted case, although the peak stress was slightly higher, the overall stress-time integral – representing accumulated fatigue – was reduced. This demonstrates that the optimisation not only limits instantaneous stress but also mitigates long-term material degradation. Importantly, the optimisation did not rely on a uniform slowing down of the load change. Instead, the algorithm exploited the plant's internal flexibility – such as the thermal inertia of the primary loop and the buffering capacity of the steam pressure – to decouple the fast electrical response from the slower thermal dynamics. This allowed the control rods to move more gradually, reducing stress, while the steam valve compensated for the required power output.

The optimised trajectories also showed improved smoothness and coordination between actuators, which is beneficial for mechanical wear and control stability. The results confirm that the proposed method provides a non-trivial improvement over conventional control strategies, offering a viable path toward extending component lifetime while maintaining operational flexibility.

Although the study focused on a specific reactor configuration and nozzle geometry, the methodology is generalisable. The derived stress transfer function and optimisation framework can be adapted to other critical components and reactor types. Future work may include the integration of fuel behaviour models and the extension of the optimisation to multi-unit coordination.

7. Conclusions

This study demonstrated that the practically required load-following operation of nuclear power plants can be realised in a way that significantly reduces reactor-side thermal stresses – thus mitigating material ageing – while still fulfilling the grid operator's dynamic power delivery requirements.

By integrating a dynamical system model with a detailed stress-strain analysis of the reactor pressure vessel, the proposed

optimisation framework enables a balance between fast electrical load manoeuvring and reactor-side structural integrity. In numerical simulations, thermal stress peaks were reduced by up to almost 50% compared to a standard linear control approach. Importantly, this reduction was achieved without exceeding acceptable deviations in steam pressure.

The resulting actuator trajectories adhered to physical constraints, including control rod speed limits (≤ 0.2 m/s), and showed smoother, more predictable behaviour, minimising high-frequency mechanical load changes. This confirms that stress-aware optimisation not only supports flexible plant operation but also contributes to long-term component reliability.

This method provides a practical and scalable solution for adapting existing nuclear power plants to modern grid demands with a high share of intermittent renewables. It supports lifetime considering control and paves the way for health-aware optimisation strategies, offering a path toward more resilient, sustainable nuclear power operation in the future energy mix.

Acknowledgements

The kind help of Dr. Á. Horváth and DSc. J. Gadó, as well as L. Tatár and D. Antók, are gratefully acknowledged.

References

- [1] Marusic, A., Loncar, D., Batelic, J., & Frankovic, V. (2016). Increasing flexibility of coal power plant by control system modifications. *Thermal Science*, 20(4), 1161–1169. doi: 10.2298/TSCI160314159M
- [2] Raskovic, P.O., Cvetanovic, G., Vujanovic, M., Schneider, D.R., Guzovic, Z., Duic, N., & Oka, S.N. (2022). The progress toward more sustainable energy, water and environmental systems approaches and applications. *Thermal Science*, 26(5B), 4057–4066. doi: 10.2298/TSCI2205057R
- [3] Ludwig, H., Salnikova, T., Stockman, A., & Waas, U. (2011). Load cycling capabilities of German nuclear power plants (NPP). *VGB Powertech*, 91(5), 38–44.
- [4] Grünwald, R., & Caviezel, C. (2017). *Load-following capability of German nuclear power plants. Summary*. KIT Library. <https://publikationen.bibliothek.kit.edu/1000137922> [accessed 17 Jan. 2025].
- [5] Szentannai, P., Szűcs, T., Pudleiner, B., & Fekete, T. (2024). Transient phenomena and their consideration in load-following control of nuclear power plants. *Thermal Science*, 28(4B), 3183–3193. doi: 10.2298/TSCI230805052S
- [6] Li, Y., Jin, T., Wang, Z., & Wang, D. (2020). Engineering critical assessment of RPV with nozzle corner cracks under pressurized thermal shocks. *Nuclear Engineering and Technology*, 52(11), 2638–2651. doi: 10.1016/j.net.2020.04.019
- [7] Liu, R., Huang, M., Peng, Y., Wen, H., Huang, J., Ruan, C., Ma, H., & Li, Q. (2018). Analysis for crack growth regularities in the nozzle-cylinder intersection area of reactor pressure vessel. *Annals of Nuclear Energy*, 112, 779–793. doi: 10.1016/j.anucene.2017.10.021
- [8] Trampus, P. (2024). Materials challenges of the new nuclear power plant in Hungary. *Periodica Polytechnica Mechanical Engineering*, 68(4), 328–335. doi: 10.3311/PPme.37668
- [9] Rodriguez-Reyes, V.I., Abundez-Pliego, A., Petatan-Bahena, K.E., Miranda-Acatitlan, K.R., Colin-Ocampo, J., & Blanco-Ortega, A. (2024). Comparative evaluation of fatigue life estimation under variable amplitude loading through damage models based

- on entropy. *Fatigue & Fracture of Engineering Materials & Structures*, 47(6), 1584–1601. doi: 10.1111/ffe.14262
- [10] Ray, A., Wu, M.K., Carpino, M., & Lorenzo, C.F. (1993). Damage-mitigating control of mechanical systems: Part I – Conceptual development and model formulation. *Proceedings of the American Control Conference*, 1172–1176, 2-4 June, San Francisco, USA. doi: 10.23919/acc.1993.4793051
- [11] Walter, H. & Epple, B. (Eds.) (2017). *Numerical simulation of power plants and firing systems* (1st ed.). Springer. doi: 10.1007/978-3-7091-4855-6
- [12] Corriou, J.-P. (2004). Dynamic modelling of chemical processes. In *Process Control: Theory and Applications* (pp. 3–75). Springer, London. doi: 10.1007/978-3-319-61143-3_1
- [13] Rossi, I., Traverso, A., & Tucker, D. (2019). SOFC/Gas turbine hybrid system: A simplified framework for dynamic simulation. *Applied Energy*, 238, 1543–1550. doi: 10.1016/j.apenergy.2019.01.092
- [14] Szentannai, P., Szűcs, T., Pudleiner, B., & Fekete, T. (2024). Transient phenomena and their consideration in load-following control of nuclear power plants. *Thermal Science*, 28(4B), 3183–3193. doi: 10.2298/TSCI230805052S
- [15] Cheng, Y.T., Huang, M., Li, Y.D., Liu, W.Y., Wang, B., Meng, X., & Ouyang, X.P. (2023). Influence of interaction among multiple cracks on crack propagation in nozzle corner of reactor pressure vessel. *Annals of Nuclear Energy*, 194, 110142. doi: 10.1016/j.anucene.2023.110142
- [16] Rabazzi, S.M., Albanesi, E.A., Nervi, J.E.R., & Signorelli, J.W. (2024). Highly detailed structural integrity assessment of the reactor pressure vessel nozzle of Atucha-I during a pressurized thermal shock event. *Nuclear Engineering and Design*, 418, 112905. doi: 10.1016/j.nucengdes.2024.112905
- [17] The European Commission (2016). *Commission Regulation (EU) 2016/631 of 14 April 2016 establishing a network code on requirements for grid connection of generators*. Official Journal of the European Union. <https://eur-lex.europa.eu/eli/reg/2016/631/oj> [accessed 17 Jan. 2025].
- [18] Bemporad, A., Ricker, N.L., & Morari, M. (2024). *Model predictive control toolbox User's guide. Release 2024b*. MathWorks Help Center. <https://www.mathworks.com/help/mpc/index.html> [accessed 17 Jan. 2025].
- [19] Eslami, M. R., Hetnarski, R., Ignaczak, J., Noda, N., Sumi, N., & Tanigawa, Y. (2015). *Theory of elasticity and thermal stresses: explanations, Problems and solutions*. Springer, Dordrecht. doi: 10.1080/01495739.2013.846763

The influence of flow resistance in the main pipelines of secondary circuit on the power of EPR and AP1000 nuclear units

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Received: 03.04.2025; revised: 13.07.2025; accepted: 12.08.2025

Abstract

The article presents the influence of hydraulic resistance in the main pipelines of EPR (evolutionary power reactor) and AP1000 (advanced passive) pressurised water nuclear reactors secondary circuit on the electric power of these units. The change of hydraulic resistance in the passage pipelines of the high-pressure and low-pressure turbine parts and in the pipelines supplying the low-pressure and high-pressure regenerative heaters was taken into account. These two types of units were selected due to the different values of live steam parameters and different values of pressure drops in the turbine passage and regeneration system pipelines. The analysis shows that the flow resistance in the turbine passage pipelines has the greatest impact on the power and efficiency of the unit, followed by high-pressure regeneration pipelines, while low-pressure regeneration pipelines have the least impact.

Keywords: EPR; AP1000; Pipeline pressure drop; Power and nuclear unit efficiency

Vol. 46(2025), No. 4, 49–57; doi: 10.24425/ather.2025.156836

Cite this manuscript as: Laskowski, R.M., Smyk, A., Wolowicz, M., & Szymczyk, J. (2025). The influence of flow resistance in the main pipelines of secondary circuit on the power of EPR and AP1000 nuclear units. *Archives of Thermodynamics*, 46(4), 49–57.

1. Introduction

The understanding and mastery of the processes occurring in a nuclear reactor enabled the construction of the first nuclear units with relatively low thermal and electrical power. Some of the oldest nuclear units connected to the electrical network were built, among others, in Russia in Obninsk [1] with a thermal power of 36 MW_{th} and electrical power of 5 MW_{el}, and in Great Britain in Calder Hall [2] with a thermal power of 268 MW_{th} and electrical power of 49 MW_{el}. Over time, due to the minimisation of the unit cost of building 1 MW_{el} of power, nuclear units with higher thermal and electrical power were built. Here, we can distinguish nuclear units of the class 400 MW_{el} (VVER – Rus. *vodo-vodyanoi enynergeticheskij reaktor* – water-water energetic reactor [3]), 600 MW_{el} (AP600 – advanced passive [4]),

1000–1200 MW_{el} (AP1000 [5–8], EPR – evolutionary power reactor [8–11], VVER [8,12,13]) and even those reaching the power of 1400 MW_{el} (APR1400 – advanced power reactor [14]) or 1600–1700 MW_{el} (EPR [8–11]). As can be seen, all of the aforementioned nuclear units, i.e. VVER, AP600, AP1000, APR 1400, EPR, belong to the PWR (pressurised water reactor) type nuclear units, which constitute the largest group among the nuclear reactors built in the world. According to the IAEA PRIS (International Atomic Energy Agency Power Reactor Information System) statistics [15], there are currently 417 operating nuclear reactors in the world with a total capacity of 373.735 MW_{el}. The largest number of PWR-type nuclear reactors is 306, with a total capacity of 293.147 MW_{el}, which is 73.7% of the number of PWR reactors of all types, and the share of PWR reactors is 78.4%.

Nomenclature

dp – pressure drop, $= p_{HE} - p_T$, bar
 h – specific enthalpy, kJ/kg
 $\dot{m}$ – mass flow rate, kg/s
 p – pressure, bar
 t – temperature, °C

Greek symbols

$\Delta\dot{m}$ – difference in mass flow rates, $= \dot{m}_{doc} - \dot{m}_{ebs}$, kg/s
 $\delta\dot{m}$ – mass flow rate relative error, $= (\dot{m}_{doc} - \dot{m}_{ebs})/\dot{m}_{doc}$, %

Subscripts and Superscripts

doc – data from the documentation
 ebs – data from the Ebsilon program
 HE – steam at the inlet to the heat exchanger
 T – steam in the turbine bleed

Abbreviations and Acronyms

AP600/1000 – advanced passive
 APR1400 – advanced power reactor

CON – steam condenser
 DE – deaerator
 EPR – evolutionary power reactor
 EPR OL3– evolutionary power reactor in Olkiluoto unit 3
 EPR F3– evolutionary power reactor in Flamanville unit 3
 G – electric generator
 HPC – high-pressure cooler
 HPRH – high-pressure regenerative heat exchangers
 HPT – the high-pressure part of the steam turbine
 IAEA PRIS – International Atomic Energy Agency Power Reactor Information System
 LPC – low-pressure cooler
 LPRH – low-pressure regenerative heat exchangers
 LPT – low-pressure part of the steam turbine
 M – electric motor
 P – pipes
 PWR – pressurised water reactor
 R – nuclear reactor
 SG – steam generator
 SH – superheater, first and second stages
 VVER – water-water energetic reactor

At the same time as the thermal and electrical power of nuclear units increased, efforts were made to increase their efficiency, which can be achieved by using classical methods that are also used in conventional steam power plants. The increase in the efficiency of nuclear power plants is achieved by increasing the pressure and temperature of steam at the inlet to the steam turbine and by reducing the pressure of condensing steam in the condenser [16–21]. In turn, the pressure of condensing steam in the condenser is influenced by the design of the condenser [22–24], the load of the unit [25], and the temperature [26–30] and mass flow of cooling water at the inlet to the condenser [31–33]. With the increase in the thermal power of the reactor, it is possible to produce more steam in the generators. The above parameters (pressure and temperature, the mass flow of steam at the inlet to the turbine, and the pressure of condensing steam at the inlet from the turbine) affect the power generated by the steam turbine, which affects the efficiency of the entire nuclear unit.

The second method of increasing the power and efficiency of nuclear units involves changes in the structure of the thermal system, e.g. by using low-pressure and high-pressure regeneration [16–19]. Thermal systems of the nuclear unit designed for cogeneration operation are also analysed, i.e. systems that simultaneously produce electricity and district heating [34–38]. Systems of nuclear units for hydrogen production [39,40] and seawater desalination [41,42] are increasingly being proposed and tested. In order to increase the efficiency and power, combined thermal systems of the nuclear unit with gas turbines [43–46] are also analysed, as well as in combination with conventional systems coupled by means of superheaters [47,48]. Combined systems with the nuclear unit significantly increase the power and efficiency of such systems, but significantly increase the construction costs due to significant interference in the structure of the thermal system.

The influence of the main parameters determining the efficiency of nuclear units, including the parameters of steam generated in generators and pressure in the condenser, has been the subject of a number of analyses, including the ones mentioned above. However, considering that nuclear units operate for 40–50 years with the possibility of extending to 60–80, even a slight increase in efficiency provides significant economic benefits. Therefore, due attention should be paid to the entire thermal system of the nuclear unit, and possible places in the system should be sought, the careful selection of which would allow for an increase in the power and efficiency of the system. One of such possibilities is the proper selection of pressure drops in the pipelines connecting the steam bleedings in the turbine with the regenerative exchangers and in the passage between the high-pressure and low-pressure parts of the turbine, which is the subject of this article.

During the design of a nuclear unit, the designers' main focus is on maintaining the required power and efficiency of the unit by ensuring the values of the main parameters, i.e. steam pressure and temperature at the turbine inlet, steam mass flow (obtained based on the required heat flow transferred in the steam generators), condenser pressure, feed water temperature at the steam generator inlet. It happens that during the design of a nuclear unit, pressure drops in the pipelines are assumed as a certain percentage of the steam pressure at the inlet. This practice is also found in conventional power plants. Therefore, it seems that they are not determined with due accuracy during their design. For a nuclear unit built with such adopted assumptions regarding flow resistance in the pipelines supplying the regenerative exchangers and in the passageway, there may be significant differences that affect the power and efficiency. The significant impact of these assumptions on the performance of the nuclear unit is presented in this paper. The EPR-type nuclear unit with the highest electrical power was selected for analysis,

as well as the AP1000 nuclear unit, which was selected as the first nuclear unit to be built in Poland.

2. List of pressure drops in pipelines to regeneration exchangers

Based on the thermodynamic and economic analysis, the appropriate number of low-pressure and high-pressure regenerative exchangers was determined for the thermal systems of EPR and AP1000 units. Considering only thermodynamic considerations, the increase in the number of regenerative exchangers causes the increase in the temperature of the feed water at the inlet to the steam generators and the increase in the efficiency of the entire system [16,49,50]. However, with the increase in the number of regenerative exchangers, this increase is smaller and smaller, and taking into account economic considerations (costs of building additional exchangers), the optimal number of high-pressure and low-pressure regenerative exchangers can be determined. In conventional units, there are 2–3 high-pressure regenerative exchangers and 4–5 low-pressure regenerative exchangers [16,18]. For nuclear units, the number of high-pressure regenerative exchangers is usually 2 and the number of low-pressure regenerative exchangers is usually 4 [7,11,34,35,40]. Table 1 shows the steam pressure in the turbine bleeds (p_T) and at the inlet to the exchangers (p_{HE}), pressure drops in the steam pipelines (dp) supplying the low-pressure and high-pressure regenerative exchangers and the percentage share of the pressure drop in relation to the pressure in the turbine extraction for conventional units, and in Table 2 for EPR and AP1000 nuclear units, also taking into account the pressures in the passage.

3. Models of the analysed thermal systems

The secondary thermal cycles of the EPR and AP1000 units operate according to the Rankine cycle, are similar to each other and have the same main elements, e.g. the high-pressure and low-pressure parts of the steam turbine, a two-stage steam superheater after the high-pressure part of the turbine, a condenser, a deaerator, low- and high-pressure regenerative exchangers, cooling water, condensate and feed water pumps and an electric generator. There are also some differences between them, e.g. in the number of turbine stage groups for the high-pressure and low-pressure parts, an additional water heater for high-pressure regeneration for the EPR and a different method of condensate flow from low-pressure regenerative exchangers. The difference also concerns the heat flux generated in the reactor and the pressure of live steam at the outlet of the steam generator, which causes them to have different generated electrical power and efficiency.

The EPR and AP1000 nuclear power unit models were developed in the Ebsilon program [51]. The Ebsilon program has been used to model conventional power units [31,52–54] as well as nuclear power units [21,30,33,44,55–57]. Both models refer to the steady state of the thermal system operation. The Ebsilon program is dedicated to thermal-fluid calculations of complex thermal systems [51]. Models of EPR and AP1000 units were created in Ebsilon based on their balance thermal diagrams. In the con-

Table 1. Steam pressure in turbine extractions and at the inlet to exchangers, pressure drops in steam pipelines supplying low-pressure and high-pressure regenerative exchangers, and percentage pressure drop for conventional units.

Unit (MW _{el})	Heat exchanger	p_T (bar)	p_{HE} (bar)	dp (bar)	dp/p_T (%)
360					
	LPRH1	0.382	0.378	0.004	1.0
	LPRH2	1.061	1.049	0.012	1.1
	LPRH3	3.14	3.08	0.06	1.9
	LPRH4	5.46	5.4	0.06	1.1
	HPRH1	21.41	21.05	0.36	1.7
	HPRH2	44.81	43.2	1.61	3.6
500					
	LPRH1	0.137	0.134	0.003	2.2
	LPRH2	0.924	0.897	0.027	2.9
	LPRH3	2.946	2.758	0.188	6.4
	HPRH1	13.63	13.27	0.36	2.6
	HPRH2	21.05	20.8	0.25	1.2
	HPRH3	43.57	42.91	0.66	1.5
1000					
	LPRH1	0.175	0.166	0.009	5.1
	LPRH2	0.553	0.525	0.028	5.1
	LPRH3	2.65	2.53	0.12	4.5
	LPRH4	5.98	5.8	0.18	3.0
	LPRH5	11.6	10.7	0.9	7.8
	HPRH1	24.5	23.3	1.2	4.9
	HPRH2	58	56.2	1.8	3.1
	HPRH3	104.3	99.1	5.2	5.0

struction of the models, ready-made components of thermal system elements contained in the program's database were used. The components are interconnected to form the primary and secondary layout of the nuclear unit, consistent with their thermal scheme. The models consist of components, such as turbine stage groups, regenerative exchangers, a deaerator, steam condenser, steam superheaters, a moisture separator, pipelines, pumps, valves, electric motors to drive pumps, an electric generator, and distribution and mixing nodes. For each component, the relevant design or balance data should be entered, e.g. for the turbine stage group – efficiency, for the regenerative exchanger – terminal temperature difference (TTD), and for the valves and pipelines – nominal pressure drops. In the heat exchanger model, the heat transfer equation according to Péclet's law is used and for turbine stage groups the Flugel-Stodola equation is used. In the online documentation of the Ebsilon program, it is possible to reach detailed descriptions of the models and the calculation formulas used, but this is laborious and beyond the scope of this article. According to the authors of the paper, the Ebsilon program is one of the most advanced programs for steady-state thermal-fluid balance calculations and is therefore used in scientific and research work at the Institute of Heat Engineering of Warsaw University of Technology.

In the model, it is also necessary to enter parameters at characteristic points, such as pressures in the turbine bleedings. In the developed models, the number of entered data for the design

Table 2. Steam pressure in turbine extractions and at the inlet to exchangers, pressure drops in steam pipelines supplying the low-pressure and high-pressure regenerative exchangers, and percentage pressure drop for EPR and AP1000 nuclear units, also taking into account the pressures in the passage.

Unit	Heat exchanger	p_T (bar)	p_{HE} (bar)	dp (bar)	dp/p_T (%)
EPR OL3	LPRH1	0.1311	0.1245	0.0066	5.0
	LPRH2	0.615	0.584	0.031	5.0
	LPRH3	2.13	2.02	0.11	5.2
	LPRH4	4.43	4.21	0.22	5.0
	HPRH1	19.96	18.96	1	5.0
	HPRH2	29.37	27.9	1.47	5.0
	Turbine passage	10	9.5	0.5	5.0
EPR F3	LPRH1	0.1703	0.1619	0.0084	4.9
	LPRH2	0.855	0.8123	0.0427	5.0
	LPRH3	3.451	3.215	0.236	6.8
	LPRH4	6.37	6.051	0.319	5.0
	HPRH1	20.81	19.78	1.03	4.9
	HPRH2	30.27	28.76	1.51	5.0
	Turbine passage	11.328	10.958	0.37	3.3
AP1000	LPRH1	0.405	0.393	0.0124	3.1
	LPRH2	0.866	0.839	0.0269	3.1
	LPRH3	2.565	2.482	0.0827	3.2
	LPRH4	4.268	4.137	0.1310	3.1
	LPRH5	17.858	17.306	0.5516	3.1
	HPRH1	28.270	27.373	0.8963	3.2
	HPRH2	0.405	0.393	0.0124	3.1
	Turbine passage	11.328	10.958	0.37	3.3
	Turbine passage	11.328	10.958	0.37	3.3

model for one unit is about 120. The Ebsilon program, on the basis of the created thermal system with the entered required parameters using mass and energy balances, determines mass flow rates at characteristic points of the thermal system for the nominal (design) condition. In the case of analysis of load changes, calculations are performed under the changed conditions of operation (off-design). In this case, the number of additional data is small (less than 10). It increases significantly in the case when the own characteristics of elements under changed operating conditions are introduced. For the analysed models of the EPR and AP1000 nuclear units, the values of pressure drops in the high-pressure to low-pressure (HP-LP) passage and in the pipelines (P1-P4) feeding the regenerative exchangers were given as input data to the model in the pipeline component. For the assumed input data, including pressure drops for these pipelines and in the HP-LP passage, the Ebsilon program determined the mass flow rate of the fluids and the performance of the unit, i.e. power and efficiency of the unit.

3.1. Model of the thermal system of the EPR nuclear unit

The diagram of the EPR nuclear unit system with parameters was taken from the data presented by Framatome in [11]. Based on the available thermal diagram, a thermal-flow model of the EPR nuclear unit was created in the Ebsilon program, as shown in Fig. 1. In the analysed EPR nuclear unit, the steam turbine consists of a high-pressure part (HPT) and three low-pressure parts. A condenser is located under each of the low-pressure turbine parts. In the model presented in Fig. 1, due to the identity of the three low-pressure turbine parts, they were aggregated into a single part (LPT), and the steam condensers (CON) were treated similarly. In the secondary circuit, there is a two-stage steam superheater (SH) located between the outlet from the HP part and the inlet to the LP part of the turbine. The high-pressure regeneration system consists of two regenerative exchangers (HPRH1-2) and a high-pre-

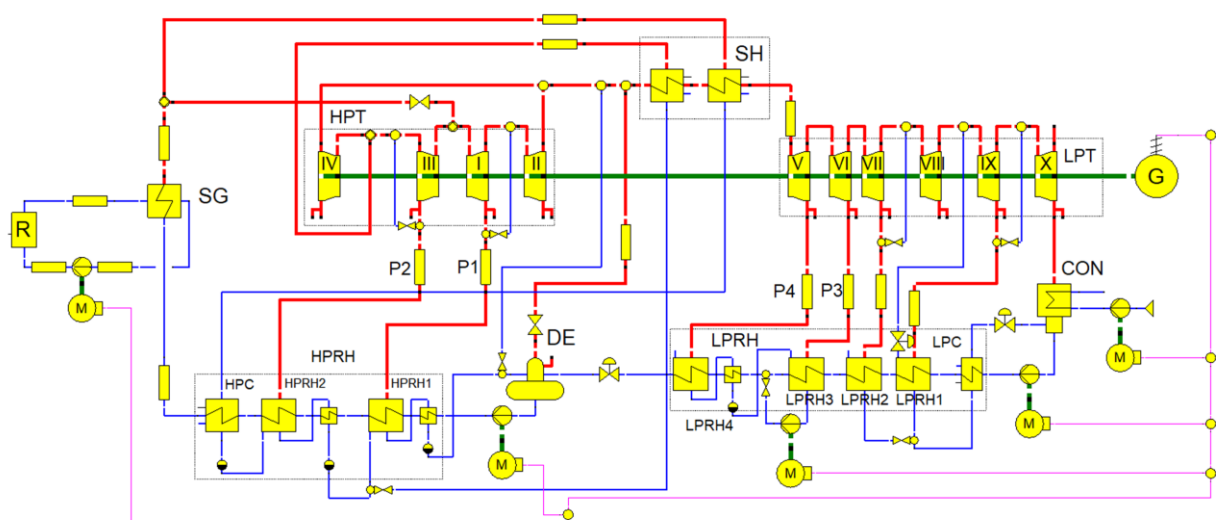


Fig. 1. Schematic diagram of the thermal system of the EPR nuclear power plant: CON – steam condenser, DE – deaerator, G – electric generator, HPC – high pressure cooler, HPRH – high-pressure regenerative heat exchangers, HPT – the high-pressure part of the steam turbine, LPC – low pressure cooler, LPRH – low-pressure regenerative heat exchangers, LPT – the low-pressure part of the steam turbine, M – electric motor, P – pipes, R – nuclear reactor, SG – steam generator, SH – superheater, first and second stages; blue line – water, red line – steam, green line – shaft, pink colour – electric lines.

ssure cooler (HPC). The low-pressure regeneration system consists of four regenerative exchangers (LPRH1–4) and a low-pressure cooler (LPC). The power of the analysed EPR nuclear unit is about 1700 MW_{el} and the gross efficiency is about 39% [9–11]. A detailed description of the model, as well as a list of parameters (temperatures, pressure, mass flow) at characteristic points of the thermal system, are presented in [21,30]. A detailed description of the system and individual components, as well as the thermal cycle, is presented in [21]. A description of the EPR and AP1000 reactors can also be found in the literature, e.g. in [58].

3.2. Thermal system model of the AP1000 nuclear unit

The AP1000 nuclear unit consists of a high-pressure turbine (HPT) part, a two-stage steam superheater (SH) and three low-pressure turbine (LPT) parts. The low-pressure regeneration system consists of four regenerative exchangers and a low-pressure cooler. The high-pressure regeneration system consists of two regenerative exchangers. The power of the analysed AP1000 nuclear unit is about 1200 MW_{el} and the gross efficiency is about 35% [1–7]. The input data for the model were taken from available documentation [7]. Based on the established structure and parameters, a unit model was created in the Ebsilon program (Fig. 2). Table 3 presents selected parameters (pressure, temperature, mass flow and specific enthalpy) for characteristic points of the system (1–14). The symbols in Fig. 2 are the same as in Fig. 1.

For the given pressures and temperatures at selected points in the thermal system, mass flows corresponding to the nominal power of the unit were determined in the Ebsilon program. Table 4 presents a comparison of mass flows and relative error at characteristic points of the system according to data from the documentation [7] and data obtained from the model in the Ebsilon program.

The maximum mass flow rate differences are 4–6 kg/s, but they occur for large mass flow rates, for which the relative error is equal to or below 0.52%. The maximum relative error does not exceed 1%, which can be considered a satisfactory model validation, considering also the fact that the Ebsilon model uses simplifications in the steam system from the glands and turbine seals.

Table 3. Pressure (p), temperature (t), mass flow ($\dot{m}$) and enthalpy (h) at characteristic points of the thermal system of the AP 1000 nuclear unit.

No.	p (bar)	t (°C)	$\dot{m}$ (kg/s)	h (kJ/kg)
1	57.61		1886.9	
2	55.71	270.72	1824.1	2785.62
3			61.3	2785.62
4	28.27		89.9	2685.60
5	11.33		1450.8	2539.99
6	34.13		82.9	2718.40
7	17.86		71.7	2610.47
8		226.67		975.76
9	4.27		43.1	2773.52
10	2.56		74.8	2686.30
11	0.84	94.78	47.8	
12	0.41		81.8	2472.54
13			1034.5	
14		42.61	1285.6	180.73

Table 4. Comparison of mass flows at characteristic points of the thermal system for the AP1000 nuclear unit according to data from [7] and the Ebsilon program.

No.	$\dot{m}_{doc}$ (kg/s)	$\dot{m}_{ebs}$ (kg/s)	$\Delta\dot{m}$ (kg/s)	$\delta\dot{m}$ (%)
1	1886.91	1882.78	4.13	0.22
2	1824.06	1820.79	3.28	0.18
3	61.32	61.79	-0.46	-0.76
4	89.91	89.83	0.08	0.09
5	1450.85	1446.07	4.77	0.33
6	82.94	83.58	-0.65	-0.78
7	71.72	71.86	-0.15	-0.21
8	125.08	126.30	-1.22	-0.98
9	43.07	43.41	-0.34	-0.80
10	74.76	75.11	-0.35	-0.46
11	47.84	48.20	-0.36	-0.76
12	81.79	81.09	0.71	0.86
13	1034.51	1034.30	0.22	0.02
14	1285.62	1278.88	6.74	0.52

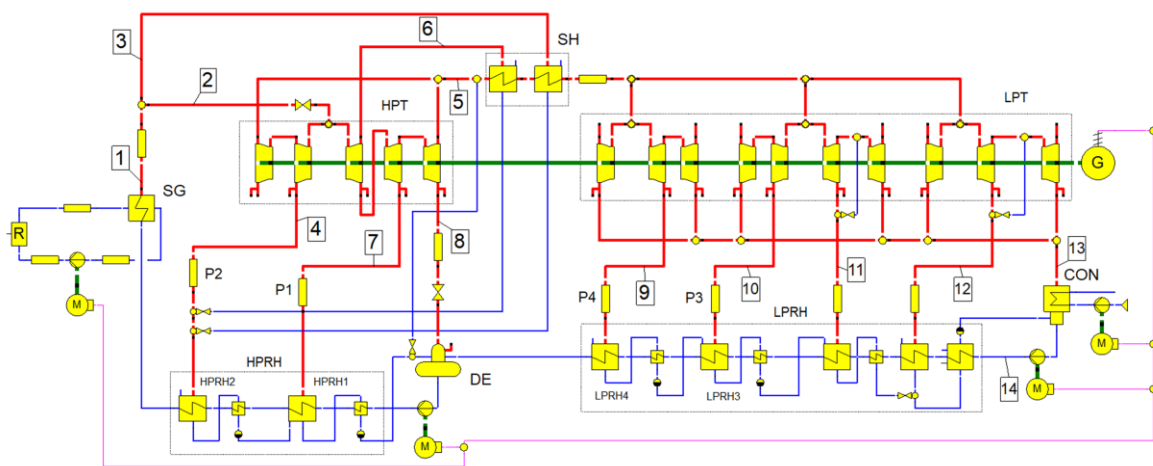


Fig. 2. Schematic diagram of the thermal system of the AP1000 nuclear power plant: 1–14 – characteristic points of the system (for legend see Fig. 1).

4. The influence of pressure drop change in the steam supply line on the performance of the regenerative heat exchanger

Regenerative exchangers can be divided into three zones (steam cooling zone, steam condensation zone and condensate cooling zone) [59–63]. High-pressure regenerative exchangers in the analysed nuclear units (HPRH1 and HPRH2) are supplied with wet steam, therefore, they do not have a zone of steam cooling to saturation. Low-pressure regenerative exchangers (LPRH3 and LPRH4) are supplied with superheated steam; therefore, they have three heat exchange zones. The temperature distribution in the high-pressure regenerative heat exchanger (HPRH1) together with its unit diagram from the Epsilon program is shown in Fig. 3, and the temperature distribution for the low-pressure regenerative heat exchanger (LPRH4) is shown in Fig. 4. The temperature distribution applies to the EPR nuclear unit for the nominal system parameters (the pressure drop in the pipeline is equal to 5% of the pressure in the steam bleeding – according to Table 2) and pressure drops in the pipelines of 3% of the steam pressure in the steam bleeding.

Reducing the steam pressure drop in the pipeline supplying the high-pressure exchanger causes an increase in steam pressure and, at the same time, an increase in steam temperature at the inlet to the exchanger (the exchanger is supplied with wet steam). Higher steam temperature and greater steam mass flow cause more heat to be transferred in the exchanger, which results in an increase in water temperatures at the exchanger outlet.

Reducing the pressure drop in the steam pipeline supplying the regenerative high-pressure exchanger (HPRH1) from 5% to

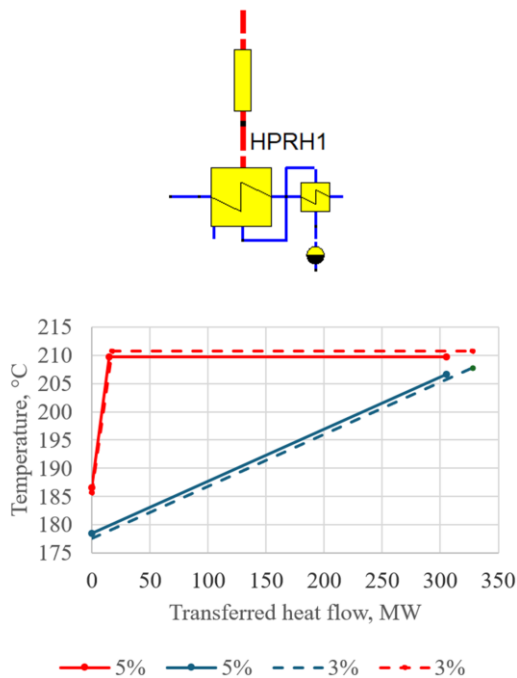


Fig. 3. Temperature distribution in the high-pressure regenerative heat exchanger (HPRH1) with its unit diagram from the Epsilon program, for 5% and 3% pressure drop in the pipeline.

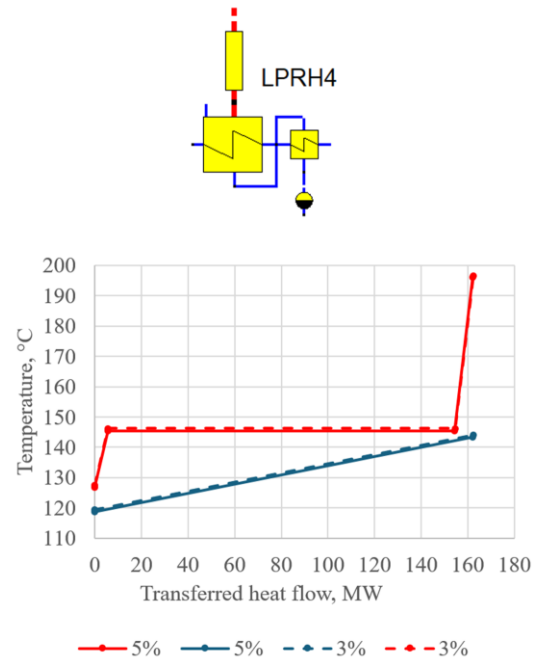


Fig. 4. Temperature distribution in the low-pressure regenerative heat exchanger (LPRH4) along with its unit diagram from the Epsilon program, for 5% and 3% pressure drop in the pipeline.

3% causes an increase in the water temperature at the exchanger outlet by 1°C.

Reducing the pressure drop in the steam pipeline supplying the low-pressure exchanger (LPRH4) from 5% to 3% causes an increase in water temperature at the exchanger outlet by 0.7°C. This increase is mainly due to the increase in saturation temperature resulting from the higher steam pressure at the exchanger inlet.

5. Results

An analysis of the influence of changes in resistance in the pipelines connecting steam extractions with regenerative exchangers, for the EPR and AP1000 nuclear units, was carried out for two high-pressure regenerative heaters and two low-pressure regenerative heaters, as well as for the passage between the high-pressure and low-pressure turbine sections.

For the EPR nuclear unit, for the nominal gross power of the unit, the pressure drops in the pipelines for the analysed regenerative exchangers and in the passageway amount to 5% of the pressure value at the inlet (Table 2). The analysis was performed for reduced pressure drops to the levels of 4% and 3%, respectively. The results of calculations of the change in the unit power are presented in Table 5. Reducing pressure drops to 4% of the inlet pressure results in a power increase of 2.31 MW_{el} (0.65 MW_{el} results from the reduction of pressure drops in the pipelines to the exchangers and 1.66 MW_{el} results from the reduction of pressure drops in the passageway). Reducing pressure drops to 3% of the inlet pressure results in a power increase of 4.58 MW_{el} (1.28 MW_{el} results from the reduction of pressure drops in the pipelines to the exchangers and 3.31 MW_{el} results from the reduction of pressure drops in the passageway).

Table 5. Increase in the power of the EPR nuclear unit due to reduction of pressure drops in the pipelines to the regenerative exchangers and in the passage by 4% and 3%.

Heat exchanger	dp/p_T (%)	Power increase (MW _{el})	dp/p_T (%)	Power increase (MW _{el})
LPRH3	4	0.204	3	0.39
LPRH4	4		3	
HPRH1	4	0.446	3	0.89
HPRH2	4		3	
Exchangers in total		0.65		1.28
Passage	4	1.66	3	3.31
Unit in total		2.31		4.58

For the AP1000 nuclear unit, for the rated gross unit power, the pressure drops in the pipelines to the analysed regenerative exchangers and in the passage are 3% of the inlet pressure (Table 2). An analysis was made of how the change in pressure drops affects the unit power for pressure drops of 4% and 2%, respectively. The calculation results of the change in the unit power are presented in Table 6. For resistances of 4%, there is an increase in pressure drops compared to the basic variant, in which these drops were equal to 3%; therefore, in this case, we have a loss of electrical power of -1.63 MW_{el} (a power loss of -0.39 MW_{el} on the pipelines to the regenerative exchangers and a power loss of -1.24 MW_{el} in the passage). Reducing the pressure drops to 2% results in a power increase of 2.47 MW_{el} (a power increase of 0.51 MW_{el} resulting from the reduction of the pressure drops in the pipelines to the regenerative exchangers and a power increase of 1.97 MW_{el} resulting from the reduction of the pressure drop in the passage).

Table 6. Change in the power of the AP1000 nuclear unit due to changes in pressure drops in the pipelines to the regenerative exchangers and in the passage for 4% and 2%.

Heat exchanger	dp/p_T (%)	Power drop (MW _{el})	dp/p_T (%)	Power increase (MW _{el})
LPRH3	4	-0.13	2	0.16
LPRH4	4		2	
HPRH1	4	-0.26	2	0.34
HPRH2	4		2	
Exchangers in total		-0.39		0.51
Passage	4	-1.24	2	1.97
Unit in total		-1.63		2.47

6. Conclusions

In the EPR unit, reducing resistance from 5% to 3% in the pipelines supplying two low-pressure regenerative exchangers (LPRH3 and LPRH4) and two high-pressure regenerative exchangers (HPRH1 and HPRH2), and in the passage, gives a power increase of 4.58 MW_{el} , which is 0.266% of the nominal power. In the AP1000 unit, reducing resistance by 2% gives a power increase of 4.1 MW_{el} , which is 0.342% of the nominal power.

The greatest impact (for the pipelines analysed) on the power and efficiency of the whole unit is exerted by the flow resistance in the pipelines in the turbine passageway, less in the pipelines in the high-pressure regeneration and the least in the pipelines in the low-pressure regeneration. An increase in the power of the unit by 1 MW_{el} gives about 3.5 million PLN in revenue from the sale of additional electricity (assuming that the annual operating time of the unit is equal to 7 000 hours and the price of electricity is PLN 500/MWh). For a 4 MW_{el} unit, this profit amounts to about 14 million PLN. Therefore, a careful analysis of the influence of resistance in the live steam, interstage steam and regeneration system pipelines should be the subject of research and analysis. Among other things, it is necessary to take into account increasing the diameter of the pipelines and shortening their length, especially for high-pressure regeneration. These suggestions can be taken into account for newly designed PWR-type nuclear power plants, including those envisioned for implementation in Poland.

The flow resistance from the outlet of the steam generators to the inlet to the high-pressure section of the turbine can still be analysed, with special attention paid to pressure drops on valves upstream of the high-pressure part of the steam turbine. To be able to assess the possibility of reducing pressure drops along this section, one needs to know the geometry of the pipelines (pipeline diameters and lengths), the geometry of elbows, the characteristics of valves, and pressure drops at the stop and control valves. The authors of the paper do not currently have access to such detailed data. If such data can be obtained, the analysis will be continued in the near future.

References

- [1] APS 1 Obninsk (2025). <https://world-nuclear.org/nuclear-reactor-database/details/APS-1-Obninsk> [accessed 25 March 2025].
- [2] Calder Hall 1 (2025). <https://world-nuclear.org/nuclear-reactor-database/details/calder%20hall-1> [accessed 25 March 2025].
- [3] VVER (2025). <https://en.wikipedia.org/wiki/VVER> [accessed 25 March 2025].
- [4] Westinghouse AP600 advanced nuclear plant design (2025). https://inis.iaea.org/collection/NCLCollectionStore/_Public/31/007/31007051.pdf [accessed 25 March 2025].
- [5] AP1000 Pressurized Water Reactor (2025) <https://westinghouse-nuclear.com/energy-systems/ap1000-pwr/> [accessed 25 March 2025].
- [6] Issued design certification - Advanced passive 1000 (AP1000) (2025). <https://www.nrc.gov/reactors/new-reactors/large-lwr/design-cert/ap1000.html> [accessed 25 March 2025].
- [7] Westinghouse AP1000 design control document (2025) <https://www.nrc.gov/docs/ml1117/ml11171a500.html> [accessed 25 March 2025].
- [8] Kubowski, J. (2014). *Nuclear power plants* (2nd ed.). WNT, Warsaw (in Polish).
- [9] EPR design description (2025) <https://www.nrc.gov/docs/ML0522/ML052280170.pdf> [accessed 25 March 2025].
- [10] Large projects (2025). <https://www.framatome.com/EN/businessnews-136/framatome-large-projects--management-and-execution-of-nuclear-reactor-new-build-projects.htm> [accessed 25 March 2025].
- [11] Jurkowski, R. (2019). *EPR circuit – overview*. Lecture at Institute of Heat Engineering at Warsaw University of Technology, Warsaw, Nov. 29 – Dec. 3 (unpublished).

- [12] *Modern reactors of Russian design* (2025). <https://www.rosatom.ru/en/rosatom-group/engineering-and-construction/modern-reactors-of-russian-design/> [accessed 25 March 2025].
- [13] *The VVER today, Evolution, design, safety.* (2025). <https://rosatom.ru/upload/iblock/0be/0be1220af25741375138ecd1afb18743.pdf> [accessed 25 March 2025].
- [14] *APR1400 Reactor* (2025). <https://home.kepco.co.kr/kepco/EN/G/htmlView/ENGBHP00102.do?menuCd=EN07030102> [accessed 25 March 2025].
- [15] Power Reactor Information System (2025). <https://pris.iaea.org/PRIS/WorldStatistics/OperationalReactorsByType.aspx> [accessed 25 March 2025].
- [16] Laudyn, D., Pawlik, M., & Strzelczyk, F. (2007). *Power plants*. WNT, Warsaw (in Polish).
- [17] Cengel, Y.A., & Boles, M.A. (2002). *Thermodynamics: An engineering approach* (4th ed.). McGraw-Hill, New York.
- [18] Nag, P.K. (2002). *Power plant engineering*. Tata McGraw-Hill, New York.
- [19] Szargut, J. (2000). *Thermodynamics*. PWN, Warsaw (in Polish).
- [20] Wang, C., Yan, C., Wang, J., Tian, C., & Yu, S. (2017). Parametric optimization of steam cycle in PWR nuclear power plant using improved genetic-simplex algorithm. *Applied Thermal Engineering*, 125, 830–845. doi: 10.1016/j.applthermaleng.2017.07.045
- [21] Laskowski, R., Smyk, A., Jurkowski R., Ancé, J., Wołowicz, M., & Uzunow, N. (2022). Selected aspects of the choice of live steam pressure in PWR nuclear power plant. *Archives of Thermodynamics*, 43(3), 85–109. doi: 10.24425/ather.2022.143173
- [22] Salij, A., & Stepień, J.C. (2013). *Performance of turbine condensers in power units of thermal systems*. Kaprint, Warsaw (in Polish).
- [23] Rusowicz, A. (2013). *Issues concerning mathematical modelling of power condensers*. Warsaw University of Technology, Warsaw (in Polish).
- [24] Saari, J., Kaikko, J., Vakkilainen, E., & Savolainen, S. (2014). Comparison of power plant steam condenser heat transfer models for on-line condition monitoring. *Applied Thermal Engineering*, 62(1), 37–47. doi: 10.1016/j.applthermaleng.2013.09.005
- [25] Laskowski, R., Smyk, A., Lewandowski, J., & Rusowicz, A. (2015). Cooperation of a steam condenser with a low-pressure part of a steam turbine in off-design conditions. *American Journal of Energy Research*, 3(1), 13–18. doi: 10.12691/ajer-3-1-3
- [26] Gañán, J., Al-Kassir, A.R., González, J.F., Macías, A., & Diaz, M.A. (2005). Influence of the cooling circulation water on the efficiency of a thermo nuclear plant. *Applied Thermal Engineering*, 25(4), 485–494. doi: 10.1016/j.applthermaleng.2004.07.001
- [27] Laković, M.S., Stojiljković, M.M., Laković, S.V., Stefanović, V.P., & Mitrović, D.D. (2010). Impact of the cold end operating conditions on energy efficiency of the steam power plants. *Thermal Science*, 14(Suppl. 1), S53–S66. doi: 10.2298/TSCI100415066L
- [28] Durmayaz, A., & Sogut, O.S. (2006). Influence of cooling water temperature on the efficiency of a pressurized-water reactor nuclear-power plant. *International Journal of Energy Research*, 30, 799–810. doi: 10.1002/er.1186
- [29] Attia, S.I. (2015). The influence of condenser cooling water temperature on the thermal efficiency of a nuclear power plant. *Annals of Nuclear Energy*, 80, 371–378. doi: 10.1016/j.anucene.2015.02.023
- [30] Laskowski, R., Smyk, A., & Uzunow, N. (2021). Influence of cooling water temperature on performance of EPR nuclear power plant. *Rynek Energii*, 152 1, 57–62.
- [31] Laskowski, R., Smyk, A., Rusowicz, A., & Grzebielec, A. (2021). Optimization of the cooling water mass flow rate under variable load of a power unit. *Applied Thermal Engineering*, 191, 116874. doi: 10.1016/j.applthermaleng.2021.116874
- [32] Dobkiewicz-Wieczorek, E. (2022). Cooling water flow influence on power plant unit performance for various condenser configurations setup. *Archives of Thermodynamics*, 43(1), 141–167. doi: 10.24425/ather.2022.140929
- [33] Wang, X., Zhao, G., Qu, X., Yang, X., Wang, J., & Wang, P. (2023). Influence of cooling water parameters on the thermal performance of the secondary circuit system of a modular high-temperature gas-cooled reactor nuclear power plant. *Energies*, 16(18), 6560. doi: 10.3390/en16186560
- [34] Smyk, A. (1999). *The influence of thermodynamic parameters of a heat cogeneration system of the nuclear heat power plant on fuel saving in energy system*, PhD thesis, Warsaw University of Technology, Warsaw (in Polish).
- [35] Jaskólski, M., Reński, A., & Minkiewicz, T. (2017). Thermodynamic and economic analysis of nuclear power unit operating in partial cogeneration mode to produce electricity and district heat. *Energy*, 141, 2470–2483. doi: 10.1016/j.energy.2017.04.144
- [36] Lipka, M., & Rajewski, A. (2020). Regress in nuclear district heating. The need for rethinking cogeneration. *Progress in Nuclear Energy*, 130, 103518. doi: 10.1016/j.pnucene.2020.103518
- [37] Jaskólski, M., Reński, A., Duzinkiewicz, K., & Kaczmarek-Kacprzak, A. (2014). Profitability criteria of partial cogeneration in nuclear power plant. *Rynek Energii*, 5(114), 141–147.
- [38] Safa, H. (2012). Heat recovery from nuclear power plants. *International Journal of Electrical Power & Energy Systems*, 42(1), 553–559. doi: 10.1016/j.ijepes.2012.04.052
- [39] Verfondern, K., Yan, X., Nishihara, T., & Allelein, H.J. (2017). Safety concept of nuclear cogeneration of hydrogen and electricity. *International Journal of Hydrogen Energy*, 42(11), 7551–7559. doi: 10.1016/j.ijhydene.2016.04.239
- [40] Milewski, J., Kupecki, J., Szczęśniak, A., & Uzunow, N. (2022). Hydrogen production in solid oxide electrolyzers coupled with nuclear reactors. *International Journal of Hydrogen Energy*, 46(72), 35765–35776. doi: 10.1016/j.ijhydene.2020.11.217
- [41] Ansari, K., Sayyaadi, H., & Amidpour, M. (2010). Thermo-economic optimization of a hybrid pressurized water reactor (PWR) power plant coupled to a multi effect distillation desalination system with thermo-vapor compressor (MED-TVC). *Energy*, 35(5), 1981–1996. doi: 10.1016/j.energy.2010.01.013
- [42] Aziz, M.A.B., Callaway, C.T., Brown, N.R., & Brooks, C. (2025). Review and modeling of integrated energy systems with nuclear reactor coupled desalination and district heating review and modeling of integrated energy systems with nuclear reactor coupled desalination and district heating. *Nuclear Science and Engineering*, 199(2), 295–313. doi: 10.1080/00295639.2024.2357394
- [43] Darwish, M.A., Awadhi, F.M., & Amer, A.O.B. (2010). Combining the nuclear power plant steam cycle with gas turbines. *Energy*, 35(12), 4562–4571. doi: 10.1016/j.energy.2010.04.031
- [44] Oziemski, M. (2019). Topping nuclear power plants steam cycles with gas turbines as the way of enhancing their efficiency. *5th International Conference on Power Generation Systems and Renewable Energy Technologies (PGSRET)*, 26-27 Aug., Istanbul, Turkey. doi: 10.1109/PGSRET.2019.8882708
- [45] Seyyedi, S.M., Hashemi-Tilehnoee, M., & Rosen, M.A. (2018). Exergy and exergoeconomic analyses of a novel integration of a 1000 MW pressurized water reactor power plant and a gas turbine cycle through a superheater. *Annals of Nuclear Energy*, 115, 161–172. doi: 10.1016/j.anucene.2018.01.028
- [46] Veszely, K. (2002). Hybrid combined cycle power plant. *International Conference on Nuclear Energy for New Europe 2002*, 9-12, Sept. Kranjska Gora, Slovenia.
- [47] Wibisono, A.F., & Shwageraus, E. (2016). Thermodynamic performance of pressurized water reactor power conversion cycle combined with fossil-fuel superheater. *Energy*, 117(2), 190–197. doi: 10.1016/j.energy.2016.10.060
- [48] Zaryankin, A., Lyskov, M., Arianov, S., & Rogalev, A. (2011). Super powerful steam superheaters and turbines for hybrid nuclear power plants. *Journal of Power Technologies*, 91(4), 191–197.

- [49] Oyedepo, S.O., Fakeye, B.A., Mabinuori, B., Babalola, P.O., Leramo, R.O., Kilanko, O., Dirisu, J.O., Udo, M., Efemwenkikie, U.K., & Oyebanji, J.A. (2020). Thermodynamics analysis and performance optimization of a reheat -regenerative steam turbine power plant with feed water heaters. *Fuel*, 280, 118577. doi: 10.1016/j.fuel.2020.118577
- [50] Srinivas, T., Gupta, A.V., & Reddy, B.V. (2007). Generalized thermodynamic analysis of steam power cycles with 'n' number of feedwater heaters. *International Journal of Thermodynamics*, 10 (4), 177–185. doi: 10.5541/ijot.201
- [51] EBSILON Professional (2025). https://www.ebsilon.com/uploads/pics/Brochure_EBSILON_Simulation_Software_eng_01.pdf [accessed 25 March 2025].
- [52] Li, P.F., Ge, Z., Yang, Z., Chen, Y., & Yang, Y. (2014). District heating mode analysis based on an air-cooled combined heat and power station. *Entropy*, 16(4), 1883–1901. doi: 10.3390/e16041883
- [53] Ge, Z., Zhang, F., Sun, S., He, J., & Du, X. (2018). Energy analysis of cascade heating with high back-pressure large-scale steam turbine. *Energies*, 11(1), 119. doi: 10.3390/en11010119
- [54] Zhao, S., Wang, W., & Ge, Z. (2019). Energy and exergy evaluations of a combined heat and power system with a high back-pressure turbine under full operating conditions. *Energies*, 13(17), 4484. doi: 10.3390/en13174484
- [55] Dudek, M., Jaszczur, M., & Kolenda, Z. (2019). Thermodynamic analysis of modular high-temperature nuclear reactor coupled with the steam cycle for power generation. *Archives of Thermodynamics*, 40(4), 49–66. doi: 10.24425/ather.2019.130007
- [56] Laskowski, R., Theibech, M., & Uzunow, N. (2021). The effect of cooling water temperature on the performance of a BWR nuclear power plant. *Modern Engineering*, 3, 5–13.
- [57] Zhang, X., Zhang, H., Hou, J., Xie, J., He, X., & Si, H. (2024). Development of online validation and monitoring system for the thermal performance of nuclear power plant in service. *Annals of Nuclear Energy*, 195, 110173. doi: 10.1016/j.anucene.2023.110173
- [58] Nowicki, J. (2020). Nuclear generation plant with AP1000 power reactor. Design solutions and application perspectives. *Energetyka*, 8, 380–391 (in Polish).
- [59] Hussaini, I.S., Zubair, S.M., & Antar, M.A. (2007). Area allocation in multi-zone feedwater heaters. *Energy Conversion and Management*, 48(2), 568–575. doi: 10.1016/j.enconman.2006.06.003
- [60] Antar, M.A., & Zubair, S.M. (2007). The impact of fouling on performance evaluation of multi-zone feedwater heaters. *Applied Thermal Engineering*, 27(14-15), 2505–2513. doi: 10.1016/j.applthermaleng.2007.02.006
- [61] Laskowski, R., & Wawrzyk, K. (2011). Comparison of two simple mathematical models for feed water heaters. *Journal of Power Technologies*, 91(1), 14–22.
- [62] Barszcz, T., & Czop, P. (2011). A feedwater heater model intended for model-based diagnostics of power plant installations. *Applied Thermal Engineering*, 31(8-9), 1357–1367. doi: 10.1016/j.applthermaleng.2010.12.012
- [63] Hossienalipour, S.M., Karbalaee, S.M., & Fathiannasab, H. (2017). Development of a model to evaluate the water level impact on drain cooling in horizontal high pressure feedwater heaters. *Applied Thermal Engineering*, 110, 590–600. doi: 10.1016/j.applthermaleng.2016.08.203

Light pollution – not just an energy problem

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Received: 28.01.2025; revised: 14.11.2025; accepted: 19.11.2025

Abstract

Light pollution is a growing problem in developed countries. One of its categories, often associated with this concept, is night sky glow. This glow is caused by scattering light from artificial ground light sources on atmospheric particles. The paper indicates that street lighting may be the primary source of light scattered on these particles. This is caused by often outdated luminaires or their installation, which does not protect the surroundings from light pollution. The second, even more important factor increasing the brightness of the sky glow may be the reflection of incident light upwards by the street surface. Satellite images of the Earth's surface allow for determining the radiance from a given area, determined by the amount of light scattered in the atmosphere. However, the part of this light associated with street lighting is difficult to separate from the background of other artificial ground light sources. In 2020, due to the COVID-19 pandemic and reduced traffic at night, the authorities of several municipalities in the Małopolska province in Poland decided to switch off street lighting at night. This was the first time street lighting was switched off for part of the year in large areas and entire cities. Analysis of satellite data allowed us to determine that the luminance from the analysed localities decreased by almost 70% after switching off city lighting. The analysis also allowed us to estimate the energy losses of city lighting resulting from lighting fixtures that do not protect the surroundings from light pollution.

Keywords: Light pollution; Radiance; Artificial sky glow; Street lighting; Energy loss

Vol. 46(2025), No. 4, 59–73; doi: 10.24425/ather.2025.156837

Cite this manuscript as: Ścieżor, T. (2025). Light pollution – not just an energy problem. *Archives of Thermodynamics*, 46(4), 59–73.

1. Introduction

1.1. The history of the problem

At the end of the 19th century, it was noticed that city street lighting was the leading cause of the formation of the night sky glow, which hurts the quality of astronomical observations [1,2]. Of course, many natural light sources brighten the night sky (the Moon, planets, stars, etc.) [3], but the anthropogenic component always dominates in urban conditions. This is why the term "light pollution" is often treated as synonymous with the artificial glow of the night sky. The observation of the effect of city

street lighting on brightening the night sky led to light pollution in the form of the artificial night sky glow being treated as another example of human pollution of the natural environment. Initially, it was important only for astronomers. Until the beginning of the 20th century, astronomical observatories (Greenwich, Mt Wilson, Mt Palomar, Kraków-Botanical Garden) were located on the outskirts of cities, most often university centres, which were dictated by logistical reasons. Modernising urban lighting, especially its electrification, meant that these observatories were frequently moved to new locations, far from civilisation (Hawaii, the Canary Islands, Czarnohora) [4,5]. Starting mainly from the end of the 20th century, other light sources have

Nomenclature

L – radiance, $\text{nW}/(\text{cm}^2 \cdot \text{sr})$

Abbreviations and Acronyms

BREEAM – building research establishment environmental assessment method

CCT – correlated colour temperature

CO – commune office

DNB – day/night band

ISS – international space station

LAADS DAAC – Level-1 and Atmosphere Archive & Distribution System Distributed Active Archive Center

LPTT – Light Pollution Think Tank

NOAA – National Oceanic and Atmospheric Administration

PCA – principal component analysis

SQM – sky quality meter

ULOR – upper light output ratio

UM – City Office

UMiG – City and Commune Office

VIIRS – visible infrared imaging radiometer suite

significantly contributed to brightening the sky: illuminated advertisements, lighting of sports facilities, monuments, parking spaces or commercial facilities, etc. [6–10]. These and other light sources make cities the primary source of light pollution in the night sky.

The sky glow is caused by light scattering on aerosols and atmospheric molecules. Light scattered in this way also affects the environment further away from the cities that are its source. To determine the potential impact of such scattered light on the surroundings of cities, several theoretical models have been developed to describe the distribution of night sky brightness in these areas. Walker created the first simple models in the 1970s based on data on population density [11–13]. The basic model describing the effect of cities on the night sky brightness was developed in 1973 by Treanor [14] and then improved in 1976 by Berry [15]. Based on these models, the Garstang model [16–18] was developed, which is still used today and is still being developed by correlating with measurement data. Currently, the most commonly used model of city light scattering on various types of atmospheric aerosols is Illumina [19], a new improved version of which was published in September 2020 [20]. Since 1972, it has been possible to quantitatively analyse light emissions from cities (radiance) based on satellite images taken by the Defense Meteorological Satellite Program (DMSP) satellites. DMSP images were used to verify the correctness of the previously mentioned models [21,22], which enabled the development of an atlas showing the model distribution of the brightness of the night, cloudless sky, observed from the ground (The First World Atlas of the Artificial Night Sky Brightness) [23].

The emergence of cheap sky brightness meters such as Sky Quality Meter (SQM) has facilitated field measurements and enabled the measurement of overcast sky brightness, which is especially important from an ecological perspective. Measurement campaigns based on SQM meters were conducted in the Zselic Landscape Protection Area in Hungary in 2007–2010 [24], in Hong Kong and its surroundings in 2008–2009 [25] and 2010–2013 [26], and in Kraków and its environs in 2008–2010 [27–29], in the latter case by the team of the Light Pollution Monitoring Laboratory (LPML) operating at the Kraków University of Technology.

In 2011, the Suomi National Polar-orbiting Partnership (SNPP) satellite was launched into Earth's orbit [30]. The satellite, made by the US National Oceanic and Atmospheric Administration (NOAA), carried the visible infrared imaging radiometer suite (VIIRS) camera. The ground resolution of cameras on

previous DMSP series satellites was 3 km, while in the case of VIIRS, it is only a few hundred meters [31]. Images taken in the day/night band (DNB) allow for determining the radiance from a given area (wavelength $0.7 \mu\text{m}$), both coming directly from light sources and reflected from the ground [32]. VIIRS data were also used to create an artificial sky glow model [33] and a new atlas of the night sky brightness [34].

Night photos of cities (Warsaw, Kraków) taken by astronauts staying on the International Space Station (ISS) allow for categorising the types of urban lighting used (Fig. 1) [35].



Fig. 1. Night photograph of the city centre of Kraków (Poland) taken from the International Space Station on March 28, 2017 [35].

These images can distinguish all types of communication routes and, based on differences in their colours, identify areas where metal halide or high-pressure sodium (HPS) lighting has been replaced by LED lighting.

1.2. Street lighting as a source of light pollution

The impact of street lighting in cities on the brightness of the artificial sky glow and on the environment has been known for a long time [36–40]. Many attempts have been made to determine the contribution of particular types of urban lighting to the brightness of the night sky's glow. The assessment of this contribution is facilitated by local shutdowns of part or all of the municipal lighting. Analysis of the total radiance of Flagstaff (Arizona, USA) conducted in 2009 showed that about 8.3% of the city light "escapes" into space, of which 33% comes from the lighting of sports facilities. After excluding this type of light-

ing, commercial and industrial lighting constituted 62%, residential lighting 14%, and road lighting 12% [41]. A similar analysis was conducted in 2006 for Reykjavik (Iceland), where all street lighting was switched off for several minutes in connection with the preparations for the International Film Festival [42]. It was only found that street lighting significantly contributes to light pollution of the night sky. In 2019, an experiment was conducted in Tucson (Arizona, USA) to obtain information about the share of light emission from street lighting and private property lighting. Additionally, the related energy consumption was estimated. The intensity of street lighting was deliberately changed for 10 days, and during this time, the change in satellite radiance was studied. It was found that municipal street lighting was responsible for only 13% of the total radiance emitted from the Tucson area after midnight [43]. When this lighting was not switched off, its contribution to the total radiance was determined to be 18%. When street lighting operated by other suppliers was considered, these values increased to 16% and 21%, respectively. Simultaneous measurements of the zenith brightness of the sky during this period, when the power for street lighting decreased from 90% before midnight to 30% after midnight, showed that the brightness decreased by 5.4% near the city centre and 3.6% in the adjacent, suburban location [44]. The light source distribution model, assuming that 26% are street lamps and 74% other sources, suggests that street lamps contribute about 14% to the light emission into the sky, causing the formation of a sky glow over the city. Direct measurements, however, suggested that the contribution of street lighting to the zenithal sky brightness is only 2–3% [43]. The difference between the modelled and experimentally determined values was explained by an underestimation of the total light emission in the city, which did not take into account the changing numbers of light sources emitting light horizontally, such as indoor lighting of residential premises and others, emitting light through uncovered building windows, car headlights or illuminated road signs and advertisements. This difference may also be related to the level of air pollution in the studied areas with suspended particulate matter [45,46].

The declaration of the COVID-19 pandemic in 2020 resulted in an increased burden on healthcare, reduced population mobility and a related decline in tourism, as well as various unexpected effects on the natural environment [47]. As a result of the reduction in transport and industrial activity, the overall carbon dioxide emission decreased [48]. Improvements in air quality and reduced water pollution were also noted in many cities worldwide [49]. The effect of the pandemic was also visible through the reduction of anthropogenic seismic noise (vibrations of the ground) from train traffic, aircraft, industrial processes, etc. [50]. To limit population mobility and thus prevent virus transmission, municipal lighting shutdowns were also introduced in many European cities, which provided an unprecedented opportunity to study the effect of this type of lighting on the radiance and the brightness of the night sky glow. Studies taking into account both ground-based measurements of the sky glow brightness and satellite radiance values, performed from March 14 to May 31, 2020, for Granada (Spain), showed a significant reduction in light pollution related both to a decrease in

light emissions from the city area and to the reduction in the content of anthropogenic aerosols in the atmosphere, which are the source of light scattering [45].

In 2019, a photometric map of the distribution of artificial sky glow was developed for Berlin and its surroundings, both in cloudless and overcast skies [46]. This map shows that during the pandemic, the sky's brightness at its zenith dropped by 20% in the city centre and by more than 50% at a distance of 58 km from it. At the same time, based on satellite data, it was found that the radiance from this area increased. It was suggested that the main reason for the reduction in the artificial sky glow brightness is the improvement in air quality due to the decrease in air and road traffic.

A similar analysis was conducted for several communes in the Małopolskie Voivodeship [50,51]. It aimed to research the impact of street lighting reduction on the radiance emitted from a given area with a variable degree of urbanisation. It was found that in the case of poorly urbanised communes, changes in radiance are caused exclusively by switching off street lighting in rural areas. However, in the case of urban communes, such changes are caused primarily by switching off city lighting. It was also found that switching off local road lighting is mainly responsible for reducing radiance, perhaps due to the poorer quality of lamp fittings on these streets.

In April 2020 and the following months, the authorities of Kraków, as well as several other municipalities in the Małopolskie Voivodeship, adopted a law to entirely or partially switch off street lighting during night hours, starting from midnight (or from 1:00 CEST - Central European Summer Time) until dawn (Fig. 2).



Fig. 2. Switching off street lights in Kraków. Photos were taken 12 minutes before midnight (left, a) and 15 minutes after midnight (right, b) on the night of April 26-27, 2020. The only light sources still active were a gas station (left), windows of several apartments, security lighting and advertisements (in the distance) [Photo: Author].

This allowed us to investigate the contribution of this type of lighting both to the radiance of the entire city and to the changes in the surface brightness of the night sky. This was facilitated because the VIIRS/DNB images of the study area were acquired after 1:00 CEST, i.e. after the street lights had been switched off. The subject of this publication is an attempt to determine what part of the energy emitted into the sky comes from municipal street lighting.

1.3. Impact of artificial lighting on ecosystems and human health

Artificial light at night (ALAN) introduces several disturbances

in the functioning of the natural environment and is harmful to human health. The literature devoted to this issue is very abundant [52].

In particular, the following negative impacts of light pollution on the environment can be mentioned, which are especially important in the urban environment:

- disturbance of nocturnal bird migration by city lights [53],
- collision of nocturnal birds with bright office windows [54],
- disturbance of reproductive cycles of several insect species [55],
- disturbance of the hunter-prey balance in various environments, including water bodies on the outskirts of cities [56].

Night lighting also affects human health by inhibiting the pineal gland's production of melatonin. It leads to an increased risk of developing sleep disorders [57], a number of mental illnesses [58], and somatic diseases, including some cancers [59].

Given the negative impact of night lighting on humans, it is all the more important to ensure proper light emission in cities, especially in housing estates, which are currently the "bed-rooms" for millions of people in Poland.

1.4. Legal regulations and standards

1.4.1. Standards specifying street lighting

The basic lighting requirements for roads intended mainly for motor traffic at high and medium speeds are based on criteria related to the level and uniformity of the luminance of the road itself, the lighting of its immediate surroundings and the appropriate limitation of glare. These criteria correspond to the relevant classes. When analysing street lighting in cities, it is necessary to consider primarily such spaces as municipal streets, residential streets, and pavements. The situation of each space mentioned above is regulated by the appropriate guidelines of the lighting standard PN-EN 13201:2016 concerning road lighting [60,61]. By this standard, classes M, C and P have been established. Class M concerns streets with increased motor traffic [62]. It concerns the area planned for motor traffic while pedestrian traffic occurs in specially designated, separate zones. Both traffic areas are treated as separate traffic zones, subject to different legal conditions. Class C concerns conflict areas, where car traffic meets pedestrian and cyclist traffic. These are areas particularly prone to collisions, which makes them dangerous and requires special solutions. This class also includes pedestrian crossings, intersections and areas with variable geometry. Class P was created mainly for road areas where pedestrians and cyclists move. Car traffic is limited in this zone, and drivers must maintain low speeds.

The requirements for street and sidewalk lighting in individual classes are limited only to the level of average illuminance and uniformity of illuminance. In particular, low-power street luminaires (usually P4 or P5 class) can be used to illuminate residential roads and sidewalks, which, when the appropriate distance between poles of 3-4 m high is maintained, illuminate the surface of sidewalks or residential streets very well, ensuring

compliance with the requirements of the standard mentioned above [63]. For streets running near housing estates, the M classes apply, which include requirements for routes with higher motorised traffic [62].

All the listed classes of street lighting, regulated by the standard mentioned above, only define the minimum level of luminance on the street surface and its uniform distribution without regulating the escape of light to the surroundings. In particular, they do not define the ULOR (Upper Light Output Ratio) parameter, which specifies the percentage of light sent to the upper hemisphere, and therefore to the sky, when the lighting fixture is set horizontally to the ground (without tilt).

However, it should be remembered that street lighting should not be burdensome for the surroundings, i.e. it cannot be a source of light pollution. In particular, it should not illuminate adjacent areas, especially windows of nearby buildings (trespass effect), and it should not blind drivers and passers-by (glare effect). Incorrect design of luminaires and/or their installation may also contribute to the night sky glow generated by the city [9,64–66].

1.4.2. Standards determining the level of light pollution

In some countries, the adverse effects of artificial night lighting on the environment have been noticed. In these countries, various actions have been taken, from legal regulations on a national, regional, or local scale to the formulation of technical standards for correct lighting parameters to guide educating the general public. Supported by researchers and reports on the risks associated with artificial light at night, governments and citizens are beginning to realise that the effects of light pollution are widespread and intervention is desirable. In Poland, an example of such grassroots actions is the establishment of the Light Pollution Think Tank (LPTT), which published a multidisciplinary guide on this issue in 2022 [67] and a report on the state of light pollution in Poland in 2023 [68].

In 2022, a review of the state of knowledge about light pollution in various European countries and the measures taken against it was published [69]. In some countries, legal acts at the national level (France, Croatia) or regional level (Spain) establish the principles of protection against light pollution. In other countries (Slovenia), decrees are issued for this purpose. In a few countries, there are (Austria) or are planned (Czech Republic) technical standards regulating the selection of luminaires with parameters for reducing scattered light, choosing the correct correlated colour temperature (CCT) or others. However, these are non-binding documents, which only legal documents refer to, making the standard mandatory.

Most European countries are already taking action to address light pollution at the national level. These include regulations, strategies, technical standards, voluntary actions aimed at awareness-raising manuals or guidelines, research projects or dark sky areas.

Furthermore, light pollution is starting to be addressed at the European level, not only from an energy efficiency perspective but also from the perspective of its adverse impacts on human health and biodiversity.

A voluntary Green Public Procurement Criteria for Street Lighting and Traffic Signals, issued in 2018 and updated in 2019, guides public policymakers and lighting professionals to procure lighting equipment based on technical criteria for the design, installation and operation phases [70]. Light pollution is cited as one of the three main environmental impacts of street lighting and traffic signals, alongside energy consumption and durability. Criteria proposed to reduce energy consumption (such as dimming during low road use and selecting the lowest necessary road lighting class to prevent unnecessary over-lighting) also help reduce light pollution, along with criteria to control up and down light output factors, CCT and blue light output. The document introduces the requirement to set lighting at ALARA levels (as low as reasonably achievable), which comply with the standard class EN 13201 (Road Lighting Standards) and do not endanger road safety.

In the Zero Pollution Action Plan, a key element of the European Green Deal adopted by the European Commission in 2021, light pollution is identified as a pollutant of emerging concern and research on it will be supported by the Horizon Europe programme [71]. Pollutants of emerging concern, including light pollution, will be included in the emerging pollutant monitoring and forward-looking framework and thus translated into policy recommendations – EU criteria for green public procurement of road lighting and traffic signals.

The impact of light pollution on ecosystems has also been noted. In particular, light pollution has been mentioned as one of the factors of pollinator decline, which will be addressed in the revised EU Pollinator Initiative [72].

In the document [69], the legislation, standards and manuals published in each of the 27 European Union countries, as well as in the United Kingdom and the EFTA states (Iceland, Liechtenstein, Norway and Switzerland), were analysed. Poland is listed and described as a country without a national policy on light pollution. However, there are awareness-raising initiatives connected to light pollution, in particular, the activities of the LPTT or the parks and dark sky protection areas operating in Poland. The aforementioned PN-EN 13201:2016 standard does not constitute legal regulations. Moreover, as already stated, it only describes the lighting of road surfaces, saying nothing about reducing light pollution.

Environmental protection issues in Poland are specified in the Environmental Protection Law (Dz.U., 2022, 2556). According to its provisions, ecological protection means taking or omitting actions that enable maintaining or restoring natural balance. On the other hand, the Act defines "pollution" as an emission that may be harmful to human health or the state of the environment, may cause damage to material goods, may deteriorate the aesthetic values of the environment or may interfere with other, justified ways of using the environment. The above definition could also include light pollution. The consequences of omitting light in the Environmental Protection Law are significant. Light emissions have not been assigned reference measurement methods or emission standards, which makes it challenging to monitor the intensity of artificial light effectively. Since there is no "emission standard", it is impossible to determine if it has been

exceeded. Therefore, any emission of artificial light, regardless of intensity, cannot be considered pollution under the law.

The qualification of light as a possible environmental pollution is unambiguous in European Union legislation. Directive 2011/92/EU of the European Parliament of 2011 obliges Member States to prepare an environmental impact assessment of specific investments (in Polish regulations, their nature is specified in the rules on projects that may significantly impact the environment). The directive speaks directly about assessing the type and quantity of expected residues and emissions (water, air and soil pollution, noise, vibrations, light, heat, radiation, etc.) resulting from the operation of the proposed project. Light is mentioned here alongside radiation.

The provision mentioned above was introduced into the legislation of the European Union in 1985 by Council Directive 85/337/EEC (OJ L 175, 5.7.1985, p. 40) and was retained in later legislative amendments. The implementation of the directive in Poland took place through the Act of 3 October 2008, which provided information on the environment and its protection, public participation in environmental protection, and environmental impact assessments (Dz.U. 2023, item 1094, as amended).

However, Polish legislation does not list a catalogue of possible forms of pollution like the European Union Directive. Still, it only refers to the provisions of the Environmental Protection Law and the definitions contained therein (omitting light emissions). Moreover, the EU Directive and the Polish Act cover a closed catalogue of large-scale investments. These circumstances make the regulations related to environmental protection and the provision of information on the environment an ineffective tool for counteracting light pollution.

Two legal acts provide real, although limited, legal grounds for reducing light pollution within the framework of applicable law. The first is the aforementioned Act on Spatial Planning and Development. It gives communes a tool to protect the cultural and natural landscape through a so-called landscape resolution. Through the document, the commune council specifies the rules for placing advertising media. Some of them are illuminated or backlit in a way that generates light pollution. Although the resolution does not contain provisions directly relating to light pollution, the consequence of its adoption may be a reduction in the number of light-emitting sources or a decrease in the intensity of this light.

The second option is to apply the provisions of the Regulation of the Minister of Infrastructure on the technical conditions to be met by buildings and their location (Dz.U., 2022, item 1225). It contains a provision regulating the maximum permissible lighting intensity, among others, on the façades of buildings (including residential buildings). The provision states that lighting devices, including advertisements, placed outside the building or in its surroundings may not cause a nuisance to its users, passers-by, or drivers. Suppose the light is directed towards a building façade containing windows. In that case, the lighting intensity on that façade must not exceed 5 lux for white light and 3 lux for coloured light or light with varying intensity, flashing, or pulsating (par. 293, point 6).

It is also worth mentioning that many new investments in Poland are currently trying to obtain the building research establishment environmental assessment method (BREEAM) certificate [73]. This method of building certification indicates the extent to which the assessed facility is friendly to the natural environment and, on the other hand, how much comfort it creates for its users. This certificate takes into account light pollution as one of the factors influencing the sustainable development of buildings. The BREEAM rating system assesses how a building's design affects the natural environment, including light pollution. Unfortunately, this only applies to the lighting of buildings and their surroundings, not addressing the street lighting problem.

1.4.3. Street lighting as a significant source of light pollution

Road lighting is by far the most widespread and essential in the context of light pollution. This lighting covers practically every road and street, so its impact is tremendous, especially in cities with extensive infrastructure. Of course, light advertising and decorative lighting are also sources of light pollution. Still, they are more local and often concentrated in specific parts of the city (e.g. shopping centres and tourist streets), so their impact on light pollution is minor on a city-wide scale.

As mentioned earlier, the lighting standards in Poland concern the level and uniformity of luminance at the street level. In this aspect, an essential element in the process of designing both indoor [74] and outdoor [75,76] lighting is the "maintenance factor", which takes into account the reduction in the "efficiency" of the lighting system over time, mainly due to the dirtiness of lighting fixtures and the ageing of light sources. In the initial phase of use (operation), road lighting will have a higher "intensity" (will emit a greater luminous flux) than required by standards to compensate for the loss of "power" over time. It is determined at the stage of designing the lighting installation to ensure that the lighting parameters meet the minimum standard requirements throughout the entire period of operation. As a standard, the maintenance factor assumes values below 1.0. This means the designed installations initially provide a surplus of lighting parameters (oversizing). Thanks to this, in the later period of operation (e.g. after a few years), the actual lighting parameters will still meet the minimum standard requirements. In the case of LEDs, the maintenance factor may be higher (closer to 1.0) due to, for example, the possibility of adjusting the luminous flux during use (dynamic control). On the other hand, for traditional luminaires with discharge lamps (e.g. sodium HPS), lower values are assumed, which in practice leads to significant oversizing of the initial road lighting.

Road lighting in Poland is designed following applicable standards (e.g. PN-EN 13201), which specify the minimum required luminance or illuminance parameters (depending on the road lighting category). During the technical acceptance of lighting installations, the correctness of lighting is checked from this angle, considering the road's specificity and surroundings. However, even well-designed installations emit light, which, through reflection from the surface (and other objects), reaches the atmosphere, generating secondary light pollution. Light re-

flected from the illuminated surface is a significant source of pollution, often underestimated in the design process. This is most noticeable in winter when the reflective properties of the surface change significantly. Failure to consider the ULOR (upward light output ratio) parameter, especially in older installations, causes the direct escape of light into the upper hemisphere. It should be noted that regular, detailed measurements of road lighting parameters during operation are not standard practice in Poland. Moreover, street lighting design still sometimes uses luminaires whose sole purpose is attractive appearance, not considering even the mentioned ULOR parameter and directing most of the light into the upper hemisphere (Fig. 3).



Fig. 3. Lighting of Babina Str. in Kalisz (Poland) implemented using spherical luminaires with a high ULOR coefficient
[Photo: Author, 14.07.2023].

1.4.4. Impact of street lighting on the natural environment

The significant impact of light pollution, the source of which is street lighting, on the natural environment has long been noticed. This applies to insects [36–40] or migratory birds [77,78]. Attention was drawn to the increasingly common use of LED luminaires, replacing earlier ones, most often high-pressure sodium (HPS) lamps. In most cases, LED luminaires ensure proper ground lighting, providing the ULOR parameter close to zero. These luminaires usually have a high CCT value, which means a significant share of the blue part in the lamp's spectrum. As has been repeatedly shown, this type of light negatively impacts ecosystems [79–81]. Additionally, it is much more effectively reflected from road surfaces (asphalt, concrete) than in the case of luminaires with a low CCT value, thus becoming a factor that strongly influences the brightness of the night sky glow [82,83].

1.4.4. Switching off street lighting as a method of determining its contribution to the night sky glow

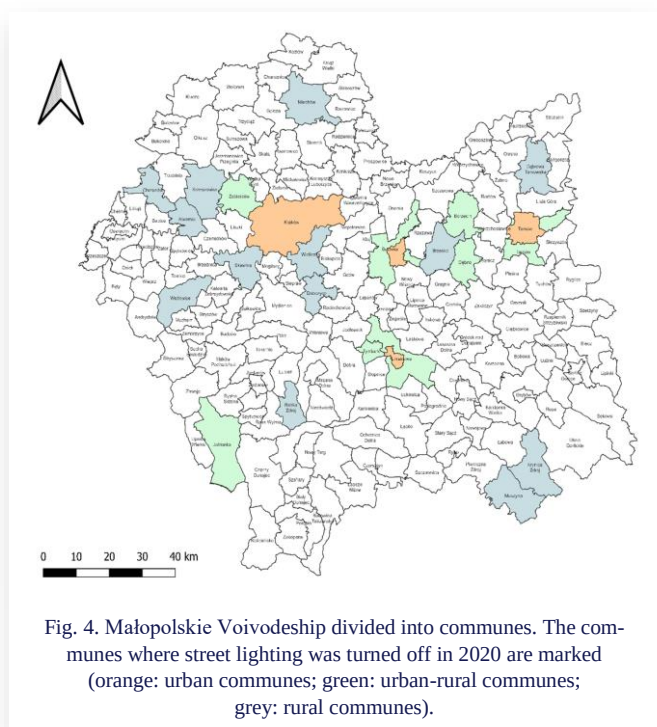
Switching off street lighting in cities, especially during the COVID-19 pandemic, has been used previously to determine the impact of this type of lighting on light pollution [41–46,84]. Of course, such infrastructure elements as illuminated advertisements, large LED screens, decorative lighting and illuminated signs have an impact on increasing the brightness of the light

glow. However, these elements constitute a negligible percentage of the light sources escaping into the sky. Moreover, as part of landscape protection, such elements have been removed in many locations, especially in Kraków, as of July 1, 2020.

2. Research area and methods

2.1. Research area

The research was conducted in the Małopolskie Voivodeship in Poland (Fig. 4). The Małopolskie Voivodeship includes 182 communes in 14 urban communes (including the city of Kraków, which is a separate commune), 48 urban-rural communes and 120 rural communes [85]. Information on switching off lighting in Kraków and other locations in the Małopolskie Voivodeship was analysed from among these communes. Based on the monthly averaged VIIRS/DNB images analysis, 22 locations were selected where a significant reduction in radiance was observed in some months of 2020. The study of media reports, as well as information received from local authorities of the surveyed locations, confirmed that in the case of 21 locations in the Małopolskie Voivodeship in 2020, due to the COVID-19 pandemic, municipal lighting was indeed switched off for at least one month. These are Alwernia, Bochnia, Borzęcin, Brzesko, Chrzanów, Dąbrowa Tarnowska, Dębno, Dobczyce, Jabłonka, Kraków, Krynica Zdrój, Krzeszowice, Limanowa, Muszyna, Rabka Zdrój, Skawina, Tarnów, Tymbark, Wadowice, Wieliczka and Zabierzów (Fig. 4).



Of the selected locations, four are seats of urban communes (Bochnia, Kraków, Limanowa, Tarnów), eight are rural communes (Bochnia, Borzęcin, Dębno, Jabłonka, Limanowa, Tarnów, Tymbark, Zabierzów), the remaining are seats of urban-rural communes with varying degrees of urbanisation (Table 1).

Table 1. Localities in Małopolskie Voivodeship, where municipal lighting was switched off in 2020. UG – Commune Office, UM – City Office, UMiG – City and Commune Office). The pause means that there is no data on the restoration (or not) of street lighting in a given locality.

Commune	Year 2020		Source of data
	beginning	end	
Alwernia	November	—	UG
Bochnia	May 1	September 1	daily press
Borzęcin	May 1	—	UG
Brzesko	April 30	June 30	UG
Chrzanów	June 25	September	UMiG
Dąbrowa Tarnowska	June 1	December	daily press
Dębno	May 11	—	UG
Dobczyce	May 20	—	UMiG
Jabłonka	May 1	August 31	UG
Kraków	April 15	June 1	UM
Krynica Zdrój	mid April	—	VIIRS/DNB
Krzeszowice	April 23	July	UMiG
Limanowa	May 15	August 31	UM, UG
Muszyna	mid April	July	UMiG
Rabka Zdrój	May 14	June 12	UM
Skawina	May 13	September 14	UMiG
Tarnów	April 18	September 25	UG
Tymbark	June 1	—	UG
Wadowice	June 1	September 14	UM
Wieliczka	May 1	September 1	UMiG
Zabierzów	June 1	December 31	UG

2.2. Sources of light pollution

As shown by the analysis of VIIRS/DNB images taken in March 2020, which light sources are also sources of light pollution depends on the nature of the municipality being studied:

- in urban communes, the most light is emitted into space from city centres (e.g. Bochnia, Limanowa), extra-urban economic centres and warehouses (e.g. Bochnia, Limanowa), railway stations (e.g. the Main Railway Station in Kraków), housing estates and academic campuses (e.g. the AGH University campus in Kraków), or some industrial plants (e.g. the ArcelorMittal steelworks in Kraków or the Grupa Azoty company in Tarnów),
- in urban-rural communes, the most light is emitted into space from city centres (in each case), suburban warehouses of various companies (Alwernia, Skawina, Wieliczka), urban and extra-urban industrial plants (Brzesko, Chrzanów, Skawina, Tarnów), quarries (Krzeszowice), summer resorts (Krynica Zdrój, Muszyna, Rabka Zdrój), communication hubs (Rabka Zdrój), and heat and power plants (Skawina),
- in rural communes, the main emitters of light are, first of all, towns that are the seats of communes, and also, most often located nearby, industrial plants and warehouses (Bochnia, Borzęcin, Jabłonka, Limanowa, Tarnów, Tymbark), communication arteries (Dębno), shopping centres (Limanowa), greenhouse complexes (Zabierzów), airports (Zabierzów).

It should be noted that VIIRS/DNB images are taken at approximately 1:00 a.m. local time when residential lighting is no longer active. This means that the main emitters of light are now

industrial lighting, advertising lighting, and traffic lights, both pedestrian, road and rail, as shown in [50].

This work aims to research the radiance from municipal lighting. Often, switching off night lighting only concerned localities that were seats of communes. Moreover, it did not affect various industrial plants, warehouses, greenhouses, etc., located in the commune. Therefore, further analysis was limited only to the study of light emission from the areas of localities that were seats of communes (Table 1).

2.3. Turning off the lights in the analysed municipalities

In April 2020, the authorities of many municipalities of the Małopolska Voivodeship, starting with Kraków (the capital of the voivodeship), decided to completely or partially turn off street lighting at night in their areas due to the reduction in street traffic during the developing COVID-19 pandemic. Lighting was switched off automatically at set times (usually at midnight official time). This process was controlled by clocks placed in lighting control cabinets in different lighting zones.

The most widespread action was to switch off city lighting in Kraków [50]. On April 8th, lighting in parks was completely switched off. From midnight on April 15th, street lighting within the city limits, subordinate to the Kraków City Roads Authority, was completely switched off. Street lighting was switched off at midnight and switched on again at 4:00 CEST. From April 22nd, park lighting was synchronised with street lighting. From May 19th, the switching off of street lighting in Kraków was postponed to 1:00 CEST, and this state was maintained until dawn. The only active sources of artificial lighting were fixtures located on private properties, fixtures subordinate to housing communities, some advertising lighting and the lighting of the key junctions (Fig. 2). From midnight on June 1st, standard lighting was restored in the city.

2.4. Analysed communes

The radiance from all municipalities of the Małopolska Voivodeship was analysed. In some cases (Fig. 4, marked in colours), street lighting was turned off at night, most often between midnight and 4:00 CEST. Street lighting is controlled by astronomical clocks placed in each lighting control cabinet in a given area, switching off the lighting in the city successively within a few minutes. Some clocks (e.g. in the Wieliczka commune) were not changed from the so-called winter time (i.e. Central European Standard Time, CET) to Central European Summer Time (CEST), as a result of which the lighting was turned off at 1:00 CEST in the areas served by them. For all the towns that are the seats of these municipalities, a radiance analysis was done based on VIIRS/DNB images.

2.5. Used units

Radiometry is a branch of physics and metrology that deals with quantitative measurements of optical radiation, i.e. electromagnetic radiation in the frequency range of $3 \cdot 10^{11}$ and $3 \cdot 10^{16}$ Hz. The only real difference between radiometry and photometry is that radiometry covers the entire spectrum of optical radiation.

In contrast, photometry is limited to the visible spectrum in terms of its influence on visual impressions in the human eye (taking into account the eye's spectral sensitivity) [86]. From the VIIRS/DNB satellite data, the radiance value is obtained, i.e. the radiant flux per unit area per unit solid angle. The symbol of radiance is L , while the standard SI unit is $W/(m^2 \cdot sr)$ [56]. This publication gives radiance values in derived units $nW/(cm^2 \cdot sr)$, in which the VIIRS/DNB data are provided.

2.6. Satellite remote sensing

Images obtained from the Suomi NPP satellite were used to determine the total radiance from the study areas [30]. The photos taken in the DNB (Day Night Band) spectral range [32] are helpful for this work. The centre of this band is at $700 \mu m$; it is $400 \mu m$ wide with an imaging resolution of $0.8 km$. Raw DNB data require careful processing and consideration of many factors, such as clouds or the angle of the image. Therefore, previously processed data were used in the analysis. In the monthly mean radiance values analysis, ready-made Monthly Cloud-free DNB Composites were used [32,87].

To perform a comparative analysis of the radiance values for specific nights before and after the start date of the lights outage in a given location, daily VIIRS/DNB images downloaded from the Level-1 and Atmosphere Archive & Distribution System Distributed Active Archive Center at Goddard Space Flight Center (LAADS DAAC) [88,89] were also analysed. During the research period (April – June 2020), VIIRS/DNB images of the study area were taken around 0:10 UTC, i.e. 2:10 CEST. Because the clocks turning off street lighting at midnight in Poland at that time were operating according to CEST, VIIRS/DNB images were taken at a time when the lighting was turned off. It should be noted that for the latitudes of the Małopolskie Voivodeship, there is no daily data from June 2020 from the period near the summer solstice.

2.7. Information about street lighting shutdowns

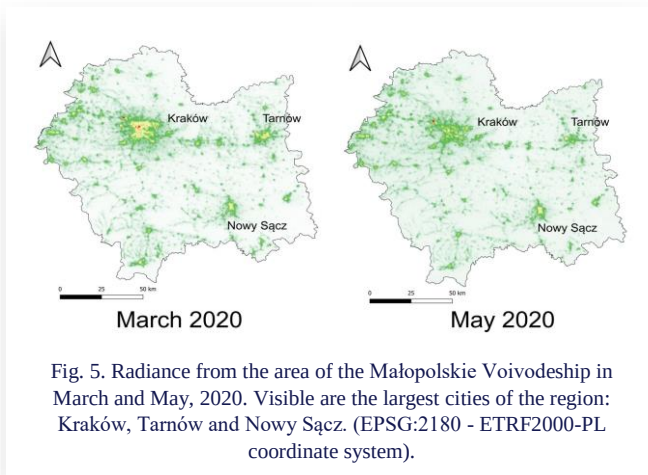
The local government offices that took these actions provided information on the spatial and temporal scope of street lighting shutdowns in 2020. In addition, information from the local press and residents of the analysed communes was also used.

3. Results and discussion

3.1. Measurement results

Average monthly radiance values obtained from VIIRS/DNB data were used. The radiance value in March 2020 was selected as typical for the period just before the announcement of restrictions related to the COVID-19 pandemic (data from January and February were omitted due to highly variable weather conditions and variable snow cover in those months).

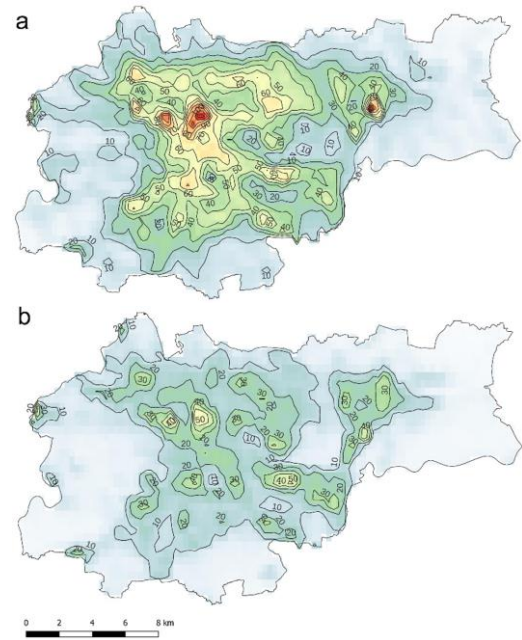
To select the communes where the action of switching off municipal lighting was taken, the VIIRS/DNB image of the Małopolskie Voivodeship from March 2020 was compared with similar images in the following months of 2020. Figure 5 presents VIIRS images for the entire Małopolskie Voivodeship in March and May 2020.



Based on this analysis, 21 communes were initially selected where a significant reduction in radiance was observed in various months of 2020 compared to March of this year. A question was sent to the authorities of all communes selected in this way, asking whether these shutdowns had taken place, and if so, they were asked to provide details of this action. Positive responses, including the dates and times of switching municipal lighting on and off, as well as the territorial scope of the action, were received from 12 communes; in the remaining cases, this data was recreated based on information from the local press or from communal orders publicly available on the Internet. No confirmation could be obtained from only two communes, Krynica Zdrój and Muszyna – they were included in the analysis because the reduction in radiance on VIIRS/DNB images in these cases is apparent and cannot raise doubts.

Radiance changes in some selected communes in Małopolskie Voivodeship have already been analysed in other publications [50,51]. As part of these studies, it was found that the results of these analyses were often influenced by light sources associated with industrial plants, warehouses, shopping centres, greenhouses, etc., located within the commune, most often outside the areas of dense construction. These sources, which contribute significantly to the total radiance from the location of a given commune and are not subject to local authorities, were not excluded as part of the described actions to reduce municipal lighting. This means that the quantitative conclusions drawn from our analyses may sometimes not be entirely correct. To avoid this effect, this analysis was limited to measuring radiance only from the area of localities that are the seats of municipal authorities.

In some cases, when there are strong light sources within the boundaries of the locality which are not municipal lighting (e.g. industrial plants, greenhouses, etc.), it was limited to areas where these sources do not occur (e.g. Tarnów). Table 2 presents the average monthly radiance values obtained from analysing VIIRS/DNB images for 21 selected localities in the Małopolskie Voivodeship in March (pre-pandemic radiance) in the month when the most significant number of municipal lighting was turned off (minimum pandemic radiance) (Fig. 6). The population of these localities, the area covered by the analysis, and the absolute and relative change in radiance in this area are also given.

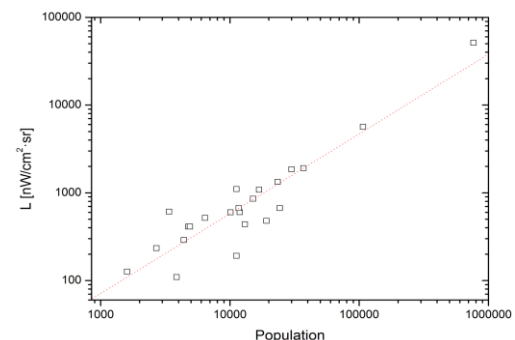


3.2. Discussion

3.2.1. Analysis of radiance measurements

As can be seen, of the analysed locations, the city of Kraków emits the most light into space (over $26\,000 \text{ nW}/(\text{cm}^2 \cdot \text{sr})$), while the village of Borzęcin emits the least light (only $33 \text{ nW}/(\text{cm}^2 \cdot \text{sr})$).

Figure 7 shows the dependence of radiance on population. A clear linear dependence (with the correlation coefficient $R=0.9403$) was found between the logarithms of these quantities. This dependence is consistent with the proposed models [13,15].



The pie chart (Fig. 8) shows the distribution of localities in the Małopolskie Voivodeship depending on the size of the reduction in relative radiance resulting from switching off municipal lighting. The communes in which the lockdown caused the greatest reduction in radiance (60-80%) were by far the most numerous.

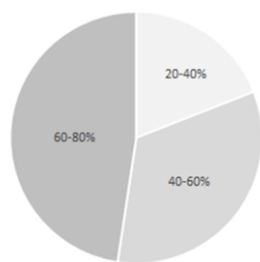


Fig. 8. Distribution of the number of localities in the Małopolskie Voivodeship depending on the size of the reduction in relative radiance as a result of switching off municipal lighting.

A statistical cluster analysis was performed to obtain a more complete picture of the impact of municipal lighting shutdowns on the radiance of individual locations. Three clusters of locations can be distinguished based on the elbow analysis and the analysis of the obtained dendrogram (Fig. 9). Due to the size of the relative reduction in radiance, these clusters concentrate around the average value of $(67 \pm 5)\%$ (cluster I), $(48 \pm 3)\%$ (cluster II) and $(36 \pm 4)\%$ (cluster III). The group represented by cluster I includes locations where all street lighting was switched off. Cluster II contains locations where part of the street lighting was left on, which is considered essential for the proper functioning of the city. In the case of Kraków, the fact that advertising lighting, some parking spaces, etc., were left on is undoubtedly significant here. In the locations covered by cluster III, only the illumination of peripheral, less essential streets was switched off. In the case of Wadowice, the lighting of the two main through streets, the City Park and the city's central square (Plac Jana Pawła II) was left. As can be seen, these facilities significantly contribute to this town's total radiance.

The above analyses show that the contribution of all municipal lighting to the total radiance of a given locality is approximately 67%. Comparing the dendrogram data with Table 2, it

was found that in the case of cluster I, the most significant reduction in radiance, amounting to even 80%, occurred for small towns (Dębno), where the dominant source of night lighting is street lighting. In comparison, the remaining 20% can be related to the illumination of private properties. Large cities in this cluster (Bochnia, Tarnów, Chrzanów, Wieliczka) recorded a reduction in radiance by approx. 67%. The remaining 33% is probably due to the lighting of housing cooperatives (not subject to municipal authorities), single-family private houses, workplaces, advertising banners, etc. This effect is even more visible for Kraków, the capital of the Małopolskie Voivodeship, which, together with Skawina, with a reduction in radiance by almost 50%, was included in cluster II.

Clustering Dendrogram of Percentage Reduction in Radiance

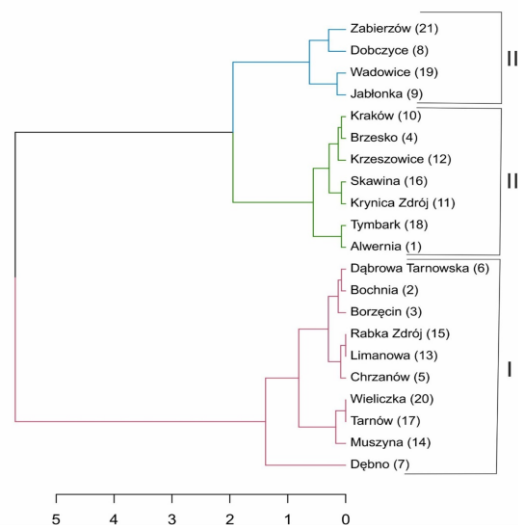
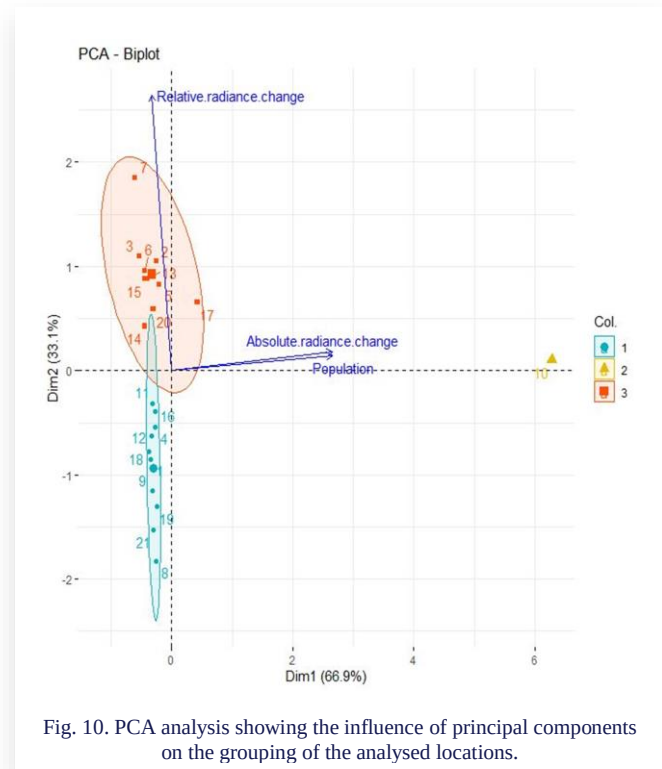


Fig. 9. Dendrogram showing the grouping of the analysed localities in terms of the percentage reduction in radiance during the street lighting shutdown period.

Table 2. Summary of radiance measurements in selected communes of the Małopolskie Voivodeship in March 2020. Krynica Zdrój and Muszyna, for which no confirmation of switching off municipal lighting was received, are marked in italics.

No.	Locality	Area, km ²	Population	Pre-pandemic radiance, nW/(cm ² ·sr)	Minimal pandemic radiance, nW/(cm ² ·sr)	Absolute change in radiance, nW/(cm ² ·sr)/km ²	Relative change in radiance, %
1	Alwernia	19.64	3380	609	217	265	44
2	Bochnia	20.55	29992	1856	584	1273	69
3	Borzęcin	3.37	3865	109	33	76	70
4	Brzesko	9.92	16827	1085	559	526	48
5	Chrzanów	20.61	37123	1913	656	1257	66
6	Dąbrowa Tarnowska	8.35	11924	602	195	406	68
7	Dębno	2.80	1601	126	25	100	80
8	Dobczyce	12.45	6425	518	360	158	31
9	Jabłonka	18.74	4767	412	247	164	40
10	Kraków	520.98	766683	51472	26311	25161	49
11	<i>Krynica Zdrój</i>	28.00	11243	1101	539	563	51
12	Krzeszowice	7.88	10090	600	317	283	47
13	Limanowa	10.62	15132	855	280	575	67
14	<i>Muszyna</i>	8.08	4909	412	161	251	61
15	Rabka Zdrój	6.75	13052	438	143	295	67
16	Skawina	5.43	24325	669	332	337	50
17	Tarnów	32.55	107045	5647	2098	3549	63
18	Tymbark	5.04	2700	235	123	112	45
19	Wadowice	2.92	19149	479	297	181	38
20	Wieliczka	8.90	23395	1334	494	840	63
21	Zabierzów	3.93	4400	291	190	100	35

To identify the factor determining the reduction in radiance, principal component analysis (PCA) was performed (Fig. 10).



This analysis shows that the dominant factor influencing the variance for Kraków (point 10) is the absolute radiance change and population. The remaining locations are divided into two clear groups, in which the variance is caused by the relative radiance change: 1 (Alwernia, Brzesko, Dobczyce, Jabłonka, Krynica Zdrój, Krzeszowice, Skawina, Tymbark, Wadowice, Zabierzów) and 2 (Bochnia, Borzęcin, Chrzanów, Dąbrowa Tarnowska, Dębno, Limanowa, Muszyna, Rabka Zdrój, Tarnów, Wieliczka). I believe that the share of municipal lighting in the total radiance of the location is responsible for this division.

3.2.2. Energy losses related to municipal lighting

Energy losses due to lighting the night sky should be understood as the part of electricity related to municipal lighting, which is directed directly into space and through reflection from the ground. In the latter case, it should be noted that the reflection of light from the ground does not always mean a loss of energy in the physical sense. Energy losses in lighting result mainly from emission of light in undesirable directions (e.g. upwards) and not necessarily from the reflection itself. If light were not reflected from the illuminated surface, it would not be visible to people. One should only strive for the luminance of such illuminated surfaces to be optimal, not causing a burden to the sense of sight, while ensuring the safety of users of the space.

In 1999, based on data from DMSP satellites, daily energy losses for city lighting were determined for all major cities in Japan and for several cities worldwide. Among them, Polish cities included Warsaw (with the result of $8.81 \cdot 10^6$ kWh) and Kraków (with the result of $4.40 \cdot 10^6$ kWh) [60]. It is difficult to re-

late these numbers to the current situation; one can only estimate how much electricity is currently lost for lighting the sky.

In 2018–2019, the average annual electricity consumption in Kraków, excluding households, amounted to $2.74 \cdot 10^6$ MWh [90], which gives $7.50 \cdot 10^6$ kWh per day. Street lighting costs constitute 38% of the total electricity consumption costs in the city [91], costing 29.5 million PLN per year. According to an analysis conducted by the International Dark Sky Association, approx. 35% of electricity is wasted on lighting the sky [91]. In the case of Kraków, this gives energy losses of around 10 million PLN (approx. 2.3 million EUR) per year. At the same time, on this basis, it is possible to determine the amount of Kraków's daily energy losses on lighting the sky at approx. $2.85 \cdot 10^6$ kWh. This significantly lower value than that given for 1999 is probably related to the systematic replacement of energy-intensive HPS lighting with energy-saving LED lighting. In 2017, approximately 3 000 sodium HPS fixtures were replaced with LED fixtures [92]. Currently, HPS fixtures constitute approximately 36% of all street fixtures in Kraków [93].

4. Conclusions

Satellite data analysis has shown that turning off municipal lighting significantly reduces radiance. When this type of lighting was turned off, in locations where it is the dominant source of artificial light at night, the radiance decreased by almost 70% (sometimes even 80%) compared to the pre-pandemic value. In large cities, where there are other light sources, such as industrial plants or advertisements, the radiance decreased by almost 50% in addition to street lighting. Turning off only the lighting of some peripheral streets reduced the radiance by only about 36%. A reduction in radiance by about 50% was also noted in smaller towns, where the lighting of the main streets was left on. It can be concluded from this that reducing the lighting of even just residential or local streets significantly reduces the emission of artificial light into the sky. Of course, it is not possible to permanently turn off this type of lighting. Still, it is possible to consider permanently leaving the lighting on only the main streets while installing motion sensors on low traffic, e.g. residential or local streets. The presented analysis shows that reducing light emissions into the sky by several dozen per cent is possible, which brings significant energy savings. A systematic replacement of street lighting fixtures in Kraków with energy-saving LED lighting is being carried out. Annual monitoring of radiance values from the Kraków area is planned to verify the thesis about the decrease in radiance.

Of course, the most beneficial solution would be implementing more advanced street lighting management systems, such as intelligent lighting control depending on vehicle traffic intensity. A similar system was analysed by the author of this paper in 2018 in Zselic Starpark (Hungary). Inside the park are localities where street lighting emits light of minimal intensity in the idle state, allowing only to distinguish objects on the ground. When a pedestrian or a car moves, the luminaires brighten only in the wake of the moving object. As a result, the lighting of the locality has a minimal impact on both local ecosystems and astronomical observations conducted within the Zselic Observatory [94].

Of course, it should be remembered that the basic task of street lighting at night is to ensure the safety of street users. However, it is necessary to implement solutions consistent with the principle of sustainable development – street lighting should provide life comfort for its users while minimising the impact on the natural environment. This is possible thanks to adopting standards that consider light pollution. It is often argued that such standards will cause living in the dark. Replacing lighting fixtures usually leads to better ground illumination while limiting light emission into the environment [95]. In 2013, a survey was conducted in Finland on a sample of 2053 people, aiming to present the harmful aspects of lighting and how the public perceives pollution [96]. The results show that residential and street lighting are considered the most common sources of light pollution, while commercial lighting is the most irritating form of light use. Respondents universally regarded light pollution as an interference with outdoor recreation and relaxation. The survey results indicate that light pollution's ecological and health effects are generally weakly reported, while aesthetic effects are emphasised.

In Poland, in 2020, a study was conducted on the impact of changing the illuminance of streets and sidewalks in the Żywieckie Housing Estate in Kraków after replacing HPS luminaires with LED lighting [97]. During the study, several randomly encountered residents, most often walking their dogs, were asked about their opinion on the impact of these lighting changes on the quality of their lives. The interviews showed that the new luminaires illuminate the ground better while limiting the penetration of light into the interior of the apartments. The surveyed residents noted an improvement in the visibility of the ground, which is essential, for example, during evening walks with the dog. On the other hand, they stated that the housing estate had become "darker". In general, the opinion of residents was that the lighting should be such as to ensure safety during evening and night walks. Of course, such statements do not constitute the basis for any statistical analysis; they are only signals from a selected group of sidewalk users. This type of analysis requires broader studies covering all areas where this type of lighting replacement occurred. Such studies are planned.

It is also important to note the apparent conflict between lighting professionals, with a background in lighting design, who often criticize calling light "pollution," and light pollution experts, with a background primarily in astronomy and the environment. In 2019, a survey was completed by 205 participants, identified as "lighting professionals" and "light pollution experts" [98]. Analysis of the survey revealed that, despite their seemingly conflicting interests, lighting professionals and light pollution experts largely agree on the definition of the problem and approaches to solving it.

This paper aims to show the current status of the described problem in Poland, where no standards or laws regarding protection against light pollution have been introduced yet. This is important when the Light Pollution Think Tank is trying to introduce such legal provisions [68].

Acknowledgments

The author would like to thank the city and commune officials of the Małopolskie Voivodeship for providing information on the spatial and temporal scope of switching off the street lighting: Krzysztof Płaza (Kraków), Ilona Aniończyk-Kępa (Alwernia); Agnieszka Piekarska (Borzęcin); Otylia Haber (Brzesko); Krzysztof Bilski and Jolanta Zubik (Chrzanów); Stanisław Molik (Czernichów); Anna Łanocha (Dębno); Tomasz Suś (Dobczyce); Bolesław Wójcik (Jabłonka); Ewa Baranowska (Krzeszowice); Rafał Woźniak and Władysław Bieda (Limanowa); Eugeniusz Kozioł (Liszki); Andrzej Banaśkiewicz (Miechów); Włodzimierz Tokarczyk (Muszyna); Leszek Świder (Rabka Zdrój); Grzegorz Kozioł (Tarnów). Agnieszka Kiełbasa (Tymbark); Krzysztof Grajny (Wadowice); Elżbieta Kłapie (Wieliczka) and Dariusz Hajto (Zabierzów).

References

- [1] Guillemin, A. (1866). *Le Ciel*, L.Hachette, Paris (pp. 384–385).
- [2] Oliver, J.A.W. (ed.) (1888). *Astronomy for Amateurs*, Longmans, Green (p. 295).
- [3] Ścieżor, T. (2018). *Natural and anthropogenic factors of the night sky glow*. Monografie Politechniki Krakowskiej. Kraków (in Polish).
- [4] Massey, P., Gronwall, C., & Pilachowski, C.A. (1990). The spectrum of the Kitt Peak night sky. *Publications of the Astronomical Society of the Pacific*, 102, 1046. doi: 10.1086/132733
- [5] Ścieżor, T. (2020). Light pollution at the Botanical Garden of Jagiellonian University – preliminary analysis. In *Ecological and astronomical aspects of light pollution* (pp. 75–91). Wydawnictwo Politechniki Krakowskiej, Kraków.
- [6] Brons, J., Bullough, J., & Rea, M. (2008). Outdoor site-lighting performance: A comprehensive and quantitative framework for assessing light pollution. *Lighting Research & Technology*, 40, 201–224. doi: 10.1177/1477153508094059
- [7] Guanglei, W., Ngarambe, J., & Kim, G.A. (2019). Comparative Study on Current Outdoor Lighting Policies in China and Korea: A Step toward a Sustainable Nighttime Environment. *Sustainability*, 11, 3989. doi: 10.3390/su11143989
- [8] Zielińska-Dabkowska, K.M., Xavia, K., & Bobkowska, K. (2020). Assessment of Citizens' Actions against Light Pollution with Guidelines for Future Initiatives. *Sustainability*, 12, 4997. doi: 10.3390/su12124997
- [9] Helbich, M., Browning, M.H.E.M., & Huss, A. (2020). Outdoor light at night, air pollution and depressive symptoms: A cross-sectional study in the Netherlands. *Science of the Total Environment*, 744, 140914. doi: 10.1016/j.scitotenv.2020.140914
- [10] Pothukuchi, K. (2021). City Light or Star Bright: A Review of Urban Light Pollution, Impacts, and Planning Implications. *Journal of Planning Literature*, 088541222098642. doi: 10.1177/0885412220986421
- [11] Walker, M.F. (1970). The California Site Survey. *Publications of the Astronomical Society of the Pacific*, 82, 672. doi: 10.1086/128945
- [12] Walker, M.F. (1973). Light Pollution in California and Arizona. *Publications of the Astronomical Society of the Pacific*, 85, 508. doi: 10.1086/129496
- [13] Walker, M.F. (1977). The effects of urban lighting on the brightness of the night sky. *Publications of the Astronomical Society of the Pacific*, 89, 405. doi: 10.1086/130142
- [14] Treanor, P.J. (1973). A Simple Propagation Law for Artificial Night-Sky Illumination. *Observatory*, 93, 117–120.
- [15] Berry, R.L. (1976). Light Pollution in Southern Ontario. *Journal of the Royal Astronomical Society of Canada*, 70, 97–115.

- [16] Garstang, R.H. (1984). Improved Scattering Formula for Calculations of Artificial Night-Sky Illumination. *Observatory*, 104, 196–197.
- [17] Garstang, R.H. (1991). Light Pollution Modeling. *International Astronomical Union Colloquia*, 112, 56–69. doi: 10.1017/S0252921100003705
- [18] Garstang, R.H. (1986). Model for Artificial Night-Sky Illumination. *Publications of the Astronomical Society of the Pacific*, 98, 364. doi: 10.1086/131768
- [19] Aube, M., Franchomme-Fosse, L., Robert-Staehler, P., & Houle, V. (2005). Light pollution modelling and detection in a heterogeneous environment: Toward a night-time aerosol optical depth retrieval method. *Proceedings of the Atmospheric and Environmental Remote Sensing Data Processing and Utilization: Numerical Atmospheric Prediction and Environmental Monitoring*. 1–4 August 2005, San Diego, CA, USA.
- [20] Aubé, M., Simoneau, A., Munõz-Tunõn, C., Díaz-Castro, J., & Serra-Ricart, M. (2020). Restoring the night sky darkness at Observatorio del Teide: First application of the model Illumina version 2. *Monthly Notices of the Royal Astronomical Society*, 497, 2501–2516. doi: 10.1093/mnras/staa2113
- [21] Cinzano, P., Falchi, F., Elvidge, C.D., & Baugh, K.E. (2000). The artificial night sky brightness mapped from DMSP satellite Operational Linescan System measurements. *Monthly Notices of the Royal Astronomical Society*, 318, 641–657. doi: 10.1046/j.1365-8711.2000.03562.x
- [22] Cinzano, P., & Elvidge, C.D. (2004). Night sky brightness at sites from DMSP-OLS satellite measurements. *Monthly Notices of the Royal Astronomical Society*, 353, 1107–1116. doi: 10.1111/j.1365-2966.2004.08132.x
- [23] Cinzano, P., Falchi, F., & Elvidge, C.D. (2001). The first World Atlas of the artificial night sky brightness. *Monthly Notices of the Royal Astronomical Society*, 328, 689–707. doi: 10.1046/j.1365-8711.2001.04882.x
- [24] Kolláth, Z. (2008). The effects of artificial lights at the Zselic Landscape Protection Area. *Przegląd Elektrotechniczny*, 84(8), 76–79.
- [25] Pun, C.S.J., & So, C.W. (2012). Night-sky brightness monitoring in Hong Kong. A city-wide light pollution assessment. *Environmental Monitoring and Assessment*, 184, 2537–2557. doi: 10.1007/s10661-011-2136-1
- [26] Pun, C.S.J., So, C.W., Leung, W.Y., & Wong, C.F. (2014). Contributions of artificial lighting sources on light pollution in Hong Kong measured through a night sky brightness monitoring network. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 139, 90–108. doi: 10.1016/j.jqsrt.2013.12.014
- [27] Kubala, M., Ścieżor, T., Dworak, T.Z., & Kaszowski, W. (2009). Artificial Sky Glow in Cracow Agglomeration. *Polish Journal of Environmental Studies*, 18, 194–199.
- [28] Ścieżor, T., Kubala, M., Kaszowski, W., & Dworak, T.Z. (2010). *Light pollution of the night sky in Cracow agglomeration: data analysis of the artificial sky glow measurements*. Monografia 388, Politechnika Krakowska, Kraków (in Polish).
- [29] Ścieżor, T., & Kubala, M. (2014). Particulate matter as an amplifier for astronomical light pollution. *Monthly Notices of the Royal Astronomical Society*, 444, 2487–2493. doi: 10.1093/mnras/stu1577
- [30] Wolfe, R.E., Lin, G., Nishihama, M., Tewari, K.P., Tilton, J.C., & Isaacman, A.R. (2013). Suomi NPP VIIRS prelaunch and on-orbit geometric calibration and characterization. *Journal of Geophysical Research: Atmospheres*, 118, 11508–11521. doi: 10.1002/jgrd.50873
- [31] *VIIRS Maps*. <https://www.lightpollutionmap.info> [accessed 17 Jan. 2025].
- [32] Elvidge, C.D., Baugh, K.E., Zhizhin, M., & Hsu, F.-C. (2013). Why VIIRS data are superior to DMSP for mapping nighttime lights. *Proceedings of the Asia-Pacific Advanced Network Consortium*, 35, 62. doi: 10.1071/25/APAN.35.7
- [33] Duriscoe, D.M., Anderson, S.J., Luginbuhl, C.B., & Baugh, K.E. (2018). A simplified model of all-sky artificial sky glow derived from VIIRS Day/Night band data. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 214, 133–145. doi: 10.1016/j.jqsrt.2018.04.028
- [34] Falchi, F., Cinzano, P., Duriscoe, D., Kyba, C.C.M., Elvidge, C.D., Baugh, K., Portnov, B.A., Rybnikova, N.A., & Furgoni, R. (2016). The new world atlas of artificial night sky brightness. *Science Advances*, 2, e1600377. doi: 10.1126/sciadv.1600377
- [35] *Gateway to Astronaut Photography of Earth*. <https://eol.jsc.nasa.gov/> [accessed 17 Jan. 2025].
- [36] Gaston, K.J., Davies, T.W., Bennie, J., & Hopkins, J. (2012). Reducing the ecological consequences of night-time light pollution: options and developments. *Journal of Applied Ecology*, 49, 1256–1266. doi: 10.1111/j.1365-2664.2012.02212.x
- [37] Schulte-Römer, N., Meier, J., Söding, M., & Dannemann, E. (2019). The LED Paradox: How Light Pollution Challenges Experts to Reconsider Sustainable Lighting. *Sustainability*, 11, 6160. doi: 10.3390/su11216160
- [38] Boyes, D.H., Evans, D.M., Fox, R., Parsons, M.S., & Pocock, M.J.O. (2021). Street lighting has detrimental impacts on local insect populations. *Science Advances*, 7, eabi8322. doi: 10.1126/sciadv.abi8322
- [39] de Miguel, A.S., Bennie, J., Rosenfeld, E., Dzurjak, S., & Gaston, K.J. (2022). Environmental risks from artificial nighttime lighting widespread and increasing across Europe. *Science Advances*, 8, eabl6891. doi: 10.1126/sciadv.abl6891
- [40] Evans, D.M. (2023). Mitigating the impacts of street lighting on biodiversity and ecosystem functioning. *Philosophical Transactions of the Royal Society*, B37820220355. doi: 10.1098/rstb.2022.0355
- [41] Luginbuhl, C.B., Lockwood, G.W., Davis, D.R., Pick, K., & Sellers, J. (2009). From The Ground Up I: Light Pollution Sources in Flagstaff, Arizona. *Publications of the Astronomical Society of the Pacific*, 121, 185–203. doi: 10.1086/597625
- [42] Hiscocks, P.D., Gudmundsson, S., Hiscocks, P.D., & Gudmundsson, S. (2010). The Contribution of Street Lighting to Light Pollution. *Journal of the Royal Astronomical Society of Canada*, 104, 190–192.
- [43] Kyba, C., Ruby, A., Kuechly, H., Kinzey, B., Miller, N., Sanders, J., Barentine, J., Kleinodt, R., & Espey, B. (2020). Direct measurement of the contribution of street lighting to satellite observations of nighttime light emissions from urban areas. *Lighting Research & Technology*. doi: 10.1177/1477153520958463
- [44] Barentine, J.C., Kundracik, F., Kocifaj, M., Sanders, J.C., Esquerdo, G.A., Dalton, A.M., Foott, B., Grauer, A., Tucker, S., & Kyba, C.C.M. (2020). Recovering the city street lighting fraction from skyglow measurements in a large-scale municipal dimming experiment. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 253, 107120. doi: 10.1016/j.jqsrt.2020.107120
- [45] Bustamante-Calabria, M., de Miguel, A.S., Martín-Ruiz, S., Ortiz, J.L., Vilchez, J.M., Pelegrina, A., García, A., Zamorano, J., Bennie, J., & Gaston, K.J. (2021). Effects of the COVID-19 lockdown on urban light emissions: Ground and satellite comparison. *Remote Sensing*, 13, 258. doi: 10.3390/rs13020258
- [46] Jechow, A., & Hölker, F. (2020). Evidence that reduced air and road traffic decreased artificial night-time skyglow during COVID-19 lockdown in Berlin, Germany. *Remote Sensing*, 12, 3412. doi: 10.3390/rs12203412
- [47] Saadat, S., Rawtani, D., & Hussain, C.M. (2020). Environmental perspective of COVID-19. *Science of the Total Environment*, 728, 138870. doi: 10.1016/j.scitotenv.2020.138870
- [48] Le Quéré, C., Jackson, R.B., Jones, M.W., Smith, A.J.P., Abernethy, S., Andrew, R.M., De-Gol, A.J., Willis, D.R., Shan, Y., & Canadell, J.G. (2020). Temporary reduction in daily global CO₂ emissions during the COVID-19 forced confinement. *Nature Climate Change*, 10, 647–653. doi: 10.1038/s41558-020-0797-x

- [49] Lokhandwala, S., & Gautam, P. (2020). Indirect impact of COVID-19 on environment: A brief study in Indian context. *Environmental Research*, 188, 109807. doi: 10.1016/j.envres.2020.109807
- [50] Ścieżor, T. (2021). Effect of street lighting on the urban and rural night-time radiance and the brightness of the night sky. *Remote Sensing*, 13, 9. doi: 10.3390/rs13091654
- [51] Ścieżor, T. (2022). Impact of the COVID-19 pandemic on light pollution of the night sky. In *Artificial light at night: between threat and development*, Politechnika Łódzka, pp.161–199 (in Polish).
- [52] Rich, C., & Longcore, T. (eds.) (2006). *Ecological Consequences of Artificial Night Lighting*, Island Press
- [53] Van Doren, B.M., Horton, K.G., Dokter, A.M., Klinck, H., Elbin, S.B., & Farnsworth, A. (2017). High-intensity urban light installation dramatically alters nocturnal bird migration. *Proceedings of the National Academy of Sciences of the United States of America*, 114(42), 11175–11180. doi: 10.1073/pnas.1708574114
- [54] Loss S.R., Will, T., Loss, S.S., & Marra, P.P. (2014). Bird–building collisions in the United States: Estimates of annual mortality and species vulnerability, *The Condor*, 116(1), 1, 8–23. doi: 10.1650/CONDOR-13-090.1
- [55] Owens, A.C.S., Cochard, P., Durrant, J., Farnworth, B., Perkin, E.K., & Seymoure, B. (2020). Light pollution is a driver of insect declines. *Biological Conservation*, 241, 108259, doi: 10.1016/j.biocon.2019.108259
- [56] Moore, M.V., Pierce, S.M., Walsh, H.M., Kvalvik, S.K., & Lim, J.D. (2001). Urban light pollution alters the diel vertical migration of *Daphnia*. *Verhandlungen des Internationalen Verein Limnologie*, 27, 1–4.
- [57] Patel, P.C. (2019). Light pollution and insufficient sleep: Evidence from the United States. *American Journal of Human Biology*, 31(6):e23300. doi: 10.1002/ajhb.23300
- [58] Tancredi, S., Urbano, T., Vinceti, M., & Filippini, T. (2022). Artificial light at night and risk of mental disorders: A systematic review. *Science of The Total Environment*, 833, 155185. doi: 10.1016/j.scitotenv.2022.155185
- [59] Walker, W.H., Bumgarner, J.R., Walton, J.C., Liu, J.A., Meléndez-Fernández, O.H., Nelson, R.J., & DeVries, A.C. (2020). Light Pollution and Cancer. *International Journal of Molecular Sciences*, 21(24), 9360. doi: 10.3390/ijms21249360
- [60] Górczewska, M. (2015), Simplified procedures for selecting road lighting classes in accordance with the new version of the CEN/TR 13 201-1:2014 report, *Wiadomości Elektrotechniczne*, LXXXIII, 10, 27–30. doi:10.15199/74.2015.10.7 (in Polish).
- [61] Górczewska, M. (2017), LED lighting – not „all clear”, 12th Conference "Road lighting – ways of managing lighting systems across the country", Jachranka, 19–20 April 2017. *Energia elektryczna*, 5 (in Polish).
- [62] Trawiński, A. Lighting classes of luminaires, PERFAND LED. <https://perfandle.pl/klassy-oswietlenia-opraw-oswietleniowych/> [accessed 30 Apr. 2025] (in Polish).
- [63] Piątkowski, P. (2023). Outdoor lighting in housing estates – design and selection of luminaires, *Murator. Obiekty mieszkalne, instalacje*, 22 March 2023. <https://obiekty mieszkalne.murator-plus.pl/instalacje/oswietlenie-zewnetrzne-na-osiedlach-mieszkaniowych-projektowanie-oswietlenia-zewnetrznego-i-wybor-opraw-aa-eywB-tSu2-VZcZ.html> [accessed 30 Apr. 2025] (in Polish).
- [64] Du, J., Zhang, X., & King, D. (2018). An Investigation into the Risk of Night Light Pollution in a Glazed Office Building: The Effect of Shading Solutions. *Building and Environment*, 145. doi: 10.1016/j.buildenv.2018.09.029
- [65] Küçü, A.K., & Ekren, N. (2021). Light Pollution and Smart Outdoor Lighting. *Balkan Journal of Electrical and Computer Engineering*, 9(2), 191–200. doi: 10.17694/bajece.874343
- [66] Remande, C. (2001). Light Pollution: How High-Performance Luminaires Can Reduce It. *Preserving the Astronomical Sky, IAU Symposium*, 196, 49–59. doi: 10.1017/S007418090016382X
- [67] *Light Pollution: Identification and Prevention: A Multidisciplinary Guide* (2022). Light Pollution Think Tank, <https://poradniklptt.gr8.com/> [accessed 30 Apr. 2025] (in Polish).
- [68] *Light pollution in Poland. 2023 Report* (2023). https://lptt.org.pl/zasoby/lptt_raport_2023.pdf [accessed 30 Apr. 2025] (in Polish).
- [69] *Light pollution reduction measures in Europe* (2022). Ministry of the Environment of the Czech Republic, [https://mzp.gov.cz/C1257458002F0DC7/cz/news_20221027-/\\$FILE/Light_pollution_reduction_measures.pdf](https://mzp.gov.cz/C1257458002F0DC7/cz/news_20221027-/$FILE/Light_pollution_reduction_measures.pdf) [accessed 30 Apr. 2025].
- [70] Donatello, S., Rodriguez Quintero, R., De Oliveira Gama Caldas, M., Wolf, O., Van Tichelen, P., Van Hoof, V., & Geerken, T. (2019). Revision of the EU Green Public Procurement Criteria for Road Lighting and traffic signals, EUR 29631 EN, *Publications Office of the European Union*, Luxembourg. JRC115406. doi:10.2760/372897
- [71] COM/2019/640 final. (2019). *The European Green Deal*. Communication from the Commission to the European Parliament, the European Council, the Council, the European Economic and Social Committee and the Committee of the Regions.
- [72] COM/2018/395 final. (2018). *EU Pollinators Initiative*. Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions.
- [73] BREEAM. <https://breeam.com/> [accessed 30 Apr. 2025].
- [74] Duplákóvá, D., & Sloboda, P. (2024). The Maintenance Factor as a Necessary Parameter for Sustainable Artificial Lighting in Engineering Production – A Software Approach. *Applied Sciences*, 14, 8158. doi: 10.3390/app14188158
- [75] Cuvalci, O., & Ertas, B. (2000). Roadway lighting design methodology and evaluation. *Journal of Integrated Design and Process Science*, 4(1), 1–23. doi:10.3233/JID-2000-4101
- [76] Valetti, L., Piccablotto, G., Taraglio, R., & Pellegrino, A. (2023). Long-Term Monitoring Campaign of LED Street Lighting Systems: Focus on Photometric Performances, Maintenance and Energy Savings. *Sustainability*, 15, 16910. doi: 10.3390/su152416910
- [77] Cabrera-Cruz, S.A., Smolinsky, J.A., & Buler, J.J. (2018). Light pollution is greatest within migration passage areas for nocturnally-migrating birds around the world. *Scientific Reports*, 8, 3261. doi: 10.1038/s41598-018-21577-6
- [78] Adams, C.A., Fernández-Juricic, E., Bayne, E.M., & St. Clair, C.C. (2021). Effects of artificial light on bird movement and distribution: a systematic map. *Environmental Evidence*, 10(37). doi: 10.1186/s13750-021-00246-8
- [79] Pawson, S.M., & Bader, M.K. (2014). LED lighting increases the ecological impact of light pollution irrespective of color temperature. *Ecological Applications*, 24, 1561–1568. doi: 10.1890/14-0468.1
- [80] Supronowicz, R., & Fryc, I. (2019). Urban park lighting as a source of botanical light pollution, *Photonics Letters of Poland*, 11(3), 90–92. doi: 10.4302/plp.v11i3.935
- [81] Sakhno, T.V., Kozhushko, G.M., & Nazarenko, V.I. (2024). Light pollution from led lighting systems and ways of reducing its environmental consequences. *Ukrainian Journal of Occupational Health*, 20(2). doi: 10.33573/ujoh2024.02.138
- [82] Robles, J., Zamorano, J., Pascual, S., Sánchez de Miguel, A., Gallego, J., & Gaston, K.J. (2021). Evolution of Brightness and Color of the Night Sky in Madrid. *Remote Sensing*, 13, 1511. doi: 10.3390/rs13081511
- [83] Fryc, I., Listowski, M., Martinsons, C., Fan, J., & Czyżewski, D. (2024). A Paradox of LED Road Lighting: Reducing Light Pollution Is Not Always Linked to Energy Savings. *Energies* 17, 5727. doi: 10.3390/en17225727
- [84] Green, R.F., Luginbuhl, C.B., Wainscoat, R.J., & Duriscoe, D. M. (2022). The growing threat of light pollution to ground-based observatories. *The Astronomy and Astrophysics Review*, 30(1). doi: 10.1007/s00159-021-00138-3

- [85] *Małopolska Voivodeship 2019 – Subregions, Poviats, Municipalities*. <https://krakow.stat.gov.pl/publikacje-ifoldery/roczniki-statystyczne/wojewodztwo-malopolskie-2019-podregiony-powiaty-gminy,7,16.html> [accessed 17 Jan. 2025] (in Polish).
- [86] Teikari, P. (2007). *Light pollution: Definition, Legislation, Measurement, Modeling and Environmental Effects*. Polytechnic University of Catalonia. Barcelona. Spain.
- [87] *Earth Observation Group*. <https://eogdata.mines.edu/products/vnl/> [accessed 17 Jan. 2025].
- [88] *EarthData. LAADS DAAC, LAADS Archive*. <https://ladsweb.modaps.eosdis.nasa.gov/archive/allData/5000/?process=ftpAsHttp&path=allData%2f5000> [accessed 17 Jan. 2025].
- [89] Isobe, S., & Hamamura, S. (2001). Light energy loss and its application to estimate a global energy usage. In *Hyperspectral Remote Sensing of the Land and Atmosphere, Proceedings of SPIE* (pp.237-245), 4151, Society of Photographic Instrumentation Engineers. doi: 10.1117/12.417013
- [90] *Kraków in numbers 2020* (2021). <https://www.bip.krakow.pl/plik.php?zid=316550&wer=0&new=t&mode=shw> [accessed 17 Jan. 2025] (in Polish).
- [91] *Light pollution costs money and wastes resources* https://www.darkskysociety.org/handouts/IDA_Brochure_Energy_waste.pdf [accessed 17 Jan. 2025].
- [92] Płaza, K. (2024). *Track and Energy Infrastructure Maintenance Department*. Kraków City Hall [private communication].
- [93] *The city pays millions for electricity. Street lighting consumes the most money*. https://lovekrakow.pl/aktualnosci/miasto-placi-miliony-za-prad-najwiecej-pieniedzy-pochlania-oswietlenie-uliczne_50103.html [accessed 17 Jan. 2025] (in Polish).
- [94] Zoltán, K. (2010). Measuring and modelling light pollution at the Zselic Starry Sky Park. *Journal of Physics Conference Series*, 218(1), 012001. doi: 10.1088/1742-6596/218/1/012001
- [95] Ścieżor, T. (2022). Lighting of pedestrian and pedestrian-driving routes in a housing estate as a source of light pollution. *Technical Transactions*, 119(1), 1–17. doi: 10.37705/TechTrans/ e2022012
- [96] Lyytimäki, J., & Rinne, J. (2013). Voices for the darkness: online survey on public perceptions on light pollution as an environmental problem. *Journal of Integrative Environmental Sciences*, 10(2), 127–139. doi: 10.1080/1943815X.2013.824487
- [97] Ścieżor, T. (2024) Modernisation of the lighting installation of a housing estate – a case study, *INSTAL*, 464(7/8), 51–57. doi: 10.36119/15.2024.7-8.7 (in Polish).
- [98] Schulte-Römer, N., Meier, J., Dannemann, E., & Söding, M. (2019). Lighting Professionals versus Light Pollution Experts? Investigating Views on an Emerging Environmental Concern. *Sustainability*, 11, 1696. doi: 10.3390/su11061696

Impact of thermal modernisation including heat source replacement on the heating load of an existing single-family building in Poland

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Received: 20.10.2025; revised: 24.11.2025; accepted: 25.11.2025

Abstract

This study presents a comprehensive analysis of the impact of thermal retrofit, including the heat source modernisation, on the heating load of an existing single-family building located in Szczecin, Poland (Climate Zone I). The building, constructed in 2002, originally lacked thermal insulation in the walls, roof and ceiling, resulting in high operating costs and difficulties in maintaining thermal comfort. The aim of the project was to reduce heat losses, improve energy efficiency and enhance user comfort. The scope of work included thermal insulation of external walls, roof insulation, insulation of the intermediate floor slab and exterior wall section adjacent to the door opening and to the fenestration opening. The analysis conducted using the Audytor OZC 7.0 Edu Pol software showed that the total heat load of the building decreased from 25 710 W to 9476 W, representing a reduction of over 63%. Special attention was given to the analysis of thermal bridges, which are significant sources of energy loss. Using Therm 7.8.77 software, solutions were designed to reduce heat losses in critical areas. The heat transfer coefficients (U -values) for these thermal bridges were significantly reduced – by more than 50% in some cases. As part of the heat source modernisation, the solid fuel boiler and liquefied gas tank were removed and replaced with a condensing gas boiler powered by natural gas from the municipal network. To support the domestic hot water preparation system, flat-plate solar collectors were designed to work in conjunction with the existing dual-coil storage water heater.

Keywords: Thermal retrofit; Thermal modernisation; Heat source modernisation; Heating load; Thermal bridges

Vol. 46(2025), No. 4, 75–87; doi: 10.24425/ather.2025.156838

Cite this manuscript as: Bernat W., & Zwarycz–Makles, K. (2025). Impact of thermal modernisation including heat source replacement on the heating load of an existing single-family building in Poland. *Archives of Thermodynamics*, 46(4), 75–87.

1. Introduction

Thermal retrofitting of existing buildings is one of the key measures aimed at reducing energy consumption and improving the energy efficiency of the residential sector. The scope of thermal retrofit depends on the specific conditions and needs of a given building and typically includes thermal insulation of external walls, roofs, and slab-on-grade floors, replacement of windows and doors, implementation of renewable energy sources and modernisation of heating and domestic hot water sys-

tems. The use of appropriate thermal insulation materials enables achieving low values of the overall heat transfer coefficient of building envelope components [1].

According to the Polish Act of Law of November 21, 2008, on supporting thermal modernisation and renovations, as well as on the central register of building emissions (Journal of Laws 2024, item 1446, as amended [2]), thermal retrofitting projects aim to reduce the demand for energy supplied to buildings, limit primary energy losses in local heating networks and heat sources, lower energy acquisition costs through heat source mo-

Nomenclature

A – area, m^2
 b – correction factor
 c_p – specific heat, $J/(kg \cdot K)$
 e_k, e_l – correction factors for location
 f – correction factor
 G_w – correction factor for groundwater influence
 l – length, m
 t – temperature, $^{\circ}C$
 U – coefficient of heat transfer, $W/(m^2 \cdot K)$
 $\dot{V}_i$ – volume flow rate, m^3/s

Greek symbols

θ – temperature, $^{\circ}C$
 ρ – density at temperature $\theta_{int,i}$, kg/m^3
 ψ – linear thermal transmittance coefficient, $W/(m \cdot K)$
 Φ – heat loss/heat power, W
 Φ_{HL} – heat load, W

Subscripts

e – external
 $equiv$ – equivalent
 g – ground
 inf – infiltration
 int, i – internal
 i, j – heated space
 k – building element
 l – thermal bridge
 RH – reheating
 T – transmission
 u – unheated space
 V – ventilation

Abbreviations and Acronyms

COP – coefficient of performance
 DHW – domestic hot water
 EPS – expanded polystyrene
 PVC – polyvinyl chloride

dermisation or energy recovery and shift to renewable heat sources. Therefore, uninsulated buildings should undergo thermal retrofitting due to economic, environmental and social considerations. The main reasons include energy and cost savings during operation, reduction of CO₂ emissions generated during heat production for space heating and improvement of residential comfort [3].

A significant improvement in energy efficiency, as well as enhanced thermal comfort for occupants, was documented in the study by Krause [4]. Following a comprehensive thermal retrofit of a single-family house, including insulation of external walls, the basement and the flat roof, replacement of windows and doors, installation of a new boiler and the implementation of a fireplace system with heat distribution, the seasonal heat demand was reduced by 87%, reaching a level of 11 430 kWh/year.

In a next study focused on optimising the retrofit of a single-family house built in 2007 in Warsaw, the authors (Zawada and Rucińska [5]) reported a 23% reduction in primary energy consumption and a 50% decrease in the thermal discomfort index following the proposed thermal modernisation. These results demonstrate that insulating the walls and roof, along with replacing windows with units featuring lower heat thermal coefficients, improves the building's heat balance during winter and enhances user comfort in summer.

The authors of publication [6], which evaluates the effects of thermal retrofit of single-family buildings, note that replacing window joinery alone results in a 3% reduction in heating energy consumption. Insulating external walls and the roof enables a 26% decrease, while comprehensive thermal modernisation, including upgrading the heat source, sealing building partition components, and improving ventilation, can reduce energy consumption by up to 59%. These improvements lead to a tangible reduction in heating costs and enhanced thermal comfort for occupants.

In the book edited by Koczyk and Antoniewicz [7], it is indicated that thermal retrofitting measures involving the insulation of all building envelope components and the reduction of ventilation rates can lead to significant energy savings ranging from 34% to 50% in terms of building heat load and heat de-

mand. These findings demonstrate that thermal retrofit efforts positively influence building usability and contribute to a reduction in operational costs.

Contemporary challenges in building energy efficiency increasingly draw designers' attention to the issue of thermal bridges, which remain a significant source of heat loss in buildings. According to the study by Kurtz and Gawin [8] on the energy certification of residential buildings, thermal bridges are defined and exemplified as areas within the external envelope structure where a reduction in the internal surface temperature and an increase in heat flux density (heat loss) are observed relative to the surrounding components. Thermal bridges are categorised as linear or point-type, and both can lead to water vapour condensation and mould growth inside the building.

Roof thermal bridges are a critical source of heat loss in buildings [9], typically located along roof edges and around structural elements. These areas contribute to localised drops in interior surface temperatures and increased energy demand to compensate for the losses. One method for detecting thermal bridges is thermographic measurement using drones, which enables the identification of even minor existing thermal bridges.

The significance of thermal bridges in the context of building thermal modernisation, particularly their impact on heat loss and indoor microclimate, is discussed in [10]. Thermal bridges are a major source of heat loss, substantially contributing to reductions in internal temperatures. Therefore, effective thermal retrofit should include not only insulation of building envelopes but also of structural joints of the building. Thermal bridges account for approximately 16% of heat loss through building partitions, corresponding to around 7.8 kWh/m² of annual heat loss. To mitigate these losses, insulation connections must be consciously designed, which leads to cost reductions and improved comfort in residential buildings [11].

The issue of improving the energy performance of buildings and meeting legal requirements in this area is discussed in the study by Źarski [12], where the author emphasises that a key condition for effective building modernisation is a comprehensive approach encompassing both the structure of the building envelope and technical installations. The study highlights the

importance of including thermal bridges in the building's heat balance, in accordance with the guidelines of the PN-EN ISO 14683 standard, as even with high levels of envelope insulation, linear losses can significantly affect energy performance indicators.

The inclusion of thermal bridges in the assessment of building energy efficiency is thoroughly addressed by Pawlak [13], who emphasises that thermal bridges represent one of the key sources of heat energy loss and should be mandatorily considered in calculations of a building's thermal performance. The author points out that neglecting their impact may lead to significant discrepancies between the theoretical energy balance and the actual heat demand.

Study [14] investigates user activity in residential buildings through surveys and interviews, highlighting its impact on thermal-flow processes and energy consumption. Six behavioural profiles were developed and implemented in hourly dynamic simulations using DesignBuilder software to assess heat demand and indoor microclimate in selected Norwegian cities. Internal heat gains from occupants, appliances and solar radiation accounted for 20–35% of the energy balance, confirming the need for precise behavioural modelling. The authors advocate for hourly calculation methods over simplified monthly approaches, especially in the context of climate change. Accurate input data and increased public awareness are essential for improving energy efficiency and retrofit planning.

In Poland, the use of renewable energy systems in building design has grown due to regulatory requirements limiting primary energy consumption. A hybrid system combining ground-source and air-source heat pumps powered by photovoltaic electricity was installed in a facility near Kraków to evaluate performance and optimise energy use [15]. Both heat pumps showed comparable efficiency during the heating season, with coefficient of performance (COP) values of 2.69 and 2.63, respectively, suggesting that air-source heat pumps may be a cost-effective alternative. However, summer operation revealed reduced efficiency due to standby energy consumption. Integration with photovoltaic systems significantly improved ecological and economic performance, with self-consumption rates exceeding national averages. The findings support the broader adoption of renewable energy sources in construction and highlight the importance of accurate system design and climate considerations.

This study presents the impact of thermal retrofit on the heat load of a single-family building located in Szczecin (Polish Climate Zone I), which, due to the lack of thermal insulation, generated high operating costs and was associated with difficulties in maintaining the interior spaces, as reported by the occupants. The analysis included the influence of thermal bridges on heat losses. Additionally, the previously non-habitable attic was converted into a heated and residential space. The heat source was modernised by removing the solid fuel boiler and propane tank and replacing them with a condensing gas boiler connected to the municipal natural gas network. The project was complemented by the installation of flat-plate solar collectors for the domestic hot water (DHW) system.

2. Building performance characteristics prior to thermal retrofit

The study analysed a detached single-family house without a basement, built in 2002 in Szczecin (Climate Zone I, with a design outdoor temperature of $\theta_e = -16^\circ\text{C}$ [16]). The building has a large-scale form, without a habitable attic and includes an integrated garage.

Figure 1 presents the ground floor plan, including the layout and areas of the rooms as well as the design temperatures. The non-usable attic extends over the entire ground floor. The clear height of the ground floor is 2.6 m, and the maximum height of the attic is also 2.6 m. The knee wall height in the attic is 80 cm. Figure 2 shows an external view of the building.

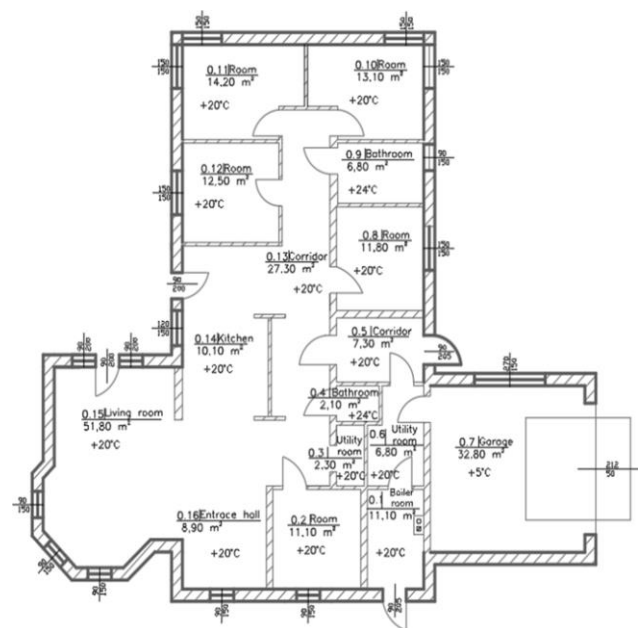


Fig. 1. Ground floor plan showing the layout and areas of the rooms, as well as the design temperatures for the analysed building.



Fig. 2. View of the analysed building.

The usable floor area is 225.80 m², and the building volume is 643.42 m³. The main entrance and garage are located on the western side. The ground floor comprises 16 rooms, including a boiler room with a separate external entrance. Due to the lack of thermal insulation in the external building partitions, heating costs were very high, prompting the decision to implement substantial thermal and energy upgrades. The attic was unheated and uninhabitable. The building's airtightness was assumed to

be $n_{50} = 6.0 \text{ h}^{-1}$. Ventilation was provided by gravity systems using existing masonry exhaust chimneys.

Prior to retrofit, the roof was covered with ceramic tiles and insulated with loosely laid mineral wool in the attic floor, with a thickness of 0.15 m. The external walls of the building were constructed from lightweight expanded clay concrete blocks with a thickness of 0.39 m, without internal insulation, and finished with gypsum board and plaster. Internal walls were made of hollow bricks measuring $120 \text{ mm} \times 250 \text{ mm} \times 63 \text{ mm}$, also without insulation. The intermediate floor slab between the ground floor and the unheated attic was made of reinforced concrete with a thickness of 0.10 m, covered with cement-bonded particle board, without insulation. The slab-on-grade ground floor structure consisted of sand, a concrete foundation layer, followed by 0.10 m of expanded polystyrene insulation, and finished with a floor compound and plywood. The windows were double-glazed, tilt-type, with polyvinyl chloride (PVC) frames, including balcony windows and two pairs of balcony doors.

The overall heat transfer coefficients for the building partitions were calculated using the Audytor OZC 7.0 Edu Pol software [17]. By comparing the calculated U -values with the maximum allowable values (U_{max}) specified in the Journal of Laws 2022 [18] on the technical conditions to be met by buildings and their location, it was observed that the roof, the intermediate floor slab below the unheated attic and the external wall did not meet current regulatory requirements due to the lack of insulation. Specifically, the roof's U -value exceeded the limit by $1.182 \text{ W}/(\text{m}^2 \cdot \text{K})$, the ceiling by $0.426 \text{ W}/(\text{m}^2 \cdot \text{K})$, and the external wall by $0.253 \text{ W}/(\text{m}^2 \cdot \text{K})$. The total heat load prior to retrofit was $25\,710 \text{ W}$, with $14\,817 \text{ W}$ attributed to the ground floor storey and $10\,893 \text{ W}$ to the non-habitable attic.

In its original state, the building did not meet the requirements in force at that time due to the absence of insulation in the external walls, roof and ceiling. This is confirmed by the thermal transmittance (U -value) prior to thermal retrofit, which was the main rationale for undertaking modernisation measures. The significant reduction in heat demand resulted from the poor initial condition of the building, not due to structural defects, but

rather the lack of insulation in key partitions, which was the primary factor leading to very high heat losses.

2.1. Heat source and central heating system prior to modernisation

The boiler room has the following dimensions: a floor area of 6.90 m^2 and a volume of 17.53 m^3 . It featured both internal and external doors opening outward. Ventilation was provided via a $14 \text{ cm} \times 14 \text{ cm}$ exhaust grille installed in the chimney shaft and a $14 \text{ cm} \times 14 \text{ cm}$ supply grille mounted in the external wall, positioned 30 cm above the floor level.

The heat sources included a solid fuel boiler (wood-fired) and a liquefied gas boiler supplied from an underground propane tank located on the property. Switching between heat sources for the central heating system was performed manually by opening and closing the appropriate shut-off valves in the boiler room. During the autumn-winter heating season, the house was heated using a wood-fired boiler operating on the principle of generator gasification. The boiler was equipped with an exhaust fan to facilitate combustion. It had a nominal power output of 40 kW , a water capacity of 80 litres , and a fuel hopper capacity of 140 dm^3 . The seasonal wood consumption was approximately 40 m^3 . The system was designed to maintain a minimum return water temperature of 65°C , and the boiler efficiency ranged from 81% to 90% (according to the manufacturer technical specification). The boiler was connected via a flue outlet to a $150 \times 150 \text{ mm}$ steel chimney duct, which was routed through a shaft and extended above the roof.

In the spring-summer period, the house was heated using a liquefied gas boiler due to its lower fuel consumption for the domestic hot water system and occasional heating. The gas boiler was a single-function, wall-mounted unit with a power range of $8.9\text{--}21 \text{ kW}$ and a closed type combustion chamber. It featured built-in weather-compensated control. The air/flue gas duct was made of steel with a diameter of $60/100 \text{ mm}$. The propane gas was supplied from an underground tank located on the property. A technical scheme of the heat source configuration prior to modernisation is presented in Fig. 3.

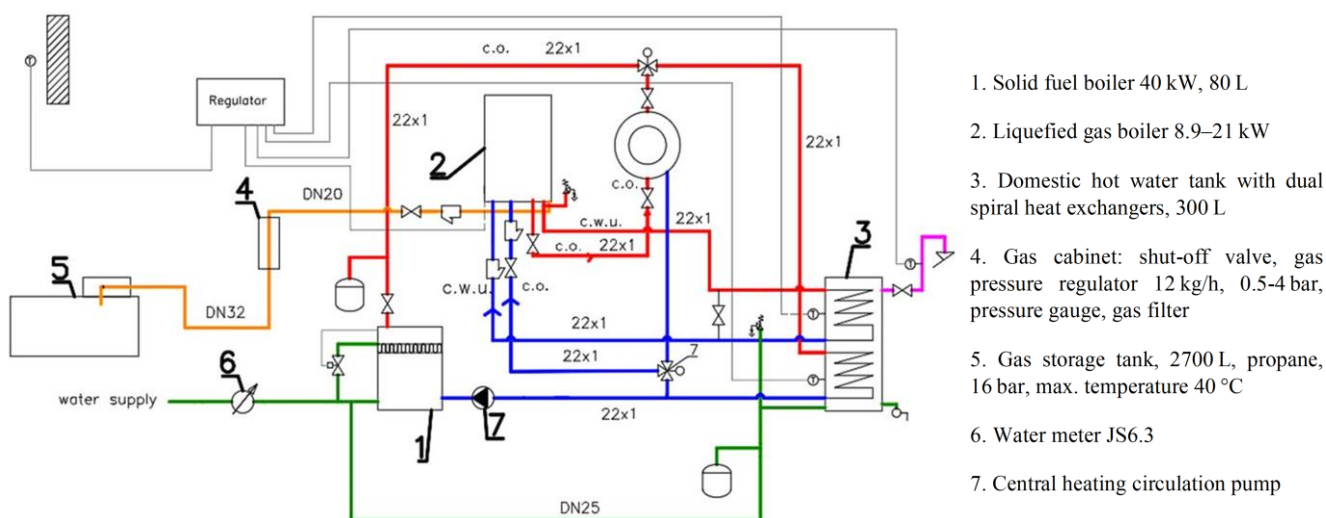


Fig. 3. Technical scheme of the heat source prior to retrofit.

The design temperatures of the heating water in the central heating system are 75°C/65°C. The system is a two-pipe installation embedded in the floor, with bottom connections made of copper pipes. Panel radiators type 22, featuring profiled heating plates and convective elements, equipped with thermostatic control valve inserts, were installed in the rooms. In the bathrooms, vertical pipe type radiators, made of non-alloy low-carbon steel, were used for heating.

During construction of the building, the dimensions of the radiators were selected and installed in such a way that their heating capacities were insufficient from the beginning to meet the high thermal loads of the rooms. As a result, it was difficult to maintain the design indoor temperature throughout the building. Additionally, the non-heated attic, lack of thermal insulation of the external walls, and the intermediate floor slab between the ground floor and attic contributed to significant heat losses from the heated spaces to the surroundings. The rooms cooled down rapidly, and users reported thermal discomfort as well as a noticeable increase in air humidity.

3. Building envelope components, thermal bridges, heat load

To ensure thermal comfort in the heated rooms, it is necessary to determine the heat flux that must be supplied to the space in order to compensate for all heat losses. The heat load calculations for the building's rooms were carried out in accordance with the PN-EN 12831:2006 Polish Standard [16], using the Audytor OZC 7.0 Edu Pol software [17]. The determination of the design heat load required to maintain the specified indoor design temperature under standardised design conditions, for the purpose of selecting an appropriate heat source, was performed as follows [14]:

$$\Phi_{HL} = \sum \Phi_{T,i} + \sum \Phi_{V,i} + \sum \Phi_{RH,i}, \quad (1)$$

where: $\Phi_{T,i}$ – transmission heat losses [W], $\Phi_{V,i}$ – ventilation heat losses [W], $\Phi_{RH,i}$ – additional heating power required to compensate for reduced heating performance (reheat) [W].

In the calculation of the design heat loss due to transmission, the following components are considered: direct heat losses to the external environment, heat losses through unheated spaces to the outside, heat losses to the ground, and heat transfer between heated zones with different indoor temperatures [16]:

$$\begin{aligned} \Phi_{T,i} = & [\sum_k (A_k U_k e_k) + \sum_l (\psi_l l_l e_l) + \sum_k (A_k U_k b_u) + \\ & + \sum_l (\psi_l l_l b_u) + f_{g1} f_{g2} \sum_k (A_k U_{equiv,k}) G_w + \\ & + \sum_k f_{ij} A_k U_k] (\theta_{int,i} - \theta_e), \end{aligned} \quad (2)$$

where: A_k – area of building element (k) [m^2], U_k – heat transfer coefficient of the building envelope components (k) [$W/(m^2 \cdot K)$], e_l , e_k – correction factors for orientation, taking into account climatic influences, ψ_l – linear thermal transmittance coefficient (l) [$W/(m \cdot K)$], l_l – length (l) of the linear thermal bridge between the internal and external environments [m], b_u – temperature reduction coefficient accounting for the difference between the temperature of the unheated space and the design external tem-

perature [$^\circ C$], f_{g1} – correction factor accounting for the influence of annual fluctuations in outdoor temperature, f_{g2} – temperature reduction factor accounting for the difference between the annual average outdoor temperature and the design outdoor temperature, $U_{equiv,k}$ – equivalent heat transfer coefficient of a building element (k) [$W/(m^2 \cdot K)$], G_w – correction factor for ground-water influence, f_{ij} – temperature reduction factor accounting for the difference between the temperature of the adjacent space and the design outdoor temperature, $\theta_{int,i}$ – design indoor temperature of the heated space (i) [$^\circ C$], θ_e – design outdoor temperature [$^\circ C$].

The ventilation heat loss is determined using the following equation [16]:

$$\Phi_{V,i} = \dot{V}_i \rho c_p (\theta_{int,i} - \theta_e), \quad (3)$$

where: $\dot{V}_i$ – volumetric flow rate of ventilation air supplied to the heated space [m^3/s].

The reheat amount includes the correction factor f_{RH} , which depends on the heating-up time and the assumed indoor temperature setback during reduced heating periods [16]:

$$\Phi_{RH,i} = A_i f_{RH}, \quad (4)$$

where: A_i – area of the heated space [m^2], f_{RH} – correction factor accounting for heating-up dynamics and temperature setback [W/m^2]. In the presented analysis, this component of the building's thermal load balance was not considered, as the occupants do not utilise this function. According to standard [16], the reheat amount is not mandatory.

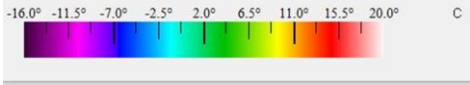
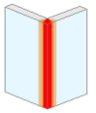
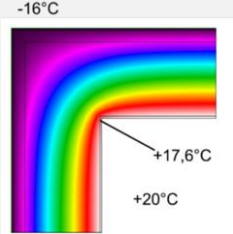
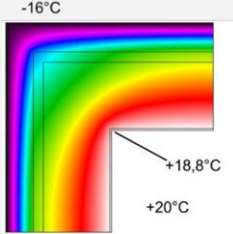
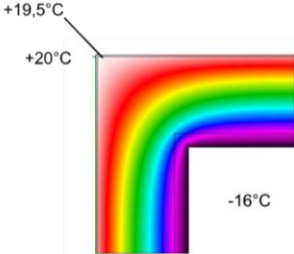
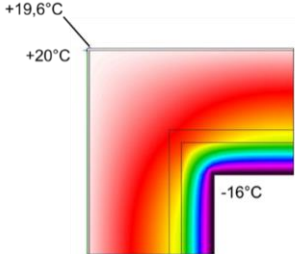
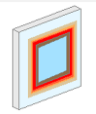
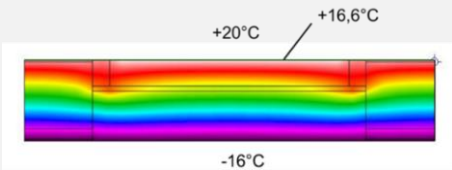
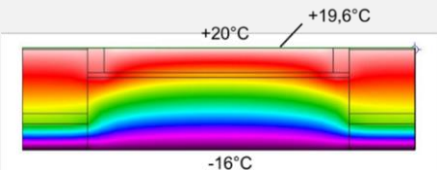
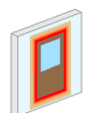
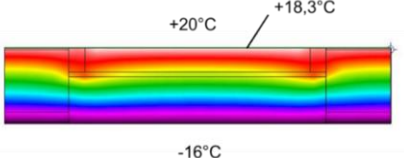
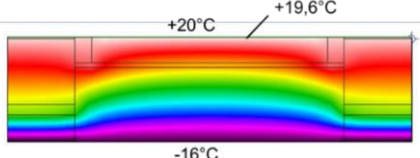
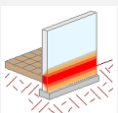
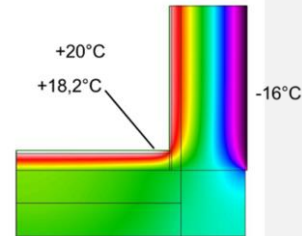
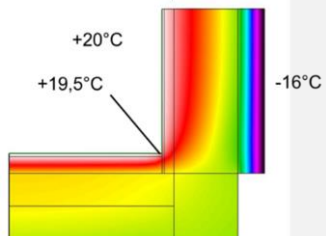
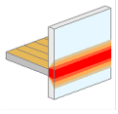
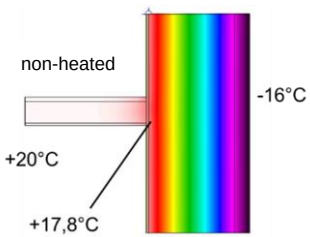
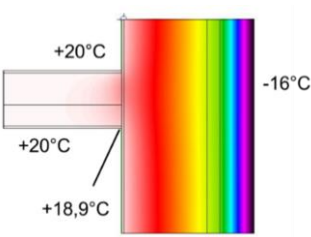
As part of the preparation for the building's thermal retrofit, the initial step involved an analysis of heat losses through linear thermal bridges. The assessment of temperature distribution within thermal bridge models, along with the determination of heat flux passing through these elements, enabled the proposal of design solutions aimed at reducing linear heat losses. Surface thermal bridges, such as glazed area and door leaf surface area, were incorporated into the heating load calculations using the software referenced in [17]. Point thermal bridges were not included in the analysis.

The lengths of thermal bridges in the presented analysis were modelled using Therm Software 7.8.77, which is based on the PN-EN ISO 10211-2 standard [19]. According to this standard, the geometric model of a thermal bridge must be created under strictly defined thermal and geometric boundary conditions. The lengths of thermal bridges are assumed to be three times the total thickness of the adjoining partitions or at least 1 m from the central element.

Table 1 presents the models of linear thermal bridges in the building, developed in accordance with the PN-EN ISO 10211:2008P standard [19], using Therm Software version 7.8.77 [20]. The first column provides a visual representation and designation of each thermal bridge type. The second and third columns illustrate the temperature distribution models for each thermal bridge in two scenarios: prior to and after the building retrofit.

Table 1 is supplemented with a colour scale legend and boundary temperature data for the building envelope components.

Table 1. Temperature distribution in building thermal bridges – models generated using Therm Software 7.8.77 [20].

Thermal bridge	Temperature distribution in building thermal bridges	
	Prior to thermal retrofit	After thermal retrofit
		
 External corner (a)		
 Internal corner (b)		
 Exterior wall section adjacent to fenestration opening (c)		
 Exterior wall section adjacent to door opening (d)		
 Slab-on-grade floor system / exterior wall interface (e)		
 Intermediate floor structure / exterior wall junction (f)		

For the external corner (convex corner) thermal bridge (a), the analysis of temperature distribution within the building envelope component prior to and after insulation revealed a significant improvement in thermal performance following the retrofit. Prior to insulation, low-temperature isotherms penetrated deeply into the structure, indicating substantial heat losses and a heightened risk of water vapour condensation. After the application of thermal insulation, a distinct outward shift of the isotherm lines was observed, demonstrating the effectiveness of the implemented insulation strategy. The total heat transfer coefficient (U -value) for this thermal bridge, under a temperature gradient of 36 K (-16°C exterior and $+20^{\circ}\text{C}$ interior), was reduced from $0.556 \text{ W}/(\text{m}^2\cdot\text{K})$ to $0.111 \text{ W}/(\text{m}^2\cdot\text{K})$.

The analysis of the internal corner (concave corner) thermal bridge (b) revealed similar phenomena, albeit with greater intensity. Prior to insulation, the temperature on the interior surface of the envelope at the corner was significantly lower than in other areas of the partition, indicating intense heat transfer and a high risk of water vapour condensation. Following the application of thermal insulation, the isotherms shifted toward the outer edge of the envelope, and the local temperature minimum increased. The total heat transfer coefficient (U -value) for this thermal bridge decreased from $0.537 \text{ W}/(\text{m}^2\cdot\text{K})$ to $0.141 \text{ W}/(\text{m}^2\cdot\text{K})$.

The next component of the analysis was the exterior wall section adjacent to the fenestration opening (c). Before the detail was improved, significant cooling of the envelope was observed in the area where the window frame connects to the masonry wall. Temperatures near the window reveal dropped locally to critical levels. After implementing an improved installation method and adding thermal insulation in the reveal zone, the low-temperature isotherms shifted toward the outer part of the envelope. The temperature at critical points increased significantly, indicating effective elimination of the local linear thermal bridge and improved energy performance of the entire construction detail. The U -value was reduced from $0.241 \text{ W}/(\text{m}^2\cdot\text{K})$ to $0.095 \text{ W}/(\text{m}^2\cdot\text{K})$.

In the analysis of the exterior wall section adjacent to door opening, and thermal bridge occurring between the door assembly and the external wall (d), the initial condition revealed significant thermal anomalies. The zone surrounding the door, particularly near the ground level, exhibited pronounced cooling and intense heat flux, contributing to localised energy losses and a reduction in thermal comfort. Following the implementation of additional thermal insulation, these adverse phenomena were substantially mitigated. The isotherms in this region became more uniform, and the temperature values within the building envelope increased, indicating improved thermal conditions and reduced potential for energy loss. The total modelled U -value decreased from $0.208 \text{ W}/(\text{m}^2\cdot\text{K})$ to $0.096 \text{ W}/(\text{m}^2\cdot\text{K})$.

The next analysed thermal bridge, slab-on-grade floor system/exterior wall interface, was located at the junction between the internal wall and the ground-bearing floor slab (e). This area is particularly susceptible to thermal bridging due to its direct contact with the subsoil, which typically maintains a significantly lower temperature.

In the pre-retrofit condition, the simulation revealed a distinct temperature drop in the internal corner, posing a risk of surface condensation. After applying additional insulation in the plinth zone, the low-temperature isotherms shifted deeper into the external envelope, and the temperature within the corner increased to a level ensuring both thermal and hygienic safety. The overall heat transfer coefficient was reduced from $0.336 \text{ W}/(\text{m}^2\cdot\text{K})$ to $0.162 \text{ W}/(\text{m}^2\cdot\text{K})$.

The final case analysed involved a thermal bridge at the junction between the exterior wall and the intermediate floor slab separating the building storeys (f). In the initial scenario – prior to thermal retrofitting, when the attic was uninsulated and unheated, the simulation clearly indicated heat losses through the uninsulated portion of the floor slab, which structurally penetrated the thermal insulation layer of the external wall. This area exhibited significant cooling, with the minimum local temperature dropping to a level conducive to water vapour condensation. After the application of additional insulation in the floor slab region, the isotherms became smoother and shifted, indicating a substantial improvement in thermal conditions. The thermally upgraded building envelope ensured continuity of the insulation layer, thereby minimising energy losses and enhancing the durability of the building structure. In the thermal simulation, the total U -value coefficient for this thermal bridge, under a temperature gradient of 36 K, was reduced from $0.225 \text{ W}/(\text{m}^2\cdot\text{K})$ to $0.165 \text{ W}/(\text{m}^2\cdot\text{K})$.

4. Building performance characteristics after thermal retrofit. Result analysis.

A thermal retrofit project was developed for the building, based on the following interventions:

- insulation of external walls using expanded polystyrene (EPS) with a thermal conductivity of $k = 0.040 \text{ W}/(\text{m}\cdot\text{K})$ and a thickness of 0.15 m,
- roof insulation with mineral wool $k = 0.052 \text{ W}/(\text{m}\cdot\text{K})$, applied in a layer of 0.35 m thickness,
- replacement of the roof covering from ceramic tiles to metal roofing sheets $k = 50 \text{ W}/(\text{m}\cdot\text{K})$, aimed at reducing the structural load on the roof,
- insulation of the intermediate floor slab between the ground floor and the attic using EPS $k = 0.040 \text{ W}/(\text{m}\cdot\text{K})$ with a thickness of 0.15 m,
- sealing of window and door frames to improve airtightness and reduce uncontrolled infiltration.

Based on literature data indicating the limited impact of window and door replacement on the overall heat demand reduction estimated at approximately 3% according to [6] and considering the high investment costs associated with such measures, the investors opted not to include fenestration replacement in the scope of thermal retrofit. Additionally, a design was developed to convert the attic into a habitable and heated space, including the layout of new rooms and the integration of a heating system. New stairs were also planned to enable users to move between floors, replacing the previously existing ladder that provided access only to the non-usable attic. To enhance thermal comfort

on the external façade, the installation of electrically operated external window shutters, integrated into the window frames, was proposed as part of the building insulation works. In the case of the garage, it was designated to be heated to an indoor temperature of $t_i = +5^\circ\text{C}$. Consequently, the internal garage walls were insulated in accordance with the envelope specifications outlined in [18]. The building occupants chose to retain natural (gravity-based) ventilation. Airtightness of the building envelope was improved to the design level of $n_{50} = 2.0 \text{ h}^{-1}$, achieved through sealing of window and door frames, insulation of external partitions and sealing of service penetrations. For retrofitted buildings without mechanical ventilation, the typical airtightness level after modernisation falls within the range of $n_{50} = 2.0\text{--}3.5 \text{ h}^{-1}$ [6]. In the analysed case, a value of

$n_{50} = 2.0 \text{ h}^{-1}$ was adopted, reflecting the effect of careful insulation of building partitions, improved continuity of insulation at structural joints, and the proposed use of external roller shutters by occupants. According to available data on airtightness measurements for retrofitted buildings, such measures can enable achieving n_{50} values below 3.0 h^{-1} .

For the proposed thermal retrofit measures, calculations of the overall heat transfer coefficients were performed using the Audytor OZC 7.0 software Edu Pol [17] in accordance with the standard [16]. The results are presented in Table 2. Due to the modernisation of the building's heat source, the total heating load of the building was also calculated and summarised in Table 3. All calculations were carried out for both analysed scenarios, prior to and after thermal retrofit.

Table 2. Values of the overall heat transfer coefficient (U -value) for the building partitions, calculated using Audytor OZC 7.0. Edu Pol [17].

Building Partition type	Area of partitions [m^2]	Overall heat transfer coefficient U [$\text{W}/(\text{m}^2\cdot\text{K})$]		
		Prior to thermal retrofit	After thermal retrofit	$U_{c(\max)}$ [18] [$\text{W}/(\text{m}^2\cdot\text{K})$]
Roof	269.01	1.332	0.143 (reduction by 89,3%)	0.150 ($t_i \geq 16^\circ\text{C}$)
External door	33.75	1.300	1.300	1,300
External window	41.40	0.900	0.900	0.900 ($t_i \geq 16^\circ\text{C}$)
Intermediate floor slab	218.73	0.576	0.180 (reduction by 68.8%)	1.000 ($\Delta t_i \geq 8^\circ\text{C}$)
Slab-on-grade floor	224.96	0.200	0.196 (reduction by 2%)	0.300 ($t_i \geq 16^\circ\text{C}$)
Internal wall	412.04	0.968	0.911 (reduction by 5.9%)	1.000 ($\Delta t_i \geq 8^\circ\text{C}$)
External wall	696.51	0.453	0.168 (reduction by 63.3%)	0.200 ($t_i \geq 16^\circ\text{C}$)

Table 3. Heat loads for rooms and the building, and the design indoor temperatures, calculated using Audytor OZC 7.0 Edu Pol [16,17].

Type of room	Temperature $\theta_{int, i}$ [$^\circ\text{C}$]		Heat load Φ_{HL} [W]	
	Prior to thermal retrofit	After thermal retrofit	Prior to thermal retrofit	After thermal retrofit
Ground floor				
Boiler room	20	20	871	333
Room	20	20	703	338
Utility room without window	20	20	18	9
Bathroom without window	24	24	200	112
Corridor	20	20	446	233
Utility room without window	20	20	403	232
Garage	non-heated	5	2320	825
Room	20	20	632	274
Bathroom	24	24	701	377
Room	20	20	1247	452
Room	20	20	1377	557
Room	20	20	716	345
Corridor	20	20	448	396
Kitchen	20	20	837	410
Living room	20	20	3051	1317
Entrance hall	20	20	847	289
Attic (residential)				
Room	non-heated	20	10893	845
Bathroom without window	non-heated	24		621
Corridor	non-heated	20		381
Room	non-heated	20		1130
Sum			25710 (25.7 kW)	9476 (9.5 kW)

A heat load value is obtained for the unheated attic because heat transfer from the heated zones to adjacent unheated spaces must be considered in accordance with standard calculation procedures (EN 12831 [16]). The attic, although not heated, receives heat through the ceiling and thermal bridges, and its temperature is typically lower than that of the conditioned space. This results in a heat flow that contributes to the overall heating demand of the building.

The proposed thermal retrofit measures resulted in the following changes in the calculated thermal transmittance (U -values) for individual building components:

- roof: $U = 0.143 \text{ W/(m}^2\cdot\text{K)}$, representing a reduction of $1.189 \text{ W/(m}^2\cdot\text{K)}$ due to the implemented improvements,
- intermediate floor slab (between ground floor and the non-heated attic): $U = 0.180 \text{ W/(m}^2\cdot\text{K)}$, corresponding to a decrease of $0.396 \text{ W/(m}^2\cdot\text{K)}$,
- slab-on-grade floor: $U = 0.196 \text{ W/(m}^2\cdot\text{K)}$, with a marginal reduction of $0.004 \text{ W/(m}^2\cdot\text{K)}$.

The total design heat loss for the thermally modernised building, based on characteristic parameters from Journal of Laws 2022 [18], amounted to 9476 W, indicating a reduction of 16 234 W. This includes a decrease for the ground floor storey from 14 817 W to 5814 W, and on the attic level from 10 893 W to 3662 W (from the uninhabitable attic to the residential attic).

To improve the quality conclusions and enable a comprehensive assessment of the thermal retrofit, two tables (Table 4 and Table 5) presenting the components of the building's heat load

have been added. Additional demand from converting an unused attic into living spaces is presented in Table 3. In the presented analysis, the reheat component of the building's thermal load balance was not considered, as the occupants do not utilise this function. According to standard [16], the reheat amount is not mandatory. Since the gas boiler is a single-function unit operating in conjunction with a domestic hot water heater, and the number of occupants (four persons) as well as the presence of two bathrooms remain unchanged before and after thermal retrofitting, the domestic hot water demand also remains constant at 20 kW, assuming a high level of domestic hot water thermal comfort.

Tables 4 and 5 present the distribution of individual components to the building's heat balance before and after thermal retrofitting. Heat transmission losses were divided into two categories:

- (1) losses through external envelope components, calculated as the sum of products $A_k \cdot U_k \cdot e_k$,
- (2) losses through partitions adjacent to unheated spaces (e.g. attic or garage), in accordance with the methodology of PN-EN 12831 [16].

Additionally, (3) linear losses caused by thermal bridges were considered separately, calculated using the relation $\psi_l \cdot l_l \cdot e_l$. Losses due to minimum ventilation ($V_{\min}$) are lower than infiltration ventilation losses ($V_{\inf}$) and, therefore, were not included in accordance with standard [16]. After thermal retrofitting, losses associated with unheated spaces were eliminated, as all rooms are heated following the modernisation.

Table 4. Distribution of heat load components in the building prior to thermal retrofit.

Type of room	Heat losses by transmission - external partitions [W]	Heat losses by transmission to unheated spaces [W]	Linear heat losses - thermal bridges [W]	Ventilation heat losses - infiltration [W]	Total heating load [W]
Boiler room	643	120	31	77	871
Room	354	152	24	173	703
Utility room without window	0	6	0	9	18
Bathroom without window	0	33	0	167	200
Corridor	202	100	31	113	446
Utility room without window	0	297	0	106	403
Garage	1813	0	77	430	2320
Room	268	159	22	183	632
Bathroom	456	109	19	117	701
Room	820	179	44	204	1247
Room	923	188	45	221	1377
Room	334	156	32	194	716
Corridor	0	224	0	224	448
Kitchen	359	249	72	157	837
Living room	1675	402	169	805	3051
Entrance hall	445	247	17	138	847
Attic (non-heated)	6666	0	2069	2160	10893
Total	14958	2621	2652	5478	25710

Table 5. Distribution of heat load components in the building after thermal retrofit.

Type of room	Heat losses by transmission - external partitions [W]	Heat losses by transmission to unheated spaces [W]	Linear heat losses-thermal bridges [W]	Ventilation heat losses - infiltration [W]	Total heating load [W]
Boiler room	238	0	31	64	333
Room	141	0	24	173	338
Utility room without window	0	0	0	9	9
Bathroom without window	76	0	0	36	112
Corridor	89	0	31	113	233
Utility room without window	126	0	0	106	232
Garage	169	0	59	597	825
Room	69	0	22	183	274
Bathroom	241	0	19	117	377
Room	204	0	44	204	452
Room	292	0	44	221	557
Room	119	0	32	194	345
Corridor	72	0	0	324	396
Kitchen	203	0	50	157	410
Living room	368	0	144	805	1317
Entrance hall	134	0	17	138	289
Attic (residential)					
Room	199	0	169	481	845
Bathroom without window	169	0	128	324	621
Corridor	180	0	53	148	381
Room	215	0	8	807	1130
Total	3304	0	875	5201	9476

The tables provide a detailed breakdown of heat loss components for individual rooms in the building before and after thermal retrofitting. Comparing these data allows identification of the elements of the heat balance that contributed most to the achieved energy savings. (1) Insulation of external partitions. The greatest impact on reducing the heat load resulted from the decrease in transmission losses through external envelope components, which dropped from 14 958 W to 3304 W, representing a 78% reduction. (2) Elimination of losses to unheated spaces. Prior to modernisation, a significant portion of losses originated from heat transfer from the ground floor to the unheated attic (2621 W). After retrofitting, the attic became a heated space, eliminating these losses entirely. (3) Reduction of thermal bridges, insulating structural details and improving window reveals and wall junctions reduced linear losses from 2652 W to 875 W, a 67% decrease, highlighting the substantial role of thermal bridges in the overall heat balance. (4) Ventilation losses decreased from 5478 W to 5201 W (a 5% reduction), primarily due to improved airtightness and reduced uncontrolled infiltration. Collectively, these measures resulted in a 63% reduction in the building's total heat load.

4.1. Heat source and central heating system after modernisation

The location of the boiler room remains unchanged. The solid fuel central heating boiler, along with all associated equipment, has been removed from the room. The chimney previously used

for flue gas discharge remains in place but will no longer be in use. The gas-fired boiler continues to operate in its original location without modification. The burner of this boiler can operate not only on liquefied petroleum gas, but also, after appropriate adjustment, on natural gas. Additionally, the heat source has been supplemented with flat-plate solar collectors and associated equipment, which have been connected to the existing bi-valent domestic hot water heater. For the purpose of automatic regulation of the modernised heat source, a new controller has been installed in the boiler room. Furthermore, the central heating circulation pump has been replaced with a modern single-speed circulating pump.

The burner of the existing gas boiler has been adapted for operation with network-distributed natural gas GZ50; however, its factory settings will be adjusted due to the change in the type of fuel used (modified power output range 11–23 kW). The fuel will be delivered to the boiler from the existing gas distribution network via a gas connection routed across the property. The underground liquefied gas tank has been decommissioned, as it became feasible to design a direct gas connection to the network, thereby reducing the inconvenience associated with fuel delivery and tank maintenance. The tank had been in use for approximately 20 years, which would soon necessitate its replacement, incurring additional costs. A gas connection has been installed from the gas network to a cabinet mounted on the building wall. The distance between the gas cabinet and the street is 12 meters. A 1.5-meter section of the connection from the cabinet was constructed using steel piping, while the remaining section was

made of PE pipe. The pipes were laid 0.6 meters below the ground level with a slope of 4%. Inside the gas cabinet mounted on the building, a self-actuating pressure regulator with two-stage pressure reduction has been installed. The regulator includes an integrated shut-off valve, an overpressure quick-closing valve at the inlet, and a mesh filter. Additionally, a gas meter with a maximum flow rate of 4 m³/h and a stainless steel shut-off valve have been installed.

To support the domestic hot water (DHW) system, flat-plate solar collectors with a total surface area of 3×2.06 m² were se-

lected. The selected collector area covers 100% of the average daily DHW demand of the building's occupants during the summer period. The collectors were installed on the roof, facing southwest, due to optimal solar exposure on that side of the building (see Fig. 4). The building is located in Szczecin, at a latitude of 53°23'N, where the average annual total sunshine duration is approximately 1700–1800 hours/year, and the annual average solar radiation on a horizontal surface amounts to 1000–1100 kWh/m²/year [21].

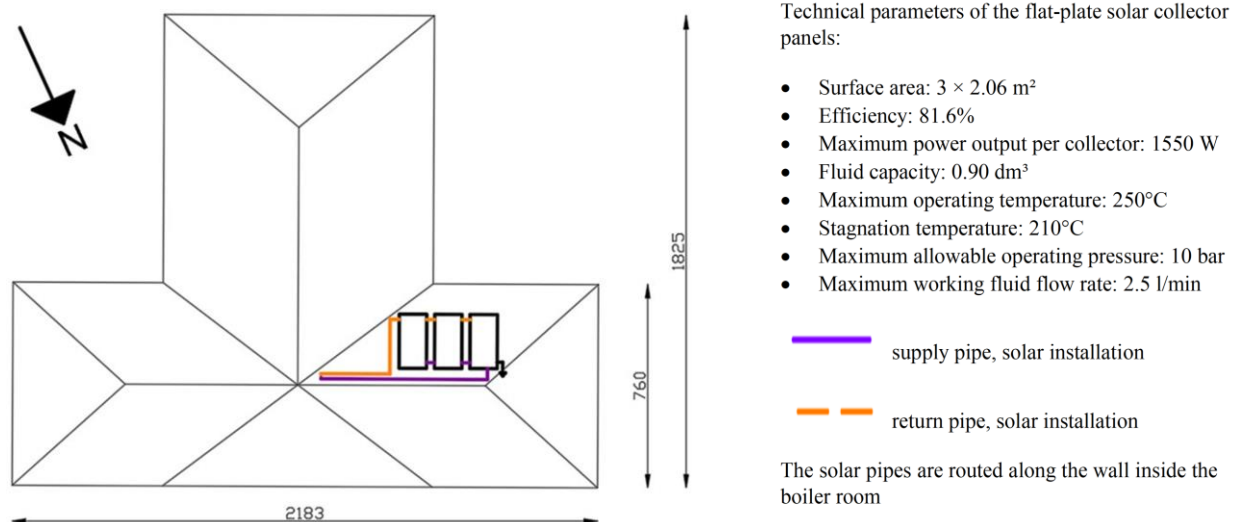


Fig. 4. Arrangement of flat-plate solar collectors on the building roof.

The existing dual-coil domestic hot water (DHW) heater with a capacity of 300 dm³, previously used in conjunction with a solid fuel boiler, has been retained and integrated with the flat-plate solar collector system. Overheating protection for the DHW system has been designed and is illustrated in the heat source schematic (see Fig. 5).

Cold water is routed and connected to the DHW line via a three-way valve, allowing cold water to be introduced into the system when the DHW temperature exceeds the maximum permissible level, thereby cooling the medium. The solar installation is protected by a safety valve and a closed expansion vessel

with a volume of 18 litres. Additionally, a vent has been installed on the roof to facilitate air removal from the system during fluid replacement or refilling.

The design temperatures of the heating system water are 55°C / 45°C. Due to insufficient thermal output of the existing radiators and piping, a complete replacement of all heaters and distribution lines was designed. The new hydronic heating system utilises modern plastic piping technology, consisting of multilayer polyethylene pipes with an aluminium diffusion barrier. The distribution network is routed through individual rooms within the thermal insulation layer of the structural floor

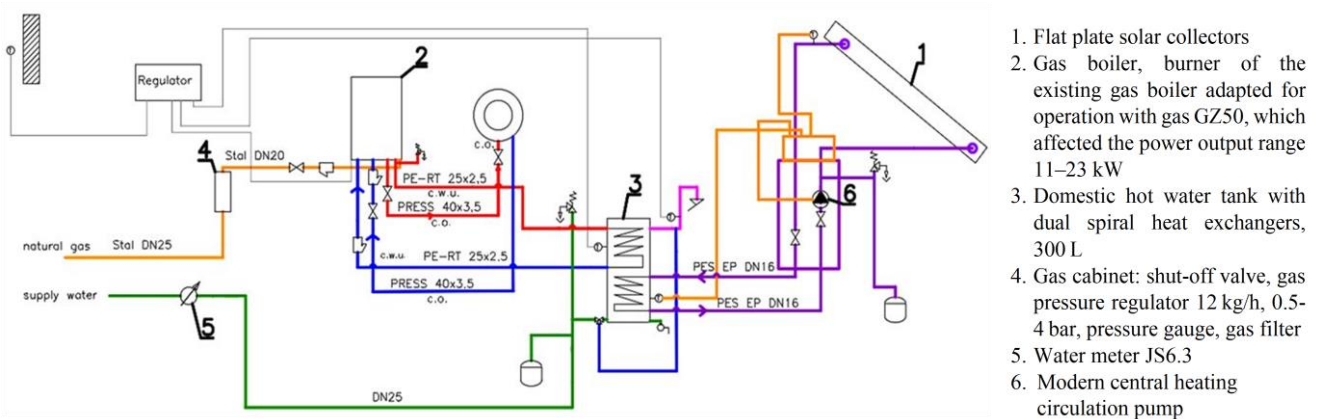


Fig. 5. Technical scheme of the heat source after modernisation.

slab and connected to the designated radiators using a tee-branch configuration. Panel radiators were specified for general spaces. For the bathroom zone, modern bathroom type radiators were selected, characterised by flat heating surfaces, providing both functional and aesthetic integration with the interior.

5. Conclusions

The integrated thermal retrofit project for the single-family building located in Szczecin was undertaken due to the absence of thermal insulation in the external walls and roof structure. This deficiency resulted in high operating costs for the users, caused by excessive heat losses, rapid cooling of the building, and the inconvenience of restoring indoor thermal comfort during the winter season. The lack of building envelope insulation led to increased energy demand for space heating in winter and reduced passive protection against overheating in summer, thereby compromising thermal comfort and energy efficiency.

Calculations and analysis of the overall heat transfer coefficient (U -values) of the building envelope components in their existing condition, including thermal bridges and heat source performance, enabled the formulation of targeted thermal and technical modernisation measures. The comparative assessment of the building's thermal performance before and after retrofitting demonstrated that comprehensive thermal retrofit would significantly reduce the total heating load, from 25 710 W to 9476 W. This reduction of over 63% provided a reduction in the useful heat output of the heat source and a shorter operating time, which is relevant in the context of using fossil fuels, for example natural gas. The substantial decrease in thermal demand highlights the high energy-saving potential resulting from the following interventions: thermal insulation of external walls, insulation of the intermediate floor slab, roof insulation, and replacement of the roof covering.

Based on the analysis of temperature distribution in building thermal bridge models conducted using the Therm 7.8.77 software [18], it was determined that existing linear thermal bridges contribute significantly to the overall heat losses, with the corresponding thermal heat transfer values being relatively high. Thermal retrofitting measures, including additional insulation and airtightness improvements at thermal bridge locations, were proposed. The simulation results indicated that, for selected construction details, the application of supplementary insulation led to a reduction in the U -value by more than 50%. This highlights the necessity of accounting for thermal bridges in energy retrofit projects, not only from the perspective of energy efficiency but also in terms of thermal comfort, hygienic conditions, and the durability of building envelopes. The computational results and temperature distribution visualisations confirm that thermal retrofitting effectively eliminates local zones of reduced surface temperature in the analysed thermal bridge areas, thereby reducing the risk of surface condensation and mould growth in practice.

The modernisation of the building's existing heat source, involving the replacement of an outdated solid fuel boiler system with a new central heating installation powered by a natural gas-fired condensing boiler, along with the proposed integration of flat-plate solar collectors for domestic hot water (DHW) sup-

port, contributed to a reduction in CO₂ emissions and decreased dependence on solid fuels and local fuel storage systems.

As a result of the design solutions prepared for the building owner, the boiler room was upgraded, flat-plate solar collectors were installed on the roof, and the DHW system was modernised. Furthermore, a decision was made to prepare an application for external state funding to support the thermal retrofitting of the building envelope in the near future. Further research, including thermographic inspections and in-situ measurements of the actual U -values of the building envelope components, is planned to be conducted prior to, during and after the implementation of the envelope modernisation works.

The findings presented in this study can be extrapolated to a broader category of buildings located within Polish Climate Zone I and regions exhibiting comparable thermal load characteristics. The case study building represents a typical structural configuration of single-family houses constructed in the early 2000s, characterised by uninsulated external walls made of lightweight aggregate concrete blocks, a reinforced concrete floor slab without thermal insulation, and insufficient roof insulation. The observed 63% reduction in heating load, achieved through thermal insulation of building envelopes and mitigation of linear thermal bridges, may serve as a reference benchmark for buildings with similar geometric, material and operational parameters. The post-retrofit overall heat transfer coefficients (U -values) comply with the requirements specified in [18] for residential buildings, indicating that the adopted insulation strategy can be considered a model configuration for structures located in Climate Zone I. Furthermore, the analysis of thermal bridges confirms that, under temperature differentials of approximately 36 K, reductions in linear thermal transmittance of up to 50% are attainable. This finding is particularly relevant for buildings situated in northern regions of the country, where the influence of thermal bridges on energy losses is amplified by the extended heating season.

Acknowledgements

The authors would like to express their sincere gratitude to Ms Jadwiga Wawer-Bernat, the owner of the analysed building, for her kind cooperation and for granting access to the property for the measurements necessary to conduct this research. Her trust and support were significant to the successful completion of this study.

This article is a revised and expanded version of a paper entitled: Impact of thermal modernisation including heat source upgrade on heat load of an existing single-family building (in Polish), which was presented at the 17th Heat and Mass Transfer Symposium, Kielce, Poland, September 8–10, 2025 [22].

References

- [1] *Thermal modernization of a building – what is it and what does it include*. Izolacje. <https://www.izolacje.com.pl/artykul/sciany-stropy/186640,termomodernizacja-budynku-co-to-jest-i-co-obejmuje-1> [Accessed 20 Jun. 2025] (in Polish).

- [2] Dz. U. 2024, item 1446. *Act of 21 November 2008 on Supporting Thermal Modernization and Renovations and on the Central Register of Building Emissions* (consolidated text). Journal of Laws of the Republic of Poland, 2024 (in Polish).
- [3] Pawłowski, K. (2020). Thermal modernization of buildings considering heat and moisture requirements from January 1, 2021. *Izolacje*, 2, 19–31 (in Polish).
- [4] Krause, P. (2010). Energy effects of thermal modernization of a residential building. *Fizyka Budowli w Teorii i Praktyce*, 5(2), 45–49 (in Polish).
- [5] Zawada, B., & Rucińska, J. (2021). Optimization of modernization of a single-family building in Poland including thermal comfort. *Energies*, 14(10), 2925. <https://doi.org/10.3390/en14102925>
- [6] Oleniacz, R., Kosietczak, M., & Rzerzutek, M. (2014). Assessment of thermal modernization effects in single-family buildings. 1. Reduction of heat consumption and pollutant emissions to air. *Czasopismo Inżynierii Lądowej, Środowiska i Architektury*, 23(61/3/I), 183–196. doi: 10.7862/rb.2014.55 (in Polish).
- [7] Koczyk, H., & Antoniewicz, B. (2000). *Heating: fundamentals of thermal design and building thermal modernization*. Wydawnictwo Politechniki Poznańskiej, Poznań (in Polish).
- [8] Kurtz, K., & Gawin, D. (2009). *Energy certification of residential buildings with examples*. Wrocławskie Wydawnictwo Naukowe Atla 2, Wrocław (in Polish).
- [9] Mayer, Z., Kahn, J., Götz, M., Hou, Y., Beiersdörfer, T., Blumenröhr, N., Volk, R., Streit, A., & Schultmann, F. (2023). Thermal bridges on building rooftops. *Scientific Data*, 10(268), 1–8. doi: 10.1038/s41597-023-02140-z
- [10] Pawłowski, K. (2018). Thermal modernization and thermal bridges in construction. *Przewodnik Projektanta*, 2, 0–14 (in Polish).
- [11] Levinskytė, A., Banionis, K., & Geležiūnas, V. (2016). The influence of thermal bridges for buildings energy consumption of “A” energy efficiency class. *Journal of Sustainable Architecture and Civil Engineering*. 15(2), 47–58. doi: 10.5755/j01.sace.15.2.15351
- [12] Żarski, K. (2008). Possibilities of meeting energy performance requirements of buildings in light of new legal regulations. *INSTAL*, 12, 6–11 (in Polish).
- [13] Pawlak, F. (2016). Impact of thermal bridges on heat losses and energy performance of buildings. Methods of accounting for thermal bridges in calculations. *INSTAL*, 6, 24–29 (in Polish).
- [14] Wiącek, A., Werle, S., & Ruszel, M. (2025). The impact of the behaviour of individual users in single-family households on the values of internal heat gains in a building. *Archives of Thermodynamics*, 46(2), 173–183. doi: 10.24425/ather.2025.154916
- [15] Turoń, M.K. (2024). Operation of a hybrid heating system based on heat pumps using a photovoltaic installation. *Archives of Thermodynamics*, 45(4), 153–162. doi: 10.24425/ather.2024.152004
- [16] PN-EN 12831:2006. *Heating systems in buildings – Method for calculating design heat load* (in Polish).
- [17] Audytor OZC 7.0. *Pro Edu, Sankom*. <https://pl.sankom.net/oprogramowanie/edukacyjne> [Accessed 15 Mar. 2025].
- [18] Dz. U. 2022, item 1225. *Ministry of Development and Technology. Announcement of 15 April 2022 on the publication of the consolidated text of the Regulation of the Minister of Infrastructure on the technical conditions to be met by buildings and their location*. Journal of Laws. 2022 (in Polish).
- [19] PN-EN ISO 10211:2008P. *Thermal bridges in buildings – Heat flows and surface temperatures – Detailed calculations* (in Polish).
- [20] *THERM Software 7.8.77. Berkeley Lab*. <https://windows.lbl.gov/therm-software-downloads> [Accessed 20 Mar. 2025].
- [21] Masłowski, A. *Solar energy in Poland – insolation. Poradnik Projektanta*. <https://poradnikprojektanta.pl/energia-sloneczna-w-polsce-naslonecznienie/> [Accessed 25 Mar. 2025] (in Polish).
- [22] Bernat, W., Zwarycz-Makles, K. (2025). Impact of thermal modernization including heat source upgrade on heat load of an existing single-family building. *17th Heat and Mass Transfer Symposium*. 8–10 September, Kielce, Poland.



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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Extended Verhoff-Banchero acid dew point model

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Received: 11.06.2025; revised: 01.10.2025; accepted: 21.10.2025

Abstract

This paper proposes a modification to the Verhoff-Banchero relation describing the acid dew point temperature, based on the evaluation of a generic equation that revealed two major limitations. First, the original relation's narrow application range was identified through comparison with the H_2O - H_2SO_4 vapour-liquid equilibrium dataset from the literature. Second, testing of the function's limits showed that the acid dew point temperature does not approach the water dew point temperature as the sulfuric acid concentration approaches zero. To address these limitations, the Verhoff-Banchero relation is extended into a bivariate quadratic function. Regression parameters, along with their estimation errors, are derived using the aforementioned dataset and the least squares method. Statistical test (p -value < 0.01) confirmed the significance of the introduced quadratic terms. The maximum estimation error for the modified relation is found to be $\pm 10^\circ\text{C}$ across a broad application range, with water partial pressures from 1013.25 Pa to 101325 Pa and sulfuric acid liquid mass fractions ranging from 0.1 to 0.98. Additionally, two semi-empirical relations are proposed, incorporating liquid-phase composition as an independent variable. The proposed extended Verhoff-Banchero model was evaluated using an experimental dataset for exhaust gas. Data points fall within an acceptable deviation range, with exceptions from two specific datasets. One of the observed deviations is explained by the partial derivative of the acid dew point temperature with respect to sulfuric acid partial pressure. The limitations of the proposed relations are discussed. Finally, the conclusions with potential future research directions in modelling the H_2O - H_2SO_4 system in exhaust gas are outlined.

Keywords: Acid dew point temperature; Vapour-liquid equilibrium; Sulfuric acid; Thermodynamic modelling

Vol. 46(2025), No. 4, 89–94; doi: 10.24425/ather.2025.156839

Cite this manuscript as: Przybyła, M. (2025). Extended Verhoff-Banchero acid dew point model. *Archives of Thermodynamics*, 46(4), 89–94.

1. Introduction

During the early stages of technological development aimed at meeting ecological and energy efficiency regulations (e.g. IMO (International Maritime Organization) Annex VI), there is a growing need to estimate exhaust gas acid dew point (ADP) temperature, especially at lower H_2SO_4 vapour pressures resulting from reduced sulfur concentrations in fuels.

The problem of ADP temperature calculation falls within the broader topic of H_2O - H_2SO_4 vapour-liquid equilibrium modelling, which is widely applied in various research areas. One key

objective of ADP temperature estimation is to prevent low-temperature corrosion on heat transfer surfaces, a critical concern in sensible waste heat recovery systems, as discussed in [1,2]. This objective has evolved as new technologies operating below this limit, such as flue gas desulfurisation using the wet sulfuric acid method, studied by Nielsen et al. [3], latent waste heat recovery from exhaust gas [4] and exhaust gas recirculation (EGR) [5].

Many researchers focus on cylinder liner wear and tribology problems [6–8], where the partial pressures of water, sulfuric acid and sulfur trioxide are relatively high and influenced by engine cycle parameters.

Nomenclature

a	– activity coefficient in the vapour phase
A	– ratio of vaporisation entropy and universal gas constant
b	– regression parameter in Verhoff-Banchero model
B	– ratio of vaporisation enthalpy and universal gas constant
L	– vaporisation enthalpy, J kmol ⁻¹
p	– partial pressure, Pa
P	– saturation pressure, Pa
R	– universal gas constant, J kmol ⁻¹ K ⁻¹
S	– vaporisation entropy, J kmol ⁻¹ K ⁻¹
t	– empirical acid dew point temperature, °C
T	– absolute acid dew point temperature, K
w	– mass fraction
x	– mole fraction

Greek symbols

β, λ, ψ – regression parameters

Subscripts and Superscripts

- 1 – water
- 2 – sulfuric acid

Abbreviations and Acronyms

ADP – acid dew point
 VB – Verhoff-Banchero
 VLE – vapour-liquid equilibrium

One of the most commonly employed ADP models is the Verhoff-Banchero (VB) formulation [9] with regression parameters determined by Jia and Ling in [10]. This formulation is given by the formula:

$$\frac{1}{T} = b - b_1 \lg p_1 - b_2 \lg p_2 + b_{12} \lg p_1 \lg p_2, \quad (1)$$

where: p_1 – partial pressure of water, p_2 – partial pressure of sulfuric acid, $b = 2.9882 \times 10^{-3}$, $b_1 = 1.3761 \times 10^{-4}$, $b_2 = 2.6740 \times 10^{-4}$, $b_{12} = 3.2870 \times 10^{-5}$.

Another widely used empirical relation was published by Okkes and it can be found in [11]. Recently, the Okkes's regression parameters were recalculated by Tang et al. [12].

To evaluate these ADP models, a set of ADP temperatures and partial pressures of the constituents was interpolated using the tabulated data from Vermeulen et al. [13]. A simplified graphical representation of this dataset is shown in Fig. 1 and the deviations of

the temperatures estimated using Eq. (1) and the equations given in [11] and [12] are presented in Table 1.

Two major drawbacks were observed from the results shown in Table 1:

- narrow application range of the tested models,
- ADP temperature does not approach the water dew point as the H₂SO₄ concentration tends to zero.

To address these limitations, a modification of the Verhoff-Banchero acid dew point relation is proposed, extending it into a bivariate quadratic function. Furthermore, two alternative semi-empirical relations are proposed, based on thermodynamic principles: the August relation derived from the Clausius-Clapeyron equation and a modified Raoult's law. The liquid-phase composition is added as an independent variable in these alternative relations due to its observed importance in previous studies.

The research objectives of this study are twofold:

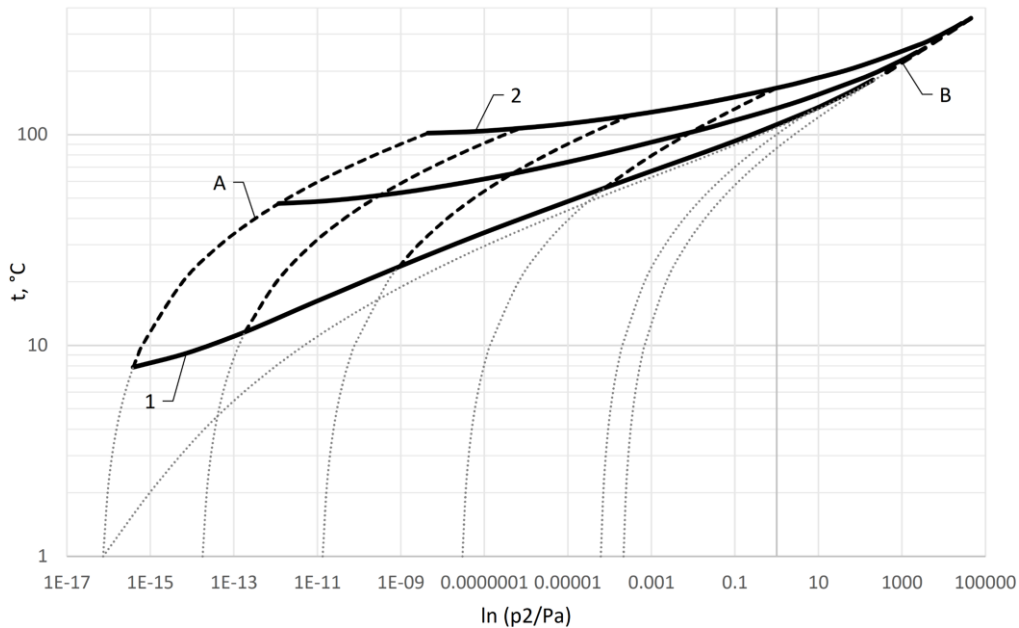


Fig. 1. Graphical representation of reference dataset interpolated from Vermeulen et al. [13]: 1, 2 – limitation curves computed at constant H₂O pressure equal to 1013.25 Pa and 101325 Pa, A-B – limitation curves computed at constant H₂SO₄ mass fraction equal to 0.1 and 0.98.

Table 1. Deviation of calculated acid dew point temperatures.

H ₂ SO ₄ mass fraction	$p_1 = 1013.25 \text{ Pa}$				$p_1 = 10132.5 \text{ Pa}$				$p_1 = 101325 \text{ Pa}$			
	ADP (°C)	Deviation Eq. (1)	Deviation Eq. from [11]	Deviation Eq. from [12]	ADP (°C)	Deviation Eq. (1)	Deviation Eq. from [11]	Deviation Eq. from [12]	ADP (°C)	Deviation Eq. (1)	Deviation Eq. from [11]	Deviation Eq. from [12]
0	7	A	A	A	46	A	A	A	100	A	A	A
0.1	8	88 ^B	A	C	47	74 ^B	A	C	102	58 ^B	A	C
0.2	9	81 ^B	A	C	49	65 ^B	A	C	104	48 ^B	A	C
0.3	12	73 ^B	A	C	52	57 ^B	A	C	107	39 ^B	25 ^B	C
0.4	16	64 ^B	A	C	57	47 ^B	A	C	113	30 ^B	17 ^B	-86 ^B
0.5	24	53 ^B	A	C	65	35 ^B	12 ^B	C	123	21 ^B	10	-21 ^B
0.6	36	40 ^B	A	C	79	22 ^B	3	-74 ^B	140	11 ^B	5	23 ^B
0.7	56	23 ^B	5	C	103	8	-1	6	166	5	6	37 ^B
0.8	88	4	2	14 ^B	137	-5	2	24 ^B	204	5	18 ^B	19 ^B
0.9	132	21 ^B	29 ^B	47 ^B	188	-4	24 ^B	7	266	27 ^B	53 ^B	-10
0.92	142	-8	19 ^B	19 ^B	199	-2	30 ^B	2	278	32 ^B	61 ^B	-14 ^B
0.94	154	-7	25 ^B	16 ^B	214	3	39 ^B	-2	296	42 ^B	75 ^B	-16 ^B
0.96	171	-5	35 ^B	13 ^B	233	9	52 ^B	-6	319	57 ^B	93 ^B	-17 ^B
0.98	198	2	53 ^B	10 ^B	267	24 ^B	77 ^B	-7	364	86 ^B	C	-13 ^B

^A Numerical solution does not exist.^B Deviation of ADP temperature exceeding the assumed application limit equal to $\pm 10^\circ\text{C}$.^C Deviation of ADP temperature exceeding $\pm 100^\circ\text{C}$ omitted in the table.

- 1) To improve the accuracy of ADP temperature estimation across a broad range of H₂O and H₂SO₄ partial pressures by developing a model that meets these criteria.
- 2) To create ADP temperature relations that are reliable and can be employed in ongoing studies on low-temperature EGR cooling systems for dual-fuel marine engines installed on LNG carriers and LNG-fuelled ships.

The importance of the developed ADP model is significant, especially for marine dual-fuel engines, due to the low sulfur content in LNG and specifically in pilot fuel oil.

2. Materials and methods

The ternary system of H₂O-SO₃-H₂SO₄ in vapour-liquid equilibrium was studied, as this is a typical configuration of components in fuel combustion products formed in the oxidation of hydrogen and sulfur. In the acid dew point model, the focus was on the binary system of H₂O (1)-H₂SO₄ (2), considering that SO₃ has not been converted into sulfuric acid as one of the remaining constituents of dry exhaust gas.

To describe the relationship between the observed variable (ADP temperature) and the independent variables (partial pressures of H₂O and H₂SO₄, regression analysis using the least squares method was employed, along with two-sided interval estimation. These methods were used to determine the regression function parameters and their standard errors. A *p*-value test of the regression parameters was performed, with a significance level set below 0.01 to ensure statistical robustness.

For the estimation of the regression parameters of the H₂O-H₂SO₄ system, a dataset consisting of 576 observations was used. The regression parameters for each pure component were determined from separate datasets. Specifically, for pure sulfuric acid, 36 observations were taken from the dataset of Vermeulen et al. [13]. The water input dataset, consisting of 177 observations, was sourced from the work of Wagner and Pruß [14].

3. Acid dew point model

The generic Verhoff-Banchero acid dew point model, presented in Eq. (1), can be considered a purely empirical relation, represented as a bivariate quadratic regression function with quadratic terms initially set to zero.

In this study, the model was extended by adding quadratic terms to improve its accuracy. The extended model is given by

$$1/T = \beta - \beta_1 \ln p_1 - \beta_2 \ln p_2 + \beta_{12} \ln p_1 \ln p_2 - \beta_{11} (\ln p_1)^2 - \beta_{22} (\ln p_2)^2, \quad (2)$$

where: p_1 – partial pressure of water, p_2 – partial pressure of sulfuric acid, with the coefficients $\beta = (2.776 \pm 0.01) \times 10^{-3}$, $\beta_1 = (1.150 \pm 0.17) \times 10^{-5}$, $\beta_2 = (9.500 \pm 0.15) \times 10^{-5}$, $\beta_{12} = (4.300 \pm 0.16) \times 10^{-6}$, $\beta_{11} = (2.930 \pm 0.16) \times 10^{-6}$, $\beta_{22} = (1.100 \pm 0.04) \times 10^{-6}$.

These quadratic terms are crucial for improving the model's performance, especially in regions of the parameter space where the relationship between the ADP temperature and partial pressures deviates from a simple linear approximation.

3.1. Alternative semi-empirical relations

An alternative acid dew point temperature relation was derived in the specific form for each component. Both presented formulas are based on the well-known August equation with a regression parameter newly introduced in this paper, denoted by ψ for water and λ for sulfuric acid.

For water, the following formula was derived

$$\frac{1}{T} = (A_1 - \ln p_1) \left[B_1 + R \left(\psi - \frac{1}{2} x_1 \right) \right]^{-1}, \quad (3)$$

where: $A_1 = 24.58 \pm 0.046$, $B_1 = 4902 \pm 20$, $\psi = \left\{ \begin{array}{l} 0.456 \pm 0.01 \\ x_1/2 \end{array} \right.$.

The first condition of the empirical parameter ψ is applied in the concentration region $0.1 < x_1 \leq 0.9$, where interaction between components becomes significant. The second condition elimi-

notes the last term from the denominator, which leads to the rearranged August equation and is valid for the H₂O mole fraction range $0.9 < x_1 \leq 1$.

A similar formula was derived for sulfuric acid:

$$\frac{1}{T} = (A_2 - \ln p_2)[B_2 + R(1 - \lambda w_2)]^{-1}, \quad (4)$$

where: $A_2 = 25.65 \pm 0.20$, $B_2 = 9204 \pm 80$, $\lambda = \begin{cases} 1.055 \pm 0.018 \\ 1.024 \pm 0.020 \end{cases}$.

The first condition of the empirical parameter lambda should be applied in the region $0.1 \leq w_2 < 0.94$, while the second one for $0.94 \leq w_2 < 1$.

In Eqs. (3) and (4), the universal gas constant, equal to $R = 8314.5 \text{ J kmol}^{-1}\text{K}^{-1}$, was used and considered free of error.

Alternative ADP temperature relation was derived by means of a binary vapour-liquid equilibrium condition having the form of modified Raoult's rule

$$p_i = a_i P_i, \quad (5)$$

where: p_i – i -th constituent partial pressure, a_i – activity coefficient of the i -th component in vapour phase, P_i – i -th pure component vapour pressure over saturated liquid.

Activity coefficient of constituents in the vapour phase was based on integrated vapour-liquid equilibrium (VLE) of Clausius-Clapeyron with respect to temperature

$$\ln a_i = -\frac{\Delta L_i}{R} \frac{1}{T}, \quad (6)$$

where ΔL_i is the difference between the vaporisation enthalpy of a given component as a pure constituent and constituent of the considered two-phase system. It is dependent on the composition and temperature. Here, averaged values over the considered temperature range were used.

Pure component vapour pressure over saturated liquid was described similarly:

$$\ln P_i = \frac{S_i}{R} - \frac{L_i}{R} \frac{1}{T}. \quad (7)$$

Using Eqs. (5)–(7), the following relation for the i th component was derived:

$$\frac{1}{T} = \left(\frac{S_i}{R} - \ln P_i \right) \left(\frac{L_i}{R} + \frac{\Delta L_i}{R} \right)^{-1}, \quad (8)$$

where the well-known notation was added: $A_i = S_i/R$ – ratio of the pure component vaporisation entropy and universal gas constant, $B_i = L_i/R$ – ratio of the pure component vaporisation enthalpy and universal gas constant.

In order to describe $\Delta L_i/R$, a specific one-parameter empirical relation was introduced for each component, as can be seen in Eqs. (3) and (4).

3.2. Error of estimation

The general regression function that was selected is suitable for the expression of Eqs. (2), (3) and (4):

$$Y = f(X, \theta) \pm \epsilon, \quad (9)$$

where: Y – dependent variable, X – set of independent variables, θ – regression parameters, ϵ – error of estimated value.

Employing the given standard error of regression parameters, one can calculate the estimation error of ADP temperature using a widely employed statistical methodology. This problem is omitted here and will be examined in the next paper.

4. Results and discussion

In Table 2, the deviations of acid dew point temperature calculated using the proposed Eqs. (2), (3) and (4) are presented. From the observed deviations, it is clear that the best fit to the curves presented in Fig. 1 is obtained for Eqs. (2) and (3). The deviations of Eq. (4) show good accuracy in the highest partial pressure of water.

Acceptable deviations for ten percent of sulfuric acid mass fraction are observed for all of the relations. This indicates that the calculated temperature tends to the water dew point as the sulfuric mass fraction tends to zero.

The superior feature of Eq. (4) is the possibility of water dew point calculation at zero mass fraction of sulfuric acid.

Table 2. Deviation of acid dew point temperature calculated using the proposed Eqs. (2), (3) and (4).

H ₂ SO ₄ mass fraction	$p_1=1013.25 \text{ Pa}$			$p_1=10132.5 \text{ Pa}$			$p_1=101325 \text{ Pa}$		
	Deviation Eq. (2)	Deviation Eq. (3)	Deviation Eq. (4)	Deviation Eq. (2)	Deviation Eq. (3)	Deviation Eq. (4)	Deviation Eq. (2)	Deviation Eq. (3)	Deviation Eq. (4)
0	^A	3	^A	^A	0	^A	^A	–2	^A
0.1	–6	3	9	–1	1	7	7	–1	4
0.2	–2	5	11 ^B	1	3	9	7	1	4
0.3	1	7	14 ^B	3	6	12 ^B	7	5	6
0.4	4	7	15 ^B	4	5	13 ^B	6	4	8
0.5	6	3	14 ^B	4	1	12 ^B	4	0	7
0.6	6	1	12 ^B	3	–2	8	1	–3	3
0.7	4	2	5	0	0	2	–4	–4	–4
0.8	–3	4	–4	–7	0	–6	–10	–6	–8
0.9	^A	1	36 ^B	–9	–3	–2	–7	–7	0
0.92	–8	–2	1	–9	–7	1	–5	–12 ^B	5
0.94	–8	–4	–6	–6	–9	–5	0	–15 ^B	–2
0.96	–6	–5	–3	–3	–10	–1	7	–14 ^B	6
0.98	–1	2	3	7	1	7	^A	3	17 ^B

^A Numerical solution does not exist.

^B Deviation of ADP temperature exceeding the assumed application limit equal to $\pm 10^\circ\text{C}$.

4.1. Model evaluation with empirical data

In Fig. 2, the experimental data on the acid dew point in exhaust gas reported by selected authors at a partial pressure of water equal to 10132.5 Pa are presented. The solid curve is computed using the extended VB model. Significant deviation of data points marked with numbers 2 and 3 from the computed curve can be observed in the figure.

This discrepancy was identified as potentially stemming from measurement challenges reported in [16] and [17]. This measurement problem by means of a conductive dew point meter in the situation of sulfuric acid low concentration in exhaust gas is described by Vasanthakumar [18].

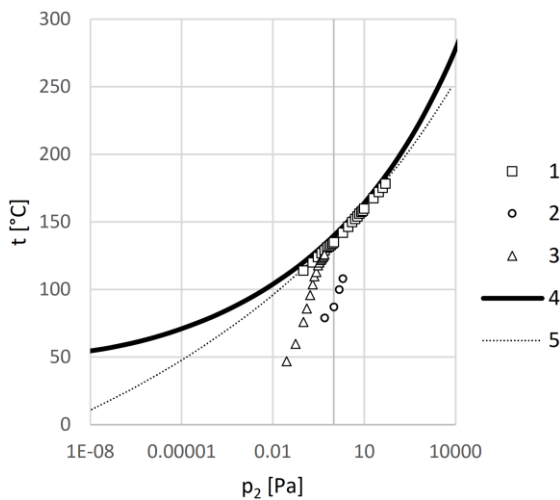


Fig. 2. Comparison of experimental data with evaluated acid dew point model: 1 – ZareNezhad et al. [15], 2 – Xiang et al. [16], 3 – Blanco et al. [17], 4 – extended VB (Eq. (2)), 5 – VB (Eq. (1)).

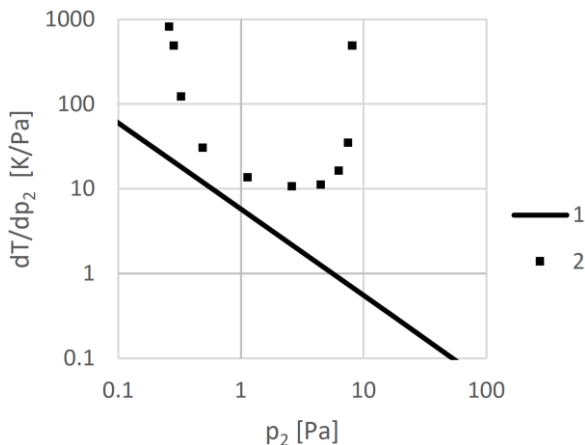


Fig. 3. Derivative of acid dew point temperature with respect to H_2SO_4 partial pressure: 1 – extended Verhoff-Banchero model (Eq.(2)), 2 – Li et al. [19].

In Fig. 3, the derivative of ADP temperature with respect to sulfuric acid partial pressure is displayed. The points were calculated using data published in [19] and the black solid curve was computed based on the modified VB model (Eq. (2)). From

the graph, we can conclude that during the experiment reported in [19], the condensation did not proceed according to the equilibrium curve that represents a reversible process.

4.2. Model limitation

Despite satisfactory estimation accuracy of acid dew point temperature, the calculated partial pressure from rearranged Eq. (2) exhibits excessive deviation from the data points of Vermeulen et al. [13], which were taken as an input to build the model. Moreover, the behaviour of the rearranged function indicates numerical instability. Thus, it is advised to use the extended VB equation only for ADP temperature estimation. The problem of estimation error of partial pressure in the considered system was studied and described in the published note by Verhoff and Banchero [20], thus, it is omitted here.

Equation (2) is the recommended primary model. The limitations of the derived Eqs. (3) and (4) require further validation.

5. Conclusions

This study presents a significant advancement in the modelling of the acid dew point temperature by extending the Verhoff-Banchero equation to include quadratic terms. The proposed modification enhances the accuracy of ADP temperature estimations, particularly in the range of sulfuric acid mass fractions where previous models showed limitations. The introduction of semi-empirical relations for water and sulfuric acid provides further flexibility and applicability across a wider range of operating conditions.

The primary contribution of this research lies in the improved understanding of ADP temperature behaviour, especially in systems with low sulfuric acid concentrations. The extended Verhoff-Banchero model addresses the issue of ADP temperature approaching the water dew point as sulfuric acid concentration decreases.

While the extended model demonstrates improved accuracy for ADP temperature estimation, challenges remain in the numerical instability and deviations in partial pressure calculations.

The results of this work contribute to the development of more reliable models for the H_2O - H_2SO_4 system, with potential applications in environmental engineering, energy recovery and industrial processes that involve sulfuric acid and water vapour.

The VLE (vapour-liquid equilibrium) modelling of the H_2O - H_2SO_4 system in exhaust gases offers several avenues for future research. This study has focused on ADP temperature as a function of partial pressures and liquid phase composition. Future work should investigate the following relations with two independent variables:

- liquid phase composition as a function of ADP temperature and vapour phase composition,
- vapour phase composition as a function of ADP temperature and liquid phase composition.

By developing these relations and incorporating the impact of condensation nuclei on mist formation, a more realistic H_2O - H_2SO_4 condensation model in exhaust gas can be created for practical applications.

References

- [1] Przybyła, M., & Adamkiewicz, A. (2020). Quantity and quality of marine dual-fuel engine's exhaust sensible waste heat considering different sulfur content in fuels. *Archives of Thermodynamics*, 41(4), 147–168. doi: 10.24425/ather.2020.135857
- [2] Ouyang, T., Su, Z., Gao, B., Pan, M., Chen, N., & Huang, H. (2019). Design and modeling of marine diesel engine multi-stage waste heat recovery system integrated with flue-gas desulfurization. *Energy Conversion and Management*, 196, 1353–1368. doi: 10.1016/j.enconman.2019.06.065
- [3] Nielsen, R.F., Haglind, F., & Larsen, U. (2014). Design and modeling of an advanced marine machinery system including waste heat recovery and removal of sulphur oxides. *Energy Conversion and Management*, 85, 687–693. doi: 10.1016/j.enconman.2014.03.038
- [4] Kornienko, V., Radchenko, M., Radchenko, R., Pavlenko, A., & Radchenko, A. (2025). A new trend in combustion engine's deep waste heat recovery by application of condensing economizers in exhaust boilers. *Applied Thermal Engineering*, 261, 125150. doi: 10.1016/j.applthermaleng.2024.125150
- [5] Yao, Y., Han, Z., Tian, W., Wu, X., Wu, Y., Yan, Y., Xia, Q., Fang, J., Luo, L., & De Weireld, G. (2023). Three condensation paths of exhaust and its five effects on exhaust gas recirculation (EGR) cooler fouling and thermal performance: A review. *Case Studies in Thermal Engineering*, 47, 103099. doi: 10.1016/j.csite.2023.103099
- [6] Pang, K.M., Karvounis, N., Walther, J.H., Schramm, J., Glarborg, P., & Mayer, S. (2017). Modelling of temporal and spatial evolution of sulphur oxides and sulphuric acid under large, two-stroke marine engine-like conditions using integrated CFD-chemical kinetics. *Applied Energy*, 193, 60–73. doi: 10.1016/j.apenergy.2017.02.020
- [7] Karvounis, N., Pang, K.M., Mayer, S., & Walther, J.H. (2018). Numerical simulation of condensation of sulfuric acid and water in a large two-stroke marine diesel engine. *Applied Energy*, 211, 1009–1020. doi: 10.1016/j.apenergy.2017.11.085
- [8] Kjemtrup, L., Cordtz, R.F., Jensen, M.V., & Schramm, J. (2020). An experimental investigation of the corrosive influence of SO₂ relative to H₂SO₄ on marine engine cylinder liners. *Lubrication Science*, 32(3), 131–144. doi: 10.1002/lis.1492
- [9] Verhoff, F.H., & Banchero, J.T. (1974). Predicting dew points of flue gases. *Chemical Engineering Progress*, 70(8), 71–72.
- [10] Jia, M., & Ling, Ch. (2003). Factors of affecting the flue gas acid dew point temperature and its way of calculation. *Industrial Boilers*, 6(82), 31–35.
- [11] Okkes, A.G. (1987). Get acid dew-point of flue-gas. *Hydrocarbon Process*, 66(7), 53–55.
- [12] Tang, Z., Li, Z., Ji, B., & Song, Q. (2025). Study on sulfuric acid condensation on planar and particle surfaces in flue gas by thermodynamic equilibrium analysis. *ACS Omega*, 10(19), 19625–19635. doi: 10.1021/acsomega.5c00382
- [13] Vermeulen, T., Dong, J., Robinson, S., & Nguyen, T. (1982). Vapor-liquid equilibrium of the sulfuric acid-water system. *AIChE Meeting*, Anaheim, California.
- [14] Wagner, W., & Pruß, A. (2002). The IAPWS formulation 1995 for the thermodynamic properties of ordinary water substance for general and scientific use. *Journal of Physical and Chemical Reference Data*, 31(2), 387–535. doi: 10.1063/1.1461829
- [15] ZareNezhad, B., & Aminian, A. (2011). Accurate prediction of the dew points of acidic combustion gases by using an artificial neural network model. *Energy Conversion and Management*, 52(2), 911–916. doi: 10.1016/j.enconman.2010.08.018
- [16] Xiang, B., Tang, B., Wu, Y., Yang, H., Zhang, M., & Lu, J. (2016). Predicting acid dew point with a semi-empirical model. *Applied Thermal Engineering*, 106, 992–1001. doi: 10.1016/j.applthermaleng.2016.06.040
- [17] Blanco, J.M., & Peña, F. (2008). Increase in the boiler's performance in terms of the acid dew point temperature: Environmental advantages of replacing fuels. *Applied Thermal Engineering*, 28(7), 777–784. doi: 10.1016/j.applthermaleng.2007.06.024
- [18] Vasanthakumar, A.L.S. (1991). The dew points of acidic vapors in simulated power station flue gases. MSc thesis, Middlesex University, London.
- [19] Li, Y., Zhu, Q., Yi, Q., Zuo, W., Feng, Y., Chen, S., & Dong, Y. (2019). Experimental method for observing the fate of SO₃/H₂SO₄ in a temperature-decreasing flue gas flow: Creation of state diagram. *Fuel*, 249, 449–456. doi: 10.1016/j.fuel.2019.03.130
- [20] Verhoff, E.H., & Banchero, J.T. (1972). A note on the equilibrium partial pressures of vapors above sulfuric acid solutions. *AIChE Journal*, 18(6), 1265–1268. doi: 10.1002/aic.690180630

Impact of finite Larmor radius on the onset of Rayleigh-Taylor instability of stratified magnetised Jeffrey plasma fluid

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Received: 19.03.2025; revised: 09.08.2025; accepted: 12.08.2025

Abstract

This study explores the stabilising interplay of finite Larmor radius corrections and quantum pressure on the Rayleigh-Taylor instability in a non-Newtonian, magnetised fluid. The investigation is motivated by the need to understand how quantum and magnetohydrodynamic effects jointly influence instability behaviour in complex fluid systems. The governing magnetohydrodynamic equations are linearised using normal mode analysis and appropriate boundary conditions to derive a general dispersion relation for Rayleigh-Taylor instability under the Jeffrey fluid model. Numerical results show that the combined presence of finite Larmor radius corrections and quantum pressure suppresses the growth rate of Rayleigh-Taylor instability modes. In contrast, the Jeffrey parameter amplifies instability, while quantum and finite Larmor radius effects reduce both the cut-off and critical wavenumbers.

Keywords: Rayleigh-Taylor instability; Jeffrey model; Finite Larmor radius; Magnetic field; Quantum pressure

Vol. 46(2025), No. 4, 95–103; doi: 10.24425/ather.2025.156840

Cite this manuscript as: Thakur, A., Sharma, V., Rana, G.C., & Gautam, P. (2025). Impact of finite Larmor radius on the onset of Rayleigh-Taylor instability of stratified magnetised Jeffrey plasma fluid. *Archives of Thermodynamics*, 46(4), 95–103.

1. Introduction

The Rayleigh-Taylor instability (RTI) arises at the interface between fluids of differing densities when a heavier fluid is placed above a lighter one in the presence of a vertical gravitational field [1]. This type of instability plays a crucial role in various physical and astrophysical phenomena, including laser-plasma interactions, Z-pinch implosions and magnetised target fusion, as well as in systems such as inertial confinement fusion, galaxy clusters and dense plasma focus devices. Goldston and Rutherford [2] extended the analysis of RTI to fluids with continuously varying density profiles and demonstrated that angular velocity, mass concentration and stress relaxation time have a stabilising influence on the instability.

The significant potential of quantum plasma in diverse fields – such as laser fusion, white dwarfs, semiconductor devices and

dense astrophysical plasmas – has recently drawn extensive research interest. Quantum plasma consists of charged particles exhibiting distinct behaviour at moderate temperatures, differing from classical plasmas. This quantum effect arises when the de Broglie wavelength of charged particles becomes comparable to or larger than the system's characteristic length [3]. The pressure term in the equations of motion is divided into two terms, $p = p^c + p^q$ (p^c – classical pressure, p^q – quantum pressure), by using Wingen principle and Schrödinger wave equation in the momentum equations, $(-\nabla p)$ and $\mathbf{Q} = \frac{\hbar^2}{2m_e m_i} \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$.

A lot of work has been done in this direction to evaluate the growth rates of RTI in various processes with quantum pressure by many researchers, both theoretically and experimentally.

Gardner [4] noticed that the quantum hydrodynamic conservation laws have the same form as the classical hydrodynamic

Nomenclature

d	– thickness of layer, m
e_{ij}	– rate of strain tensor
g	– gravitational acceleration, m/s ²
$\hbar$	– Planck constant, $6.62607015 \times 10^{-34}$ m ² kg/s
H	– magnetic field, A/m
k	– wave number, 1/m
k_{max}	– cut-off wave number, 1/m
L_D	– length scale, m
m_e	– mass of electron, amu
m_i	– mass of ion, amu
n	– growth rate, 1/s
n_i	– growth rate of RTI, 1/s
n_q	– quantum parameter, kg m/s
p	– pressure, Pa
p^C	– classical pressure, Pa
p^Q	– quantum pressure, Pa
Q	– quantum pressure, Pa
t	– time, s
T	– temperature, K
T_{ij}	– stress tensor, Pa
$\mathbf{u}$	– velocity vector of fluid, m/s

u_j	– velocity vector components, m/s
u, v, w	– velocity vector components, m/s
x, y, z	– Cartesian coordinates, m

Greek symbols

δ_{ij}	– Kronecker delta
λ	– Jeffrey parameter
λ_0	– stress relaxation time parameter
λ_1	– strain retardation time parameter
μ	– dynamic viscosity, Pa·s
μ_e	– magnetic permeability, H/m
ν	– kinematic viscosity, m ² /s
ν_{FR}	– finite Larmor radius correction, kg/(m ² s ²)
ρ	– density, kg/m ³
τ_{ij}	– viscous stress tensor
Ω	– ion gyro-frequency, rad/s

Abbreviations and Acronyms

FLR	– finite Larmor radius
ICF	– inertial confinement fusion
MHD	– magnetohydrodynamics
QMHD	– quantum magnetohydrodynamics
RTI	– Rayleigh-Taylor instability

equations. Haas [5] employed the quantum hydrodynamics model to analyse charge mobility in dense quantum plasmas. Elena et al. [6] investigated the influence of quantum effects on RTI and internal waves in plasmas, while Bychkov et al. [7] studied RTI in incompressible, stratified quantum plasmas, establishing that quantum pressure at low temperatures stabilises perturbation growth. Sharma et al. [8] explored the combined effects of angular velocity and surface tension on the stability of two superimposed fluids embedded with dust particles. Hoshoudy [9–11], and Hoshoudy and Awasthi [12] analysed RTI in stratified quantum plasmas under the influence of vertical magnetic fields and viscosity, concluding that quantum pressure, in conjunction with the magnetic field, plays a crucial role in stabilising the system and suppressing RTI. Additionally, Hoshoudy and Prajapati [13] demonstrated the impact of suspended particles on RTI, observing a stabilising effect due to both quantum pressure and dust particle mass concentration.

Sharma and Gupta [14] examined the stability of an elasto-viscous fluid under the influence of magnetic field and rotation. Dolai and Prajapati [15] analysed RTI in strongly coupled, dusty, rotating plasmas and observed significant suppression of instability due to the combined influence of shear and angular velocities. Micro-level instabilities in plasmas are often described by models incorporating collisionless dissipative effects, finite Larmor radius (FLR) corrections and other non-ideal behaviours. Although the length and time scales associated with micro-instabilities typically match those of transport coefficients and turbulence – leading to a common practice of neglecting FLR effects – this assumption breaks down when the Larmor radius becomes comparable to the hydromagnetic wavelength or when the ion gyrofrequency approaches the wave frequency. Under such conditions, the FLR must be considered. Consequently, the time and spatial scales at which magnetohydrodynamics (MHD) fails align with the scales of ion gyration and the ion Larmor frequency.

The effects of finite Larmor radius have drawn significant attention due to their applicability in a range of astrophysical environments, including mirror machines, the solar corona, and both interplanetary and interstellar plasmas, particularly in the presence of gyro-viscous forces. Rosenbluth et al. [16] demonstrated the stabilising influence of FLR on RTI by employing plasma fluid equations. Roberts and Taylor [17] extended MHD theory by incorporating finite Larmor radius effects. Jukes [18] showed that FLR can suppress RTI, though with some constraints introduced by the presence of a sheared magnetic field. Following these foundational studies, numerous investigations explored the effects of FLR, dust particles, rotation and viscosity on RTI in continuously stratified magnetofluids [19–22]. Notably, Tiwari et al. [23] were the first to establish the influence of FLR corrections on the RTI in an inviscid, stratified plasma, incorporating both magnetic fields and quantum pressure. They concluded that FLR corrections suppress the instability more effectively than quantum pressure alone. Sun et al. [24] highlighted the RTI in magnetised fluids relevant to inertial confinement fusion (ICF), emphasising that viscosity, rather than electrical resistivity, plays a dominant role in determining the interfacial dynamics.

It is important to note that most of the aforementioned studies have been conducted in the context of Newtonian plasma fluids. However, in recent years, the mechanisms and applications of non-Newtonian fluids described by various rheological models have gained considerable interest due to their relevance in both industrial (e.g. chemical processing) and astrophysical settings. A fluid is characterised as Newtonian or non-Newtonian based on the nature of the relationship between stress and strain rate, expressed through its constitutive equation. Among non-Newtonian models, the Jeffrey fluid model [25] is of particular interest in this study. It exhibits linear viscoelastic behaviour, yield stress, shear-thinning properties and high shear viscosity,

making it well-suited for modelling complex plasma dynamics with non-Newtonian characteristics.

In Jeffrey fluid, the two-time parameters – namely, the strain retardation time and stress relaxation time – play a significant role in understanding wave propagation phenomena, such as those occurring within the Earth's mantle. The strain retardation time relates to the delay in deformation under stress, while the stress relaxation time represents the time a fluid takes to return from a disturbed state to its original equilibrium. Fluids exhibiting such time-dependent behaviour are particularly relevant in environmental and biomedical applications, including polypropylene coalescence sintering, geological flows and blood flow dynamics.

Yadav [26–28] investigated the effects of anisotropy, electric fields, and thermal non-equilibrium on the onset and instability mechanisms of Jeffrey fluid convection in various porous media configurations and found that rotation and anisotropy in thermal diffusivity delay the onset of Jeffrey fluid convection, whereas the Jeffrey parameter and permeability anisotropy exhibit dual effects under rotation. Increased electric field and internal heating parameters reduce system stability. Gautam et al. [29] investigated the influence of an electric field on thermal convection in a nanofluid-saturated porous medium, whereas Sharma et al. [30] analysed the Rayleigh-Taylor instability of superposed dusty Jeffrey fluids in a porous medium, considering interfacial surface tension. Yadav et al. [31–32] analysed the Horton-Rogers-Lapwood problem and convective flow of ethylene glycol-silver Jeffrey nanofluid in a Hele-Shaw cell under the influence of a magnetic field.

Prajapati [33] investigated RTI in a strongly coupled viscoelastic fluid, considering the effects of non-homogeneous magnetic fields, uniform rotation and density gradients, and observed a substantial suppression of the instability. Garai et al. [34] examined RTI in viscoelastic plasma embedded with dust particles and magnetic fields, highlighting the significant stabilising influence of non-Newtonian properties. Dey [35] explored Jeffrey fluid flow in the presence of suspended particles and angular velocity, including the Hall effect and volume fraction, and concluded that non-Newtonian parameters tend to have a retarding influence on the evolution of instability in the system. Garai et al. [36] analysed RTI in strongly coupled quantum plasma with shear velocity and found that shear effects could either suppress or trigger instability, depending on the direction of the shear velocity gradient. Adak et al. [37] studied RTI in inhomogeneous pair-ion plasma and derived the instability criteria for the classical case.

Das et al. [38] investigated the collective behaviour of strongly coupled dusty plasma, whereas Dharodi and Das [39] performed numerical simulations on gravity-driven instabilities in such plasmas. The result indicates that increasing the coupling strength of the medium leads to suppression of these instabilities. Dharodi [40–41] conducted numerical investigations on gravity-driven instabilities in strongly coupled dusty plasmas, focusing on hetero-interactions between a rising bubble and a falling droplet, as well as homo-interactions between pairs of rising or falling bubbles/droplets. The study revealed that, under gravity, the formation of counter-rotating vorticity lobes causes bubbles to ascend and droplets to descend. In viscoelastic fluids, besides the initial separation, shear waves generated by rotating

vortices were found to play a key role in bringing two droplets or bubbles closer together.

Motivated by the work presented above, this study focuses on investigating the influence of FLR corrections on the Rayleigh-Taylor instability in a non-Newtonian plasma fluid arranged in horizontal stratification, incorporating quantum pressure effects. The RTI is a fundamental plasma instability that plays a critical role in astrophysical plasmas, ICF and space weather phenomena. Understanding the onset and control of RTI under the influence of quantum and magnetic effects is crucial for enhancing the stability of fusion plasmas and improving performance in high-energy density systems. The inclusion of FLR corrections and non-Newtonian (Jeffrey) fluid behaviour provides a more realistic model for magnetised plasmas found in laboratory and astrophysical environments, such as solar prominences, planetary magnetospheres and accretion disks. The novelty of this study lies in the combined consideration of quantum pressure effects, FLR corrections and non-Newtonian fluid dynamics (via the Jeffrey model) in analysing RTI. While previous studies have independently examined RTI in Newtonian or classical fluids with either quantum effects or magnetic field influences, this is the first comprehensive analysis that integrates all three aspects simultaneously in a stratified magnetised plasma system. Moreover, the use of the Jeffrey fluid model introduces a new viscoelastic framework that captures memory effects absent in earlier studies. The non-Newtonian behaviour of the plasma is modelled using the Jeffrey fluid framework, which effectively captures viscoelastic features and complex rheological behaviour.

2. Mathematical models

The equation that governs the behaviour of fluid flow in the context of Jeffrey fluid, as formulated by [26–32], can be expressed as follows:

$$T_{ij} = -p\delta_{ij} + \tau_{ij} \quad (1)$$

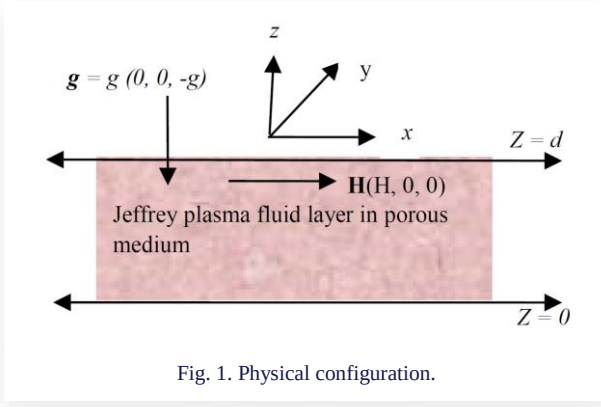
and

$$\tau_{ij} = -p\delta_{ij} + \frac{2\mu}{1+\lambda} \left[1 + \lambda_1 \left\{ \frac{\partial(2e_{ij})}{\partial t} + \frac{\partial u_i}{\partial x_j} \right\} \right] e_{ij}. \quad (2)$$

For incompressible fluids, the above relation reduces to

$$\tau_{ij} = -p\delta_{ij} + \frac{2\mu}{1+\lambda} e_{ij}. \quad (3)$$

Here, an incompressible, heterogeneous, infinitely extending and infinitely electrically conducting viscoelastic Jeffrey fluid of finite thickness d , comprised of the planes $z = 0$ and $z = d$, is organised in a horizontal layer of electrons and immobile ions saturating homogeneous, isotropic porous media. A uniform gravitational field $\mathbf{g}(0, 0, -g)$ acts vertically downward, thereby influencing the buoyancy-driven motion within the system. In addition, the plasma is subjected to a uniform externally applied magnetic field $\mathbf{H}(H, 0, 0)$ directed along the horizontal axis, introducing magnetohydrodynamic (MHD) effects into the flow. The presence of a quantum pressure term $\mathbf{Q}$, originating from quantum mechanical effects associated with electron degeneracy, further modifies the momentum balance and contributes to



stabilising or destabilising tendencies in the convective motion. The schematic representation of the physical system, along with the associated external forces and boundary constraints, is illustrated in Fig. 1.

The equations of momentum balance in quantum hydrodynamics (QHD) are [33–37]:

$$\rho \left[\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right] \mathbf{u} = -\nabla p + \rho \mathbf{g} + \nabla \cdot \mathbf{P} + \mathbf{Q} + \frac{\mu}{1+\lambda} \nabla^2 \mathbf{u} + \frac{\mu_e}{4\pi} (\nabla \times \mathbf{H}) \times \mathbf{H}, \quad (4)$$

where $\mathbf{u}$, ρ , p , μ , λ , and μ_e represent the velocity of fluid, density, pressure, dynamic viscosity, Jeffrey parameter and magnetic permeability, respectively. Here, $\mathbf{P}$ represents the pressure tensor that anticipates the finite Larmor radius effects.

The mass balance equation of incompressible plasma fluid is

$$\nabla \cdot \mathbf{u} = 0. \quad (5)$$

The condition of compressibility implies that the fluid density is not constant and may vary with pressure and temperature. Accordingly, the continuity equation takes the general form as:

$$\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla) \rho = 0. \quad (6)$$

In various circumstances of physical intrigue in ICF, instability occurs at velocities much smaller than that of local sound speed. Consequently, accelerations in flow are too weak to vary the fluid flow density appreciably, and the fluid moves without expanding or compressing, meaning that elements of fluid move with constant density, as seen in Eq. (6).

Maxwell's equations, due to the presence of a magnetic field, are:

$$\nabla \cdot \mathbf{H} = 0, \quad (7)$$

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}). \quad (8)$$

Furthermore, the initial steady state is characterised by no flow motion and each physical variable varies along the vertical, i.e. z -axis, only. Therefore, the steady state solutions are given as

$$\mathbf{u} = (0, 0, 0), \quad p = p(z), \quad \rho = \rho(z), \quad \mathbf{Q} = \mathbf{Q}(z). \quad (9)$$

To investigate the stability of the hydrodynamic motion, infinitesimal perturbations are superimposed on each of the physical quantities of the initial state solutions as

$$\begin{aligned} \rho &= \rho_0 + \rho', \quad p = p_0 + p', \quad \mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \\ \mathbf{u} &= \mathbf{u}_0 + \mathbf{u}', \quad \mathbf{Q} = \mathbf{Q}_0 + \mathbf{Q}', \end{aligned} \quad (10)$$

where p' , ρ' , $\mathbf{h}(h_x, h_y, h_z)$, $\mathbf{u}'(u, v, w)$, $\mathbf{Q}'(Q_{x1}, Q_{y1}, Q_{z1})$ represent the perturbations in the pressure, the density of fluid, the magnetic field, the plasma fluid velocity and the quantum force, respectively. The subscript '0' denotes the equilibrium state.

Utilising the disturbance equations (10) and the linear theory, Eqs. (4)–(8) take linear form as

$$\nabla \cdot \mathbf{u}' = 0, \quad (11)$$

$$\rho \frac{\partial \mathbf{u}'}{\partial t} = -\nabla p' + \rho \mathbf{g}' - \nabla \cdot \mathbf{P}' + \mathbf{Q}' + \frac{\mu}{1+\lambda} \nabla^2 \mathbf{u}' + \frac{\mu_e}{4\pi} [(\nabla \times \mathbf{h}) \times \mathbf{H}], \quad (12)$$

$$\frac{\partial \rho'}{\partial t} + (\mathbf{u}' \cdot \nabla) \rho = 0, \quad (13)$$

$$\nabla \cdot \mathbf{h} = 0, \quad (14)$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u}' \times \mathbf{H}), \quad (15)$$

where

$$\mathbf{Q}' = \frac{\hbar^2}{2m_e m_i} \left[\begin{aligned} &\frac{1}{2} \nabla (\nabla^2 \rho') - \frac{1}{2\rho} \nabla \rho' \nabla^2 \rho - \frac{1}{2\rho} \nabla \rho \nabla^2 \rho' \\ &+ \frac{\rho'}{2\rho^2} \nabla \rho \nabla^2 \rho - \frac{1}{2\rho} \nabla (\nabla \rho \nabla \rho') \\ &+ \frac{\rho'}{4\rho^2} \nabla (\nabla \rho)^2 + \frac{1}{2\rho^2} (\nabla \rho)^2 \nabla \rho' \\ &+ \frac{1}{\rho^2} (\nabla \rho \nabla \rho') \nabla \rho - \frac{\rho'}{\rho^3} (\nabla \rho)^3 \end{aligned} \right]. \quad (16)$$

For the horizontal magnetic field, the stress tensor in the component form is [23]:

$$\begin{aligned} P_{xx} &= 0, \quad P_{yy} = -\rho v_{FR} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial x} \right), \quad P_{zz} = \rho v_{FR} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial x} \right), \\ P_{xy} &= P_{yx} = -2\rho v_{FR} \left(\frac{\partial u}{\partial z} \right), \quad P_{xz} = P_{zx} = 2\rho v_{FR} \left(\frac{\partial u}{\partial z} \right), \\ P_{yz} &= P_{zy} = \rho v_{FR} \left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial x} \right), \end{aligned} \quad (17)$$

where $v_{FR} = \frac{a^2 \Omega}{4}$ represents the FLR correction, with a being the ion Larmor radius and Ω denoting the ion gyrofrequency.

3. Methodology adopted

To investigate the stability of a system, perturbations in physical variables are analysed in terms of modes by ascribing a horizontal wave number, depending on y , z and time (t), are supposed to vary as

$$f'(y, z, t) = f(z) \exp[i(ky + nt)], \quad (18)$$

where i is the imaginary unit, and the term $\exp[i(ky + nt)]$ represents a **wave-like disturbance** varying sinusoidally in space

(in the y -direction with the wave number k) and in time (with the growth rate n).

Using Eq. (18), Eqs. (11)–(15) transform to Cartesian components as:

$$ikv + Dw = 0, \quad (19)$$

$$\rho inv = -ikp' + \rho v_{FR}(D^2 - k^2)w + 2v_{FR}D\rho Dw + \frac{\mu}{1+\lambda}(D^2 - k^2)v + Q_{y1}, \quad (20)$$

$$\rho inw = -Dp' - g\rho' - v_{FR}D\rho(ikw + Dw) - \rho v_{FR}(D^2 - k^2)v + \frac{\mu}{1+\lambda}(D^2 - k^2)w + Q_{z1}, \quad (21)$$

$$in\rho' + wD\rho = 0, \quad (22)$$

$$h_x = h_y = h_z = 0, \quad (23)$$

where

$$Q_{y1} = \frac{k_y}{k_x} \frac{\hat{h}^2}{2\epsilon n m_e m_i} \left[\frac{1}{2} D\rho D^2 w + \left\{ D^2 \rho - \frac{1}{2\rho} (D^2 \rho)^2 \right\} Dw + \left\{ \frac{1}{2} D^3 \rho - \frac{1}{\rho} D\rho D^2 \rho - \frac{k^2}{2} D\rho + \frac{1}{2\rho^2} (D\rho)^3 \right\} w \right], \quad (24)$$

$$Q_{z1} = \frac{\hat{h}^2}{2\epsilon n m_e m_i} \left[\frac{1}{2} D\rho D^3 w + \left\{ \frac{3}{2} D^2 \rho - \frac{1}{\rho} (D^2 \rho)^2 \right\} D^2 w + \left\{ \frac{1}{2} D^3 \rho - \frac{1}{\rho} D\rho D^2 \rho - \frac{k^2}{2} D\rho + \frac{3}{2\rho^2} (D\rho)^3 \right\} Dw + \left\{ \frac{1}{2} D^4 \rho - \frac{1}{\rho} D\rho D^3 \rho - \frac{k^2}{2} D^2 \rho - \frac{1}{\rho} (D^2 \rho)^2 + \frac{5}{2\rho^2} (D\rho)^2 D^2 \rho + \frac{k^2}{2\rho} (D\rho)^2 - \frac{1}{\rho} (D\rho)^4 \right\} w \right]. \quad (25)$$

In the above, D stands for $\frac{d}{dz}$, ϵ is the medium porosity, and k_x and k_y are the horizontal and vertical components of the wave number, respectively.

On eliminating v , ρ' , and p' from Eq. (17) and using Eqs. (15)–(16) and Eqs. (18)–(21), we get the general differential equation for w as

$$\left[\rho n^2 + 2v_{FR}knD\rho + \frac{\mu kn}{1+\lambda}D\rho + k^2 A \right] D^2 w + [n^2 D\rho + 2v_{FR}knD^2 \rho + k^2 B] Dw - \left[\rho n^2 k^2 + \left(2v_{FR}kn + \frac{\mu kn}{1+\lambda} + g - C \right) k^2 D\rho \right] w = 0. \quad (26)$$

Here

$$A = -\frac{\hat{h}^2}{4m_e m_i} \frac{1}{\rho} (D\rho)^2, B = -\frac{\hat{h}^2}{4m_e m_i} \frac{1}{\rho} D\rho [(D\rho)^2 - 2\rho D^2 \rho],$$

$$C = -\frac{\hat{h}^2}{4m_e m_i} \frac{k^2}{\rho} D\rho.$$

The density of fluid is assumed to vary exponentially with respect to z and depends on L_D —a length scale. Thus, the density distribution at $z = 0$ is described as

$$\rho(z) = \rho \exp\left(\frac{z}{L_D}\right). \quad (27)$$

Equation (26), using Eq. (23), after algebraic simplification yields

$$\left[n^2 + \frac{2v_{FR}kn}{L_D} + \frac{vkn}{(1+\lambda)L_D} - n_q^2 k^2 \right] D^2 w + \left[n^2 + \frac{2v_{FR}kn}{L_D} + \frac{vkn}{(1+\lambda)L_D} - n_q^2 k^2 \right] Dw - k^2 \left[n^2 + \frac{2v_{FR}kn}{L_D} + \frac{vkn}{(1+\lambda)L_D} + \frac{g}{L_D} - n_q^2 k^2 \right] w = 0, \quad (28)$$

where $\nu = \frac{\mu}{\rho}$ and $n_q^2 = \frac{\hat{h}^2 k^2}{2\epsilon n m_e m_i}$ represent the kinematic viscosity and the quantum parameter accounting for quantum pressure.

The plasma fluid velocity diminishes at $z = 0$ and $z = d$, irrespective of the nature of the bounding surface. Therefore, the solution of Eq. (24) is taken as

$$w = w_1 \sin\left(\frac{m_1 \pi}{d} z\right) \exp(\lambda z), \quad (29)$$

where m_1 is a positive integer, $\lambda = \frac{1}{2L_D}$ and w_1 is constant.

Using the solutions given in Eq. (29), Eq. (28) gives

$$n^2 + \left(\frac{2v_{FR}k}{L_D} + \frac{vk}{(1+\lambda)L_D} \right) n - n_q^2 k^2 + \frac{4gk^2 d^2 L_D}{d^2 + 4m_1^2 \pi^2 L_D^2 + 4m_1^2 k^2 L_D^2} = 0. \quad (30)$$

Now introducing the non-dimensional variables

$$n^* = \frac{n_1}{n_{pe}}, \quad n_q^{*2} = \frac{\hat{h}^2}{4m_e m_i L_D^4 n_{pe}^2},$$

$$\nu^* = \frac{\nu}{L_D^2 n_{pe}^2}, \quad \nu_{FR}^* = \frac{\nu_{FR}}{L_D^2 n_{pe}^2},$$

$$d^{*2} = \frac{d^2}{L_D^2}, \quad k^{*2} = k^2 L_D^2, \quad g^* = \frac{g}{n_{pe} L_D}. \quad (31)$$

Equation (30) (the asterisks are omitted for the sake of convenience) transforms to

$$n^2 + \left(2\nu_{FR}k + \frac{vk}{1+\lambda} \right) n - n_q^2 k^2 + \frac{4gk^2 d^2}{d^2 + 4m_1^2 \pi^2 + 4d^2 k^2} = 0. \quad (32)$$

It is noteworthy from Eq. (32) that stability/instability of the Rayleigh-Taylor configuration is modified in the presence of FLR corrections, quantum pressure and Jeffrey parameter. Also, it is observed that RTI remains uninfluenced due to the presence of the magnetic field.

Substituting $n = n_r + in_i$ (n_r and n_i are real numbers) in Eq. (32) and separating real and imaginary parts, we get

$$(n_r^2 - n_i^2) - \left(2\nu_{FR}k + \frac{vk}{1+\lambda} \right) n_i - n_q^2 k^2 + \frac{4gk^2 d^2}{d^2 + 4m_1^2 \pi^2 + 4d^2 k^2} = 0 \quad (33)$$

and

$$2n_r n_i + \left(2\nu_{FR}k + \frac{vk}{1+\lambda} \right) n_r = 0,$$

which implies that $n_r = 0$.

Putting $n_r = 0$ in Eq. (32), one gets

$$n_i^2 + \left(2\nu_{FR}k + \frac{vk}{1+\lambda} \right) n_i + n_q^2 k^2 - \frac{4gk^2 d^2}{d^2 + 4m_1^2 \pi^2 + 4d^2 k^2} = 0, \quad (34)$$

which is the required relation between n and k to examine the Rayleigh-Taylor instability in a stratified non-Newtonian plasma

fluid with FLR corrections, quantum pressure and Jeffrey parameter.

3.1. Special cases

For non-viscous Newtonian fluid, i.e. $\nu = 0$, $\lambda = 0$, Eq. (34) condenses to

$$n_i^2 + 2v_{FR}kn_i + n_q^2k^2 - \frac{4gk^2d^2}{d^2 + 4m_1^2\pi^2 + 4d^2k^2} = 0, \quad (35)$$

which coincides with the earlier result of Tiwari et al. [23].

4. Numerical results and discussion

To investigate the influence of the Jeffrey parameter, FLR and quantum pressure on both classical and non-classical cases of stratified RTI, the growth rate frequency of the most unstable mode has been numerically computed from the dispersion relation given in Eq. (29), using Mathematica (version 12). The permissible experimental values of all the involved parameters used by Tiwari et al. [23] and Dey [35], and many others, are taken as

$v_{FR} = 0.5$, $n_q = 0.6$, $\lambda = 0.3$, $m_1 = 1$, $d = 1$ and $g = 10$, respectively. The growth rate (n_i) of RTI against wave number (k) is plotted to analyse the effect of distinct values of FLR correction, $v_{FR} = 0.3, 0.5, 0.9$ in Figs. 2 and 3 for both classical and quantum cases, respectively. The plots in Figs. 2 and 3 clearly show that increasing the FLR correction parameter leads to a notable reduction in the growth rate of RTI in a stratified plasma fluid. This suppression weakens the formation of Rayleigh-Taylor structures and significantly dampens the instability dynamics, particularly in dense plasma systems. Thus, the FLR correction plays a stabilising role by effectively mitigating RTI in both classical and non-classical regimes. Additionally, it is observed that the cut-off wave number k_{max} corresponding to the maximum growth rate shifts to lower values under the influence of quantum pressure, further contributing to the suppression of the instability.

Figures 4 and 5 illustrate the impact of distinct values of the non-dimensional quantum parameter accounting for quantum pressure, $n_q = 0.3, 0.5, 0.7$, on the growth rate of RTI versus wave number, with and without FLR corrections, respectively. The curves indicate a decrease in the frequency and growth rate of RTI with increasing quantum pressure. Consequently, both

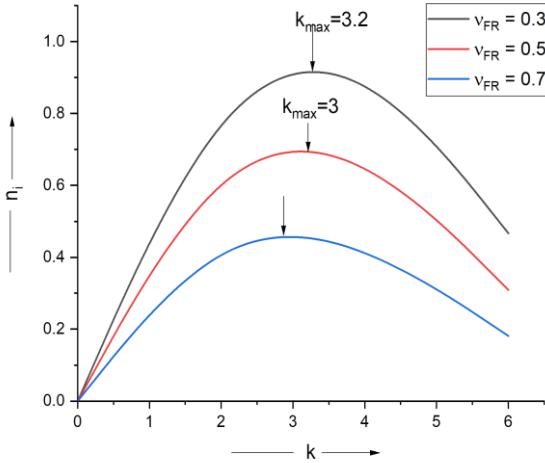


Fig. 2. Impact of FLR correction (v_{FR}) on the growth rate of RTI (n_i) against the wave number (k), with quantum pressure.

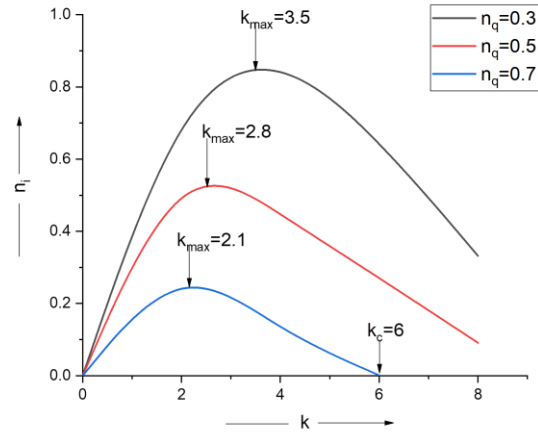


Fig. 4. Impact of the quantum pressure on the growth rate of RTI (n_i) against the wave number (k), with FLR correction.

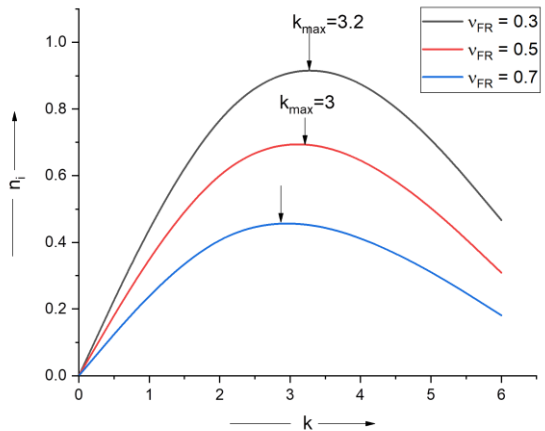


Fig. 3. Impact of FLR correction (v_{FR}) on the growth rate of RTI (n_i) against the wave number (k), with no quantum pressure.

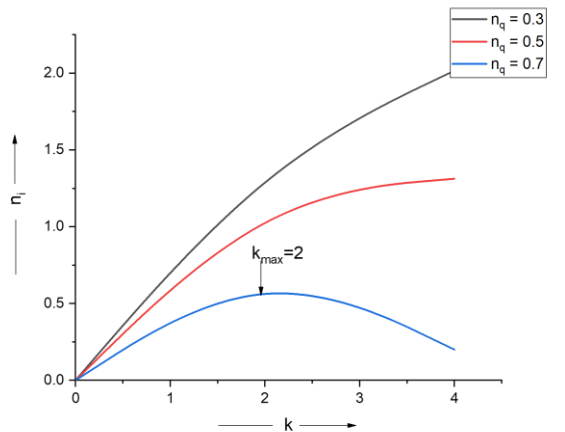


Fig. 5. Impact of the quantum pressure on the growth rate of RTI (n_i) against the wave number (k), with no FLR correction.

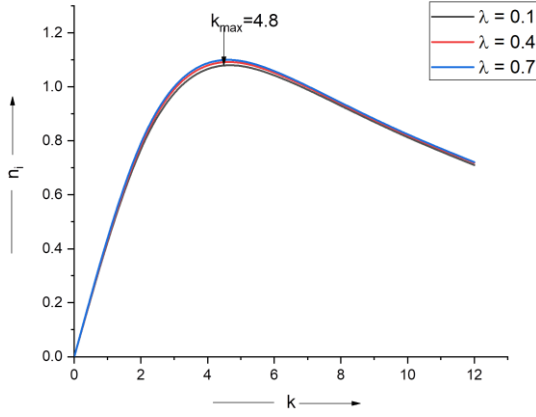


Fig. 6. Impact of the Jeffrey parameter (λ) on the growth rate of RTI (n_i) against the wave number (k), with no quantum pressure (classical case).

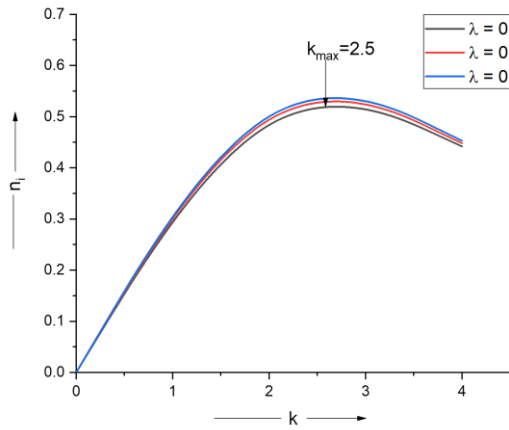


Fig. 7. Impact of the Jeffrey parameter (λ) on the growth rate of RTI (n_i) against the wave number (k), with quantum pressure.

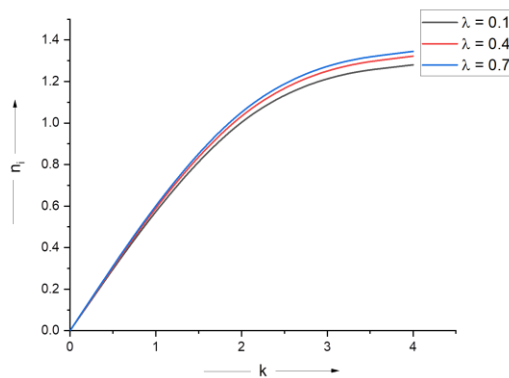


Fig. 8. Impact of the Jeffrey parameter on the growth rate (n_i) of RTI against the wave number (k), with quantum pressure and no FLR corrections.

stability occurs at $k_c = 6.0$ in lieu of both quantum pressure and FLR corrections.

In Figs. 6 and 7, the influence of various values of the Jeffrey parameter, $\lambda = 0.1, 0.4, 0.7$, on the growth rate of Rayleigh-Taylor instability for both quantum and classical cases, is displayed. The curves illustrate that the growth rate of RTI increases with the rise in the Jeffrey parameter. Hence, the Jeffrey parameter tends to promote the onset of RTI in stratified plasma fluid for both cases. However, it is visualised from the curves in Figs. 7 and 8 that the value of k_{max} gets decreased in the simultaneous presence of quantum pressure and FLR corrections.

Figure 8 illustrates the effect of the Jeffrey parameter on the growth rate of RTI in the presence of quantum pressure and the absence of FLR correction. The curves indicate that an increase in the Jeffrey parameter results in a higher amplitude of the RTI growth rate under the influence of quantum pressure. It is noteworthy to observe from the curves that k_{max} is not attained in the absence of FLR corrections.

Figures 9 and 10 depict the influence of kinematic viscosity on the growth rate of RTI, with and without the inclusion of the Jeffrey parameter, respectively. The plots reveal that kinematic viscosity reduces the instability region in both cases. However, k_{max} is higher for a non-Newtonian fluid than for a Newtonian

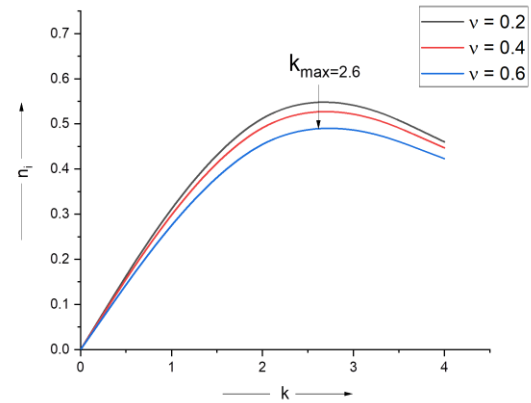


Fig. 9. Impact of kinematic viscosity (v) on the growth rate of RTI (n_i) against wave number (k), with Jeffrey parameter.

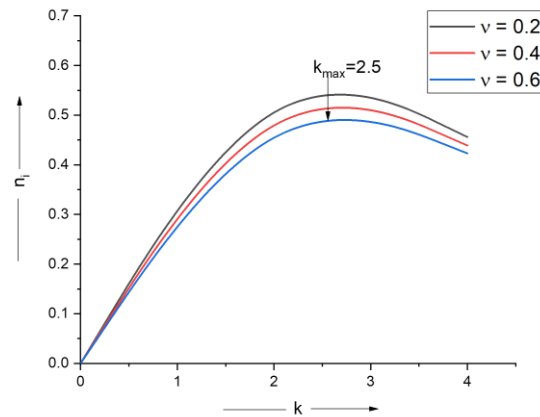


Fig. 10. Impact of the kinematic viscosity (v) on the growth rate of RTI (n_i) against the wave number (k) in the absence of Jeffrey parameter.

quantum pressure and FLR corrections demonstrate a stabilising influence by effectively suppressing the growth of instability in non-Newtonian stratified plasma fluids. It is noticed from Fig. 3 that the growth rate starts to decrease for $k > k_{max}$ and complete

fluid in the present case. These results are in strong agreement with the findings reported by various authors [18–23].

5. Conclusions

The combined effects of quantum pressure and FLR corrections on the RTI in a laminar Jeffrey model plasma fluid are investigated. An explicit dispersion relation for the RTI growth rate is derived, highlighting its dependence on various physical parameters.

- The classical Rayleigh-Taylor instability undergoes significant modification with the inclusion of finite Larmor radius corrections and quantum pressure, resulting in enhanced stability and effective suppression of Rayleigh-Taylor instability in the plasma fluid system.
- The Jeffrey parameter has a destabilising effect on the Rayleigh-Taylor instability of stratified plasma fluid.
- The viscosity of the plasma fluid enhances the stabilisation of the Rayleigh-Taylor instability configuration under the influence of finite Larmor radius corrections and quantum pressure, regardless of the presence of the Jeffrey parameter.
- An important facet of the present study is the demonstration that quantum pressure and finite Larmor radius corrections play a crucial role in various non-Newtonian astrophysical systems, particularly in inertial confinement fusion capsules and white dwarfs.
- The findings offer valuable insights into the non-Newtonian behaviour of astrophysical systems characterised by low temperatures and high densities, enhancing our understanding of their underlying dynamics.

Examining the effects of compressibility, magnetic shear and temperature gradients on the role of finite Larmor radius in the onset of Rayleigh-Taylor instability in stratified magnetised Jeffrey plasma could offer a deeper understanding of the interplay between kinetic effects and macroscopic plasma parameters, thereby improving stability predictions in both astrophysical and laboratory settings.

References

- [1] Chandrasekhar, S. (1961). *Hydrodynamic and hydromagnetic stability*. Oxford University Press, Oxford.
- [2] Goldston, R.J., & Rutherford, P.H. (1997). *Introduction to plasma physics*. Institute of Physics, London.
- [3] Jung, Y.-D. (2001). Quantum-mechanical effects on electron-electron scattering in dense high-temperature plasmas. *Physics of Plasmas*, 8, 3842–3844. doi: 10.1063/1.1386430
- [4] Gardner, C.L. (1994). The quantum hydrodynamic model for semiconductor devices. *SIAM Journal on Applied Mathematics*, 54(2), 409–427. <https://www.jstor.org/stable/2102225>
- [5] Haas, F. (2005). A magnetohydrodynamic model for quantum plasma. *Physics of Plasmas*, 12, 062117. doi: 10.1063/1.1939947
- [6] Zaitseva, E., Levashenko, V., & Matiaško, K. (2008). Decomposition and estimation of multi-state systems by dynamic reliability indices. *Journal of KONBiN*, 4(1), 53–361. doi: 10.2478/v10040-008-0026-6
- [7] Bychkov, V., Marklund, M., & Modestov, M. (2008). The Rayleigh-Taylor instability and internal waves in quantum plasmas. *Physics Letters A*, 372(17), 3042–3045. doi:10.1016/j.physleta.2007.12.065
- [8] Sharma, V., Kumari, R., Gupta, S., & Shyam, R. (2017). Stability of stratified viscoelastic Walters' (model B') fluid/plasma in the presence of quantum pressure. *Research Journal of Science and Technology*, 9(1), 201–206. doi: 10.5958/2349-2988.2017.00034.1
- [9] Hoshoudy, G.A. (2011). Rayleigh-Taylor instability in quantum magnetized viscous plasma. *Plasma Physics Reports*, 37, 775–784. doi: 10.1134/S1063780X11080046
- [10] Hoshoudy, G.A. (2012). External magnetic field effects on the Rayleigh-Taylor instability in an inhomogeneous rotating quantum plasma. *Journal of Modern Physics*, 3, 1792–1801. doi: 10.4236/jmp.2012.311224
- [11] Hoshoudy, G.A. (2014). Rayleigh-Taylor instability in magnetized plasma. *World Journal of Mechanics*, 4, 260–272. doi: 10.4236/wjm.2014.48027
- [12] Hoshoudy, G.A., & Awasthi, M.K. (2020). Compressibility effects on the Kelvin-Helmholtz and Rayleigh-Taylor instabilities between two immiscible fluids flowing through a porous medium. *The European Physical Journal Plus*, 135, 169. doi: 10.1140/epjp/s13360-020-00160-x
- [13] Hoshoudy, G.A., & Prajapati, R.P. (2016). Quantum effects on the Rayleigh-Taylor instability of stratified plasma in the presence of suspended particles. *Pramana – Journal of Physics*, 87, 99. doi: 10.1007/s12043-016-1297-4
- [14] Sharma, V., & Gupta, U. (2010). Stability of stratified elasto-viscous Walters' (model B') fluid in the presence of horizontal magnetic field and rotation. *Studia Geotechnica et Mechanica*, 32(2), 41–54.
- [15] Dolai, B., & Prajapati, R.P. (2018). The rotating Rayleigh-Taylor instability in a strongly coupled dusty plasma. *Physics of Plasmas*, 25(8), 083708. doi:10.1063/1.5041088
- [16] Rosenbluth, M.N., Krall, N.A., & Rostoker, N. (1961). *Finite Larmor radius stabilization of 'weakly' unstable confined plasmas*. *Nuclear Fusion, Supplement*, GA-2371.
- [17] Roberts, K.V., & Taylor, J.B. (1962) Magnetohydrodynamic equations for finite Larmor radius. *Physical Review Letters*, 8, 197. doi: 10.1103/PhysRevLett.8.197
- [18] Jukes, J.D. (1964). Gravitational resistive instabilities in plasma with finite Larmor radius. *Physics of Fluids*, 7(1), 52–58. doi: 10.1063/1.1711054
- [19] Surindar, S., & Harikrishan, H. (1966). Magnetic viscosity and the stability of superposed fluids in a magnetic field. *Nuclear Fusion*, 6(1), 6. doi: 10.1088/0029-5515/6/1/002
- [20] Sanghvi, R.K., & Chhajlani, R.K. (1987). Finite Larmor-radius effects on Rayleigh-Taylor instability of a dusty magnetized plasma. *Astrophysics and Space Science*, 132, 57–64. doi: 10.1007/BF00637781
- [21] Sharma, P.K., & Chhajlani, R.K. (1998). Effect of rotation on the Rayleigh-Taylor instability of two superposed magnetized conducting plasma. *Physics of Plasmas*, 5(6), 2203–2209. doi: 10.1063/1.872893
- [22] Sayed, A.H. (2010). *Fundamentals of adaptive filtering*. Wiley India, Delhi.
- [23] Tiwari, A., Argal, S., Khan, N., & Sharma, P.K. (2018). Quantum and FLR effects on the Rayleigh Taylor instability of stratified plasmas. *Physics of Plasmas*, 25(1), 012109. doi: 10.1063/5.0057762
- [24] Sun, Y.B., Gou, J.N., & Wang, C. (2021). Combined effects of viscosity and a vertical magnetic field on Rayleigh-Taylor instability. *Physics of Plasmas*, 28(9), 092707. doi: 10.1063/5.0057762
- [25] Joseph, D.D., & Preziosi, L. (1989). Heat waves. *Reviews of Modern Physics*, 61(1), 41–73. doi: 10.1103/RevModPhys.61.41
- [26] Yadav, D. (2021). Influence of anisotropy on the Jeffrey fluid convection in a horizontal rotary porous layer. *Heat Transfer*, 50(5), 4595–4606. doi: 10.1002/htj.22090

- [27] Yadav, D. (2022). Effect of electric field on the onset of Jeffery fluid convection in a heat-generating porous medium layer. *Pramana – Journal of Physics*, 96, 19 doi: 10.1007/s12043-021-02242-6
- [28] Yadav, D. (2022). Thermal non-equilibrium effects on the instability mechanism in a non-Newtonian Jeffrey fluid saturated porous layer. *Journal of Porous Media*, 25(2), 1–12. doi: 10.1615/JPorMedia.2021038392
- [29] Gautam, P.K., Rana, G.C., & Saxena, H. (2020). Stationary convection in the electrohydrodynamic thermal instability of Jeffrey nanofluid layer saturating a porous medium: free-free, rigid-free, and rigid-rigid boundary conditions. *Journal of Porous Media*, 23(11), 1043–1063. doi: 10.1615/JPorMedia.2020035061
- [30] Sharma, V., Thakur, A., Devi, J., & Rana, G.C. (2023). Rayleigh-Taylor instability of superposed dusty Jeffrey fluids through porous medium with interfacial surface tension. *Acta Physica Polonica A*, 143(3), 238–245. doi: 10.12693/APhysPolA.143.238
- [31] Yadav, D., Mohamad, A.A., & Awasthi, M.K. (2021). The Horton-Rogers-Lapwood problem in a Jeffrey fluid influenced by a vertical magnetic field. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*. 235(6), 2119–2128. doi: 10.1177/09544089211031108
- [32] Yadav, D., Al-Balushi, S., Awasthi, M.K., Al-Hadi, T., Al-Abri, R., Al-Wahaibi, J., Al-Nasseri, F., Al-Siyabi, S., Ragoju, R., & Bhattacharyya, K. (2023). Convective flow of ethylene glycol-silver Jeffery nanofluid in a Hele-Shaw cell with an influence of external magnetic field. *Asia-Pacific Journal of Chemical Engineering*, 18(3), 2884. doi: 10.1002/apj.2884
- [33] Prajapati, R.P. (2016). Rayleigh-Taylor instability in non-uniform magnetized rotating strongly coupled viscoelastic fluid. *Physics of Plasmas*, 23(2), 022106. doi: 10.1063/1.4941593
- [34] Garai, S., Ghose-Choudhury, A., & Guha, P. (2020). Rayleigh Taylor like instability in presence of shear velocity in a strongly coupled quantum plasma. *Physica Scripta*, 95(10), 105605. doi: 10.1088/1402-4896/abb697
- [35] Dey, D. (2018). Dusty Jeffrey fluid flow in a rotating system with volume fraction and Hall effect: An analytical approach. *Advances in Modelling and Analysis A*, 55(2), 70–75. doi: 10.18280/ama_a.550205
- [36] Garai, S., Banerjee, D., Janaki, M.S., & Chakrabarti, N. (2015). Stabilization of Rayleigh-Taylor instability in a non-Newtonian incompressible complex plasma. *Physics of Plasmas*, 22, 033702. doi: 10.1063/1.4916126
- [37] Adak, A., Ghosh, S., & Chakrabarti, N. (2024). Rayleigh-Taylor instability in an equal mass plasma. *Physics of Plasmas*, 21, 09120. doi: 10.1063/1.4896714
- [38] Das, A., Dharodi, V., & Tiwari, S. (2014). Collective dynamics in strongly coupled dusty plasma medium. *Journal of Plasma Physics*, 80(6), 855–861. doi: 10.1017/S0022377814000506
- [39] Dharodi, V.S., & Das, A. (2021). A numerical study of gravity-driven instability in strongly coupled dusty plasma. Part 1. Rayleigh-Taylor instability and buoyancy-driven instability. *Journal of Plasma Physics* 87(2), 905870216. doi: 10.1017/S0022377821000349
- [40] Dharodi, V.S. (2021). A numerical study of gravity-driven instability in strongly coupled dusty plasma. Part 2. Hetero-interactions between a rising bubble and a falling droplet. *Journal of Plasma Physics*, 87(4), 905870402. doi: 10.1017/S0022377821000684
- [41] Dharodi, V.S. (2024). A numerical study of gravity-driven instability in strongly coupled dusty plasma. Part 3. Homo-interaction between a pair of rising/falling bubbles/droplets. *Journal of Plasma Physics*, 90(4), 905900408. doi: 10.1017/S0022377824001144



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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Comparative Analysis of Cylinder-to-Cylinder Variation of Parameters of an Automotive CRDI Engine

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Received: 07.05.2025; revised: 09.09.2025; accepted: 12.09.2025

Abstract

In the present work, an automotive three cylinder compression ignition engine employed with a common rail direct injection system, fuelled with diesel was investigated to analyse its performance and emission characteristics, and also cylinder-to-cylinder variation of important parameters of combustion. Variation of cylinder pressure, peak cycle pressure, rate of maximum pressure rise, rate of maximum heat release and combustion duration between the cylinders was presented as a function of brake mean effective pressure, torque and speed of the engine. Variation of carbon monoxide, unburnt hydrocarbon, oxides of nitrogen and brake thermal efficiency for the tested engine was presented as a function of brake mean effective pressure. It can be seen that measured combustion parameters show a significant variation between the cylinders, and the variation of the magnitude of average cylinder pressure was marginal. The peak pressure and combustion duration in cylinder 2 are higher by approx. 11% and 9%, respectively, than those of the other two cylinders. At a 40 Nm torque and at the lowest operated speed, the peak pressure in cylinder 2 is higher by 10% than that of the other two cylinders. This trend was also observed at the highest operated speed, with the peak pressure higher by approx. 14%. The maximum combustion duration in cylinder 2 was observed to be 9% higher than in the other two cylinders at identical comparable brake mean effective pressure values. It can also be seen that emission parameters were varied considerably with brake mean effective pressure and a trade-off point was obtained between NO_x -CO, NO_x -UBHC and NO_x -BTE.

Keywords: Common rail direct injection; Cylinder to cylinder; Emission; Combustion; Torque; Speed

Vol. 46(2025), No. 4, 105–115; doi: 10.24425/ather.2025.156841

Cite this manuscript as: Subramani, S., Kumar, P.D., Govindasamy, R., & Iyamperumal, M.S. (2025). Comparative Analysis of Cylinder-to-Cylinder Variation of Parameters of an Automotive CRDI Engine. *Archives of Thermodynamics*, 46(4), 105–115.

1. Introduction

Compression ignition (CI) engines are very common in medium and heavy duty automotive vehicles due to their high thermal efficiency, durability and robustness in nature. In a conventional direct injection (DI) multi-cylinder CI engine, it is very difficult to attain identical combustion in all the engine cylinders. Engine performance and emission with respect to speed and load are hampered by uneven combustion performance of the worst cylinders, which leads to misfiring and too early combustion phas-

ing. The temperature of the inlet air, fuel supply, fuel injection timing, injection delay, ignition delay, air-fuel ratio, compression ratio, port geometry, gas exchange, exhaust gas recirculation supply, and cylinder cooling are the most important factors which can differ from cylinder to cylinder. The difference in any or in combinations of these factors is responsible for load variations and combustion phasing between the cylinders, which in turn may lead to power loss, increased mechanical vibration, cylinder-to-cylinder torque imbalance and engine emissions [1–3].

Nomenclature

CA – crank angle, °
 p – pressure, bar
 S – estimated factor (governed by different self-governing variables)
 $x_1, x_2, x_3, \dots, x_n$ – variables

Abbreviations and Acronyms

BMEP – brake mean effective pressure
 BTE – brake thermal efficiency
 CD – combustion duration

CI – compression ignition
 CO – carbon monoxide
 CRDI – common rail direct injection (system)
 DI – direct injection
 ECU – electronic control unit
 MHRR – maximum heat release rate
 MROPR – maximum rate of pressure rise
 NO_x – oxides of nitrogen
 SOC – start of combustion
 TDC – top dead centre
 UBHC – unburnt hydrocarbon

To save the environment and to improve the fuel economy, very stringent emission norms were developed, which cannot be attained by a conventional mechanical fuel injection system. This paved the way for the development of a high pressure injection system called common rail direct injection (CRDI) system [4,5]. In modern CI engines, an accumulator, simply called as common rail, is the main component in the CRDI system, which accumulates the fuel at high pressure in the range of 1000 bar to 2000 bar, and the fuel is injected into the cylinders through the respective fuel injectors. Injecting the fuel at higher pressure makes the fuel droplets to travel a longer distance inside the cylinder by penetrating the hot compressed air, and also atomises the fuel droplets into very fine particles. These two factors make the fuel vaporise at a much faster rate, which thereby improves the fuel-air mixing, and in turn leads to better combustion characteristics. In the CRDI system, by regulating the accumulator pressure, the fuel injection pressure can be controlled.

Since the fuel injection in the CRDI system is monitored by an electronic control unit (ECU), the required fuel quantity can be supplied at an exact time by either a single injection or multiple injections. Injecting a small quantity of fuel in advance and a subsequent injection to the regular injection refers to as a pilot and post injection, respectively. Pilot injection reduces the combustion noise by the gradual progress of combustion reactions in the cylinder [6,7]. The residence time of the pilot injected fuel's vapour phase will be comparatively longer, which allows the proper mixing with the air trapped in the cylinder. The pilot injected fuel burns after undergoing the pre-combustion reactions, resulting in greater cylinder temperature, so that the primary injected fuel begins to combust after causing a short delay period. This results in a reduced maximum rate at which the pressure increases, resulting in lower combustion noise. Hence, an overwhelming interest was created among the researchers and scientists to investigate the combustion features of the CRDI engine.

Cyclic and cylinder variations can be effectively controlled by suitably adjusting the injection timing and amount of exhaust gas recirculation (EGR). Many researchers have analysed the diesel engine characteristics with diesel and its alternatives [8–13]. Attempts were made on a single cylinder CI engine with different vegetable oils like karanja [14], jatropha [15], sunflower [16], cottonseed [17], rape seed [18], palm [19], rice bran [20], corn [21] and neem [22] in their blended forms with diesel and also in the converted forms of biodiesel.

It was inferred that vegetable oils in their pure forms and their derivatives have the capability to be used as a partial repla-

cement of diesel in CI engines without any engine modifications, and the performance characteristics were comparable with those of diesel.

Attempts were also made to explore the outcome of changing engine design considerations on the engine characteristics with diesel and its alternatives [23–25]. In all these investigations, a single cylinder stationary engine [26–29] or an automotive engine employing mechanical fuel injection [30–35] was utilised. The CRDI concept was also attempted on a one cylinder stationary CI engine by increasing the fuel injection pressure [36–39]. Investigations have been carried out by modifying a single cylinder engine with the CRDI setup [38,39].

In all earlier research on the CRDI diesel engine, the concept of injecting fuel at a relatively higher pressure than the conventional injection pressure was attempted [36–45] and an actual automotive engine was not tested to analyse its performance, emission and combustion characteristics. Investigations carried out with automotive engines were also restricted to constant speed, and fuel injection characteristics were not controlled by ECU. To understand the behaviour of the CRDI engine, it is necessary to test a real time automotive engine to analyse its characteristics. In the earlier work, the authors have compared the combustion characteristics of an automotive CRDI engine with a conventional high-power compression ignition (HCV) engine through the measurement of combustion parameters in the first cylinder of the engine [46].

After analysing a wide range of literature, a certain gap was observed in the area of CRDI engine where the investigations of variations among cylinders of an automotive CRDI engine were not sufficiently addressed in the previous research works. Most of the reported works on multi-cylinder engines were restricted to investigate the characteristics of the engines through the measurement of combustion parameters of one of the cylinders.

Hence, to overcome these limitations, an attempt was made on an automotive passenger car CRDI engine to study the variation of combustion characteristics among the cylinders by measuring the cylinder pressure of all three cylinders using a separate pressure transducer for individual cylinders. The attempt was also extended to study the performance and emission characteristics of the engine at different operating conditions. The key objectives of this investigation are outlined as:

- to analyse the cylinder-to-cylinder variation of different combustion parameters of an automotive CRDI engine by operating the engine at different speeds and torque values,

- to investigate the performance and emission characteristics of an automotive CRDI engine with respect to different brake mean effective pressures (BMEP).

2. Experimental methodology

2.1. Experimental setup

An automotive, four stroke, three cylinder CRDI diesel engine manufactured for use in a passenger car was tested in the present investigation. Table 1 shows the details of the three cylinder engine used for testing, and the line diagram seen in Fig. 1 displays the setup used to conduct the experiments.

Table 1. Specifications of engine.

Arrangement	Inline, 3 cylinder
Volume (l)	0.98
Bore x stroke (mm)	73 x 78
Rated power (kW)	40 kW @ 4000 rpm
Peak torque (Nm)	130 Nm @ 1750–2750 rpm
Fuel system	unit pump common rail, 1600 bar system
Engine aspiration	turbocharged and cooled

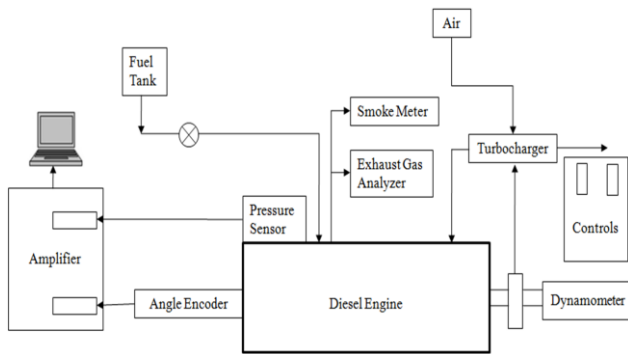


Fig. 1. Layout of experimental setup.

A dry gas type eddy current dynamometer ECB-120 HP was attached to the test engine as a loading device. Each of the three cylinders is fitted with a separate pressure transducer to record the pressure generated inside the individual cylinder and instrumented with an angle encoder to note down the pressure inside the cylinder at various crank angles. The pressure signals were amplified by their respective charge amplifier and then fed to an AVL data acquisition system to interpret the recorded in-cylinder pressure and stored in a personal computer. An AVL exhaust gas analyser and AVL smoke meter were employed in engine emission measurements. Table 2 shows the range, accuracy and percentage uncertainties of the instruments used for emission measurement.

2.2. Test conditions

As per the standards of the modified Indian duty cycle, the small sized passenger cars were allowed to operate around 50 per cent of the maximum rated speed and torque to simulate the normal driving conditions and further lower range for city driving con-

ditions. Hence, the test conditions of the engine were selected to represent the same. The test matrix selected for the investigation is shown in Table 3, and Table 4 shows the properties of diesel.

Table 2. Range, accuracy and percentage uncertainties of instruments.

Instrument	Measured quantity	Range	Accuracy	Uncertainties, %
AVL DIGAS 444 Exhaust gas analyzer	NO _x	0–5000 ppm	±10 ppm	0.2
	UBHC	0–20000 ppm n-hexane	±50 ppm	
	CO	0–20%	±5%	
AVL 437C Smoke meter	Smoke opacity	0–100 %	±2 %	1.0

Table 3. Test matrix.

S. No.	Speed (rpm)	Torque (Nm)
1	1850	65
2	2000	30
3	1550	40
4	1300	45
5	1850	50
6	1000	55
7	1250	60
8	1750	65
9	1050	40
10	1100	40
11	1250	40
12	2000	40
13	1250	30
14	1250	45
15	1250	50
16	1250	60

Table 4. Properties of diesel [46].

Property of fuel	Method	Diesel
Viscosity at 40°C (mm ² /sec)	ASTM D445	3.522
Flash point (°C)	ASTM D93	70
Calorific value (kJ/kg)	ASTM D240	43000
Distillation temperature T90 (°C)	ASTM D86	335
Specific gravity	ASTM D4052	0.8

To explore the deviation of combustion parameters as an element of BMEP, the engine was tested at different operating conditions by varying both the engine speed and torque as specified in Table 3. To investigate the effect of speed and torque on combustion parameters, tests were conducted by maintaining the engine torque at 40 Nm for different speeds and also by changing the torque at a uniform speed of 1250 rpm as shown in Table 3.

2.3. Test procedure

In order to stabilise the engine operating conditions, the test engine was operated for a short span of time at idle speed with no

load condition. After the engine was stabilised, it was operated at different operating conditions as specified in the test matrix given in Table 3. The test engine was also made to run at different speeds by maintaining a steady torque of 40 Nm and dissimilar torques by retaining an identical speed of 1250 rpm. At each operating condition, performance, emission and combustion parameters were documented and analysed.

2.4. Uncertainty analysis

The errors linked with the measurements of different combustion parameters were calculated. The maximum potential errors in the measurement of pressure and crank angle were predicted by employing the technique recommended by Moffat [47]. From the least measured values and the measuring instrument's accuracy, the possible errors were calculated.

If an estimated quantity S depends on multiple independent variables ($x_1, x_2, x_3, \dots, x_n$), the associated error in S can be expressed as shown in Eq. (1):

$$\frac{\partial S}{S} = \left\{ \left(\frac{\partial x_1}{x_1} \right)^2 + \left(\frac{\partial x_2}{x_2} \right)^2 + \dots + \left(\frac{\partial x_n}{x_n} \right)^2 \right\}^{\frac{1}{2}}, \quad (1)$$

where $(\partial x_1/x_1)$, $(\partial x_2/x_2)$, etc. are the errors in the independent variables, ∂x_1 depicts the accuracy of the measuring instrument, and x_1 depicts the least measured output value.

2.4.1. Calculation of errors in measured parameters

The pressure inside the combustion chamber was measured by commissioning a pressure transducer, and the electrical signal received was amplified with the help of a charge amplifier. The crank angle degree was measured with the help of a crank angle encoder. The accuracy of the pressure measurement instrument employed is 0.01, and that of the crank angle measurement is 0.02. Hence, the values of maximum possible errors for pressure and crank angle measurements are calculated as shown below:

Cylinder pressure

The minimum value of cylinder pressure measured while conducting the investigations is 0.72 bar with an accuracy of 0.01. Therefore, the maximum possible error during the measurement of pressure is calculated using Eq. (2):

$$\left(\frac{\partial p}{p} \right)_{EXP} = \left(\left(\frac{0.01}{0.72} \right)^2 \right)^{\frac{1}{2}} \cdot 100 = 1.38\%. \quad (2)$$

Crank angle

The minimum degree of measured crank angle during the experimentation is 1 degree, and the measurement accuracy is 0.02. Therefore, the calculated maximum possible error in the measurement of crank angle is calculated using Eq. (3):

$$\left(\frac{\partial CA}{CA} \right)_{EXP} = \left(\frac{0.02}{1} \right) \cdot 100 = 2\%. \quad (3)$$

3. Results and discussion

3.1. Cylinder pressure and heat release rate

Figures 2 and 3, respectively, display the deviation of pressure inside the cylinder and the variation of heat release rate for all the three cylinders of the test engine based on crank angle degree ($^{\circ}\text{CA}$) for a total range of 60°CA (-30°CA to 30°CA with respect to top dead centre (TDC) position) for an operating speed of 1850 rpm and torque 65 Nm.

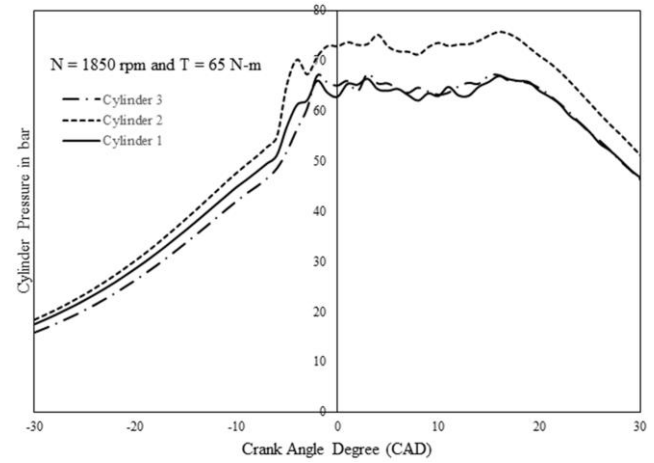


Fig. 2. Variation of cylinder pressure.

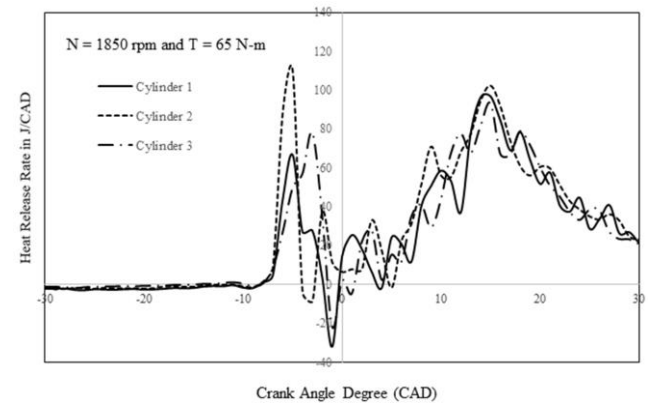


Fig. 3. Variation of heat release rate.

From Fig. 2, it can be perceived that the magnitude of cylinder pressure increases steadily for all three cylinders as the piston moves towards TDC, and the pressure of cylinder 1 lies between those of cylinder 2 and cylinder 3. Until the piston reaches 6°CA before TDC (bTDC), the variation in pressure between cylinder 1 and cylinder 2 is marginal (cylinder 1 has a higher value), and the variation in pressure between cylinder 1 and cylinder 3 is also around 3 bar (cylinder 3 has a higher value). At 5°CA bTDC, the pressure in cylinder 2 increases steeply, and its variation with other cylinders increases, and almost the same difference is maintained throughout the remaining cycle as shown in Fig. 2. This may be attributed to the marginal variation of the injected fuel quantity and its compression, which may result in a higher pressure for cylinder 2 when compared with the

other cylinders. It was also observed that the peak pressure for all three cylinders occurred at 16°CA after TDC (aTDC).

From Fig. 2, it can also be seen that after the completion of ignition delay, in-cylinder pressure variation between the different phases of combustion was marginal. It can also be seen that the difference between the magnitude of average cylinder pressure (measured during the uncontrolled and controlled combustion phase) and peak pressure was also marginal. This may be due to the influence of pilot fuel injection employed in the test engine. As the injection is carried out in two phases as pilot injection and main injection, the peak pressure is occurred at two positions, one bTDC due to pilot injection and the other aTDC due to main injection and the former was lower than the latter. This may be due to the fact that more fuel was injected through the main injection when compared with that of the pilot injection, and the injected fuel will be burned immediately after entering the combustion space, which leads to a larger peak pressure aTDC. Higher pressure of fuel injection in the test engine enhances the fuel atomisation and vaporisation by providing smaller fuel particles, which in turn improves the combustion process. This results in smooth combustion during pilot and main injections, which leads to a marginal variation in pressure during the combustion.

From Fig. 3, it can be witnessed that after the completion of the delay period, the rate of heat release for all cylinders was higher due to the combustion of the accumulated charge through pilot injection. The maximum heat release due to pilot injection was obtained at 5°CA bTDC for cylinder 1 and cylinder 2, and at 3°CA bTDC for cylinder 3 whose magnitude of heat release lies between that of cylinder 1 and cylinder 2. The maximum heat released at this condition was obtained in cylinder 2. This may also be due to the increased charge pressure as a result of a marginal increase in compression of injected fuel, which may improve the combustion and, in turn, increase the combustion temperature. Likewise, the heat release rate increased in the course of the expansion stroke, and the maximum heat release obtained in all three cylinders occurred at 15°CA aTDC, the value for cylinder 2 being the highest and the value for cylinder 1 lying between those of cylinder 3 and cylinder 2. As discussed earlier, the volume of the main injected fuel is larger, which leads to a greater heat release rate than that obtained during the pilot injection period. A negative heat release rate is also observed at a location close to that of TDC (bTDC). This is due to the delay in the start of combustion of injected fuel during the primary injection, which absorbed the heat generated due to combustion of the pilot injection.

3.2. Combustion parameters as a function of BMEP

In this section, the variation of different combustion parameters is presented as a function of BMEP. Torque and speed of the engine given in Table 3 were chosen to represent the combustion parameters as a function of BMEP. The characteristics of the pilot injection and main injection at different operating conditions employed in the engine are depicted in Table 5.

Injecting a small portion of fuel as pilot injection will drastically reduce the combustion noise, combustion temperature, and thereby reduce the formation of nitrogen oxides. Table 5 clearly

shows that 10 per cent of the fuel was injected through pilot injection and the remaining 90 per cent of fuel was injected through main injection. It also shows that the start of pilot injection occurred between 7.5°CA bTDC and 10°CA bTDC, and the main injection occurred very close to TDC. The fuel injection timing and quantity vary as the engine operates at different speeds and torques, which is also presented in Table 5.

Table 5. Characteristics of pilot injection and main injection.

Speed (rpm)	Torque (Nm)	Pilot injection			Main injection		
		Start of injection	Fuel (mg/str.)	Inj. pulse duration (μs)	Start of injection	Fuel (mg/str.)	Inj. pulse duration (μs)
2000	30	-8.2	0.682	29	1.74	6.827	287
1550	40	-9.05	0.797	35	1.01	7.972	345
1300	45	-9.3	0.954	42	0.76	9.542	415
1850	50	-7.8	1.065	39	2.3	10.65	387
1000	55	-9.32	1.216	52	0.69	12.17	524
1250	60	-9.3	1.190	46	0.69	11.91	463
1750	65	-8.02	1.296	43	1.88	12.96	432
1050	40	-9.2	0.769	39	0.69	7.696	386
1100	40	-9.2	0.780	38	0.69	7.801	384
1250	40	-9.3	0.811	38	0.69	8.116	381
2000	40	-7.5	0.905	35	2.54	9.058	347
1250	30	-10	0.539	28	0.03	5.391	279
1250	45	-9.2	0.979	43	0.69	9.795	430
1250	50	-0.93	1.167	48	0.69	11.67	484
1250	60	-9.2	1.422	54	0.69	14.22	538

The variation of peak pressure is shown in Fig. 4(a) as a function of BMEP for all three cylinders. It is found that the peak pressure for all cylinders increases with an increase in BMEP. This might be the effect of increased volume of charge burned, releasing more heat, which in turn increases the cylinder pressure and therefore results in a higher peak pressure as BMEP increases. Engine operation was also found to be smoother for all three cylinders as a result of the peak pressure varying almost linearly with BMEP.

Variation of the maximum rate of pressure rise (MROPR) as a function of BMEP is shown in Fig. 4(b). It is observed that the rate of pressure rise shows an erratic trend with BMEP. As the start of combustion is advanced with BMEP, the delay period will be shorter, resulting in lower MROPR at higher BMEP. An increase in MROPR with BMEP may be due to the change in the start of combustion with BMEP.

Figure 4(c) displays the manner in which the start of combustion (SOC) varies with BMEP for cylinder 1, cylinder 2 and cylinder 3. It is observed that with an increase in BMEP, SOC advances. The earlier SOC at a higher BMEP may be due to an increase in the fuel quantity burned and combustion chamber temperature, which in turn enriches fuel vaporisation and shortens the ignition delay period as BMEP increases. The same trend is observed for all three cylinders. It is also seen that at lower BMEP, SOC of cylinder 2 lies between those of cylinder 1 and

cylinder 3, but as BMEP increases, SOC of cylinder 2 is slightly advanced than for the other two cylinders. A delay in SOC is also observed at a higher BMEP for all three cylinders. This may

be due to the change in the pilot injection period with respect to the engine operating conditions.

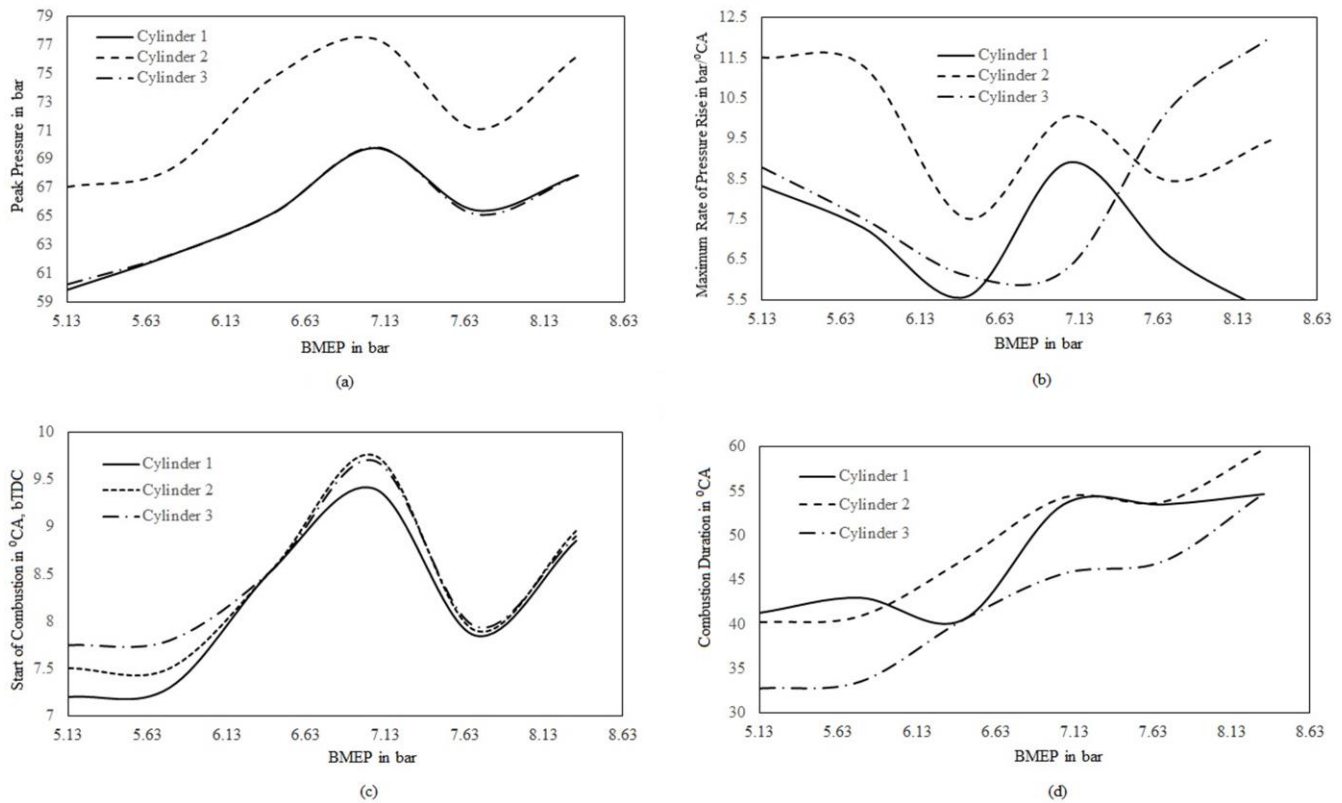


Fig. 4. Variation of combustion parameters with BMEP.

Figure 4(d) displays the deviation of combustion duration (CD) with respect to BMEP. It is inferred that an increase in the value of BMEP escalates CD. Cylinder 2 and cylinder 3 show a similar trend, and cylinder 2 shows a marginally longer CD than the other cylinders. This may be the effect of injecting a higher volume of fuel as BMEP is increased, which in turn may increase the period of fuel burning. Cylinder 1 deviates from the other two cylinders, showing an erratic trend.

3.3. Influence of speed and torque on combustion parameters

The influence of engine speed and torque on engine combustion parameters like SOC, CD, peak cylinder pressure, maximum heat release rate 1 (MHRR1) due to secondary injection, maximum heat release rate 2 (MHRR2) due to main injection and maximum rate of pressure rise (MROPR) are presented and discussed in this chapter. The combustion parameters are recorded and investigated at two different engine running conditions:

- (a) different torques at a uniform speed, 1250 rpm,
- (b) different speeds at a uniform torque, 40 Nm.

3.3.1. Influence of speed on combustion parameters

Here, the variation of combustion features as a function of speed for all three cylinders is presented and discussed. It is observed in Fig. 5(a) that the cylinder peak pressure is less affected for the

individual cylinders when the engine is operated at different speeds by maintaining a constant torque, but the peak pressure of cylinder 2 is marginally higher than those of cylinder 1 and cylinder 3. From Fig. 5(b), it is observed that MROPR is increased with respect to speed till it attains its maximum value and after that, MROPR is found to be decreased at higher speeds. The same trend is followed for all three cylinders, but for cylinder 2, MROPR at lower speeds is much higher than for the other cylinders.

Figure 5(c) shows that as the engine speed increases, SOC is slightly advanced in order to provide adequate time for complete combustion. This may be due to the fact that the combustion duration remains the same in terms of milliseconds, but it slightly increases in terms of crank angle degree, and hence SOC is advanced at higher speeds to compensate for this effect. All three cylinders follow a similar trend. The deviation of combustion duration for different speeds is shown in Fig. 5(d). It is observed that at moderate speeds, the combustion duration is slightly lower, but the duration of combustion is found to increase when there is a rise in the engine speed, and the same trend is followed for all three cylinders.

3.3.2. Effect of torque on combustion parameters

The variation of combustion parameters as a function of torque for all three cylinders is presented in Fig. 6.

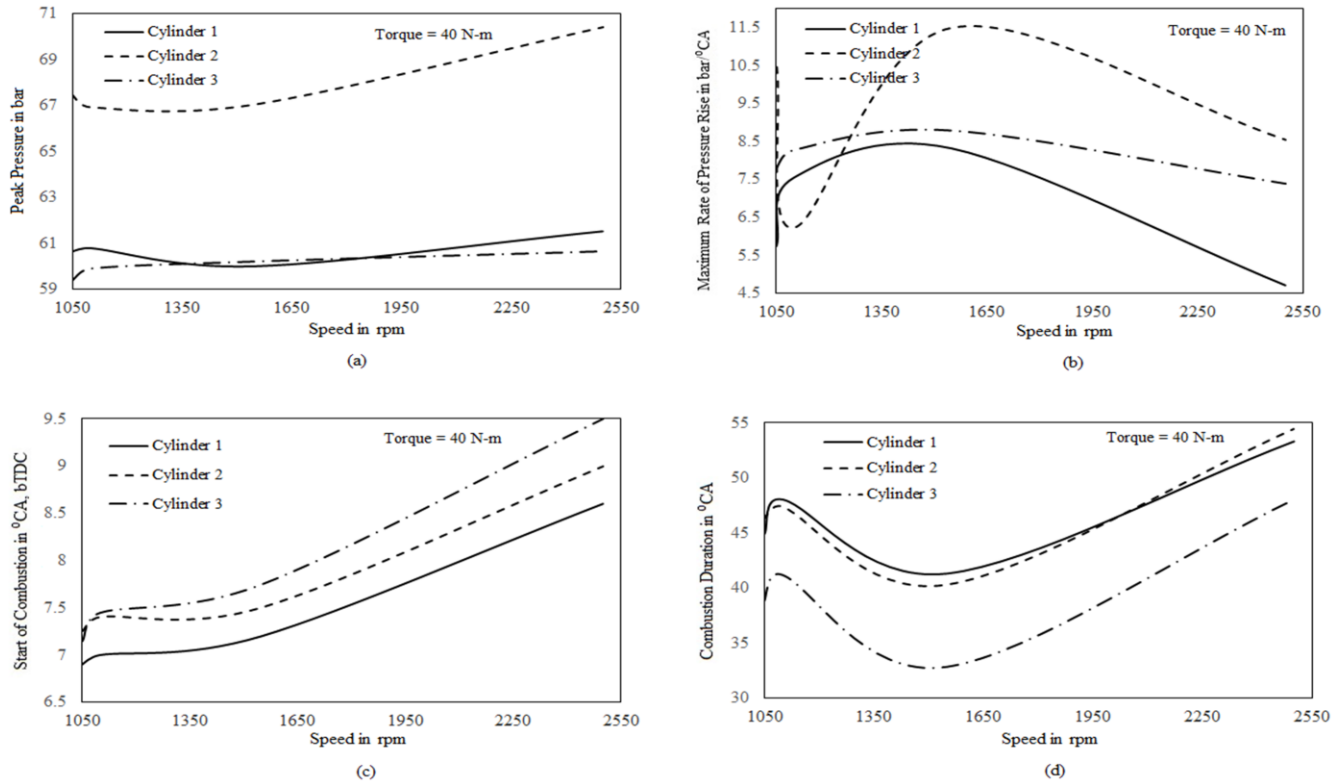


Fig. 5. Variation of combustion parameters with speed.

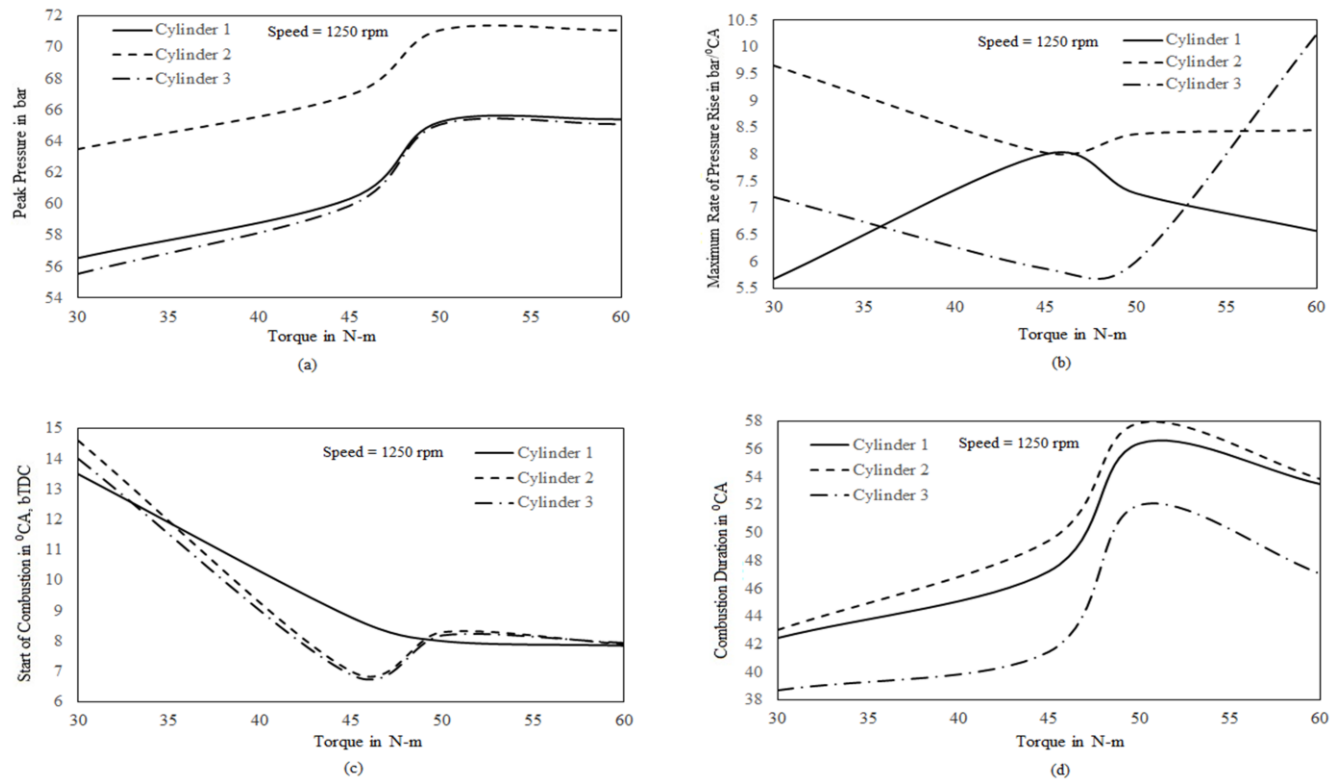


Fig. 6. Variation of combustion parameters with torque.

It was observed from Fig. 6(a) that the peak pressure of all three cylinders varies considerably when the engine is operated at various torques by maintaining a constant speed. As the torque

is increased, the cylinder peak pressure also increases due to increased fuel burning at a higher torque, and a similar trend is found for all cylinders. Also, a similar trend of variation is ob-

served for the variation of peak pressure for the cylinders as reported in Fig. 5(a).

Figure 6(b) displays the manner in which the maximum rate of pressure rise varies as a function of torque. From the figure, it is clear that the variation between the cylinders shows an erratic trend. Figure 6(c) presents the fashion in which the start of combustion varies as a function of torque. It can be seen that the start of combustion is advanced as the engine is operated at higher torque values. This may be due to the enhanced fuel vapourisation at higher torques, and hence, an earlier start of combustion. Figure 6(d) shows the manner in which the combustion

duration varies with respect to torque. An increasing trend is observed in the combustion duration for increased torque values, and all cylinders follow a similar trend. This is primarily because of the increased volume of charge burned as the torque increases.

3.3.3. Effect of speed and torque on maximum heat release rate

Variation of the maximum heat release rate resulting from fuel combusted through the duration of pilot injection and main injection as a function of speed is shown in Fig. 7.

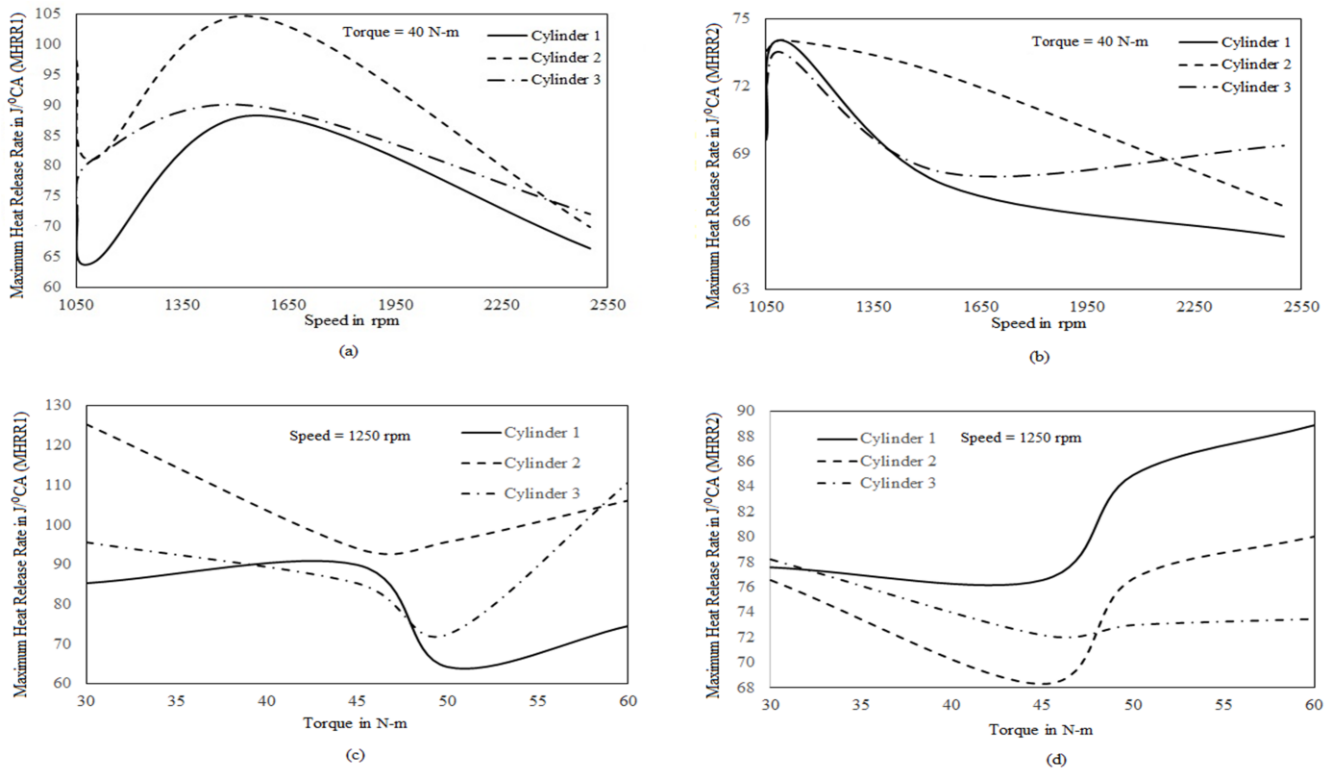


Fig. 7. Variation of MHRR1 and MHRR2 with speed and torque.

It is observed from Fig. 7(a) that an increase in speed leads to a rise in the value of MHRR1 initially, and further increasing the speed causes a decrease in MHRR1. At higher speeds, the duration of pilot injection will be short, and resulting in a lower MHRR1 when compared to MHRR1 recorded at lower speeds, and a similar trend is charted for all cylinders. Figure 7(b) shows that MHRR2 seems to be higher at lower speeds and tends to decrease slightly for higher speeds.

It is also observed in Fig. 7(c) that an increase in torque increases MHRR1 because of burning a higher fuel quantity at an increased torque. Figure 7(d) depicts that as the torque increases, MHRR2 resulting from the main injection also increases owing to an increase in volume of fuel burned at an increased torque.

3.4. Performance and emission parameters as a function of BMEP

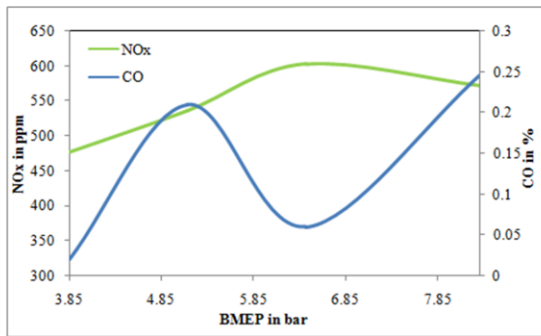
In this section, the variations of carbon monoxide (CO), unburnt hydrocarbon (UBHC), oxides of nitrogen (NO_x) and brake ther-

mal efficiency (BTE) are presented as a function of BMEP as a trade-off curve between NO_x-CO, NO_x-UBHC and NO_x-BTE.

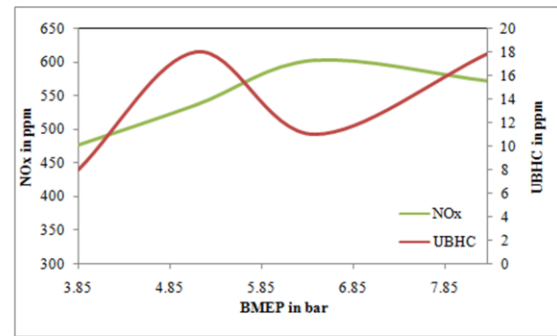
From Fig. 8, it can be predicted that an increase in BMEP results in an increased quantity of NO_x. The value of BMEP is increased by burning an additional quantity of fuel to compensate for the increase in load, which results in a higher temperature inside the combustion chamber. It is obvious that higher temperatures favour the NO_x formation inside the combustion chamber. The value of NO_x reaches a maximum and tends to reduce due to the non-availability of oxygen at a higher BMEP. It can also be predicted that at a slightly increased BMEP, the quantity of UBHC and CO increases, because of overfuelling, a small portion of fuel remains unburned, and decreases as the combustion temperature is increased further. At a higher BMEP, due to an increased crevice volume and reduced quantity of oxygen, the amount of UBHC and CO will increase. A similar trend of variation is observed for BTE as for CO and UBHC. As BMEP increases, a slight reduction in BTE is observed at the

middle range of BMEP, and a further increase in BMEP results in an increasing trend for BTE with BMEP. This may be due to the increase in fuel consumption with torque. A decrease in BTE

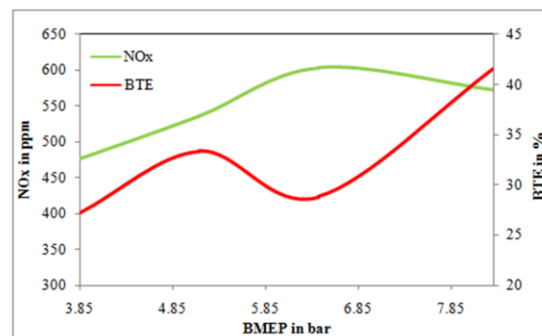
with BMEP during the middle range of BMEP may be due to the change in the engine speed when compared to that for the lower BMEP.



(a)



(b)



(c)

Fig. 8. Variation of performance and emission parameters with BMEP.

A trade-off between NO_x -CO, NO_x -UBHC and NO_x -BTE is observed at a higher BMEP close to 8 bar, where the two curves intersect each other. This point may be considered as an optimised condition for both the parameters presented in the graph. It is also observed that NO_x -UBHC and NO_x -CO curves intersect during the middle range of operation too, which indicates the operating condition for simultaneous control of NO_x , UBHC and CO emissions.

4. Conclusions

In the present work, performance and emission parameters and cylinder-to-cylinder variation of combustion parameters of a medium duty 3-cylinder CRDI passenger car diesel engine were investigated by testing the engine at different operating conditions. The following conclusions were made based on the findings:

- The combustion parameters vary considerably between the cylinders.
- The combustion parameters of cylinder 2 differ considerably from those of the other two cylinders. Its uncontrolled and controlled phase of combustion differs noticeably from those of the other two cylinders.
- The consistently elevated in-cylinder pressure in cylinder 2 was primarily attributed to a slight advance in the start of injection, resulting in earlier combustion and a higher peak pressure, as well as an increase in cumulative heat release.

Minor compression differences, likely caused by carbon deposits on the piston crown, were also observed but considered secondary contributors.

- The extended combustion duration in cylinder 2 may be attributed to factors such as uneven air-fuel mixture distribution, variations in injector flow rate or spray pattern, and thermal variations between the cylinders. These factors, while not directly measured in this study, require further examination, recommended to identify the root cause of the observed asymmetry.
- The maximum heat release rate, maximum rate of pressure rise and cycle peak pressure for cylinder 2 were higher when compared with the other two cylinders.
- Cylinder 2 shows a marginally longer CD than the other cylinders.
- A trade-off point was obtained at a higher BMEP for simultaneous control of NO_x , CO and UBHC at a higher brake thermal efficiency.
- During the engine operation at a medium BMEP, it is possible to simultaneously control the NO_x , CO and UBHC emissions.

From this investigation, it can be concluded that a significant variation in the magnitude of combustion parameters was observed between the three cylinders of a passenger car engine. It can also be concluded that the engine emissions can be simultaneously controlled.

This work can be further extended to investigate the influence of biodiesel and higher alcohols on the performance, combustion and emission characteristics of the test engine, and the results can be compared with those of diesel. To maintain the combustion uniformity in CRDI engines, regular injector flow calibration and matching are recommended. A theoretical model can be developed, and the simulated results can be validated with the experimental results.

References

- [1] Hyönen, J., Haraldsson, G., & Johansson, B. (2004). Balancing Cylinder-to-Cylinder Variations in a Multi-Cylinder VCR-HCCI Engine. *SAE Technical Paper 2004-0-1897*. doi: 10.4271/2004-01-1897
- [2] Ohtsubo, H., Yamauchi, K., Nakazono, T., Yamane, K., & Kawasaki, K. (2007). Influence of Compression Ratio on Performance and Variations in Each Cylinder of Multi Cylinder Natural Gas Engine with PCCI Combustion. *SAE Technical Paper 2007-01-1877*. doi: 10.4271/2007-01-1877
- [3] Johansson, B., & Einewall, P. (2000). Cylinder to Cylinder and Cycle to Cycle Variations in a Six Cylinder Lean Burn Natural Gas Engine. *SAE Technical Paper 2000-01-1941*. doi: 10.4271/2000-01-1941
- [4] Stumpp, G., & Ricco, M. (1996). Common Rail – An Attractive Fuel Injection System for Passenger Car DI Diesel Engines. *SAE Technical Paper 960870*. doi: 10.4271/960870
- [5] Guerrassi, N., & Dupraz, P. (1998). A Common Rail Injection System For High Speed Direct Injection Diesel Engines. *SAE Technical Paper 980803*. doi: 10.4271/980803
- [6] Torregrosa, A., Broatch, A., Garcá, A., & Mónico, L. (2013). Sensitivity of combustion noise and NO_x and soot emissions to pilot injection in PCCI diesel engines. *Applied Energy*, 104, 149–157. doi: 10.1016/j.apenergy.2012.11.040
- [7] Huang, H., Liu, Q., Yang, R., Zhu, T., Zhao, R., & Wang, Y. (2015). Investigation on the effects of pilot injection on low temperature combustion in high-speed diesel engine fueled with n-butanol–diesel blends. *Energy Conversion and Management*, 106, 748–758. doi: 10.1016/j.enconman.2015.10.031
- [8] Barsic, N., & Humke, A.L. (1981). Performance and Emissions Characteristics of a naturally aspirated diesel engine with vegetable oil fuels. *Transaction of SAE, Technical Paper 810262*, doi: 10.4271/810262
- [9] Hemmerlein, N., Korte, V., Richter, H., & Schröder, G. (1991). Performance, Exhaust Emissions and Durability of Modern Diesel Engines Running on Rapeseed Oil. *Transaction of SAE, Technical Paper 910848*. doi: 10.4271/910848
- [10] Huang, H., Zhou, C., Liu, Q., Wang, Q., & Wang, X. (2016). An experimental study on the combustion and emission characteristics of a diesel engine under low temperature combustion of diesel/gasoline/n-butanol blends. *Applied Energy*, 170, 219–231. doi: 10.1016/j.apenergy.2016.02.126
- [11] Saravanan, S., Nagarajan, G., & Lakshmi Narayana, R.G. (2009). Comparison of Combustion Characteristics of Crude Rice Bran Oil Methyl Ester with Diesel as a CI Engine Fuel. *Journal of Biobased Materials and Bioenergy*, 3, 32–36. doi: 10.1166/jbmb.2009.1003
- [12] Singh, A.K., Palia, H.S., & Khan, M.M. (2024). Experimental investigations of CI engine performance using ternary blends of n-butanol/biodiesel/diesel and n-octanol/biodiesel/diesel, *Archives of Thermodynamics*, 45, 109–118. doi: 10.24425/ather.2024.150443
- [13] Kumara, D., Kumar, N., & Chaudhary, R. (2024). Use of butanol, pentanol and diesel in a compression ignition engine: A review, *Archives of Thermodynamics*, 45, 27–35. doi: 10.24425/ather.2024.151994
- [14] Kolloi, A. Reddy, N., Paruvada, S., & Murugan, S. (2019). Experimental studies of a diesel engine run on biodiesel n-butanol blends. *Renewable Energy*, 135, 687–700. doi: 10.1016/j.renene.2018.12.011
- [15] Sayon, S., Joel, B., Tizane, D., Gilles, V., & Jean, K. (2020). Comparative study of three ways of using Jatropha curcas vegetable oil in a direct injection diesel engine. *Scientific African*, 7, e00290. doi: 10.1016/j.sciaf.2020.e00290
- [16] Canakci, M., Ozsazen, A.N., & Turkcan, A. (2009). Combustion analysis of preheated crude sunflower oil in an IDI diesel engine. *Biomass and Bioenergy*, 33(5), 760–767. doi: 10.1016/j.biombioe.2008.11.003
- [17] Kandasamy, S.K., Selvaraj, A.S., & Rajagopal, T.K.R. (2019). Experimental investigations of ethanol blended biodiesel fuel on automotive diesel engine performance, emission and durability characteristics. *Renewable Energy*, 141, 411–419. doi: 10.1016/j.renene.2019.04.039
- [18] Qi, D.H., Lee, C.F., Jia, C.C., Wang, P.P., & Wu, S.T. (2014). Experimental investigations of combustion and emission characteristics of rapeseed oil–diesel blends in a two cylinder agricultural diesel engine. *Energy Conversion and Management*, 77, 227–232. doi: 10.1016/j.enconman.2013.09.023
- [19] Ali, O.M., Mamat, R., Abdullah, N.R., & Abdullah, A.A. (2016). Analysis of blended fuel properties and engine performance with palm biodiesel–diesel blended fuel. *Renewable Energy*, 86, 59–67. doi: 10.1016/j.renene.2015.07.103
- [20] Jayaprabakar, J., Karthikeyan, A., & Rameshkumar, V. (2017). Effect of injection timing on the combustion characteristics of rice bran and algae biodiesel blends in a compression-ignition engine. *International Journal of Ambient Energy*, 38, 116–121. doi: 10.1080/01430750.2015.1048901
- [21] Rama Krishna Reddy, E., Subbalakshmi, Y., Dhana Raju, V., Appa Rao, K., Harun Kumar, M., Rami Reddy, S., & Tharun Sai, P. (2019). Assessment of performance, combustion and emission characteristics of the diesel engine powered with corn biodiesel blends. *International Journal of Ambient Energy*, 40, 435–443. doi: 10.1080/01430750.2019.1636867
- [22] Ganesan, S., Devaraj, A., & Devarajan, Y. (2020). Emission characteristics on single cylinder diesel engine using biofuels. *International Journal of Ambient Energy*, 41, 1613–1616. doi: 10.1080/01430750.2018.1517694
- [23] Jindal, S., Nandwana, B.P., Rathore, N.S., & Vashistha, V. (2010). Experimental investigation of the effect of compression ratio and injection pressure in a direct injection diesel engine running on Jatropha methyl ester. *Applied Thermal Engineering*, 30(5), 442–448. doi: 10.1016/j.applthermaleng.2009.10.004
- [24] Pradeep, V., & Sharma, R.P. (2005). Evaluation of Performance, Emission and Combustion Parameters of a CI Engine Fuelled with Bio-Diesel from Rubber Seed Oil and its Blends. *Transactions of SAE, Technical Paper 2005-26-353*. doi: 10.4271/2005-26-353
- [25] Ganapathy, T., Gakkhar, R.P., & Murugesan, K. (2011). Influence of injection timing on performance, combustion and emission characteristics of Jatropha biodiesel engine. *Applied Energy*, 88, 4376–4386. doi: 10.1016/j.apenergy.2011.05.016
- [26] Hirkude, J.B., & Padalkar, A.S. (2014). Performance optimization of CI engine fuelled with waste fried oil methyl ester-diesel blend using response surface methodology. *Fuel*, 119, 266–273. doi: 10.1016/j.fuel.2013.11.039

- [27] Bari, S., Yu, C.W., & Lim, T.H. (2004). Effect of fuel injection timing with waste cooking oil as a fuel in a direct injection diesel engine. *Proceedings of the Institution of Mechanical Engineers; part D, Journal of Automobile Engineering*, 18, 93–104. doi: 10.1243/095440704322829209
- [28] Agarwal, A.K., Srivastava, D.K., Dhar, A., Maurya, R.K., Shukla, P.C., & Singh, A.P. (2013). Effect of fuel injection timing and pressure on combustion, emissions and performance characteristics of a single cylinder diesel engine. *Fuel*, 111, 374–383. doi: 10.1016/j.fuel.2013.03.016
- [29] Payri, F., Benajes, J., Arrègle, J., & Riesco, J.M. (2006). Combustion and Exhaust Emissions in a Heavy-Duty Diesel Engine with Increased Premixed Combustion Phase by Means of Injection Retarding. *Oil & Gas Science and Technology – Rev. IFP*, 61(2), 247–258. doi: 10.2516/ogst:2006018x
- [30] Tauzia, X., Maiboom, A., & Shah, S.R. (2010). Experimental study of inlet manifold water injection on combustion and emissions of an automotive direct injection diesel engine. *Energy*, 35, 3628–3639. doi: 10.1016/j.energy.2010.05.007
- [31] Saravanan, S., Nagarajan, G., & Lakshmi Narayana, R.G. (2009). Feasibility analysis of crude rice bran oil methyl ester blend as a stationary and automotive diesel engine fuel. *Energy for Sustainable Development*, 13(1), 52–55. doi: 10.1016/j.esd.2009.03.001
- [32] Saravanan, S., Nagarajan, G., & Sampath, S. (2011). Investigation on combustion characteristics of crude rice bran oil methyl ester blend as a heavy duty automotive engine fuel. *International Journal of Oil, Gas and Coal Technology*, 4(3), 282–295. doi: 10.1504/IJOGCT.2011.040840
- [33] Sidibe, S., Blin, J., Daho, T., Vaitilingom, G., & Koulidiati, J. (2020). Comparative study of three ways of using Jatropha curcas vegetable oil in a direct injection diesel engine, *Scientific African*, 7, e00290. doi: 10.1016/j.sciaf.2020.e00290
- [34] Kandasamy, S.K., Selvaraj, A.R., & Rajagopal, T.K.R. (2019). Experimental investigations of ethanol blended biodiesel fuel on automotive diesel engine performance, emission and durability characteristics, *Renewable Energy*, 141, 411–419. doi: 10.1016/j.renene.2019.04.039
- [35] Devarajan, Y., Munuswamy, D., Nagappan, B., & Subbiah, G. (2019). Experimental assessment of performance and exhaust emission characteristics of a diesel engine fuelled with Punnai biodiesel/butanol fuel blends, *Petroleum Science*, 16, 1471–1478. doi: 10.1007/s12182-019-00361-9
- [36] Roy, S., Banerjee, R., Das, A.K., & Bose, P.K. (2014). Development of an ANN based system identification tool to estimate the performance-emission characteristics of a CRDI assisted CNG dual fuel diesel engine. *Journal of Natural Gas Science and Engineering*, 21, 147–158. doi: 10.1016/j.jngse.2014.08.002
- [37] Roy, S., Ghosh, A., Das, A.K., & Banerjee, R. (2014). A comparative study of GEP and an ANN strategy to model engine performance and emission characteristics of a CRDI assisted single cylinder diesel engine under CNG dual-fuel operation. *Journal of Natural Gas Science and Engineering*, 21, 814–821. doi: 10.1016/j.jngse.2014.10.024
- [38] Hwang, J., Qi, D., Jung, Y., & Bae, C. (2014). Effect of injection parameters on the combustion and emission characteristics in a common-rail direct injection diesel engine fueled with waste cooking oil biodiesel. *Renewable Energy*, 63, 9–17. doi: 10.1016/j.renene.2013.08.051
- [39] Syed Aalam, C., Saravanan, C.G., & Kannan, M. (2015). Experimental investigations on a CRDI system assisted diesel engine fuelled with aluminium oxide nanoparticles blended biodiesel. *Alexandria Engineering Journal*, 54, 351–358. doi: 10.1016/j.aej.2015.04.009
- [40] Basavarajappa, D.N., Banapurmath, N.R., Khandal, S.V., & Manavendra, G. (2015). Performance Evaluation of Common Rail Direct Injection (CRDI) Engine Fuelled with Uppage Oil Methyl Ester (UOME), *International Journal of Renewable Energy Development*, 4(1), 1–10. doi: 10.14710/ijred.4.1.1-10
- [41] Ashok, B., Nanthagopal, K., Chaturvedi, B., Sharma, S., & Thundil Karuppa Raj, R. (2018). A comparative assessment on Common Rail Direct Injection (CRDI) engine characteristics using low viscous biofuel blends, *Applied Thermal Engineering*, 145, 494–506. doi: 10.1016/j.applthermaleng.2018.09.069
- [42] Choi, B., Kim, Y.K., Jung, G., Lee, C., Jiang, X., & Choi, I. (2014). Effect of Diesel Fuel Blend with Biobutanol on the Emission of Turbocharged CRDI Diesel Engine. *Energy Procedia*, 61, 2145–2148. doi: 10.1016/J.EGYPRO.2014.12.096
- [43] Choi, B., & Jiang, X. (2015). Individual hydrocarbons and particulate matter emission from a turbocharged CRDI diesel engine fuelled with n-butanol/diesel blends. *Fuel*, 154, 188–195. doi: 10.1016/J.FUEL.2015.03.084
- [44] Babu, J.M., Prasad, K.S., Ganji, P.R., Ravikiran, Ch., & Velu, R. (2022). Analysis on the effect of pilot injection strategies on combustion and emission characteristics of palm-munja biodiesel/diesel blend on CRDI diesel engine. *International Journal of Ambient Energy*, 43(1), 618–621. doi: 10.1080/01430750.2019.1663368
- [45] Senthil Kumar, D., & Thirumalini, S. (2022). Effect of injection parameters and EGR on performance and emissions in a CRDI engine fuelled with cashew nut shell biodiesel blend. *International Journal of Ambient Energy*, 43, 3085–3095. doi: 10.1080/01430750.2020.1796786
- [46] Saravanan, S., Durai, K.P., & Sundaram, I.M. (2019). Comparison of combustion characteristics of an automotive CRDI engine with conventional HCV engine. *International Journal Oil, Gas and Coal Technology*, 21(3), 390–405. doi: 10.1504/IJOGCT.2019.10021964
- [47] Moffat, R.J. (1985). Using uncertainty analysis in the planning of an experiment. *Transaction of ASME, Journal Fluids Engineering*, 107(2), 173–178.

Machine Learning Optimisation of CI Engine Characteristics with WSA Algorithm and Prosopis Juliflora Biofuel

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Received: 17.09.2025; revised: 03.11.2025; accepted: 16.11.2025

Abstract

Machine learning, a division of artificial intelligence (AI), empowers systems to gain knowledge from information and refine their capabilities over time. It uses algorithms to identify patterns and make prophecies or decisions. *Prosopis juliflora* is becoming gradually notorious as an optimistic substitute in biofuel inquiry. The mixing of *Prosopis juliflora* with diesel for use in combustion engines has been the subject of growing study in recent years. However, limited research has explored the impact of *Prosopis juliflora* on the compression ignition (CI) engine exhaust employing data-driven optimisation highlighting the need for new inquiries to address this shortfall. We aim to explore the cutting-edge and proficient machine learning driven weighted superposition attraction algorithm to optimise the efficiency and exhaust of CI engines powered with *Prosopis juliflora* biodiesel – diesel blends. Regression modelling is employed to define the relationships between factors such as the blend percentage and brake mean effective pressure (bar), and responses like the brake thermal efficiency (%), brake specific fuel consumption (g/kWh), smoke opacity (%), NO_x (g/kWh), CO (g/kWh), and HC (g/kWh). The data-driven weighted superposition attraction algorithm is subsequently employed to determine the best factor levels. Validation of the results demonstrates that the brake thermal efficiency is enhanced, while the other response variables are effectively reduced, showcasing the effectiveness of this methodology.

Keywords: Neural network; Machine learning; Juliflora; CI engine

Vol. 46(2025), No. 4, 117–125; doi: 10.24425/ather.2025.156842

Cite this manuscript as: Narayanan, S., Subramanian, V., Rajkumar, T., & Nadarajan, V. (2025). Machine Learning Optimisation of CI Engine Characteristics with WSA Algorithm and *Prosopis Juliflora* Biofuel. *Archives of Thermodynamics*, 46(4), 117–125.

1. Introduction

The scientific community is intensifying investigations into sustainable fuel technologies, responding to mounting environmental challenges and the critical imperative of reducing anthropogenic greenhouse gas contributions [1]. Emerging biofuel studies are utilising complex mathematical modelling and optimisation techniques – specifically Taguchi methods, genetic algorithm, response surface methodology (RSM), and integrated algorithmic frameworks like RF-NSGA III-TOPSIS – to systematically investigate and optimise combustion efficiency, engine

performance, and emission characteristics [2]. *Prosopis juliflora*-based diesel biofuel represents a compelling technological solution, demonstrating remarkable capabilities in addressing critical environmental challenges through reduced greenhouse gas generation, enhanced air quality and decreased fossil fuel consumption [3]. The complex challenges of oil market volatility are catalysing transformative approaches to energy production, with *Prosopis juliflora* biomass-derived biofuels representing a technologically sophisticated strategy for reducing reliance on traditional fossil fuel resources [4]. *Prosopis juliflora* is produced from the seed of the *Prosopis juliflora* tree, commonly

Nomenclature

$d_{ij}(t)$ – the i^{th} solution movement direction in the j^{th} dimension
 $sl(t)$ – step size
 $x_{ij}(t)$ – value for the i^{th} solution in the j^{th} dimension
 X – feature
 Y – outcome variable

Greek symbols

β – coefficient

ε – residual error term

Abbreviation and Acronyms

BMEP – brake mean effective pressure
 BTE – brake thermal efficiency
 CI – compression ignition
 HC – hydrocarbon
 NO_x – nitrogen oxides
 RSM – response surface methodology
 WSA – weighted superposition attraction

known as mesquite. The improved energy efficiency of engines powered by Prosopis juliflora mineral diesel renewable fuel has drawn significant interest from the freight industry, which is striving to reduce fuel usage and cut operating costs [5].

Prosopis juliflora mineral diesel renewable fuel holds significant potential as a sustainable energy solution for the future. Its properties closely align with those of standard diesel, making it a viable substitute for fossil fuels. Additionally, its ability to function seamlessly in mineral diesel or compression ignition (CI) engines excluding major adjustments has further fuelled research into its application [6]. These benefits support the position of Prosopis juliflora diesel biofuel as a promising alternative for complying with increasingly strict vehicle emission regulations. Several studies have shown that biofuel delivers a power output comparable to traditional diesel. The strong compatibility of Prosopis juliflora with diesel fuel provides a prospect to formulate a refined diesel – Prosopis juliflora blend with characteristics akin to pure diesel while effectively resolving phase separation issues [7]. Researchers have utilised both experimental methods and numerical simulations to probe the application of Prosopis juliflora in CI engines. The research examines the performance and emission traits of Prosopis juliflora biodiesel in diesel engines, emphasising key insights. The B25 blend demonstrated the most favourable results, offering thermal efficiency comparable to diesel while reducing emissions. Higher biodiesel percentages led to increased CO, HC and smoke emissions, but NO_x emissions dropped by 17.71%. Biodiesel was produced via transesterification using methanol and alkaline catalysts, showcasing its potential as an eco-friendly fuel alternative. The research identified B20 and B25 blends as optimal for achieving a balance between engine performance and emission control [8]. Additionally, an experimental study investigated the impact of adding diethyl ether (DEE) to Prosopis juliflora biodiesel blends on engine efficiency and pollutants. While the brake thermal efficiency (BTE) of biodiesel mixtures was slightly lower than that of pure diesel oil, the addition of DEE improved combustion efficiency. For instance, the B20DEE10 blend (20% biodiesel with 10% DEE) achieved BTE of 31.4% at 50% load compared to diesel's 36.7%. Moreover, the DEE additives reduced emissions such as CO₂, HC and CO, along with a decrease in NO_x emissions, attributed to DEE's elevated cetane grade and heat of vaporisation [9]. The investigation examines the efficiency of juliflora oil biodiesel in a compression ignition engine, showing a 5–7% increase in BTE and a 15–20% cutback in CO and HC exhausts compared to diesel. NO_x emissions were slightly higher by 10–12%, a common trade-off with biodiesel. Overall, the findings highlight juliflora

biodiesel's potential for cleaner energy, with further optimisation needed for emission control [10].

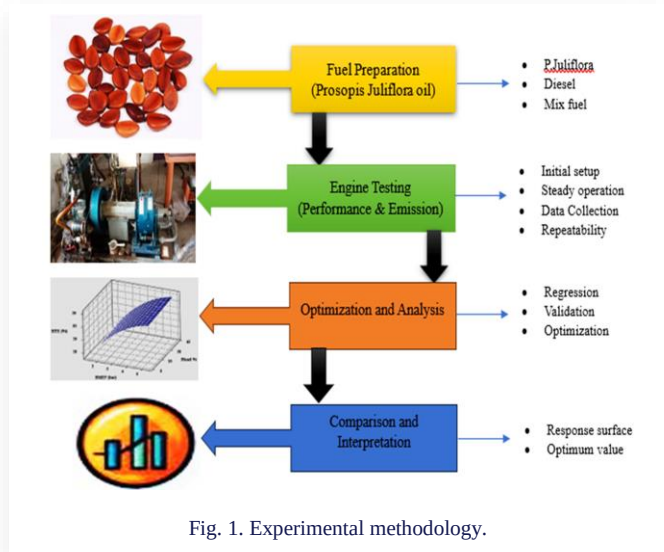
This comprehensive research explores the adaptation of juliflora biodiesel and its variants as clean fuels in CI engines through a combination of simulation and experimental approaches. Studies demonstrate that juliflora biodiesel offers comparable engine performance with reduced emissions, establishing its viability as a sustainable fuel option [11]. Further investigation into H₂ boosting with Prosopis juliflora biofuel highlights performance and emission improvements using modern optimisation practices like RSM and artificial neural networks (ANN) [12]. Additionally, research on Prosopis juliflora methyl ester emphasises the role of transesterification in enhancing fuel properties, such as viscosity, to improve engine compatibility and efficiency [13]. Machine learning in engine research applies artificial intelligence (AI) practices to analyse data and past experiences in the domain of biofuels and engines. Through the application of algorithms and computational frameworks, it examines experimental or simulated data concerning engine efficiency, exhaust characteristics and Prosopis juliflora fuel properties. This methodology uncovers trends, enables forecasting and provides critical insights, aiding researchers in activities like determining ideal biofuel mixtures, forecasting engine behaviour and refining butanol production techniques. The rising significance of data-driven ANN has noticeably enhanced forecasting and optimisation initiatives. Extensive investigations have been conducted on biofuel applications in CI engines. Recent efforts have particularly examined the potential of Prosopis juliflora in diesel engines, leveraging machine learning techniques to determine optimal outcomes.

This research applies regression modelling for quantitative analysis and utilises the weighted superposition attraction (WSA) for optimising multiple objectives. The WSA technique is used to identify the optimal concentrations of Prosopis juliflora (vol. %) and brake mean effective pressure (BMEP) (bar) for multiple performance factors, such as BTE (%), brake specific fuel consumption (BSFC) (g/kWh), NO_x (g/kWh), HC (g/kWh), CO (g/kWh) and smoke opacity (%). The concurrent optimisation of these parameters using WSA is an innovative feature of this research, which seeks to integrate the WSA approach into biofuel studies within the context of CI engines.

2. Juliflora performance optimisation

The experimental approach described provides a structured method for studying how Prosopis juliflora biofuel blends perform in a diesel engine. The study involves adjusting the fuel

blend and engine settings, and using cutting edge techniques like the weighted sum approach to analyse the results. The experimental setup and process are shown in Fig. 1.



The study used different blends of Prosopis juliflora biofuel mixed with diesel as the trial fuels. The distinct proportions and readiness techniques for these blends are outlined in earlier research. Typically, the biofuels are made by combining pure Prosopis juliflora with conventional diesel at varying levels. The blends tested in the study include pure diesel, JB10 (10% Prosopis juliflora, 90% diesel), JB20 (20% juliflora, 80% diesel), JB30 (30% juliflora, 70% diesel), and JB40 (40% juliflora, 60% diesel), with their properties summarised in Table 1.

Table 1. Properties of mineral diesel and biofuel.

Features	Juliflora oil	JB10	JB20	JB30	JB40	Diesel
Density (g/cm ³)	0.811	0.872	0.876	0.879	0.846	0.830
Kinematic viscosity (mm ² /s)	3.707	3.963	4.019	4.515	3.819	43.96
Cetane number	61.97	59.72	54.15	50.21	59.10	46 - 55
Sulphur content (mg/kg)	59.344	58.11	84.30	111.79	65.93	6800
Higher heating value	36.763	39.457	39.60	39.94	38.279	42,500
Oxidation stability (110°C)	12.661	15.84	16.630	6.75	11.198	–
Oxygen content	8.6	8.4	12.2	14.02	10.49	–

Each fuel blend was carefully mixed and tested to ensure uniformity before being utilised in the trials. Subsequently, various operational conditions were applied to assess the engine's performance with each fuel blend. The key stages of the evaluation process are as follows:

(a) Primary setup.

The engine was heated to ensure consistent working temperatures throughout all the tests, and reference measurements were obtained using conventional diesel fuel to set a benchmark for comparison.

(b) Stable working conditions.

The engine was evaluated under different stable working conditions by adjusting BMEP. Typically, tests were performed at several load points, including 1.35, 2.70, 4.8 and 5.8 bar.

(c) Gathering data.

Essential engine metrics like BSFC, BTE and exhaust emissions were measured, alongside burning factors such as the heat release rate and in-cylinder pressure using sensors.

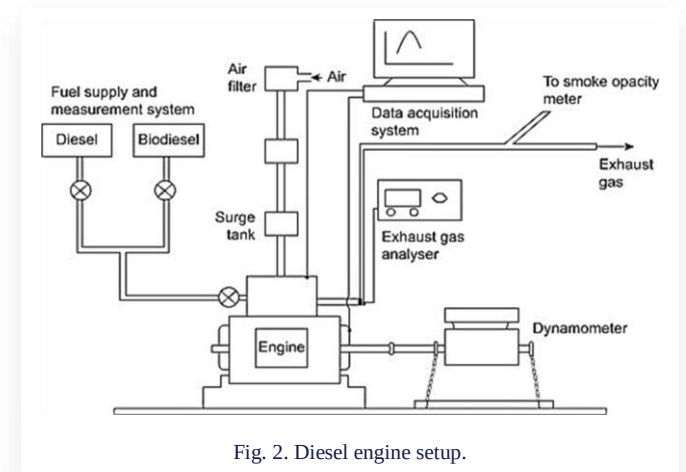
(d) Consistency and validation.

Each trial was conducted multiple times to confirm reliable and precise data.

The trial data were evaluated to determine the effects of various Prosopis juliflora diesel mixtures on engine efficiency and exhausts. The WSA approach was enforced to optimise the blends, granting the detection of the most potent mixture for elevating fuel efficiency and minimising emissions. Factors such as fuel blend ratio, engine load and speed were considered in the optimisation procedure. The results for the Prosopis juliflora blends were compared to the reference diesel fuel characteristics. The inquiry centred on detecting patterns in efficiency and exhausts, assessing the burning characteristics of Prosopis juliflora blends, and exploring the potential of Prosopis juliflora as a clean alternative to diesel fuel.

2.1. Test setup

The tests were carried out using a mono-cylinder CI engine, with the configurations and setup provided in Fig. 2.



2.2. WSA approach

The WSA algorithm is a modern optimisation method rooted in the broader field of metaheuristic algorithms and multi-objective optimisation. It extends the traditional weighted sum approach, which consolidates several objectives into one scalar value through a process of weighted summation. The emergence of WSA was driven by the increasing complexity of engineering problems, particularly in fields like energy systems, material sciences and automotive research, where advanced optimisation tools were needed. Recent developments have refined WSA to improve its efficiency and adaptability, making it suitable for

high-dimensional datasets and complex applications. In engine research, WSA has been applied to optimise multiple parameters, such as performance metrics and emissions, addressing the dual challenge of enhancing efficiency while meeting environmental standards. Its ability to balance multiple objectives simultaneously has made WSA a valuable tool for solving real-world problems, with its adoption steadily growing in engineering and environmental studies. The WSA algorithm directs the keys at repetition t near their designated search paths using the equation:

$$x_{ij}(t+1) = x_{ij}(t) + sl(t) \cdot d_{ij}(t) \cdot \|x_{ij}(t)\|. \quad (1)$$

In the WSA algorithm, $x_{ij}(t)$ represents the value of the j^{th} dimension for the i^{th} solution, while $sl(t)$ denotes the step size. The solution's movement direction is defined by $d_{ij}(t) \in \{1, 0, -1\}$ for dimension j . The term $\|x_{ij}(t)\|$ indicates the distance between the origin and the j^{th} dimension of solution i at the t^{th} iteration. The algorithm updates the solution along its dimensions in the specified search directions using the second term, $sl(t) \cdot d_{ij}(t) \cdot \|x_{ij}(t)\|$ as indicated on the opposite side of Eq. (1) [16].

3. Outcomes and analysis: diesel engine operated with juliflora biofuel and machine learning

Twenty experimental trials were carried out according to the experimental design. The findings from these trials are illustrated in Fig. 3 and Fig. 4. Regression models were created for uncoded factor levels, utilising the outcomes such as NO_x (g/kWh), Smoke Opacity (%), CO (g/kWh), HC (g/kWh), BSFC (g/kWh) and BTE (%).

The graphs in Fig. 3 illustrate the performance of diesel (D100) and biodiesel blends (JB10, JB20, JB30, JB40) in terms of BTE (%) and BSFC (g/kWh) as functions of BMEP (bar).

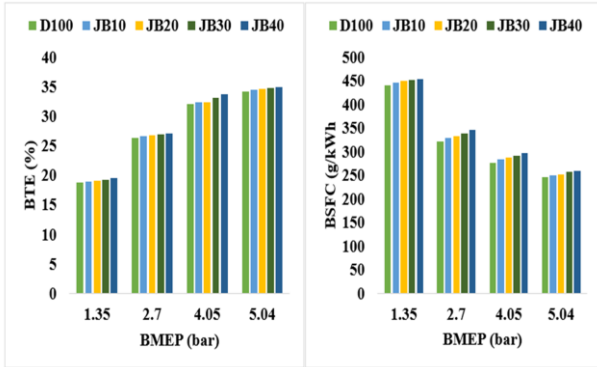


Fig. 3. History of BTE (%) and BSFC (g/kWh) vs. BMEP (bar).

BTE increases with BMEP for all fuels, peaking at approximately 38% at 5.04 bar. D100 consistently exhibits the highest BTE, while higher biodiesel blends show slightly reduced efficiency due to their lower calorific value and higher viscosity. At 1.35 bar, BTE ranges from 15–17%, with JB40 showing the most significant drop. By 4.05 bar, BTE rises to 30–33%, with JB10 and JB20 performing closest to D100, indicating that

lower biodiesel blends maintain efficiency while incorporating renewable fuel. Conversely, BSFC decreases with increasing BMEP, from ~450–470 g/kWh at 1.35 bar to ~200–220 g/kWh at 5.04 bar. D100 achieves the lowest BSFC, reflecting its superior fuel efficiency. Biodiesel blends, particularly JB30 and JB40, show higher BSFC due to biodiesel's minimal energy density and elevated viscosity, which can affect combustion. However, JB10 and JB20 closely match the BSFC of D100, demonstrating a favourable balance between efficiency and renewable fuel use. Improved in-cylinder conditions at higher BMEP reduce the performance gap between diesel and biodiesel blends, making JB10 and JB20 optimal choices for maintaining efficiency while integrating biodiesel.

The graphs in Fig. 4 illustrate the emission characteristics of diesel fuel (D100) and biodiesel blends (JB10, JB20, JB30, JB40) across various brake mean effective pressures (BMEP).

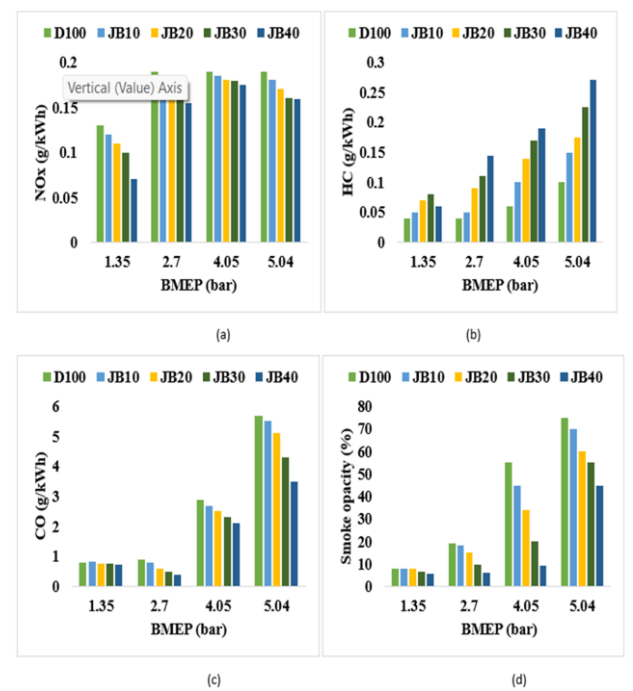


Fig. 4. History of Exhaust of (a) NO_x , (b) HC, (c) CO, and (d) Smoke opacity vs. BMEP (bar).

NO_x emissions increase steadily with BMEP for all fuels, peaking at 0.18–0.2 g/kWh at 5.04 bar. Biodiesel blends, particularly JB30 and JB40, exhibit higher NO_x emissions than D100 owing to their higher oxygen presence, which improves combustion but raises in-cylinder temperatures, promoting NO_x formation. At low load (1.35 bar), NO_x exhausts range from 0.05–0.1 g/kWh, with D100 showing the lowest levels due to the absence of oxygen and lower combustion temperatures. Hydrocarbon (HC) emissions are lowest at 1.35 bar, with biodiesel blends emitting less than D100, reflecting better combustion. At 2.7 bar, emissions increase slightly, with JB40 reaching about 0.1 g/kWh. At higher loads (4.05 and 5.04 bar), HC emissions rise significantly, peaking at 0.3 g/kWh for JB40 due to poor atomisation and reduced volatility in higher biodiesel blends. JB10 and JB20 demonstrate lower HC emissions, indicating bet-

ter combustion efficiency. Carbon monoxide (CO) emissions remain minimal at 1.35 and 2.7 bar, staying below 1 g/kWh for all blends, showing efficient combustion at low loads. At 4.05 bar, CO emissions increase, with JB40 emitting approximately 3 g/kWh. At 5.04 bar, CO emissions peak, with JB40 reaching 5.5 g/kWh, highlighting incomplete combustion and lower oxidation efficiency. D100 and JB10 perform better, keeping CO emissions below 5 g/kWh. Smoke opacity is minimal at 1.35 bar (<10%) for all fuels, indicating clean combustion. It increases slightly at 2.7 bar, with JB40 reaching 15%. At 4.05 bar, opacity rises significantly, with JB30 and JB40 reaching ~45%. At 5.04 bar, smoke opacity peaks at 70% for JB40 owing to elevated viscosity and inferior atomisation, which hinder burning. D100 consistently outperforms biodiesel blends across all parameters due to its higher volatility and better combustion characteristics.

In this study, WSA is used for optimisation. The optimisation process consists of three stages: (i) performing the trials, (ii) developing models for regression, and (iii) executing the optimisation of WSA. The factor points are provided in Table 2.

The experimental process involves 20 trials, as outlined in Table 3, utilising the specified factor values. In the second phase, regression equations are formulated for uncoded factor levels. Coded models are essential for the optimisation phase, while the uncoded models (original models) are also constructed to illustrate the true mathematical relationships for the readers.

Consequently, the data provided in Table 4 are applied in the second phase for model development.

Table 2. Engine specifications.

Narrative	Standard
Model	Kirloskar, 4- stroke
Cooling	Water
Bore × stroke	78 mm × 62 mm
Displacement	325 cm ³
No. of cylinders	Mono
CR	18.5
Maximum speed	2600 rpm
Maximum power	4.1 kw
Fuel IT	14 ±1° bTDC
Fuel IP	19.6 MPa

Table 3. Level of factors.

Features	Notation	Levels			
		L1	L2	L3	L4
Biodiesel blends (vol. %)	X_1	B10	B20	B30	B40
BMEP (bar)	X_2	1.35	2.7	4.05	5.04

Table 4. Experimental runs.

Run	Uncoded factors		Responses					
	Biodiesel blend (vol. %)	BMEP (bar)	BTE (%)	BSFC (g/kWh)	NO _x (g/kWh)	HC (g/kWh)	CO (g/kWh)	Smoke (%)
1	0	1.35	18.75	441	0.13	0.04	0.8	8.12
2	10	1.35	18.9	446	0.12	0.05	0.82	7.95
3	20	1.35	19.12	450	0.11	0.07	0.78	7.92
4	30	1.35	19.26	452	0.1	0.08	0.76	6.5
5	40	1.35	19.55	454	0.07	0.06	0.74	5.5
6	0	2.7	26.35	322	0.19	0.04	0.9	19.21
7	10	2.7	26.56	329	0.18	0.05	0.8	18.45
8	20	2.7	26.75	332	0.17	0.09	0.6	15.1
9	30	2.7	26.9	338	0.165	0.11	0.5	9.94
10	40	2.7	27.1	345	0.155	0.145	0.4	6.2
11	0	4.05	32.1	276	0.19	0.06	2.9	55
12	10	4.05	32.36	284	0.185	0.1	2.7	45
13	20	4.05	32.42	288	0.18	0.14	2.5	34
14	30	4.05	33.1	292	0.179	0.17	2.3	20.12
15	40	4.05	33.69	297	0.175	0.19	2.1	9.45
16	0	5.04	34.1	245	0.19	0.1	5.7	75
17	10	5.04	34.48	249	0.18	0.15	5.5	70
18	20	5.04	34.59	251	0.17	0.175	5.1	60
19	30	5.04	34.78	257	0.16	0.225	4.3	55
20	40	5.04	34.89	259	0.159	0.271	3.5	45

Equation (2) illustrates the full quadratic model used for the regression analysis:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{12} X_1 X_2 + \varepsilon. \quad (2)$$

In this expression, Y signifies the outcome variable, X represents the features, β stands for the coefficients, and ε denotes the residual error term [14,15]. Following several preliminary efforts, the model is established using linear terms for Y and com-

plete quadratic models for Y. Regression analysis and review of the model are conducted using Minitab, a commonly used analytical tool. The models related to both base factor levels and

base outcomes are presented in Table 5. Figure 5 illustrates the surface plots for the outcomes, providing a clear sight of the parameter spaces.

Table 5. Level of factors.

BTE, %	$7.617 + 9.205 \text{ BMEP (bar)} + 0.0125 \text{ Blend \%} - 0.7931 \text{ BMEP (bar)} \cdot \text{BMEP (bar)} + 0.000171 \text{ Blend \%} \cdot \text{Blend \%} + 0.00136 \text{ BMEP (bar)} \cdot \text{Blend \%}$
BSFC, g/kWh	$580.7 - 121.40 \text{ BMEP (bar)} + 0.494 \text{ Blend \%} + 11.001 \text{ BMEP (bar)} \cdot \text{BMEP (bar)} - 0.0020 \text{ Blend \%} \cdot \text{Blend \%} + 0.0052 \text{ BMEP (bar)} \cdot \text{Blend \%}$
NO_x, g/kWh	$0.0390 + 0.08241 \text{ BMEP (bar)} - 0.001336 \text{ Blend \%} - 0.010425 \text{ BMEP (bar)} \cdot \text{BMEP (bar)} - 0.000002 \text{ Blend \%} \cdot \text{Blend \%} + 0.000165 \text{ BMEP (bar)} \cdot \text{Blend \%}$
HC, g/kWh	$0.0410 - 0.00871 \text{ BMEP (bar)} + 0.000486 \text{ Blend \%} + 0.00346 \text{ BMEP (bar)} \cdot \text{BMEP (bar)} - 0.000013 \text{ Blend \%} \cdot \text{Blend \%} + 0.000816 \text{ BMEP (bar)} \cdot \text{Blend \%}$
CO, g/kWh	$1.315 - 0.991 \text{ BMEP (bar)} + 0.0308 \text{ Blend \%} + 0.3374 \text{ BMEP (bar)} \cdot \text{BMEP (bar)} - 0.000279 \text{ Blend \%} \cdot \text{Blend \%} - 0.01256 \text{ BMEP (bar)} \cdot \text{Blend \%}$
Smoke, %	$7.29 - 2.50 \text{ BMEP (bar)} + 0.279 \text{ Blend \%} + 2.977 \text{ BMEP (bar)} \cdot \text{BMEP (bar)} - 0.00358 \text{ Blend \%} \cdot \text{Blend \%} - 0.2121 \text{ BMEP (bar)} \cdot \text{Blend \%}$

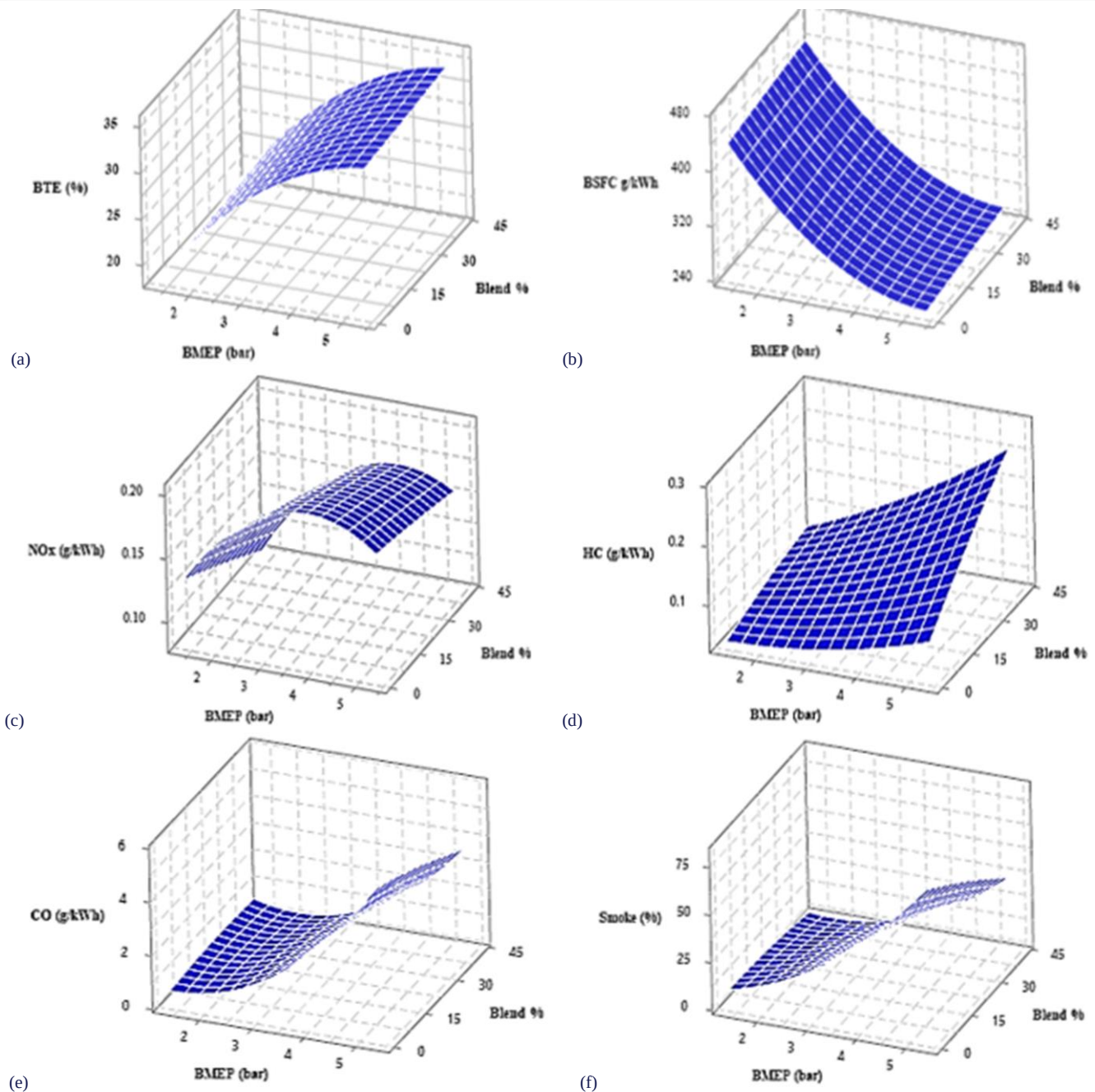


Fig. 5. Surface plots for responses (a) BTE, (b) BSFC, (c) NO_x, (d) HC, (e) CO (g/kWh) and (f) smoke opacity (%) vs. BMEP (bar).

The series of 3D surface plots provides a comprehensive analysis of the effects of BMEP and biodiesel blend percentage on various efficiency and exhaust features. BTE increases with BMEP, peaking at around 35% for moderate biodiesel blends (20–30%) at 5 bar, but remains below 25% at light loads (below 2 bar) due to inefficient combustion. Higher blends (above 40%) slightly reduce BTE due to lower calorific value, while D100 peaks at 33%, and JB30 achieves a maximum of 35%, showcasing the efficiency advantage of moderate blending. The brake specific fuel consumption (BSFC) decreases with increasing BMEP, dropping from 450 g/kWh at 1.5 bar to around 240 g/kWh at 5 bar, but increases with higher biodiesel blends due to lower calorific value, with JB40 showing 10–15% higher BSFC than D100 at all BMEP levels. NO_x emissions rise with both BMEP and blend percentage, peaking at 0.2 g/kWh for JB40 at 5 bar due to higher in-cylinder temperatures, while moderate blends like JB20 offer a balanced performance, producing

0.15–0.18 g/kWh at higher loads, and D100 shows the lowest NO_x emissions, particularly at light loads. Hydrocarbon (HC) emissions increase with BMEP, and are the lowest for the 0% blend, reaching around 0.05 g/kWh at 2 bar but rising with higher loads and blends. Carbon monoxide (CO) emissions also increase with BMEP, exceeding 6 g/kWh for higher blends at 5 bar, while remaining negligible at low BMEP for minimal blends. Similarly, smoke emissions intensify with rising BMEP and blend percentage, surpassing 50% at 4 bar and peaking near 75% for a 45% blend, emphasising the compounding impact of engine load and biodiesel blending on particulate formation and burning features.

Table 6 presents the R² values for these models. To be suitable for optimisation, the R² values should approach 1 (100%). In this case, it is concluded that the factors included in the model sufficiently explain the variation in the response.

Table 6. Computed values for the coefficient of determination.

Determination of coefficient	Outcomes					
	BTE (%)	BSFC (g/kWh)	NO _x (g/kWh)	HC (g/kWh)	CO (g/kWh)	Smoke (%)
R ² (%)	99.71	99.17	96.23	98.24	98.43	97.34
R ² (%) predicted	99.41	98.49	89.48	95.53	96.29	93.37
R ² (%) adjusted	99.61	98.87	94.88	97.61	97.87	96.39

Table 6 presents the computed coefficients of determination (R², adjusted R², and predicted R²) obtained using Minitab. To ensure the models' validity, their significance must be evaluated. This is achieved through ANOVA (analysis of variance), a statistical method that uses the F-test to assess the consequence of

regression models. As shown in Table 7, the p-values for all models are 0.000, confirming the statistical significance of the regression models. Table 8 provides the tuning factors of the WSA approach along with their description [15].

Table 7. Regression models for all responses.

	BTE (%)	BSFC (g/kWh)	F-value	p-value
Cause	F-value	p-value		
Model	978.80	0.000	332.97	0.000
Linear	2306.71	0.000	771.34	0.000
BMEP (bar)	4597.96	0.000	1531.31	0.000
Blend %	15.46	0.002	11.37	0.002
Square	140.24	0.000	61.08	0.000
BMEP (bar) × BMEP (bar)	280.37	0.000	122.13	0.000
Blend (%) × Blend (%)	0.11	0.745	0.03	0.859
2 way interaction	0.11	0.741	0.00	0.952
BMEP (bar) × Blend %	0.11	0.741	0.00	0.952

	NO _x (g/kWh)		HC(g/kWh)		CO(g/kWh)		Smoke (%)	
Cause	F-value	p-value	F-value	p-value	F-value	p-value	F-value	p-value
Model	71.43	0.000	156.10	0.000	175.99	0.000	102.39	0.000
Linear	115.49	0.000	356.37	0.000	73.61	0.000	231.41	0.000
BMEP (bar)	181.30	0.000	424.88	0.000	716.97	0.000	397.00	0.000
Blend %	49.68	0.002	287.87	0.002	30.25	0.000	65.82	0.000
Square	61.00	0.000	4.33	0.000	55.82	0.000	14.55	0.000
BMEP (bar) × BMEP (bar)	121.96	0.000	7.74	0.000	111.00	0.438	28.75	0.000
Blend (%) × Blend (%)	0.04	0.745	0.93	0.859	0.64	0.000	0.35	0.563
2 way interaction	4.20	0.741	59.07	0.952	21.10	0.000	20.02	0.001
BMEP (bar) × Blend %	4.20	0.741	59.07	0.952	21.10		20.02	0.001

Table 8. Control parameters of WSA algorithm.

Elements	Description	Standard
MaxIter	Max. iteration number	40,000
Ns	Solutions generated in each cycle	40
τ	Adjustable elements	0.7
Φ	Adjustable elements	0.0001
Λ	Adjustable elements	0.755
Sl ₀	Primary step size	0.001

The WSA approach determined the tuned variable levels to be $X_1 = 40$ (unscaled variable level: 1) and $X_2 = 1.648056$ (unscaled variable level: 0.944899). However, since $X_2 = 1.648056$ is not practical, it has been adjusted to 1.6 (unscaled variable

level: 0.96) by rounding to the nearest suitable value.

The given dataset in Table 9 represents optimal engine settings due to its balanced performance and emissions characteristics.

Table 9. Validation of response optimisation.

Blend (%)	BMEP (bar)	BTE %	BSFC g/kWh	NO _x g/kWh	HC g/kWh	CO g/kWh	Smoke (%)
40	2.74091	27.812	347.72	0.1480	0.1311	0.5448	4.9815

A BMEP of 2.74 bar reflects a moderate torque output, ensuring sufficient engine performance without overstressing components or increasing fuel consumption excessively. The brake thermal efficiency (BTE) of 27.81% indicates effective energy conversion, striking a good balance between the fuel efficiency and power output. While BSFC of 347.72 g/kWh suggests relatively high fuel consumption, it reflects a compromise necessary to maintain lower emissions and engine reliability. The low NO_x emissions (0.148 g/kWh) highlight effective control of combustion temperatures, reducing harmful pollutants to meet environmental standards. Similarly, moderate HC (0.1311 g/kWh) and CO (0.5448 g/kWh) levels demonstrate reasonably complete combustion while preserving engine efficiency. Finally, the low smoke percentage of 4.98% indicates minimal particulate emissions, ensuring compliance with air quality standards.

4. Conclusions

- Diesel (D100) achieves the ultimate BTE of 38% at 5.04 bar, with BSFC around 200 g/kWh, while biodiesel blends like JB10 and JB20 follow closely with BTE values of 37–38% and BSFC of 210–220 g/kWh. At a lower BMEP (1.35 bar), BTE for all fuels ranges between 15–17%, with biodiesel blends like JB40 showing the largest drop and higher BSFC (450–470 g/kWh).
- NO_x and HC emissions rise with BMEP and biodiesel content, with biodiesel blends (especially JB30 and JB40) showing higher NO_x emissions due to increased combustion temperatures.
- CO emissions and smoke opacity increase significantly at a higher BMEP, with JB40 peaking at 5.5 g/kWh for CO and 70% for smoke opacity, indicating incomplete combustion in higher biodiesel blends.

- The optimal engine settings with a balanced performance profile achieve a moderate torque (BMEP of 2.74 bar) and effective energy conversion (BTE of 27.81%).
- The relatively high BSFC (347.72 g/kWh) is balanced by low emissions of CO (0.5448 g/kWh), NO_x (0.148 g/kWh), and smoke opacity (4.98%), ensuring both efficiency and environmental compliance. These settings optimise engine performance while adhering to emissions standards.
- The WSA algorithm was applied to determine the optimal factor levels based on the regression models. Subsequent confirmation tests validated these optimised settings, showing that BTE was maximised while emissions and other response parameters were effectively minimised.

References

- [1] Stančin, H., Mikulcic, H., Wang, X., & Duic, N. (2020). A review on alternative fuels in future energy system. *Renewable and Sustainable Energy Reviews*, 128:109927. doi: 10.1016/j.rser.2020.109927
- [2] Zhang, Z., Wang, S., Pan, M., Lv, J., Lu, K., Ye, Y., & Tan, D. (2024). Utilization of hydrogen-diesel blends for the improvements of a dual-fuel engine based on the improved Taguchi methodology. *Energy*, 292, 130474. doi: 10.1016/j.energy.2024.130474
- [3] Debella, H.A., Ancha, V.R., Atnaw, S.M., & Zeleke, D.S. (2024). Experimental study on performance and emissions from Prosopis juliflora biodiesel blends with diethyl ether additives. *Energy Conversion and Management*: X, 22, 100581. doi: 10.1016/j.ecmx.2024.100581
- [4] Jaiganesh, J., Prakash, R., & Krishnan, M.G. (2024). Exploring the synergistic potential of prosopis juliflora and waste plastic oil biodiesel through investigation of performance, combustion, and emission in a CRDI engine: A shift towards Sustainable Fuel. *Process Safety and Environmental Protection*, 184, 720–735. doi: 10.1016/j.psep.2024.02.036
- [5] Kukana, R., & Jakhar, O.P. (2021). Synthesis of biodiesel from Prosopis juliflora and using MCDM analytical hierarchy process

- technique for evaluating with different biodiesels. *Cogent Engineering*, 8(1). doi: 10.1080/23311916.2021.1957291
- [6] Sasikumar, R., Sankaranarayanan, G., & Karthikeyan, R. (2022). Investigation characteristics of Prosopis juliflora biodiesel blended with diesel fuel in a DI diesel engine. *Australian Journal of Mechanical Engineering*, 20(3), 692–697. doi: 10.1080/14484846.2020.1740021
- [7] Kumar, S., Ningegowda, B.A., & Raju, V.D. (2022). The combined influence of injection pressure and exhaust gas recirculation on the characteristics of the diesel engine fuelled with Juliflora Biodiesel. *International Journal of Ambient Energy*, 43(1), 7952–7961. doi: 10.1080/01430750.2022.2075929
- [8] Selvan, P.T., & Goteti, G.S. (2020). Experimental analysis of VCR engine operated with Prosopis juliflora biodiesel blends. *International Journal of Renewable Energy Research*, 10(2). doi: 10.20508/ijrer.v10i2.10776.g7965
- [9] Abbas, A.M., Alomran, M.M. Alharbi, N.K. & Novak, S.J. (2023). Suppression of Seedling Survival and Recruitment of the Invasive Tree Prosopis juliflora in Saudi Arabia through Its Own Leaf Litter: Greenhouse and Field Assessments. *Plants*, 12(4). doi: 10.3390/plants12040959
- [10] Vinodraj, S., Arularasan, R., Karthikeyan, D., Sethuraman, N., & Suryavarman, K. (2023). Impact of non-edible biodiesel (Juliflora oil seed) on CI engine – combustion, performance and emission features. *Materials Today: Proceedings*. doi: 10.1016/j.matpr.2023.05.306
- [11] Musthafa, B., Saravanan, B., Vaibhav, B., Muhammad, P., Sam, F., & Asokan, M.A. (2023). Performance and emission analysis of a diesel engine fuelled with Juliflora Biodiesel: A simulation and experimental study. *Australian Journal of Mechanical Engineering*, 23(1), 220–234. Doi: 10.1080/14484846.2023.2274999
- [12] Musthafa, B., & Asokan, M.A. (2024). Numerical and experimental investigation of performance and emission characteristics of hydrogen enrichment with prosopis juliflora biodiesel in CI engine using RSM and ANN optimization. *International Journal of Hydrogen Energy*, 66, 326–336. doi: 10.1016/j.ijhydene.2024.04.083
- [13] Duraisamy, B., Velmurugan, K., & Venkatachalapathy, V.S. (2022). Analysis of prosopis juliflora methyl ester as a fuel in the CI engine. *International Journal of Research and Review*, 9(7), 112–118. doi: 10.52403/ijrr.20220713
- [14] Mason, R.L., Gunst, R.F., & Hess, J.L. (2003). *Statistical design and analysis of experiments: with applications to engineering and science* (2nd ed.). John Wiley & Sons.
- [15] Simsek, D., Ozyurek, D., Ileri, E., Akpınar, S., Karaoglan, A.D. (2022). Optimizing of Wear Performance on Elevated Temperature of ZrO₂ Reinforced AMCs Using Weighted Superposition Attraction Algorithm. *Journal of Scientific & Industrial Research*, 81, 462–474. doi: 10.56042/jsir.v81i05.47027
- [16] Baykasoğlu, A., & Akpınar, S. (2015). Weighted Superposition Attraction (WSA): A swarm intelligence algorithm for optimization problems–Part 2: Constrained optimization. *Applied Soft Computing*, 37, 396–415. doi: 10.1016/j.asoc.2015.08.052

Experimental and numerical research on the impact of design solutions on the internal heat exchanger operating parameters

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Received: 25.03.2025; revised: 27.08.2025; accepted: 31.08.2025

Abstract

Research aimed at increasing the efficiency of heat exchangers used in car air conditioning systems may lead to modifications in the design of refrigeration systems. One such modification involves the use of smaller gas coolers, which directly reduces the overall production costs. This study presents an experimental investigation into the impact of constructional solutions on the performance of an internal heat exchanger using propylene glycol as a refrigerant. Two internal heat exchangers with different outer channel geometries were analysed. The experimental study was complemented by numerical simulations using the computational fluid dynamics method. The results showed that modifying the outer channel geometry from a parallel to a twisted configuration led to a 5.06% increase in cooling efficiency, a reduction of the outlet temperature by 2.2°C and a nearly 72 W increase in the total heat exchange rate. Numerical simulations confirmed the experimental findings, demonstrating good agreement with the measured data.

Keywords: Heat exchanger; IHX; Cooling efficiency; Numerical fluid mechanics; Numerical geometry

Vol. 46(2025), No. 4, 127–133; doi: 10.24425/ather.2025.156843

Cite this manuscript as: Janus, J., Skotniczny, P., Richert, M., & Długopolski, J. (2025). Experimental and numerical research on the impact of design solutions on the internal heat exchanger operating parameters. *Archives of Thermodynamics*, 46(4), 127–133.

1. Introduction

An internal heat exchanger (IHX) is a device that transfers heat between a high-pressure liquid stream and a low-pressure vapour stream. The liquid refrigerant exiting the condenser flows through the inner tube, while the vapour from the evaporator circulates in the outer channel. By lowering the temperature of the liquid before it enters the expansion devices, the IHX improves the overall performance of the refrigeration cycle.

With the progressive miniaturisation of refrigeration systems, improving the effectiveness of heat exchangers has become essential. This challenge has encouraged researchers to explore different approaches to enhance internal heat recovery. Methods include modifications of heat transfer mechanisms, the

use of alternative refrigerants, and the redesign of exchanger geometry. Previous studies [1–5] demonstrated that altering the geometry of plate and internal heat exchangers can significantly improve heat transfer, enabling the development of more energy-efficient heating, ventilation and air conditioning (HVAC) systems. Similar solutions are increasingly applied in automotive refrigeration as well.

Figure 1 illustrates the layout of an automotive air-conditioning cycle. The installation comprises a compressor, a condenser and an evaporator connected in a closed refrigerant loop. After being compressed, the working fluid flows to the condenser, where it is cooled and condensed into liquid form with the assistance of forced air provided by fans. The liquid refrigerant then

Nomenclature

c_p	– specific heat, J/(kg·K)
L	– curve total length, m
q	– total heat transfer rate, W
Q	– mass flow rate, kg/s
r	– cylinder radius, m
s	– distance of the forming cylinder between the nearest points of the helix, m
t	– parameter of moving along the trajectory from 0 to 1
T	– temperature, °C

y	– Cartesian coordinate, m
z	– Cartesian coordinate, position in the axis, m

Greek symbols

θ	– rotation angle of the pull-out surface, deg
η	– heat exchanger efficiency, %

Abbreviations and Acronyms

IHX	– internal heat exchanger
CFD	– computational fluid dynamics
FPGA	– field programmable gate array

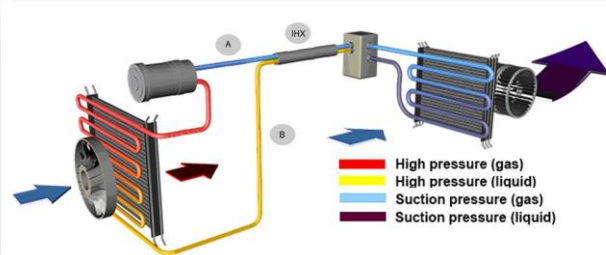


Fig. 1. Scheme of the car air-conditioning system [6].

passes through the IHX on its way to the evaporator, where a sharp temperature drop occurs during the expansion process. The function of the IHX is to improve system efficiency by transferring heat between the liquid stream in tube B and the colder vapour stream in tube A, thereby subcooling the liquid before expansion.

Recent studies also focus on innovative heat exchanger designs. Among the proposed solutions are the microjet configurations [6] and technologies based on mini- and microchannels [7,8]. These approaches enable a significant enhancement of the heat transfer coefficient within the exchanger.

Computational tools, particularly numerical simulations based on the finite volume methods (FVMs), have become a powerful instrument in heat exchanger research and design. Computational fluid dynamics (CFD) makes it possible to analyse complex processes, including flow behaviour, temperature field and stress distributions, even under challenging conditions, such as underground mining environments [9–12]. One of the key strengths of CFD is its ability to represent phenomena in both two- and three-dimensional domains. A comprehensive review of literature on CFD applications in the heat exchanger design is provided in [13], where the authors, after examining 66 publications, emphasised the cost-effectiveness and versatility of such simulations. They demonstrated that CFD can support the design process at various stages – from preliminary flow predictions to final optimisation of exchanger geometry. Numerical studies [2,14] apply CFD specifically to internal heat exchangers in refrigeration systems using CO₂, while other investigations address microchannel radiators in automotive air conditioning applications [15].

In recent years, significant progress has been made in the development of methods aimed at increasing the efficiency of heat

transfer systems, particularly through the implementation of innovative designs, advanced simulations and the use of alternative working media. In [16], the use of machine learning was proposed for reconstructing multiphase fluid structures based on capacitive sensor data, showing the potential of intelligent diagnostics and control in thermal systems.

Experimental studies presented in [17] focused on the performance of a pumping engine in a micro organic Rankine cycle (micro-ORC) system utilising a low-boiling working fluid, demonstrating the influence of thermodynamic cycle design on the overall system efficiency. The study in [18] analysed the operation of a hybrid heating system based on heat pumps integrated with a photovoltaic installation, highlighting the growing role of renewable energy sources in thermal system optimisation.

In [19], a coupled heat, mass, and momentum transfer model was developed for a refrigerated storage chamber, providing valuable insights into the thermal behaviour of complex cooling environments. Furthermore, numerical simulations presented in [20] compared the effects of different baffle configurations in rectangular heat exchanger channels, confirming that modifications in internal geometry can significantly enhance heat transfer performance.

These studies emphasise the importance of combining experimental measurements with numerical modelling and intelligent analysis methods to design more efficient heat exchangers. The current research continues this trend by evaluating the impact of twisted outer channel geometry on the thermal efficiency of an internal heat exchanger operating with propylene glycol.

2. Geometry of internal heat exchangers and refrigerant selection

The simulations were performed for two tube-in-tube counter-flow heat exchanger configurations, as illustrated in Fig. 2. In the first variant (model I), sixteen outer channels of the cooled medium were arranged in parallel to the central heating channel. In the second variant (model II), those outer channels were helically wound around the inner tube. In both variants, the exchanger design includes a central channel with a diameter of 16.4 mm, surrounded by sixteen outer channels. The cross-sectional shape and dimensions of these outer channels are presented in Fig. 3. Each considered heat exchanger type had an external radius of 12.5 mm and a total length of 1.0 m.

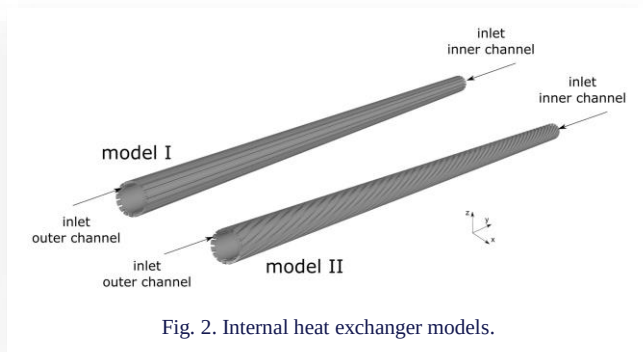


Fig. 2. Internal heat exchanger models.

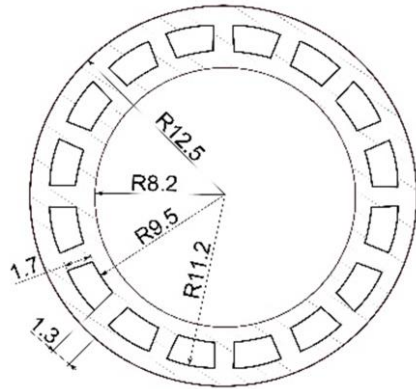


Fig. 3. Heat exchanger cross-section.

In model I, the outer channels were designed as straight prismatic channels. In contrast, model II employed helically wound outer channels, generated according to a parametric curve equation. The trajectory of this curve was implemented through a surface-extraction script in the pre-processing software:

$$s = 2\pi r \text{ctg}(\theta), \quad (1)$$

$$\theta = \frac{L}{s} t \cdot 360, \quad (2)$$

$$z = tL, \quad (3)$$

where: s – distance of the forming cylinder between the nearest points of the helix, r – radius of the cylindrical surface defining the outer channel on which the helical trajectory is generated, θ – rotation angle of the pull-out surface, L – total length of the curve, t – parameter of moving along the trajectory from 0 to 1, z – position in the axis.

The refrigerant used in research was the antifreeze propylene glycol InnovaTherm P/P-ALU (Table 1). It is an environmentally friendly liquid that is safe for humans. Additionally, it contains corrosion inhibitors for aluminium installations, extending their service life.

3. Laboratory research

The main purpose of laboratory tests [22] was to record the medium temperature at the inlet and outlet of the exchanger channels in various geometries. The assumptions for the laboratory station were to guarantee the possibility of testing the flow of liquid inside the IHX in the opposite direction.

Table 1. Selected properties of InnovaTherm P/P-ALU liquid [21].

Fluid	Crystallisation temperature (°C)	Boiling point (°C)	pH scale	Specific weight (kg/m ³)	Dynamic viscosity at 20 °C (m Pa·s)	Thermal conductivity (W/(m K))
propylene glycol	–15	103	7.5–9.5	1020	3.15	0.457

To enable liquid temperature measurements in outer channels for both exchanger models, it was decided to use the connector, which was made by 3D printing technology. The connector is the end element of the exchanger, which directs the flow of liquid to the measuring point. Also, it performs a sealing function. Design of a measuring station is shown in Fig. 4. Thermostats with parameters shown in Table 2 were used as the source of heat and cold.

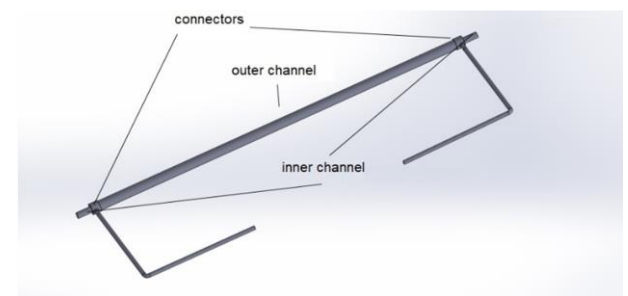


Fig. 4. Design of a measuring station.

Table 2. Parameters of thermostats.

Thermostat	Temperature range (°C)	Pump pressure (bar)	Max. pump flow (l/min)
Lauda A100	–20–100	0.4	17
Lauda WKL 1000	–10–40	1.0	30

The ends of the inner aluminium tube were threaded and then metal ends were placed at their ends to allow the system to be connected by a quick coupling system. Short conductive aluminium tubes were mounted on the ends of the outer tube to allow temperature reading. Temperature sensors were placed in selected locations. During the tests, the influence of external temperature on the measurements was limited by applying insulation foams.

To ensure the simultaneous temperature measurement at the inlet and outlet of the exchanger, it was decided to use digital DS18B20 temperature sensors produced by MAXIM-DALLAS Company. The temperature measurement range of the sensors was from –55°C to 125°C, with a measurement accuracy declared by the manufacturer of 0.5%. A programmable system logic type FPGA (field programmable gate array) Cyclone III manufactured by INTEL Company [22] was used. The extension

of the laboratory station enabled testing of two internal heat exchangers simultaneously. Both stations were connected in parallel, which enabled testing both heat exchangers under the most similar conditions. The complex set is shown in Fig. 5. Operating parameters of the thermostats during laboratory test are presented in Table 3.



Fig. 5. Laboratory station for two internal heat exchangers (IHXs).

Table 3. Thermostat operating parameters during laboratory test.

Channel type	Pump flow (l/min)	Velocity (m/s)	Mass flow rate (kg/s)	Temperature (°C)
Inner channel	20	1.58	0.35	19.60
Outer channel	12	2.69	0.013	58.60

4. Numerical calculations

The next stage of research on the IHX cooling efficiency for various geometries was numerical tests using computational fluid dynamics. In order to obtain correct numerical calculation results, it was necessary to apply a dense calculation mesh. It was decided to use the mesh size function, which allows control of the size of the numerical mesh around the selected point, edge or surface [23]. A three-dimensional model of the internal heat exchanger was digitised with an unstructured tetrahedral mesh,

which then, due to the large size of the model and selection of a dense computing mesh, was converted into a polyhedral mesh. The advantage of using this type of mesh is to obtain more accurate results by converting bad cells and obtaining the final results faster due to a smaller number of cells compared to the tetrahedral mesh. Mesh conversion was made using the Ansys Fluent 2020 software. The mesh consisted of 2.15 million elements for each geometric model. For numerical calculations, the $k-\varepsilon$ turbulent model was used [23].

In the studied issue, there are two media: a liquid in the form of propylene glycol and a solid in the form of aluminium alloy. Their properties, presented in Table 4, were defined based on the Ansys Fluent software material database.

Boundary conditions for inner and outer channels were defined as a flat inlet velocity profile. The value of velocity was selected according to the pump flow parameters (Table 5).

Table 4. Properties of used materials.

Material	Density (kg/m ³)	Thermal conductivity coefficient (W/(m·K))	Kinematic viscosity (kg/(m·s))	Specific heat (J/(kg·K))
Propylene glycol	1040	0.457	0.0157	2510
Aluminium alloy	2719	202.4	–	871

Table 5. Boundary conditions for numerical simulation.

Channel type	Pump flow (l/min)	Velocity (m/s)	Mass flow rate (kg/s)	Temperature (°C)
Inner channel	20	1.58	0.35	19.60
Outer channel	12	2.69	0.013	58.60

5. Results of laboratory tests and numerical calculations

The results of temperature measurements at the inlet and outlet of the channels in models I and II, carried out on a laboratory model, are presented as averaged temperature values from 30 measurement obtained after the temperature stabilisation, as shown in Tables 6 and 7, where: T_{R1} – refrigerant temperature at the hot inlet of the outer channel, T_{R1}' – refrigerant temperature at the hot outlet of the outer channel in model I, T_{R1}'' – refrigerant temperature at the hot outlet of the outer channel in model II, T_{R2} – refrigerant temperature at the cold inlet of the inner channel, T_{R2}' – refrigerant temperature at the cold outlet of the inner channel in model I, T_{R2}'' – refrigerant temperature at the cold outlet of the inner channel in model II.

The results of laboratory measurements allow us to notice changes in the temperature value at the outlet of the outer channel, caused by a change in the heat exchanger geometry. Twisting the outer channels, in which the heating medium flows, caused the decrease in temperature at the outlet by 2.2°C. The obtained temperature at the outlet of the outer channel in model I is 43.0°C (Table 6), in model II this temperature drops to 40.80°C (Table 7).

The results of numerical calculations of the refrigerant temperature distribution in models I and II of the IHX are presented

Table 6. Laboratory measurement results – refrigerant temperature at the inlet and outlet of model I channels.

Inner channel	Outer channel
cold inlet	hot inlet
$T_{R2} = 19.60^{\circ}\text{C}$	$T_{R1} = 58.60^{\circ}\text{C}$
cold outlet	hot outlet
$T_{R2}' = 40.20^{\circ}\text{C}$	$T_{R1}' = 43.00^{\circ}\text{C}$

Table 7. Laboratory measurement results – refrigerant temperature at the inlet and outlet of model II channels

Inner channel	Outer channel
cold inlet	hot inlet
$T_{R2} = 19.60^{\circ}\text{C}$	$T_{R1} = 58.60^{\circ}\text{C}$
cold outlet	hot outlet
$T_{R2}'' = 39.20^{\circ}\text{C}$	$T_{R1}'' = 40.80^{\circ}\text{C}$

in the temperature diagram in the inner and outer channels. In addition, the results are presented in the form of temperature contours. The initial flow parameters at the inlet of the inner channel, outer channel and calculated parameters at the outlet are presented in tabular form [24].

Figure 6 shows the temperature distribution at the exchanger cross-section $y = 0$ m. Due to the counter-current flow configuration, this plane corresponds to the inlet of the outer channel and the outlet of the inner channel. The figure illustrates the difference in refrigerant temperature between model I and model II. As the medium flows through the outer channels from $y = 0$ m to $y = 1$ m, the influence of the lower temperature in the inner channels becomes evident, leading to a reduction in the refrigerant temperature in the outer channel. At the outlet of the outer channel ($y = 1$ m), the influence of the twisted outer channels (model II) is noticeable, resulting in a lower refrigerant temperature compared to model I. The heating effect of refrigerant flowing through the inner channel in model II is slightly larger compared to model I. During the temperature contours analysis, a higher temperature of refrigerant was observed near the channel wall compared to the temperature in the central part of the channel cross-section (Fig. 7). This is due to the way of medium flows. During the fluid flow in closed channels, velocity and temperature profiles are created in subsequent cross-sections. The maximum value of velocity is located in the middle flow

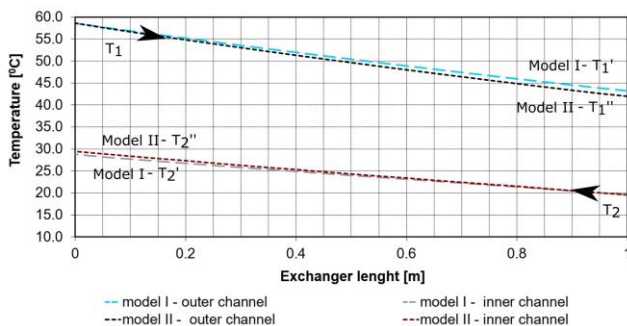


Fig. 6. Numerical simulation results – temperature distribution in the inner and outer channels.

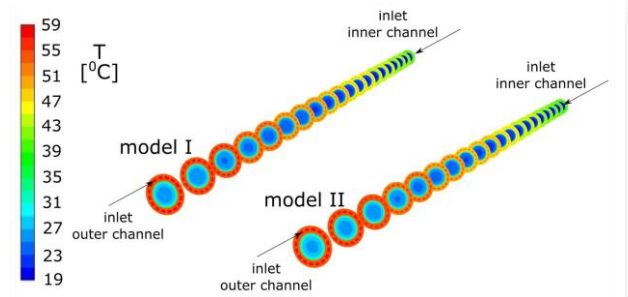


Fig. 7. Numerical simulation results – temperature contours for model I and II.

zone, which is a potential core, within which the lowest heat exchange occurs. For both models I and II, the biggest changes in temperature values occur at the end cross-section of the inner and outer channels. It is caused by the occurrence of the largest temperature gradient in these cross-sections.

The changes observed in the potential core flow region in the inner channel indicate an increase in the cooling effect in model II. These differences are visible in Tables 8 and 9, where: T_1 – refrigerant temperature at the hot inlet of the outer channel, T_1' – refrigerant temperature at the hot outlet of the outer channel in model I, T_1'' – refrigerant temperature at the hot outlet of the outer channel in model II, T_2 – refrigerant temperature at the cold inlet of the inner channel, T_2' – refrigerant temperature at the cold outlet of the inner channel in model I, T_2'' – refrigerant temperature at the cold outlet of the inner channel in model II. For the same boundary conditions, the difference in temperature obtained at the inner channel outlet between model I and model II is 0.64°C . In the outer channel, the temperature at the outlet decreased due to a geometry change from 43.23°C to 41.94°C .

The temperature distribution in the outer channel, obtained using the numerical method, corresponds to the temperature obtained during the laboratory test. For the same boundary conditions, at the outlet of the outer channel in model I, the measured temperature is 43.00°C , while the temperature obtained by the numerical method is 43.23°C . In the case of results for model II, the difference between the obtained results from numerical calculation and measurements increases to 1.14°C .

Table 8. Numerical calculation results – refrigerant temperature at the inlet and outlet of model I channels.

Inner channel	Outer channel
cold inlet	hot inlet
$T_2 = 19.60^{\circ}\text{C}$	$T_1 = 58.60^{\circ}\text{C}$
cold outlet	hot outlet
$T_2' = 28.81^{\circ}\text{C}$	$T_1' = 43.23^{\circ}\text{C}$

Table 9. Numerical calculation results – refrigerant temperature at the inlet and outlet of model II channels.

Inner channel	Outer channel
cold inlet	hot inlet
$T_2 = 19.60^{\circ}\text{C}$	$T_1 = 58.60^{\circ}\text{C}$
cold outlet	hot outlet
$T_2'' = 29.45^{\circ}\text{C}$	$T_1'' = 41.94^{\circ}\text{C}$

Much larger discrepancies between the results of laboratory tests and numerical calculations were obtained for the inner channel. For model I, the measured temperature at the outlet of the inner channel is 40.20°C, while the temperature obtained by numerical calculations is 28.81°C. For model II, the measured temperature at the outlet of the inner channel is 39.20°C, while the numerical calculations show a temperature of 29.45°C.

Such significant differences in temperature indications for the inner channel can be caused by the temperature sensors' location in the channel cross-section. Analysing the graph in Fig. 8, which shows the temperature distribution in the inner channel cross-section in model II, the lower temperature (26.00°C) around the channel axis can be observed. However, near the inner channel wall, this temperature increases to around 55.00°C.

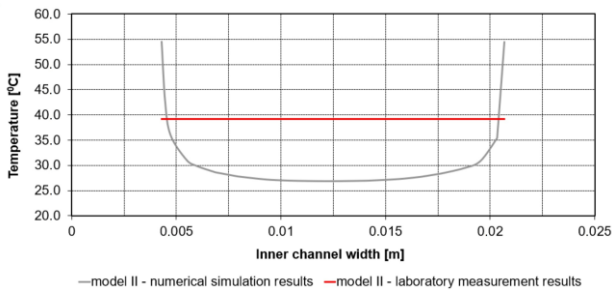


Fig. 8. Temperature distribution in the inner channel cross section, model II.

The temperature obtained from numerical simulations is the average temperature for the entire cross-section of the inner channel, while the measurement results indicate the temperature at one point in the cross-section. This temperature varies depending on the selection of the measuring point.

The heat exchanger efficiency for models I and II was calculated using the following formulas:

$$\begin{aligned}\eta_{RI} &= \frac{T_{R1} - T_{R1'}}{T_{R1} - T_{R2}} \cdot 100\%, \\ \eta_{RII} &= \frac{T_{R1} - T_{R1}''}{T_{R1} - T_{R2}} \cdot 100\%, \\ \eta_I &= \frac{T_1 - T_1'}{T_1 - T_2} \cdot 100\%, \\ \eta_{II} &= \frac{T_1 - T_1''}{T_1 - T_2} \cdot 100\%,\end{aligned}\quad (4)$$

where η_{RI} and η_{RII} are the heat exchanger efficiency for model I and model II, respectively, both calculated based on the laboratory measurements, and η_I and η_{II} represent the heat exchanger efficiency for model I and model II, respectively, calculated based on numerical simulation results.

The heat transfer rate may be obtained from the overall energy balance for the hot fluid. The total heat transfer rates for models I and II were calculated using the following formulas:

$$\begin{aligned}q_{RI} &= Q_m c_p (T_{R1} - T_{R1}'), \\ q_{RII} &= Q_m c_p (T_{R1} - T_{R1}''), \\ q_I &= Q_m c_p (T_1 - T_1'), \\ q_{II} &= Q_m c_p (T_1 - T_1''),\end{aligned}\quad (5)$$

where: $Q_m = 0.013 \text{ kg/s}$, $c_p = 2510 \text{ J/(kg}\cdot\text{K)}$; q_{RI} is the total heat transfer rate for model I, calculated based on laboratory measurements; q_{RII} – total heat transfer rate for model II, calculated based on laboratory measurements; q_I – total heat transfer rate for model I, calculated based on numerical calculation results; q_{II} – total heat transfer rate for model II, calculated based on numerical calculation results.

The calculated heat exchanger efficiency and total heat transfer rate are presented in Table 10, where: RI – results of laboratory measurements for model I, RII – results of laboratory measurements for model II, I – results of numerical calculations for model I, II – results of numerical calculations for model II.

Table 10. Summary of the calculated exchanger efficiency and total heat transfer rate for model I and II.

	Heat exchanger efficiency (%)	Total heat transfer rate (W)
RI	40.00	509.03
RII	45.06	580.81
I	39.41	501.52
II	42.74	543.62

The results show that the changes in geometry of the heat exchanger outer channels increase the efficiency of heat exchange by 5.06%. This efficiency was calculated based on laboratory measurements. As regards the numerical simulation results, the difference in the calculated efficiency between models I and II was 3.33%.

For the operating conditions presented above, the calculated heat transfer rate based on laboratory measurements is 509.03 W for model I and 580.81 W for model II. In the case of numerical simulation, the calculated heat transfer is equal to 501.52 W for model I and 543.62 W for model II.

4. Conclusions

This paper presents the results of laboratory tests and numerical simulations investigating the impact of constructional modifications on the cooling efficiency of an internal heat exchanger using propylene glycol as a refrigerant. Two heat exchanger models with different outer channel geometries were designed and analysed. In model I, the outer channels were arranged parallel to the inner channels, while in model II, the outer channels were helically twisted around the inner channel.

The experimental results indicate that modifying the geometry of the outer channels led to a 5.06% increase in cooling efficiency when compared to the conventional parallel design. This improvement is attributed to the elongation of the outer channels, which enhanced the heat exchange surface area and improved thermal performance. Additionally, the modification resulted in a 2.2°C lower temperature at the inner channel outlet, further demonstrating the effectiveness of the twisted-channel configuration. The total heat exchange rate also increased by nearly 72 W, which translates into improved efficiency of the car air conditioning system.

The findings from laboratory tests were further validated through numerical simulations, which provided a detailed representation of heat transfer characteristics in both models. The numerical calculations closely aligned with the experimental results, confirming the reliability of the applied methodology. The boundary conditions used in the simulations were consistent with laboratory assumptions, ensuring the accuracy of the computational models.

These results highlight the potential benefits of implementing twisted outer channel designs in heat exchangers for automotive applications. By enhancing heat transfer performance, such modifications contribute to the development of more efficient and cost-effective refrigeration systems, reducing energy consumption and improving overall system reliability.

Acknowledgements

This paper is financed from the statutory funds of the Strata Mechanics Research Institute of the Polish Academy of Sciences.

References

- [1] Furberg, R., Palm, B., Li, S., Toprak, M., & Muhammed, M. (2009). The use of a nano- and microporous surface layer to enhance boiling in a plate heat exchanger. *Journal of Heat Transfer*, 131(10), 101010. doi: 10.1115/1.3180702
- [2] Sanchez, D., Patino, J., Llopis, R., Cabello, R., Torrella, E., & Fuentes, F.V. (2014). New positions for an internal heat exchanger in a CO₂ supercritical refrigeration plant: Experimental analysis and energetic evaluation. *Applied Thermal Engineering*, 63(1), 129–139. doi: 10.1016/j.applthermaleng.2013.10.061
- [3] Wajs, J., & Mikieliewicz, D. (2014). Effect of surface roughness on thermal-hydraulic characteristics of a plate heat exchanger. *Key Engineering Materials*, 597, 63–74. doi: 10.4028/www.scientific.net/KEM.597.63
- [4] Wajs, J., & Mikieliewicz, D. (2016). Influence of a metallic porous microlayer on pressure drop and heat transfer of a stainless steel plate heat exchanger. *Applied Thermal Engineering*, 93, 1337–1346. doi: 10.1016/j.applthermaleng.2015.08.101
- [5] Zhang, F.Z., Jiang, P.X., Lin, Y.S., & Zhang, Y.W. (2011). Efficiencies of subcritical and transcritical CO₂ inverse cycles with and without an internal heat exchanger. *Applied Thermal Engineering*, 31(4), 432–438. doi: 10.1016/j.applthermaleng.2010.09.018
- [6] Wajs, J., Mikieliewicz, D., Fornalik-Wajs, E., & Bajor, M. (2015). Recuperator with microjet technology as a proposal for heat recovery from low-temperature sources. *Archives of Thermodynamics*, 36(4), 49–64. doi: 10.1515/aoter-2015-0032
- [7] Mikieliewicz, D., & Wajs, J. (2017). Possibilities of heat transfer augmentation in heat exchangers with minichannels for marine applications. *Polish Maritime Research*, 24(s1), 133–140. doi: 10.1515/pomr-2017-0031
- [8] Rahman, M.M., Kariya, K., & Miyara, A. (2018). An experimental study and development of new correlation for condensation heat transfer coefficient of refrigerant inside a multiport minichannel with and without fins. *International Journal of Heat and Mass Transfer*, 116, 50–60. doi: 10.1016/j.ijheatmasstransfer.2017.09.010
- [9] Dziurzyński, W., Pałka, T., & Krach, A. (2014). Computer simulation of the propagation of heat in abandoned workings insulated with slurries and mineral substances. *Archives of Mining Sciences*, 59(1), 3–23. doi: 10.2478/amsc-2014-0001
- [10] Janus, J., & Krawczyk, J. (2017). Modeling of the propagation of methane from the longwall goaf, performed by means of a two-dimensional description. *Archives of Mining Sciences*, 59(4), 851–868. doi: 10.2478/amsc-2014-0059
- [11] Janus, J., & Krawczyk, J. (2014). The numerical simulation of a sudden inflow of methane into the end segment of a longwall with Y-type ventilation system. *Archives of Mining Sciences*, 59(4), 941–957. doi: 10.2478/amsc-2014-0065
- [12] Śliwa, T., Golaś, A., Wołoszyn, J., & Gonet, A. (2012). Numerical model of borehole heat exchanger in Ansys CFX software. *Archives of Mining Sciences*, 57(2), 375–390. doi: 10.2478/v10267-012-0024-3
- [13] Mahmood, M., Bhutta, A., Hayat, N., Bashir, M.H., Khan, A.R., Ahmad, K.N., & Khan, S. (2012). CFD applications in various heat exchanger designs: A review. *Applied Thermal Engineering*, 32, 1–12. doi: 10.1016/j.applthermaleng.2011.09.001
- [14] Ituna-Yudonago, J.F., Belman-Flores, J.M., Elizalde-Blancas, F., & García-Valladares, O. (2017). Numerical investigation of CO₂ behavior in the internal heat exchanger under variable boundary conditions of the transcritical refrigeration system. *Applied Thermal Engineering*, 115, 1063–1078. doi: 10.1016/j.applthermaleng.2017.01.042
- [15] Janus, J., Skotniczny, P., & Richert, M. (2019). Modeling of the impact of construction solutions on operating parameters of the internal heat exchanger with refrigerant R744. *Archives of Thermodynamics*, 40(1), 145–160. doi: 10.24425/ather.2019.128295
- [16] Omowski, R., Lackowski, M., & Kwidzinski, R. (2024). Application of machine learning for reconstruction of multiphase fluid structure measured by a capacitance multi-electrode sensor. *Archives of Thermodynamics*, 45(4), 5–12. doi: 10.24425/ather.2024.151993
- [17] Kaczmarczyk, T.Z. (2024). Experimental research of a pumping engine in a micro-ORC system with a low-boiling medium. *Archives of Thermodynamics*, 45(4), 125–140. doi: 10.24425/ather.2024.152002
- [18] Turoń, M.K. (2024). Operation of a hybrid heating system based on heat pumps using a photovoltaic installation. *Archives of Thermodynamics*, 45(4), 153–162. doi: 10.24425/ather.2024.152004
- [19] Kołodziejczyk, M., Śmierciw, K., & Gagan, J., Butrymowicz, D., Jakończuk, P., Pawłowski, M. (2024). Coupled heat, mass and momentum transfer model of a napa cabbage refrigerated storage chamber. *Archives of Thermodynamics*, 45(4), 223–235. doi: 10.24425/ather.2024.152012
- [20] Zidani, C., Benyoucef, B., Didi, F., & Guendouz, N. (2020). Simulation and numerical analysis of a rectangular pipe with transversal baffle – comparison between zigzag and plane baffles. *Archives of Thermodynamics*, 41(4), 269–283. doi: 10.24425/ather.2020.135988
- [21] <http://innova-therm.pl/zabezpieczenie-przed-zamarzaniem/innova-therm-p-p-alu.html> [accessed 24 March 2025].
- [22] Długopolski, J., & Richert, M. (2019). Naturally parallel measuring system based on FPGA hardware. *Journal of Machine Construction and Maintenance*, 113(2), 113–119.
- [23] Ansys Inc. (2016). *Ansys Fluent Users Guide*.
- [24] Li, M.J., Zhang, H., Zhang, J., Mu, Y.T., Tian, E., Dan, D., Zhang, X.D., & Tao, W.Q. (2018). Experimental and numerical study and comparison of performance for wavy fin and a plain fin with radiantly arranged winglets around each channel in fin-and-channel heat exchangers. *Applied Thermal Engineering*, 133, 298–307. doi: 10.1016/j.applthermaleng.2018.01.01



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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Numerical Studies on Symmetrically and Asymmetrically Heated Vertical Channels with Isothermal Walls

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Received: 12.06.2025; revised: 16.09.2025; accepted: 19.09.2025

Abstract

Numerical studies on combined free and forced convection together with radiation in a channel comprising two plates that are vertical and parallel are made. The channel here addresses two cases, viz. symmetric, and asymmetric isothermal walls. The cooling agent could be any gaseous medium that is assumed transparent to thermal radiation. The present work considers air for the job. The equations of conservation of mass, momentum and energy are solved through the finite volume method and an explicit code is prepared for this. The prime independent parameters considered are: aspect ratio, wall emissivity, Richardson number, Grashof number and Reynolds number. The effects exhibited by these parameters on the average convection Nusselt number, and the contributory roles enacted by convection and radiation in discarding the heat in the channel have been elucidated. The correlations for the mean convection Nusselt number and mean radiation Nusselt number are deduced through regression analysis using a large set of data gathered from the computer code that is prepared as part of the present work. The gross errors that occur in the cooling load calculations upon ignoring radiation in problems of this kind are clearly demonstrated.

Keywords: Vertical channel; Isothermal walls; Mixed convection; Radiation

Vol. 46(2025), No. 4, 135–140; doi: 10.24425/ather.2025.156844

Cite this manuscript as: Rao, C.G. (2025). Numerical Studies on Symmetrically and Asymmetrically Heated Vertical Channels with Isothermal Walls. *Archives of Thermodynamics*, 46(4), 135–140.

1. Introduction

Interaction of mixed (combined free and forced) convection with either (or both) of the other two modes of heat transfer, viz. conduction and radiation, continues to attract the attention of the researchers working in heat transfer. This is the case with all kinds of geometries (plates, channels, cavities, and so on). Carpenter et al. [1] reported their numerical studies on the role enacted by radiation along with developing laminar natural convection between vertical plates possessing asymmetric and uniform heat fluxes. Sparrow et al. [2] came out with their numerical calculations on coupled radiation and developing natural

convection in a channel having one isothermal and one adiabatic wall. Anand et al. [3] performed numerical studies on the role exhibited by conduction and natural convection between asymmetrically heated plates subjected to uniform surface temperature. Almost simultaneously, Kim et al. [4] carried out numerical studies on a similar problem for the configuration of uniform surface heat flux. The governing equations were solved by them through the implicit finite difference method. Kim and Anand [5] performed a numerical study on developing laminar flow of a fluid and the accompanying transfer of heat between a string of parallel and conducting plates comprising the heat generating blocks mounted on their walls, with air taken as the coolant.

Nomenclature

A – aspect ratio of the vertical channel, $[L/S]$
 $\bar{C}_f$ – mean friction coefficient
 F_{ij} – view factor from the i^{th} to the j^{th} element of an enclosure
 Gr_S – Grashof number, $(g\beta(T_L - T_\infty)S^3/\nu^2)$
 g – local acceleration due to gravity, 9.81 m/s^2
 G – irradiation, W/m^2
 J – radiosity, W/m^2
 L – channel height, m
 $\bar{Nu}_C(\text{Left})$ – mean convection Nusselt number over the left wall,
 $-(1/A) \int_0^A (\partial\theta/\partial Y)_{Y=0} dX$
 $\bar{Nu}_C(\text{Right})$ – mean convection Nusselt number over the right wall,
 $-(1/A) \int_0^A (\partial\theta/\partial Y)_{Y=1} dX$
 $\bar{Nu}_C$ – mean convection Nusselt number over both walls,
 $\bar{Nu}_C(\text{Left}) + \bar{Nu}_C(\text{Right})$
 $\bar{Nu}_R$ – mean radiation Nusselt number over both walls,
 $\bar{Nu}_R(\text{Left}) + \bar{Nu}_R(\text{Right})$
 Pe_S – Peclet number based on channel spacing
 Pr – Prandtl number, (ν/α)
 Q – total rate of heat transfer from the channel, $(Q_C + Q_R)$, W/m

Q_C – net convection heat transfer rate from the channel, W/m
 Q_R – net radiation heat transfer rate from the channel, W/m
 Re_S – Reynolds number, $[u_\infty S/\nu]$
 Ri_S – Richardson number, $[Gr_S/Re_S^2]$
 S – channel width (spacing), m
 T – temperature at a given location, K
 T_L – left wall temperature of the channel, K
 T_R – right wall temperature of the channel, K
 T_∞ – entry temperature of air, K
 t – wall thickness, m
 U, V – non-dimensional vertical and horizontal velocities
 u_∞ – entry velocity of air, m/s
 x, y – axial and transverse distances, respectively, m
 X, Y – normalised axial and normalised transverse distances, $[x/S]$ and $[y/S]$, respectively

Greek symbols

ε – channel wall emissivity
 θ – normalised local temperature, $[(T - T_\infty)/(T_L - T_\infty)]$
 σ – Stefan-Boltzmann constant, $5.6697 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)$
 ψ, ω – non-dimensional stream and vorticity functions

Hacohen et al. [6] documented experimental as well as numerical findings on the formation of the boundary layer together with heat transfer coefficients in several thermally simulated electronic components catering to both forced and free convection environments in a channel. Watson et al. [7] worked on numerical simulations on laminar combined natural and forced convection between several plates that are parallel and vertically disposed, carrying planar heating elements.

Gururaja Rao [8] explored coupled conduction-mixed convection-radiation from the configuration of a plate that is vertically disposed and has an embedded discrete heater that traverses along the plate, and elucidated, quantitatively as well as qualitatively, the roles enacted by the two surface heat transfer modes, viz. convection and radiation. Multi-mode heat transfer connecting free convection and internal thermal conduction speaking about a vertical heat generating plate has been probed by Mamun et al. [9]. A theoretical investigation narrating mixed convection flow of a nanofluid consisting of water and alumina for the geometry of a vertical micro channel has been credited to Malavandi and Ganji [10]. Londhe and Gururaja Rao [11] delved into the interplay between combined natural and forced convection, conduction and radiation catering to a channel held vertically and heated discretely. They handled the channel that had three identical and symmetrically positioned embedded discrete heaters in the left wall of the channel. Fairly recently, Shah and Gururaja Rao [12] extended the above work by replacing the identical and symmetrically positioned heaters with five discrete heaters of progressively decreasing sizes in the left wall of the channel. The results of a conjugate study on heat transfer by the combined modes of natural convection and radiation from an array of fins have been documented by de Araujo et al. [13], making use of a semitransparent fluid medium as the cooling agent.

A review of the various findings documented in multi-mode heat transfer from the geometry of a channel indicated that a detailed probe into the channel geometry that comprises (i) sym-

metrically and (ii) asymmetrically heated isothermal walls is not explicitly covered in the literature. Thus, the same has been explored in the current paper. The finite volume method (FVM) is used for this purpose. It is also proposed to prepare an appropriate computer program to tackle the present problem. Along with the simulation studies, an attempt is made here to arrive at the correlations for (i) mean convection Nusselt number and (ii) mean radiation Nusselt number through a huge numerical data set obtained from the computer code written here.

2. Problem description and solution methodology

Figure 1 shows the vertical parallel wall channel of height L and spacing (or width) S .

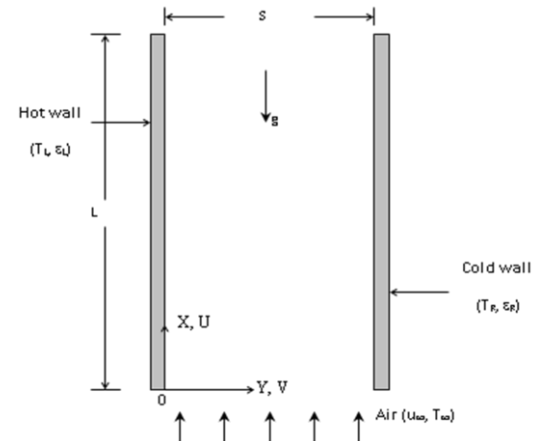


Fig. 1. Symmetrically or asymmetrically heated vertical channel considered in the present study.

The aspect ratio (A) of the channel is defined as (L/S) . For a given height (L), a larger A implies a narrower channel, and a smaller A pertains to a broader channel. The left wall is hotter (T_L) than the right wall (T_R) for the case of “asymmetric wall heating”, while both walls acquire identical temperatures

($T_R = T_L$) for the case of “symmetric wall heating”. The left and right wall emissivities are, respectively, ε_L and ε_R , and these too may (or may not) be equal. The cooling agent (air) is seen entering the channel from the bottom end at a uniform velocity [u_∞] and a uniform temperature [T_∞]. The basic equations of conservation of mass, momentum, and energy, after their initial conversion into vorticity-stream function form and an accompanying normalisation, are:

$$U \frac{\partial \omega}{\partial X} + V \frac{\partial \omega}{\partial Y} = -\text{Ri}_S \frac{\partial \theta}{\partial Y} + \frac{1}{\text{Re}_S} \left[\frac{\partial^2 \omega}{\partial X^2} + \frac{\partial^2 \omega}{\partial Y^2} \right], \quad (1)$$

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\omega, \quad (2)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Pe}_S} \left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right]. \quad (3)$$

The Richardson number (Ri_S), based on the channel spacing (S), and defined as ($\text{Gr}_S/\text{Re}_S^2$), appears in the vorticity-transport equation (Eq. (1)). It marks the distinction between different regimes of convection, viz. forced convection dominant ($\text{Ri}_S \ll 1$), pure mixed convection ($\text{Ri}_S = 1$) and free convection dominant ($\text{Ri}_S \gg 1$) regimes. The “enclosure method of analysis” comprising the radiosity (J) – irradiation (G) formulation is used for the calculations involving radiation.

2.1. Boundary conditions used

The various boundary conditions used in the current study are shown in Fig. 2.

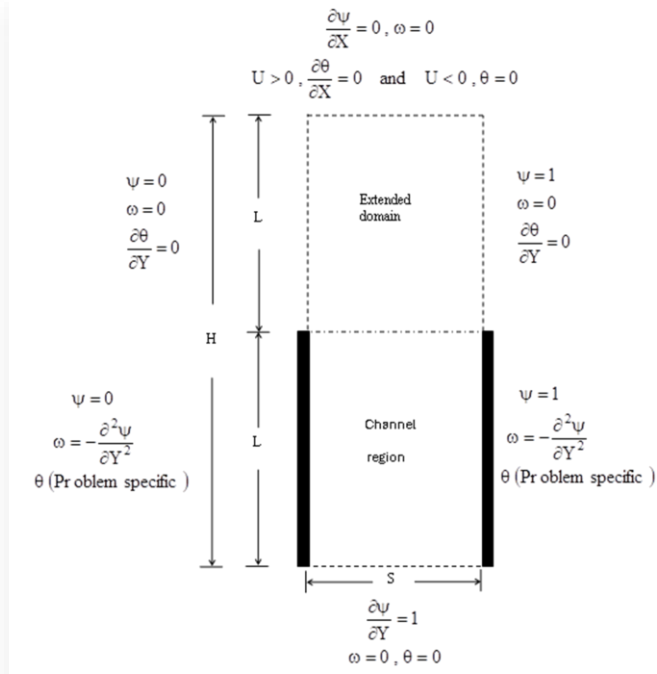


Fig. 2. Various boundary conditions used in the present study.

2.2. Discretisation of the computational domain and the grid sensitivity test

The computational domain has been discretised adhering to the condition that a specific number of finer grids always exist (i) at

the bottom end of the channel from where the fluid (air) begins its flow and (ii) near each channel wall, where the gradients of velocity and temperature are comparatively steeper. This led to using the “cosine function” to generate the grids in Y-direction, and a “semi-cosine function” is used in X-direction along the channel walls. In the domain beyond the channel exit, uniformly spaced grids are preferred in axial direction, without any change in the grid pattern in Y-direction.

A “grid sensitivity test” is conducted for a typical case with $T_L = T_R = 100^\circ\text{C}$, $A = 8$, $\text{Ri}_S = 1$, $\text{Re}_S = 100$ and $\text{Gr}_S = 10000$. The grid sensitivity is tested in two stages – first with M fixed and later with N fixed. The results of the first stage show that the difference in $\overline{\text{Nu}}_C$ between the grid sizes 81×141 and 81×161 is 0.008%, while the difference in $\overline{C}_f$ between the same two grid sizes is 0.0002%. The results of the second stage of the study with N fixed indicate that the difference in $\overline{\text{Nu}}_C$ between grid sizes 81×141 and 101×141 is 0.72%, while for $\overline{C}_f$, the difference between these two grid sizes is 0.55%. In view of these results, M and N have been fixed as 81 and 141, respectively.

It is to be noted that, in the above study, the number of grid elements along the channel is 90 (i.e. the number of nodes along the channel $N_1 = 91$), which is based on another study for the same typical case as above. It can be noticed from the table that the difference in $\overline{\text{Nu}}_C$ between $N_1 = 71$ and 91 is 4.39%, while that between $N_1 = 91$ and 111 is 0.51%. Regarding $\overline{C}_f$, the difference between $N_1 = 71$ and 91 is 0.083% and that between $N_1 = 91$ and 111 is 0.029%. Thus, N_1 is fixed as 91. Hence, for all the subsequent calculations, a grid pattern with $M = 81$, $N_1 = 91$ and $N = 141$ is used. Figure 3 illustrates one such typical grid system used in the current work.

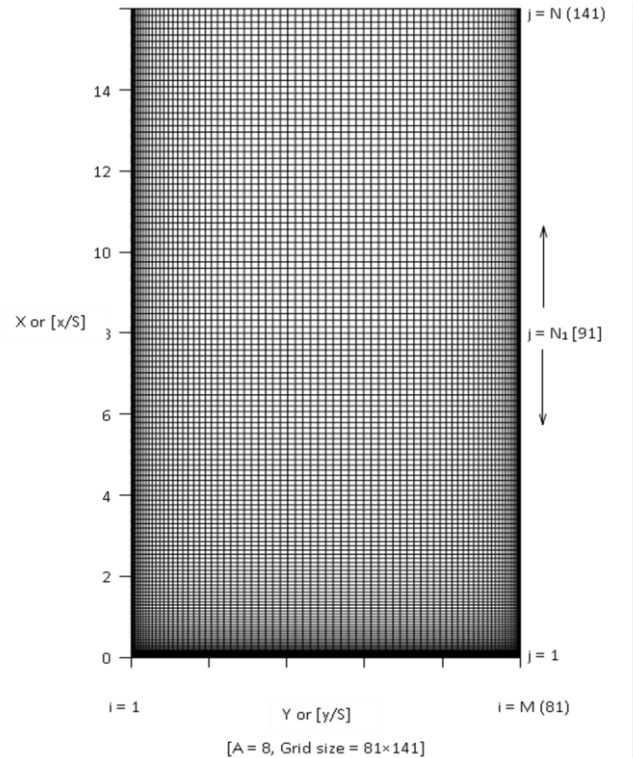


Fig. 3. Grid pattern used for one of the cases of the present work.

2.3. Calculations

The velocities and temperatures everywhere in the computational domain are the initial results in the work. The local convection Nusselt numbers along the two walls (both hot and cold) are evaluated by using a second degree Lagrangian polynomial, while the mean convection Nusselt number ($\overline{Nu}_C$) based on each of the two walls is calculated by numerical integration (the Simpson's 1/3 rule). The mean convection Nusselt number based on both walls ($\overline{Nu}_C$) is then obtained by adding algebraically the mean convection Nusselt numbers based on each channel wall, thus calculated. On similar lines, the mean radiation Nusselt number ($\overline{Nu}_R$) based on both walls is the algebraic sum of the mean radiation Nusselt numbers on the two walls. All the calculations are made employing air (cooling agent) that is treated as non-participating in radiation. The height (L) of the channel is taken equal to 23.34 cm.

3. Results and discussion

In this section, some prominent results obtained in the present study will be presented.

3.1. Dependence of mean convection Nusselt number on other parameters

Figure 4 shows a family of curves depicting variations of the average convection Nusselt number ($\overline{Nu}_C$) with Richardson number (Ri_S). The curves are plotted for four values of aspect ratio ($A = 4, 8, 12$ and 16) for a typical case of symmetric wall heating ($\theta_R = 1$), with $T_L = T_R = 100^\circ\text{C}$.

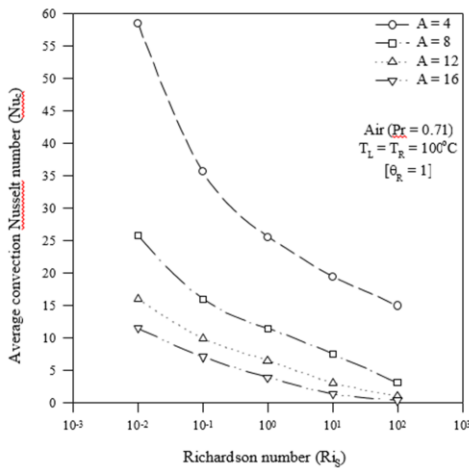


Fig. 4. Dependence of mean convection Nusselt number on Richardson number for different aspect ratios of the channel.

The figure reveals that, for a given aspect ratio (A), $\overline{Nu}_C$ decreases as Ri_S increases from 0.01 to 100. This is because, in this example, Gr_S is held fixed, and thus Re_S decreases as Ri_S increases. This means that the flow shifts from the regime of forced convection to that of natural convection, which brings down the heat transfer rate and thus the Nusselt number in the channel. However, the amount of decrease in $\overline{Nu}_C$ is higher in the forced convection regime ($0.01 \leq Ri_S < 1$) than in the natural

convection regime ($1 < Ri_S \leq 100$). Calculations indicate that, for $A = 4$, $\overline{Nu}_C$ decreases by 55% between $Ri_S = 0.01$ and 1, while it decreases only by about 34% between $Ri_S = 1$ and 100. Also, beyond $Ri_S = 100$, the variation in $\overline{Nu}_C$ tends to be asymptotic with reference to Ri_S , especially towards larger aspect ratios. The figure further indicates that $\overline{Nu}_C$ diminishes as the aspect ratio (A) increases between 4 and 16, for a given Ri_S . This is because, with the channel height (L) remaining unaltered as A increases, the channel spacing (S) decreases. Since $\overline{Nu}_C$ is based on S , there is an understandable decrease in $\overline{Nu}_C$ with the increasing A . Looking into the example here, for $Ri_S = 1$ (mixed convection), $\overline{Nu}_C$ is found to decrease by as much as 84% as the aspect ratio increases from 4 to 16. Trends of a similar kind are perceived in the case of asymmetric wall heating also.

3.2. Roles taken up by convection and radiation in channel heat dissipation

In the current problem, both convection and radiation contribute to dissipation of heat from the channel. The heat transferred by convection (free and forced) and that transferred by radiation are independently calculated. The net heat transfer in the channel is then obtained as the algebraic sum of the rates of convection and radiation heat transfer. The design of any cooling system (e.g. for electronic equipment) that uses a gaseous medium (say air) as the coolant should necessarily take radiation into account, because a sizable amount of heat may get transferred by radiation in all such applications. The current study is taken up to address this issue adequately.

The role radiation plays in heat transfer from the channel can be best delineated by considering a typical case with $A = 4$, $T_L = T_R = 100^\circ\text{C}$ ($\theta_R = 1$), $T_\infty = 25^\circ\text{C}$ and $Gr_S = 10^4$. Five values of Ri_S ($= 0.01, 0.1, 1, 10$ and 100) and three values of wall emissivity ($\varepsilon_L = \varepsilon_R = 0.05, 0.45$ and 0.85) are considered. In this study, convection heat transfer is not a function of emissivity (ε), while radiation heat transfer is not a function of Richardson number (Ri_S). Figure 5 shows a family of curves depicting the relative roles enacted by convection and radiation in channel heat dissipation plotted against Ri_S for the three values of emissivity opted above.

It is noticeable that the role exhibited by convection tapers down as Ri_S increases from 0.01 to 100 for a given ε , while the contribution of radiation continuously increases. In the example here, for $\varepsilon = 0.45$, the radiation heat transfer (Q_R) remains constant at 126.54 W/m for all values of Ri_S . However, the convection heat transfer (Q_C) decreases from 313.99 W/m to 60.35 W/m, as Ri_S increases from 0.01 to 100. The net result is that the total heat transfer (Q) decreases from 440.53 W/m to 186.89 W/m. Thus, the convection contribution in the total heat transfer decreases from 71.28% to 32.29%, with a mirror-image increment in radiation heat transfer from 28.72% to 67.71%. If one were to design a cooling system overlooking radiation, the heat dissipation rate in the channel for $Ri_S = 100$ would have been reported as only 60.35 W/m ($= Q_C$). However, there is an accompanying radiation heat transfer of 126.54 W/m (for $\varepsilon_L = \varepsilon_R = 0.45$), which is more than two times the convection heat transfer rate.

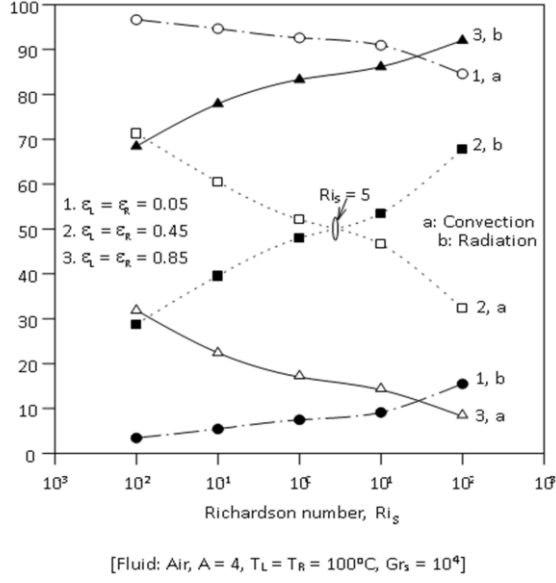


Fig. 5. Contributory roles enacted by convection and radiation in heat transfer.

It can also be noticed that the curves belonging to convection and radiation cross each other for the case where $\varepsilon_L = \varepsilon_R = 0.45$, implying equal contributions to heat transfer at $Ri_s \approx 5$. The figure also reveals that for a given Ri_s , there is a continuous decrease in the percentage of convection heat transfer, with a proportionate increase in its radiation counterpart, as ε (ε_L or ε_R , which are equal in this specific example) increases from 0.05 to 0.85. In the current example, for $Ri_s = 1$, the heat transfer by convection (Q_C) remains constant at 137.3 W/m for all values of ε (due to its non-dependence on ε). The radiation heat transfer (Q_R), on the other hand, increases from 11.03 W/m to 671.32 W/m, as ε increases from 0.05 to 0.85. In effect, the share of convection in total heat transfer drops down from 92.56% to 16.9%, with the contribution from radiation increasing from 7.44% to 83.1%. In this context, if a cooling system is designed ignoring radiation, and if the channel walls, for example, have an emissivity (ε) of 0.85, the heat transfer rate would have been calculated as 137.3 W/m ($= Q_C$) only. However, the radiation heat transfer for this case is 671.32 W/m. Thus, there is a total heat transfer of 808.62 W/m here, which is about 5.9 times the heat transfer (Q_C) that would have been reported had radiation been ignored. The above exercise thus highlights the importance of the role radiation plays in this class of problems.

3.3. Correlations for the case of symmetric wall heating

The following ranges of different independent parameters have been used here:

$$\begin{aligned} 1 &\leq A \leq 16, \\ 0.05 &\leq \varepsilon_L, \quad \varepsilon_R \leq 0.85, \\ 0 &\leq \theta_R \leq 1, \\ 0 &< T_{R1} \leq 1, \\ 0 &< T_{R2} \leq 1, \\ 0.01 &\leq Ri_s \leq 100, \\ 1 &\leq Re_s \leq 10000, \\ 10 &\leq Gr_s \leq 10000. \end{aligned}$$

A correlation has been deduced for the mean convection Nusselt number ($\overline{Nu}_C$) based on both walls of the channel:

$$\overline{Nu}_C = 2.84 A^{-0.65} T_{R1}^{0.49} Re_s^{0.46} (1 + Ri_s)^{0.09}. \quad (4)$$

It is to be noted that $\overline{Nu}_C$, correlated above, is based on the channel spacing (S). Equation (4) has the coefficient of correlation equal to 0.991 and the error band equal to $\pm 4.3\%$. For obtaining the correlation above, $(1 + Ri_s)$ has been chosen as the most appropriate form for Ri_s . This is due to the reason that, in the situation where $Ri_s = 0$, $\overline{Nu}_C$ would not be zero since it still possesses the forced convection component.

For the left wall of the channel, based on a set of 45 data points, a correlation for $\overline{Nu}_R$ turns out to be:

$$\overline{Nu}_R(\text{Left}) = 187.46 A^{-1.12} (1 - T_{R1}^4)^{-0.09} \varepsilon_L^{1.55} \varepsilon_R^{-0.07}. \quad (5)$$

The coefficient of correlation given by Eq. (5) is 0.998, and it has the error band $= \pm 4.9\%$. For the same 45 data points as used for Eq. (5), the correlation for $\overline{Nu}_R$, based on the right wall of the channel, having the coefficient of correlation equal to 0.993 and the error band equal to $\pm 5.1\%$, turned out to be:

$$\overline{Nu}_R(\text{Right}) = 171.89 A^{-1.18} (1 - T_{R1}^4)^{-0.49} \varepsilon_L^{0.03} \varepsilon_R^{1.64}. \quad (6)$$

3.4. Correlations for the case of asymmetric wall heating

The mean convection Nusselt number ($\overline{Nu}_C$) based on both channel walls (left as well as right), for asymmetric wall heating, has been correlated, using 151 data points, as:

$$\overline{Nu}_C = 1.72 A^{-0.71} (1 + \theta_R)^{0.77} Re_s^{0.45} (1 + Ri_s)^{0.08}. \quad (7)$$

Equation (7) has the coefficient of correlation equal to 0.993 and the error band equal to $\pm 4.8\%$. The form $(1 + \theta_R)$ has been chosen to be appropriate for θ_R , because θ_R varies from 0 to 1, and $\overline{Nu}_C$ would be finite even when $\theta_R = 0$. The form $(1 + Ri_s)$ is retained for Ri_s following the reasons already explained.

The correlation for $\overline{Nu}_R$, based on the left (hot) wall, evolved using 116 data that has the coefficient of correlation equal to 0.996 and the error band equal to $\pm 5.6\%$, is:

$$\begin{aligned} \overline{Nu}_R(\text{Left}) &= 48.52 A^{-1.16} (1 - T_{R1}^4)^{-2.75} \times \\ &\times (1 - T_{R2}^4)^{-0.02} \varepsilon_L^{1.59} \varepsilon_R^{-0.02}. \end{aligned} \quad (8)$$

The correlation for $\overline{Nu}_R$ for the right (cold) wall, based on 105 data points and having the coefficient of correlation equal to 0.995 and the error band equal to $\pm 5.7\%$, is:

$$\begin{aligned} \overline{Nu}_R(\text{Right}) &= 1.77 A^{-0.82} (1 - T_{R1}^4)^{-2.77} \times \\ &\times (1 - T_{R2}^4)^{-1.36} \varepsilon_L^{-0.39} \varepsilon_R^{1.57}. \end{aligned} \quad (9)$$

In the above correlations for $\overline{Nu}_R$, $(1 - T_{R1}^4)$ and $(1 - T_{R2}^4)$ have been chosen to be the appropriate forms for the two temperature ratios $T_{R1} (= T_\infty/T_L)$ and $T_{R2} (= T_R/T_L)$, respectively.

4. Conclusions

Some of the prominent conclusions derivable from the current numerical study include:

- The aspect ratio (A) plays a prominent role in the current problem. An increasing aspect ratio, and thus the progressively narrower channels, are found to bring down the mean convection Nusselt number ($\overline{Nu}_C$). This happens in all regimes of mixed convection. For example, in the pure mixed convection regime ($Ri_s = 1$), $\overline{Nu}_C$ decreases by a maximum of about 80 – 85% (depending on other parameters) with the aspect ratio rising from 4 to 16.
- For a given aspect ratio (A), $\overline{Nu}_C$ generally decreases with Ri_s . This trend is found to be more apparent in the region of forced convection dominance (viz. $0.01 \leq Ri_s < 1$) as against the region of free convection dominance (viz., $1 < Ri_s \leq 100$).
- The fact that the heat load calculation in any application that makes use of a gaseous medium (like air) as the coolant turns out to be erroneous if one does not take radiation into account has been adequately established. A detailed graphical narration has been given in this context.
- Correlations for the mean convection Nusselt number ($\overline{Nu}_C$) and mean radiation Nusselt number ($\overline{Nu}_R$) evolved making use of a very large number of data points obtained using the computer program exclusively written in the current work.

References

- [1] Carpenter, J.R., Briggs, D.G., & Sernas, V. (1976). Combined radiation and developing laminar free convection between vertical flat plates with asymmetric heating. *ASME Journal of Heat Transfer*, 98(1), 95–100. doi: 10.1115/1.3450476
- [2] Sparrow, E.M., Shah, S., & Prakash, C. (1980). Natural convection in a vertical channel: 1. Interacting convection and radiation; 2. The vertical plate with and without shrouding. *Numerical Heat Transfer*, 3(3), 297–314. doi: 10.1080/01495728008961760
- [3] Anand, N.K., Kim, S.H., & Aung, W. (1990). Effect of wall conduction on free convection between asymmetrically heated vertical plates: uniform wall temperature. *International Journal of Heat and Mass Transfer*, 33(5), 1025–1028. doi: 10.1016/0017-9310(90)90083-7
- [4] Kim, S.H., Anand, N.K., & Aung, W. (1990). Effect of wall conduction on free convection between asymmetrically heated vertical plates: uniform wall heat flux. *International Journal of Heat and Mass Transfer*, 33(5), 1013–1023. doi: 10.1016/0017-9310(90)90082-6
- [5] Kim, S.H., & Anand, N.K. (1994). Laminar developing flow and heat transfer between a series of parallel plates with surface-mounted discrete heat sources. *International Journal of Heat and Mass Transfer*, 37(15), 2231–2244. doi: 10.1016/0017-9310(94)90366-2
- [6] Hachohen, J., Chiu, T.W., & Wragg, A.A. (1995). Forced and free convective heat transfer coefficients for a model printed circuit board channel geometry. *Experimental Thermal and Fluid Science*, 10(3), 327–334. doi: 10.1016/0894-1777(94)00073-H
- [7] Watson, J.C., Anand, N.K., & Fletcher, L.S. (1996). Mixed convective heat transfer between a series of vertical parallel plates with planar heat sources. *ASME Journal of Heat Transfer*, 118(4), 984–990. doi: 10.1115/1.2822601
- [8] Gururaja Rao, C. (2004). Buoyancy-aided mixed convection with conduction and surface radiation from a vertical electronic board with a traversable discrete heat source. *Numerical Heat Transfer, Part A: Applications*, 45(9), 935–956. doi: 10.1080/10407780490439202
- [9] Mamun, A.A., Chowdhury, Z.R., Azim, M.A., & Maleque, M.A. (2008). Conjugate heat transfer for a vertical flat plate with heat generation effect. *Nonlinear Analysis: Modelling and Control*, 13(2), 213–223. doi: 10.15388/NA.2008.13.2.14581
- [10] Malvandi, A., & Ganji, D.D. (2014). Mixed convective heat transfer of water/alumina nano fluid inside a vertical microchannel. *Powder Technology*, 263, 37–44. doi: 10.1016/j.powtec.2014.04.084
- [11] Londhe, S.D. & Gururaja Rao, C. (2014). Interaction of surface radiation with conjugate mixed convection from a vertical channel with multiple discrete heat sources. *Heat and Mass Transfer*, 50, 1275–1290. doi: 10.1007/s00231-014-1333-1
- [12] Shah A.P., & Gururaja Rao, C. (2020). Buoyancy-aided conjugate mixed convection with surface radiation from a vertical channel with multiple non-identical discrete heat sources. *International Journal for Computational Methods in Engineering Science and Mechanics*, 21(1), 12–25. doi: 10.1080/15502287.2020.1718799
- [13] de Araujo, M.E.V., Barbosa, E.G., Martins, M.A., & Corrêa, P.C. (2022). Conjugated analysis of heat transfer by natural convection and radiation in a fin array using a semitransparent fluid medium. *Heat and Mass Transfer*, 58, 1119–1132. doi: 10.1007/s00231-021-03163-1



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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A theoretical investigation of magnetohydrodynamics, couple stress and viscosity variation on squeeze film lubrication between a cylinder and rough porous flat surface

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Received: 14.03.2025; revised: 16.09.2025; accepted: 29.10.2025

Abstract

Squeeze film lubrication involving couple stress fluids, viscosity variation and magnetohydrodynamics is critically important for high-performance applications in automotive systems, aerospace mechanisms and biomedical devices. This study aims to examine the combined influence of these effects on lubrication performance between a cylinder and a rough porous flat surface. To achieve this, we developed a generalised modified Reynolds equation by integrating Christensen's stochastic model for surface roughness, Darcy's law for porous media flow, and Stokes flow theory. This equation was solved with appropriate boundary conditions to derive analytical expressions for pressure, enabling the calculation of load-carrying capacity and squeeze film time for both longitudinal and transverse roughness patterns. Key findings demonstrate that couple stress, magnetohydrodynamics effects, and viscosity variation significantly enhance pressure distribution, increasing load-carrying capacity by up to 85.29% and squeeze film time by up to 85.34% under optimal conditions. Notably, longitudinal roughness pattern degrades tribological performance, whereas transverse roughness pattern enhances it. Furthermore, three-dimensional analysis demonstrates that optimal tuning of roughness and porosity parameters maximises the system performance. These findings, validated against established results like Lin et al. [31], provide valuable design guidelines for optimising tribological systems in fields requiring precise and durable lubrication, particularly in precision manufacturing and microelectromechanical systems.

Keywords: Magnetohydrodynamics; Porous, Rough surface; Viscosity variation

Vol. 46(2025), No. 4, 141–156; doi: 10.24425/ather.2025.156845

Cite this manuscript as: Ramareddy, S., Rudrappa, A., Hanumagowda, B.N., & Raghavendra, S. (2025). A theoretical investigation of magnetohydrodynamics, couple stress, and viscosity variation on squeeze film lubrication between a cylinder and rough porous flat surface. *Archives of Thermodynamics*, 46(4), 141–156.

1. Introduction

Surface roughness in combination with a porous medium influenced by magnetohydrodynamics (MHD) holds significant practical applications, particularly in industries such as aircraft design. However, the presence of surface roughness is crucial for understanding how components interact with their surround-

ings. Nevertheless, excessive surface roughness may not yield beneficial outcomes. Surface roughness serves as an appropriate indicator of physical component efficiency, as irregularities on surfaces can lead to areas of nucleation for cracking, rusting or breakdown, and rough surfaces tend to exhibit higher coefficients of friction compared to smoother surfaces. In practical scenarios, all bearing surfaces inherently exhibit some degree of

Nomenclature

B_0 – magnetic field applied externally, Wb/m²
 c – roughness factor
 C – dimensionless roughness
 D – cylinder's length, mm
 $g(h_s)$ – probability density function
 G – viscosity parameter
 h – thickness of the film, mm
 h_m – minimal film thickness, mm
 h_{m0} – initial minimal thickness of the film, mm
 h_s – stochastic film thickness, mm
 k – porous medium factor
 l^* – dimensionless couple stress, $l^* = 2l/h_{m0}$
 m – magnetic moment of the particle, A m²
 M_0 – Hartmann number
 p^* – pressure in the porous area, Pa
 P – pressure, Pa
 P^* – dimensionless pressure
 R – cylinder's radius, mm
 R_{T^*} – relative frequency of squeeze film time

R_{W^*} – relative frequency of load-carrying capability
 T^* – dimensionless squeeze film time
 u, w – velocity factor in the x and z direction, mm/s
 u^*, w^* – Darcy's velocity factor in the permeable area, mm/s
 V – velocity of squeezing, mm/s
 W^* – dimensionless load-carrying capability

Greek symbols

α – viscosity coefficient
 β – microstructure to the size of pore ratio, $\beta = (\eta/\mu)/k$
 δ – porous layer thickness, mm
 η – material constant, N s
 μ – dynamic viscosity, cP
 μ_0 – initial velocity, mm/s
 σ – lubricant electrical conductivity, S/m
 $\bar{\sigma}$ – standard deviation
 ψ – porous factor

Abbreviations and Acronyms

MEMS – Microelectromechanical systems
MHD – Magnetohydrodynamics

roughness, including porous bearings, which are typically designed for low-load operations. Surface roughness significantly affects lubrication, influencing critical performance parameters such as load-bearing capability, pressure and response time. Recent studies have explored various factors influencing squeeze film lubrication under various circumstances. Bujurke et al. [1] explored the impacts of roughness on squeeze film lubrication among curved-shaped annular surfaces, deriving a modified Reynolds equation using Christensen's stochastic method. Their findings revealed that circumferential roughness increases the load-bearing capability while shifting the pressure maximum towards the outlet edge, whereas radial roughness decreases the load capacity. Christensen [2], through a stochastic theory approach, demonstrated that surface roughness considerably impacts bearing characteristics, with its effect dependent on roughness type but not critically on the distribution function. Daliri and Jalali-Vahid [3] extended this by incorporating viscosity-pressure dependency and non-Newtonian fluid effects, showing an improved squeeze film performance for circular surface roughness patterns, while radial patterns led to a reduction in squeeze film performance. Further, Mouda et al. [4] examined MHD non-parallel lubrication with non-Newtonian fluids and identified significant interactions among MHD, surface roughness and non-Newtonian effects. Dass et al. [5] highlighted that velocity-slip and ferrofluid lubrication, combined with couple stress and surface roughness, greatly enhance pressure and load-carrying capacity under magnetic fields. Other studies, such as Ullah et al. [6] and Johnny et al. [7], explored MHD flows in porous media, emphasising how magnetic fields and porosity influence velocity profiles and load capacity, respectively. Yadav and Srivastava [8] explored immiscible fluid dynamics in porous channels, validating the use of porous materials for flow control. Patel et al. [9] examined ferrofluid lubrication in porous squeeze films, finding significant performance enhancements using specific magnetic fluid flow models. Similarly, Khan et al. [10] and

Patel and Deheri [11] demonstrated improvements in squeeze film characteristics due to magnetic effects, couple stress fluids, and surface roughness. Lin et al. [12] developed a two-dimensional Reynolds expression for non-Newtonian fluids, showing increased load capacity with couple stresses and magnetic fields. Nabhani and El Khelifi [13] further confirmed the benefits of magnetic fields, couple stress and fluid inertia on squeeze film performance. Ramasamy [14] emphasised the effect of non-Newtonian thixotropic fluids on squeeze film behaviour, with higher load capacity attributed to increased yield numbers and Hartmann parameters. Jyothi et al. [15] extended this to sphere-flat plate systems, demonstrating the detrimental effects of slip velocity despite enhancements from couple stress.

Squeeze film lubrication has advanced by integrating factors like surface roughness and porous materials, enhancing our understanding of lubrication dynamics. For instance, Prakash et al. [16] utilised stochastic theory to examine the hydrodynamic lubrication effects on rough surfaces between rotating annular discs, highlighting the interplay between porous facings and film pressure. Shah [17] extended these investigations to ferrofluid-lubricated circular disc designs under variable magnetic fields, demonstrating how circumferential roughness and radial permeability affect load-carrying capacity in different bearing configurations. Thakkar and Patel [18] provided a comprehensive analysis of the MHD-based squeeze film among porous circular surfaces, concluding that magnetisation significantly enhances load capacity, while negatively skewed roughness contributes positively to bearing efficiency. Vadher et al. [19] expanded the scope to conducting rough, porous elliptical plates, illustrating that hydromagnetic lubrication offsets the adverse effects of transverse surface roughness and porosity through the synergistic impact of conductivity and magnetisation. Deheri and Patel [20] applied a stochastic framework to study magnetic fluid-lubricated rough, porous parallel plates, revealing that porosity decreases load-carrying capacity but can be mitigated by

proper roughness and magnetic parameters. Anthony and Elamparithi [21] analysed a couple stress fluids in rough-porous Rayleigh step slide bearings under MHD, noting that roughness increased both workload and frictional forces. Ali et al. [22] investigated non-Newtonian fluids across oscillatory extending sheets in porous media, finding viscosity variations and couple stresses to amplify velocity oscillations and skin friction. Studies by Naduvinamani and Siddangouda [23] emphasised the influence of azimuthal and radial roughness patterns on squeeze film lubrication among porous circular-shaped stepped surfaces, while Vashi et al. [24] demonstrated how ferrofluid and negatively skewed roughness parameters enhance bearing performance for longitudinally rough stepped plates.

Significant progress has been made in squeeze film lubrication research, with numerous studies examining the impacts of roughness, porosity, couple stress, viscosity variation and MHD on lubrication attributes. Prakash and Gururajan [25] examined the influence of slip velocity on rough, porous journal bearings. Applying Christensen's stochastic theory for rough surfaces at the porous region, they derived modified Reynolds expressions for different roughness patterns and demonstrated that roughness-slip relations significantly alter bearing efficiency. Rahul and Rao [26] investigated the flow behaviour of Rabinowitsch fluids in a porous medium, incorporating viscosity variation and surface roughness effects on circular stepped plates. Their theoretical findings revealed that surface roughness and porosity decrease (increase) load-bearing capability and squeeze film time for radial (azimuthal) roughness, whereas viscosity enhances load-bearing capability and reduces squeeze film time in comparison with smooth, isoviscous surfaces. Sangeetha and Kesavan [27] extended these analyses to a couple stress MHD squeeze films in rough, porous rectangular-shaped surfaces, concluding that the magnetic field significantly enhances pressure compared to the Newtonian case. Similarly, Masood and Elamparithi [28] studied finite plates lubricated with non-Newtonian fluids under a transverse MHD and noted that viscosity variation and roughness significantly improve load-bearing capability, whereas porosity reduces it. Sangeetha et al. [29] explored MHD and viscosity variations effects on non-Newtonian fluids in porous triangular plates. Their results highlighted that viscosity variation enhances load capacity while reducing pressure under specific conditions. Sekar et al. [30] examined slip velocity with surface roughness and viscosity variation among porous, rough circular-shaped stepped surfaces using non-Newtonian fluids. Their findings emphasised improvements in load capacity due to roughness and viscosity variation.

Although extensive research has been conducted in the field of squeeze film lubrication, most existing studies have examined only a limited set of influencing parameters. For example, Lin et al. [31] explored how pressure-dependent viscosity affects couple stress fluids, reporting improvements in load-carrying capacity and longer film duration. Kumar and Sachidananda [32] studied the influence of surface roughness and viscosity variation in journal bearings, highlighting their combined effect on performance. Rajashekar and Kashinath [33] investigated MHD effects using a combination of couple stress theory and Christensen's stochastic roughness model for squeeze films be-

tween a sphere and a porous flat surface. Their results indicated that couple stress fluids enhance pressure and load-carrying capacity, with azimuthal roughness improving, and radial roughness diminishing, the film characteristics. Similarly, Baksh and Hanumagowda [34] analysed curved annular surfaces, demonstrating that magnetic fields and couple stress fluids can improve performance, with longitudinal and transverse roughness showing contrasting effects. However, these studies did not incorporate the full range of influential factors, namely viscosity variation, surface roughness, porosity, couple stress effects and magnetohydrodynamics into a single comprehensive model. As a result, the combined effect of these interacting factors remains largely unexplored. This gap is especially important for optimising lubrication systems in industries such as aerospace, automotive, biomedical devices and nanotechnology, where these combined effects significantly influence performance and durability.

To address existing limitations in the literature, the present study develops a comprehensive model that integrates all key influencing factors to provide a holistic understanding of squeeze film lubrication. Previous studies often examined effects such as viscosity variation, surface roughness, porosity, couple stress, and magnetohydrodynamics in isolation, resulting in an incomplete understanding of their combined influence. This work bridges that gap by introducing a unified approach supported by clearly defined research questions: How do viscosity variation, longitudinal and transverse surface roughness, porosity, couple stress and MHD forces collectively affect squeeze film characteristics such as pressure distribution, load-carrying capacity and squeeze film time? What are the distinct effects of longitudinal versus transverse roughness patterns on squeeze film characteristics? How can multi-parameter optimisation enhance the overall lubrication performance? And how is the proposed model validated for accuracy and reliability? These questions align with the study's objectives and guide a focused and coherent investigation. By integrating all major parameters into a single model, this research offers new insights into lubrication behaviour and performance. The findings contribute to the development of advanced lubrication systems and bearing designs, with practical relevance in fields such as MEMS, nanotechnology, rotating machinery and high-precision manufacturing, especially in applications involving micro- and nanoscale lubrication challenges.

2. Mathematical formulation

A cylinder with radius R is positioned above a rough, porous flat surface, with a homogeneous magnetic field B_0 employed along the x -axis. The cylinder and the flat porous plate are separated by a distance h . As the cylinder moves downward toward the plate with a velocity of V , a squeeze film effect is generated within the fluid region between them. The lubricant in this region is modelled as a couple stress fluid, which accounts for the microstructural interactions within the fluid. It is assumed that no external body forces or couples are acting on the system. The motion and continuity of the fluid are governed by the mathematical equations derived by Jyothi et al. [15], which provide the necessary framework for describing the behaviour of the

fluid in this configuration. The fundamental expression is represented as follows (see also Fig. 1):

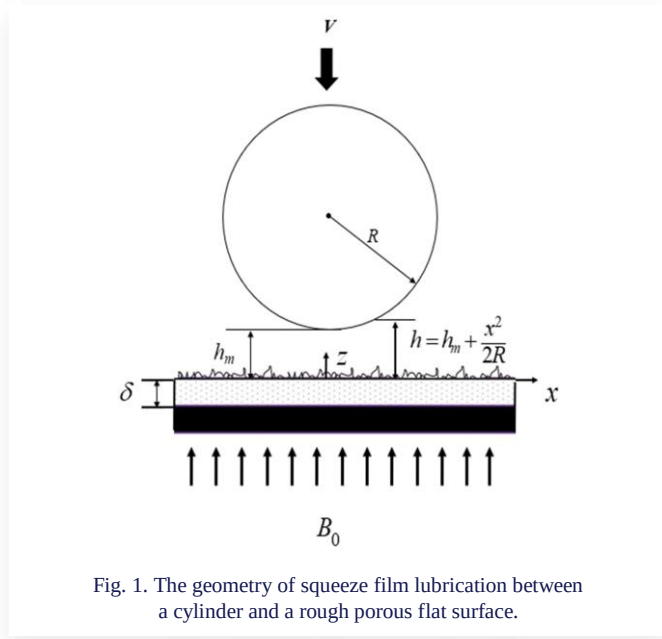


Fig. 1. The geometry of squeeze film lubrication between a cylinder and a rough porous flat surface.

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x}, \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (2)$$

$$\frac{\partial p}{\partial z} = 0, \quad (3)$$

whereby

$$M_0 = B_0 h_{m_0} (\sigma/\mu)^{1/2}.$$

Darcy's law governing the porous area under the influence of MHD is given in a reduced form as follows:

$$u^* = \frac{-k}{\mu \left(1 - \beta + \frac{k \sigma B_0^2}{\mu M}\right)} \frac{\partial p^*}{\partial x}, \quad (4)$$

$$w^* = \frac{-k}{\mu(1-\beta)} \frac{\partial p^*}{\partial z}. \quad (5)$$

Boundary conditions are:

- at the upper region $z = h$:

$$u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \text{and} \quad w = V = -\frac{\partial h}{\partial t}, \quad (6)$$

- at the lower region $z = 0$:

$$u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \text{and} \quad w = -w^*. \quad (7)$$

For the film region where x is less than R , the value of h is determined by:

$$h = h_m + \frac{x^2}{2R}.$$

By solving Eq. (1) and utilising the conditions (6-7), we obtain a solution for $u(x, z)$ as follows:

$$u(x, z) = \frac{1}{\sigma B_0^2} \frac{\partial p}{\partial x} \left[\frac{A^2}{A^2 - B^2} \frac{\cosh\{B(2z-h)/2l\}}{\cosh(Bh/2l)} - \frac{B^2}{A^2 - B^2} \frac{\cosh\left\{\frac{A(2z-h)}{2l}\right\}}{\cosh\left(\frac{Ah}{2l}\right)} - 1 \right]. \quad (8)$$

The condition discussed by Barus [35,36] as viscosity pressure dependency is described as follows:

$$\mu = \mu_0 e^{\alpha p}. \quad (9)$$

By substituting u from Eq. (8) into Eq. (2), integrating from 0 to h with the boundary conditions (6-7), and applying condition (9), we obtain a modified Reynolds expression:

$$\frac{\partial}{\partial x} \left\{ \frac{e^{-\alpha p}}{\mu_0} g(h, l, M_0) \frac{\partial p}{\partial x} \right\} = V. \quad (10)$$

Here:

$$g(h, l, M_0) = \frac{h_{m_0}^2}{M_0^2} \left\{ \frac{2l}{A^2 - B^2} \left(\frac{B^2}{A} \tanh \frac{Ah}{2l} - \frac{A^2}{B} \tanh \frac{Bh}{2l} \right) + h \right\} + \frac{k \delta M_0^2}{h_{m_0}^2 \left(1 - \beta + \frac{k M_0^2}{h_{m_0}^2 m} \right)},$$

$$A = \left\{ \frac{1 + \left(1 - 4M_0^2 l^2 / h_{m_0}^2 \right)^{1/2}}{2} \right\}^{1/2},$$

$$B = \left\{ \frac{1 - \left(1 - 4M_0^2 l^2 / h_{m_0}^2 \right)^{1/2}}{2} \right\}^{1/2}.$$

The film thickness can be divided into two portions to create a mathematical model representing the surface's roughness.

$$H = h + h_s(x, z, \xi). \quad (11)$$

Considering the stochastic mean, Eq. (10) with respect to $g(h_s)$, we get:

$$\frac{\partial}{\partial x} \left\{ \frac{e^{-\alpha E(p)}}{\mu_0} E\{g(H, l, M_0)\} \frac{\partial E(p)}{\partial x} \right\} = V, \quad (12)$$

where $E(*) = \int_{-\infty}^{\infty} (**) g(h_s) dh_s$.

The roughness function, as described by Christensen [2], is regarded as:

$$g(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3, & -c < h_s < c \\ 0, & \text{elsewhere} \end{cases}, \quad (13)$$

where $c = 3\bar{\sigma}$.

From Christensen's [2] perspective, it is employed to examine two distinct forms of one-dimensional roughness, such as longitudinal and transverse roughness patterns.

Longitudinal roughness patterns

The longitudinal roughness patterns have elongated, thin spikes and an inclined valley that runs along the x -axis. Subsequently:

$$H = h + h_s(z, \xi). \quad (14)$$

Transverse roughness patterns

The transverse roughness patterns have elongated, thin spikes and an inclined valley that runs along the z -axis. Subsequently:

$$H = h + h_s(x, \xi). \quad (15)$$

The average modified Reynolds flow equation is as follows:

$$\frac{\partial}{\partial x} \left\{ \frac{e^{-\alpha E(p)}}{\mu_0} G(H, l, M_0, c) \frac{\partial E(p)}{\partial x} \right\} = V, \quad (16)$$

where:

- for longitudinal roughness:

$$G(H, l, M_0, c) = E[g(H, l, M_0)],$$

- for transverse roughness:

$$G(H, l, M_0, c) = E \left[\frac{1}{g(H, l, M_0)} \right].$$

Introducing dimensionless quantities as shown below:

$$\begin{aligned} x^* &= \frac{x}{R}, & C &= \frac{c}{h_{m0}}, & h_m^* &= \frac{h_m}{h_{m0}}, & l^* &= \frac{2l}{h_{m0}}, \\ P^* &= -\frac{h_{m0}^3 E(p)}{\mu_0 R V}, & \delta^* &= \frac{\delta}{h_{m0}}, & G &= -\frac{\alpha \mu_0 R V}{h_{m0}^2}, \\ \beta &= \frac{h_{m0}}{R}, & h^* &= \frac{h}{h_{m0}} = h_m^* + \frac{x^{*2}}{2\beta}, & h_s^* &= \frac{h_s}{h_{m0}}, & \psi &= \frac{k\delta}{h_{m0}^3}, \end{aligned}$$

and substituting the specified dimensionless parameters into Eq. (16) yields:

$$\frac{d}{dx^*} \left\{ e^{-GP^*} G^*(H^*, l^*, M_0, C, \psi) \frac{dP^*}{dx^*} \right\} = -\frac{1}{\beta}, \quad (17)$$

where:

- for longitudinal roughness:

$$G^*(H^*, l^*, M_0, C, \psi) = E[g^*(H^*, l^*, M_0, \psi)],$$

- for transverse roughness:

$$G^*(H^*, l^*, M_0, C, \psi) = E \left[\frac{1}{g^*(H^*, l^*, M_0, \psi)} \right],$$

$$E[g^*(H^*, l^*, M_0, \psi)] =$$

$$= \frac{35}{32c^7} \int_{-C}^C g^*(H^*, l^*, M_0, \psi) (c^2 - h_s^{*2})^3 dh_s^*,$$

$$E \left[\frac{1}{g^*(H^*, l^*, M_0, \psi)} \right] = \frac{35}{32c^7} \int_{-C}^C \frac{(c^2 - h_s^{*2})^3}{g^*(H^*, l^*, M_0, \psi)} dh_s^*,$$

$$g^*(H^*, l^*, M_0, \psi) =$$

$$\begin{aligned} &= \frac{1}{M_0^2} \left\{ \frac{l^*}{A^{*2} - B^{*2}} \left(\frac{B^{*2}}{A^*} \tanh \frac{A^* H^*}{l^*} - \frac{A^{*2}}{B^*} \tanh \frac{B^* H^*}{l^*} \right) + H^* \right\} + \\ &\quad + \frac{\psi M_0^2}{\left(1 - \beta + \frac{\psi M_0^2}{\delta^* m} \right)}, \end{aligned}$$

$$A^* = \left\{ \frac{1 + \sqrt{1 - (l^* M_0)^2}}{2} \right\}^{1/2},$$

$$B^* = \left\{ \frac{1 - \sqrt{1 - (l^* M_0)^2}}{2} \right\}^{1/2}.$$

The following are the pressure boundary conditions:

$$\frac{dP^*}{dx^*} = 0 \text{ at } x^* = 0, \quad (18)$$

$$P^* = 0 \text{ at } x^* = 1. \quad (19)$$

By solving Eq. (17) and incorporating Eqs. (18–19), we get:

$$P^* = -\frac{1}{G} \ln \left\{ 1 + \frac{1}{\beta} \int_1^{x^*} \frac{Gx^*}{G^*(H^*, l^*, M_0, C, \psi)} dx^* \right\}. \quad (20)$$

The load-bearing capacity is:

$$W = D \int_{-R}^R p dr.$$

The non-dimensional load-carrying capability is:

$$\begin{aligned} W^* &= \frac{Wh_{m0}^3}{\mu_0 R^2 \left(\frac{dh}{dt} \right)} = \\ &= -\frac{1}{G} \int_{-1}^1 \left[\ln \left\{ 1 + \frac{1}{\beta} \int_1^{x^*} \frac{Gx^*}{G^*(H^*, l^*, M_0, C, \psi)} dx^* \right\} \right] D dx^*. \end{aligned} \quad (21)$$

The dimensionless time is attained by integrating Eq. (21):

$$\begin{aligned} T^* &= \frac{th_{m0}^2}{B\mu_0 R^2} = \\ &= -\frac{1}{G} \int_{h_1^*}^1 \left\langle \int_{-1}^1 \left[\ln \left\{ 1 + \frac{1}{\beta} \int_1^{x^*} \frac{Gx^*}{G^*(H^*, l^*, M_0, C, \psi)} dx^* \right\} \right] D dx^* \right\rangle dh_m^*. \end{aligned} \quad (22)$$

3. Methodology

This study investigates squeeze film lubrication between a cylinder and a rough, porous flat surface through a comprehensive theoretical and numerical methodology, integrating multiple physical phenomena into a unified model. The lubricant, modelled as a couple stress fluid using Stokes' theory, is assumed to exhibit isothermal, incompressible flow without external body forces or couples, with pressure-dependent viscosity governed by Barus' law. Surface roughness is characterised via Christensen's stochastic model for longitudinal and transverse patterns, and porosity is incorporated using Darcy's law. The governing equations of motion and continuity are solved under appropriate boundary conditions to derive a velocity field, leading to a modified Reynolds equation that integrates roughness effects and magnetohydrodynamic influences via the Hartmann number. The model is non-dimensionalised for the generalised analysis, yielding dimensionless pressure distributions, and analytical expressions for load-carrying capacity and squeeze film time are established where feasible. Solving the modified Reynolds equation is not merely a mathematical exercise, but a crucial step that enables accurate prediction of lubrication performance under complex conditions. This understanding is essential for optimising system design in high-precision applications, including advanced manufacturing. To address the complexity of interdependent parameters (Hartmann number, surface roughness, viscosity variation, couple stress and porosity), the lubrication domain is discretised with a uniform rectangular mesh, optimised through convergence studies for computational accuracy. Numerical integration is performed using Simpson's one-third rule, which is well-suited for capturing the smooth variations inherent in the system. This numerical approach not only demonstrates

the capability to solve the modified Reynolds equation but also provides practical benefits by enabling detailed analysis of lubrication performance across a wide range of operating conditions. Comprehensive data collection is achieved through systematic simulations across a wide range of realistic parameter values: surface roughness ($C = 0, 0.3, 0.5$), viscosity variation ($G = 0.001, 0.003, 0.005$), couple stress parameter ($l^* = 0, 0.2, 0.4$), Hartmann number ($M_0 = 0, 2, 4$), and porosity ($\psi = 0.001, 0.002, 0.004$). Other parameters are held constant at $m = 0.1$, $\alpha = 0.2$, $\delta = 0.1$, $h_m^* = 0.8$, and $\beta = 0.04$. For each parameter combination, simulations are performed to compute the pressure distribution, load-carrying capacity and squeeze film time. The simulation results are presented in tabular form and visualised through 2D and 3D plots to illustrate parameter sensitivity, interdependencies and performance trends. The interpretation emphasises both the magnitude and direction of each parameter's effect on lubrication performance, ensuring the derivation of meaningful engineering insights. Then, error analysis is conducted by comparing numerical results with analytical solutions, achieving strong agreement with relative errors typically below a very small percentage, confirming the model's robustness. Discrepancies are quantified using standard metrics such as mean absolute error to ensure reliability. Finally, the model's validity is confirmed by comparing numerical results with analytical solutions, showing strong agreement. Crucially, unlike previous studies, which often analysed parameters in isolation (e.g. Lin et al. [31]), this methodology integrates all relevant physical factors – couple stress, roughness, porosity and viscosity variation – within a single cohesive framework, providing a more accurate and holistic understanding of lubrication performance for enhanced predictive capabilities in aerospace, biomedical, and micro/nano-scale applications.

4. Results and discussion

This study examines how surface roughness (C), viscosity changes (G), couple stress (l^*), Hartmann number (M_0) and porosity (ψ) affect squeeze film lubrication performance, focusing on dimensionless pressure (P^*), load carrying capacity (W^*) and squeeze film time (T^*). By solving governing equations, we understand how these factors impact fluid flow, helping to optimise lubrication systems for better efficiency and durability in engineering fields like bearings and seals in the automotive and aerospace industries. This detailed parametric analysis, which explores the interaction of various factors, sets this work apart from previous studies and offers practical design recommendations.

4.1. Pressure

The plot of P^* with x^* for various C , with $\alpha = 0.2$, $M_0 = 2$, $l^* = 0.4$, $\psi = 0.001$, $\beta = 0.04$, $h_m^* = 0.8$, $G = 0.002$, $\delta = 0.1$ and $m = 0.1$ is displayed in Fig. 2. It is witnessed that the configuration of the one-dimensional roughness pattern coincides at $C = 0$, indicating no pressure variation. As C increases, irregularities in the surface lead to higher flow resistance, resulting in less effective pressure forces in the flow direction. Consequently, P^* declines for the longitudinal roughness pattern. This

is because higher roughness increases friction and impedes fluid flow, leading to a diminished pressure. In contrast, the transverse roughness pattern enhances P^* as C rises, as the fluid interacts with additional obstacles or irregularities perpendicular to the flow direction, enhancing the pressure.

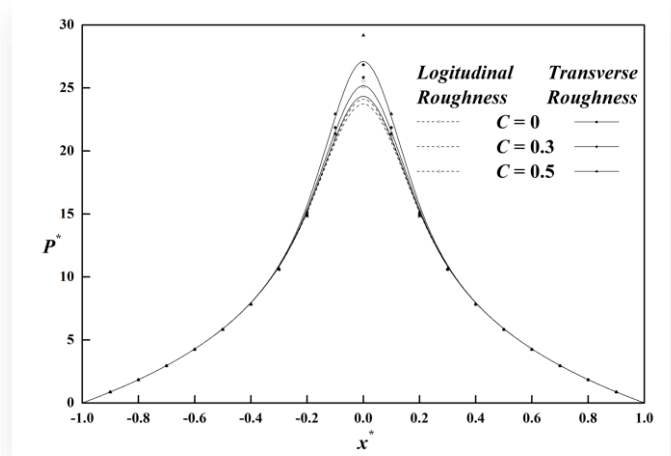


Fig. 2. Plot of P^* versus x^* for different C .

In Fig. 3, P^* vs. x^* for different M_0 is illustrated, with $\alpha = 0.2$, $m = 0.1$, $h_m^* = 0.8$, $G = 0.002$, $C = 0.3$, $\delta = 0.1$, $\psi = 0.001$, $l^* = 0.4$ and $\beta = 0.04$. It is revealed that as M_0 enhances, the influence of MHD stabilises the fluid flow through surface irregularities, resulting in enhanced P^* . This stabilisation reduces turbulence and allows the fluid to navigate irregularities more efficiently, leading to a higher pressure.

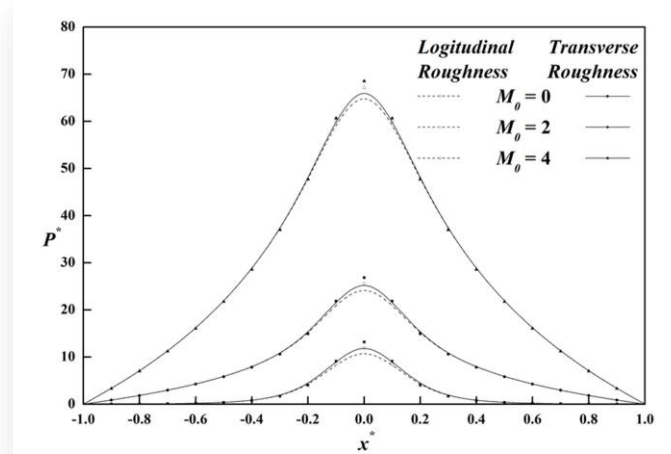


Fig. 3. Plot of P^* with respect to x^* for different M_0 .

Figure 4 shows the plot of P^* against x^* for distinct l^* . It is noticed that as l^* increases, there is increased fluid restriction within the roughness region, leading to an enhancement in pressure P^* . Higher couple stress results in greater fluid resistance to deformation, which improves the pressure by causing the fluid to exert more force against the rough surfaces.

In Fig. 5, P^* versus x^* for different ψ is presented. It is observed that increasing ψ improves fluid absorption through the porous regions. However, the delay of flow due to pores reduces the capacity of fluids to establish pressure within the surfaces,

resulting in a decline in pressure as the fluid's pressure is compromised by the delayed flow through the porous medium.

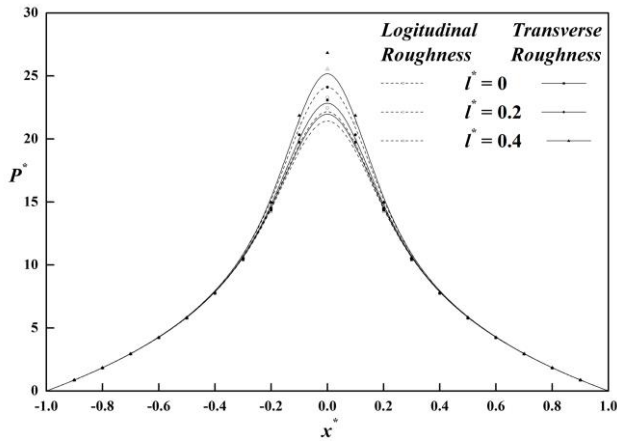


Fig. 4. Graph of P^* versus x^* for various l^* .

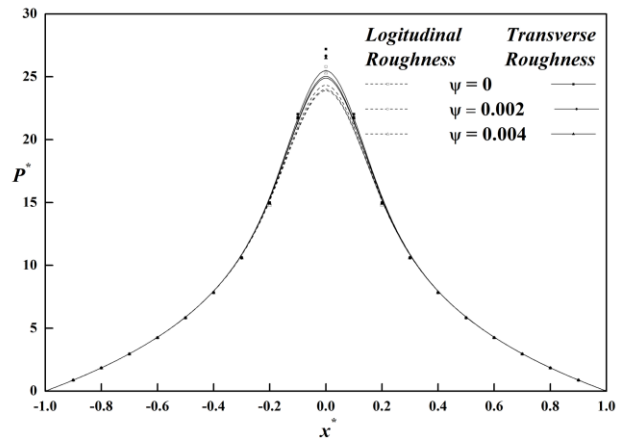


Fig. 5. Plot of P^* versus x^* for different ψ .

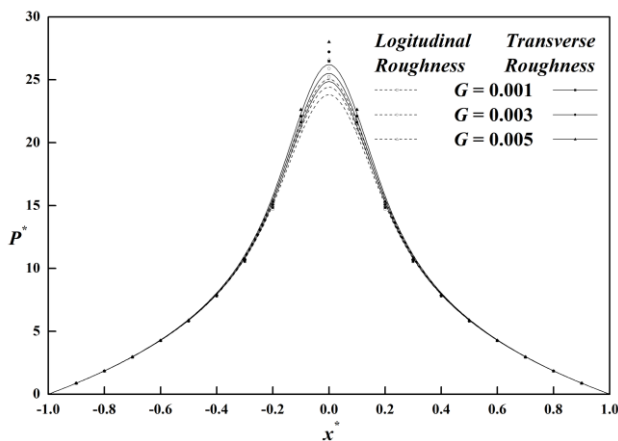


Fig. 6. Plot of P^* versus x^* for various G .

The function P^* against x^* for distinct G is shown in Fig. 6. It is revealed that as G rises, P^* also enhances due to changes in fluid behaviour. Higher viscosity variation makes the fluid more

resistant to flow changes, which helps maintain pressure within the squeeze film by better managing the fluid's interaction with surface irregularities. Overall, the observed variations in pressure profiles can be linked to how surface roughness, magnetic fields, fluid restrictions, porosity and viscosity affect fluid dynamics. Smoother surfaces, magnetic stabilisation and higher couple stress generally enhance pressure, while increased roughness, higher porosity and viscosity variations can either improve or impede pressure depending on their specific effects on fluid flow and interaction with the surface. These insights are valuable for designing more efficient lubrication systems in engineering applications, where maintaining optimal pressure and flow characteristics is crucial.

4.2. Load-carrying capacity

The graph of W^* versus h_m^* for distinct C , with $G = 0.002$, $\psi = 0.001$, $M_0 = 2$, $\beta = 0.04$, $l^* = 0.4$, $\alpha = 0.2$, $\delta = 0.1$, $m = 0.1$, is presented in Fig. 7.

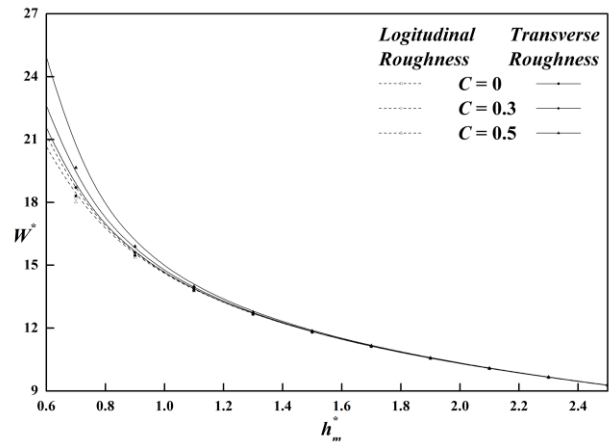


Fig. 7. Graph of W^* versus h_m^* for different C .

It is noticed that at $C = 0$, the configuration of the one-dimensional roughness pattern coincides. Moreover, as the C enhances, W^* declines for the longitudinal roughness pattern due to increased surface interactions, resulting in higher friction and reduced efficiency in load-bearing capacity. The rougher surface increases resistance, diminishing the film's load-carrying capacity. Conversely, W^* enhances as C rises for the transverse roughness pattern. The increased surface roughness enhances fluid interaction vertical to the flow direction, improving the film's ability to support loads by increasing fluid retention and film stability.

Figures 8 and 9 depict W^* vs. h_m^* for distinct M_0 and l^* , with $\alpha = 0.2$, $\beta = 0.04$, $G = 0.002$, $\delta = 0.1$, $C = 0.3$, and $\psi = 0.001$. It is found that as M_0 and l^* increase, fluid behaviour becomes more responsive to the magnetic field and couple stress. This enhanced responsiveness leads to improved fluid confinement within the roughness regions, which enhances W^* . The magnetic field and couple stress increase the fluid's resistance to distortion and stabilise its flow, allowing for better load support within the film.

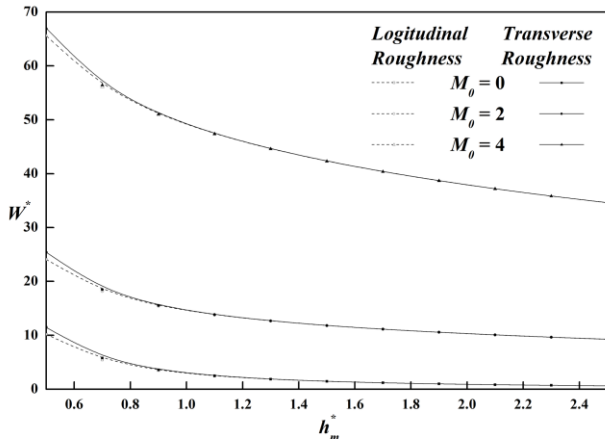


Fig. 8. Graph of W^* versus h_m^* for different M_0 .

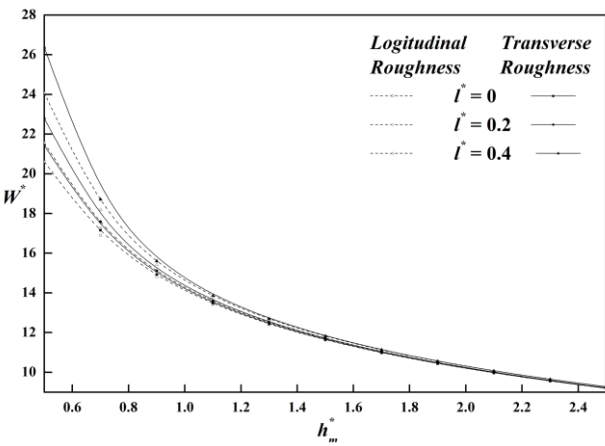


Fig. 9. Plot of W^* with h_m^* for distinct l^* .

Figure 10 demonstrates W^* against h_m^* for different ψ , with $M_0 = 2$, $\alpha = 0.2$, $l^* = 0.4$, $m = 0.1$, $G = 0.002$, $C = 0.3$, $\beta = 0.04$, and $\delta = 0.1$. It is noticed that as ψ enhances, fluid penetration through porous regions enhances, but this enhanced fluid absorption results in reduced load-bearing capacity. The increased porosity delays fluid flow and decreases the film's ability to es-

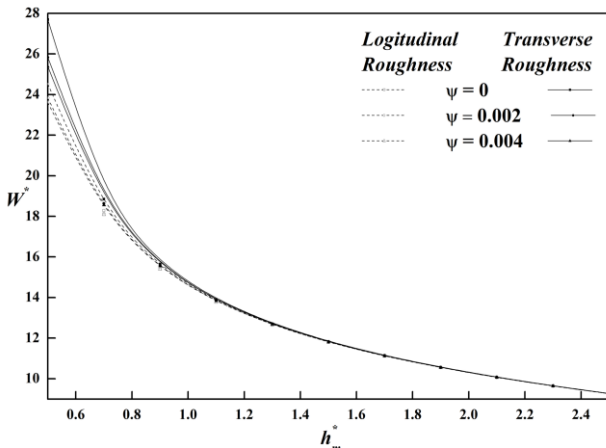


Fig. 10. Graph of W^* versus h_m^* for different ψ .

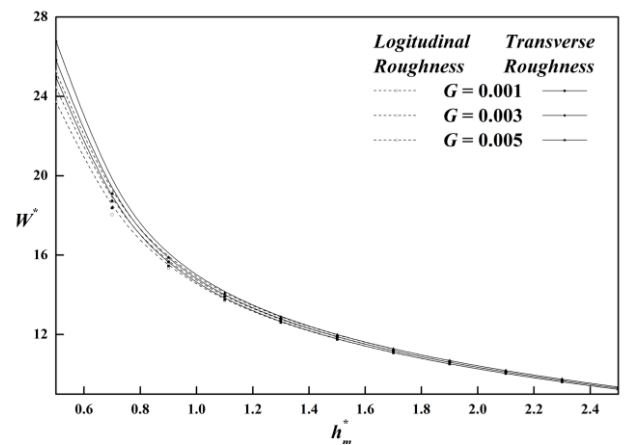


Figure 11. Graph of W^* versus h_m^* for different G .

tablish and maintain effective pressure, leading to a decrease in W^* .

The function W^* against h_m^* for different G is displayed in Fig. 11. It is seen that as G improves, W^* also improves. Higher viscosity variation alters fluid behaviour, enhancing its resistance to flow changes and increasing the fluid's load-bearing capability. This change helps the fluid maintain its support and stability within the roughness regions, leading to an improvement in W^* . In summary, the variations in W^* are linked to how roughness, magnetic fields, couple stress, porosity and viscosity affect fluid dynamics within the squeeze film. Increased roughness in the longitudinal direction generally decreases load-bearing capacity due to higher friction, while transverse roughness can improve it by enhancing fluid support. Magnetic fields and couple stress improve fluid confinement and load support, higher porosity reduces the film's load-bearing capacity, and increased viscosity variation stabilises fluid behaviour, thereby enhancing load support. These findings are crucial for optimising lubrication systems and designing effective bearing surfaces in engineering applications.

4.3. Squeeze film time

Figure 12 illustrates the graph of T^* vs. h_m^* for various C , with $\psi = 0.001$, $G = 0.002$, $M_0 = 2$, $l^* = 0.4$, $\delta = 0.1$, $m = 0.1$, $\alpha = 0.2$, $\beta = 0.04$, $G = 0.002$. It is noted that at $C = 0$, the configuration of the one-dimensional roughness pattern coincides. Moreover, as C rises, T^* declines for the longitudinal roughness pattern. This decline is due to increased fluid permeation through the rough surface, which reduces the film's ability to sustain a longer squeeze time. Conversely, T^* increases for the transverse roughness pattern as C rises. In this case, the roughness enhances fluid confinement and retention, which prolongs the squeeze film time by improving the stability and duration of the fluid layer.

Figures 13 and 14 depict the function T^* vs. h_m^* for distinct M_0 and l^* , with $\delta = 0.1$, $\psi = 0.001$, $G = 0.002$, $C = 0.3$, $m = 0.1$, $\beta = 0.04$, $\alpha = 0.2$. It is found that as M_0 and l^* increase, fluid motion becomes more responsive to the magnetic field and couple stress, leading to improved fluid containment within the roughness regions. This results in an increase in T^* , as the enhanced fluid behaviour supports a longer film duration.

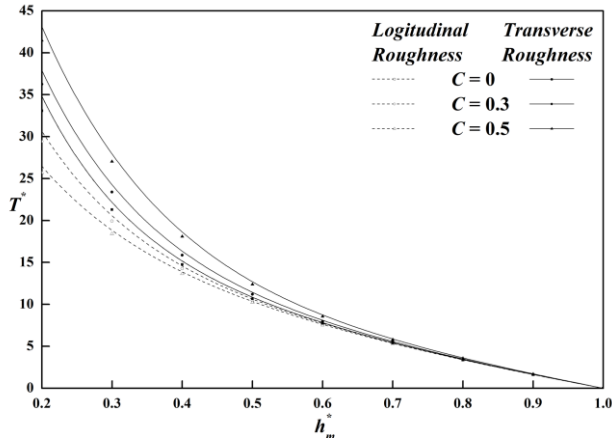


Fig. 12. T^* with h_m^* for various C .

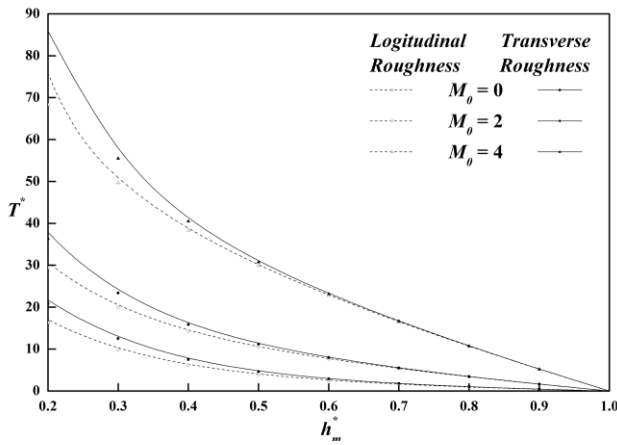


Fig. 13. T^* with h_m^* for various M_0 .

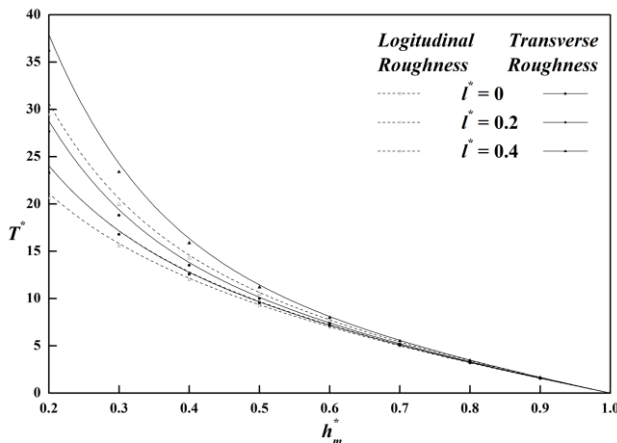


Fig. 14. T^* with h_m^* for different l^* .

The graph of T^* versus h_m for various ψ is presented in Fig. 15, with $\alpha = 0.2$, $l^* = 0.4$, $m = 0.1$, $C = 0.3$, $\delta = 0.1$, $\beta = 0.04$, $G = 0.002$, $M_0 = 2$. It is shown that as ψ rises, higher porosity reduces the fluid's ability to remain contained within the film. The increased porosity allows more fluid to escape through the

porous regions, leading to a decrease in T^* as the fluid's ability to maintain a stable film is diminished.

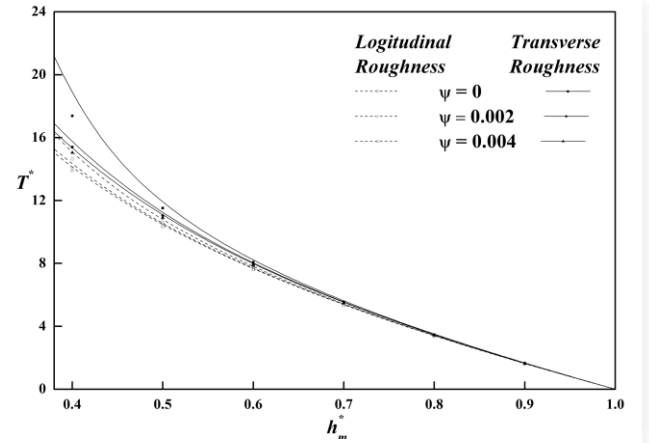


Fig. 15. T^* with h_m^* for different ψ .

Figure 16 describes T^* changing with h_m^* for different G , showing that T^* enhances as G rises. Higher viscosity variation alters the fluid's behaviour, enhancing its resistance to flow changes and improving its ability to remain within the film for a longer period. These consequences occur in an extended squeeze film time, as the fluid's stability is better maintained despite changes in surface roughness.

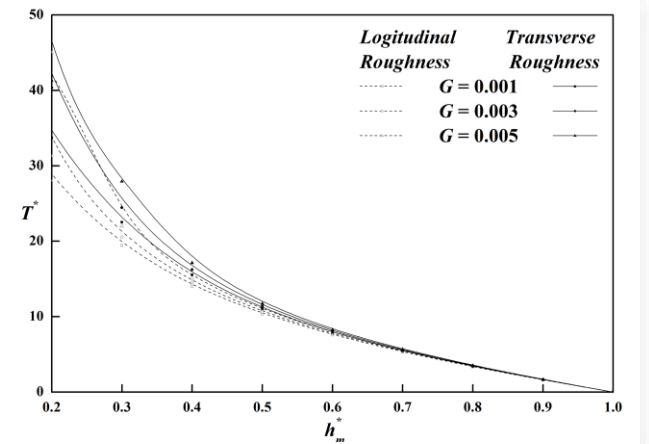


Fig. 16. T^* versus h_m^* for various G .

Overall, the observed variations in T^* are related to how different factors affect fluid behaviour within the squeeze film. Increased roughness in the longitudinal direction leads to reduced squeeze film time due to greater fluid permeation, while transverse roughness improves fluid retention and prolongs squeeze time. Enhanced magnetic fields and couple stress, along with higher viscosity variation, contribute to improved fluid stability and extended squeeze time, whereas higher porosity diminishes the film's ability to sustain a longer film duration. These findings are crucial for designing lubrication systems where maintaining optimal squeeze film time is essential for reliable performance.

4.4. Combined effect of roughness and porosity, viscosity variation, Hartmann number and couple stress

To comprehensively evaluate the interdependence of parameters affecting squeeze film performance, this section presents an extensive three-dimensional parametric analysis examining the combined effects of surface roughness with porosity, viscosity variation, Hartmann number and couple stress parameter on the dimensionless pressure, load-carrying capacity and squeeze film time. The study considers both longitudinal and transverse roughness patterns, revealing complex interactions that provide valuable insights for optimising lubrication system performance. To enhance clarity and address the need for varied graphical representation, three-dimensional surface plots are utilised. The results are further contextualised with practical applications in engineering domains that demand precise fluid dynamic control.

4.4.1. Combined effect of roughness and porosity

Figures 17–22 present the three-dimensional interaction between surface roughness and porosity on dimensionless pressure, load-carrying capacity and squeeze film time, under both longitudinal and transverse roughness configurations. The analysis was performed using the parameters: $M_0 = 1$, $G = 0.04$, $h_m^* = 0.7$, $\alpha = 0.2$, $l^* = 0.3$, $\delta = 0.1$, $m = 0.1$, and $\beta = 0.3$. For longitudinal roughness, the results reveal that a simultaneous reduction in roughness pattern and porosity leads to a significant enhancement in all three performance indicators: P^* , W^* and T^* . This suggests that smoother surfaces, coupled with lower

fluid absorption through the porous structure, facilitate more effective pressure build-up and stronger load-bearing capabilities. From a lubrication design perspective, this combination minimises energy losses due to flow resistance and ensures prolonged squeeze film support, which is essential for high-precision applications such as aerospace bearings and engine liners. On the other hand, under transverse roughness conditions, a different trend emerges. An increase in surface roughness combined with a decrease in porosity results in a corresponding improvement in pressure, load carrying capacity and squeeze film duration. This can be attributed to the transverse orientation introducing flow-interrupting features that trap the lubricant more efficiently. As the porosity decreases, less fluid is lost through absorption, allowing for a thicker, more stable film. This behaviour is particularly advantageous in industrial settings where components are exposed to heavy loads or intermittent shocks, such as in heavy-duty gears or piston-cylinder arrangements.

Overall, the combined effect of C and ψ highlights a dual design strategy: reducing both parameters under longitudinal roughness improves efficiency by reducing flow resistance, while increasing roughness under transverse configurations enhances lubricant retention and structural support. This duality underscores the importance of tailoring surface geometry and material porosity according to the orientation of the roughness pattern and specific operational demands. These findings not only contribute to the theoretical understanding of textured porous bearings but also provide a robust design framework for optimising tribological performance across a range of real-world applications.

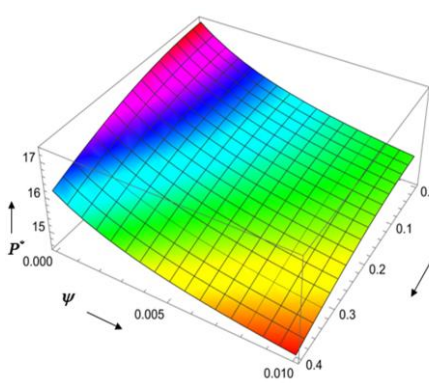


Fig. 17. P^* vs. C (longitudinal roughness) and ψ .

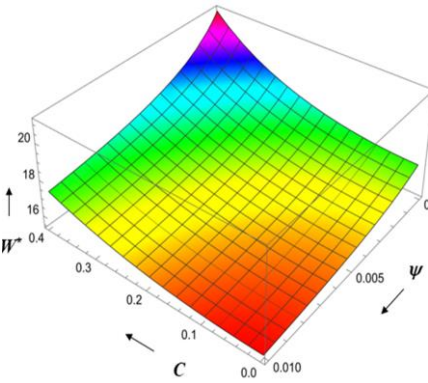


Fig. 18. W^* vs. C (longitudinal roughness) and ψ .

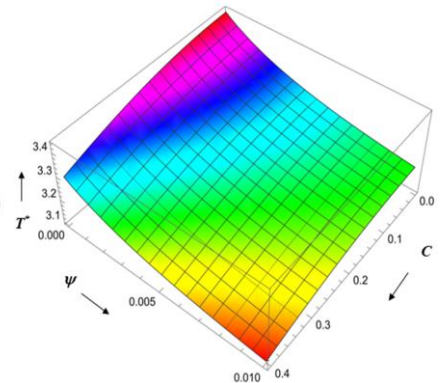


Fig. 19. T^* vs. C (longitudinal roughness) and ψ .

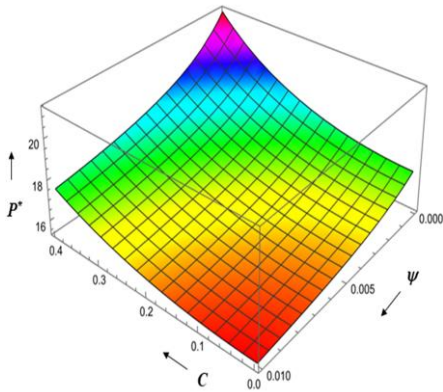


Fig. 20. P^* vs. C (transverse roughness) and ψ .

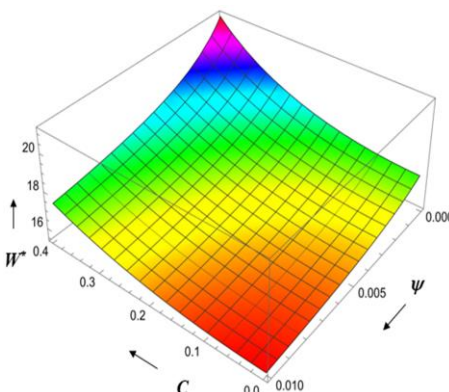


Fig. 21. W^* vs. C (transverse roughness) and ψ .

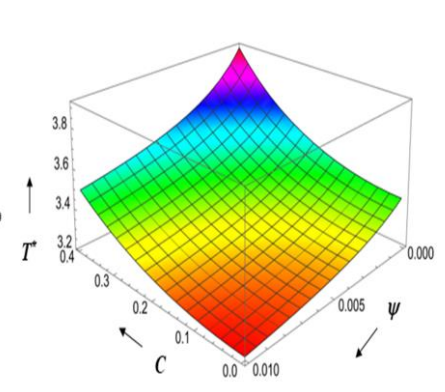


Fig. 22. T^* vs. C (transverse roughness) and ψ .

4.4.2. Combined effect of roughness and viscosity variation

Figures 23–28 provide a comprehensive three-dimensional analysis of how the combined influence of surface roughness and viscosity variation affects P^* , W^* and T^* . The simulations are conducted using the following parameters: $m = 0.1$, $M_0 = 1$, $l^* = 0.3$, $\beta = 0.3$, $h_m^* = 0.7$, $\alpha = 0.2$, $\psi = 0.001$ and $\delta = 0.1$. For longitudinal roughness patterns, the results clearly show that a decrease in roughness combined with an increase in viscosity variation leads to significant improvements in P^* , W^* and T^* . This behaviour is physically intuitive: smoother surfaces reduce mechanical friction and irregularities that disrupt fluid flow, while enhanced viscosity variation improves the lubricant's resistance to shear deformation and allows it to better conform to surface features. The resulting increase in pressure and load-carrying capability, coupled with an extended squeeze film time, makes this combination highly advantageous in applications where maintaining a continuous and stable lubrication layer is critical, such as in precision mechanical systems, microbearings, and journal bearings in automotive and aerospace sectors. In contrast, for transverse roughness patterns, the study reveals that simultaneous increases in both C and G produce optimal performance. Unlike longitudinal patterns, transverse roughness intro-

duces geometric features perpendicular to the flow direction, creating zones of increased fluid entrapment. When combined with higher viscosity variation, the fluid exhibits greater resistance to motion, while being retained more effectively between the rough textures. This synergy leads to improved pressure distribution, greater film thickness and prolonged film stability. Such behaviour is especially valuable in high-load, high-friction environments—such as heavy-duty gear systems, off-road vehicle suspensions and industrial press components—where maintaining lubricant integrity under variable stress conditions is essential.

Overall, the results highlight the nonlinear interplay between surface topography and fluid rheology, demonstrating that optimal performance is not solely dependent on minimising roughness or maximising viscosity variation but rather on the appropriate pairing of both based on surface orientation. For longitudinal roughness, smoother surfaces benefit more from increased viscosity gradients, while transverse configurations leverage roughness to trap and stabilise high-viscosity fluids. These findings contribute meaningful insights into the design of advanced lubrication systems, offering guidance on how to strategically manipulate surface and fluid parameters to improve tribological performance across diverse engineering applications.

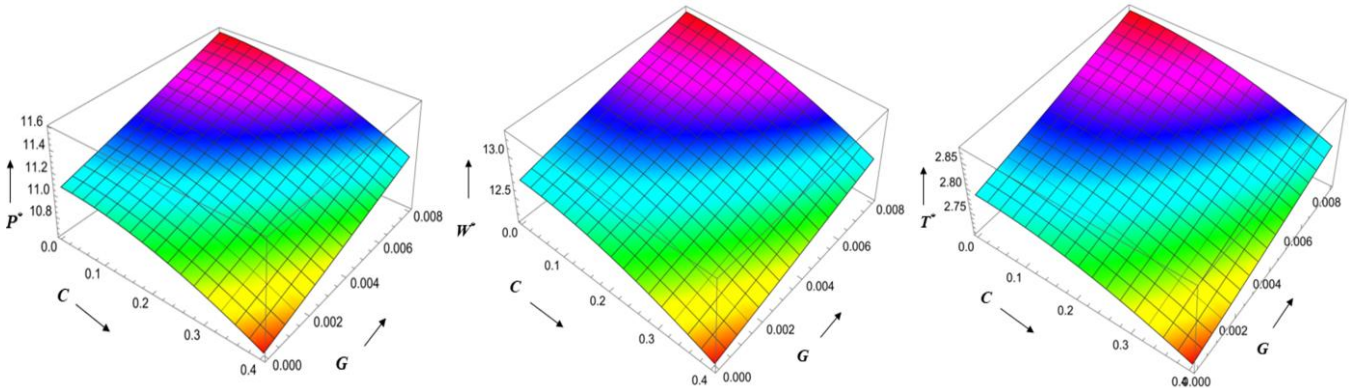


Fig. 23. P^* vs. C (longitudinal roughness) and G . Fig. 24. W^* vs. C (longitudinal roughness) and G . Fig. 25. T^* vs. C (longitudinal roughness) and G .

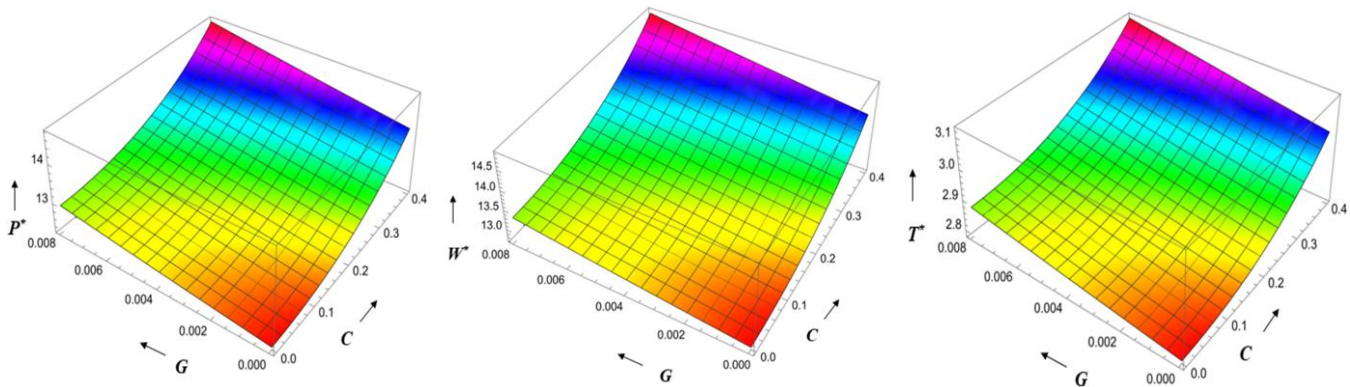


Fig. 26. P^* vs. C (transverse roughness) and G . Fig. 27. W^* vs. C (transverse roughness) and G . Fig. 28. T^* vs. C (transverse roughness) and G .

4.4.3. Combined effect of roughness and Hartmann number

Figures 29–34 illustrate the combined influence of surface roughness and Hartmann number on P^* , W^* and T^* . The simulations are conducted using the following fixed parameters:

$\psi = 0.001$, $\beta = 0.3$, $G = 0.04$, $h_m^* = 0.7$, $l^* = 0.3$, $\alpha = 0.2$, $\delta = 0.1$ and $m = 0.1$. In the case of longitudinal roughness, the results reveal that decreasing surface roughness while increasing the Hartmann number yields pronounced improvements in all three performance indicators: P^* , W^* and T^* . The enhancement in

performance can be attributed to the MHD effect, where the application of a magnetic field induces Lorentz forces within the conductive fluid. These forces act to dampen velocity gradients, reduce turbulence and align fluid motion, thereby stabilising the flow through irregular rough regions. As a result, the fluid film becomes more resilient, better able to maintain pressure and capable of supporting higher loads over longer durations. This mechanism is particularly beneficial in lubrication systems involving magnetorheological or ferrofluid lubricants, where external magnetic fields can be strategically employed to enhance operational efficiency and reduce mechanical wear. For transverse roughness patterns, a different trend emerges. The data show that simultaneously increasing both roughness and Hartmann number leads to optimal performance. The underlying physics can be explained by the unique role of transverse roughness in introducing geometric barriers perpendicular to the flow. When combined with a strong magnetic field, these barriers enhance the retention of the fluid within the microcavities of the surface. The magnetic field dampens fluid motion in regions where escape is likely, improving fluid stability and resistance to leakage. Consequently, the lubricant film exhibits higher

pressure and greater durability, which translates to increased load-bearing capacity and prolonged squeeze film time. From a practical standpoint, these results have significant engineering implications. In applications where magnetic fields can be externally controlled – such as in magnetorheological dampers, electric actuators and aerospace bearing systems – the deliberate tuning of surface roughness and magnetic intensity offers a valuable tool for real-time control of lubricant film behaviour. For longitudinal surfaces, smoothness should be prioritised alongside MHD effects to minimise friction and improve responsiveness. Conversely, for transverse patterns, enhanced surface roughness can be leveraged to synergise with magnetic stabilisation, ensuring film integrity even under dynamic and harsh operating conditions. Overall, the combined effect of roughness and Hartmann number illustrates the importance of aligning fluid control strategies with surface design. The MHD effect not only stabilises the fluid flow but also amplifies the positive or negative effects of roughness depending on its orientation. These findings bridge the gap between theoretical modelling and field applications, offering actionable insights for optimising lubrication systems in magnetically active environments.

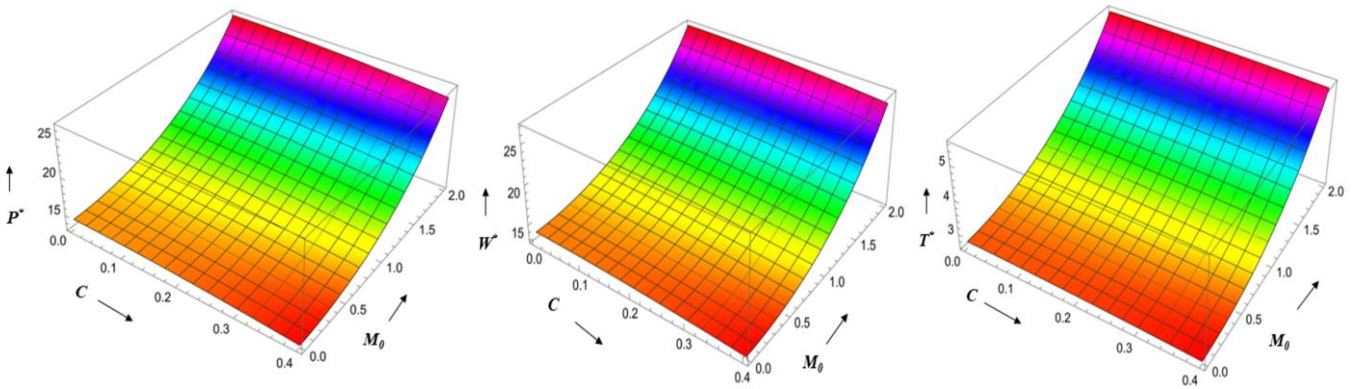


Fig. 29. P^* vs. C (longitudinal roughness) and M_0 . Fig. 30. W^* vs. C (longitudinal roughness) and M_0 . Fig. 31. T^* vs. C (longitudinal roughness) and M_0 .

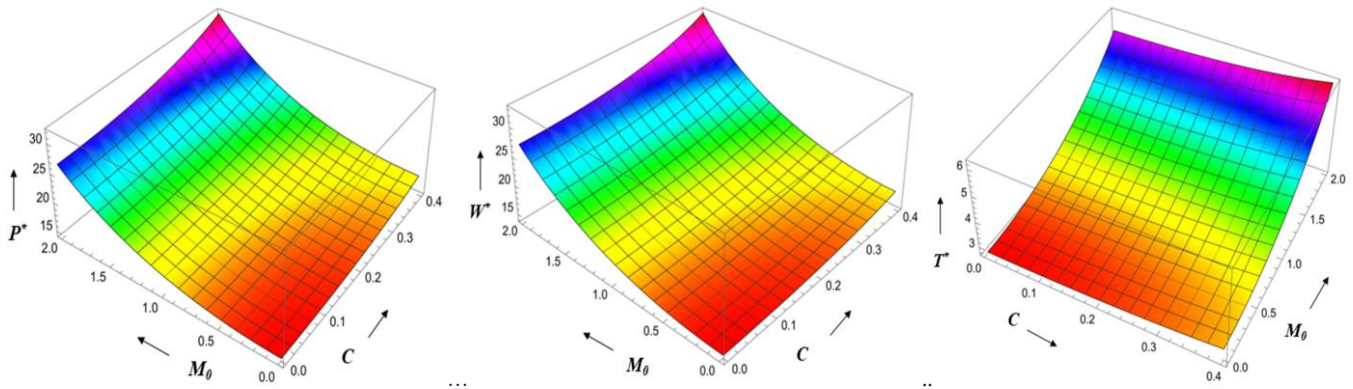


Fig. 32. P^* vs. C (transverse roughness) and M_0 . Fig. 33. W^* vs. C (transverse roughness) and M_0 . Fig. 34. T^* vs. C (transverse roughness) and M_0 .

4.4.4. Combined effect of roughness and couple stress

Figures 35–40 present the combined influence of the surface roughness parameter and the couple stress parameter on the P^* , W^* and T^* , under the conditions: $G = 0.04$, $\beta = 0.3$, $M_0 = 1$, $\psi = 0.001$, $h_m^* = 0.7$, $\alpha = 0.2$, $\delta = 0.1$ and $m = 0.1$. The analysis is conducted for both longitudinal and transverse roughness patterns to uncover nuanced behaviours in complex fluid-structure

interactions. For longitudinal roughness configurations, the study reveals that a decrease in surface roughness, when accompanied by an increase in the couple stress parameter, significantly enhances P^* , W^* and T^* . This trend can be attributed to the nature of couple stress fluids, which incorporate microstructural effects that resist velocity gradients and shear deformation. An elevated l^* value signifies stronger internal resistance within the fluid due to the presence of suspended particles, additives or

molecular chain entanglements. These microstructural effects stabilise the film, improve fluid retention and resist squeeze-induced thinning, ultimately resulting in higher pressure build-up and prolonged film duration. In contrast, under transverse roughness conditions, the combined increase of both C and l^* leads to noticeable improvements in performance. This synergy likely stems from the transverse roughness acting as a micro-reservoir system that works in tandem with the couple stress

properties of the fluid. The enhanced surface texture traps the fluid more effectively, while the couple stress interactions prevent its rapid deformation or escape. As a result, the squeeze film remains stable over extended periods, even under cyclic or impact loading. This outcome is particularly relevant in high-load, high-frequency environments such as those found in microelectromechanical systems (MEMS), aerospace bearings and biomedical implants.

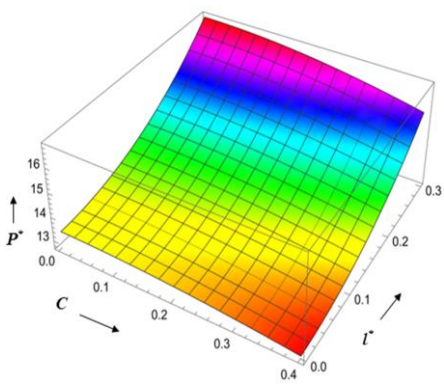


Fig. 35. P^* vs. C (longitudinal roughness) l^* .

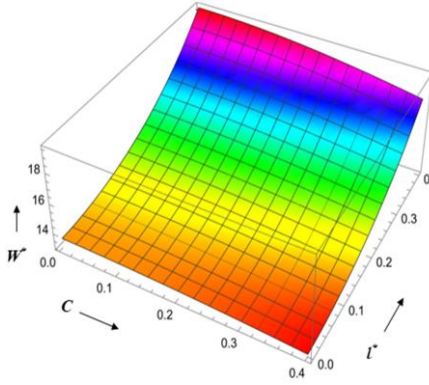


Fig. 36. W^* vs. C (longitudinal roughness) and l^* .

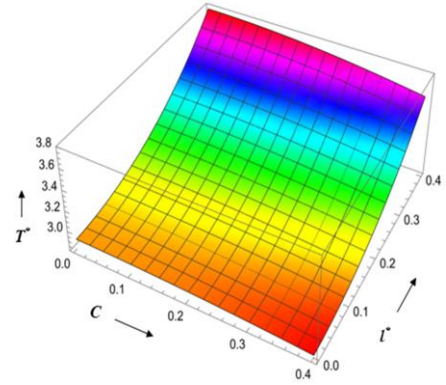


Fig. 37. T^* vs. C (longitudinal roughness) and l^* .

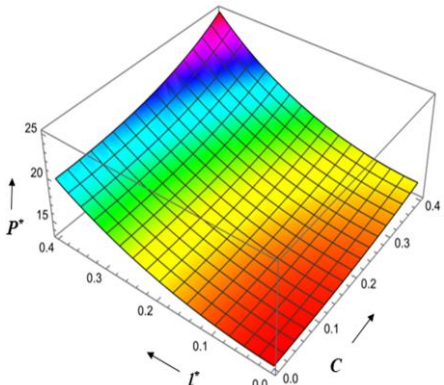


Fig. 38. P^* vs. C (transverse roughness) and l^* .

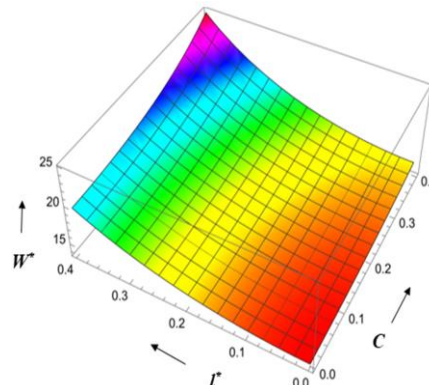


Fig. 39. W^* vs. C (transverse roughness) and l^* .

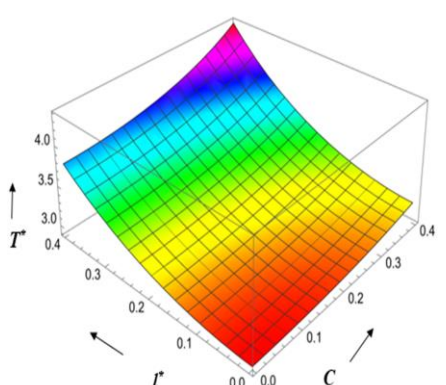


Fig. 40. T^* vs. C (transverse roughness) and l^* .

These findings provide meaningful design implications. For lubrication systems operating under high stress or involving non-Newtonian or microstructured lubricants, fine-tuning both surface texture and couple stress properties can lead to marked performance enhancements. In longitudinal roughness patterns, minimising surface roughness is beneficial to reduce friction and allow the microstructural fluid to respond uniformly. Meanwhile, in transverse orientations, increased roughness, when properly engineered, offers mechanical interlocking with the fluid's internal structure, increasing pressure, load support and operational life. Across all studied configurations, variations in P^* , W^* and T^* were found to be strongly linked to the fluid's microstructural resilience and its interaction with the topography of the bounding surfaces. Smoother surfaces in the direction of motion reduce energy losses, while controlled roughness in the transverse direction enhances entrapment and stability. These interactions are more profound in couple stress fluids compared to classical Newtonian lubricants, as they exhibit additional degrees of freedom due to intrinsic angular momentum and size-dependent effects. This multidimensional investigation

advances existing literature by moving beyond isolated parametric studies. Previous works have typically employed 2D analyses or focused solely on either surface geometry or fluid rheology. In contrast, this study utilises 3D surface plots and parametric combinations to deliver a more comprehensive, visually intuitive, and practically applicable understanding of lubrication dynamics. Plots summarise optimal combinations, enabling quick reference for design engineers. Overall, achieving optimal performance in squeeze film systems requires a strategic combination of key design and operating parameters. Specifically, reducing longitudinal roughness and porosity enhances pressure buildup and fluid film stability, while increasing viscosity variation, couple stress effects and Hartmann number improves the fluid's resistance to deformation and external magnetic influences. Additionally, transverse roughness, when carefully optimised, can serve as a functional advantage by enhancing fluid entrapment and load support. Together, these combined effects contribute to improved pressure distribution, greater load-carrying capacity, and extended squeeze film time, making the lubri-

cation system more efficient and durable under a wide range of engineering applications.

The present investigation demonstrates strong alignment with the analytical results of Lin et al. [31], particularly when the effects of magnetohydrodynamics, viscosity variation, and porosity are excluded from the current model. Under such simplified conditions, the numerical outcomes for pressure, load-carrying capacity, and squeeze film time in the current study closely mirror those reported by Lin et al. [31], particularly for a cylinder on a flat surface configuration. This validation, presented in Table 1, confirms the accuracy and consistency of the proposed numerical framework. The excellent agreement further reinforces the model's credibility in simulating complex physical phenomena across various parametric settings.

Table 1. Current analysis numerically compared with Lin et al. [31] using parameter $C = 0.2$, $G = 0.003$, $M_0 = 2$, $\psi = 0.002$, $\delta = 0.1$, $m = 0.1$.

Analysis		Lin et al [31] analysis	Present analysis			
Parameter		$C = 0$, $M_0 = 0$, $\psi = 0$, $G = 0$	$C = 0$, $M_0 = 0$, $\psi = 0$, $G = 0$	$C = 0.2$, $G = 0.003$, $\psi = 0.002$, $M_0 = 2$, $\delta = 0.1$, $m = 0.1$		
			LR	TR	LR	TR
P^*	$l^* = 0$	0.483621	0.483579	0.483579	5.62156	5.62291
	$l^* = 0.2$	0.485617	0.485619	0.485618	5.64031	5.64168
	$l^* = 0.4$	0.491384	0.491385	0.491385	5.67947	5.68086
W^*	$l^* = 0$	0.483621	0.483579	0.483579	5.62156	5.62291
	$l^* = 0.2$	0.485617	0.485619	0.485618	5.64031	5.64168
	$l^* = 0.4$	0.491384	0.491385	0.491385	5.67947	5.68086
T^*	$l^* = 0$	0.483621	0.483579	0.483579	5.62156	5.62291
	$l^* = 0.2$	0.485617	0.485619	0.485618	5.64031	5.64168
	$l^* = 0.4$	0.491384	0.491385	0.491385	5.67947	5.68086

LR - Longitudinal roughness, TR - Transverse roughness

The relative percentage enhances R_{W^*} , R_{T^*} with respect to W^* and T^* , respectively are:

$$R_{W^*} = \left(\frac{W_{magnetic}^* - W_{non-magnetic}^*}{W_{non-magnetic}^*} \right) \times 100,$$

$$R_{T^*} = \left(\frac{T_{magnetic}^* - T_{non-magnetic}^*}{T_{non-magnetic}^*} \right) \times 100.$$

To evaluate the enhancement in squeeze film performance, Table 2 presents a comparative analysis of the relative percentage increases in W^* and T^* , expressed through dimensionless ratios R_{W^*} and R_{T^*} across various combinations of Hartmann number and viscosity variation, with $\alpha = 0.2$, $C = 0.3$, $h_m^* = 0.8$, $\beta = 0.04$, $\psi = 0.001$, $\delta = 0.1$, $m = 0.1$ and $l^* = 0.2$. For instance, at $M_0 = 2$ and $G = 0.004$, the model shows significant improvements of approximately 85.29% in load-carrying capacity and 85.34% in squeeze film time, respectively. These remarkable enhancements underscore the synergistic benefit of combining magnetic field influence with fluid viscosity variation. The results suggest that magnetic stabilisation not only improves fluid confinement but also complements the stabilising effect of non-Newtonian fluid behaviour in high-performance lubrication regimes.

Table 2: Relative percentage of R_{W^*} and R_{T^*} .

M_0	G	R_{W^*}	R_{T^*}
0	0.002	0.024321522	0.024243271
	0.004	0.072964567	0.073140716
	0.006	0.121607611	0.121832709
1	0.002	0.090454811	0.090926625
	0.004	0.273019095	0.273330945
	0.006	0.456134932	0.456286336
2	0.002	0.282115348	0.282318323
	0.004	0.852940023	0.853367247
	0.006	1.432319049	1.433143993

These findings carry profound engineering significance. In automotive and aerospace systems – such as engine liners, thrust bearings and landing gear – where high loads, high speeds and varying environmental conditions prevail, the combined presence of couple stress effects, viscosity gradients and magnetic fields can substantially enhance performance. Improved pressure generation and increased load-bearing capacity reduce mechanical wear, enhance energy efficiency and extend component life. Similarly, in microelectromechanical systems, where surface-to-volume ratios are high and the influence of surface interactions is dominant, the inclusion of MHD and couple stress effects can enhance the reliability of micro-lubrication. Such enhancements support stable and leak-resistant operation even under high-frequency conditions, which is critical in micro-pumps, micro-turbines and other precision components. In the biomedical domain, especially in the design of artificial joints and implantable microfluidic devices, increased squeeze film time plays a vital role in providing improved shock absorption, smoother articulation and enhanced operational lifespan. The influence of surface roughness and porosity becomes particularly relevant here, as material selection and structural permeability are key factors in biocompatible lubrication. Importantly, the observed performance reduction due to increased porosity emphasises the necessity of carefully selecting materials with an optimal balance between permeability and structural strength. For porous bearings and filters, too much porosity can impair pressure generation and reduce the system's overall effectiveness. This highlights the importance of tuning design parameters based on specific application demands – whether the goal is longer operation, higher load support or improved damping. Overall, the insights gained from this study extend beyond theoretical analysis. They provide practical design guidance for optimising lubrication systems across a wide array of applications. By quantitatively illustrating the interplay between surface roughness, porosity, viscosity variation, magnetic fields and microstructural fluid effects, the results offer a comprehensive framework that engineers can apply to design more robust, efficient and application-specific fluid film systems.

5. Conclusions

The current study investigates the performance of squeeze lubrication between a cylinder and a rough, porous flat surface em-

ploying Christensen's stochastic model. Several important observations are:

- ❖ The effect of surface roughness for longitudinal surface roughness tends to decrease the dimensionless pressure, load-carrying capacity, and squeeze film time when compared to a smooth surface. In contrast, transverse roughness improves these performance metrics due to enhanced fluid retention.
- ❖ The presence of the Hartmann number, couple stress and viscosity variation positively influences the system. These factors significantly increase the load-bearing capacity, pressure and squeeze film time, highlighting their crucial roles in boosting the overall efficiency of the squeeze film.
- ❖ Porosity negatively impacts the system, resulting in a decline in pressure, squeeze film time and load-bearing ability for one-dimensional roughness patterns, owing to fluid seepage through the porous medium. This underscores the importance of minimising porosity in high-performance applications.
- ❖ The 3D parametric analysis shows that decreasing the roughness parameter in longitudinal roughness patterns and increasing it in transverse patterns, along with reduced porosity, produces maximum values of P^* , W^* and T^* . Additionally, higher values of G , M_0 and I^* consistently lead to improved P^* , W^* and T^* across both roughness orientations, confirming the benefits of multi-parameter optimisation.
- ❖ A comparative numerical analysis with the findings of Lin et al. [31] demonstrates strong agreement, validating the accuracy and reliability of the proposed model and numerical framework.
- ❖ Combining magnetic field effects ($M_0 = 2$) with viscosity variation ($G = 0.004$) results in significant enhancement of squeeze film lubrication, achieving relative percentage improvements of approximately 85.29% in load-carrying capacity and 85.34% in squeeze film time under optimal conditions.

Despite the robustness of the current theoretical model, several limitations should be recognised. The model is developed under idealised assumptions, such as isothermal, incompressible flow with no external forces, and the analysis is limited to its reliance on numerical simulations without experimental validation, which may not fully capture real-world dynamics. These findings offer valuable insights for designing efficient lubrication systems. Future research should focus on experimental validation using micro-tribological setups, integration of thermal and dynamic effects, and adoption of higher-order or adaptive numerical schemes to improve accuracy and extend the model's applicability to more complex, real-world lubrication scenarios.

6. Novelty

This study presents a novel investigation into the impact of roughness patterns and the role of porosity on squeeze film performance, uniquely integrating factors like viscosity variation, Hartmann number and couple stress, which are often examined separately in existing research. Specifically, it introduces an in-depth three-dimensional graph analysis, revealing how precise

optimisation of roughness and porosity can enhance pressure, load-carrying capability and squeeze film time. This approach provides fresh insights into maximising performance in squeeze film applications. The study introduces these novel factors and demonstrates a strong alignment with Lin et al. [31], showcasing significant advancements over previous models. This integration and comparative analysis underline the practical implications for optimising engineering designs, making a meaningful contribution to the field.

References

- [1] Bujurke, N.M., Naduvanamani, N.B., & Basti, D.P. (2007). Effect of surface roughness on the squeeze film lubrication between curved annular plates. *Industrial Lubrication and Tribology*, 59(4), 178–185. doi: 10.1108/00368790710753572
- [2] Christensen, H. (1969). Stochastic models for hydrodynamic lubrication of rough surfaces. *Proceedings of the institution of mechanical engineers*, 184(1), 1013–1026. doi: 10.1243/PIME_PROC_1969_184_074_02
- [3] Daliri, M., & Jalali-Vahid, D. (2015). Combined effects of piezo-viscous coupled stress lubricant and rotational inertia upon squeeze film performance of rough circular discs. *Industrial Lubrication and Tribology*, 67(6): 564–571. doi: 10.1108/ILT-01-2015-0009
- [4] Mouda, M., Nabhani, M., & El Khelifi, M. (2021). Surface roughness effects on non-Newtonian MHD non-parallel squeeze film bearing. *Industrial Lubrication and Tribology*, 73(1), 45–51. doi: 10.1108/ILT-02-2020-0071
- [5] Dass, T.D., Gunakala, S.R., & Comissiong, D.M. (2022). Combined effect of variable-viscosity and surface roughness on the squeeze film characteristics of infinitely wide rectangular plate with couple stress fluid, velocity-slip and ferrofluid lubricant. *Materials Today: Proceedings*, 56, 1717–1725. doi: 10.1016/j.matpr.2021.10.355
- [6] Ullah, I., Rahim, M.T., Khan, H., & Qayyum, M. (2016). Analytical analysis of squeezing flow in porous medium with MHD effect. *University of Bucharest Scientific Bulletin Series A. Applied Mathematics and Physics*, 78(2), 281–292.
- [7] Johny, A., Sujatha, E., & Sree, M.G. (2024). Effect of MHD on Porous Circular Stepped Plate Lubricated With Micropolar Fluid. *IAENG International Journal of Applied Mathematics*, 54(9), 1847–1854.
- [8] Yadav, P.K., & Srivastava, P. (2024). Impact of porous material and slip condition on the MHD flow of immiscible Couple stress-Newtonian fluids through an inclined channel: Head loss and pressure difference. *Chinese Journal of Physics*, 89, 1198–1221. doi: 10.1016/j.cjph.2024.03.046
- [9] Patel, N.C., Patel, J.R., & Deheri, G.M. (2023). An effect of a porous structure, slip velocity and Rosensweig's viscosity on the ferrofluid based squeeze film in porous curved annular plates. *Journal of Nanofluids*, 12(2), 498–505. doi: 10.1166/jon.2023.1906
- [10] Khan, M.S., Shah, R.A., & Khan, A. (2019). Effect of variable magnetic field on the flow between two squeezing plates. *The European Physical Journal Plus*, 134(5), 219. doi: 10.1140/epjp/i2019-12595-0
- [11] Patel, S.J., & Deheri, G. (2023). Slip effect on Shliomis model based magnetic fluid lubrication of a squeeze film in circular cylinder near a plane. *Acta Technica Corviniensis – Bulletin of Engineering*, 16(4).
- [12] Lin, J.R., Chu, L.M., Hung, C.R., & Wang, P.Y. (2013). Derivation of two-dimensional couple-stress hydromagnetic squeeze

- film Reynolds equation and application to wide parallel rectangular plates. *Meccanica*, 48(1), 253–258. doi: 10.1007/s11012-012-9613-7
- [13] Nabhani, M., & El Khelifi, M. (2016). Non-Newtonian inertial magnetohydrodynamic porous squeeze film lubrication between circular discs. *Tribology International*, 94, 373–382. doi: 10.1016/j.triboint.2015.09.047
- [14] Ramasamy, P. (2020). Effects of magnetic force and non-Newtonian characteristics on squeeze film bearings. *Asia-Pacific Journal of Chemical Engineering*, 15(5), e2510. doi: 10.1002/apj.2510
- [15] Jyothi, V., Hanumagowda, B.N., & Verma, A. (2023). Analysis of pressure load-carrying capacity and squeezing time on magnetohydrodynamic couple stress fluid flow between sphere and flat plate with slip velocity. *International Journal of Modern Physics B*, 38(26), 2450349. doi: 10.1142/S02179792245 03491
- [16] Prakash, J., & Tiwari, K. (1985). Effect of surface roughness on the squeeze film between rotating porous annular discs with arbitrary porous wall thickness, *International Journal of Mechanical Sciences*, 27(3), 135–144. doi: 10.1016/0020-7403(85)90054-2
- [17] Shah, R.C. (2022). Ferrofluid lubrication of porous-rough circular squeeze film bearings. *The European Physical Journal Plus*, 137(2), 1–16. doi: 10.1140/epjp/s13360-022-02344-z
- [18] Thakkar, P.L., & Patel, H.C. (2020). Effect of surface roughness on characteristics of magnetic fluid based squeeze film between porous circular plates. *Journal of the Serbian Society for Computational Mechanics*, 14(1), 90–98. doi: 10.24874/jsscm.2020.14.01.08
- [19] Vadher, P.A., Deheri, G., & Patel, R.M. (2008). The effect of surface roughness on the performance of hydromagnetic squeeze films between two conducting rough porous elliptical plates. *Turkish Journal of Engineering and Environmental Sciences*, 32(4), 219–227 (<https://www.researchgate.net/publication/228653661>).
- [20] Deheri, G.M., & Patel, J.R. (2011). Magnetic fluid based squeeze film in a rough porous parallel plate slider bearing. *Annals of the Faculty of Engineering Hunedoara*, 9(3), 443–448.
- [21] Anthony, J., & Elamparithi, S. (2023). Effect of MHD and surface roughness on porous step-slider bearing lubricated with couple-stress fluid. *International Journal of Heat and Technology*, 41(1), 135–142. doi: 10.18280/ijht.410114
- [22] Ali, N., Khan, S.U., Sajid, M., & Abbas, Z. (2016). MHD flow and heat transfer of couple stress fluid over an oscillatory stretching sheet with heat source/sink in porous medium. *Alexandria Engineering Journal*, 55(2), 915–924. doi: 10.1016/j.aej.2016.02.018
- [23] Naduvanamani, N.B., & Siddangouda, A. (2007). Combined effects of surface roughness and couple stresses on squeeze film lubrication between porous circular stepped plates. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, 221(4), 525–534. doi: 10.1243/13506501JET204
- [24] Vashi, Y., Patel, R.M., & Deheri, G.M. (2018). Ferrofluid Based Squeeze Film Lubrication between Rough Stepped Plates with Couple Stress Effect. *Journal of Applied Fluid Mechanics*, 11(3), 597–612. doi: 10.29252/jafm.11.03.27854
- [25] Prakash, J., & Gururajan, K. (1999). Effect of velocity slip in an infinitely long rough porous journal bearing. *Tribology Transactions*, 42(3), 661–667. doi: 10.1080/1040200 9908982267
- [26] Rahul, A.K., & Rao, P.S. (2021). Rabinowitsch fluid flow with viscosity variation: Application of porous rough circular stepped plates. *Tribology International*, 154, 106635. doi: 10.1016/j.triboint.2020.106635
- [27] Sangeetha, S., & Kesavan, S. (2018). Pressure distribution with surface roughness for effect between porous infinitely long rectangular plates with MHD couple stress squeeze film lubrication. *IOP Conference Series: Journal of Physics: Conference Series*, 1000, 012006. doi: 10.1088/1742-6596/1000/1/012006
- [28] Masood, F.A., & Elamparithi, S. (2023). Effects of Surface Roughness and MHD on Squeeze Film Characteristics for Various Finite Porous Plate Geometries with Couple-Stress Fluid. *International Journal of Heat and Technology*, 41(3), 730–736. doi: 10.18280/ijht.410328
- [29] Sangeetha, S., Govindarajan, A., & Sujatha, E. (2020). Effect of roughness on porous triangular plates with MHD and viscosity variation. *AIP Publishing AIP Conference Proceedings*, 2277, 030019. doi: 10.1063/5.0025774
- [30] Sekar, S., Nathan, S.L., & Saravanathan, L.P. (2022). Velocity Slip with Viscosity Variation for Rough Porous Circular Stepped Plates Using Couple Stress Fluid. *Mathematical Modelling of Engineering Problems*, 9(6), 1631–1638. doi: 10.18280/mmep.090624
- [31] Lin, J.R., Liao, W.H., & Hung, C.R. (2004). The effects of couple stresses in the squeeze film characteristics between a cylinder and a plane surface. *Journal of Marine Science and Technology*, 12(2), 7. doi: 10.51400/2709-6998.2228
- [32] Kumar, S., & Sachidananda, S. V. (1979). Effects of viscosity variation and surface roughness in short journal bearings. *Wear*, 52(2), 341–346. doi: 10.1016/0043-1648(79)90071-1.
- [33] Rajashekar, M., & Kashinath, B. (2012). Effect of surface roughness on MHD couple stress squeeze-film characteristics between a sphere and a porous plane surface. *Advances in Tribology*, 2012, 935690. doi: 10.1155/2012/935690
- [34] Baksh, S.A., & Naganagowda, H.B. (2022). Study of surface roughness with MHD and couple stress fluid on porous curved annular plates. *Acta Polytechnica*, 62(6), 574–588. doi: 10.14311/AP.2022.62.0574
- [35] Barus, C. (1891). Note on the Dependence of Viscosity on Pressure and Temperature, *Proceedings of the American Academy of Arts and Sciences*, 27, 13–18. doi:10.2307/20020462
- [36] Barus, C. (1893). Isothermals, Isopiestic and Isometrics relative to Viscosity. *American Journal of Science*, S3–45(266), 87–96. doi: 10.2475/ajs.s3-45.266.87



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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Optimisation of heat pipe heat exchangers with deep cooling of boiler flue gas based on entropy generation minimisation method

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Received: 14.07.2024; revised: 09.10.2025; accepted: 21.10.2025

Abstract

The paper presents the results of numerical simulations and optimisation of boiler heat exchanger parameters under conditions of deep cooling of combustion products. The specific features of heat and mass transfer calculations, particularly when combustion products are cooled below the dew point with water vapour condensation, are discussed. Experimental results are provided for a large-scale (1.5 m) heat pipe filled with a water/ammonia mixture. These results are utilised in mathematical modelling and optimisation of heat exchanger performance involving heat pipes. The heat transfer process is simulated in a two-stage heat exchanger, where different sections of heat pipes are filled with different working fluids depending on the temperature zones. The optimisation is performed using the minimum entropy production method. The optimal distribution of heat flux density and temperature is determined, considering both design and operational parameters. Numerical examples are provided to demonstrate the performance of the heat recovery system. The results of numerical modelling and optimisation of the design and operating parameters of a two-stage heat exchanger of heat pipes filled with various liquids are presented. The optimal parameters of the heat exchanger are determined using the criterion of minimum entropy production.

Keywords: Heat exchanger; Heat pipes; Deep cooling; Thermodynamic efficiency; Entropy generation; Boiler combustion product

Vol. 46(2025), No. 4, 157–164; doi: 10.24425/ather.2025.156846

Cite this manuscript as: Redko, A., Ujma, A., Redko, O., Redko, I., Zadiranov, V., & Zaika, V. (2025). Optimisation of heat pipe heat exchangers with deep cooling of boiler flue gas based on entropy generation minimisation method. *Archives of Thermodynamics*, 46(4), 157–164.

1. Introduction

Heat exchangers are critical components of boiler systems, contributing significantly to their energy efficiency, environmental performance, and economic viability. The scientific literature extensively discusses the theoretical foundations and criteria for selecting working fluids and components [1–5]. Applications of

heat exchangers based on heat pipes are well documented in various industries, including steel [6] and ceramics [7]. Thermosiphon heat pipes, owing to their high thermal capacity, have also been explored for use in city gate station heaters [8,9]. Combustion product heat recovery can increase the thermal power output of boilers by 10–12%. Combustion gases typically contain 10–20% water vapour. When cooled below 50–55°C, more than

Nomenclature

$A_{0,ev}$ – internal ribbed surface area of the heat pipe in the evaporation zone, kW/K
 $A_{r,ev}$ – external ribbed surface area of the heat pipe in the evaporation zone, kW/K
 $A_{0,c}$ – internal/external surface area of the heat pipe in the condensation zone, kW/K
 c_{pg} – specific heat of combustion products, J/(kg K)
 D – outer diameter of the pipe ribbing, m
 E – rib efficiency factor
 h_p – rib height, m
 K_{ir} – irrigation density, kg/(m² h)
 l – length of thermosiphon, m
 m – weight of the device, kg
 $\dot{m}_g$ – exhaust gas consumption, m³/h
 N – number of rows of the HP
 Pr – Prandtl number
 P_w/P_g – relative volume concentration of water vapour at the interface
 Δp – pressure losses in the gas duct, Pa
 ΔP – aerodynamic resistance of the heat exchanger for gas, Pa
 Q – the amount of heat transferred, kW
 Q_g^* – amount of heat transferred by convection, kW
 Re – Reynolds number
 r_v – relative volume concentration of water vapour in the combustion product flow, %
 S_p – rib pitch on the heat pipes, m
 ΔS – change in the entropy of heat carriers, kW/K
 ΔS_{heat} – entropy change due to heat transfer, kW/K
 $\Delta S_{\Delta p}$ – entropy change due to viscous friction, kW/K
 ΔS_c – change in entropy due to condensation of water vapour on the surface of heat pipes in the evaporation zone, kW/K
 t_{c1c} – temperature of the inner surface of the thermosiphon in the condensation zone, °C
 t_{c2c} – temperature of the surface of the thermosiphon in the condensation zone, °C
 t_g – temperature of gas, °C
 t_s – temperature of the working fluid in the heat pipe, °C

t_w – temperature of water, °C
 T_1, T_2 – heat carrier temperatures, K
 W_1, W_2 – heat capacity rate, kW/K
 $X_1, \dots, X_5$ – coded dimensionless variables that take values of ± 1
 Z – distance between the edges of the adjacent heat pipes, m

Greek symbols

α_{1ev} – heat transfer coefficient from the inner surface of the heat pipe wall to the working heat carrier, W/(m² K)
 α_{1c} – heat transfer coefficient from the working fluid to the heat pipe inner wall in the condensation zone, W/(m² K)
 α_{2av} – average heat transfer coefficient of the ribbed pipe bundle when washing effluent gases by the cross-flow, W/(m² K)
 α_{2c} – heat transfer coefficient from the heat pipe wall to the heated water in the condensation zone, W/(m² K)
 α_{2ev} – heat transfer coefficient from gas to the ribbed ‘dry’ evaporation zone surface, W/(m² K)
 α_r – heat transfer coefficient on the rib surface, W/(m² K)
 δ – fin thickness, m
 η_{opt} – efficiency of the optimum heat exchanger, %
 λ – thermal conductivity, W/(m K)
 ρ – density, kg/m³
 σ – entropy generation, kW/K
 φ – ribbing surface ratio
 ψ – additional geometric coefficient

Subscripts and Superscripts

c – condensation
 ev – evaporation
 g – gas
 in – inlet
 out – outlet
 r – rib
 w – water

Abbreviations and Acronyms

HP – heat pipe

70–80% of this moisture condenses. The resulting dehumidified combustion gases exit the chimney with increased operational reliability, as there is minimal risk of condensation forming within the flue.

The temperature at the outlet of a heat exchanger is limited by economic considerations. For optimal operation, the efficiency of the heat exchanger should range between 0.55 and 0.65, depending on the gas flow rate and pressure drop. The complexity of heat and mass transfer processes in condensate-based heat exchangers necessitates more advanced modelling and optimization techniques. While literature offers a variety of methods for designing condensation surface heat exchangers, practical applications often rely on empirical relationships [10]. However, these relationships require adjustments when applied to deep flue gas cooling in vertical heat pipes filled with low-temperature working fluids. Under specific conditions, the influence of non-analogous heat and mass transfer mechanisms can be neglected, enabling simplified engineering models.

Studies on condensation of water vapour in combustion gases containing non-condensable components (e.g. CO₂, NO_x)

assume that cross vapour flow effects are minimal. Furthermore, the thermal resistance of the vapour film is significantly greater than that of the gas mixture, and can thus be neglected. This allows for using the film condensation mechanism in modelling and optimization. Consequently, the external heat transfer coefficients in both the dry and condensation zones of ribbed heat pipes can be calculated using different correlations. A comparison of various empirical relationships is presented in [11–14].

In the mathematical model developed for this study, water flows longitudinally in a multi-pass configuration, while combustion gases flow counter-currently in a single pass. The heat pipes have vertically oriented evaporation and condensation zones with different lengths ($l_{ev} > l_c$), and are equipped with spiral cross ribs. The heat recovery system is of the ‘liquid-gas’ type, heating water from 5–10°C to 55–65°C.

2. Materials and methods

Figures 1 and 2 show a diagram of the heat exchanger. The heat pipes are fixed into the exchanger’s tube sheet and arranged in

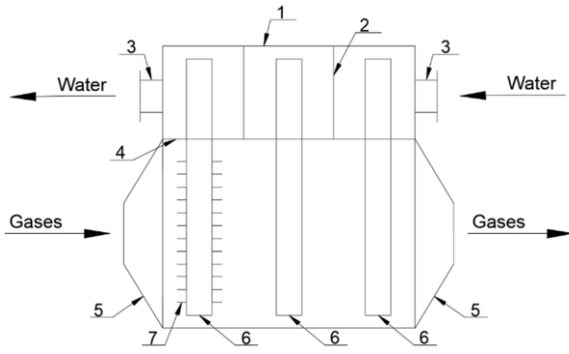


Fig. 1. Longitudinal section of the heat-pipe heat exchanger: 1 – water-side baffle, 2 – subsequent water-side baffle (forming alternating cross-counterflow passes), 3 – water nozzles (inlet/outlet), 4 – tube sheet, 5 – shell (flue-gas duct), 6 – heat pipes (thermosiphons), 7 – fins on the evaporator section of the heat pipe.

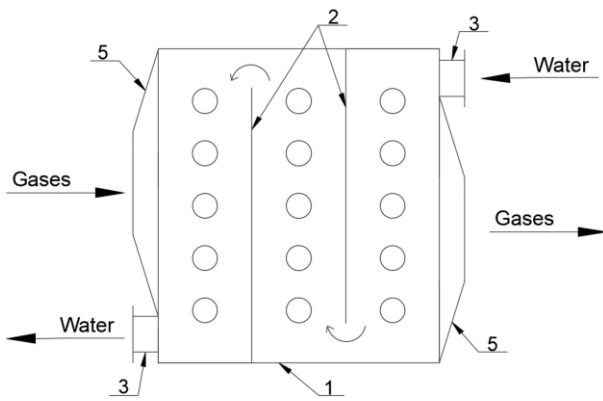


Fig. 2. Longitudinal section of the heat-pipe heat exchanger (tube-sheet/front view): 1 – water-side baffle, 2 – subsequent water-side baffle (forming alternating cross-counterflow passes), 3 – water nozzles (inlet/outlet), 5 – shell (flue-gas duct).

rows along the flue gas flow. The number of tubes was varied both across the width (6–8) and along the length (10–15), depending on the required thermal duty. The condenser sections of the heat pipes are washed by water in a cross-counterflow, multi-pass arrangement formed by baffles. The condenser section of each heat pipe is shorter than its evaporator section. The evaporator sections are finned with annular (or helical-ribbon) fins.

The mathematical model of a heat exchanger employing heat pipes includes the overall energy balance equation, as well as individual heat transfer equations corresponding to each elementary stage of the heat exchange process from gas to liquid. These stages include:

- the heat transfer from gas to liquid – the working heat carrier of the heat pipe in the evaporation zone:

$$Q = \frac{(t_g - t_s)}{R_{ev}(t_s, Q)}, \quad (1)$$

$$R_{ev}(t_s, Q) = \frac{1}{\alpha_{2ev} A_{r, ev}} + \frac{\delta_w}{\lambda_w A_{0, ev}} + \frac{1}{\alpha_{1ev} A_{0, ev}}, \quad (2)$$

- the heat transfer from the heat-carrier fluid of the heat pipe to the inner wall of the pipe in the condensation zone:

$$Q = \alpha_{1c} A_{0, c} (t_s - t_{c1c}), \quad (3)$$

- the heat conduction through the wall of the heat pipe in the condensation zone:

$$Q = \frac{\lambda_w}{\delta_w} A_{0, c} (t_{c1c} - t_{c2c}), \quad (4)$$

- the heat transfer from the outer wall of the heat pipe to the heated water:

$$Q = \alpha_{2c} A_{0, c} (t_{c2c} - t_w), \quad (5)$$

where t_{c1c} and t_{c2c} represent the temperatures of the inner and outer surfaces of the thermosiphon in the condensation zone, respectively, Q denotes the amount of heat transferred, t_s is the temperature of the working fluid inside the heat pipe, t_g and t_w are the temperatures of gas and water, respectively, $A_{0, ev}$ and $A_{r, ev}$ are the internal and external ribbed surface area of the heat pipe in the evaporation zone, $A_{0, c}$ is the surface area of the heat pipe (both internal and external) in the condensation zone, α_{2c} and α_{2ev} are the heat transfer coefficients from the heat pipe wall to the heated water in the condensation zone and from the combustion gases to the ribbed surface in the evaporation zone, respectively, α_{1c} stands for the heat transfer coefficient from the working fluid to the heat pipe inner wall in the condensation zone, α_{1ev} represents the heat transfer coefficient from the inner surface of the heat pipe wall to its working fluid.

The system of Eqs. (1)–(5) includes four unknown variables (t_{c1c} , t_{c2c} , t_s , Q). The system is nonlinear due to the dependence of the heat transfer coefficient on the heat flow rate.

The system of Eqs. (3)–(5) can be transformed to the form

$$t_{c1c} - \frac{Q \delta_w}{\lambda_w A_{0, c}} = t_w + \frac{Q}{\alpha_{2c} A_{r, c}}, \quad (6)$$

$$t_{c1c} = t_s - \frac{Q}{\alpha_{1c} A_{0, c}}, \quad (7)$$

where Q is determined iteratively using Eq. (1), while temperatures t_s and t_{c1c} are obtained from Eqs. (6) and (7), respectively.

The total heat flux transferred to the heat pipes and directed into the thermosiphon is given by

$$Q = Q_g + Q_c. \quad (8)$$

The heat balance equation for the evaporation zone of the heat pipe (convective heat transfer component) is given as follows:

$$Q_g = \alpha_{2ev} A_{2ev} (t_g - t_{av}). \quad (9)$$

To determine the heat transfer coefficient α_{2ev} , the following empirical correlations are used:

$$\alpha_{2ev} = \alpha_{2av} \frac{A_r E \psi + A_0}{A_r}, \quad (10)$$

where E is the rib efficiency factor, A_0 is the area of a smooth surface, α_{2av} is the average heat transfer coefficient for a ribbed pipe bundle subjected to cross-flow of exhaust gases [15]:

$$\alpha_{2av} = 0.33 c_z c_s \left(\frac{\lambda_g}{l} \right) \varphi^{-0.5} \text{Re}^{0.6} \text{Pr}^{0.07} \text{Pr}_c^{0.33} \left(\frac{\text{Pr}_g}{\text{Pr}_{2c}} \right)^{0.25}, \quad (11)$$

where c_z and c_s are correction factors depending on the number of transverse rows of pipes in the bundle and on the pipe pitch in the bundle, l the length of thermosiphon, Re represents the Reynolds number, Pr is the Prandtl number, Pr_g and Pr_{2c} stand for Prandtl number for gas and condensate, respectively, and ϕ is the ribbing surface ratio. The latent heat released during water vapour condensation is calculated using the following formula:

$$Q_c = \alpha_{2av} A_{2ev} r_v \frac{R_v}{R_g} \left(r_v - \frac{P_v}{P_g} \right) \frac{1}{c_{pg}}, \quad (12)$$

where P_v/P_g is the relative volumetric concentration of water vapour at the gas-liquid interface, r_v is the relative volumetric concentration of water vapour in the combustion product flow (with $r_v < 20\%$), R_v and R_g denote the gas constants for vapour and gas, respectively, and c_{pg} denotes the specific heat capacity of the combustion products. A filmwise condensation mechanism for water vapour – representing an approximate analogy between heat and mass transfer – is assumed in the computations. This regime is ensured by the geometric parameters of the ribbed pipe bundles. Specifically, the rib height, thickness and material are selected to satisfy the following conditions:

$$D < 57 \times 10^{-3} \text{ m}, S_p > (3.5-4.0) \times 10^{-3} \text{ m}, \phi < 14,$$

$$\left(\frac{2\alpha_r}{\delta_r \lambda_r} \right)^{\frac{1}{2}} D \leq 3.2, \quad (13)$$

where α_r is the heat transfer coefficient on the rib surface, S_p is the rib pitch and D is the outer diameter of the pipe. The design ensures a rib efficiency factor $E > 0.9$. The critical heat fluxes across the pipe's cross-sectional area were determined according to the method described in [16]. The optimisation technique is described in detail in [17]. The calculation procedure, based on a sequential arrangement of pipe rows, allows for a zone-by-zone evaluation of parameters along the heat exchanger. These parameters include temperature, pressure, the amount of heat transferred, heat pipe power, the quantity of condensate formed, and others. The following parameters were treated as variables: the length of the heat pipe and its evaporation zone, the outer and inner diameters of the heat pipe, the number of pipes per row, the spacing between pipes, the height, thickness and pitch of the ribs, as well as the flow rate and temperature of the combustion products at the inlet, and the flow rate and temperature of the inlet water. The total number of heat pipes (i.e. the overall heat transfer surface area), the temperatures of the heat carriers, the wall temperature and the temperature of the intermediate working fluid inside the heat pipes, as well as the aerodynamic resistance of the combustion gases, the hydraulic resistance on the water side and the total mass of the heat recovery unit, were determined as outcomes of the computational experiment. Irreversible energy losses in the heat recovery unit arise from heat transfer across finite temperature differences, mass transfer during the condensation of water vapour and viscous friction in the flow of the heat carrier.

An experimental setup was developed to validate the analytical relationships used in the mathematical model (Fig. 3). The experimental rig, as shown in the figure, comprises heat pipes

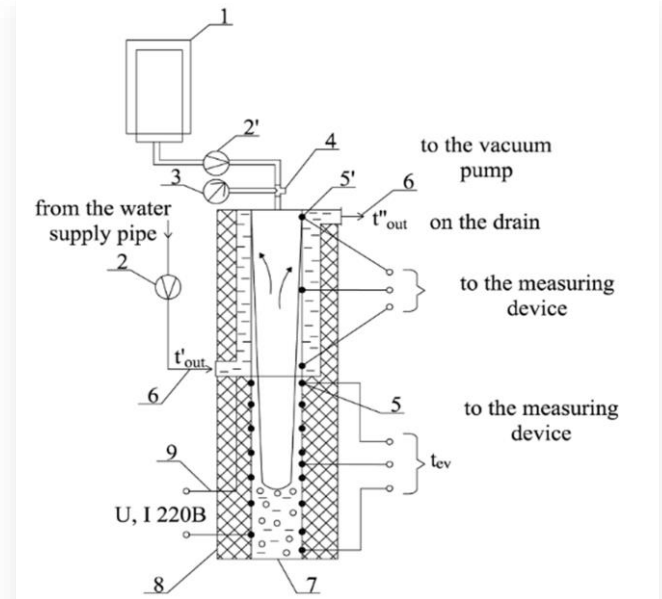


Fig. 3. Scheme of the experimental setup to determine thermal heat pipe capacity: 1 – calibrated measuring tank (graduated vessel) for water flow-rate determination, 2 (2') – flow meters, 3 – manometer, 4 – shut-off valves, 5 (5') – copper-constantan thermocouples, 6 – thermometer; 7 – heat pipe, 8 – insulation, 9 – heating element.

with an evaporator section length of 1000 mm, a condenser section length of 500 mm and an outer diameter of 50 mm. Thermocouples were affixed to the external surface of each heat pipe. An ammonia-water working fluid was employed at concentrations of 10%, 25%, and 40%. The objective of the experiments was to determine the thermal power (heat transfer rate) of the heat pipe of this design. Heat was supplied by an electric heater. The useful thermal power was determined by measuring the water mass flow rate and the temperature difference between the shell outlet and inlet in the water-circulation loop. It was found that the thermal power of the heat pipe of the studied design is 5–6 kW. The experimental results were used as estimation inputs for modelling the heat exchanger parameters.

Experimental studies were conducted to evaluate the thermal capacity of a heat pipe (HP) filled with ammonia-water solution (25% NH_3) at surface temperatures ranging from 40°C to 120°C. The heat is supplied by an electric heater (9) mounted on the surface of the HP in the evaporation zone of the working fluid. In the condensation zone, heat is removed by water flowing through a casing mounted around the HP. The water temperature at the inlet and outlet of the casing is measured using a thermometer (6), while the water flow rate is monitored with a flow meter (2). The surface temperature of the thermosiphon in both the evaporation and condensation zones is recorded using copper-constantan thermocouples fixed on the respective surfaces (positions 5 and 5'). The pressure of the working fluid is measured using a manometer (3), and its temperature is determined based on the saturated vapour pressure.

Figure 4 presents a comparison between the experimental results and the calculated thermal power output of the HP. The thermal power of the heat pipe (HP) was measured as a function

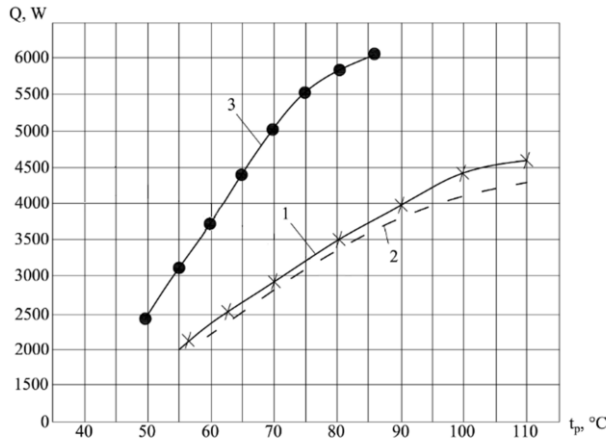


Fig. 4. Heat pipe power as a function of the working fluid temperature: 1 – ammonia-water with a 25% NH_3 concentration (experimental data), 2 – ammonia-water with a 25% NH_3 concentration (calculated data), 3 – ammonia-water with a 40% NH_3 concentration (calculated data).

of the vapour temperature of the working fluid (t_p). A comparison between the experimental and calculated values confirmed the qualitative accuracy of the modelled HP power functions at different ammonia concentrations. In particular, the HP filled with an ammonia-water solution containing 40% NH_3 demonstrated higher thermal efficiency within the temperature range of 40–120°C compared to heat pipes with other concentrations.

A parametric study was carried out to investigate the influence of input variables on the number of rows in the apparatus (assuming 6–8 thermosiphons per row), aerodynamic resistance and the total mass of the heat recovery unit. Pareto-optimal solutions were obtained using a multi-criteria optimisation algorithm. The optimisation was performed with a program based on the Sobol sequences method (also called the LP τ -sequence method or (t,s) sequences in base 2), which systematically probes the factor space through uniformly distributed experimental design points. The optimisation methodology, described in detail in [17–19], employs LP τ -search [20] to generate structured multifactor simulation plans. This technique, belonging to the class of Monte Carlo methods, enables the construction of stable regression models and helps reduce estimate variance and standard error. The computational experiments were designed using the Multicrit software package. The system as a whole is intended to identify efficient configurations of the heat exchanger by regenerating sets of input parameters within specified bounds. The variation ranges for the key factors are as follows: thermosiphon length $l = 1.0\text{--}1.1$ m; rib pitch on the HP $S_p = 0.004\text{--}0.008$ m, rib height $h_p = 0.025\text{--}0.035$ m, exhaust gas flow rate $\dot{m}_g = 8200\text{--}9200$ m³/h, distance between adjacent HPs $Z = 0.006\text{--}0.012$ m.

The solution to the multi-criteria optimisation problem yields a set of effective Pareto-optimal points, which cannot be directly compared with one another across all criteria. Therefore, the final decision must be made subjectively, based on engineering judgment or project-specific priorities. In the optimisation of the heat recovery device, the goal is to minimise the

following criteria: N is the number heat pipe rows; ΔP is pressure difference of the heat exchanger to gas flow, m is the weight of the device, ΔS is the entropy change of the heat carriers.

The energy balance equation is extended by incorporating an entropy balance equation, in accordance with the methodology outlined in [21]:

$$\sigma = \Delta S_{\text{heat}} + \Delta S_c + \Delta S_{\Delta p}, \quad (14)$$

where ΔS_{heat} is the entropy change associated with heat transfer, $\Delta S_{\Delta p}$ is the entropy change resulting from viscous friction in the flow of the heat carrier and ΔS_c is the change in entropy due to vapour condensation on the surface of the heat pipes in the evaporation zone.

Equation (14) can be written as

$$\sigma = Q_g \left(\frac{1}{T_1} - \frac{1}{T_2} \right) + Q_c \left(\frac{1}{T_1} - \frac{1}{T_2} \right) + \frac{\dot{m}_1 \Delta p}{\rho_0 T_1}, \quad (15)$$

where T_1 and T_2 are the heat carrier temperatures, Δp represents pressure losses in the gas duct, $\dot{m}_1$ is the mass flow rate and ρ_0 is the density of combustion products.

3. Results and discussion

The regression equations are derived from the results of a computational experiment based on a three-level Hartley design, involving five factors and a total of 27 experimental runs.

The experimental plan includes a core structure in the form of a fractional factorial design of type ‘25-1’ (16 basic experiments, 10 ‘star’ points and 1 ‘zero’ point).

As a result of the computational experiment, the following regression models were developed:

$$N = 9.3333 + 0.222222X_1 + 1.83333X_2 - 0.333333X_3 + 0.222222X_4, \quad (16)$$

$$\Delta P = 94.22 - 6.61111X_1 - 8.32222X_2 - 34.0444X_3 - 12.25X_4 + 7.61111X_5 + 6.96X_2^2 + 10.36X_3^2 + 2.825X_1X_3 + 6.1625X_3X_4, \quad (17)$$

where X_1, X_2, X_3, X_4, X_5 are coded dimensionless variables that take values of ± 1 :

$$X_1 = \frac{l_{ev} + 1.05}{0.05}, X_2 = \frac{S_p - 0.006}{0.002}, X_3 = \frac{h_p - 0.03}{0.005},$$

$$X_4 = \frac{Z - 0.009}{0.003}, X_5 = \frac{\dot{m}_g - 8700}{500}.$$

The influence of various input parameters on the thermal power output, aerodynamic resistance and total mass of the device has been evaluated.

Designing the heat exchanger requires establishing quantitative relationships between the selected heat and mass transfer indicators and the initial parameters, as well as optimising the thermal characteristics of the system. The objective of the computational experiment was to evaluate the influence of input parameter variations on the required heat transfer surface area and the aerodynamic resistance of the system, under the constraint of a fixed thermal power and the condition of minimum entropy production. A multi-criteria optimisation program based on the

probing of the factor space using uniformly distributed sequences was applied [20].

The results show that the thermal performance of the heat exchanger is strongly influenced by the choice of working fluid in each stage of the device. The selection of the working fluid determines the temperature distribution of the heat carrier along the apparatus, the local heat transfer coefficient and the resulting heat flux density. At combustion gas temperatures of 150–130°C, optimal working fluids were found to be: water in the first stage and a water-ammonia mixture in the second stage.

Simulation results indicate that the condensation of water vapour significantly increases the heat flux density. In particular, the share of condensation heat in the total heat transfer increases to 64%, while the water vapour content in the combustion gases decreases from 18% to 10–12%.

Figure 5 shows the variation of entropy production as a function of the heat capacity rate of the combustion products. As it can be seen, at a constant power of a heat exchanger the decrease in the temperature of combustion products and the increase in their flow rate result in reducing the entropy production.

Numerical simulations were carried out for a two-stage heat

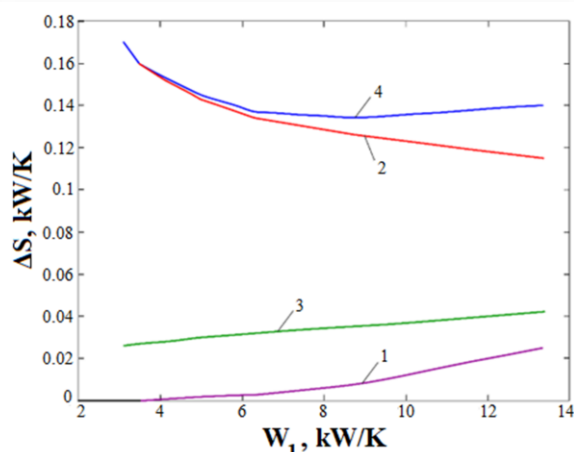


Fig. 5. Entropy production vs. gas heat capacity rate: 1 – mechanical, 2 – thermal, 3 – at the condensation of water vapour, 4 – total.

recovery unit utilising heat pipes (thermosiphons) filled with two different working fluids in the first and second stages. Water, ammonia-water solution, methanol and isobutane (C_4H_{10}) were considered as potential working fluids for the heat pipes. The results show that the heat exchange surface area for a heat recovery unit with heat pipes filled with water is significantly smaller (by approximately 39–40%) compared to heat pipes filled with isobutane. However, the thermal power increases in a heat recovery unit designed according to a two-stage thermal scheme: the first stage consists of 3 rows of heat pipes (thermosiphons) filled with methanol and the second stage consists of 18 rows of thermosiphons filled with isobutane, or, the first stage consists of 3 rows of heat pipes filled with methanol and the second stage consists of 10 rows of thermosiphons filled with ammonia water. The distribution of heat pipes thermal power along the length of the heat recovery unit is shown in Fig. 6.

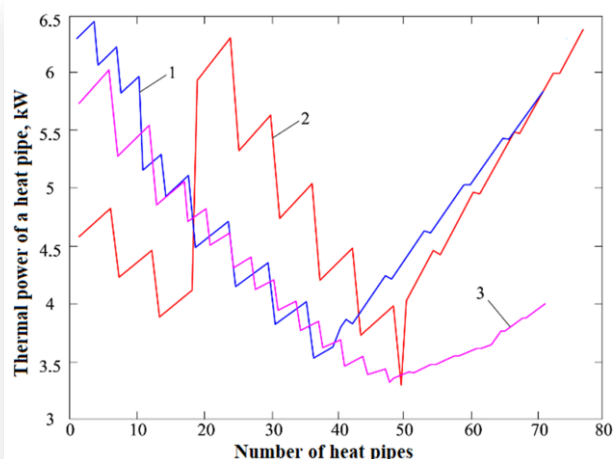


Fig. 6. Distribution of thermal power of heat pipes along the length of the heat recovery unit: 1 – up to the 19th thermosiphon – methanol and in others – isobutane, 2 – up to the 19th thermosiphon – methanol and in others – ammonia-water, 3 – ammonia-water in all thermosiphons.

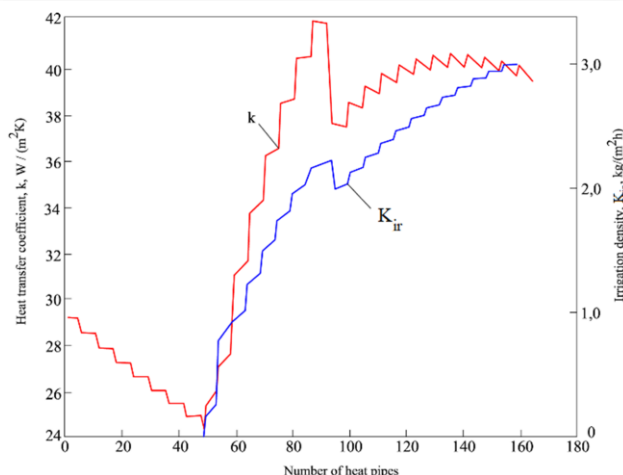


Fig. 7. Distribution of the heat transfer coefficient and irrigation density along the length of the heat recovery unit.

Figure 7 presents the variation of the heat transfer coefficient and water vapour concentration along the length of the heat recovery unit. The observed increase in thermal power output of the heat pipes in the second stage is attributed to the condensation of water vapour.

The accuracy of the computational model was verified using experimental data for the irrigation density (K_{ir}) defined as the amount of water vapour condensed per square meter of heat exchange surface per hour. Figure 7 also shows the distribution of the irrigation density K_{ir} along the length of the heat exchanger with a total heat exchange surface area of 443.52 m². At a combustion gas flow rate of 10×10^3 m³/h, the irrigation coefficient varies from 0 to 2.8 kg/(m² h).

According to experimental data [22], typical values of K_{ir} range from 2 to 4 kg/(m² h). For a steam DE-type boiler, the measured mass of condensate collected in the apparatus is 260–

Table 1. Results of heat exchanger parameter optimisation considering water vapour condensation.

Gas temperature at the inlet, °C	Gas flow rate, m ³ /h	Heat capacity rate, kW/K	Thermal power/ Q_g^* , kW/kW	Water temperature at the inlet, °C	Gas temperature at the outlet, °C	Heat transfer capacity ($k \times A$), kW/K	Pressure losses, Pa	Entropy production, kW/K (therm./mech./tot.)
130	8000	3.10	301/244	41.0	50.6	4.64	59	0.170(0.026)/0.00/0.170
120	9000	3.48	298/237	40.6	51.2	4.86	72	0.160(0.027)/0.00/0.160
100	13000	4.99	299/230	40.7	53.6	5.57	133	0.143(0.030)/0.002/0.145
90	16500	6.32	299/223	40.6	54.6	6.04	200	0.134(0.032)/0.003/0.137
80	23000	8.79	299/214	40.7	55.6	6.70	352	0.126(0.035)/0.008/0.134
70	35000	13.34	298/192	40.5	55.6	7.61	721	0.115(0.042)/0.025/0.140

Q_g^* is the amount of heat being transferred by convection.

Table 2. Results of heat exchanger parameter optimisation without water vapour condensation.

Gas temperature at the inlet, °C	Gas flow rate, m ³ /h	Heat capacity rate, kW/K	Thermal power, kW	Water temperature at the inlet, °C	Gas temperature at the outlet, °C	Heat transfer capacity ($k \times A$), kW/K	Pressure losses, Pa	Entropy production, kW/K (therm./mech./tot.)	Entropy production, kW/K*
140	8200	3.18	302	41.0	44.2	4.68	124	0.189/0.001/0.190	0.181
130	9400	3.64	300	40.8	47.0	4.78	157	0.181/0.002/0.183	0.174
120	11300	4.36	300	40.8	50.7	4.93	215	0.176/0.003/0.178	0.169
110	14200	5.47	300	40.9	54.7	5.10	318	0.170/0.005/0.175	0.164
100	19000	7.30	299	40.7	58.8	5.29	525	0.163/0.011/0.174	0.159
90	30000	11.50	300	40.8	63.9	5.56	1156	0.158/0.036/0.194	0.147
98	20700	7.95	300	40.8	60.1	5.35	609	0.163/0.013/0.176	0.152
104	16650	6.00	299	40.7	57.0	5.2	418	0.166/0.007/0.173	0.161

*Calculated according to the formula: $\Delta S = W_1 \ln(T_1^{out}/T_1^{in}) + W_2 \ln(T_2^{out}/T_2^{in})$

340 kg/h. The proposed method yields a calculated value of 240 kg/h, confirming high accuracy of the developed model. In this simulation, the working fluid is water up to the 97th thermosiphon and ammonia-water mixture beyond that point (60/40% split). The increase in entropy production due to viscous friction corresponds to a minimum total entropy generation $dS_{total} = 0.134$ kW/K (as shown in Table 1) at a combustion gas inlet temperature of $T_1 = 80^\circ\text{C}$ and a heat capacity rate $W_1 = 8.79$ kW/K. As W_1 increases, the required heat exchange surface area multiplied by the overall heat transfer coefficient (i.e. the $k \times A$ value) increases from 4.68 to 5.56 kW/K. As can be seen, in the absence of water vapour condensation, entropy production becomes more intensive – by up to 31% – due to increased gas flow rate required to maintain the same thermal power output (Table 2).

The calculated results for the heat exchanger parameters without water vapour condensation are presented in Table 2.

When modelling and optimising thermal parameters of the heat exchanger, the entropy production criterion σ allows for an increase in the outlet temperature T_2 of the heated medium by selecting appropriate values of heat capacity rates W_1 and W_2 , as well as by choosing an optimal heat exchange configuration for a given heat transfer surface area. Numerical investigations were conducted to analyse the effect of varying the heat consumption of the heated medium (W_2) on the efficiency of the optimised heat exchanger (η_{opt}).

4. Summary and conclusions

The results of numerical simulation and optimisation of two-stage heat exchanger parameters using heat pipes, with consid-

eration of entropy change, demonstrate the following: operating modes that minimise entropy production lead to reduced irreversible losses in the heat transfer process and enhance the overall energy efficiency of both the heat exchanger and the heat-generating unit. However, the implementation of such modes requires a thorough technical and economic justification – particularly when selecting the heat capacity rate of the combustion products (W_1) – with consideration of relevant economic criteria.

References

- [1] Baskakov, A.P., & Il'ina, E.V. (2003). Heat and mass exchange in the deep cooling of the combustion products of natural gas. *Journal of Engineering Physics*, 16(2), 335–342.
- [2] Verinchuk, E. (2004). *Modelling of heat-and-mass transfer processes in recuperative condensing heat-utilizers*. PhD thesis, Moscow Energy Institution, Moscow (in Russian).
- [3] Redko, I., Redko, A., Pavlovskiy, S., Redko, O., Burda, Y., & Ujma, A. (2021). Energy efficiency of buildings in the cities of Ukraine under the conditions of sustainable development of centralized heat supply system. *Energy and Buildings*, 247, 110947. doi: 10.1016/j.enbuild.2021.110947
- [4] Zhang, Y. (Ed.) (2018). *Heat Pipes: Design, Applications and Technology*. Nova Science Publishers.
- [5] Zohuri, B. (2016). *Heat pipe design and technology: Modern applications for practical thermal management* (2nd ed.). Springer. doi: 10.1007/978-3-319-29841-2
- [6] Jouhara, H., Almahmoud, S., Chauhan, A., Delpech, B., Bianchi, G., Tassou, S.A., Llera, R., Lago, F., & Arribas, J.J. (2017). Experimental and theoretical investigation of a flat heat pipe heat exchanger for waste heat recovery in the steel industry. *Energy*. 141. 1928–1939. doi: 10.1016/j.energy.2017.10.142

- [7] Jouhara, H., Bertrand, D., Axcell, B., Montorsi, L., Venturelli, M., Almahmoud, S., Milani, M., Ahmad, L., & Chauhan, A. (2021). Investigation on a full-scale heat pipe heat exchanger in the ceramics industry for waste heat recovery. *Energy*, 223, 120037. doi: 10.1016/j.energy.2021.120037
- [8] Rastegar, S., Kargarsharifabad, H., Behshad Shafii, M., & Rahbar, N. (2020) Experimental investigation of the increasing thermal efficiency of an indirect water bath heater by use of thermosiphon heat pipe. *Thermal Science*, 24(6B), 4277–4287. doi: 10.2298/TSCI190428054R
- [9] Azizi, S.H., Rashidmardani, A., & Andalibi, M.R. (2014). Study of preheating natural gas in gas pressure reduction station by the flue gas of indirect water bath heater. *International Journal of Science and Engineering Investigations*, 3(27), 17–22.
- [10] Cherepanova, E. (2005). *Cooling combustion products of gaseous fuel in ribbed heat exchangers*. PhD thesis, Ural State Technical University, Ekaterinburg (in Russian).
- [11] Isachenko, V.P., Osipova, V.A., & Sukomel, A.S. (1975). *Heat transfer* (3rd ed.). Energiya, Moscow.
- [12] Zhukauskas, A. (1982) *Convective transfer in heat exchangers*. Nauka, Moscow (in Russian).
- [13] Mikheev, M.A., & Mikheeva, I.M. (1977). *Fundamentals of heat transfer* (2nd ed.). Energiya, Moscow (in Russian).
- [14] Labuntsov, D. (1960). Heat exchange during steam condensation on a vertical surface under conditions of turbulent runoff of a condensate film. *Journal of Engineering Physics and Thermophysics*, 3(8), 3–12.
- [15] Yudin, V.F. (1982). Heat exchange of cross-ribbed pipes (pp. 46–108). Mashinostroyenie, Leningrad (in Russian).
- [16] Bezrodnyi, M.K., Pioro, I.L., & Kostyuk, T.O. (2005). *Transfer processes in two-phase thermosiphon systems. Theory and practice* (2nd ed.). Fact Publishing House, Kiev (in Russian).
- [17] Kulikova, N., & Redko, A. (2014). Simulating of heat-transfer processes in two-phase heat-utilizer at heat pipes. *Structure and Environment*, 6(2), 35–43.
- [18] Nikolaev, G. (2012). Calculation of operating characteristics of loop heat pipes. *Journal of Young Scientist*, 3, 17–25.
- [19] Redko, A., Kulikova, N., Pavlovskiy, S., & Redko, A. (2018) Simulation and optimization of heat-exchanger parameters of heat pipes by changes of entropy. *Heat Transfer Research*, 49(16), 1545–1557. doi: 10.1615/HeatTransRes.2018019336
- [20] Sobol, I.M., & Statnikov, R.B. (1981). *Selecting optimal parameters in multicriteria problems*. Nauka, Moscow (in Russian).
- [21] Bejan, A. (2006). *Advanced engineering thermodynamics* (3 ed.) (pp. 101–144, 574–655), John Wiley & Sons, New York. doi: 10.1002/9781119245964
- [22] Kudinov, A. (2001) *Energy saving in thermal generating plants*. Ulyanovsk State Technical University, Ulyanovsk.

Thermal and entropic behaviour of unsteady nanofluid flow in a permeable-walled Couette system

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Received: 13.03.2025; revised: 25.10.2025; accepted: 29.10.2025

Abstract

This study examines the energy dissipation mechanisms in unsteady nanofluid flow confined between two permeable walls under a generalised Couette flow configuration, with a particular emphasis on entropy generation and convective heat transfer effects. Such an analysis is of growing significance in advanced thermal management applications, including industrial cooling processes, microelectronic systems, renewable energy technologies and biomedical devices, where nanofluids are increasingly employed to enhance thermal performance. The governing partial differential equations, derived from the conservation principles of mass, momentum, and energy, are solved numerically using a semi-discretisation finite-difference approach in conjunction with a Runge-Kutta-Fehlberg integration algorithm. Comprehensive results for velocity distribution, temperature field, skin friction coefficient, Nusselt number, entropy generation rate and Bejan number are presented and discussed. The findings reveal, among other trends, that increasing injection/suction rates and the Biot number reduce the Nusselt number, entropy generation rate and Bejan number at the stationary lower wall, while simultaneously enhancing these quantities at the upper moving wall. These results highlight the dual role of boundary conditions and thermal parameters in governing irreversibility, heat transfer efficiency and overall system performance. By offering insights into the minimisation of entropy-related losses, this work contributes to the design of more energy-efficient and sustainable nanofluid-based thermal systems.

Keywords: Transient Couette flow; Nanofluid; Suction/Injection; Irreversibility analysis; Numerical simulation

Vol. 46(2025), No. 4, 165–176; doi: 10.24425/ather.2025.156847

Cite this manuscript as: Mkwizu, M.H., & Makinde, O.D. (2025). Thermal and entropic behaviour of unsteady nanofluid flow in a permeable-walled Couette system. *Archives of Thermodynamics*, 46(4), 165–176.

1. Introduction

The study of thermodynamic analysis of unsteady Couette flow of nanofluids in a porous wall channel with convective cooling is significant due to its implications in heat transfer enhancement, energy efficiency and fluid dynamics in advanced engineering applications [1]. Nanofluids, which are suspensions of nanoparticles in a base fluid, exhibit superior thermal properties compared to conventional fluids, making them highly valuable in optimising heat dissipation in various systems [2].

Moreover, the properties of nanofluids can also be tailored for specific applications by adjusting the type, size and concentration of nanoparticles in the suspension. This allows for customisation of the thermal properties of nanofluids to meet the requirements of different heat transfer systems. Choi [3] introduced the concept of nanofluids, highlighting their superior thermal conductivities compared to traditional fluids. Analysis of unsteady heat and mass transfer in rotating magnetohydrodynamic (MHD) convection flow over a porous vertical plate has been conducted by Narmathaa and Kavitha. The results show

Nomenclature

a	– channel width, m
Be	– Bejan number
Bi	– Biot number
$(c_p)_f$	– specific heat of the base fluid, J/(kg K)
$(c_p)_s$	– specific heat of nanoparticles, J/(kg K)
C_f	– skin friction coefficient
Ec	– Eckert number
k_f	– thermal conductivity of the base fluid, W/(m K)
k_{nf}	– nanofluid thermal conductivity, W/(m K)
k_s	– thermal conductivity of nanoparticles, W/(m K)
N_{FFI}	– fluid friction irreversibility
N_{HTI}	– heat transfer irreversibility
Ns	– dimensionless entropy generation rate
Nu	– Nusselt number
P	– nanofluid pressure, Pa
Pr	– Prandtl number
Re	– Reynolds number
S_G	– entropy generation rate, W/(m ³ K)
t	– time, s

T	– temperature of the nanofluid, K
T_w	– lower stationary wall temperature, K
u	– nanofluid velocity in the x-direction, m/s

Greek symbols

α_{nf}	– thermal diffusivity of the nanofluid, m ² /s
μ_{nf}	– dynamic viscosity of the nanofluid, kg/(s m)
ρ_{nf}	– density of the nanofluid, kg/m ³
ρ_f	– density of the base fluid, kg/m ³
ρ_s	– density of the nanoparticle, kg/m ³
φ	– nanoparticles volume fraction

Subscripts and Superscripts

f	– base fluid
nf	– nanofluid
s	– nanoparticle

Abbreviations and Acronyms

PDE	– partial differential equation
MOL	– method of lines

that the velocity and temperature decrease as the Prandtl number increases [4]. Unsteady Couette flow, characterised by fluid motion induced by shear between two parallel plates, plays a vital role in industrial processes, such as lubrication, polymer manufacturing and microfluidics, where real-time variations in flow can significantly influence system efficiency [5]. In studies involving nanofluid dynamics within confined geometries like channels, the Couette flow configuration is frequently employed, typically involving one moving surface and one stationary. This model serves as a fundamental framework for analysing shear-driven flows in nanofluid systems, offering valuable insights into how nanoparticle size and concentration affect flow behaviour and heat transfer. Notably, researchers such as Abu-Nada and Chamkha [6] and Ali et al. [7] have demonstrated that incorporating nanoparticles not only modifies flow dynamics but also enhances thermal conductivity and overall flow performance.

The incorporation of porous walls introduces permeability effects that significantly influence thermal regulation, pressure distribution and cooling efficiency, particularly in heat exchangers, biomedical devices and energy systems [8]. The introduction of permeable walls in nanofluid systems adds a layer of complexity to the analysis, as fluid can infiltrate or be sucked through the porous surfaces, which impacts both the flow rate and heat transfer efficiency. The concept of wall permeability has been explored by Kasaeian et al. [9], who investigated nanofluid flow in porous media and found that permeability significantly affects the flow and thermal characteristics. When coupled with convective cooling, the permeable walls can act as heat sinks, enhancing the overall heat transfer by facilitating the continuous replacement of hot fluid with cooler fluid from the surroundings. Convective cooling, as highlighted by Kakac and Pramuanjaroenkij [10], plays a crucial role in such systems by promoting efficient heat dissipation. The cooling effect is further augmented in systems involving nanofluids due to the higher thermal conductivity of these fluids. Studies by Shanthy et al. [11] and Malik et al. [12] have shown that the use of porous

materials for convective cooling significantly boosts the heat dissipation rate, especially when coupled with nanofluids, making these systems highly efficient in industrial applications, such as electronic cooling. Convective cooling mechanisms play a pivotal role in thermal management technologies, such as microprocessors, fuel cells and aerospace heat shielding, ensuring optimal operating temperatures and reduced entropy generation [13].

The thermodynamic analysis of unsteady nanofluid Couette flow, particularly in a permeable wall channel, presents several challenges, especially in terms of understanding the time-dependent effects on heat transfer and energy dissipation. The unsteady flow introduces transient behaviours, which require careful modelling to account for variations in temperature and velocity profiles over time. Ndelwa et al. [14] emphasised the importance of entropy generation analysis in evaluating the thermodynamic efficiency of such systems, as it offers insights into the irreversibilities caused by viscosity, heat transfer and unsteady flow dynamics. The study of entropy generation in unsteady nanofluid systems provides a comprehensive view of the energy losses and potential for optimisation. Oztop and Abu-Nada [15] further explored how the unsteady flow conditions in a permeable channel increase entropy generation due to the time-varying nature of temperature and velocity fields. This, coupled with the effects of convective cooling, which continuously replenishes the hot fluid with cooler nanofluid, helps mitigate some of the thermodynamic inefficiencies, making the system more effective in energy dissipation and heat transfer enhancement [16,17]. Ali et al. [18] conducted a numerical investigation of the transient flow behaviour of silica nanofluids over a stretching cylinder, considering the influence of various nanoparticle shapes and Joule heating. Gorai et al. [20] carried out a theoretical study on how nanofluids affect the thermodynamic performance of flat-plate solar collectors. As the thermodynamic understanding of these systems continues to evolve, future research should focus on improving the models that de-

scribe the transient behaviours and optimising the configurations for maximum thermodynamic efficiency in nanofluid cooling applications.

This study analyses entropy generation in unsteady Couette flow of Cu-water nanofluid with convective cooling through permeable walls, providing a measure of irreversibility and energy loss. These insights support the development of more efficient thermal management and lubrication systems. The research has broad applications, including solar collectors, nuclear reactors, drug delivery systems, artificial organs, turbine cooling, aerodynamic heating control and advanced microfluidic devices. By combining nanofluid thermodynamics, porous media flow and convective cooling principles, the study offers valuable guidance for next-generation cooling technologies, sustainable energy systems and optimised industrial processes. According to the literature, this problem has not been exhaustively investigated; our objective is to fill this gap. The originality and novelty of the present study are outlined as follows:

- It provides a unique investigation of the unsteady thermal and entropic behaviour of nanofluid flow in a Couette system, emphasizing time-dependent effects and the role of nanoparticle-enhanced heat transfer, which are often neglected in existing models;
- It explores the impact of wall permeability, through suction and injection, on the boundary-layer development and entropy generation, offering fresh insights into thermodynamic irreversibility and flow regulation in advanced thermal systems.
- It establishes a comprehensive theoretical and numerical framework by employing the method of lines and adaptive Runge-Kutta-Fehlberg integration to analyse transient velocity, temperature and entropy characteristics in nanofluid-based configurations relevant to microfluidics, energy systems and heat transfer optimisation.

In the following sections, the model is formulated, analysed and numerically tackled. Pertinent results are presented graphically and discussed.

2. Mathematical model

Consider the unsteady laminar Couette flow of a viscous, incompressible water-based nanofluid containing copper nanoparticles through a parallel plate channel with permeable walls. The fluid is uniformly injected into the channel through the lower wall, while a uniform suction occurs at the moving upper plate, as illustrated in Fig. 1.

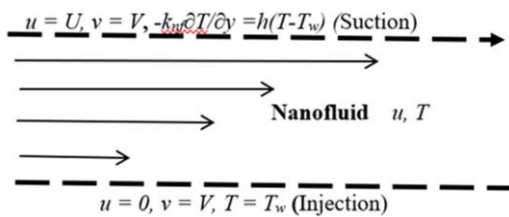


Fig. 1. Schematic diagram of the problem.

Based on the above assumptions, the governing equations for the momentum and energy of the nanofluid in one dimension can be expressed as follows.

$$\frac{\partial u}{\partial \bar{t}} + V \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2}, \quad (1)$$

$$\frac{\partial T}{\partial \bar{t}} + V \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\alpha_{nf} \mu_{nf}}{k_{nf}} \left(\frac{\partial u}{\partial y} \right)^2, \quad (2)$$

$$S_G = \frac{k_{nf}}{T_w^2} \left(\frac{\partial T}{\partial y} \right)^2 + \frac{\mu_{nf}}{T_w} \left(\frac{\partial u}{\partial y} \right)^2, \quad (3)$$

$N_{HTI} \quad N_{FFI}$

where S_G is the entropy generation rate, N_{HTI} is the heat transfer irreversibility and N_{FFI} is the fluid friction irreversibility. The appropriate initial and boundary conditions are:

$$u(y, 0) = 0, \quad T(y, 0) = T_w, \quad (4)$$

$$u(0, \bar{t}) = 0, \quad T(0, \bar{t}) = T_w, \quad (5)$$

$$u(a, \bar{t}) = U, \quad k_{nf} \frac{\partial T}{\partial y}(a, \bar{t}) = h[T(a, \bar{t}) - T_w], \quad (6)$$

where u is the nanofluid velocity in the x -direction, T is the temperature of the nanofluid, P is the nanofluid pressure, $\bar{t}$ is the time, a is the channel width, T_w is the lower stationary wall temperature, μ_{nf} is the dynamic viscosity of the nanofluid, k_{nf} is the nanofluid thermal conductivity, ρ_{nf} is the density of the nanofluid and α_{nf} is the thermal diffusivity of the nanofluid. The properties of the nanofluid are given by the formulas [6,10,13,17,19,20]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \quad (7)$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)},$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad (\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s,$$

where φ represents the nanoparticle volume fraction, ρ_f represents the density of the base fluid and ρ_s is the density of the nanoparticle, k_f is the thermal conductivity of the base fluid, and k_s is the thermal conductivity of the nanoparticles. The heat capacities of the base fluid and nanoparticles are represented by $(\rho c_p)_f$ and $(\rho c_p)_s$, respectively. Table 1 presents the thermophysical properties of water and copper nanoparticles.

Table 1. Thermophysical properties of nanofluid [6,10,13,17,19,20].

Physical properties	Fluid phase (water)	Copper (Cu)
C_p (J/kg K)	4179	385
ρ (kg/m ³)	997.1	8933
k (W/m K)	0.613	401

The following dimensionless variables and parameters are introduced, as follows:

$$\theta = \frac{T-T_w}{T_w}, \quad W = \frac{u}{U}, \quad t = \frac{\bar{t}V}{a}, \quad v_f = \frac{\mu_f}{\rho_f}, \quad \bar{P} = \frac{Pa}{\mu_f U}, \quad A = -\frac{\partial \bar{P}}{\partial x'},$$

$$X = \frac{x}{a}, \quad \eta = \frac{y}{a}, \quad Ns = \frac{S_G a^2}{k_f}, \quad Pr = \frac{\mu_f c_{pf}}{k_f}, \quad Ec = \frac{U^2}{c_{pf} T_w}, \quad (8)$$

$$\tau = \frac{(\rho c_p)_s}{(\rho c_p)_f}, \quad m = \frac{(k_s + 2k_f) + \varphi(k_f - k_s)}{(k_s + 2k_f) - 2\varphi(k_f - k_s)}, \quad Re = \frac{Va}{\nu_f}, \quad Bi = \frac{ha}{k_f}.$$

The governing equations (Eqs. (1)-(3)) in the dimensionless form can be written as:

$$\frac{\partial W}{\partial t} = \frac{A}{Re[(1-\varphi) + \varphi \frac{\rho_s}{\rho_f}]} + \frac{1}{Re[(1-\varphi) + \varphi \frac{\rho_s}{\rho_f}](1-\varphi)^{2.5}} \frac{\partial^2 W}{\partial \eta^2} - \frac{\partial W}{\partial \eta}, \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{RePr m(1-\varphi + \varphi \tau)} \frac{\partial^2 \theta}{\partial \eta^2} + \frac{Ec}{Re(1-\varphi)^{2.5}(1-\varphi + \varphi \tau)} \left(\frac{\partial W}{\partial \eta} \right)^2 - \frac{\partial \theta}{\partial \eta}, \quad (10)$$

$$Ns = \frac{k_{nf}}{k_f} \left(\frac{\partial \theta}{\partial \eta} \right)^2 + EcPr \frac{\mu_{nf}}{\mu_f} \left(\frac{\partial W}{\partial \eta} \right)^2, \quad (11)$$

with the initial and boundary conditions given as

$$\left. \begin{aligned} \theta(0, t) &= 0, & \theta(\eta, 0) &= 0, \\ W(\eta, 0) &= 0, \\ W(0, t) &= 0, \\ W(1, t) &= 1, \\ \frac{\partial \theta}{\partial \eta}(1, t) &= -mBi\theta(1, t). \end{aligned} \right\}. \quad (12)$$

Other quantities of practical interest in this study are the skin friction coefficient (C_f), the local Nusselt number (Nu), and the Bejan number (Be), which are defined as:

$$C_f = \frac{\mu_{nf}}{\mu_f} \frac{\partial W}{\partial \eta} \Big|_{\eta=0.1}, \quad Nu = -\frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial \eta} \Big|_{\eta=0.1}, \quad Be = \frac{1}{1+Q}, \quad (13)$$

where $Q = \frac{N_{FFI}}{N_{HTI}}$ is the irreversibility ratio and

$$N_{HTI} = \frac{k_{nf}}{k_f} \left(\frac{\partial \theta}{\partial \eta} \right)^2, \quad N_{FFI} = EcPr \frac{\mu_{nf}}{\mu_f} \left(\frac{\partial W}{\partial \eta} \right)^2. \quad (14)$$

3. Numerical procedures

Equations (9)–(12) form a system of nonlinear initial boundary value problems (IBVPs) that are solved numerically using a semi-discretisation finite-difference method known as the method of lines (MOL) [19] (see Fig. 2). We divide the spatial interval $0 \leq \eta \leq 1$ into N equal parts, where the grid size $\Delta\eta = 1/N$ and grid points are defined as $\eta_i = (i-1)\Delta\eta$ for $1 \leq i \leq N+1$. This discretisation utilises a linear Cartesian mesh and a uniform grid to calculate finite differences. The first and second spatial derivatives in Eqs. (9)–(10) are approximated using second-order central finite differences.

Let $W_i(t)$ and $\theta_i(t)$ represent approximations of $W(\eta_i, t)$ and $\theta(\eta_i, t)$, respectively. The semi-discrete system for the problem then becomes:

$$\frac{dW}{dt} = \frac{A}{Re[(1-\varphi) + \varphi \frac{\rho_s}{\rho_f}]} + \frac{1}{Re[(1-\varphi) + \varphi \frac{\rho_s}{\rho_f}](1-\varphi)^{2.5}} \left(\frac{W_{i+1} - 2W_i + W_{i-1}}{(\Delta\eta)^2} \right) - \left(\frac{W_{i+1} - W_{i-1}}{2\Delta\eta} \right), \quad (15)$$

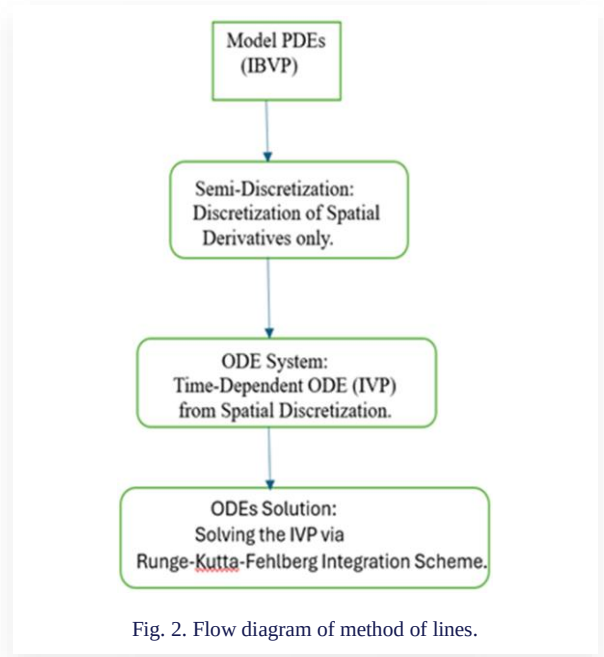


Fig. 2. Flow diagram of method of lines.

$$\frac{d\theta}{dt} = \frac{1}{RePr m(1-\varphi + \varphi \tau)} \left[\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta\eta)^2} \right] + \frac{Ec}{Re(1-\varphi)^{2.5}(1-\varphi + \varphi \tau)} \left(\frac{W_{i+1} - W_{i-1}}{2\Delta\eta} \right)^2 - \left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta\eta} \right). \quad (16)$$

Equations (15) and (16) represent a system of first-order ordinary differential equations with known initial conditions. The convergence of the MOL used to solve this initial boundary value problem with the Runge-Kutta-Fehlberg (RKF) method is assured, as it depends on the combined effects of accurate spatial discretisation and adaptive time-stepping control inherent in the RKF scheme.

In this approach, the spatial derivatives in the governing partial differential equations (PDEs) are discretised using a second-order central difference scheme, which converts the PDEs into a system of ordinary differential equations (ODEs) in time. The RKF method, an explicit Runge-Kutta algorithm of orders four and five, advances the solution in time by automatically adjusting the step size according to the estimated local truncation error between the two embedded approximations. Convergence is achieved when the spatial discretisation reliably captures the behaviour of the spatial derivatives as the mesh is refined and the time integration remains stable under the RKF's adaptive error control mechanism.

Since the method satisfies both consistency and stability conditions, the Lax equivalence theorem guarantees convergence of the overall numerical solution. Moreover, the resulting initial value problem can be solved iteratively using the Runge-Kutta-Fehlberg method, implemented using Maple software. To validate the accuracy of the numerical procedure and establish a benchmark for the present analysis, a special case of the current model is compared with previously published results for the velocity profile reported in [21]. This validation is performed under a steady-state condition for a conventional fluid ($\varphi = 0$).

Table 2. Velocity profile – comparison with the results from [21] for steady state when $\phi = 0$, $A = 1$ and $Re = 1$.

η	Ref. [21] $w(\eta)$	Present $w(\eta)$
0	0	0
0.1	0.038792	0.0387930
0.2	0.071148	0.0711488
0.3	0.096390	0.0963903
0.4	0.113769	0.1137695
0.5	0.122459	0.1224593
0.6	0.121546	0.1215460
0.7	0.110019	0.1100195
0.8	0.086763	0.0867637
0.9	0.050544	0.0505450
1.0	0	0

confined between stationary permeable walls, as presented in Table 2. The comparison demonstrates excellent agreement in both qualitative trends and quantitative values, thereby confirming the reliability of the adopted computational approach.

4. Results and discussion

This section explores the influence of emerging parameters on Cu-water nanofluid flow and its heat transfer characteristics. The results from computational analyses are comprehensively presented in Figs. 3–26. The selected parameter ranges and values are based on established literature.

4.1. Velocity profiles

The influence of key emerging parameters on the nanofluid velocity distribution is systematically illustrated in Figs. 3–7, offering insights into the coupled thermal and entropic behaviour of the unsteady nanofluid flow within a permeable-walled Couette system. Figures 3–5 demonstrate the temporal and spatial evolution of the velocity field, showing that the velocity remains zero at the stationary lower permeable wall due to the no-

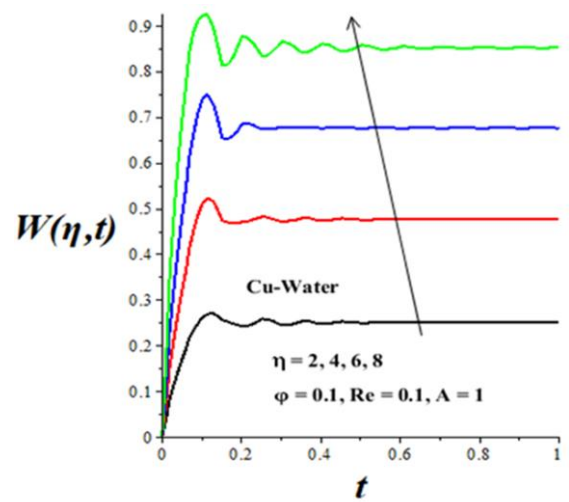


Fig. 4. Impact of η on the velocity profile.

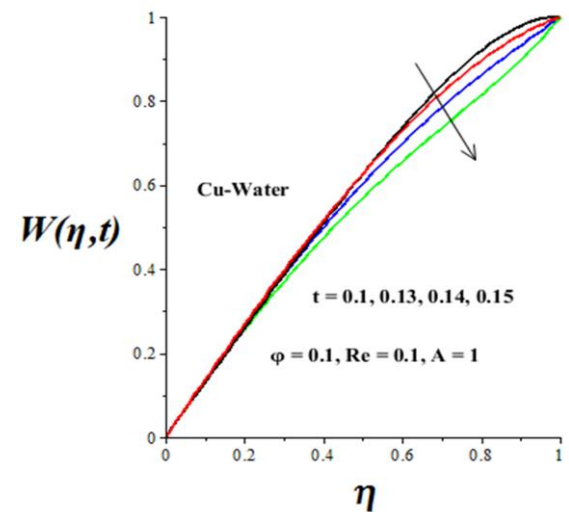


Fig. 5. Impact of t on the velocity profile.

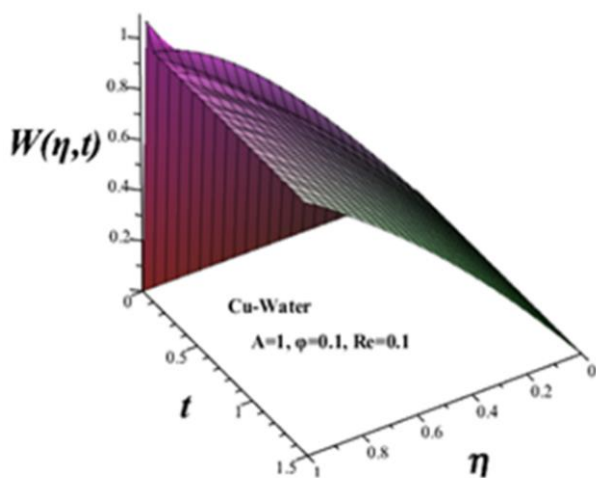


Fig. 3. Variation of the velocity profile with time and space.

slip boundary condition, then progressively increases along the channel, attaining its maximum at the upper permeable moving wall. With time, the velocity field transitions from a transient state to a stable steady-state configuration, thereby highlighting the unsteady relaxation mechanisms that govern nanofluid transport. Figure 6 illustrates the influence of nanoparticle volume fraction, where increasing Cu nanoparticle concentration enhances fluid velocity, particularly near the upper moving wall.

This enhancement stems from the superior thermal conductivity and augmented momentum diffusion imparted by the suspended nanoparticles, which in turn accelerates convective transport and facilitate more effective heat dissipation. Figure 7 emphasises the effect of Reynolds number (Re), showing that higher values intensify the velocity distribution towards the upper moving wall as a result of stronger injection at the lower wall and enhanced suction at the upper wall. This increase in Re not only accelerates the nanofluid flow, but also augments the overall convective heat transfer and entropy generation processes, underscoring the balance between improved thermal performance and irre-

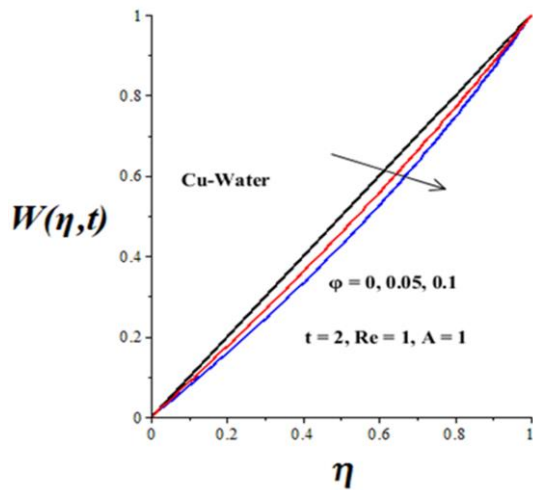
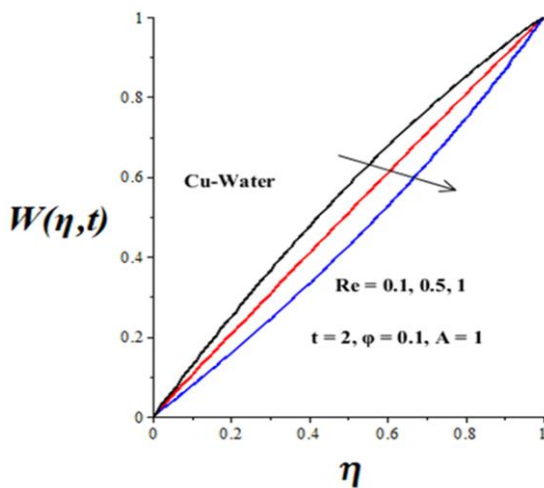
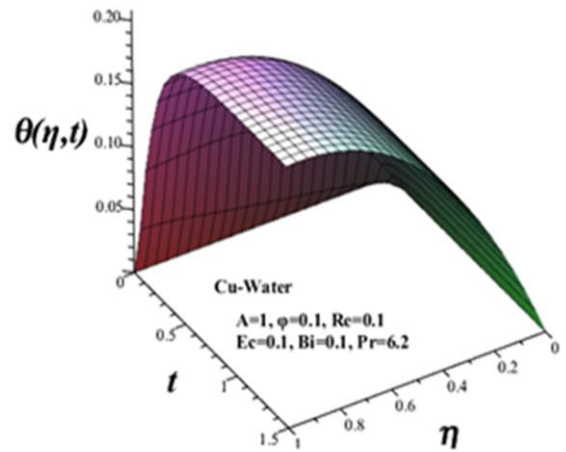
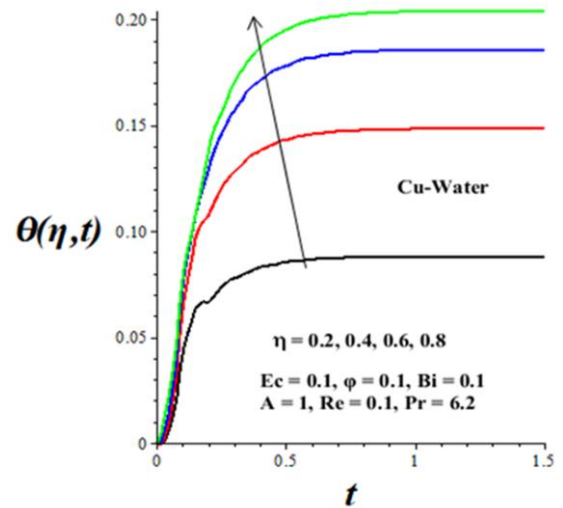
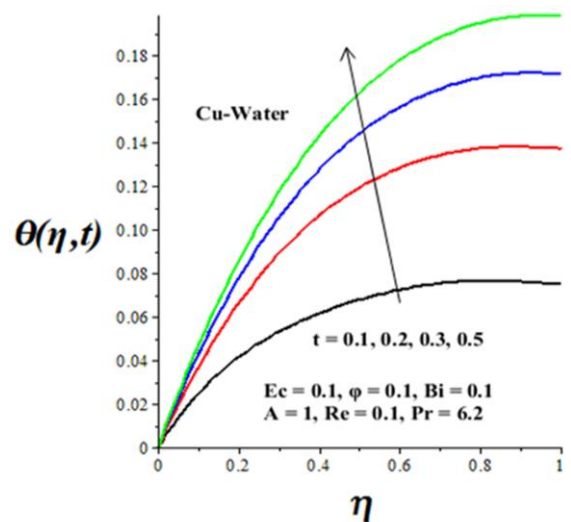
Fig. 6. Impact of ϕ on the velocity profile.Fig. 7. Impact of Re on the velocity profile.

Fig. 8. Variation of the temperature profile with time and space.

Fig. 9. Impact of η on the temperature profile.Fig. 10. Impact of t on the temperature profile.

versibility. From an engineering perspective, these findings carry important implications: the ability to modulate velocity, thermal transport and entropy generation through nanoparticle concentration, wall permeability and flow inertia, offers a pathway for optimising nanofluid-based thermal systems. Applications include advanced cooling strategies in microelectronics and industrial heat exchangers, efficiency enhancement in renewable energy systems such as solar thermal collectors, improved biomedical fluid transport in drug delivery and dialysis devices, and controlled porous filtration in chemical processing. By linking velocity field modifications to thermal and entropic outcomes, this study highlights how tailored parameter tuning in permeable-walled Couette systems can reduce irreversibility, enhance heat management and contributes to the design of energy-efficient, sustainable technologies.

4.2. Temperature profiles

Figures 8–14 present a comprehensive depiction of the thermal and entropic behaviour of unsteady nanofluid flow in a permeable-walled Couette system, highlighting the sensitivity of

temperature distributions to variations in time, space and key thermophysical parameters. Figures 8–10 demonstrate the transient evolution of the nanofluid temperature profile, where the fluid initially exhibits its lowest temperature at the stationary lower permeable wall due to the imposed boundary conditions. As the fluid migrates upward, the temperature rises progressively under the influence of convective heat transfer at the upper permeable moving wall, eventually stabilising into a steady-state profile over time for fixed parameter values. Figure 11 emphasises the role of Cu nanoparticle volume fraction, showing that higher concentrations increase nanofluid temperature owing to the superior thermal conductivity of metallic nanoparticles, which enhances energy storage and transport across the fluid domain. Similarly, Fig. 12 illustrates the effect of the Eckert number (Ec), where an increase corresponds to greater viscous dissipation and the subsequent conversion of kinetic energy into internal energy, thereby elevating the temperature field.

In contrast, Figs. 13 and 14 reveal that increasing Reynolds number and Biot number (Bi) leads to notable temperature suppression within the channel. This reduction results from enhanced convective cooling and intensified fluid suction at the upper moving wall, which promotes rapid heat dissipation into

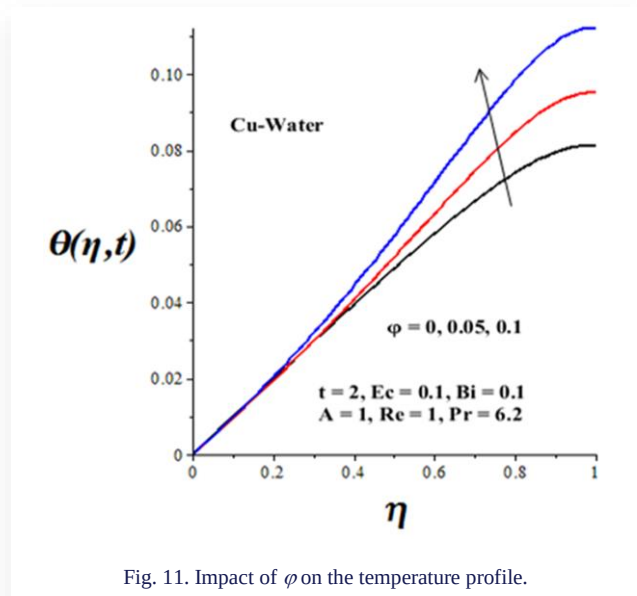


Fig. 11. Impact of ϕ on the temperature profile.

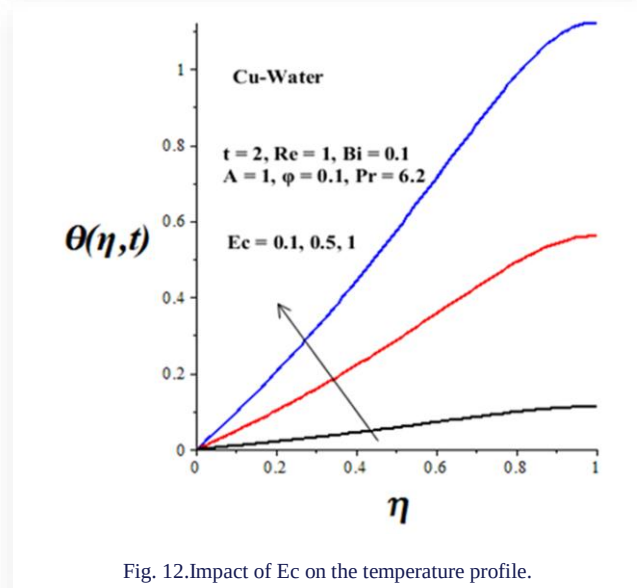


Fig. 12. Impact of Ec on the temperature profile.

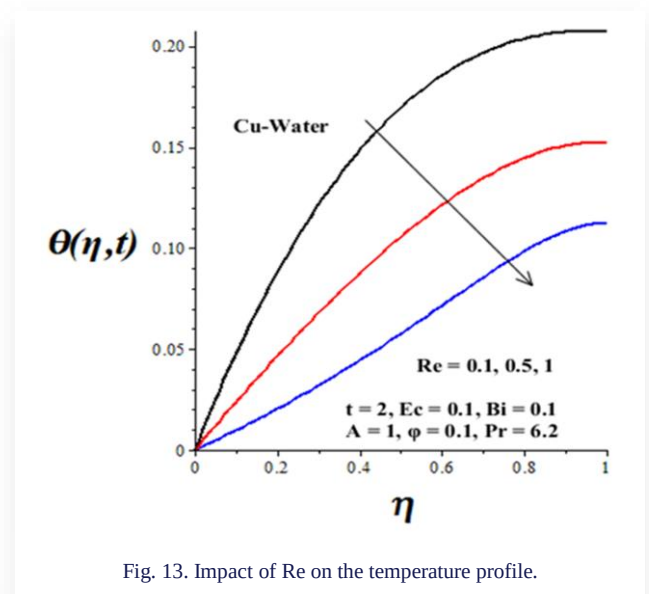


Fig. 13. Impact of Re on the temperature profile.

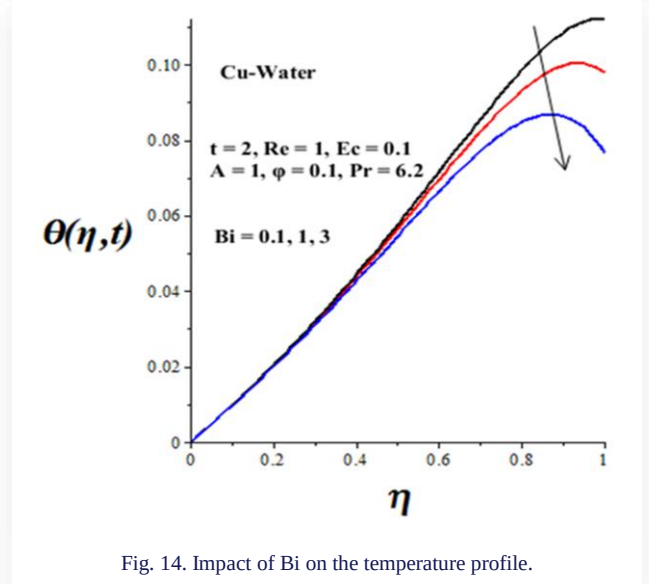


Fig. 14. Impact of Bi on the temperature profile.

the surrounding medium. From an engineering perspective, these findings carry significant implications: the tunability of nanoparticle concentration, viscous dissipation effects and convective boundary conditions provides a pathway for optimising thermal management strategies in high-performance cooling systems, porous filtration processes, microelectronic devices and biomedical applications, where precise regulation of temperature and minimisation of entropy generation are critical. In particular, systems requiring rapid heat removal, such as compact heat exchangers and lab-on-chip devices, benefit from the elevated Re and Bi values, while applications prioritising thermal energy storage or targeted heating, such as photothermal therapy or solar thermal collectors, may exploit

higher nanoparticle concentrations and viscous dissipation effects. Thus, the interplay between thermal and entropic behaviour in permeable-walled Couette systems provides a valuable design framework for developing energy-efficient, sustainable and application-specific nanofluid technologies.

4.3. Skin friction and Nusselt number

Figures 15–17 provide important insights into the coupled thermal and entropic behaviour of unsteady nanofluid flow in a permeable-walled Couette system, with direct implications for engineering applications, where heat transfer efficiency and energy dissipation are critical. The results indicate that an increase in the nanoparticle volume fraction (ϕ) significantly raises the skin friction at both channel walls, a trend linked to the higher effective viscosity and augmented momentum transport of nanofluids, which intensify wall shear stresses. Specifically, Fig. 15 shows that increasing the Reynolds number enhances skin friction at the upper permeable moving wall due to stronger suction effects, which accelerate near-wall momentum transfer, whereas the opposite trend is observed at the lower permeable

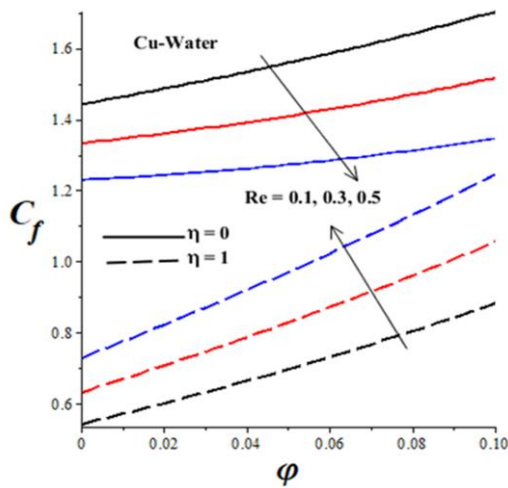


Fig. 15. Impact of Re and ϕ on skin friction.

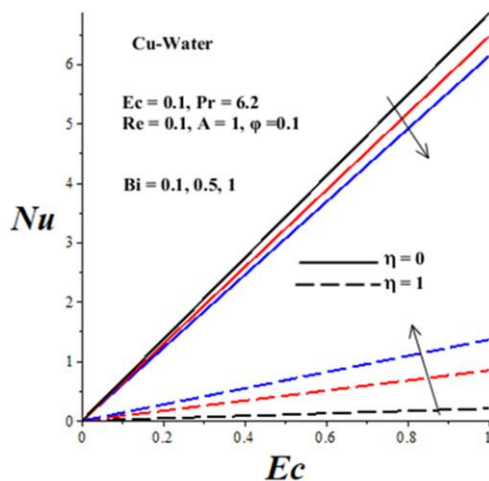


Fig. 16. Impact of Bi and Ec on the Nusselt number.

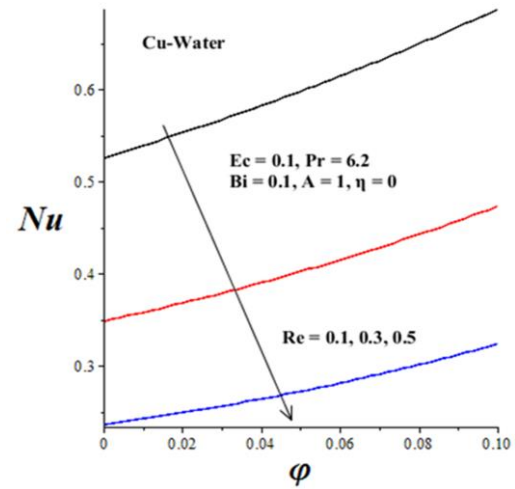


Fig. 17. Impact of Re and ϕ on the Nusselt number.

stationary wall, where injection weakens shear stress and reduces skin friction. This contrasting behaviour underscores the delicate balance between suction and injection mechanisms in modulating near-wall flow dynamics. Figures 16 and 17 further highlight the influence of thermophysical parameters on the Nusselt number, a measure of convective heat transfer.

The findings reveal that higher nanoparticle volume fraction, Eckert number and Biot number, all contribute to improved convective heat transfer rates. This enhancement is attributed, respectively, to improved thermal conductivity from nanoparticle suspension, increased viscous dissipation and internal heat generation and stronger convective coupling between the fluid and its thermal environment. In contrast, a rise in Re diminishes the Nusselt number, as intensified suction and injection dynamics facilitate stronger convective cooling that dissipates thermal energy more rapidly, thereby lowering the effective heat transfer at the walls. These observations carry important engineering implications: for instance, in microelectronic cooling systems, increasing nanoparticle concentration and optimising viscous dissipation can significantly enhance thermal management, while in porous filtration and biomedical applications, careful regulation of suction and injection is necessary to balance shear stress effects with heat transfer requirements. Similarly, in renewable energy systems, such as solar collectors or geothermal exchangers, tuning Biot and Eckert numbers can improve energy recovery efficiency while minimising entropy generation. Overall, the parametric interplay illustrated in Figs. 15–17 highlights strategies for enhancing heat transfer while mitigating irreversibility, thereby guiding the design of more efficient and sustainable nanofluid-based thermal systems.

4.4. Entropy generation rate

Figures 18–21 present a detailed exploration of the thermal and entropic behaviour of unsteady nanofluid flow in a permeable-walled Couette system, highlighting how emerging flow and thermal parameters influence entropy generation and, by extension, the efficiency of energy transport. A key observation is that

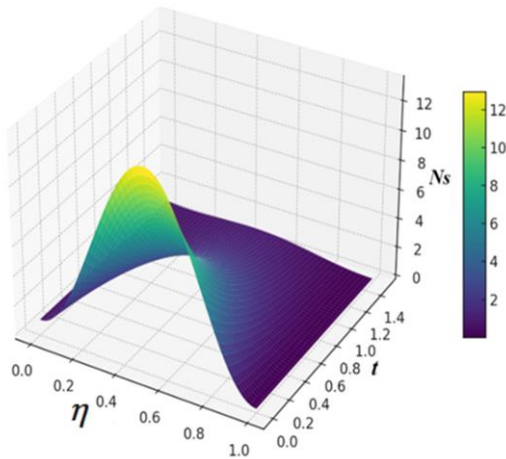


Fig. 18. The entropy generation rate (Ns) profile with time and space.

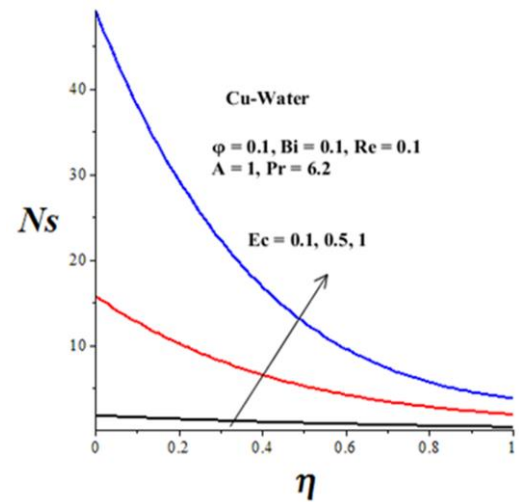


Fig. 20. Impact of Ec on Ns .

entropy generation is consistently higher near the lower stationary permeable wall compared to the upper moving wall, a disparity arising from the combined effects of fluid injection at the lower boundary and the modified thermophysical properties of the nanofluid, which accentuate velocity and temperature gradients. Figure 18 provides a three-dimensional surface plot of the entropy generation rate (Ns) in the (η, t) -space, offering an insightful visualisation of how thermodynamic irreversibility evolves both spatially and temporally. Here, entropy generation is dominated by wall shear stresses and transient start-up effects, with higher values evident at early times and near-wall regions. Over time, as the system progresses toward a quasi-steady state, irreversibility diminishes, indicating stabilisation of both thermal and hydrodynamic fields. This behaviour has significant engineering implications: in microchannel heat exchangers, micro-electromechanical systems (MEMS), energy storage devices and biomedical thermal management systems, minimising entropy generation directly translates to improved exergy efficiency, reduced operational losses and more sustainable designs. Figures 19 and 20 show that increasing the copper nanoparticle

volume fraction enhances entropy generation due to the dual effect of improved thermal conductivity and elevated viscosity.

While nanoparticles facilitate more efficient heat transfer, the associated viscous resistance increases momentum dissipation, thereby elevating irreversibility, a trade-off engineers must balance when designing nanofluid-based cooling and energy systems.

Similarly, higher Eckert numbers intensify entropy production by converting more mechanical energy into heat through viscous dissipation, underscoring the critical role of flow velocity and shear rates in determining energy efficiency in practical applications, such as turbine blade cooling or biomedical perfusion processes. Figures 21 and 22 further demonstrate the nuanced effects of Biot number and Reynolds number: increasing Bi and Re reduces entropy generation at the lower stationary permeable wall due to stronger convective cooling and fluid injection, which diminishes thermal gradients and lowers irreversibility. Conversely, the upper moving wall exhibits higher entropy production under the same conditions, driven by enhanced

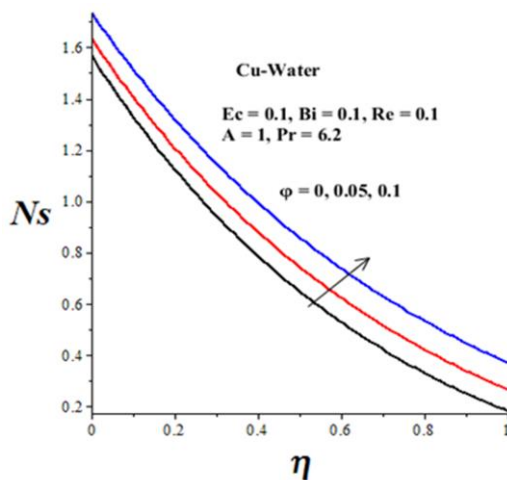


Fig. 19. Impact of ϕ on Ns .

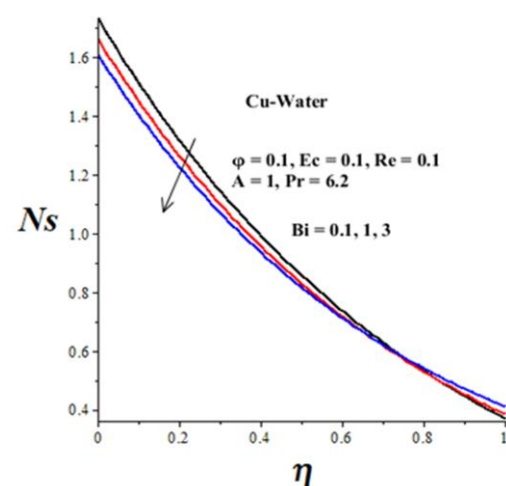
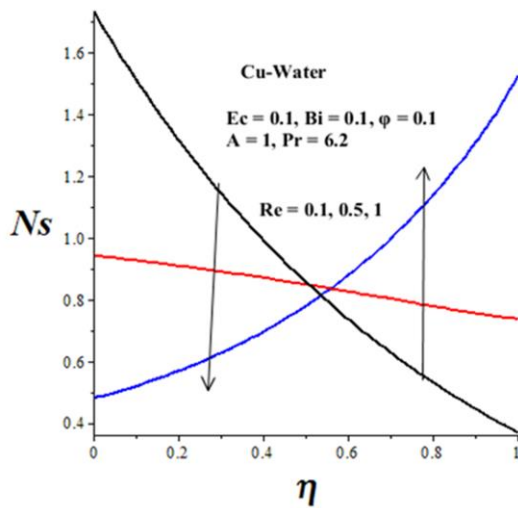
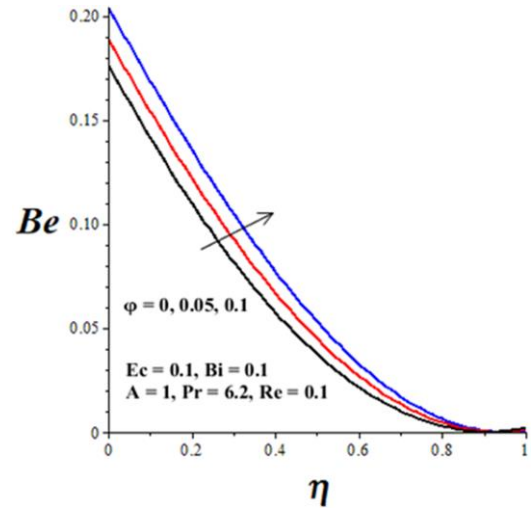


Fig. 21. Impact of Bi on Ns .

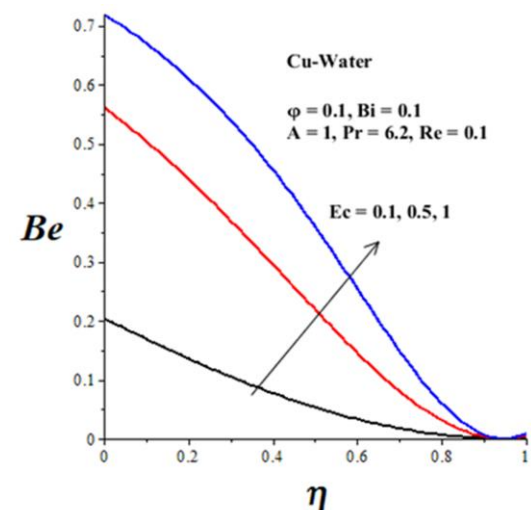
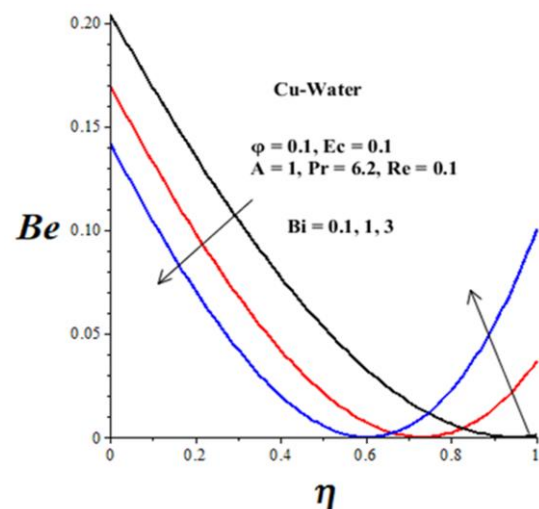
Fig. 22. Impact of Re on Ns .Fig. 23. Impact of ϕ on Be .

suction effects and amplified convective heat exchange, which strengthen local energy dissipation.

These results emphasise that while parameters, such as nanoparticle concentration and the Eckert number uniformly increase entropy generation, the Biot and Reynolds numbers introduce spatially contrasting behaviours, making entropy distribution highly dependent on wall conditions and flow regulation. From an engineering standpoint, this tunability is advantageous, allowing for tailored optimisation of system performance through strategic manipulation of boundary conditions. For instance, in industrial cooling loops, porous filtration systems or renewable energy collectors, controlling suction/injection rates and wall thermal properties can minimise irreversibility in critical regions, thereby improving efficiency and lowering energy costs. Overall, the thermal and entropic analysis not only deepens the understanding of nanofluid transport in permeable Couette systems, but also provides practical guidance for designing advanced thermal-fluid systems, in which irreversibility hotspots must be identified, controlled and mitigated to achieve sustainable and high-performance energy management.

4.5. Bejan number

Figures 23–26 illustrate the complex influence of key thermo-physical and flow parameters on the Bejan number (Be), offering critical insights into the dominant mechanisms of entropy generation within the unsteady nanofluid flow in a permeable-walled Couette system. The Bejan number serves as a vital metric for distinguishing between the relative contributions of heat transfer irreversibility and fluid friction irreversibility to the total entropy generation rate. A higher Bejan number indicates that entropy production is primarily driven by thermal gradients and heat transfer processes, whereas a lower value reflects a predominance of fluid friction and viscous effects. Across the channel, a consistent observation is that the Bejan number is generally higher near the lower stationary permeable wall compared to the upper moving wall, suggesting that thermal irreversibilities are more pronounced in regions influenced by fluid injection and significant temperature gradients. The analysis shows that increasing the Cu

Fig. 24. Impact of Ec on Be .Fig. 25. Impact of Bi on Be .

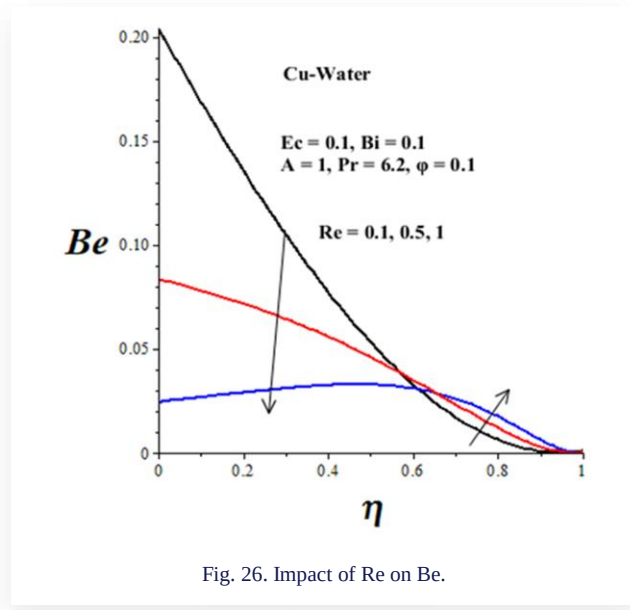


Fig. 26. Impact of Re on Be.

nanoparticle volume fraction and the Eckert number leads to a marked elevation in the Bejan number. Higher nanoparticle concentrations enhance the effective thermal conductivity of the nanofluid, facilitating more efficient heat transport and consequently increasing the dominance of heat transfer-related entropy generation. Similarly, larger Eckert numbers, representing the conversion of kinetic energy into internal energy through viscous dissipation, further amplify thermal irreversibility, elevating the Bejan number throughout the channel.

From an engineering perspective, these findings indicate that carefully tuning nanoparticle concentration and controlling flow velocities to modulate viscous heating can enhance thermal management efficiency in systems, such as microchannel heat exchangers, electronic cooling devices and solar thermal collectors, where minimising entropy generation is critical for performance optimisation. In contrast, the Biot number and the injection/suction Reynolds number exhibit spatially dependent effects on the Bejan number. At the lower stationary wall, increasing Bi and Re results in a decrease in the Bejan number, signalling that fluid friction irreversibility becomes more dominant. This reduction arises from enhanced convective cooling and intensified fluid injection, which reduce local temperature gradients and shift the primary source of entropy generation towards shear-induced losses.

Conversely, at the upper moving wall, increasing the Biot and Reynolds numbers elevates the Bejan number, reflecting an intensified role of thermal irreversibility due to stronger fluid suction and augmented convective heat transfer at this boundary. This spatial variation in entropy generation mechanisms underscores the importance of localised flow and thermal control strategies for optimising system performance, particularly in applications involving porous surfaces, microfluidic devices, or biomedical cooling systems, where precise thermal regulation is critical. Overall, these results highlight that while parameters, such as nanoparticle volume fraction and Eckert number, consistently enhance heat transfer irreversibility across the channel, parameters like Bi and Re exert location-specific effects, producing complex

distributions of entropy generation. Understanding these trends is essential for engineering design strategies aimed at reducing energy losses, improving heat transfer efficiency, and developing more sustainable and high-performance nanofluid-based flow systems.

6. Conclusions

This study presents a detailed numerical investigation of unsteady Cu-water nanofluid flow in a permeable-walled Couette system with convective heat transfer, employing a semi-discretisation finite-difference method coupled with a Runge-Kutta-Fehlberg integration scheme. Entropy generation was analysed through velocity and temperature gradients, providing insights into the interplay between thermal and viscous irreversibilities. The influence of key thermophysical and flow parameters on velocity, temperature, skin friction, Nusselt number, entropy generation rate and Bejan number was systematically examined, revealing several notable trends with direct relevance to engineering applications. The key conclusions from this study are as follows:

- The nanofluid velocity and temperature fields evolve dynamically over time, ultimately stabilising into steady-state profiles within the channel. This behaviour is critical for engineering systems, where predictable thermal and flow performance is essential, such as in microchannel heat exchangers and cooling devices for electronics.
- Increasing the nanoparticle volume fraction and the injection/suction Reynolds number enhances the velocity distribution toward the upper moving permeable wall. This indicates that manipulating nanoparticle concentration and flow intensity can effectively control fluid transport and shear rates, which is beneficial for applications requiring targeted convective enhancement, such as industrial cooling channels and biomedical microfluidic systems.
- The nanofluid temperature rises with higher solid volume fraction and Eckert number, reflecting improved thermal conductivity due to nanoparticles and increased viscous dissipation, while it decreases with higher Reynolds and Biot numbers, highlighting the influence of flow-induced cooling and convective heat transfer. Understanding these trends is vital for thermal management in high-heat-flux devices, solar collectors and other energy systems, where precise temperature control is required.
- Higher nanoparticle volume fraction leads to increased skin friction, Nusselt number, entropy generation rate and Bejan number, emphasising that enhanced heat transfer is accompanied by greater thermal irreversibility. Similarly, higher Eckert numbers amplify heat transfer and entropy production, underlining the trade-offs between viscous heating and thermal efficiency. These insights are important for designing systems where maximising heat transfer must be balanced against minimising energy losses, such as in advanced cooling technologies and energy-efficient nanofluid-based heat exchangers.

- The effects of the Reynolds and Biot numbers are spatially dependent: increasing the Reynolds number reduces skin friction, the Nusselt number, the entropy generation and the Bejan number at the lower stationary wall, but increases them at the upper moving wall. Likewise, a higher Biot number decreases these quantities at the lower wall while enhancing them at the upper wall. These variations underscore the critical role of boundary-layer control, fluid injection/suction and convective conditions in optimising both thermal performance and energy efficiency in engineering applications, including porous channels, microfluidic devices and industrial thermal management systems.

Finally, understanding the spatial and parametric dependencies of entropy generation and heat transfer provides a framework for optimising nanofluid-based systems, enabling engineers to maximise thermal performance while minimising irreversibility, ultimately contributing to more sustainable and energy-efficient technologies. For the future, the model may be extended to include the combined effects of other physical factors, such as magnetic field, thermal radiation, buoyancy forces, Joule heating, thermophoresis and Brownian motion.

References

- [1] Bejan, A. (1995). *Entropy generation minimization: The method of thermodynamic optimization of finite-size systems and finite-time processes*. CRC Press, Boca Raton.
- [2] Das, S.K., Choi, S.U.S., & Patel, H.E. (2006). Heat transfer in nanofluids – A review. *Heat Transfer Engineering*, 27(10), 3–19. doi: 10.1080/01457630600904593
- [3] Choi, S.U.S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. In *Developments and applications of non-Newtonian flows*. ASME FED, 231/MD 66 (pp. 99–105), New York.
- [4] Narmatha, M., & Kavitha, R. (2024). Analysis of unsteady heat and mass transfer in rotating MHD convection flow over a porous vertical plate. *Archives of Thermodynamics*, 45(4), 179–187. doi: 10.24425/ather.2024.152007
- [5] Batchelor, G.K. (2000). *An introduction to fluid dynamics*. Cambridge University Press.
- [6] Abu-Nada, E. & Chamkha, A.J. (2010). Mixed convection flow in a lid-driven inclined square enclosure filled with a nanofluid. *European Journal of Mechanics B/Fluids*, 29(6), 472–482. doi: 10.1016/j.euromechflu.2010.06.008
- [7] Ali, K., Rajashekar Reddy, Y., & Balla, C.S. (2021). Numerical solution of transient Fe₃O₄-EG nanofluid past the Couette channel associated with radiation. *International Journal of Ambient Energy*, 43(1), 5673–5686. doi: 10.1080/01430750.2021.1970623
- [8] Nield, D.A., & Bejan, A. (2006). *Convection in porous media*. Springer. doi: 10.1007/978-3-319-49562-0
- [9] Kasaeian, A., Daneshazarian, R., Mahian, O., Kolsi, L., Chamkha, A.J., Wongwises, S., & Pop, I. (2017). Nanofluid flow and heat transfer in porous media: A review of the latest developments. *International Journal of Heat and Mass Transfer*, 107, 778–791. doi: 10.1016/j.ijheatmasstransfer.2016.11.074
- [10] Kakac, S. & Pramuanjaroenkij, A. (2009). Review of convective heat transfer with nanofluids. *International Journal of Heat and Mass Transfer*, 52(13–14), 3187–3196. doi: 10.1016/j.ijheatmasstransfer.2009.02.006
- [11] Shanthi, R., Anandan, S.S., & Ramalingam, V. (2012). Heat transfer enhancement using nanofluids: An overview. *Thermal Science*, 16(2), 423–444. doi: 10.2298/TSCI110201003S
- [12] Malik, Z.U., Mallawi, F.O., & Haq, R.U. (2021). Thermal energy performance of nanofluid flow and heat transfer vertical channel with permeable surface. *Case Studies in Thermal Engineering*, 28, 101447. doi: 10.1016/j.csite.2021.101447
- [13] Mkwizu, M.H., & Makinde, O.D. (2015). Entropy generation in variable viscosity channel flow of nanofluids with convective cooling. *Comptes Rendus Mecanique*, 343(1), 38–56. doi: 10.1016/j.crme.2014.09.002
- [14] Ndelwa, E.J., Mkwizu, M.H., Matofali, A.X., & Ali, A.O. (2025). Thermodynamic analysis of entropy generation in MHD hybrid nanofluid channel flow under Navier slip conditions. *International Journal of Ambient Energy*, 46(1), 2559148. doi: 10.1080/01430750.2025.2559148
- [15] Oztop, H.F., & Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, 29(5), 1326–1336. doi: 10.1016/j.ijheatfluidflow.2008.04.009
- [16] Siddabasappa, Siddheshwar, P.G. & Makinde, O.D. (2021). A study of entropy generation and heat transfer in a magnetohydrodynamics flow of a couple-stress fluid through a thermal non-equilibrium vertical porous channel. *Heat Transfer*, 50(6), 6377–6400. doi: 10.1002/htj.22176
- [17] Rikitu, E.H., & Makinde, O.D. (2024). Entropy generation and heat transfer analysis of Eyring-Powell nanofluid flow through inclined microchannel subjected to magnetohydrodynamics and heat generation. *International Journal of Thermofluids*, 22, 100640. doi: 10.1016/j.ijft.2024.100640
- [18] Ali, R., Iqbal, A., Abbass, T., Arshad, T., & Shahzad, A. (2024). Unsteady flow of silica nanofluid over a stretching cylinder with effects of different shapes of nanoparticles and Joule heating. *Archives of Thermodynamics*, 45(3), 115–126. doi: 10.24425/ather.2024.151222
- [19] Gao, X.W., Zhu, Y.-M., & Pan, T. (2023). Finite line method for solving high-order partial differential equations in science and engineering. *Partial Differential Equations in Applied Mathematics*, 7, 100477. doi: 10.1016/j.padiff.2022.100477
- [20] Gorai, V.K., Kumar, M., Singh, R., & Sahu, M.K. (2025). Theoretical investigation for optimal thermal and thermodynamic performance of flat-plate solar collector with nanofluids. *Archives of Thermodynamics*, 46(1), 155–167. doi: 10.24425/ather.2025.154189
- [21] Makinde, O.D., & Eegunjobi, A.S. (2013). Effects of convective heating on entropy generation rate in a channel with permeable walls. *Entropy*, 15(1), 220–233. doi: 10.3390/e15010220

Effects of radiative magneto-thermal and solutal stratification on heat and mass transfer over an exponentially stretching sheet with chemical reaction

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Received: 19.05.2025; revised: 20.09.2025; accepted: 08.10.2025

Abstract

A numerical investigation has been conducted on the flow of a nanofluid over an exponentially stretching surface, a configuration relevant to many industrial heat and mass transfer applications. By applying appropriate similarity transformations, the governing partial differential equations are reduced to a set of dimensionless ordinary differential equations. These nonlinear equations are solved numerically using the fourth-order Adams–Moulton method integrated with a shooting technique to ensure accuracy and stability. The validity of the computed results is established by benchmarking them against previously published findings in the literature. Key parameters influencing the flow are illustrated and discussed through graphical representations. The study's overall findings support the conclusion that increasing the Hartmann number, Schmidt number and thermal stratification parameter reduces the thermal boundary layer thickness. Moreover, enhanced solutal stratification and chemical reaction parameters are found to suppress the nanoparticle concentration. Engineering quantities like the skin friction coefficient, Nusselt number and Sherwood number are also computed and analysed, providing insights into optimising nanofluid-based thermal systems under complex boundary conditions.

Keywords: Magnetic field; Chemical reaction; Viscous dissipation; Exponential stretching sheet; Adams–Moulton method

Vol. 46(2025), No. 4, 177–186; doi: 10.24425/ather.2025.156848

Cite this manuscript as: Srisailam, B., Ramesh, P., Umakanth, M., Govardhan, K., & Narender, G. (2025). Effects of radiative magneto-thermal and solutal stratification on heat and mass transfer over an exponentially stretching sheet with chemical reaction. *Archives of Thermodynamics*, 46(4), 177–186.

1. Introduction

A nanofluid is a type of fluid that contains nanoparticles uniformly dispersed throughout a base liquid, creating a stable colloidal mixture. Typical base fluids include water, ethylene glycol and various oils, while the nanoparticles are often made of metals, metal oxides, carbides or carbon nanotubes. The term "nanofluid" was first introduced by Choi [1] to describe this novel class of fluids. Nanofluid is essentially a mixture of conventional low-thermal-conductivity fluid and nanoparticles smaller than 100 nm. It can also be defined as a suspension of

tiny particles within a base fluid. The most widely used nanoparticles in nanofluids include carbon nanotubes, metals, oxides and carbides. These fluids are synthetically engineered to exhibit enhanced thermal conductivity compared to their base fluids. The thermal conductivity of nanofluids can be further improved by incorporating nanoparticles such as gold, copper and silver. Buongiorno [2] investigated the factors contributing to this enhancement and found that the increase in thermal conductivity is primarily influenced by Brownian motion and the thermophoresis effect. Nanofluids have diverse applications across various industries. In the heavy vehicle and information technol-

Nomenclature

B_0	– magnetic flux density, T
C	– concentration, kg/m ³
C_f	– skin friction coefficient
D	– diffusion coefficient, m ² /s
Ec	– Eckert number
M	– Hartmann number
Nb	– Brownian motion parameter
Nt	– thermophoresis parameter
Nu	– Nusselt number
P	– pressure, Pa
Pr	– Prandtl number
R	– thermal radiation parameter
Sc	– Schmidt number
Sh	– Sherwood number
Sm	– thermal solutal stratification parameter
St	– thermal stratification parameter
T	– temperature of the fluid, K
u	– velocity in x -direction, m/s
v	– velocity in y -direction, m/s

x, y – Cartesian coordinates, m

Greek symbols

α	– thermal diffusivity, m ² /s
γ	– chemical reaction parameter
θ	– dimensionless temperature
κ	– thermal conductivity, W/(m K)
λ_r	– porosity parameter
μ	– fluid dynamic viscosity, Pa s
ν	– kinematic viscosity, m ² /s
ρ	– fluid density, kg/m ³
σ	– electrical conductivity, S/m
ϕ	– nanoparticle volume fraction

Abbreviations and Acronyms

KVL	– Khanafer-Vafai-Lightstone (model)
MHD	– magnetohydrodynamics
ODEs	– ordinary differential equations
OHAM	– optimal homotopy asymptotic method
PDEs	– partial differential equations

ogy sectors, they are used as coolants to improve heat dissipation. Additionally, nanofluids play a crucial role in industrial, biomedical and technological fields, making them a valuable innovation.

Other characteristics of the nanofluids, such as volume concentration, particle agglomeration, thermophoresis, Brownian motion, surface area, particle shape, etc., should be considered for the nanofluids to perform more effectively due to their numerous remarkable uses in a variety of industrial and scientific fields, including the production of rubber and plastic sheets, geothermal energy extraction, fiber glass, hot rolling, etc. The fluid dynamics community has given the study of boundary layer nanofluid flow a great deal of attention. To investigate the rotating flow of nanofluids induced by an exponentially stretched layer, Mushtaq et al. [3] carried out a numerical analysis.

The word magnetohydrodynamics originates from magneto, meaning magnetic field; hydro, referring to fluid; and dynamics, which denotes motion. These three concepts are combined to form the term magnetohydrodynamics (MHD). The magnetic characteristics of an electrically conducting fluid are examined via hydromagnetic flow. When a conducting fluid moves through a magnetic field, it generates an electric current, leading to the formation of the Lorentz force, which alters the fluid's motion. The MHD effect plays a crucial role in regulating cooling rates and achieving desired material properties. Shit et al. [4] analysed unsteady MHD nanofluid flow and heat transfer over an exponentially stretching surface with radiation and porous medium effects. Numerical results show that Brownian motion, thermophoresis and magnetic field enhance thermal boundary layers, while unsteadiness reduces them. Entropy generation is strongly influenced by magnetic field, dissipation, radiation and Biot number.

Reddy and Chamkha [5] examined the MHD nanofluid flow of Al₂O₃–water and TiO₂–water past a stretching sheet within a porous medium, accounting for radiation, chemical reaction and heat source/sink effects. Their work employed a validated

finite element approach to solve the governing equations and assessed the impact of critical parameters on velocity, temperature and concentration distributions.

Additionally, Gupta et al. [6] analysed MHD stagnation point flow of a non-Newtonian nanofluid past an inclined stretching sheet, incorporating chemical reaction, thermal radiation, Brownian motion and thermophoresis. By applying the optimal homotopy asymptotic method (OHAM), they examined the influence of various parameters on flow and heat transfer, confirming the method's superiority over conventional techniques. More recently, Gireesha et al. [7] studied unsteady nanofluid flow with dust particles over a stretching sheet under Hall current effects. Using the Khanafer-Vafai-Lightstone (KVL) model and similarity transformations, they reported that dust suspension and Hall current significantly enhance heat transfer and the Nusselt number.

For a continuous surface that is stretching exponentially, Magyari and Keller [8] outlined the features of mass and transmission of heat in the boundary layer. Other pertinent research studies were conducted by Vajravelu [9]. Cortell [10] studied how viscous dissipation via a nonlinear stretching affected the thermal boundary layer. Narender et al. [11] contemplated the radiation effects in the presence of heat generation/absorption and magnetic field on the magnetohydrodynamics (MHD) stagnation point flow over a radially stretching sheet using a Casson nanofluid. Narender et al. [12] explored the impacts of external magnetic field inclinations and viscous dissipation due to heat generation or absorption parameter on MHD mixed convective flow of Casson nanofluid. Murugesan and Kumar [13] also elucidated the same effect but with the flow of nanofluid over the exponentially stretching sheet. Hayat and Nadeem [14] examined hybrid nanofluids in a 3D rotating stretching sheet with radiation, heat generation and chemical reaction, showing that they achieve higher heat transfer rates than conventional nanofluids. This topic was then continued by Hayat et al. [15], who investigated the influence of MHD, heat generation/absorp-

tion and nanoparticle concentration on hybrid nanofluid flow over a rotating stretching surface. They analysed the effects on velocity, skin friction and heat transfer, showing that hybrid nanofluids outperform conventional nanofluids. Other researchers [16–19] have studied boundary layer flow over a stretching sheet, where the velocity increases linearly from a fixed point.

Chemical reactions are crucial components in numerous living and non-living processes. Examples of their applications include burning fuels, the production of glass and pottery, iron smelting and food processing. Chemical reactions are categorised as homogeneous and heterogeneous, with the rate at which they occur depending on the concentration of the participating reactants. At the heart of chemical engineering, chemical reactions are employed to produce high-quality substances from natural raw materials. It is crucial to streamline a reaction, thereby optimising its quality and yield while minimising the use of reactants, energy and hazardous by-products. Researchers, engineers and academicians are increasingly showing a keen interest in developing efficient and sustainable chemical reaction systems. Researchers led by Swapna examined the flow of a magneto-micropolar nanofluid above a heated, permeable sheet that stretches, incorporating nonlinear radiation, chemical reaction and viscous dissipation effects [20]. Researchers led by Reddy et al. [21] investigated the influence of an inclined magnetic parameter on the unsteady, two-dimensional flow of an incompressible, viscous fluid confined between parallel plates.

The field of heat generation/absorption is a collaborative effort, essential in managing the heat transfer performance. This has led to numerous investigations into the effects of heat generation/absorption on nanofluids. These studies have consistently shown that increased heat generation leads to a rise in the temperature profile. Notably, Thirupathi et al. [22] conducted a comprehensive study on a nonlinear stretching sheet, examining thermal radiation, viscous dissipation, heat source and chemical reaction in Casson nanofluid flow. Eid and Nafe [23] showed that $\text{Fe}_3\text{O}_4\text{-Cu/EG}$ (ethylene glycol) hybrid nanofluid enhances heat transfer over shrinking/stretching sheets, with effects influenced by nanoparticle concentration, flow type and MHD conditions. The relevant literature for these studies can be found in [24–26].

Our study examines nanofluid flow over an exponentially stretching sheet, with particular attention to the effect of chemical reaction. Since these conditions have not been studied together in prior research Hayat et al. [27], we aim to analyse the flow behaviour under this unique configuration. The series expressions for velocity, temperature and nanoparticle concentration are developed by the method of Adams-Moulton of fourth order with a shooting technique. The numerical solutions are obtained by the software FORTRAN Language program. The organisation of this article is structured as: Section 2 outlines the problem formulation and the essential physical quantities of interest. Section 3 describes the application of the Adams-Moulton method to the proposed problem. Section 4 reviews the results, whereas Section 5 is reserved for concluding observations.

1.1. Core objectives of the thermal transport analysis

The core objectives are:

- To model the flow and heat transfer characteristics of a na-

nofluid over an exponentially stretching sheet, incorporating magnetic field effects, chemical reactions and heat generation or absorption.

- Also, explore how significant physical parameters influence the velocity, temperature and concentration profiles within the boundary layer.
- Utilise the Adams–Moulton method with a shooting technique for solving the governing equations.
- Analyse critical engineering indicators, such as local skin friction and Nusselt number, to evaluate heat transfer performance and surface resistance.
- Consider the practical insights into improving thermal performance in engineering applications by evaluating the combined impact of magnetic, thermal and chemical effects on nanofluid behaviour.

1.2. Novel contributions and significance of the study

- To our knowledge, this study offers the framework of nanofluid flow over a stretching sheet in which chemical reaction and radiative heat transfer are included.
- The incorporation of chemical reactions into the model adds greater physical fidelity and enhances its applicability to real-world thermal systems where reactive processes are significant.
- This work obtains coupled ODEs from highly nonlinear governing PDEs through similar transformations and, consequently, makes numerical analyses easier to perform.
- The study is applicable to advanced technologies such as electronic cooling systems, biomedical applications, waste heat recovery and environmental systems.

2. Problem statement and mathematical formulation

Let us consider the steady fully developed incompressible nanofluid flow along a continuously exponentially stretching sheet subject to a transverse magnetic field of strength B_0 in the presence of thermal radiation and chemical reaction.

The x -axis is aligned with the sheet length, while the y -axis is normal to it. The stretching sheet velocity is $u_w = U_0 e^{x/L}$. The flow is confined to $y > 0$. Figure 1 illustrates the schematic of our study.

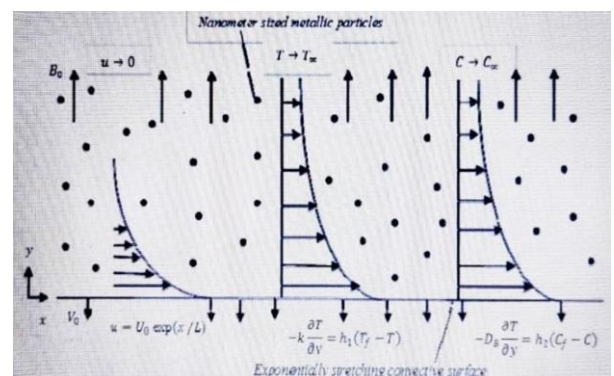


Fig. 1. Configuration of the flow.

The temperature and concentration at the wall and away from it are regarded to be exponentially stratified $T_w = T_0 + be^{x/2L}$, $C_w = C_0 + ae^{x/2L}$, $T_\infty = T_0 + ce^{x/2L}$, $C_\infty = C_0 + de^{x/2L}$, respectively (a, b, c and d constants).

The considered formulations for this system, expressed using the equations of continuity, momentum, energy and nanoparticle concentration, are as follows [17,27]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K} u - \sigma B_0^2 u, \quad (2)$$

$$\left. \begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{k}{(\rho C_p)_f} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \\ &+ \frac{(\rho C_p)_p}{(\rho C_p)_f} \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \\ &+ \frac{16\sigma^* T_\infty^3}{k^* (\rho C_p)_f} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{(\rho C_p)_f} \left(\frac{\partial u}{\partial y} \right)^2, \end{aligned} \right\} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - K_1 (C - C_\infty), \quad (4)$$

along with:

$y = 0$:

$$\left. \begin{aligned} u &= u_w = U_0 e^{\frac{x}{2L}}, \\ v &= 0, \\ T &= T_w = T_0 + b e^{\frac{x}{2L}}, \\ C &= C_w = C_0 + a e^{\frac{x}{2L}}, \end{aligned} \right\} \quad (5a)$$

$y \rightarrow \infty$:

$$\left. \begin{aligned} u &\rightarrow 0, \\ v &\rightarrow 0, \\ T &= T_\infty = T_0 + c e^{\frac{x}{2L}}, \\ C &= C_\infty = C_0 + d e^{\frac{x}{2L}}. \end{aligned} \right\} \quad (5b)$$

Equations (1) to (4) show how mass, momentum and energy are conserved in the system and include the equation for how concentration changes. In these equations, u and v are flow speeds in the x and y directions; T is temperature; ϕ - nanoparticle volume fraction; density is denoted by ρ ; electrical conductivity by σ ; the kinematic viscosity by ν ; μ - dynamic viscosity; K - porous medium permeability; $(C_p)_f$ - fluid heat capacity; $(C_p)_p$ - nanoparticle heat capacity; D_B - Brownian diffusion coefficient; D_T - thermophoresis coefficient; and T_∞ - ambient temperature.

The radiative heat flux vector can be described as:

$$q_r = \frac{-4\sigma^* \partial T^4}{3k^* \partial y}, \quad (6)$$

where σ^* and k^* denote the Stefan-Boltzmann constant and the mean absorption coefficient, respectively, and is linearised via a Taylor series expansion about T_∞ , from which we get:

$$\Rightarrow \frac{\partial T^4}{\partial y} = 4T_\infty^3 \frac{\partial T}{\partial y}. \quad (7)$$

Using Eq. (7) in Eq. (6) and differentiating, we have the following form:

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (8)$$

To find the solution of our model, we use the following similarity transformation [13,27]:

$$\left. \begin{aligned} \eta &= y \sqrt{\frac{U_0}{2\nu_f L}} e^{\frac{x}{2L}}, \\ v &= -\sqrt{\frac{\nu_f U_0}{2L}} e^{\frac{x}{2L}} [f(\eta) + \eta f'(\eta)], \\ u &= U_0 e^{\frac{x}{2L}} f'(\eta), \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_0}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_0}. \end{aligned} \right\} \quad (9)$$

Using Eq. (9), the governing equations (2-4) transform to new equations:

$$f''' - (M + \lambda)f' - 2(f')^2 + ff'' = 0, \quad (10)$$

$$\left. \begin{aligned} \frac{(1 + \frac{4R}{3})}{Pr} \theta'' + Nb\theta'\phi' + Nt(\theta')^2 + Ec + \\ -f'(St + \theta) + \theta'f = 0, \end{aligned} \right\} \quad (11)$$

$$\phi'' + \frac{Nt}{Nb} \theta'' + Sc(f\phi' - f'\phi) - Sc(Smf' - \gamma\phi) = 0, \quad (12)$$

together with:

$$\begin{aligned} f'(0) &= 1, \quad f(0) = 0, \quad f'(\infty) = 0, \\ \theta(0) &= 1 - Sm, \quad \theta(\infty) = 0, \\ \phi(0) &= 1 - St, \quad \phi(\infty) = 0. \end{aligned} \quad (13)$$

Different parameters used in the ordinary differential equations (ODEs) (10-12) and boundary conditions (13) have been listed in Table 1.

The local skin friction, Nusselt number and Sherwood number can be defined:

$$\begin{aligned} C_f &= \left[\frac{\tau_w}{\left(\frac{1}{2}\right)\rho U_0^2 e^{\frac{2x}{L}}} \right]_{y=0}, \\ Nu &= \frac{-xq_w}{k(T_w - T_\infty)}, \\ Sh &= \frac{-xq_m}{D_B(C_w - C_\infty)}, \\ \left. \begin{aligned} C_f \sqrt{\frac{Re_x}{2}} &= f''(0), \\ Nu Re_x^{-1/2} \sqrt{\frac{2L}{x}} &= -\frac{1}{1-St} \theta'(0), \\ Sh Re_x^{-1/2} \sqrt{\frac{2L}{x}} &= -\frac{1}{1-Sm} \phi'(0), \end{aligned} \right\} \quad (14)$$

where $Re_x = U_0 e^{\frac{x}{2L}} \frac{x}{\nu}$ is the local Reynolds number.

Table 1. Various dimensionless parameters utilised in the governing ODEs and their typical ranges of variation.

Model parameter	Definition	Significance	Typical ranges used
Porosity parameter (λ)	$\frac{\nu_f L}{KU_0} e^{-\frac{x}{L}}$	Impact on the material's ability to store and transport fluids	0.2 to 0.7. For example, sandstones typically have a porosity of 10–40% (0.1–0.4), while peats can exceed that of 50% (0.5).
Hartmann number (M)	$\frac{2\sigma B_0^2(x)L}{\rho_f U_0}$	The ratio of Lorentz force to viscous force in the fluid.	0 to 100, often in the range of 0.1 to 10 for many theoretical studies on MHD flows.
Prandtl number (Pr)	$\frac{\nu_f(\rho C_p)_f}{k_f}$	Momentum to thermal diffusivity	A Prandtl number of $Pr = 1.7$ corresponds to water at relatively high temperatures, such as near its boiling point
Thermal radiation parameter (R)	$\frac{4\sigma^* T_\infty^3}{k_0 k}$	Ratio of radiative heat flux to conductive heat transfer.	Often investigated in the range of 0.1 to 10, with higher values signifying more dominant radiation effects, common in high-temperature processes like those in furnaces or solar collectors.
Viscous parameter (Eckert number) (Ec)	$\frac{U_0^2 e^{\frac{x}{L}}}{(c_p)(T_f - T_\infty)}$	The relationship between a flow's kinetic energy and the boundary layer enthalpy difference.	Values can vary widely depending on the flow speed, from very small ($Ec \ll 1$) for low-speed flows to values near 1.0 for high-speed flows where viscous heating becomes significant.
Thermal stratification parameter (St)	$\frac{c}{b}$	Buoyancy forces to mixing forces in a fluid system	The effect is significant in situations where the temperature varies, with values ranging from 0.0 to 1.0 or more, with 0.0 indicating no stratification.
Thermal solutal stratification parameter (Sm)	$\frac{d}{a}$	Ratio of thermal buoyancy forces to solutal buoyancy forces	Similar to thermal stratification, it is typically investigated in the range of 0.0 to 1.0 or more, with 0.0 representing a uniform concentration
Brownian motion parameter (Nb)	$\frac{\tau D_B (C_w - C_\infty)}{\nu}$	The effect of random thermal fluctuations on the diffusion of nanoparticles in a fluid.	Typically studied within the range of 0.1 to 1.0, though some studies use values up to 5.0 to explore pronounced effects.
Thermophoresis parameter (Nt)	$\frac{\tau D_T (T_w - T_0)}{\nu(T_\infty)}$	The ratio of thermophoretic forces to viscous forces.	Generally considered in the range of 0.1 to 1.0 in most nanofluid studies.
Schmidt number (Sc)	$\frac{\nu}{D_B}$	The ratio of momentum diffusivity (viscosity) to mass diffusivity	For gases, it's typically close to 1.0 (around 0.2 to 4.0). For liquids, values are much higher, ranging from 200 to 1500, indicating that momentum diffuses much faster than mass.
Chemical reaction parameter (γ)	$\frac{K_0 L}{U}$	Ratio of reaction rate to transport rate (diffusion or convection)	Ranges from small values (approaching 0 for slow reactions) to higher values (e.g., 1.0 to 5.0) for moderate to fast reactions.

3. Implementation of the numerical scheme

The ODEs (10–12) along with boundary conditions (BCs) stated in (13) have been explored numerically via a shooting technique. After performing successive iterations, the bounded domain $[0, \eta_{\max}]$ has been considered instead of $[0, \infty]$ for an appropriate value of $\eta_{\max}$, the value after which the solution converges asymptotically. The variable f is denoted by y_1 , θ by y_4 and ϕ by y_6 . The boundary value problem (BVP) (10)–(13) in new variables is converted into an initial value problem (IVP):

$$\left. \begin{aligned}
 y_1' &= y_2, & y_1(0) &= 0, \\
 y_2' &= y_3, & y_2(0) &= 1, \\
 y_3' &= [2y_2^2 - y_1 y_3 + (M + \lambda)y_2], & y_3(0) &= s, \\
 y_4' &= y_5, & y_4(0) &= 1 - Sm, \\
 y_5' &= -\frac{3Pr}{(3+4R)} \left[y_1 y_5 + Nb y_5 y_7 + Nt y_7^2 + \right. \\
 &\quad \left. + Ec(f'')^2 - y_2(St + y_4) \right], & y_5(0) &= t, \\
 y_6' &= y_7, & y_6(0) &= 1 - St, \\
 y_7' &= \frac{Nt}{Nb} y_5' - Sc(y_1 y_7 - y_2 y_6) + ScSm y_2 + Sc \gamma y_7, & y_7(0) &= u.
 \end{aligned} \right\} \quad (15)$$

The above equations are solved using the shooting technique along with the Adams-Moulton method of order 4 with an initial

guess s , t and u . The missing initial conditions s , t , u are chosen such that:

$$y_2(\eta, s, t, u) = 0, \quad y_4(\eta, s, t, u) = 0, \quad y_6(\eta, s, t, u) = 0. \quad (16)$$

To solve the system of algebraic equations (16), we use Newton's method. The iterative process is repeated until the following criteria is met:

$$\max\{|y_2(\eta_{\max}) - 0|, |y_4(\eta_{\max}) - 0|, |y_6(\eta_{\max}) - 0|\} < \xi.$$

Throughout this work, ξ has been taken as 10^{-7} unless otherwise mentioned.

4. Analysis of results and validation

To clearly provide physical insight into the heat and mass transfer of an MHD nanofluid in the presence of viscous dissipation, thermal radiation parameter, hear generation/absorption and chemical reaction. In this complex geometry, we focused on the impact of the Hartmann number, the porosity parameter ($0 \leq M, \lambda \leq 1$), the Brownian motion parameter ($1.0 \leq Nb \leq 1.6$), the thermophores parameter ($0.1 \leq Nt \leq 0.6$), thermal stratification parameter ($0.1 \leq St \leq 0.4$), thermal radiation parameter

($0.1 \leq R \leq 0.4$), Eckert number ($0 \leq Ec \leq 1.1$), the Schmidt number ($0.9 \leq Sc \leq 1.3$), the solutal stratification parameter ($0.1 \leq Sm \leq 0.8$) and the chemical reaction parameter ($0.1 \leq \gamma \leq 0.4$) on the flow structures, velocity, temperature and concentration of the nanoparticles. For validation, the results of this study show the Schmidt number compared with previously published findings by Hayat et al. [27]. The numerical values of $-f''(0)$ are computed and compared in Table 2 for the case of $M = \lambda = 0$. We observe that the present and previous results are found to be in reasonable correlation. Thus, we guarantee that the technique, as well as the results provided in this present study, are valid and acceptable. Table 3 provides numerical res-

ults for the Nusselt number and Sherwood number for different flow impact factors. As anticipated, the Nusselt number reduces as the Schmidt number (Sc), the solutal stratification parameter (Sm) and chemical reaction parameter (γ) rise, while the opposite phenomenon is identified for the Sherwood number.

Table 2. The comparison between the previous and present results of $-f''(0)$ for $M = \lambda = 0$.

Value of	Hayat et al. [27]	Present
$-f''(0)$	1.281808	1.2818170
$f(\infty)$	0.905639	0.9050169

Table 3. Nusselt number and Sherwood number for $M = 0.1$, $\lambda = 0.1$, $Ec = 0.5$, $Pr = 1.7$, $Nt = Nb = 0.8$, $R = 0.4$ and different values of Sc , St , Sm , γ .

Sc	St	Sm	γ	$-\theta'(0)$	$-\frac{\theta'(0)}{(1-St)}$	$-\phi'(0)$	$-\frac{\phi'(0)}{(1-St)}$
1.1	0.1	0.1	0.1	0.3952224	0.4391360	0.7880001	0.8755557
1.2				0.3885755	0.4317505	0.8556961	0.9507734
1.5				0.3719586	0.4132873	1.042329	1.1581430
1.1	0.2	0.1	0.1	0.4570068	0.5712585	0.6412907	0.7125453
	0.3			0.5208401	0.7440573	0.4927663	0.5475181
	0.5			0.6548117	1.3096230	0.1901195	0.2112439
1.1	0.1	0.2	0.1	0.3322426	0.3691584	0.8856008	1.1070010
		0.3		0.2690218	0.2989132	0.9830891	1.4044130
		0.5		0.1417351	0.1574834	1.1778130	2.3556250
1.1	0.1	0.1	0.1	0.2936075	0.3262306	0.8585148	0.9539053
				0.2507791	0.2786434	0.8873966	0.9859962
				0.1275781	0.1417534	0.9683428	1.0759360
1.1	0.1	0.1	0.2	0.2398219	0.2664688	1.0439920	1.1599910
			0.3	0.2360424	0.2622693	1.1074420	1.2304910
			0.5	0.2302854	0.25587260	1.2173230	1.3525810

Figure 2 illustrates the impact of the Hartmann number (M) on the velocity field f' .

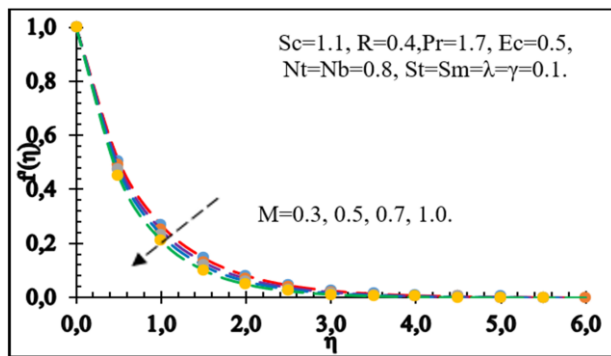


Fig. 2. Variation in $f'(\eta)$ with M .

As M increases, a noticeable decrease in the velocity profile is observed. This reduction occurs because the presence of a magnetic field generates a resistive force known as the Lorentz force. The Lorentz force acts as a damping mechanism by opposing the motion of the fluid, thereby reducing its velocity. Physically, this force arises due to the interaction between the induced electric currents within the fluid and the applied magnetic field, leading to an increase in the resistive effect. Conse-

quently, as M increases, the influence of the Lorentz force becomes more significant, further suppressing the fluid's motion and causing a more pronounced decline in the velocity profile.

Figure 3 presents the temperature field for varying values of the Brownian motion parameter (Nb) and the thermophoresis parameter (Nt).

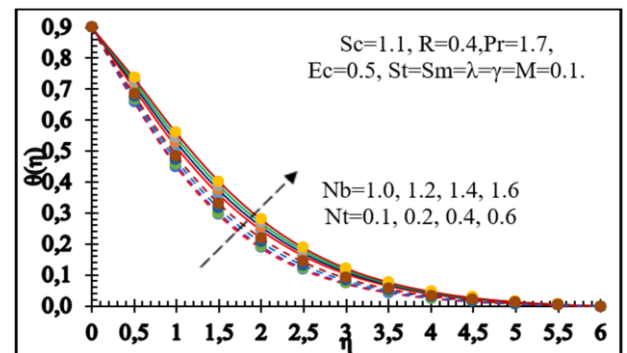


Fig. 3. Variation in $\theta(\eta)$ with Nb and Nt .

The figure reveals that as the values of Nb and Nt increase, both the temperature profile and the thermal boundary layer thickness exhibit significant enhancement. This increase occurs because Brownian motion, which describes the random movement of nanoparticles in the fluid, contributes to increased ther-

mal energy transport, thereby raising the temperature distribution. Similarly, thermophoresis, which refers to the movement of particles due to temperature gradients, leads to the migration of nanoparticles from hotter to cooler regions, further influencing heat transfer. As a result, the combined effect of higher Nb and Nt leads to a more pronounced thermal boundary layer, indicating an overall rise in the temperature profile throughout the flow region.

Figure 4 illustrates the concentration profile for different values of Nb and Nt .

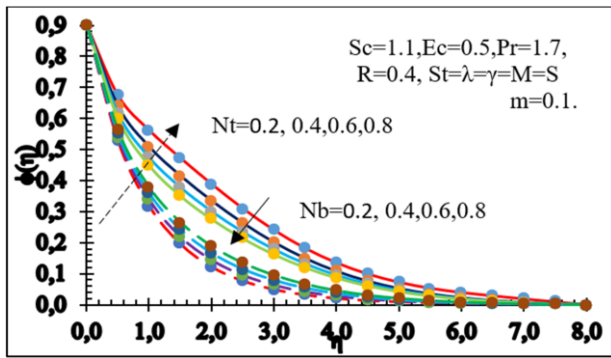


Fig. 4. Variation in $\phi(\eta)$ with Nb and Nt .

As the Brownian motion parameter (Nb) increases, the random movement of particles intensifies, enhancing diffusion and promoting a more uniform distribution. This increased dispersion reduces localised particle accumulation, leading to a lower overall concentration. Conversely, a higher thermophoresis parameter (Nt) results in greater particle accumulation in cooler regions. Thermophoresis drives particles away from hotter areas due to temperature-induced force imbalances, causing them to concentrate in lower-temperature zones. Unlike Brownian motion, which enhances mixing, thermophoresis induces directional transport, leading to localised clustering. The balance between these effects governs the overall particle distribution in the fluid.

Figure 5 illustrates the effect of the thermal stratification parameter (St) on temperature, showing that as St increases, the overall temperature decreases and the temperature difference between the surface and the surrounding fluid diminishes.

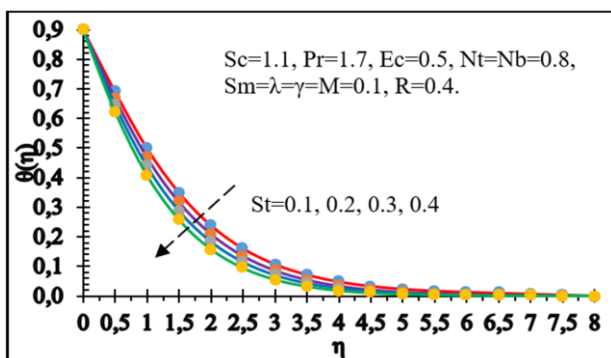


Fig. 5. Variation in $\theta(\eta)$ with St .

Physically, a higher St represents stronger thermal stratification, which enhances heat dissipation and restricts heat retention

near the surface. This leads to a more uniform temperature distribution, reducing thermal gradients and limiting heat transfer between layers.

As observed in Fig. 6, the dimensionless temperature increases with a higher radiation parameter (R). Physically, this occurs because an increase in R corresponds to a decrease in the mean absorption coefficient (k^*), which reduces the fluid's ability to absorb radiation. However, at the same time, the overall rate of radiative heat transfer to the fluid increases, thereby supplying additional thermal energy and raising the fluid temperature. The enhanced radiative heat transfer leads to a more uniform temperature distribution and thicker thermal boundary layer, reducing temperature gradients and slowing heat dissipation.

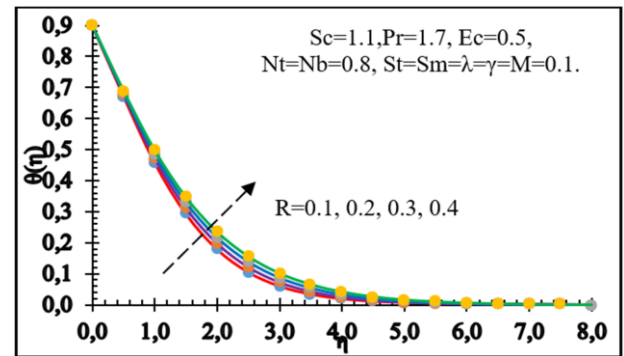


Fig. 6. Variation in $\theta(\eta)$ with R .

The fluid viscosity is highly sensitive to temperature changes, and as shown in Fig. 7, the temperature increases with the increasing Eckert number (Ec) due to the effect of resistive forces.

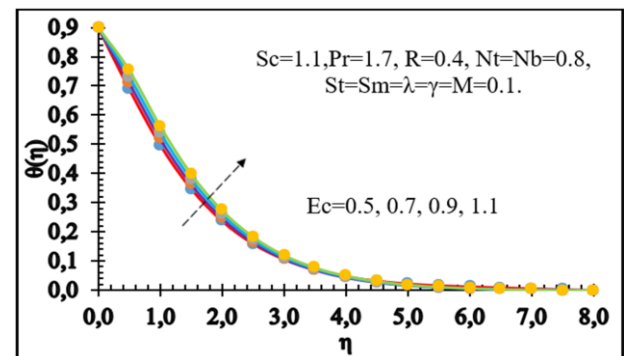


Fig. 7. Variation in $\theta(\eta)$ with Ec .

The Eckert number represents the conversion of kinetic energy into internal energy as the fluid works against viscous stresses, leading to heat generation within the system. This process, known as viscous dissipation, contributes to increasing the fluid temperature, particularly in high-speed or high-shear flows. As a result, the thickness of the thermal boundary layer increases, allowing heat to spread more effectively within the fluid. The presence of viscous dissipation plays a significant role in determining the temperature distribution, making it a crucial factor in high-speed flows, lubrication systems and thermal energy management.

Figure 8 exhibits the consequences of changing Sc . The Schmidt number (Sc) significantly affects the concentration profile by influencing the mass diffusion rate in the fluid.

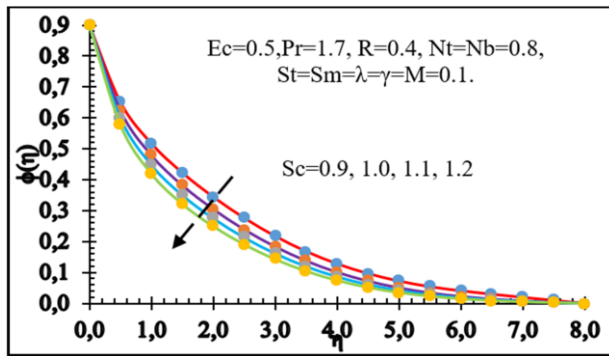


Fig. 8. Variation in $\phi(\eta)$ with Sc .

A higher Sc indicates lower mass diffusivity, meaning that particles diffuse more slowly, leading to a steeper concentration gradient and a thinner concentration boundary layer. Conversely, a lower Sc corresponds to higher diffusivity, allowing particles to spread more quickly and resulting in a more uniform concentration distribution. This effect is particularly important in applications such as chemical mixing, pollutant dispersion and mass transfer processes, where controlling the diffusion rate is essential for efficiency and effectiveness of the processes.

Figure 9 illustrates the effect of the solutal stratification parameter (S_m) on the concentration profile.

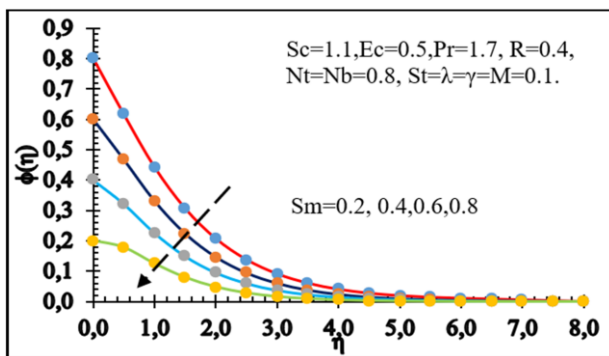


Fig. 9. Variation in $\phi(\eta)$ with S_m .

This parameter influences the concentration profile by affecting the mass transfer resistance within the fluid. As the stratification parameter increases, the difference in solute concentration between various layers of fluid becomes more pronounced, which restricts mass diffusion. This leads to a decrease in the overall concentration and the formation of a thinner solutal boundary layer. Increased stratification stabilises the concentration gradient, which is essential for ocean mixing, pollutant dispersion and chemical processing applications, where the controlled solute distribution is essential.

Figure 10 illustrates the influence of the chemical reaction parameter (γ) on the concentration profile $\phi(\eta)$. It is observed that, as γ increases, the concentration $\phi(\eta)$ decreases due to the

enhanced rate of chemical reactions. Physically, a higher γ indicates more intense chemical activity, leading to a faster conversion of reactants into products. This rapid consumption of reactants reduces the overall concentration, particularly in regions where reactions are most active, such as reaction fronts. Additionally, a larger γ limits mass diffusion, resulting in a thinner concentration boundary layer. This effect plays a crucial role in chemical and industrial processes where reaction rates significantly impact mass transfer and fluid behaviour.

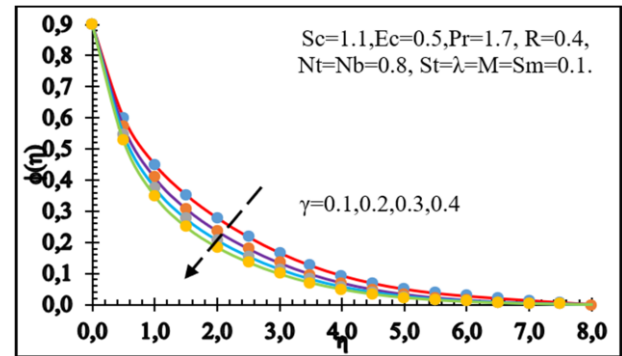


Fig. 10. Variation in $\phi(\eta)$ with γ .

Figure 11 presents the skin-friction coefficient as a function of the magnetic interaction parameter M for various values of the porosity parameter λ . An increase in M reduces the skin-friction coefficient, and a higher λ further decreases it.

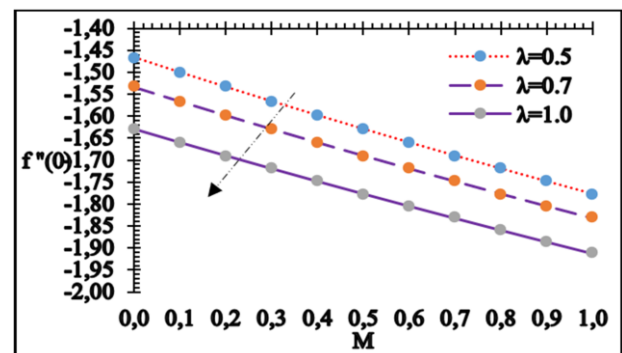


Fig. 11. Variation in skin friction with M and λ .

5. Conclusions

This research investigates, through numerical analysis, the effects of double stratification on nanofluid flow over an exponentially stretching surface. A two-dimensional, steady magnetohydrodynamic boundary layer flow of a thermo-solutally stratified nanofluid past an exponentially stretching sheet embedded in a porous medium is analysed. Using the shooting method, series-form solutions for the velocity, temperature and concentration profiles are derived. The numerical results are portrayed for several parameters. Similarity transformations and a strong numerical method – the shooting technique combined with the Adams–Moulton scheme were used to analyse the fluid behaviour.

Main results:

- An increase in the magnetic interaction parameter leads to a noticeable suppression of the velocity profile due to the enhanced Lorentz force opposing the flow.
- Elevated values of the thermal stratification parameter reduce the fluid temperature, while the solutal stratification parameter causes a decline in nanoparticle concentration, both effects attributed to the resistance against thermal and solutal diffusion.
- The thermal radiation parameter is found to negatively influence the Nusselt number, indicating reduced heat transfer efficiency in the presence of strong radiative effects.
- Heat transfer performance is enhanced with the increasing Schmidt number, Eckert number, thermophoresis parameter and Brownian motion parameter, reflecting enhanced energy transport due to mass diffusivity, viscous dissipation and nanoparticle dynamics.
- A rise in the chemical reaction parameter results in a lower nanoparticle volume fraction, which can be attributed to chemical dissipation processes occurring within the nanofluid.

References

- [1] Choi, S.U.S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME International Mechanical Engineering Congress and Exposition*, 66, 99–105. 12–17 November, San Francisco, USA.
- [2] Buongiorno, J. (2005). Convective transport in nanofluids. *Journal of Heat Transfer*, 128, 240–250. doi: 10.1115/1.2150834
- [3] Mushtaq, A., Mustafa, M., Hayat, T., & Alsaedi, A. (2016). Numerical study for rotating flow of nanofluids caused by an exponentially stretching sheet. *Advanced Powder Technology*, 27(5), 2223–2231. doi: 10.1016/j.appt.2016.08.007
- [4] Shit, G., Haldar, R., & Mandal, S. (2017). Entropy generation on MHD flow and convective heat transfer in a porous medium of exponentially stretching surface saturated by nanofluids. *Advanced Powder Technology*, 28(6), 1519–1530. doi: 10.1016/j.appt.2017.03.023
- [5] Reddy, P.S., & Chamkha, A.J. (2016). Soret and Dufour effects on MHD convective flow of Al_2O_3 –water and TiO_2 –water nanofluids past a stretching sheet in porous media with heat generation/absorption. *Advanced Powder Technology*, 27(4), 1207–1218. doi: 10.1016/j.appt.2016.04.005
- [6] Gupta, S., Kumar, D., & Singh, J. (2018). MHD mixed convective stagnation point flow and heat transfer of an incompressible nanofluid over an inclined stretching sheet with chemical reaction and radiation. *International Journal of Heat and Mass Transfer*, 118, 378–387. doi: 10.1016/j.ijheatmasstransfer.2017.11.007
- [7] Gireesha, B. J., Mahanthesh, B., Thammanna, G., & Sampathkumar, P. (2018). Hall effects on dusty nanofluid two-phase transient flow past a stretching sheet using KVL model. *Journal of Molecular Liquids*, 256, 139–147. doi: 10.1016/j.molliq.2018.01.186
- [8] Magyari, E., & Keller, B. (1999). Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *Journal of Physics D: Applied Physics*, 32(5), 577. doi: 10.1088/0022-3727/32/5/012
- [9] Vajravelu, K. (2001). Viscous flow over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 124(3), 281–288. doi: 10.1016/S0096-3003(00)00062-X
- [10] Cortell, R. (2007). Viscous flow and heat transfer over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 184(2), 864–873. doi: 10.1016/j.amc.2006.06.077
- [11] Narendar, G., Govardhan, K., & Sreedhar Sarma, G. (2020). Magnetohydrodynamic stagnation point on a Casson nanofluid flow over a radially stretching sheet. *Beilstein Journal of Nanotechnology*, 11, 1303–1315. doi: 10.3762/bjnano.11.114
- [12] Narendar, G., Govardhan, K., & Sreedhar Sarma, G. (2021). MHD Casson nanofluid past a stretching sheet with the effects of viscous dissipation, chemical reaction and heat source/sink. *Journal of Applied and Computational Mechanics*, 7(4), 2040–2048. doi: 10.22055/jacm.2019.14804
- [13] Murugesan, T., & Kumar, M.D. (2019). Viscous dissipation and Joule heating effects on MHD flow of a thermo-solutal stratified nanofluid over an exponentially stretching sheet with radiation and heat generation/absorption. *World Scientific News*, 129, 193–210.
- [14] Hayat, T., & Nadeem, S. (2017). Heat transfer enhancement with Ag–CuO/water hybrid nanofluid. *Results in Physics*, 7, 2317–2324. doi: 10.1016/j.rinp.2017.06.034
- [15] Hayat, T., Nadeem, S., & Khan, A.U. (2019). Numerical analysis of Ag–CuO/water rotating hybrid nanofluid with heat generation and absorption. *Canadian Journal of Physics*, 97, 644–650. doi: 10.1139/cjp-2018-0011
- [16] Khan, S., & Sanjayanand, K. (2005). Viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet. *International Journal of Heat and Mass Transfer*, 48, 1534–1542. doi: 10.1016/j.ijheatmasstransfer.2004.10.032
- [17] Sajid, M., & Hayat, T. (2008). Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *International Communications in Heat and Mass Transfer*, 35, 347–356. doi: 10.1016/j.icheatmasstransfer.2007.08.006
- [18] Nadeem, S., Hayat, T., Malik, M.Y., & Rajputa, S.A. (2010). Thermal radiation effects on the flow by an exponentially stretching surface: A series solution. *Zeitschrift für Naturforschung A*, 65a, 495–503. doi: 10.1515/zna-2010-6-703
- [19] Elbashbeshy, E.M.A. (2001). Heat transfer over an exponentially stretching continuous surface with suction. *Archives of Mechanics*, 53(6), 643–651.
- [20] Swapna, D., Govardhan, K., & Narendar, G. (2024). Magneto-micropolar nanofluid flow over a convectively heated sheet with nonlinear radiation, chemical reaction, and viscous dissipation. *Fusion Science and Technology*, 81(2), 132–143. doi: 10.1080/15361055.2024.2339700
- [21] Reddy, S. S., Govardhan, K., & Narendar, G. (2024). Numerical investigation of squeezing flow with mass and heat exchange between parallel plates: A study of velocity, temperature, and concentration distributions. *Radiation Effects and Defects in Solids*. doi: 10.1080/10420150.2024.2369127
- [22] Thirupathi, G., Govardhan, K., & Narendar, G. (2021). Radiative magnetohydrodynamics Casson nanofluid flow and heat and mass transfer past a nonlinear stretching surface. *Beilstein Archives Journal of Advanced Research in Numerical Heat Transfer*, 6(1), 1–21.
- [23] Eid, M.R., & Nafe, M.A. (2020). Thermal conductivity variation and heat generation effects on magneto-hybrid nanofluid flow in a porous medium with slip condition. *Waves in Random and Complex Media*, 32(2), 1–25. doi: 10.1080/17455030.2020.1810365
- [24] Narendar, G., Govardhan, K., & Sreedhar Sarma, G. (2021). MHD Casson nanofluid past a stretching sheet with the effects of viscous dissipation, chemical reaction and heat source/sink. *Journal of Applied and Computational Mechanics*, 7(4), 2040–2048. doi: 10.22055/jacm.2019.14804

- [25] Aly, E.H., & Pop, I. (2020). MHD flow and heat transfer near stagnation point over a stretching/shrinking surface with partial slip and viscous dissipation: Hybrid nanofluid versus nanofluid. *Powder Technology*, 367, 192–205. doi: 10.1016/j.powtec.2020.03.030
- [26] Mathews, J., & Hymavathi, T. (2024). Unsteady magnetohydrodynamic free convection and heat transfer flow of $\text{Al}_2\text{O}_3\text{-Cu/wa-}$ ter nanofluid over a non-linear stretching sheet in a porous medium. *Archives of Thermodynamics*, 45(1), 165–173. doi: 10.24425/ather.2024.150449
- [27] Hayat, T., Rashid, M., & Imtiaz, M. (2017). MHD effects on a thermo-solutal stratified nanofluid flow on an exponentially radiating stretching sheet. *Journal of Applied Mechanics and Technical Physics*, 58, 214–223. doi: 10.1134/S0021894417020043



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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<http://www.imp.gda.pl/archives-of-thermodynamics/>



Experimental and theoretical investigation of thermally insulated roof slabs

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Received: 10.06.2025; revised: 28.07.2025; accepted: 03.08.2025

Abstract

In this analysis, thermal performance of building roof elements subjected to periodic changes in temperature at a roof top was examined with and without insulating materials. The objective of this analysis is to reduce energy consumption and prevent heat flow through roofs by using various insulating materials on the roof slab. It aims to compare the thermal performance of roof slabs with and without insulation under different atmospheric temperature conditions. Further, the study seeks to accurately predict the roof bottom temperature using optimisation techniques in order to minimise experimental cost and time. Nine identical rooms were constructed and eight roofs were retrofitted with different insulating materials such as coconut shell, fly ash, Jhama brick bats, rice husks, glass fibre, rubber sheet, earthen pots and asbestos sheets. The roof of one room was kept untreated and was considered as a reference. The thermal behaviour of every innovative roof slab was analysed. Among the eight insulating materials considered in this study, the performance of the roof slab with fiberglass was found to be the best. Roof slab with fiberglass could provide temperature reduction in the room of about 6.17°C, whereas roof slab with earthen pot could provide temperature reduction in the room of 2.27°C. The time lag between the maximum roof top temperature and maximum room temperature was found to be maximum in the case of roof slab with fiberglass and minimum in the case of roof slab with earthen pot. The time lag between the maximum roof top temperature and maximum room temperature is 4 hours for roof slab with fiberglass. The variation of roof bottom temperature was predicted by five optimisation techniques such as cuckoo search, bacterial colony optimisation, group search optimisation, social spider optimisation and genetic algorithm. It was observed that the social spider optimisation technique predicted the temperature accurately and the error between the measured and predicted temperature values was minimum compared to the other optimisation techniques.

Keywords: Thermal insulation; Insulating materials; Thermal performance; Optimisation techniques; Time lag.

Vol. 46(2025), No. 4, 187–199; doi: 10.24425/ather.2025.156849

Cite this manuscript as: Dixit, A.K., Roul, M.K., Panda, B.C., Roul, P.P., Sahu, S., Roul, P.D., & Sahu, S.B.B.P.J. (2025). Experimental and theoretical investigation of thermally insulated roof slabs. *Archives of Thermodynamics*, 46(4), 187–199.

1. Introduction

Construction of buildings is one of the oldest established activities since the creation of human civilisation [1]. Man has transformed a lot in the construction of houses, right from caves to huts and

from huts to reinforced cement concrete (RCC) structures. RCC slabs are commonly used in roofs, walls and floors of buildings and bridges. The temperature inside houses with RCC roof slabs is required to be maintained at the desired level for the comfort of occupants. The application of insulating materials has increa-

Nomenclature

A – surface area, m^2
 d – thickness of test specimen, m
 F – inverse of calorimeter heat transfer coefficient, $1/N$
 N – heat transfer coefficient of reference calorimeter
 Q – heat flux, W/m^2
 R_s – thermal resistance of the test specimen, $m^2 K/W$
 R_{int} – interface thermal resistance, $m^2 K/W$
 T_u – temperature of upper plate, K or $^{\circ}C$
 T_m – temperature of the middle plate, K or $^{\circ}C$
 T_l – temperature of bottom heater plate, K or $^{\circ}C$
 U – overall heat transfer coefficient, $W/(m^2 K)$

Greek symbols

λ – thermal conductivity, $W/(m \cdot K)$
 ΔT_s – temperature difference across test sample, K
 ΔT_r – temperature difference across reference calorimeter, K

Subscripts and Superscripts

int – interface
 l – lower or bottom heater plate
 m – middle or test sample interface
 r – reference (calorimeter or temperature difference)
 s – sample
 u – upper plate

Abbreviations and Acronyms

BCO – bacterial colony optimisation
CS – cuckoo search
GA – genetic algorithm
GSO – group search optimisation
HVAC – heating, ventilation and air conditioning
IR – infrared
RCC – reinforced cement concrete
SSO – social spider optimisation

sed appreciably in recent years. One or more of the insulating materials may be provided in the form of a ceiling board in order to maintain thermal comfort inside the room [2].

Rock may be used as material for building walls; wood may be employed to provide roofing support; a ceiling may be provided to reduce temperature inside buildings [3]. It has been a matter of concern for the engineers to design a building that can provide thermal comfort for the occupants [4]. There exist design and structural differences between the developments in the ground-level landscape and rooftop [5]. Lundholm et al. [6] observed that buildings with green roofs use less energy in the summer. They measured the heat flux for extensive green roof systems and determined their dependence on solar radiation, temperature of the substrate and depth of the snow cover. They also verified the connections between vegetation type, snow accumulation and substrate temperature. Considering the rooftop developments, thermal expansion may be caused due to absorption of heat energy in the roof slabs that may lead to cracks in the load-bearing masonry materials [7]. Therefore, a suitable insulation system is required in the roof slabs to overcome this problem [8]. Insulation is very much essential in buildings to shield the building from the inflow of heat from outside during summer and outflow of heat from the building during winter and to attain thermal comfort for its dwellers during warm summers and cold winters [9,10]. Björk and Enochsson [11] investigated the performance and durability of thermal insulation materials under extreme environmental conditions, highlighting their impact on long-term thermal efficiency in building applications.

Gagliano et al. [12] explored the effectiveness of ventilated roofs portrayed by different insulation layers with respect to the air gap. Their results show that the aeration through roofs can decrease heat fluxes considerably in the summer season. Halwatura and Jayasinghe [13] proposed different insulation systems for roof slabs and studied experimentally their performances using small and large-scale models. Jaffal et al. [14] provided a detailed investigation of the impact of a green roof on indoor temperature. With a green roof, the temperature inside the building could be lowered by $2^{\circ}C$ in summer, with a significant decrease

in the annual requirement of energy. Dong et al. [15] examined the consequence of various factors, such as phase change material thickness, latent heat, radiation intensity, roof slope, absorption coefficient, etc., on the temperature delay, thermal comfort and energy consumption. Dixit et al. [16] used different optimisation techniques, such as bacterial colony optimisation (BCO), cuckoo search (CS) and group search optimisation (GSO) to determine the most favourable weight of the structure.

Numerous studies have been carried out on natural convection heat transfer, providing the scientific basis for experimental and theoretical investigation of thermally insulated roof slabs. Roul and Nayak [19] investigated heat transfer within vertical tubes. Nayak et al. [20–22] presented the enhancement of heat transfer in vertical tube due to internal obstacles. In their work, they considered rings of rectangular cross section as internal obstacles. Sahoo et al. [23–26] employed computational fluid dynamics techniques to analyse heat transfer from pin finned horizontal and vertical plates. Roul and Sahoo [27] investigated the pressure drop through pipes with variable cross sectional area, which is necessary for estimating the power required to pump the fluid through the pipe to create forced draft. Himrane et al. [28] investigated thermal comfort by numerical simulation of natural convection in an enclosed cavity, based on the lattice Boltzmann method, analysing the effects of Rayleigh number (Ra) and oscillation amplitude on temperature distribution and flow behaviour. Their study provides valuable insights into air-flow patterns, thermal stratification, and heat transfer characteristics under varying thermal and dynamic conditions. Belenguer et al. [29] experimentally investigated the efficiency of different methods used for reducing energy requirements in homes and achieved a reduction in energy consumption up to 14.6%. Martínez et al. [30] generated computational models to envisage the effectiveness of external walls to provide thermal comfort and validated their model experimentally using waste insulating materials. Chandra et al. [31] introduced an innovative approach to insulate the roof by employing bamboo strips and found the most favourable thickness of insulation layer, which can reduce the maximum temperature at the peak hour with the highest de-

lay in time. Koru [32] investigated some insulation materials and found that the thermal conductivity depends mostly upon density, moisture content, size and configuration of pores and environmental conditions. Alyami [33] employed a simulation tool to determine various climatic zones and observed that climatic conditions affect the performance of buildings as regards energy efficiency. Kumar et al. [34] estimated the economic viability and environmental affability of the thickness of insulation used for ducts in heating, ventilation and air conditioning (HVAC) systems to minimise the energy loss, taking into account different insulating materials. Alnahhal et al. [35] studied both computationally and analytically the insulation in walls and roofs with air cavities for variable aspect ratios and found the optimum aspect ratio for providing thermal insulation to minimise energy consumption in buildings. Alhefnawi and Abdu-Allah Al-Qahtany [36] observed that the geometry, orientation, material and type of construction of buildings affect the heat transfer to the buildings through the walls and roofs. They also examined the effectiveness of an air gap as a thermal insulator to reduce the energy requirement for air conditioning purposes. Sassine et al. [37] used hollow blocks with various void ratios so as to augment the thermal resistance of the wall and decrease the heat transfer to the room.

Thermal insulation of roof slabs plays an important role in improving the comfort of inhabitants by dropping the indoor air temperature. Heat transfer through the roof decreases significantly by providing thermal insulation under or on the roof slab. In the present analysis, the thermal performance of building roof elements subjected to periodic changes in temperature at the roof top was examined, with and without insulating materials. Nine identical rooms were constructed and eight roofs were outfitted with different insulating materials, such as coconut shell, fly ash, Jhama brick bats, rice husks, fibreglass, rubber sheet with aeration, earthen pots and asbestos chips. The roof of one room was kept untreated (no insulation) and was considered a reference. The thermal behaviour of all innovative roof slabs was analysed. For each room, the experiment examines the room temperature with an insulating material and without an insulating material, along with the roof bottom temperature and roof top temperature, atmospheric temperature and temperature inside the room.

The investigation implies that the roof model with insulating material achieves better performance than the one without insulating material. Thermal insulation reduces the heat flow through the roof, as a result of which the energy requirement of the building can be reduced and the comfort of the occupants can be improved. The variation of the roof bottom temperature was predicted by five optimisation techniques, such as bacterial colony optimisation (BCO), cuckoo search (CS), group search optimisation (GSO), genetic algorithm (GA) and social spider optimisation (SSO). To minimise experimental cost and time, these optimisation techniques can be used to predict the roof bottom temperature when various insulating materials are employed at the top of the roof. The various inputs considered for the theoretical analysis are wall area, thickness, testing temperature and time. The methodology is prepared by taking 80% of the dataset for the training function and utilising the remaining

20% of the dataset to validate the model. In the proposed numerical modelling, the objective function is formulated in agreement with the measured output parameters. The mathematical modelling with optimisation predicts results with minimum error by incorporating the optimal weights. We have effectively employed a number of optimisation techniques to establish the optimal weight of the system and to predict the roof bottom temperature. These mathematical models are found to be time and cost-effective. The MATLAB 2018 software was utilised to implement the complete procedure of different optimisation methods. A total of 285 data were used, out of which 228 were training data.

Objectives and scope of this analysis is to (i) reduce the energy consumption and prevent heat flow through the roofs by using various insulating materials on roof slab, (ii) compare the roof slab with and without insulating materials and determine the thermal performance of roof elements under various atmospheric temperature conditions, and (iii) accurately predict the roof bottom temperature by using different optimisation techniques in order to minimise cost and duration of the experiment.

In contrast to previous studies that examined the effects of singular or limited insulating materials or focused solely on simulation, the present work provides a comprehensive experimental and theoretical investigation of eight different insulating materials, many of which are locally available and environmentally sustainable, installed under real environmental conditions. The study uniquely combines experimental data with advanced optimisation algorithms (BCO, CS, GSO, GA and SSO) to predict roof bottom temperatures with high accuracy. The use of social spider optimisation as a predictive model, in particular, has not been widely reported in this context and demonstrates superior accuracy over traditional methods. This dual approach not only helps in identifying the most effective insulating materials for energy-efficient construction but also presents a cost and time-effective methodology for temperature prediction in thermally insulated roof systems. Therefore, this research significantly contributes to the design of thermally efficient buildings, particularly in hot and humid climates, and fills a gap in the literature by offering both performance comparison and predictive modelling of various insulation strategies.

2. Experimental investigations

Nine identical rooms with internal dimensions of $1\text{ m} \times 1\text{ m} \times 1\text{ m}$ were constructed.

The selection of insulating materials in this study was motivated by a combination of factors, including thermal properties, environmental sustainability, cost-effectiveness and local availability. Each material was chosen based on one or more of the following considerations. Coconut shells, rice husks, fly ash, Jhama brick bats and earthen pots are locally available agricultural or industrial by-products, often considered waste materials. Their reuse aligns with sustainable construction practices and helps reduce environmental burdens by promoting waste valorisation. These materials are economically viable, making them suitable for low-cost construction in rural and semi-urban areas. They are widely accessible and affordable, allowing for scalable application in developing regions. Materials like earthen pots

and rice husks have inherent thermal insulation properties due to their porous structure and low thermal conductivity. Fly ash and Jhama brick bats are commonly used in construction fill or waste management and have shown potential in improving thermal mass, thereby contributing to passive cooling when integrated into roofing systems. Glass fibre was selected as a benchmark commercial insulating material due to its established performance in modern construction. Including it allowed for comparison between conventional and unconventional (eco-friendly) alternatives. Rubber sheets and asbestos sheets were selected for their known insulation characteristics and availability as roofing components in various commercial and industrial applications.

By incorporating a diverse range of materials, from natural and agricultural waste to industrial by-products and commercial insulators, the study aims to provide a holistic evaluation of thermal performance and support the selection of sustainable, cost-effective roofing solutions across varied socioeconomic and climatic contexts.

Roof top surface temperatures were measured using a Fluke 62 Max infrared thermometer (which offers single laser targeting for accurate and repeatable measurements) at five different points on the exterior surface of the slab. The average of these readings was used to determine the rooftop temperature. The accuracy of measurement lies within $\pm 1.5\%$ error, which can be considered highly accurate. Similarly, the roof bottom surface temperature was recorded at the central point of the interior slab surface. Internal air temperature was estimated by holding a matte paper target at multiple positions (centre and corners) inside the test room, allowing thermal equilibrium with the surrounding air and recording its surface temperature using the infrared (IR) thermometer.

The average of these measurements was considered the room air temperature. This method was suitable due to the small volume (1 m^3) and minimal air stratification within the space. The various types of roof slabs used in this experiment are given in Table 1.

Figure 1 shows a roof slab with dry coconut shell. Above the 100 mm RCC slab, chiselled 25 mm inverted dry coconut shells were placed. The gaps were filled with asbestos paste to hold it properly. A weatherproof coarse ferrocement slab of 12.5 mm thickness, comprised of steel mesh of 10 mm square size (3 numbers) cast with rich cement mortar (1:2), was placed above the coconut shells.



Fig. 1. Coconut shell roof slab model.

Table 1. Roof slab types and insulating materials.

No.	Roof slab type	Insulating material	λ (W/m K)	U (W/m ² K)	Layers
1	Coconut shell	Dry coconut shell	0.164	3.62	100 mm RCC slab; 25 mm inverted dry coconut shell; 12.5 mm ferrocement slab
2	Glass fiber	Fiberglass	0.050	1.60	100 mm RCC slab; 25 mm glass fibre; 12.5 mm ferrocement slab
3	Earthen pots	Earthen pots	0.380	5.28	100 mm RCC slab; 25 mm inverted dry earthen pots; 12.5 mm ferrocement slab
4	Rice husk	Rice husks	0.0727	2.14	100 mm RCC slab; 25 mm rice husk; 12.5 mm ferrocement slab
5	Rubber sheet	Rubber sheet	0.200	2.66	100 mm RCC slab, 17 mm air gap; 8 mm thick rubber sheet; 12.5 mm ferrocement slab
6	Asbestos sheets	Asbestos sheets	0.147	3.40	100 mm RCC slab; 25 mm asbestos sheets; 12.5 mm ferrocement slab
7	Jhama bricks	Jhama brick bats	0.103	2.73	100 mm RCC slab; 25 mm Jhama brick bats; 12.5 mm ferrocement slab
8	Fly ash	Fly ash	0.0856	2.41	100 mm RCC slab; 25 mm fly ash; 12.5 mm ferrocement slab

The trapped air inside the inverted coconut shell acts as a very good insulator. A ferrocement slab is used above the dry inverted coconut shell to make the roof strong and safe. Similarly, the roof slab with other insulating materials is shown in Figs. 2 to 8.

The thermal conductivity of different insulating materials was measured according to ASTM E1530, using the guarded heat flow meter technique (Figs. 9 and 10). The sample of the insulating material was prepared and placed between two surfaces, which are maintained at different temperatures. The schematic diagram of the setup for measuring the thermal conductivity of a test sample is shown in Fig. 9. Fourier's law of heat con-



Fig. 2. Glass fibre roof slab model.



Fig. 3. Earthen pot roof slab model.



Fig. 4. Rice husk roof slab model.



Fig. 5. Rubber sheet roof slab.

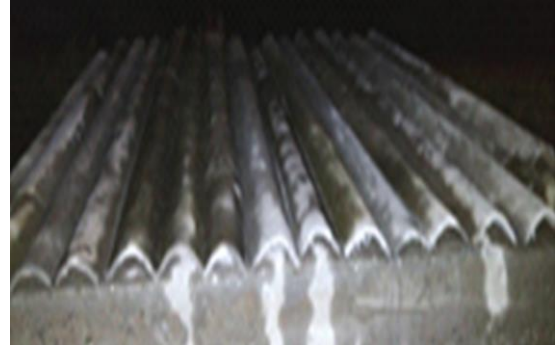


Fig. 6. Roof slab with asbestos sheet.



Fig. 7. Roof slab with Jhama brick bats.



Fig. 8. Fly ash roof slab.

duction was used to determine the thermal conductivity of the test specimen by measuring the temperature difference and the output of the heat flux transducer. From the Fourier law of heat transfer, we have

$$R_s = \frac{T_u - T_m}{Q} - R_{int}, \quad (1)$$

where: R_s – thermal resistance of the test specimen, T_u – temperature of the upper plate, T_m – temperature of the lower plate, Q – heat flux, R_{int} – total interface resistance.

The thermal resistance of the test specimen is given by

$$R_s = \frac{d}{\lambda}, \quad (2)$$

where: d – thickness of the test specimen, λ – thermal conductivity.

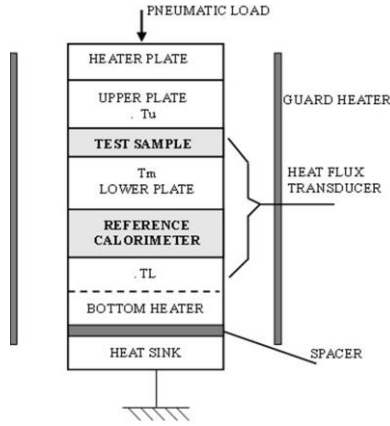


Fig. 9. Diagram of the test section for measuring thermal conductivity.



Fig. 10. Measurement of thermal conductivity according to ASTM E1530. Image courtesy: CIPET, Bhubaneswar.

The interface thermal resistance is considered in Eq. (1) because the temperature difference is measured between the upper and lower surface plates.

The heat flux is measured by calculating the temperature across the reference calorimeter:

$$Q = N(T_m - T_l), \quad (3)$$

where: N – heat transfer coefficient of the reference calorimeter, T_m – surface temperature of the lower plate, T_l – temperature of the bottom heater.

Combining Eqs. (1) and (3), we can find the thermal resistance of the test sample:

$$R_s = F \frac{T_u - T_m}{T_m - T_l} - R_{int} \quad (4)$$

or

$$R_s = F \frac{\Delta T_s}{\Delta T_r} - R_{int},$$

where: $F = 1/N$, ΔT_s – temperature difference across the sample, ΔT_r – temperature difference across the reference calorimeter.

Under ideal conditions, at steady state, the factor F is inversely proportional to the heat transfer coefficient (N) of the

reference calorimeter. Such conditions are closely approximated in the method because a guard heater surrounds the test stack. The thermal conductivity measurement device (Model 2022) is calibrated to find F and R_{int} . From Eq. (4), it yields that there is a straight-line relationship between variables R_s and $\Delta T_s/\Delta T_r$. Temperatures are measured to determine the ΔT ratio for several samples, whose thermal resistances are known. These are plotted on a graph to get a straight line, as shown in Fig. 11. The slope of this line gives F , and the ordinate axis intercept is R_{int} . Points A through D represent various calibration samples. Once the relationship between R_s and the ΔT ratio is known, following the equipment calibration, the thermal resistance of any unknown sample can be readily obtained after measuring the temperatures during the test and then calculating the ΔT ratio, as shown by the dotted lines in Fig. 11. For the test result to be accurate, the thermal resistance thus obtained must fall within the range of the samples used during calibration. Once the thermal resistance has been determined, thermal conductivity can be computed from Eq. (2).

3. Results and discussion

This study is fully based on real-time investigation under open environmental conditions. The performance of nine roof slabs (eight with insulation and one without insulation) is calculated based on atmospheric temperatures at various time periods. Correspondingly, the sample thermal conductivity, sample thermal resistivity and the mean sample temperature are computed for

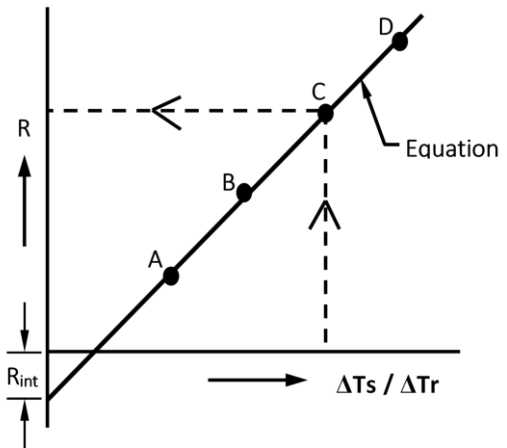


Fig. 11. Equipment calibration and data analysis.

roof sections with and without insulating materials.

The data taken from the real-time analysis were implemented in the MATLAB software.

For each case studied, the temperature between 7.30 am and 10.00 pm was analysed. The average temperature at the roof top, roof bottom and in the room was measured for the roof slab with and without insulating materials to eventually demonstrate the effect of insulating material on the room temperature.

3.1. Performance characteristics of roof slab with inverted coconut shell and the global optimisation techniques

Figure 12 illustrates the thermal performance of the roof slab with inverted coconut shells as insulating material. At 2.30 pm, the temperature of the rooftop reached the peak value, and thereafter the temperature gradually decreased. It is evident from the graph that the temperature of the rooftop with insulation attains the maximum value of 59.45°C at 2.30 pm, whereas the temperature of the roof bottom attains the maximum value of 38.76°C at 4.30 pm. The temperature of the room without insulation attains a maximum value of 41.52°C at 4.00 pm, whereas the temperature of the room with insulation attains a maximum value of 38.58°C at 5.00 pm. Thus, there is a decrease in temperature of the room for the roof slab with insulation by 2.94°C with a time lag of 1.00 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 2.30 h.

A comparison of actual values of the roof bottom temperature with those predicted by five optimisation techniques considered in this study is shown in Table 2. The data clearly indicates that the social spider optimisation (SSO) technique provides the most accurate prediction of temperature values, with the error between the measured and predicted values ranging from 0.12 to 0.91. In comparison, the cuckoo search (CS) optimisation technique yields an error range of 1.38 to 7.92, bacterial colony optimisation (BCO) shows an error between 0.69 and 4.62, group search optimisation (GSO) results in an error of 0.21 to 3.67, and the genetic algorithm (GA) gives an error range of 2.11 to 3.32. Among all these techniques, SSO demonstrates the minimum error, highlighting its superior performance and accuracy in temperature predictions.

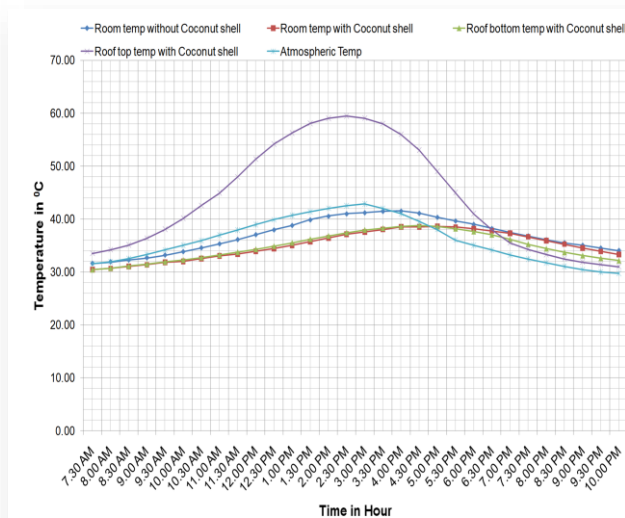


Fig. 12. Real-time performance of the roof slab with coconut shell.

Table 2. Performance characteristics of the roof slab with coconut shell.

Input		Output (temperature, °C)						
Area, m ²	Thickness, m	Testing temperature (roof top)	Actual value (roof bottom)	Predicted value				
				SSO	GA	GSO	CS	BCO
1	0.1397	33.50	30.45	30.57	32.56	30.67	31.83	31.14
		34.20	30.75	30.91	33.35	30.99	32.2	31.44
		35.10	31.11	31.29	34.02	31.32	33.35	31.83
		36.36	31.49	31.61	34.65	31.72	33.75	32.26
		38.05	31.92	32.13	35.11	32.2	34.26	32.79
		40.15	32.27	32.52	35.49	32.58	34.81	33.16
		42.52	32.74	33.02	36.03	33.09	35.98	33.92
		44.92	33.22	33.54	36.54	33.63	37.06	34.51
		47.92	33.73	34.1	36.83	34.54	37.87	35.59
		51.32	34.3	34.71	37.28	35.71	38.64	36.19
		54.15	34.87	35.34	37.59	35.99	39.63	36.97
		56.24	35.5	36.03	38.14	36.62	40.42	37.66
		58.05	36.19	36.77	38.81	38.31	41.32	38.38
		58.98	36.8	37.4	39.39	38.99	42.07	39.03
		59.45	37.4	37.52	39.94	40.52	42.78	39.66
		59.05	37.92	38.55	40.43	41.16	43.71	41.06
		58.00	38.25	38.94	40.72	41.92	44.19	41.54
		56.00	38.58	39.29	41.02	41.76	44.72	40.77
		53.00	38.76	39.5	41.17	41.94	45.43	42.94
		49.00	38.55	39.33	40.94	41.53	45.67	42.84
		45.00	38.15	39.06	40.47	40.99	45.52	42.77
		41.00	37.6	38.38	39.9	40.78	45.24	40.86
		38.00	37	37.74	39.26	39.18	44.91	40.24
		35.50	36.15	36.86	38.38	38.27	44.09	39.42
		34.30	35.25	35.93	37.35	37.33	36.67	37.48
		33.30	34.4	35.01	36.79	35.58	41.93	36.59
		32.50	33.7	34.27	36.02	34.84	39.84	35.81
		31.90	33.1	33.64	35.4	34.18	39.1	35.08
		31.40	32.6	33.11	34.86	33.59	37.94	33.87
		31.00	32.15	32.55	34.38	32.83	36.53	32.92

3.2. Performance characteristics of roof slab with fibreglass and the global optimisation techniques

Figure 13 visualises the real-time thermal performance of the roof slab with glass fibre. It can be observed from the figure that the temperature of the roof bottom with insulation attains the maximum value of 35.58°C at 6.00 pm, whereas the temperature of the room attains a maximum value of 35.35°C at 6.30 pm.

Thus, compared to the non-insulated roof slab case (peak room temperature of 41.52°C at 4.00 pm), there is a decrease in temperature of the room for the roof slab with insulation by

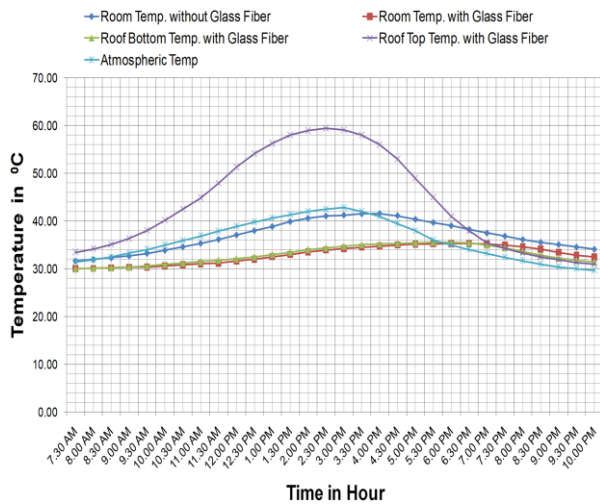


Fig. 13. Real-time performance of the roof slab – fiberglass.

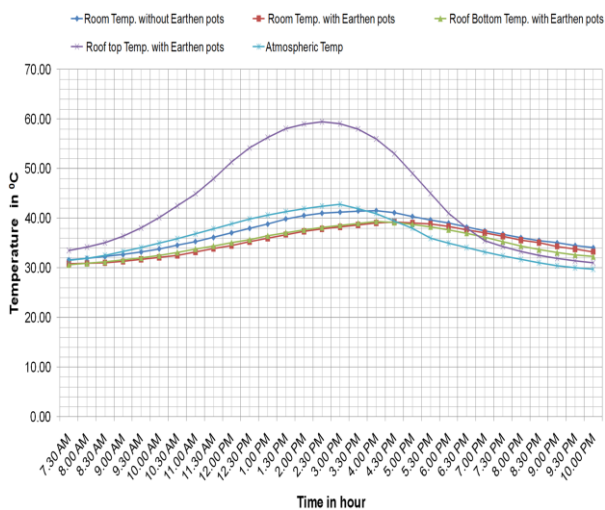


Fig. 14. Real-time performance of the roof slab with earthen pots.

6.17°C with a time lag of 2.30 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 4.00 h. Since the coefficient of emissivity of the fiberglass varies between 0.85 to 0.95 and the thermal conductivity is 0.05 W/(m K), the temperature of the room is reduced by the maximum amount.

Similarly, among the optimisation techniques used, in this case, SSO was found to demonstrate the highest accuracy, with an error range of 0.08–0.99 between measured and predicted temperatures. GSO also gave good predictions, with an error range of 0.16–4.32, followed by BCO (0.97–7.07), GA (2.10–3.30) and CS (1.34–9.34). These results show that the SSO method provides the most reliable predictions, significantly reducing the error margin compared to other optimisation methods. It was observed that the SSO technique predicts the temperature accurately, and the error between the measured and predicted value of temperature is minimum.

The fiberglass layer, encapsulated beneath a 12.5 mm thick ferrocement slab, reduces indoor temperature primarily due to

its low thermal conductivity (0.05 W/(m·K)), which significantly impedes heat transfer through the roof. The emissivity of fibreglass does not directly affect the heat exchange process in this configuration, as it is not exposed to external radiation.

3.3. Performance characteristics of the roof slab with earthen pots and the global optimisation techniques

Figure 14 depicts the real-time thermal performance of a roof slab with earthen pots. While the temperature of the rooftop with insulation attains the maximum value of 59.45°C at 2.30 pm, the temperature of the roof bottom in this case attains the maximum value of 39.35°C at 4.00 pm, and the temperature of the room reaches a maximum value of 39.25°C at 4.30 pm. Thus, there is a decrease in temperature of the room for the roof slab with insulation by 2.27°C compared to a roof without insulation, with a time lag of 0.30 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 2.00 h. The roof bottom temperature was predicted using five optimisation techniques, such as cuckoo search, bacterial colony optimisation, group search optimisation, genetic algorithm, and social spider optimisation. The error range between the measured and predicted values for various optimisation methods is: CS (1.37–7.99), BCO (0.67–4.54), GSO (0.16–3.33), GA (2.10–3.30), and SSO (0.12–0.82). Among these, the SSO technique exhibited the highest accuracy, with the minimum error range, indicating its superior performance in precisely envisaging the roof bottom temperature.

3.4. Performance characteristics of roof slab with rice husk and the global optimisation techniques

The real-time performance of the roof slab with rice husk is displayed in Fig. 15. The results reveal that the temperature of the roof bottom with insulation attains the maximum value of 36.88°C at 5.30 pm, whereas the temperature of the room attains a maximum value of 36.70°C at 6.00 pm. A decrease in

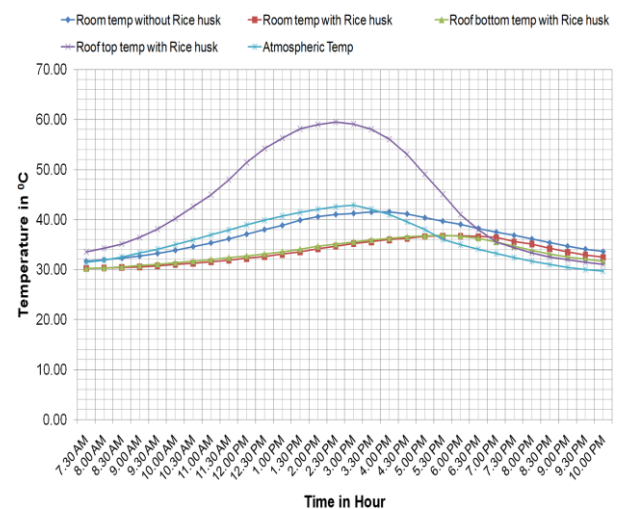


Fig. 15. Real-time performance of the roof slab with rice husk.

the room temperature for the roof slab with insulation compared to a roof without insulation is 4.82°C with a time lag of 2.00 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 3.30 h.

The error ranges for different optimisation methods were CS (1.34–8.14), BCO (0.67–4.54), GSO (0.16–3.30), GA (2.10–3.30) and SSO (0.12–0.82). This shows that, also in this case, SSO exhibited the highest accuracy with the lowest prediction error.

3.5. Performance characteristics of roof slab with rubber sheets and the global optimisation techniques

Figure 16 demonstrates the real-time thermal performance of the roof slab with rubber sheets as an insulating material.

The results show that the temperature of the roof bottom with this insulation type reaches the maximum value of 39.18°C at 4.00 pm, while the maximum room temperature of 39.05°C is reached at 4.30 pm. Considering that the room temperature without insulation attains a maximum value of 41.52°C at 4.00 pm, there is a decrease in temperature of the room for the roof slab with insulation by 2.47°C with a time lag of 0.30 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 2.00 h.

Temperature prediction errors using five optimisation methods were as follows: CS (1.46–7.91), BCO (0.32–2.11), GSO (0.67–4.29), GA (2.10–3.30) and SSO (0.12–0.78). Among all these methods, SSO demonstrated the highest accuracy with the lowest error, indicating its superior performance compared to CS, BCO, GSO and GA.

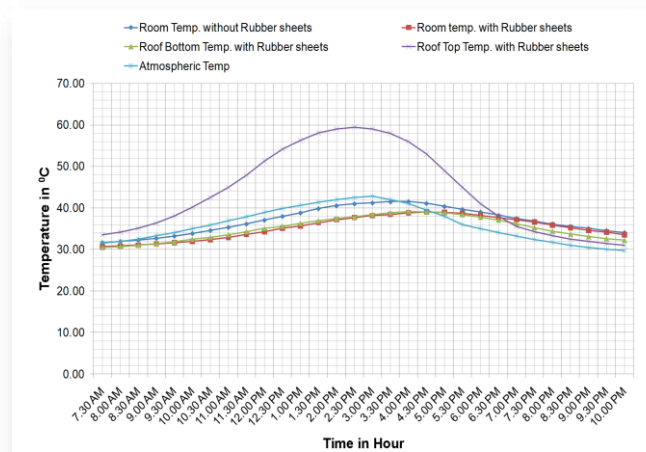


Fig. 16. Real-time performance of the roof slab – rubber sheets.

3.6. Performance characteristics of roof slab with asbestos sheet and the global optimisation techniques

Figure 17 shows the real-time thermal performance of the roof slab with an asbestos sheet. As can be seen from the figure, the temperature of the roof bottom attains a maximum of 38.33°C at 5.00 pm.

A comparison of the temperature in a room for the case of roof without insulation (a peak value of 41.52°C at 4.00 pm)

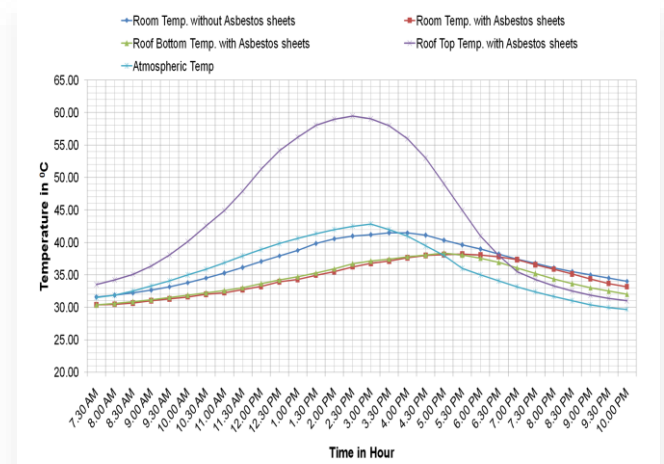


Fig. 17. Real-time performance of the roof slab with asbestos sheet.

with the one for the insulated roof case that reaches a peak value of 38.22°C at 5.30 pm, indicates that there is a decrease in temperature of the room for the roof slab with insulation by 3.30°C with a time lag of 1.30 h. The time lag between the maximum rooftop temperature and the maximum room temperature with insulation is 3.00 h.

The error ranges between measured and predicted roof bottom temperatures for different optimisation methods were as follows: 1.37–8.14 for the cuckoo search, 0.67–4.54 for the bacterial colony optimisation, 2.10–3.30 for the genetic algorithm, 0.16–3.30 for the group search optimisation and 0.12–0.82 for the social spider optimisation. Out of these five optimisation techniques, SSO was found to be the most effective technique with the highest prediction accuracy.

3.7. Performance characteristics of roof slab with Jhama brick bats and the global optimisation techniques

The real-time thermal performance of the roof slab with Jhama brick bats as insulating material is presented in Fig. 18. The measured average temperatures at the roof top, roof bottom and in the room for the insulated roof slab were compared with the corresponding temperatures for the roof slab without insulation. In this case, the maximum temperature of the roof bottom with insulation, amounting to 38.21°C , is achieved at 5.00 pm, whereas the temperature of the room attains a maximum value of 38.05°C at 5.30 pm. Since the room temperature in the case of a non-insulated roof slab reaches a maximum value of 41.52°C at 4.00 pm, it follows that there is a decrease in temperature of the room for the roof slab with insulation by 3.47°C with a time lag of 1.30 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 3.00 h.

The prediction errors between measured and estimated roof bottom temperatures using various optimisation techniques were as follows: 1.34–8.14 for the cuckoo search technique, 0.67–4.54 for the bacterial colony optimisation, 2.10–3.30 for the genetic algorithm, 0.16–3.30 for the group search optimisation and 0.12–0.79 for the social spider optimisation. Similarly, it is concluded that SSO demonstrated the highest accuracy, characterised by the lowest error range. A comparative analysis

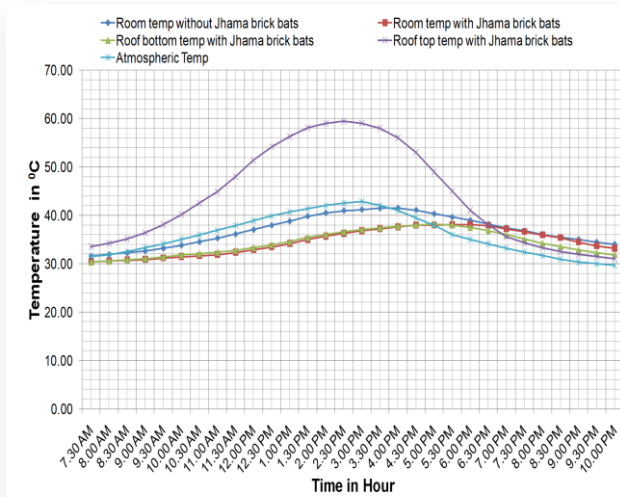


Fig. 18. Real-time performance of the roof slab – Jhama brick bats.

confirms that SSO outperforms the other four techniques in predicting temperature with greater accuracy.

3.8. Performance characteristics of roof slab with fly ash and the global optimisation techniques

Figure 19 demonstrates the real-time thermal performance of the roof slab with fly ash as an insulating material. While the temperature of the rooftop with insulation attains its maximum of 59.45°C at 2:30 pm, the peak temperature of the roof bottom in this case is 37.20°C and is reached at 5:30 pm. The room temperature reaches a maximum value of 37.08°C at 6:00 pm, which is a decrease of 4.44°C compared to the non-insulated roof case, with a time lag of 2.00 h. The time lag between the maximum rooftop temperature and the maximum room temperature is 3.30 h.

The error ranges between the measured and predicted roof bottom temperatures using various optimisation methods were as follows: 1.34–8.14 for the cuckoo search method, 0.67–4.54 for the bacterial colony optimisation, 2.10–3.30 for the genetic algorithm, 0.16–3.30 for the group search optimisation and 0.12–0.82 for the social spider optimisation. Among all the

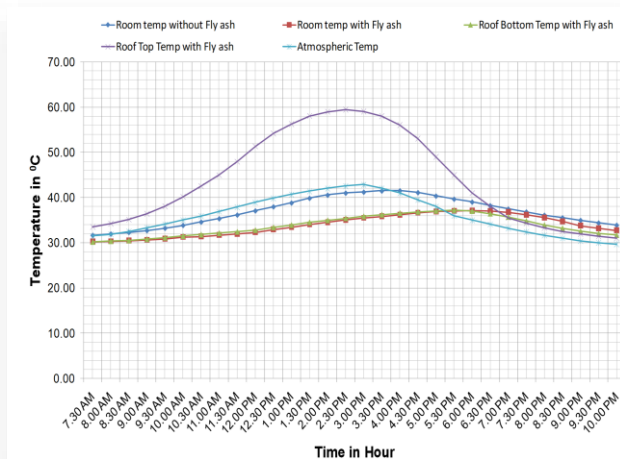


Fig. 19. Real-time performance of the roof slab – fly ash.

methods considered, SSO exhibited the highest accuracy, yielding the lowest prediction error, which confirms that SSO outperforms the other four algorithms in temperature prediction accuracy.

3.9. Performance characteristics of roof slab without insulating material and the global optimisation techniques

Figure 20 demonstrates the real-time thermal performance of the roof slab without insulation, including recorded average temperatures at the roof top, roof bottom and inside the room. The rooftop temperature reached its peak (59.45°C) at 2:30 pm and thereafter gradually decreased, whereas the temperature of the roof bottom attained the maximum value of 41.75°C at 3:30 pm. The maximum temperature in the room was 41.52°C and was obtained at 4:00 pm. The time lag between the maximum rooftop temperature and the maximum room temperature is 1.30 h.

Similarly, the variation in roof bottom temperature was predicted using five optimisation techniques – bacterial colony optimisation (BCO), social spider optimisation (SSO), group search optimisation (GSO), cuckoo search (GS) and genetic algorithm (GA), and the results were compared with actual values. The error ranges for different optimisation methods were: CS (1.37–7.95), BCO (0.69–4.36), GA (2.18–3.29), GSO (0.19–3.28) and SSO (0.12–0.96).

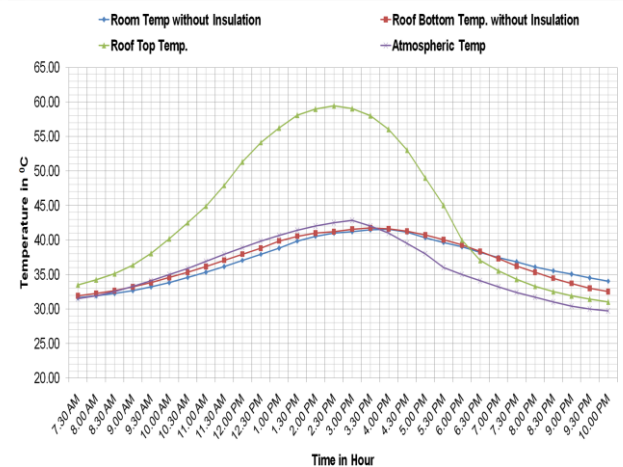


Fig. 20. Real-time performance of the roof slab without insulating material.

Out of five optimisation techniques considered, SSO was found to exhibit the highest accuracy with the lowest prediction error, thus confirming its superior performance compared to the other four methods. The effect of all the insulating materials, considered in this study, on the room temperature is plotted in Fig. 21. The performance of roof slabs with various insulating materials is summarised in Table 3. The time lag between the maximum room temperature with and without insulation is found to be maximum in case of glass fibre and minimum in the case of earthen pots. Similarly, the maximum reduction in room temperature with roof slab insulation was found to be 6.17°C in the case of glass fibre and 2.27°C in the case of earthen pots.

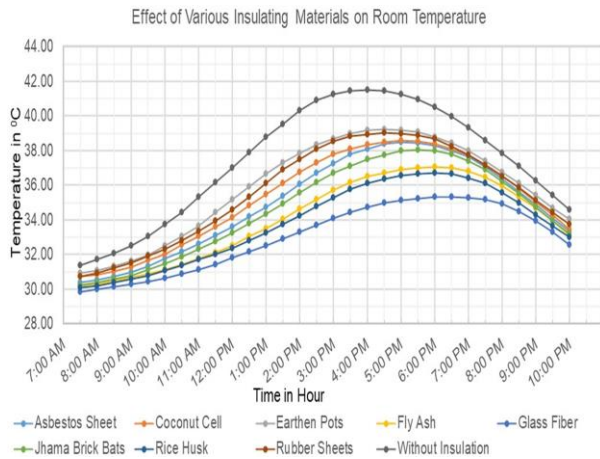


Fig. 21. Effect of various insulating materials on the room temperature.

Table 3. Comparison of the roof slab thermal performance with and without different insulation materials.

No.	Insulation material	Roof top temp., °C / time	Room temp. with insulation, °C / time	Room temp. without insulation, °C / time	Temp. reduction in the room, °C	Time lag (room vs. no insulation)	Time lag, h (roof top vs. room)
1	Inverted coconut shell	59.45 / 2:30 pm	38.58 / 5:00 pm	41.52 / 4:00 pm	2.94	1.00	2.30
2	Glass fibre	59.45 / 2:30 pm	35.35 / 6:30 pm	41.52 / 4:00 pm	6.17	2.30	4.00
3	Earthen pots	59.45 / 2:30 pm	39.25 / 4:30 pm	41.52 / 4:00 pm	2.27	0.30	2.00
4	Rice husk	59.45 / 2:30 pm	36.70 / 6:00 pm	41.52 / 4:00 pm	4.82	2.00	3.30
5	Rubber sheets	59.45 / 2:30 pm	39.05 / 4:30 pm	41.52 / 4:00 pm	2.47	0.30	2.00
6	Asbestos sheet	59.45 / 2:30 pm	38.22 / 5:30 pm	41.52 / 4:00 pm	3.30	1.30	3.00
7	Jhama brick bats	59.45 / 2:30 pm	38.05 / 5:30 pm	41.52 / 4:00 pm	3.47	1.30	3.00
8	Fly ash	59.45 / 2:30 pm	37.08 / 6:00 pm	41.52 / 4:00 pm	4.44	2.00	3.30
9	No insulation	59.45 / 2:30 pm	—	41.52 / 4:00 pm	—	—	1.30

The experimental analysis clearly demonstrates that the inclusion of insulating materials on roof slabs leads to a significant reduction in indoor temperature and delays the thermal response of the indoor environment. The most striking finding is the superior performance of fibreglass, which not only achieved the highest temperature reduction (6.17°C) but also the longest time lag (4 h). This is attributable to its low thermal conductivity 0.05 W/(m·K), which significantly impedes heat transfer through the roof.

In contrast, natural and waste-based materials such as coconut shells, rice husks and Jhama brick bats also demonstrated appreciable performance, with temperature reductions ranging

between 2.94°C and 4.82°C. These materials are highly porous, contain trapped air pockets and have relatively low thermal conductivities, making them effective as low-cost, sustainable insulators.

The time lag is a particularly important metric from a comfort and energy efficiency standpoint. For example, a delay of 2–4 hours in peak room temperature relative to peak rooftop temperature helps reduce the HVAC system loads during the hottest part of the day. This could translate to significant energy savings, particularly in tropical and subtropical regions.

Materials like earthen pots and rubber sheets, though less effective than fibreglass or rice husk, still provided substantial benefits. The relatively lower performance may be due to factors like higher density (in the case of earthen pots) or limited thickness and air gap arrangement (in rubber sheets), which reduce their insulation capacity.

The prediction models using metaheuristic optimisation methods further reinforce the experimental observations. The fact that the social spider optimisation consistently achieved the lowest error margins highlights the potential of such techniques in predicting thermal behaviour and reducing the dependency on exhaustive experimental trials. Moreover, the experimental setup using identical test rooms and real outdoor conditions adds practical relevance to the findings. Unlike simulation-only studies, this real-world data offers actionable insights for architects, builders and policy-makers seeking passive solutions for thermal management in buildings.

4. Conclusions

The thermal performances of eight roof slabs with insulating materials were compared with the roof slab without an insulating material under the same atmospheric conditions. Eight insulating materials such as coconut shell, fibreglass, fly ash, rubber sheet, asbestos sheets, rice husk, Jhama brick bats and earthen pots were utilised in this study. The variation of the roof bottom temperature was predicted using five optimisation techniques, such as bacterial colony optimisation, group search optimisation, cuckoo search, genetic algorithm and social spider optimisation. Based on the experimental test results and theoretical analysis, the following conclusions have been drawn:

- The time difference between the peak external (roof top) temperature and the corresponding peak internal (room) temperature was recorded for each type of insulation. This metric indicates the material's ability to delay heat penetration, which is critical for maintaining thermal comfort during peak summer hours. For example, the fibreglass roof showed a maximum time lag of 4 h, while the earthen pot roof showed a minimum lag of 2 h.
- The difference in indoor temperature between rooms with and without insulation was quantified. This reduction reflects the insulator's ability to mitigate indoor heat gain. Fibreglass achieved a reduction of 6.17°C, whereas for earthen pots it was 2.27°C.
- Using the ASTM E1530 guarded heat flow meter method, the thermal conductivity of each insulating material was experimentally determined. Corresponding thermal resistance values were calculated using Fourier's

law. These provide direct insight into the heat transfer inhibition through each material.

- (iv) The overall heat transfer coefficient was computed for each roof slab configuration, reflecting the heat transfer rate through the composite slab. A lower overall heat transfer coefficient indicates better insulation.
- (v) The study applied five metaheuristic algorithms to predict roof bottom temperatures, comparing them to actual experimental data. The error range between predicted and measured values served as a performance indicator of both insulation efficiency and the robustness of modelling methods. Social spider optimisation demonstrated the highest level of accuracy in prediction, with errors as low as 0.08°C.
- (vi) While the actual energy expenditure (e.g. cooling load or kilowatt-hours saved) was not measured directly, temperature reduction and time lag can be used as alternative metrics. These reductions are directly correlated with decreased energy demands for ventilation and air conditioning. Thus, the findings support energy-saving implications for passive cooling strategies.
- (vii) It was observed that the social spider optimisation technique predicted the roof bottom temperature accurately, and the error between the measured and predicted value of temperature was minimum as compared to the other four optimisation techniques.
- (viii) The performance of the roof slab using different insulating materials in descending order is found to be fibre-glass, rice husk, fly ash, Jhama brick bats, asbestos sheet, coconut shell, rubber sheet and earthen pot.

References

- [1] Rajeev, P., Sanjayan, J.G., & Seenuth, S.S. (2016): Assessment of thermal cracking in concrete roof tiles. *Journal of Materials & Design*, 107, 470–477. doi: 10.1016/j.matdes.2016.06.072
- [2] Al-Sanea, S.A. (2002). Thermal performance of building roof elements. *Journal of Building and Environment*, 37(7), 665–675. doi: 10.1016/S0360-1323(01)00077-4
- [3] Zhang, Z., Tong, S., & Yu, H. (2016). Life Cycle Analysis of Cool Roof in Tropical Areas. *Procedia Engineering*, 169, 392–399. doi: 10.1016/j.proeng.2016.10.048
- [4] Venugopal, P.R., Murugan, R., Thyla, P.R., Aithal, S., Babu, R., Perumal, C., & Raj, V.B. (2015). Experimental and numerical investigations on roof slab of a pool type sodium cooled fast reactor based on model studies. *Annals of Nuclear Energy*, 85, 1085–1095. doi: 10.1016/j.anucene.2015.06.045
- [5] Ramamurthy, P., Sun, T., Rule, K., & Bou-Zeid, E. (2015). The joint influence of albedo and insulation on roof performance. A modeling study. *Energy and Buildings*, 102, 317–327 doi: 10.1016/j.enbuild.2015.06.005
- [6] Lundholm, T.J., Weddle, B.M., & MacIvor, J.S. (2014). Snow depth and vegetation type affect green roof thermal performance in winter. *Energy and Buildings*, 84, 299–307. doi: 10.1016/j.enbuild.2014.07.093
- [7] Kumar, A., & Suman, B.M. (2013). Experimental evaluation of insulation materials for walls and roofs and their impact on indoor thermal comfort under composite climate. *Building and Environment*, 59, 635–643. doi: 10.1016/j.buildenv.2012.09.023
- [8] Jim, C.Y. (2014). Air-conditioning energy consumption due to green roofs with different building thermal insulation. *Applied Energy*, 128, 49–59. doi: 10.1016/j.apenergy.2014.04.055
- [9] Berardi, U., & Naldi, M. (2017). The impact of the temperature dependent thermal conductivity of insulating materials on the effective building envelope performance. *Energy and Buildings*, 144, 262–275 doi: 10.1016/j.enbuild.2017.03.052
- [10] D'Orazio, M.D., Perna, C.D., & Giuseppe, E.D. (2010): The effects of roof covering on the thermal performance of highly insulated roofs in Mediterranean climates. *Energy and Buildings*, 42(10), 1619–1627. doi: 10.1016/j.enbuild.2010.04.004
- [11] Björk, F., & Enochsson, T. (2009). Properties of thermal insulation materials during extreme environment changes. *Construction and Building Materials*, 23(6), 2189–2195. doi: 10.1016/j.conbuildmat.2008.12.006
- [12] Gagliano, A., Patania, F., Nocera, F., Ferlito, A., & Galesi, A. (2012). Thermal performance of ventilated roofs during summer period. *Energy and Buildings*, 49, 611–618 doi: 10.1016/j.enbuild.2012.03.007
- [13] Halwatura, R.U., & Jayasinghe, M.T.R. (2008). Thermal performance of insulated roof slabs in tropical climates. *Energy and Buildings*, 40(7), 1153–1160. doi: 10.1016/j.enbuild.2007.10.006
- [14] Jaffal, I., Ouldboukhitine, S.-E., & Belarbi, R. (2012). A comprehensive study of the impact of green roofs on building energy performance. *Renewable Energy*, 43, 157–164. doi: 10.1016/j.renene.2011.12.004
- [15] Dong, L., Yumeng, Z., Changyu, L., & Guozhong, W. (2015). Numerical analysis on thermal performance of roof contained PCM of a single residential building. *Energy Conversion and Management*, 100, 147–156. doi: 10.1016/j.enconman.2015.05.014
- [16] Dixit, A.K., Roul, M.K., & Panda, B.C. (2017). Designing an efficient mathematical model for different thermal insulation material using group search optimization. *International Journal of Intelligent Engineering and Systems*, 10(1), 28–37. doi: 10.22266/ijies2017.0228.04
- [17] Dixit, A.K., Roul, M.K., & Panda, B.C. (2019). Mathematical model using soft computing techniques for different thermal insulation materials. *Journal of Intelligent Systems*, 28(5), 821–833. doi: 10.1515/jisys-2017-0103
- [18] Dixit, A.K., Roul, M.K., & Panda, B.C. (2018). Numerical techniques for different thermal insulation materials. *International Journal of Optimization in Civil Engineering*, 8(1), 29–42. doi: joce.iust.ac.ir/article-1-323-en
- [19] Roul, M.K., & Nayak, R.C. (2012). Experimental investigation of natural convection heat transfer through heated vertical tubes. *International Journal of Engineering Research and Applications*, 2(6), 1088–1096.
- [20] Nayak, R.C., Roul, M.K., & Sarangi, S.K. (2017). Experimental investigation of natural convection heat transfer in heated vertical tubes with discrete rings. *Experimental Techniques*, 41, 585–603. doi: 10.1007/s40799-017-0201-6
- [21] Nayak, R.C., Roul, M.K., & Sarangi, S.K. (2017). Experimental investigation of natural convection heat transfer in heated vertical tubes. *International Journal of Applied Engineering Research*, 12(10), 2538–2550.
- [22] Nayak, R.C., Roul, M.K., & Sarangi, S.K. (2018). Natural convection heat transfer in heated vertical tubes with internal rings. *Archives of Thermodynamics*, 39(4), 85–111. doi: 10.1515/aoter-2018-0031
- [23] Sahoo, L.K., Roul, M.K., & Swain, R.K., (2017). CFD analysis of steady laminar natural convection heat transfer from a pin

- finned isothermal vertical plate. *Heat Transfer—Asian Research*, 46(7), 840–862. doi: 10.1002/htj.21246
- [24] Sahoo, L.K., Roul, M.K., & Swain, R.K. (2017). Natural convection heat transfer augmentation factor with square conductive pin fin arrays. *Journal of Applied Mechanics and Technical Physics*, 58, 1115–1122. doi: 10.1134/S0021894417060189
- [25] Sahoo, L.K., Roul, M.K., & Swain, R.K. (2018). CFD analysis of natural convection heat transfer augmentation from square conductive horizontal and inclined pin fin arrays. *International Journal of Ambient Energy*, 39(8), 840–851. doi: 10.1080/01430750.2017.1354317
- [26] Sahoo, L.K., Roul, M.K., & Swain, R.K. (2017). CFD analysis of heat transfer in hexagonal subchannels of super-fast reactor in upward flow. *Heat Transfer—Asian Research*, 46(8), 1399–1412. doi: 10.1002/htj.21281
- [27] Roul, M.K., & Sahoo, L.K. (2012). CFD modeling of pressure drop caused by two-phase flow of oil/water emulsions through sudden expansions. *International Journal of Engineering Research and Applications*, 2(6), 1047–1054.
- [28] Himrane, N., Ameziani, D.E., & Nasser, L. (2020). Study of thermal comfort: numerical simulation in a closed cavity using the lattice Boltzmann method. *SN Applied Sciences*, 2, 785. doi: 10.1007/s42452-020-2600-z
- [29] Belenguer, E., Garcia, N., & Sabater-Grande, G. (2019). Assessment of energy efficiency improvement methods in the residential sector through the development of economic experiments. *SN Applied Sciences*, 1, 1409. doi: 10.1007/s42452-019-1439-7
- [30] Martínez, M., Huygen, N., Sanders, J., & Atamturktur, S. (2018). Thermo-fluid dynamic analysis of concrete masonry units via experimental testing and numerical modeling. *Journal of Building Engineering*, 19, 80–90. doi: 10.1016/j.job.2018.04.029
- [31] Chandra, M.S., Nandapala, K., Priyadarshana, G. & Halwatura, R.U. (2019). Developing a durable thermally insulated roof slab system using bamboo insulation panels. *International Journal of Energy and Environmental Engineering*, 10(4), 511–522. doi: 10.1007/s40095-019-0308-x
- [32] Koru, M. (2016). Determination of thermal conductivity of closed-cell insulation materials that depend on temperature and density. *Arabian Journal for Science and Engineering*, 41, 4337–4346. doi: 10.1007/s13369-016-2122-6
- [33] Alyami, S.H. (2019). Critical analysis of energy efficiency assessment by international green building rating tools and its effects on local adaptation. *Arabian Journal for Science and Engineering*, 44, 8599–8613. doi: 10.1007/s13369-019-03972-x
- [34] Kumar, D., Ali, I., Hakeem, M., Junejo, A., & Harijan, K. (2019). LCC optimization of different insulation materials and energy sources used in HVAC duct applications. *Arabian Journal for Science and Engineering*, 44, 5679–5696. doi: 10.1007/s13369-018-3689-x
- [35] Alnahhal, M., Elnaggar, M., & Edwan, E. (2018). Thermal analysis and thickness optimization of two-cavity thermal insulator in buildings. *Arabian Journal for Science and Engineering*, 43, 1369–1381. doi: 10.1007/s13369-017-2854-y
- [36] Alhefnawi, M.A.M., & Abdu-Allah Al-Qahtany, M. (2017). Thermal insulation efficiency of unventilated air-gapped facades in hot climate. *Arabian Journal for Science and Engineering*, 42, 1155–1160. doi: 10.1007/s13369-016-2370-5
- [37] Sassine, E., Cherif, Y., Dgheim, J., & Antczak, E. (2020). Experimental and numerical thermal assessment of Lebanese traditional hollow blocks. *International Journal of Thermophysics*, 41, 47. doi: 10.1007/s10765-020-02626-7

Application of machine learning in the process of commander decision support in the military fuel distribution system

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Received: 01.09.2025; revised: 03.12.2025; accepted: 30.12.2025

Abstract

Providing energy to troops requires maintaining optimal fuel levels across all management stages, especially within Garrison Support Units and Regional Logistic Bases. The article examines the fuel distribution system supported by a program that predicts commanders' actions using input data from subordinate units. To aid decision-making, Garrison Support Units implemented neural network variants to model logistical activities, training, peacetime operations, or combat, and segment fuel supply accordingly. The Neural Network Toolbox from MATLAB (MathWorks) was used for computations. The study presents the Garrison Support Units operational assumptions, the role of commanders as agents, and factors affecting fuel distribution. It also outlines the development of the Logistic Decision Support System dashboard, which enables entering decision variables, neural network coefficients, and weights to forecast fuel consumption and plan future operations based on environmental and operational data. The article includes MATLAB simulation results, analysing neural network algorithms and neuron counts per layer to determine the most effective configuration for decision-making optimisation. Results show that the Bayesian regularisation algorithm achieved the lowest mean square error across all data sets and the highest prediction accuracy measured by the root mean squared error. The regression coefficient confirmed a strong correlation between predicted and actual outcomes, demonstrating the Bayesian regularisation algorithm's effectiveness in supporting logistical fuel management decisions.

Keywords: Machine learning, Root mean squared error; Bayesian regularisation; Decision-making process support; Energy in military logistics.

Vol. 46(2025), No. 4, 201–213; doi: 10.24425/ather.2025.156850

Cite this manuscript as: Kępczyński, A. (2025). Application of machine learning in the process of commander decision support in the military fuel distribution system. *Archives of Thermodynamics*, 46(4), 201–213.

1. Introduction

The increasing complexity of fuel supply chains and the pressure for resilience, efficiency and environmental sustainability mean that artificial intelligence and machine learning (ML) methods are rapidly being adopted in logistics (including military). In the literature of recent years, there has been a clear shift from classic optimisation problems to predictive analytics, integration of heterogeneous data, and models supporting short-term

operational decisions. This is particularly important in the context of the military fuel distribution system, where the commander's decisions are burdened by demand uncertainty, infrastructure constraints, and spatio-temporal dynamics. The concept of a modular Logistic Decision Support System (LDSS) adopted in this paper, based on multilayer neural networks and the selection of learning algorithms such as Bayesian regularisation, Levenberg-Marquardt (LM), Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton, or resilient backpropaga-

Nomenclature

BR – Bayesian regularisation
 GSU – Garrison Support Unit
 LDSS – Logistic Decision Support System
 MATLAB– Matrix Laboratory

ML – machine learning
 MSE – mean square error
 R – regression coefficient
 RLB – Regional Logistic Base
 RMSE – root mean squared error

tion (RP), remains consistent with the directions observed in research from practical implementations of short-term prediction to critical reviews of data quality, standardisation and model reliability.

The article aims to research the application of an artificial intelligence algorithm to support decision-making processes in the management of fuel resources, based on the adopted model of elements of the military logistics system. When starting the research, previous publications [1–4] characterised the logistics space within a selected scope. The need to develop solutions supporting the decision-making process of the commander (decision-maker) based on machine learning algorithms was indicated, including Broyden–Fletcher–Goldfarb–Shanno quasi-Newton, Levenberg-Marquardt, Bayesian regularisation and the resilient backpropagation algorithm.

The innovative nature of the article is manifested, among other things, by the fact that variables that have not been collected in the IT logistics system so far are indicated, as well as the method of developing the architecture of the LDSS program for implementation. The author indicated a number of dependent variables that he detected while working with the data that were analysed, namely the indicated fuel level in the Garrison Support Unit (GSU), the intensity of operations, field data, meteorological data or variables illustrating the capabilities of GSU.

The proposed model of support for the decision-making process in the fuel distribution system is based on a neural network, for which the basic input parameters and initial states of the system have been indicated, limited only to fuel distribution. The model of the modular multi-layered one-way neural network (MNN) seems to be the solution with a small and acceptable error to provide courses of action for the management staff in the indicated positions of the military logistics system. Nevertheless, in order for the indicated information technology (IT) solution to be effective, the artificial intelligence (AI) system must be taught through the experience of decision-makers and historical data, in order to present several optimal variants of fuel resource management. Finally, the thesis in which the reinforcement learning (RL) model improves the quality and accelerates the process of developing an appropriate variant of the decision-maker's action was analysed.

Bibliometric studies of logistics in Saudi Arabia [5] synthetically show the global shift towards sustainable logistics, machine learning, and the integration of technology (AI, blockchain) into traditional supply chain paradigms. The authors, analysing over 7.6 thousand publications, identify the growing importance of chain resilience and international cooperation as an accelerator of innovation. These conclusions are relevant for military logistics: the development of predictive and decision-making capabilities in the fuel system requires combining clas-

sic procedures with machine learning (ML) models and connecting multi-domain data (operational, environmental, infrastructural), which directly corresponds to the concept adopted in this paper of extending the input vectors of the network to include meteorological, terrain and operational factors.

A strong practical trend is represented by the work of Eichenseer et al. [6], in which a data-driven ML model for a five-day forecast of the number of "delivery positions" for workforce planning in logistics was developed and validated. The model, tested in the company, exceeded both practical expert forecasts and auto ML systems, especially in the short term. The value of models tailored to the specifics of the process (custom ML) in relation to universal tools is a key here from the perspective of the military GSU. This approach reinforces the decision to use a specialised multilayer perceptron (MLP) architecture in LDSS and to select training algorithms for generalisation stability with limited samples and strong operational variability. A five working days forecast horizon (week) seems to be a natural "decision window" for fuel distribution planning and personnel dispatch.

For this article, several literature items were analysed, where, among others, Nguyen et al. [7] present a critical review of ML models for estimating propulsion power and fuel consumption of marine vessels, an area inherently related to fuel and energy (power) prediction (FEP) and exposed to environmental disturbances. The authors emphasise the need for "data-centric AI": standardisation of metrics (beyond mere accuracy), benchmarks, and policies for maintaining trustworthiness in models. For military applications, this means the need to:

- unambiguous definition of quality measures (root mean squared error/mean square error (RMSE/MSE), regression coefficient (R), but also resistance to regime changes and out-of-distribution),
- monitoring of procedures and re-training,
- documenting data and model decisions (auditability), which in this article is reflected in the adopted set of metrics and comparisons of BR/LM/BFGS/RP algorithms and the conclusion about the advantage of research and development (R&D) in terms of generalisation.

The cost and decision perspective in commercial logistics was developed by Yaiprasert and Hidayanto [8], who used ensembles of ML methods to explore cost strategies on simulated data. Despite the synthetic nature of the collection, the paper shows two lessons useful for LDSS:

- ensembles increase predictive resilience to data variance and heterogeneity in operational conditions,
- It is possible to use simulations to "seal" sparse regions of the state space, which is important when real data (e.g., intensity of activities, procedural constraints) are not yet fully collected in departmental systems.

This approach can complement a trained MLP network with a simulation component or an MLP + ensemble hybrid for critical inventory thresholds.

Zhou et al. [9], on the other hand, propose a three-stage geoinformation model, geospatial information system (GIS), multiple-criteria decision making / multiple-criteria decision analysis (MCDM), or social network analysis (SNA) network, for the selection of locations for urban green logistics centres. Despite the civilian context, the construction of the method (combining environmental, economic, technological, and social criteria with network analysis) is analogous to the needs of military planning for the deployment of depots and GSU in operational space.

Finally, a broad overview of sustainable aviation fuel (SAF) supply chains by Liang et al. [10] highlights that the energy transition in transportation requires a combination of mathematical programming, ML and multi-criteria optimisation. The identified barriers: cost, complexity of multi-stage production, instability of raw material supply and regulatory discrepancies, translate into military implications:

- a) the need to take alternative fuels scenarios into account in planning (compatibility, availability, chain risk),
- b) expansion of the decision-making module with multi-criteria functions (cost-risk-environmental footprint-operational readiness),
- c) tracking policies and standards, which support the approach to building an LDSS decision-making dashboard with weights and ratios reflecting operational priorities.

In conclusion, the literature confirms the three pillars adopted in this work. First, short-term prediction on operational data (Eichenseer et al. [6]) is a key component of resource planning and should be the core of the GSU commander's support module; the choice of training algorithms with good generalisation with limited samples (Bayesian regularisation) is methodologically justified and, in our experiments brings the best compromise of accuracy/robustness, which has been demonstrated empirically. Secondly, the integration of spatial and multi-criteria data (Zhou et al. [9]) indicates that the extension of LDSS to include the GIS/MCDM/SNA component will support decisions on inventory manoeuvre and supply priorities under conditions of infrastructural and environmental constraints. Third, the trend towards sustainability and resilience (Alasmari and Alzahrani [5], Liang et al. [10]) requires that decision-making tools take into account both performance metrics (RMSE, MSE, R) and chain risk and environmental impact criteria, while maintaining the principles of "data-centric AI" and model transparency (Nguyen et al. [7]). In this context, the LDSS module, which is being developed and is based on MLP and enriched with regularisation, validation and monitoring mechanisms, is a way to authenticate operational recommendations for the GSU decision-maker, in accordance with the literature.

2. Description of the research problem

The research focused on mapping the logistics space with which agents can interact with the indication of states and transitions between individual layers of the network dedicated to the selected GSU. The agent is the commander of the selected GSU

(in our solution, we assume only one decision-making level for simplicity). He observes the changing situation of fuel security in a given area of responsibility, introducing GSU into the states through actions (decisions). The operational situation in a given area of responsibility complements the environment of operations. Actions that can be taken by the decision-maker are: receiving, dispensing, or withholding the dispensing of fuel.

The decision-maker performs actions to maintain the fuel at the required level in the unit's tanks. The agent has limited transportation resources, is constrained by the load capacity of the supplied units, and must also deliver fuel within the timeframe specified in the demand of a given military unit. The time in the proposed model is discrete, and a training set based on data from five years of GSU operation was used for prediction. Then, 70% of the data was used for training the network; 15% of all data was used to validate the network for generalisation and to stop the training process before overfitting occurred, and a further 15% was used as a test set to independently assess the network's ability to generalise.

The time is discrete, and one week of fuel management in the GSU has been taken as the time step. The prediction of fuel distribution and decision-maker actions was set ten months ahead. The purpose of the program in the MATLAB environment is to train the neural network in such a way that it is able to reduce the mean square error of the deviation of real values from those predicted in the network training process as much as possible. For the purpose of studying the degree of mapping of the prediction results of the GSU input variables, the parameters of the neural network were introduced, such as the number of neurons in the network layers and the function according to which the training was conducted. As a parameter that was predicted, it was a series of fuel level data in the GSU. In subsequent tests for the LDSS commander's desktop, the number of predicted parameters (e.g. intensity of operations, number of available personnel, availability of equipment, etc.) should be increased simultaneously on the basis of the input data. The neural network learning process involves appropriately adjusting the weights between neuronal connections in the network layers to minimise errors and improve the quality of predictions. Reducing prediction errors can be achieved by using a feedback loop (Fig. 1) backpropagation, i.e. comparison of the obtained results with the desired values. The error calculated in this way leads to a weight correction to minimise this error in subsequent steps (iterations). These processes take place in the so-called epochs, and each epoch is a single passage through the entire set of learning data. The most important steps in the network training process are: initialisation of weights, feed forward, error calculation, backpropagation, updating of weights, and repeating the process in steps from 2 to 5 over many epochs. Thanks to these measures, the network becomes more and more accurate ("learns"), and its predictions are closer to the actual results. Finally, there is a process of completing learning after reaching a certain criterion, e.g. after reaching a satisfactory level of accuracy, in our case of mapping the prediction results, the parameters of the neural network were changed, such as: internal delays, feedback delay, number of epochs, the function according to which the training was carried out, and the size of hidden lay-

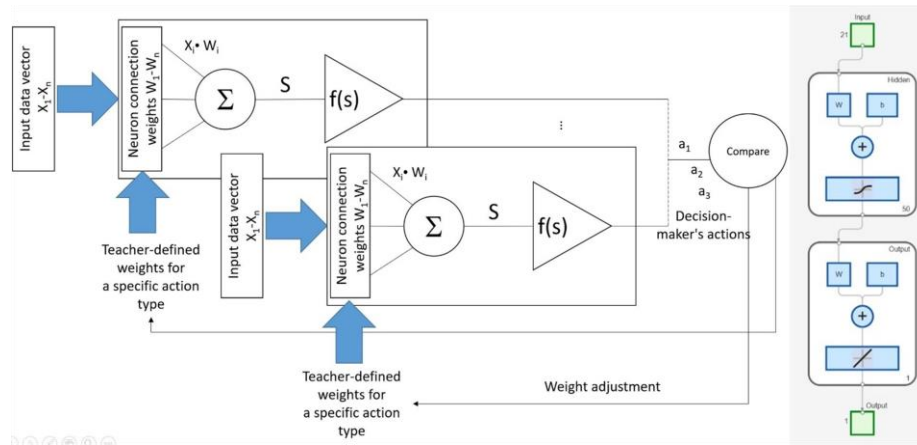


Fig. 1. Neural network model to predict fuel consumption for the selected GSU. The function according to which the network was trained was: Bayesian regularisation, Levenberg-Marquardt, BFGS quasi-Newton and resilient backpropagation.

ers. Below is a brief description of four algorithms used in the study.

2.1. Bayesian regularisation

A modification of a classical algorithm with feedback propagation that optimises both model fit and complexity by regularising weights. In this algorithm, the error function has two components – a matching error (MSE) and a penalty for heavy weights. The weights and regularisation parameter are estimated using Bayesian methods. It is known for high resistance to overfitting and excellent generalisation, especially with a large number of neurons, without needing separate cross-validation. However, it requires longer training times than non-regularised methods. This approach is chosen as a benchmark for the quality of generalisation. It is particularly effective when using many neurons and dealing with the risk of overtraining, which is important in studies with a network of 50 neurons in the hidden layer. It offers automatic selection of the regularisation parameter and reduces manual hyperparameter tuning [11].

2.2. Levenberg–Marquardt

An optimisation algorithm that is a hybrid of the Gauss–Newton method and gradient fall adjusts the learning step by switching seamlessly between the fast Gauss–Newtonian confluence (near the minimum) and the stability of the gradient method (far from the minimum). Its advantages are very fast convergence for small and medium-sized networks and high accuracy with a small number of neurons. High memory requirements. For large networks, it can become unstable or very slow. Widely recognised as the fastest algorithm for training small and medium-sized MLP networks. It handles regression problems and achieves low error in a small number of epochs very well. Ideal for comparison with BR in terms of trade-off of learning time vs. accuracy [12].

2.3. BFGS quasi-Newton

An advanced optimisation algorithm that approximates the inverse Hessian matrix (second-order derivatives) without fully

calculating it. Updates the Hessian approximation in each iteration, using gradient and weight changes. Its advantages include faster convergence than a pure straight gradient and good results for medium-sized networks and well-conditioned problems. However, it may lose efficiency with very large networks. High memory requirements. It was chosen as a classic, robust optimisation algorithm, allowing us to assess how it performs against newer and more adaptive methods [13].

2.4. Resilient backpropagation

A variation of backpropagation that ignores the size of a gradient based only on its character. A very stable algorithm in conditions where the gradient is scaled badly. Good for problems with large differences in input values. It does not use the gradient size information, so it can reach very low error values more slowly. A simple but effective gradient sign method. It is in contrast to methods that require calculations of the Hessian matrix or regularisation. In studies, it acts as a benchmark of stability at different numbers of neurons [14].

An important gap highlighted by the review authors (Nguyen et al. [7]) is the insufficient standardisation of data and benchmarks, and the lack of widely accepted measures of model confidence. In military conditions, this gap is reinforced by the limited availability of structured data on the intensity of operations, procedural constraints, or environmental parameters, which has also been identified in this study, and directly motivates the expansion of the range of collected input variables (21+ features) and the procedure for dividing the sets into training/validation/test. In addition, the literature indicates the potential for reinforcement learning (RL) for problems in which decisions and rewards are spaced over time; combined with an MLP network trained on historical data, RL can support fuel dispensing/halt-ing/receiving tactics, balancing exploration and exploitation in a dynamic environment, a direction that has been outlined as the next stage in LDSS development.

In light of the above, the GSU commander's decision support model proposed in the paper using the MLP architecture, BR/LM/BFGS/RP algorithms, an extended set of input features, and a dashboard module with weights and coefficients is in line

with the best practices identified in the literature, and responds to the key challenges:

- short-term prediction of demand and inventory levels,
- integration of multi-domain data (operational, environmental, infrastructural),
- transparency and standardisation of model evaluation,
- possibility of further hybridisation with RL and ensemble methods for resistance to data variability.

The states of the GSU logistics system were determined on the basis of historical observations from 2019–2023. It was assumed that the logistics system worked optimally and the agents acted in accordance with the adopted strategy. The number of times GSU was in a situation where the fuel level for GSU was adequate (Fig. 2) and was counted week by week.

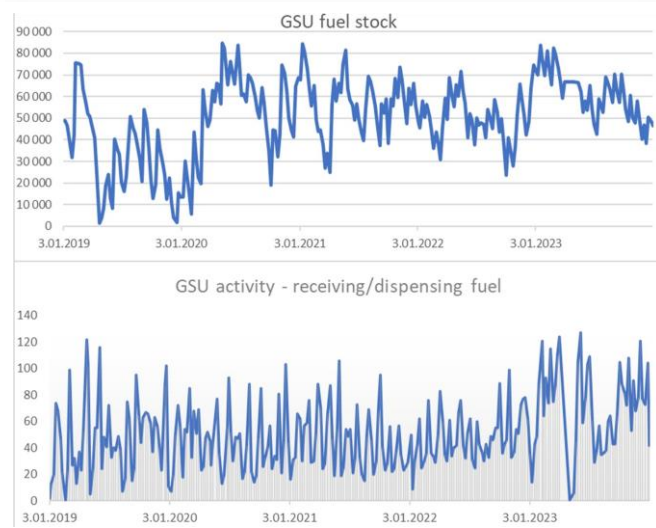


Fig. 2. Historical data on the state of diesel fuel levels and activity (fuel receiving/dispensing) in GSU in 2019–2023, every week (260 weeks – as of the first day of the week).

Using the collected historical data, it is possible to analyse changes in fuel levels for a selected fuel composition, GSU depot or a single storage facility. Changes in fuel levels reflect

many factors, such as the number and size of units supplied in a given area, their operational and training activity, or the ability to maintain and restore stocks. Therefore, the detailed information contained in the database of the Integrated Multi-Level IT System of the Ministry of National Defence (ZWSI RON – "Zintegrowany Wieloszczeblowy System Informatyczny Resortu Obrony Narodowej") on the change in the level of fuel stocks in the actual area and time for specific military units (MU) may constitute sensitive military data. While maintaining the overt nature of the study for analytical research, an "artificial" model was adopted, built from existing elements of the system, but in fact coming from various unrelated fuel supply regions. This model consists of a superior fuel depot in the Regional Logistic Base (RLB), GSU and three supplied MUs of different sizes: brigade, regiment and battalion (Fig. 3).

The task of GSU is to dispense fuel to MU in the amount and time in accordance with the demand. Of course, there are many factors affecting the quality and certainty of this task [1]. However, the basic element determining success or failure from the perspective of GSU is maintaining an appropriate level of fuel in the warehouses. Taking into account the specificity of peacetime military logistics, going below a certain minimum level should be considered a failure, even if this resource could still secure the reported needs of MU. This is related to the operational need to maintain the necessary reserves. Due to the need to manoeuvre the fuel resource between the internal GSU storage facilities, it is advisable not to exceed the specified maximum level of fuel storage capacity. Exceeding this level should be considered a failure, even if the storage capacity has not been fully used. We consider all activities leading to maintaining the state between the minimum and the maximum as a success.

To sum up, the neural network is designed to learn from historical data, taking into account the input data entered by the decision-maker in LDSS for the assumed number of weeks, how the GSU processes are carried out, and to present a prediction of the unit behaviour in terms of fuel distribution based on the decisions that the decision-maker will make for a given period of time.

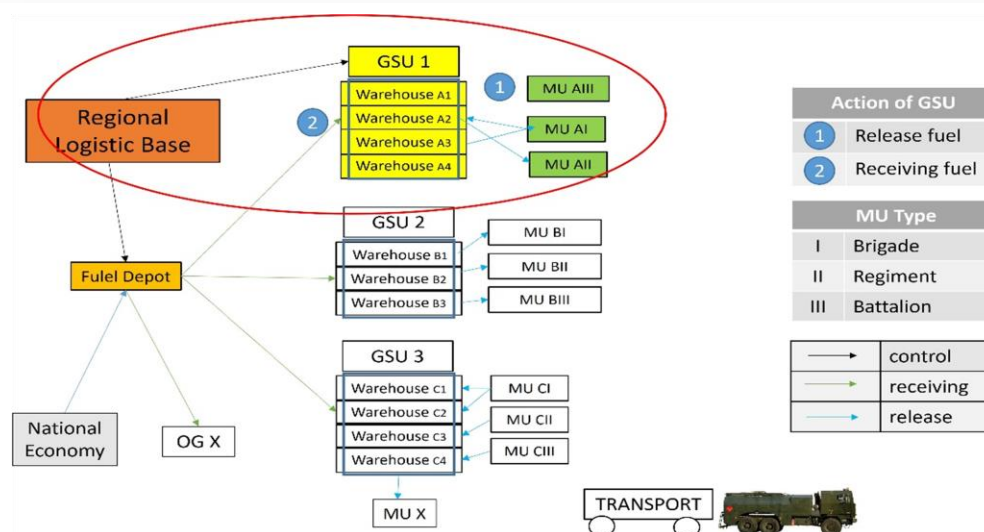


Fig. 3. The process of dispensing and receiving fuel by GSU.

The analysis covered data from selected MUs and GSU warehouses, which have been artificially linked by logistical relationships, but are a reliable model of the fuel supply system for research purposes. The probability of the initial state obtained thanks to the aggregation of historical data guarantees that, for the last five years, day by day, the tasks of supplying fuel have been performed in accordance with the adopted logistics policy of the troops. The operating environment affected the logistics system and caused successes and failures in procurement. The Logistics Decision Support System (LDSS) is a predictive module that, based on historical data and operational assumptions, e.g. exercises and policy, will allow for predicting the level of GSU fuel through the prism of the actions assumed at a given time and the operational situation. The system, based on the assumed coefficients, is to show how the decision-maker will lead GSU in the future.

The minimum and maximum level of maintaining fuel stocks was adopted, which made it possible to assess fuel inventory levels and determine the probability of the logistics system for a given week. The behaviour of the GSU decision-maker is influenced by logistical space-time – a complex concept that is superimposed by many factors, including the dynamics of actions, manoeuvres of forces, surprise and dispersion. Military logistics must keep up with the changes in this environment, and the commanders of logistics subunits must make decisions in a short time by analysing large amounts of data.

Providing fuel in the theatre of military operations is only one of the elements of the military logistics system. For the purposes of creating a decision-making model, we assume that securing the fuel needs of the operating MU requires several logistics processes based on the resources available in the operational space over time (based on GSU's resources as well as allied and civilian resources). According to this, it is necessary to overcome the logistic space-time [15] to ensure the appropriate level of fuels in time (T) in the indicated locations (X) with input data such as:

- a) Environmental:
 - temperature,
 - humidity,
 - the amount of rainfall in a given area,
 - characteristics of the terrain,
 - season,
 - altitude.
- b) Structural:
 - accessibility of land, sea, and airspace with landing strips,
 - transmission line capacity (fuel tonnage over time),
 - storage capability: equipment parameters, tonnage, dispensing capacity, and filling and distribution time.
- c) Operational (space, time, resources, and information):
 - duration of the operation,
 - the number of main equipment, infrastructure, and soldiers in the area of responsibility,
 - the dynamics of activities,
 - loss factor [1].

Based on the area of the logistics operation, we can distinguish the general function of the space-time of the operation [15].

There are several phases of the decision-making process. At the outset, it is necessary to identify the decision-making situation, which should be characterised by all factors that affect the decision-maker's verdict. Then, we will formulate the decision-making problem that the decision-maker faces. We must include a definition of: a decision-maker; decision options; factors limiting the decision-making space, and the reasons shaping the assessment of decision-making options. The first phases of a decision problem are mainly descriptive and are based on the coefficients and weights of the input variables of the neural network.

With the help of network inputs, you can determine the elements of a set of acceptable decision options, as well as indicate optimal options. Finally, you need to determine different subsets of the set of options: acceptable, satisfactory and optimal, and make a decision on this basis. The determination of different subsets of the set of options is based on the use of, among others, single- and multi-criteria optimisation methods. The Logistics Decision Support System – LDSS is to be a computerised system that supports decision-making in the area of logistics at the appropriate decision-making level of the military logistics system.

3. Neural network selection and the learning process

For the purpose of predicting the actions to be carried out by the decision-maker, a model of energy consumption prediction based on neural networks was used. The MATLAB software was used for this purpose. It includes, among other things, the Neural Network Toolbox library, which enables the construction and use of neural networks for forecasting. For this purpose, you can use ready-made tools, or use the basic command line.

To build a neural network, you need to follow these steps:

- collect data,
- create a network,
- configure the network,
- initialise scales,
- test the network,
- use the network for prediction.

We assume that the artificial neural network will generate the GSU fuel level signal taking into account the above conditions, with 260 lines of historical data set containing 21 input variables such as time, intensity of activities in the GSU operating area, including the number of fuel acceptance/dispensing operations, availability of equipment, number of personnel in the GSU system, variable specifying procedural restrictions (regulations, instructions), type of terrain, meteorological data (wind speed, temperature, humidity, variable that determines who is the recipient (size of the unit, e.g. brigade, regiment, battalion), time of fuel dispensation, execution of the movement of the GSU fuel resource, state of the GSU warehouse.

The prediction assumes the determination of 10 consecutive values of fuel levels in the GSU for the decision-maker or other

data (intensity of activities, dispensing or suspension of fuel dispensing, etc.) necessary to decide for the next 10 weeks of GSU action planning.

An array of input and output data containing 21 input variables and n-waveforms of output parameters necessary to predict the operation of GSU, which in the MATLAB environment will be used for prediction using a multilayer one-way network (multilayer perceptron).

The supervised learning method was used in the research, in which the network parameters are selected on the basis of a comparison of the values at the network output with the set values for the recorded, actual level of fuel dispensed in GSU. Training in this case consists of minimising the error function depending on the differences between the set values and the actual network output for training data.

A variant of the network with one layer of hidden neurons for different numbers of neurons was studied. It was assumed that there would be 10, 20 and 50 neurons in the hidden layer.

Four learning algorithms were adopted for the task, including three second-order algorithms:

- BFGS quasi-Newton (BFG),
- Levenberg-Marquardt (LM),
- Bayesian regularisation (BR),
- resilient backpropagation algorithm (RP).

Sigmoid neuronal activation functions in the latent layer were used.

The program randomly divided the data into three sets:

- 70% – a training set,
- 15% – a set used to validate a network in terms of its ability to generalise and stop the learning process before the phenomenon of overtraining occurs,
- 15% – a test set to perform an independent test of the network's ability to generalise.

The results were evaluated using:

- regression coefficient (R^2) measuring the correlation between the exit signal and the given target (the closer the value to 1, the better the result);
- MSE determining the quality of processing for all sets used in the study;
- RMSE mean squared error, indicating the accuracy of the forecast in a given model.

The terms dispensing or adoption cover specific decision-making processes leading to such "final" actions.

We assume that an artificial neural network model based on the algorithm of a neural network of learning with a teacher (Fig. 1) is designed to indicate, based on historical data, whether the decision-maker should perform one of three types of actions: dispense fuel, suspend dispensing fuel, or accept fuel. At the input of the neural network, to predict the decision-maker's actions, we need to collect specific data in IT systems, which are elements of the input vector of the model [1].

To train the neural network to predict fuel level changes at GSU, a group of environmental, operational and infrastructural variables were used. According to the assumptions of the LDSS system, it should be assumed that the decision-maker will introduce indicated groups of variables into the program, which will be used to create a matrix of validation data in order to predict

how the fuel level parameter in GSU will behave. An artificial intelligence algorithm will indicate a different distribution of GSU fuel levels based on historical data and the relationship between the layers of the network. The decision-maker, thanks to the illustration as in Fig. 4 or thanks to the indicators of differences between the levels assessed by the decision-maker and the one indicated by the network, will allow the decision-maker to react or take action on refuelling in a given week.

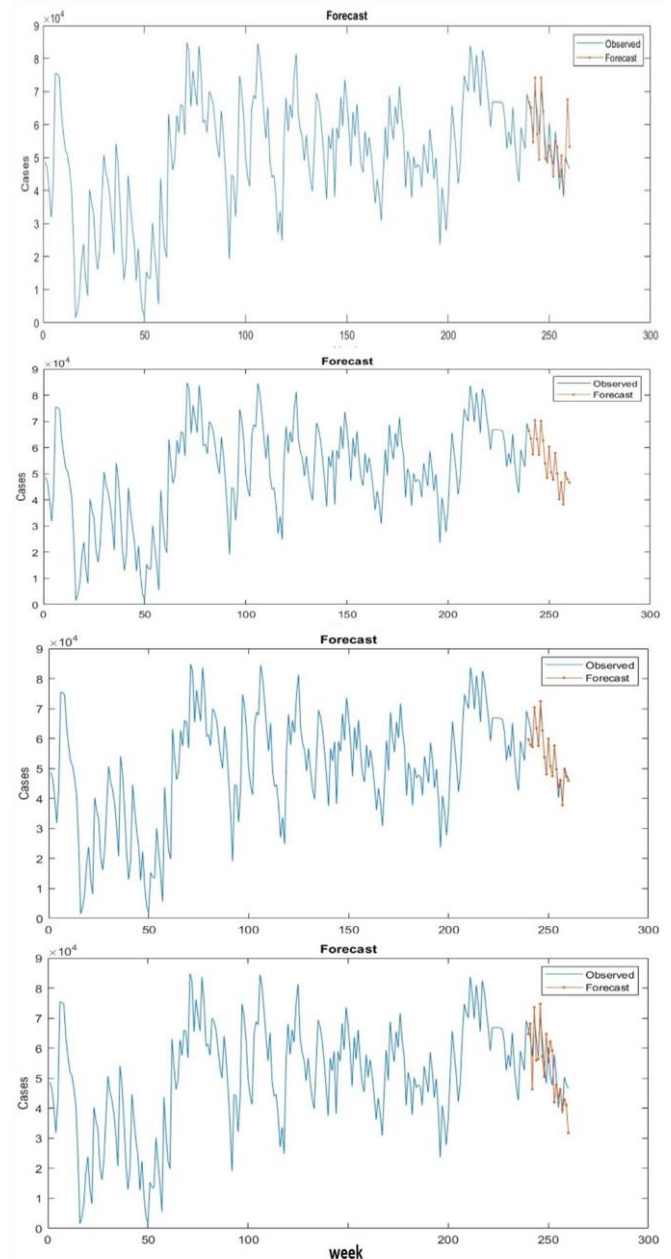


Fig. 4. Comparison of the fit plots of the simulated and measured results and the mean squared error for the study using a multilayer unidirectional network with 50 neurons using the BFG, BR, LM and RP algorithms.

A similar analysis can be carried out for other GSU parameters, such as the number of personnel (200–300 people), the intensity of activities, which consequently translates into changes in the fuel level in GSU. There are dependencies between the variables, thanks to which we can determine dependent var-

ables, such as the time of fuel dispensing from the GSU, which depends on the intensity of activities (the number of fuel trading operations per week was 0–127), the number of available personnel (accepted in the study in the range of 200–300 people) and the amount of fuel dispensed (historical data). In this case, the decision-maker will receive information whether his calculations are correct. In addition, it will be possible to determine from historical data whether an increased intensity of activities should be expected in a given week beyond that assumed by the decision-maker.

Figure 4 should be interpreted in such a way as to look for

differences between the indications of the GSU parameter from the prediction and the validation set. The regression error in this case will indicate whether these values differ so significantly that the network is trained to provide the right variants of action for the decision-maker.

Below, the results of research using the BFG, BR, LM and RP algorithms for a multilayer neural network with one hidden layer and 50 neurons are presented in the form of graphs in Figs. 5–8. Similar data analyses were carried out for networks with 10 and 20 neurons, but in this configuration, the differences in results for individual algorithms are most representative.

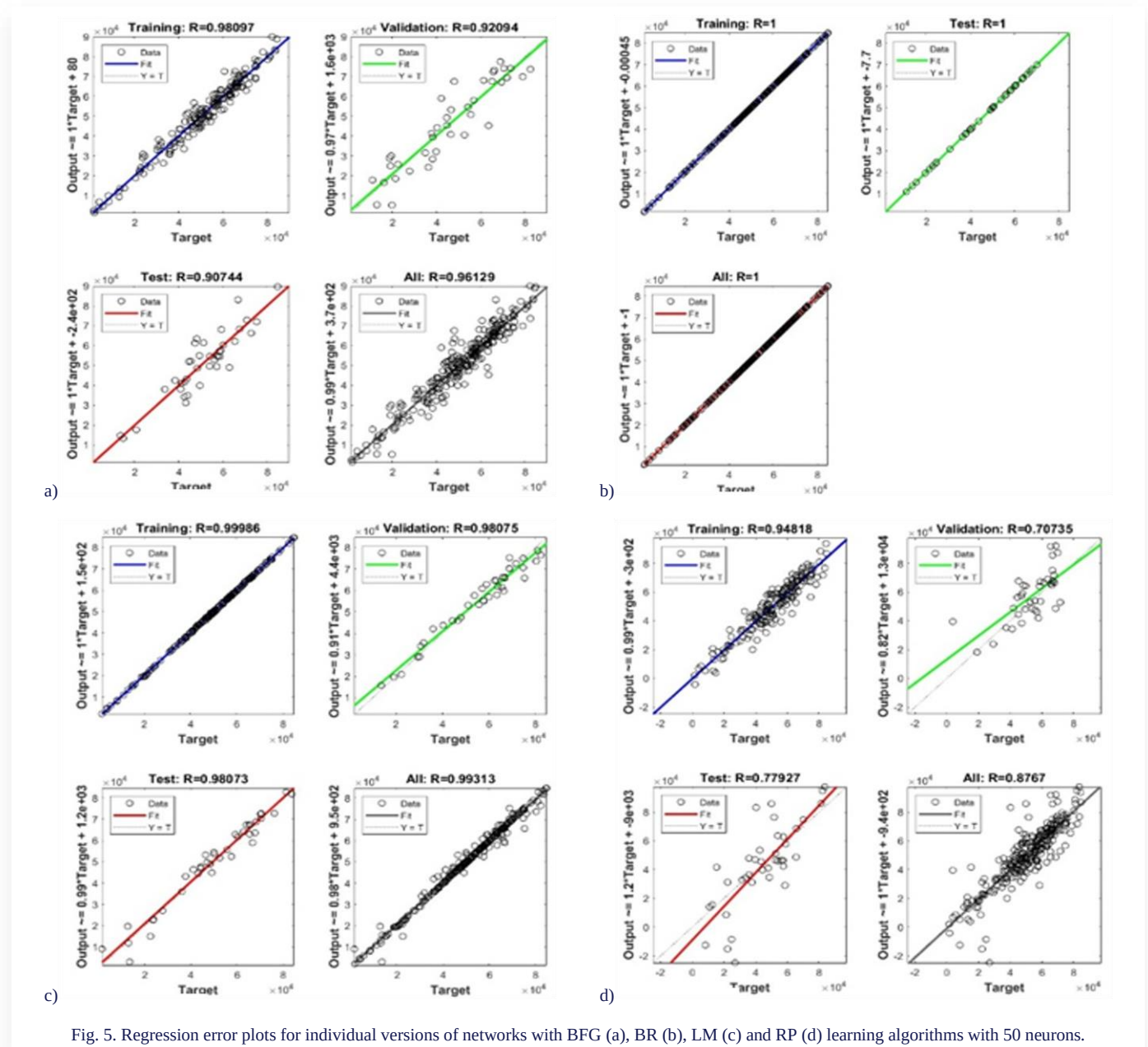


Fig. 5. Regression error plots for individual versions of networks with BFG (a), BR (b), LM (c) and RP (d) learning algorithms with 50 neurons.

In the results obtained, the BR algorithm obtained the best parameters mainly due to the specificity of its operation:

1. Regularisation mechanism minimises overlearning:
 - BR modifies the error function to simultaneously minimise the matching error and the complexity of the network scales.

- This way, the network does not learn training data "by heart", but generalises better to test and validation data.
- effect: the highest R coefficient (0.999) and the lowest MSE in validation.

2. Learning stability with a large number of neurons:

- In the configuration (50 neurons, 1 layer), the network has a very large number of weights → a high risk of overtraining with classical methods.
 - BR copes with this better than BFG, LM or RP, because it automatically adjusts the degree of regularisation based on the data.
3. Data noise immunity:
- Because there is a "penalty" component for too high weights in the target function, the BR-trained network does not over-adjust to single outliers. In practice, this

translates into the highest number of zero-error hits in the test.

4. Better fit in regression problems:

- BR is particularly effective in continuous regression because it not only optimises the MSE error, but also improves the input-output correlation.
- The results show that the differences between LM, BFG and BR are the largest in the validation MSE, suggesting that BR is better at predicting values for data that the network has not seen before.

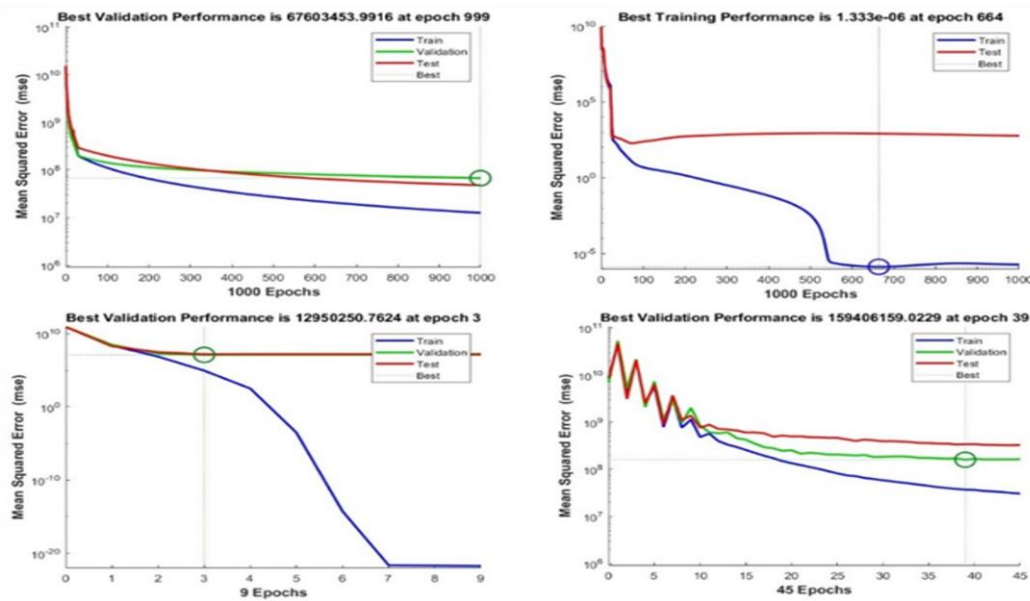


Fig. 6. Evaluation of the quality of validation for a multilayer neural network with one hidden layer and 50 neurons, using the BFG, BR, LM and RP algorithms.

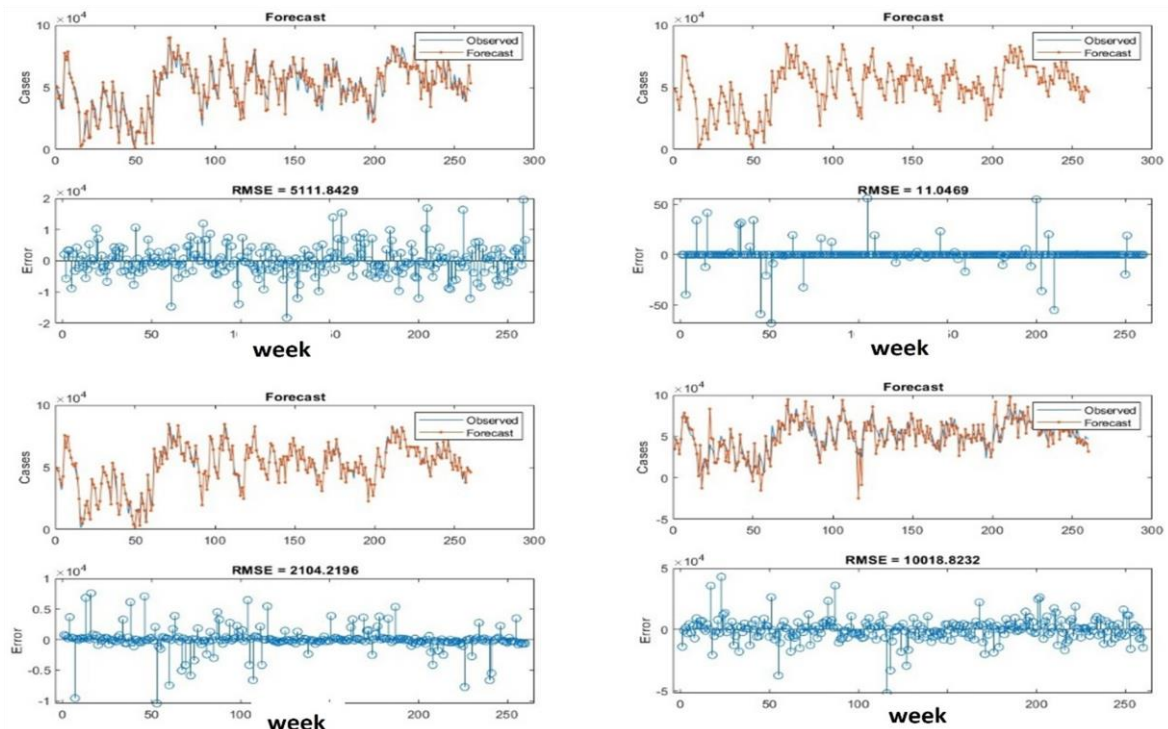


Fig. 7. Comparison of the fit plots of the simulated and measured results and the mean squared error for the study using a multilayer unidirectional network with 50 neurons, using the BFG, BR, LM and RP algorithms.

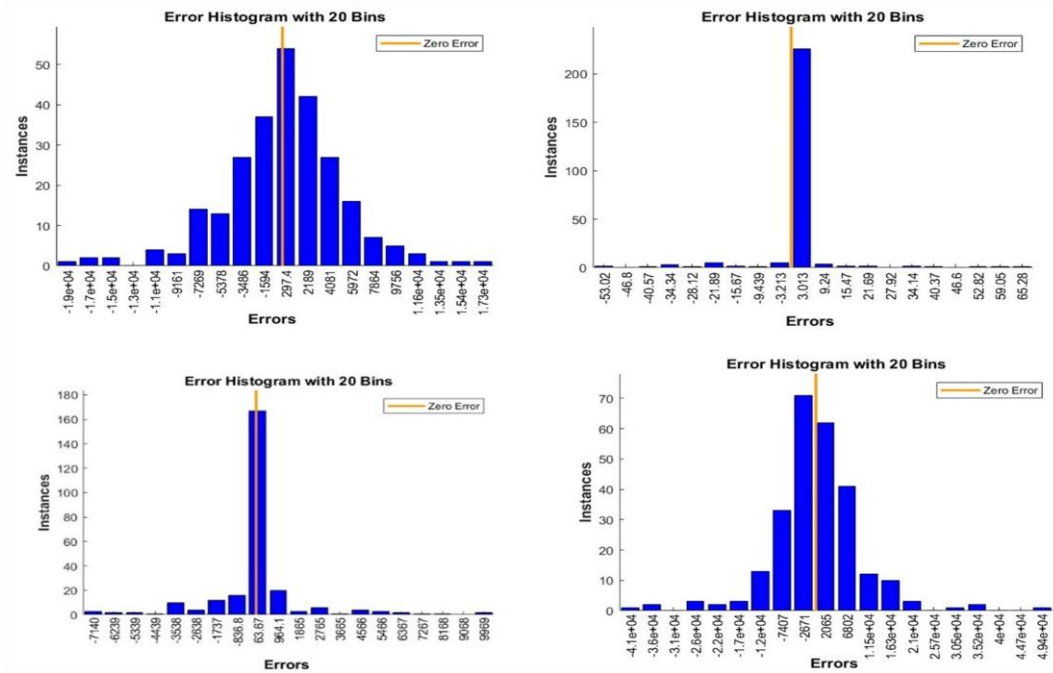


Fig. 8. Regression error histograms for consecutive neural networks with 50 neurons using the BFG, BR, LM and RP algorithms.

4. Summary

BR performed best for the network with 50 neurons, as its built-in regularisation mechanism reduced overtraining and increased generalisation. This is especially important with large numbers of neurons and potentially complex inputs.

The results obtained for different neural network structures and learning algorithms are summarised in Table 1. Based on the conducted research, Fig. 9 depicts formulated trend conclusions from data for 10, 20 and 50 neurons in BFGS, BR, LM and RP algorithms:

- Accuracy of fit (R):
 - Highest R (1.0),
 - BR and LM with 10 neurons (perfect match),
 - LM maintains $R = 1.0$ also with 20 neurons,
 - BR with 50 neurons still $R \approx 1.0$,
 - BFGS – stable around 0.98 with 10 and 20 neurons, a slight drop to 0.96 with 50 neurons,
 - RP – a marked decrease with the number of neurons: $0.97 \rightarrow 0.92 \rightarrow 0.87$.

Conclusion: BR and LM are very accurate on small and medium-sized networks; with larger networks, BR maintains an advantage over LM and other algorithms.

- Mean squared error (RMSE and MSE):

- BR – extremely low MSE in each case (in the order of $10^{-4} - 10^{-6}$), which suggests high stability and lack of overlearning,
- LM – with 10 and 20 neurons, low RMSE (1.65 and 0.36), but with 50 neurons, a significant increase in RMSE ($2.1 \cdot 10^3$) and MSE ($1.3 \cdot 10^8$), which may suggest a problem with fitting or overlearning,
- BFGS – increasing RMSE and MSE with the number of neurons,
- RP – high RMSE and MSE in any scenario, deteriorates with larger networks.

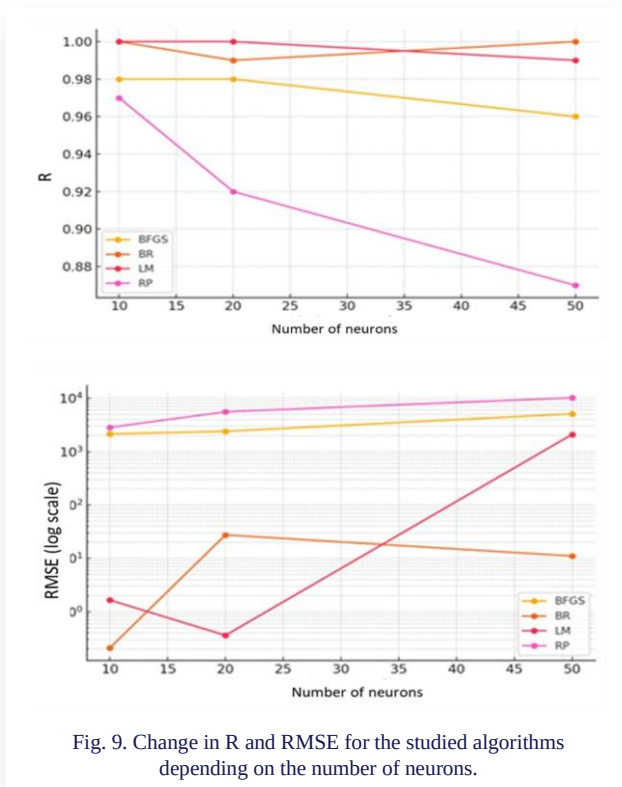
Conclusion: BR has a definite qualitative advantage, LM is good for smaller networks, and BFGS and RP lose accuracy as neurons grow.

- Influence of the number of neurons:

- BR – virtually independent of the number of neurons in terms of fit quality ($R \approx 1.0$, low MSE),
- LM – great up to 20 neurons, but at 50 neurons, there is a degradation in quality,
- BFGS – moderately stable up to 20 neurons, later larger errors,
- RP – quality decreases significantly as neurons grow, which may indicate a problem with weight propagation in large networks.

Table 1. Results for three networks and four learning algorithms (the best fit data for the BR algorithm is marked in green).

Algorithm	BFGS quasi_Newton			Bayesian regularisation			Levenberg-Marquardt			Resilient backpropagation		
Indicator	RMSE	MSE	R	RMSE	MSE	R	RMSE	MSE	R	RMSE	MSE	R
10 neurons	$2,14 \cdot 10^3$	$6,14 \cdot 10^6$	0,98	0,21	$1,13 \cdot 10^{-4}$	1	1,65	7,03	1	$2,84 \cdot 10^3$	$1,9 \cdot 10^7$	0,97
20 neurons	$2,4 \cdot 10^3$	$1,18 \cdot 10^7$	0,98	27,5	$2,6 \cdot 10^{-6}$	0,99	0,36	0,38	1	$5,58 \cdot 10^3$	$8,12 \cdot 10^7$	0,92
50 neurons	$5,11 \cdot 10^3$	$6,76 \cdot 10^7$	0,96	11,04	$1,3 \cdot 10^{-6}$	1	$2,1 \cdot 10^3$	$1,3 \cdot 10^8$	0,99	$1,02 \cdot 10^4$	$1,6 \cdot 10^8$	0,87

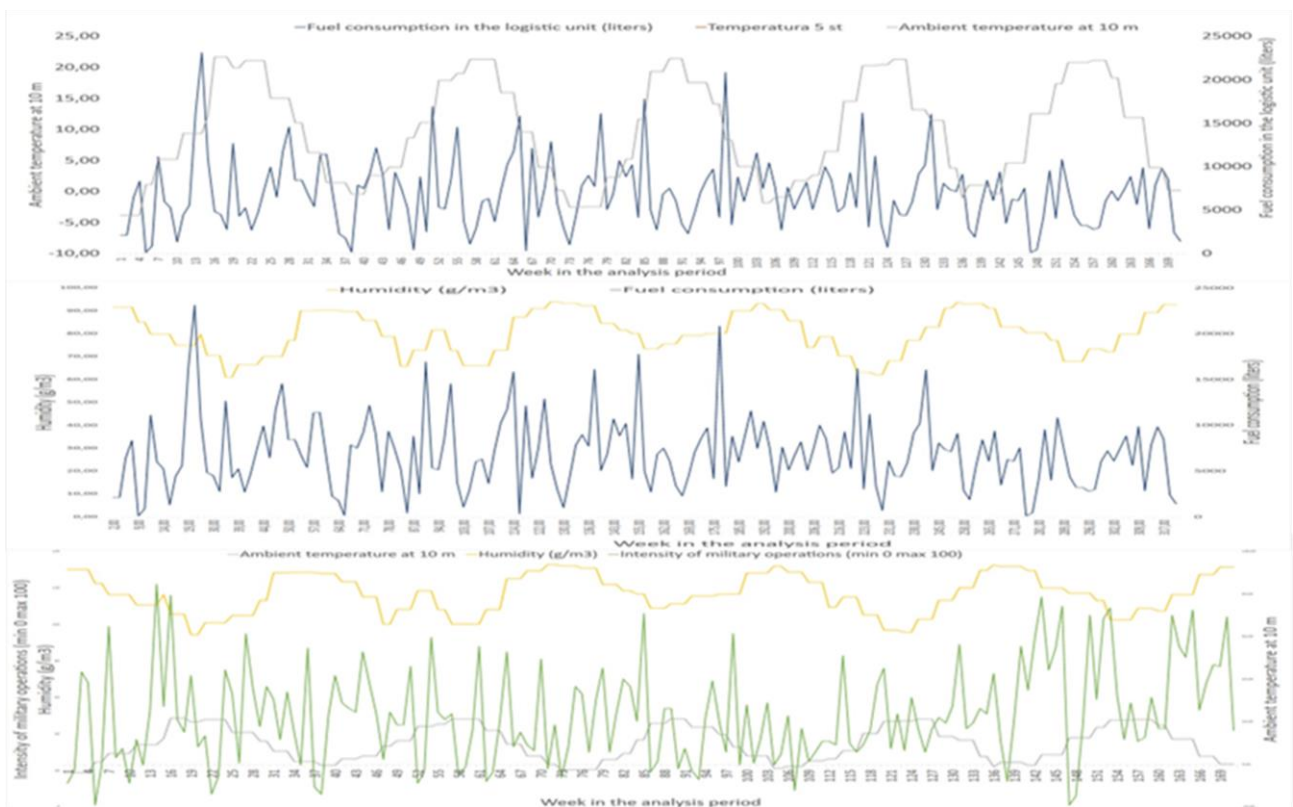


The main consumer of fuel in the adopted model of the supply region by GSU are vehicles and devices based on the operation of internal combustion engines. An analysis of the available

scientific literature [16–18] in recent years indicates a significant influence of air temperature and humidity on the fuel consumption of internal combustion engines. This relationship was the reason for including these parameters in the database to be studied. The research analyses the collected data in terms of the impact of temperature and humidity on fuel consumption in GSU. In order to determine the relationship between fuel consumption in a logistics unit and weather conditions, a correlation and regression analysis was performed, using variables: ambient temperature, relative humidity and intensity of activities measured as the number of fuel extraction operations (Fig. 10). The direct relationship was assessed using the Pearson correlation coefficient and the linear regression model.

The results clearly indicate that fuel consumption strongly depends on the intensity of operations of military units in the area of responsibility of GSU, which is confirmed by a positive correlation ($\gamma \approx 0.51$). This means that an increase in the number of fuel withdrawal operations is significantly linked to an increase in fuel consumption.

On the other hand, the correlations between fuel consumption and temperature ($r \approx -0.05$) and humidity ($r \approx 0.03$) are close to zero, indicating that there is no direct linear relationship between the variables in question. Adding temperature and humidity to the regression model did not increase its explanatory power ($R^2 \approx 0.26$), and the coefficients assigned to them turned out to be statistically insignificant ($p > 0.5$). This means that the impact of meteorological parameters on fuel consumption is not due to direct impact, but may be a secondary effect depending on planning and seasonality of activities.



Fuel consumption in the tested model unit does not directly depend on air temperature or humidity, but is strongly correlated with the intensity of logistics activities. Weather conditions may indirectly affect consumption by modifying training activities, but they are not an independent factor determining fuel consumption. Even the use of the non-linear random forest model did not allow for a satisfactory quality forecast of fuel consumption based on meteorological data. The most important predictor remains the intensity of fuel withdrawal operations, which indicates that the demand for fuel is determined mainly by training and operational activities. Weather factors, on the other hand, can play an indirect role – they affect the planning of exercises and military operations, but they are not an independent determinant of the level of consumption.

5. Conclusions

In order to create software supporting the decision of the GSU decision-maker in the future, neural networks based on the most commonly used algorithms were trained. The Levenberg-Marquardt and Bayesian regularisation algorithms within the framework of approximation tasks are considered to be the best in obtaining MSE from all the algorithms used [16]. It should be noted that the Levenberg-Marquardt algorithm, which in many studies shows its advantage over other algorithms, turned out to be less useful in this case. LM is considered the fastest, not too complex, and is used almost exclusively for training a medium-sized unidirectional network with a single neuron at the output layer. Unfortunately, the amount of data collected for the purpose of training the network to predict activities in GSU was small, mainly because so far the ZWSI RON system (Integrated Multi-Level IT System of the Ministry of National Defense) has not collected data such as the intensity of activities in the area of the GSU's operation, the type of terrain, the availability of equipment, the variable that determines the limitations related to procedures (regulations), meteorological data (wind speed, temperature, humidity) probability of moving the GSU fuel resource. The scale of the amount of data that must be implemented into the military logistics information system is significant; however, the presented article is intended to indicate the need to take actions that will lead to the expansion of the number of collected parameters of the military logistics system. In this case, probably too small a dataset to train and a decidedly small network size did not allow for fully showing the advantages of this algorithm, including its speed.

The use of the Bayesian regularisation algorithm turned out to be the most effective method in terms of prediction ability, much better than the Levenberg-Marquardt algorithm, which is slightly inferior to this algorithm. Such a favourable result for R&D was influenced by, among others, greater resistance to learning, less cross-validation [17], and better flexibility and quality of generalisation [18].

In the case of BFGS algorithms, the quasi-Newton algorithm is considered to be a more complex algorithm, but its properties are perfect for small networks, due to the complexity of the calculations it performs. The results confirmed this by placing the algorithm in third place in the study.

The resilient RP backpropagation algorithm, despite the advantage of eliminating errors in the training set, did not guarantee good results in the obtained prediction.

It turned out that the quality of learning using this algorithm, given the relatively small size of the network used for the research, gives worse results than other methods.

To sum up, it should be stated that a multilayer unidirectional perceptron with a sigmoid function of neuronal activation in the hidden layer and with the use of the Bayesian regularisation learning algorithm can be successfully used to predict fuel management in GSU.

Proper construction of a database for GSU and entering geolocation and meteorological data into the ZWSI RON system will significantly improve the ability of the neural network to generalise, and thus fit into the decision-making model in GSU by creating tools for the development of a logistic decision support system – LDSS.

Regardless of the above, during the study of the problem of optimisation of prediction using machine learning algorithms, it can be noted that the next step in optimising this type of software, mainly in terms of the quality and speed of prediction, are reinforcement learning (RL) algorithms. Based on research conducted on RL algorithms in, e.g. the game "Tetris" [19,20], the key factor in reinforcement learning is the time delay between the action and the reward. Based on the literature on the use of RL algorithms in computer games, it should be stated that the worst performance is achieved by networks trained to predict actions several dozen steps away from the next reward. It can be said that the longer the interval between the action and the reward, the game requires a much more deliberate strategy, and the shorter the pause, the more the game is reactive to the changing situation of the operating environment. RL-based learning occurs much faster [21] in reflex-based games than in games that require a lot of strategy, which makes sense and is in line with the author's research. The results show that using AI to combine an agent neural network trained on historical GSU data and a strategy-based RL algorithm is a very beneficial approach in terms of improving decision-making efficiency. Reinforcement learning differs from supervised learning in that it does not require the presentation of labelled input/output pairs and does not require direct correction of suboptimal actions. Instead, we focus on finding a balance between exploring unknown solutions and the decision-maker's experience to maximise the long-term reward, whose feedback may be incomplete or delayed [22].

Taking into account the above, the use of a neural network to predict activities in a business branch turned out to be very useful and allows decision-makers to make decisions for the next weeks of GSU's operation with the indication of variables to be predicted. On the other hand, supplementing the analysis with RL algorithms in the future will allow us to further optimise this process and significantly speed it up. The use of the proposed machine learning methods is aimed at developing a logistics decision support system – LDSS in the future, which will be designed to support decision-making processes in the area of logistics at the appropriate decision-making level of the Armed Forces logistics system.

The method of predicting fuel consumption of the logistics system in GSU described in the article can be successfully used to determine other thermodynamic parameters, such as the pressure at the inlet to the turbine of a jet engine, the distribution of air flow on the steam turbine blade depending on the parameters of the external environment, as well as to predict fuel consumption in power plants or other energy generation systems. Thanks to the use of backpropagation, and comparison of the obtained results with the desired values and continuous calculation of the error, the network leads to weight correction to minimise error in subsequent steps (iterations). These processes take place in the so-called epochs, and each epoch is a single passage through the entire set of learning data. The method described in the article can be used by readers for their own research and lead to many interesting solutions to research problems in the field of thermodynamics.

References

- [1] Kępczyński, A. (2024). A model of supporting the decision-making process for the needs of providing energy to the Armed Forces. *Perspectives of military and civilian logistics in the context of ensuring resilience to the Armed Force*, 12–14 June, Bydgoszcz, Poland (in Polish).
- [2] Prokopowicz, W., & Kępczyński, A. (2024). Application of neural networks to model decision-making processes in a military fuel distribution system. *Perspectives of military and civilian logistics in the context of ensuring resilience to the Armed Force*, 12–14 June, Bydgoszcz, Poland (in Polish).
- [3] Kępczyński, A., & Stępień, S. (2024). Optimization and AI support in military logistics. *Perspectives of military and civilian logistics in the context of ensuring resilience to the Armed Force*, 12–14 June, Bydgoszcz, Poland (in Polish).
- [4] Kępczyński, A., Lisowski, G., Prokopowicz, W., & Stępień, S. (2023). Adaptive methods in the logistics of military energy. In: *Science for Defense and the Environment* (pp. 20–28). ITWL Publishing House, Warsaw (in Polish).
- [5] Alasmari, T., & Alzahrani, A. (2025). Saudi Arabia's shifts towards green and sustainable logistics: Bibliometric and machine learning-based insights and forecasts. *Journal of Cleaner Production*, 509, 145577. doi: 10.1016/j.jclepro.2025.145577
- [6] Eichenseer, P., Hans, L., & Winkler, H. (2024). A data-driven machine learning model for forecasting delivery positions in logistics for workforce planning. *Smart Computing and Applications*, 9, 100099. doi: 10.1016/j.sca.2024.100099
- [7] Nguyen, S., Gadel, M., Wang, K., Li, J., Zhang, X., Kong, S.-C., Fu, X., & Qin, Z. (2025). Maritime decarbonization through machine learning: A critical systematic review of fuel and power prediction models. *Cleaner Logistics and Supply Chain*, 14, 100210. doi: 10.1016/j.clscn.2025.100210
- [8] Yaiprasert, C., & Hidayanto, A.N. (2023). AI-powered ensemble machine learning to optimize cost strategies in logistics business. *International Journal of Industrial and Manufacturing Engineering*, 4(1), 100209. doi: 10.1016/j.ijime.2023.100209
- [9] Zhou, J., Yang, C., Liu, D., Wang, Y., Zhong, Z., & Wu, Y. (2025). A three-stage geospatial network optimal location decision model for urban green logistics centers from a sustainable perspective. *Sustainable Cities and Society*, 128, 106481. doi: 10.1016/j.scs.2025.106481
- [10] Liang, Y.Y., Shahabuddin, M., Ahmed, S.F., Tan, J.X., & Ali, S.M. (2025). Optimizing sustainable aviation fuel supply chains: Challenges, mitigation strategies and modeling advances. *Fuel*, 402, 135972. doi: 10.1016/j.fuel.2025.135972
- [11] Burden, F.R., & Winkler, D.A. (2008). Bayesian regularization of neural networks. *Methods in Molecular Biology*, 458, 25–44. doi: 10.1007/978-1-60327-101-1_3
- [12] Hagan, M.T., & Menhaj, M.B. (1994). Training feedforward networks with the Marquardt algorithm. *IEEE Transactions on Neural Networks*, 5(6), 989–993. doi: 10.1109/72.329697
- [13] Fletcher, R. (1970). A new approach to variable metric algorithms. *The Computer Journal*, 13(3), 317–322. doi: 10.1093/comjnl/13.3.317
- [14] Riedmiller, M., & Braun, H. (1993). A direct adaptive method for faster backpropagation learning: The RPROP algorithm. *IEEE International Conference on Neural Networks*, 586–591. 28 March–01 April, San Francisco, USA. doi: 10.1109/ICNN.1993.298623
- [15] Ficoń, K. (2006). *Applied Operational Research. Models and applications*. BEL STUDIO Publishing House, Warsaw (in Polish).
- [16] Saber, H.A., Al-Barwari, R.R.I., & Talabany, Z.J. (2013). Effect of Ambient Air Temperature on Specific Fuel Consumption of Naturally Aspirated Diesel Engine. *Journal of Science and Engineering*, 1(1), 1–7. <https://www.scribd.com/document/133228366/> [accessed 22 Nov. 2025].
- [17] Bielaczyc, P.A., Szczotka, A., & Woodburn, J. (2011). The effect of a low ambient temperature on the cold-start emissions and fuel consumption of passenger cars. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 225(9), 1253–1264. doi: 10.1177/0954407011406613
- [18] Andrych-Zalewska, M., Chłopek, Z., Merksiz, J., & Pielecha, J. (2023). Impact of the Internal Combustion Engine Thermal State during Start-Up on the Exhaust Emissions in the Homologation Test. *Energies*, 16(4), 1937. doi: 10.3390/en16041937
- [19] Kayri, M. (2016). Predictive abilities of Bayesian regularization and Levenberg–Marquardt algorithms in artificial neural networks: A comparative empirical study on social data. *Mathematical and Computational Applications*, 21(2). doi: 10.3390/mca21020020
- [20] Burden, F., & Winkler, D. (2008). Bayesian regularization of neural networks. In D. J. Livingstone (Ed.), *Artificial Neural Networks: Methods in Molecular Biology*, 458. Humana Press. doi: 10.1007/978-1-60327-101-1_3
- [21] Sariev, E., & Germano, G. (2020). Bayesian regularized artificial neural networks for the estimation of the probability of default. *Quantitative Finance*, 20(2), 311–328. doi: 10.1080/14697688.2019.1633014
- [22] Groß, A., Friedland, J., & Schwenker, F. (2008). Learning to play Tetris applying reinforcement learning methods. *16th European Symposium on Artificial Neural Networks*, 23–25 April, Bruges, Belgium.
- [23] Stevens, M., & Pradhan, S. (2016). Playing Tetris with deep reinforcement learning. Stanford University. https://cs231n.stanford.edu/reports/2016/pdfs/121_Report.pdf [accessed 22 Nov. 2025].
- [24] Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A.A., Veness, J., Bellemare, M.G., Graves, A., Riedmiller, M., Fidjeland, A.K., Ostrovski, G., Petersen, S., Beattie, Ch., Sadik, A., Antonoglou, I., King, H., Kumaran, D., Wierstra, D., Legg, S., & Hassabis, D. (2015). Human-level control through deep reinforcement learning. *Nature*, 518(7540), 529–533. doi: 10.1038/nature14236
- [25] Kaelbling, L.P., Littman, M.L., & Moore, A.W. (1996). Reinforcement learning: A survey. *Journal of Artificial Intelligence Research*, 4, 237–285.

Thermodynamic modelling and experimental validation of direct carbon dioxide methylation to para-xylene

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Received: 09.06.2025; revised: 29.10.2025; accepted: 18.11.2025

Abstract

With the acceleration of global industrialisation, carbon dioxide (CO₂) emissions continue to rise, causing a series of environmental problems. The development of CO₂ conversion and utilisation technology is of great significance for achieving sustainable development. This study conducts an in-depth thermodynamic analysis of the key reactions in the CO₂ conversion and utilisation process based on the principle of thermodynamic equilibrium. By constructing a chemical reaction model, thermodynamic parameters such as equilibrium constants and Gibbs free energy are calculated for various reactions under different temperature and pressure conditions, and the feasibility and product distribution of the reactions are evaluated. Meanwhile, the influence of different reaction conditions on the distribution of equilibrium products during CO₂ conversion and utilisation is explored. The results showed that under the conditions of 370–440°C, 3 MPa, and a feed molar ratio (1:1–1:4), the conversion rate of CO₂ and toluene was over 90%, and the selectivity of xylene was over 60%. From thermodynamic analysis, the order of thermodynamic advantages of the entire reaction was: CH₃OH methylation > CO₂ methylation > CO₂ to olefin production. Based on the above data, the optimal operating parameters for the CO₂ direct methylation process were determined through thermodynamic analysis. This study can provide an important theoretical basis and practical guidance for the industrial application of CO₂ conversion and utilisation technology.

Keywords: CO₂ conversion and utilisation; Thermodynamic analysis; Key conversion reactions; Catalyst; Reaction mechanism

Vol. 46(2025), No. 4, 215–228; doi: 10.24425/ather.2025.156851

Cite this manuscript as: Du, X. (2025). Thermodynamic modelling and experimental validation of direct carbon dioxide methylation to para-xylene. *Archives of Thermodynamics*, 46(4), 215–228.

1. Introduction

With the acceleration of global industrialisation, energy-related carbon dioxide (CO₂) emissions continue to rise. The global CO₂ emissions in 2022 have reached 36.8 billion tons. If effective measures are not taken, it is expected that the global temperature rise will exceed 2°C by the end of the 21st century, triggering extreme climate and ecological crises [1]. In this context, CO₂ resource utilisation technology has become an international research hotspot, with the goal of converting CO₂ into high value-added chemicals (such as methanol, olefins, aromatics, etc.),

achieving a carbon cycle loop and benefits win-win situation. In recent years, the direct methylation of CO₂ to produce p-xylene (PX) has attracted much attention due to its strategic position in the aromatic hydrocarbon industry chain. However, this process faces thermodynamic limitations, competition from side reactions and catalyst design bottlenecks, which urgently require systematic thermodynamic analysis and process optimisation [2]. Many experts and scholars have conducted research and analysis on this. Guo et al. [3] developed a metal oxide/zeolite bifunctional catalyst to improve the efficiency of CO₂ hydrogenation to aromatics and studied it using the ZnZrOx/ZSM-5 system as an example. This sys-

Nomenclature

f_i – fugacity of the i -th substance, Pa
 f_i^0 – standard fugacity of the i -th substance, Pa
 G^t – total Gibbs free energy of the system, kJ/mol
 G_r^0 – standard Gibbs free energy of the i -th substance
 ΔG – Gibbs free energy change, kJ/mol
 ΔG_r^0 – standard molar Gibbs free energy change of reaction, kJ/mol
 ΔH – enthalpy change, kJ/mol
 ΔH_r^0 – standard molar enthalpy change of reaction, kJ/mol
 K – equilibrium constant
 K^0 – standard equilibrium constant
 n_i – number of moles of the i -th substance
 N – number of reactants
 R – ideal gas constant, = 8.3145 J/(mol K)
 ΔS – entropy change, kJ/(mol K)
 ΔS_r^0 – standard molar entropy change of reaction, kJ/(mol K)
 T – temperature, K
 T^0 – standard temperature, K

Greek symbols

η_{CO_2} – CO₂ conversion rate
 η_T – toluene conversion rate
 ψ_i – distribution ratio of the i -th substance

Abbreviations and Acronyms

C1-C4 – carbon number 1 to 4 hydrocarbons
 C⁹⁺ – carbon number ≥ 9 aromatic hydrocarbons
 GFE – Gibbs free energy
 HZSM-5 – proton-exchanged zeolite socony Mobil-5
 MOX – metal oxides
 MTA – methanol-to-aromatics
 MTO – methanol-to-olefins
 PX – p-xylene
 SX – selectivity of xylene
 ZnZrOx – zinc-zirconium oxide
 ZnCr₂O₄ – zinc chromite
 ZrCuOx – zirconium-copper oxide
 ZSM-5 – zeolite socony Mobil-5

tem could achieve a CO₂ conversion rate of over 30%, demonstrating the development potential of bifunctional catalysts in CO₂ hydrogenation to aromatics. Qi et al. [4] explored the laws and limiting factors of the CO₂ methylation reaction through thermodynamic calculations. It was found that the reaction is limited by Gibbs free energy (GFE), and low temperature (< 400°C) conditions are more conducive to the generation of PX. This achievement provided a theoretical basis for optimising the reaction conditions of CO₂ hydrogenation to aromatics from a thermodynamic perspective, and clarified the importance of temperature control for the generation of the target product PX. Wang et al. [5] found that zeolite acidity regulation can effectively inhibit the isomerisation reaction of PX, which has a positive significance for improving the selectivity of PX. However, despite acidity regulation, the selectivity of by-product light hydrocarbons (C₁-C₄) in the CO₂ hydrogenation to aromatics process was still as high as 40%. This indicated that there were still challenges in inhibiting side reactions and improving the selectivity of target products. Ma and Liu [6] compared the efficiency of different metal oxides in CO₂ activation and found that metal oxides, such as ZrCuOx, had significantly better CO₂ activation efficiency than traditional Cu/ZnO systems. Awad et al. [7] studied the reaction of CO₂ hydrogenation to aromatics under different pressure conditions and found that high-pressure operation (3–5 MPa) can improve the conversion rate of raw materials, but at the same time, energy consumption will also increase. This required a balance between the advantages and disadvantages of high-pressure operation in practical applications. On the one hand, increasing pressure could promote the reaction towards generating more aromatic products and improve the utilisation efficiency of raw materials. However, excessive pressure could lead to an increase in equipment investment costs and energy consumption, thereby affecting the economic feasibility of the entire process. Kim et al. [8] conducted process integration research and proposed a strategy for unreacted gas circulation. This strategy could increase carbon utilisation to 70%. This was of great significance for improving the

resource utilisation efficiency of the CO₂ hydrogenation to aromatics process. By recycling unreacted gases, not only was the utilisation rate of raw materials increased and waste emissions reduced, but also production costs were lowered. Zheng et al. [9] conducted an economic evaluation and found that the production cost of PX is extremely sensitive to the lifespan of the catalyst. PX production was only competitive when the lifespan of the catalyst exceeded 1000 hours. This highlighted the crucial role of catalyst stability in the industrial production of aromatics through CO₂ hydrogenation. Long-life catalysts could reduce the cost of frequent catalyst replacement, minimise production interruptions caused by catalyst deactivation, and ensure production continuity and economy. Xu et al. [10] studied the indirect pathway of CO₂ and found that although this pathway is relatively mature, it is prone to trigger side reactions in methanol-to-olefins (MTO). This meant that when using indirect pathways for CO₂ hydrogenation to produce aromatics, it was necessary to pay attention to and solve the problems caused by side reactions, such as decreased product selectivity and increased separation costs. However, due to its relatively mature technology, it still had some practical value.

The above studies show that single reaction pathway analysis cannot reveal the mechanism of side reaction competition. Pure experimental research lacks thermodynamic guidance and is difficult to systematically optimise. The dynamic model relies on reaction mechanism parameters that cannot be clearly defined. In view of this, the study adopts thermodynamic equilibrium analysis based on the GFE minimisation principle, and combines Advanced System for Process Engineering Plus (ASPEN Plus) with Python coupling calculation to construct a CO₂ conversion system model containing 17 main/side reactions. This method can systematically analyse the competition and cooperation relationships between pathways in complex reaction networks and predict the equilibrium composition by minimising the total GFE of the system. It uses ASPEN Plus's reliable physical property database and Python's flexible scalability to implement multi-condition optimisation calculations.

At the same time, the model's accuracy is verified through experimental data.

The main innovation of the research lies in the construction of a CO₂ conversion system model based on the GFE minimisation principle, which includes 17 main/side reactions. Through ASPEN Plus and Python coupling calculation, the thermodynamic parameters of each reaction under standard conditions are quantified, and the thermodynamic advantage order of 'methanol methylation > CO₂ methylation > CO₂ to olefins' is clarified. At the same time, the limitations of single factor analysis are broken through, systematically exploring the synergistic effects of temperature, pressure and feed ratio, and determining the optimal process parameters of 370–440°C, 3 MPa, CO₂/C₇H₈ = 1:1–1:4.

Compared to previous research, it has solved multiple problems. One reason is that previous studies have focused on a single reaction pathway and have not clearly defined the conditions for inhibiting side reactions. This study explains the impact of side reaction competition on PX selectivity through the 17 reaction model. The second is to solve the problem of the disconnect between catalyst design and thermodynamic optimisation. Pre-

vious studies did not relate thermodynamic characteristics to catalyst deactivation. Based on the endothermic characteristics of the toluene disproportionation side reaction, this study specifically designs a bifunctional catalyst. The third is to quantify the applicable scenarios of 'direct/indirect methylation'.

2. Methods and materials

2.1. CO₂ conversion scheme and materials

As global climate change becomes increasingly severe, reducing CO₂ emissions has become a shared responsibility of the international community. In this context, global researchers have conducted numerous studies on developing efficient and economically feasible CO₂ capture and utilisation technologies. Especially, the methylation reaction between CO₂ as a C1 source and toluene, which converts it into p-xylene, has significant value that cannot be ignored [11,12]. P-xylene is widely used in the downstream polyester industry due to its properties as a key value-added chemical. At present, the production of PX mainly involves two different routes: indirect methylation and direct methylation. The specific conversion routes are shown in Fig. 1.

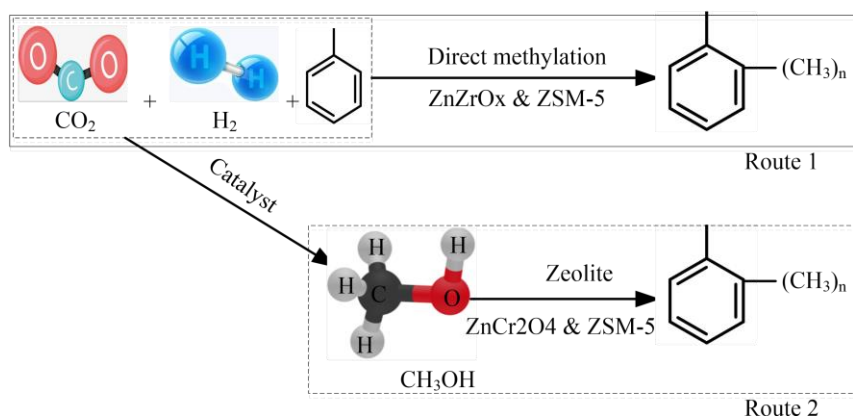
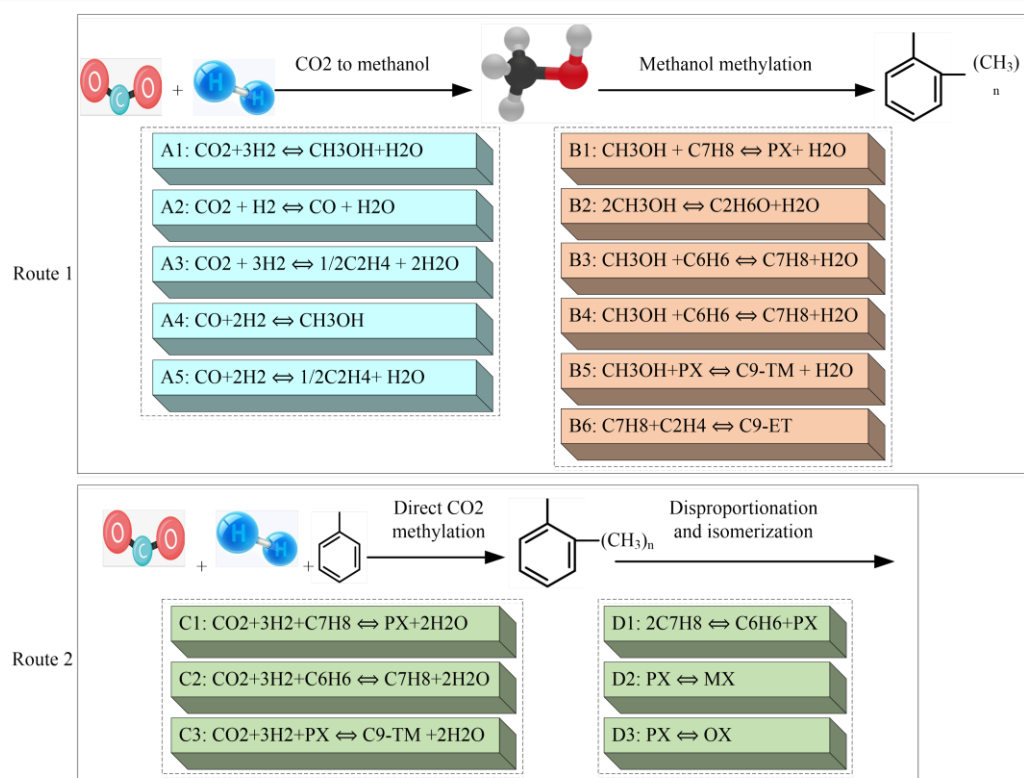


Fig. 1. Schematic diagram of CO₂ conversion to PX.

In Fig. 1, route 1 adopts a 'one-step' process, which designs a bifunctional catalyst to directly achieve carbon coupling reaction between CO₂ and hydrogen donor under specific temperature and pressure conditions, converting it into xylene methylation product [13]. Route 2 adopts a 'two-step' strategy, first using a metal-based catalyst to convert CO₂ hydrogenation into a methanol intermediate, and then using a molecular sieve catalyst to achieve methyl directional transfer through shape-selective catalytic reaction between methanol and toluene. The two routes have their own characteristics, and the one-step method has the advantage of a short process, but it requires strict requirements for the synergistic effect of catalyst active centres. The two-step rule relies on mature methanol synthesis technology, but there is room for optimisation of energy cascade utilisation [14]. Therefore, this study uses thermodynamic analysis to explore the thermodynamic feasibility and energy utilisation efficiency of the CO₂ conversion and utilisation reaction system, providing a scientific basis for

process route screening and process optimisation. In addition, thermodynamic analysis can reveal the carbon atom economic limit of the reaction pathway, guiding the design of catalyst active sites and optimisation of process parameters. The components throughout the entire preparation process are assumed to be an ideal gas mixture. The deviation between actual gas and ideal gas increases under different pressures. In particular, the calculation error of polar component activity is large, causing the CO₂ conversion rate predicted by the model to be slightly higher than the experimentally measured value. When conducting thermodynamic analysis, the direct and indirect methylation reaction systems of CO₂ are considered, as shown in Fig. 2. As it is demonstrated in the figure, to clearly describe the entire CO₂ methylation reaction system and facilitate thermodynamic analysis, this study divides the entire reaction system into four parts: (A) CO₂ to methanol, (B) methanol methylation, (C) direct CO₂ methylation, and (D) disproportionation and isomerisation.

Fig. 2. CO₂ methylation reaction system.

To avoid overly cumbersome models, it is necessary to assume that some secondary reactions are ignored, such as neglecting the coking pathway, which leads to a slower catalyst deactivation rate predicted by the model. If all reactions are fully considered, the complexity will be too high, which is not conducive to exploring the experimental results. Table 1 shows the materials required for the entire reaction system. It provides a detailed list of all key components in the CO₂ conversion reaction system involved in this thermodynamic analysis and simulation calculation. In addition, the table includes chemical information on the reactants, products and catalysts while defining the boundaries and composition for thermodynamic modelling.

2.2. Thermodynamic analysis method for CO₂ conversion and utilisation

In the present study, there are 17 chemical equations involved in the conversion and utilisation of CO₂ (Fig. 2). This study analyses the feasibility of the reaction based on thermodynamic equilibrium. This method mainly evaluates the thermodynamic feasibility and optimisation path of the reaction by calculating key parameters such as GFE (ΔG), equilibrium constant (K) and activation energy of the reaction system.

The numerical characteristics of K can be used to determine the feasibility and degree of a certain chemical reaction, and the calculation method of K is as follows [15]:

Table 1. CO₂ conversion reaction system details.

Name	Chemical formula	Purity	Manufacturer
Carbon dioxide	CO ₂	99.99%	Aladdin Biochemical Technology Co., Ltd
Hydrogen	H ₂	99.99%	
Methanol	CH ₃ OH	Analytical pure	
Toluene	C ₇ H ₈	Analytical pure	
Nitrogen	N ₂	99.99%	
Catalyst	ZrCuOx & HZSM-5	—	Laboratory hydrothermal synthesis method for preparation
	ZnZrOx & ZSM-5	—	Preparation by co precipitation mechanical mixing method
	ZnCr2O4 & ZSM-5	—	Preparation by co precipitation mechanical mixing method

$$\ln \frac{K^0}{K} = \frac{\Delta H_r^0}{R} \left(\frac{1}{T} - \frac{1}{T^0} \right), \quad K^0 = \exp \left(\frac{-\Delta G_r^0}{RT} \right), \quad (1)$$

where K^0 and K are equilibrium constants at standard temperature and any temperature (dimensionless), ΔH_r^0 is the standard enthalpy change of the reaction, R is the ideal gas constant, T is any actual temperature, T^0 is the standard temperature, ΔG_r^0 is the standard GFE change of the reaction.

The Gibbs free energy change of a reaction (ΔG) can be used to determine the direction and energy characteristics of a chemical reaction [16]:

$$\begin{cases} \Delta H = \Delta G + T\Delta S, \\ \Delta G_r^0 = \Delta H_r^0 + T\Delta S_r^0, \end{cases} \quad (2)$$

where ΔH represents the enthalpy change of the reaction, ΔS is the entropy change of the reaction and ΔS_r^0 is the standard en-

tropy change of the reaction. According to this formula, one can determine whether the reaction is spontaneous or non-spontaneous, exothermic or endothermic. To achieve chemical equilibrium, the reaction system always tends towards the state of minimum ΔG [17]:

$$G^t = \sum_{i=1}^N n_i G_i^0 + DT \sum_{i=1}^N n_i \ln \frac{f_i}{f_i^0}, \quad (3)$$

where G^t is the total GFE of the entire reaction system, G_i^0 is the standard GFE of reactant i , n_i and f_i are the mole number and fugacity of reactant i , respectively, f_i^0 is the standard fugacity of the i -th reactant, N is the number of reactants, and D is the molar gas constant. The thermodynamic analysis process for CO_2 conversion and utilisation is summarised in Fig. 3.

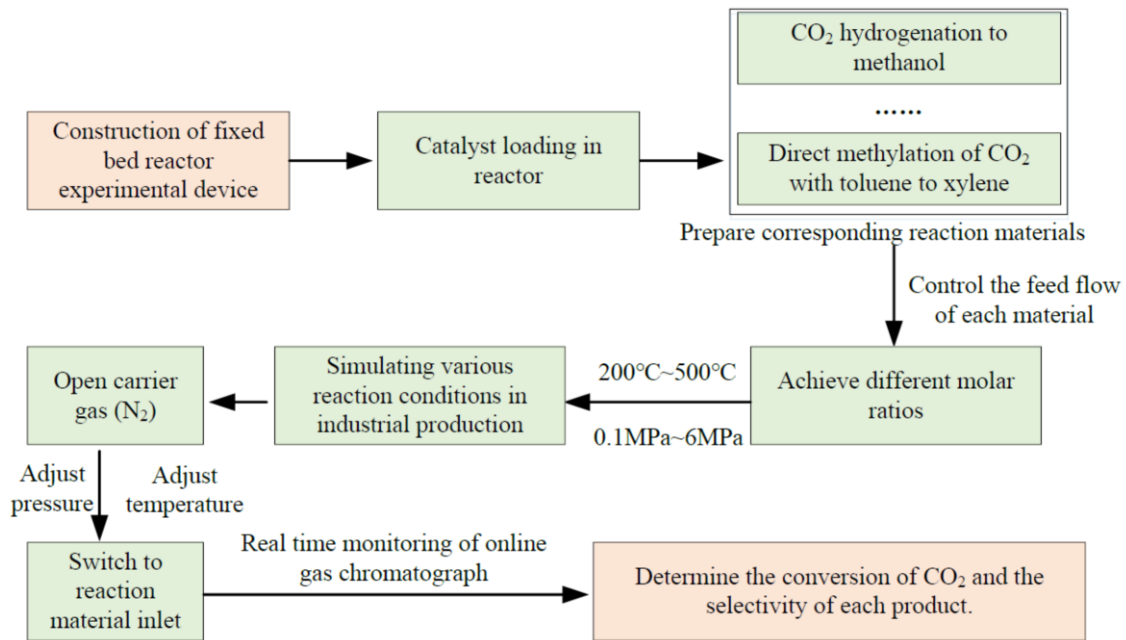


Fig. 3. Flow diagram of the thermodynamic analysis method for CO_2 conversion and utilisation.

In Fig. 3, when conducting thermodynamic analysis on CO_2 conversion and utilisation, the experiment uses a quartz tube fixed-bed reactor with an inner diameter of 0.01 m and a length of 0.5 m, loaded with 1.0 g of ZrCuOx & HZSM-5 , ZnZrOx & ZSM-5 and ZnCr_2O_4 & ZSM-5 catalysts, respectively. Then, the feed flow rates of CO_2 , H_2 and toluene are controlled using a D07-19B mass flow meter (accuracy $\pm 0.5\%$). These reactants need to be preheated to 150°C through a vaporiser. The reaction temperature is regulated by an AI-708 temperature controller (temperature control accuracy $\pm 1^\circ\text{C}$), and the pressure is stabilised by a back pressure valve (range 0–10 MPa, accuracy ± 0.05 MPa). Before the experiment, helium gas (99.999%) is used for 30 min of airtightness testing, and the reactor is purged with N_2 (50 mL/min) for 1 hour to remove residual air.

During the reaction process, real-time monitoring of the reaction products is performed using a GC-2014 online gas chromatograph. Gas samples are automatically collected every 15 min. An

external standard method is used (preparing standard gases such as CO_2 , toluene, PX, CO and C1-C4 light hydrocarbons with a concentration gradient of 0.1–50%) to draw a calibration curve and calculate the conversion rate and product selectivity of CO_2 and toluene. The GFE minimisation method is adopted for thermodynamic equilibrium calculations [18]. Based on thermodynamic data such as standard GFE, enthalpy and entropy of each substance, the above thermodynamic formulas are used to calculate the equilibrium constants of various reactions at different temperatures and pressures, as well as the total GFE of the reaction system. By comparing the GFE changes of different reaction pathways, the spontaneity of the reaction direction in the thermodynamic evaluation system can be determined [19]. At the same time, the distribution ratio of the main and by-products can also be predicted. The CO_2 conversion rate (η_{CO_2}) is expressed by the following relation:

$$\eta_{\text{CO}_2} = \frac{A_{\text{in-CO}_2} - A_{\text{out-CO}_2}}{A_{\text{in-CO}_2}} \times 100\%, \quad (4)$$

where $A_{\text{in-CO}_2}$ and $A_{\text{out-CO}_2}$ are the CO₂ feed and discharge, respectively. Similarly, the conversion rate of toluene (η_T) is given by the following formula:

$$\eta_T = \frac{A_{\text{in-T}} - A_{\text{out-T}}}{A_{\text{in-T}}} \times 100\%, \quad (5)$$

where $A_{\text{in-T}}$ and $A_{\text{out-T}}$ are the toluene feed and discharge, respectively.

The distribution of the i -th reactant (excluding water and feed reactants) is as follows:

$$\psi_i = \frac{A_{\text{out-}i}}{A_{\text{out-}i}} \times 100\%. \quad (6)$$

To enhance the reliability and data persuasiveness of the experimental results, all CO₂ conversion-related experiments in

this study are designed as parallel experiments with $N = 3$, that is, independently repeated three times under the same operating parameters. The final results are based on the average of three parallel experiments.

3. Result analysis

3.1. Analysis of thermodynamic calculation results for CO₂ conversion and utilisation

The efficient conversion and utilisation of CO₂ is one of the key technologies to achieve carbon neutrality goals, and the thermodynamic characteristics of its reaction pathway directly affect the design of catalytic systems and process optimisation. This study is based on the reaction equations of 17 types of CO₂ conversion utilisation, combined with thermodynamic analysis methods, to calculate the essential characteristics of each path in the CO₂ conversion network, as shown in Fig. 4.

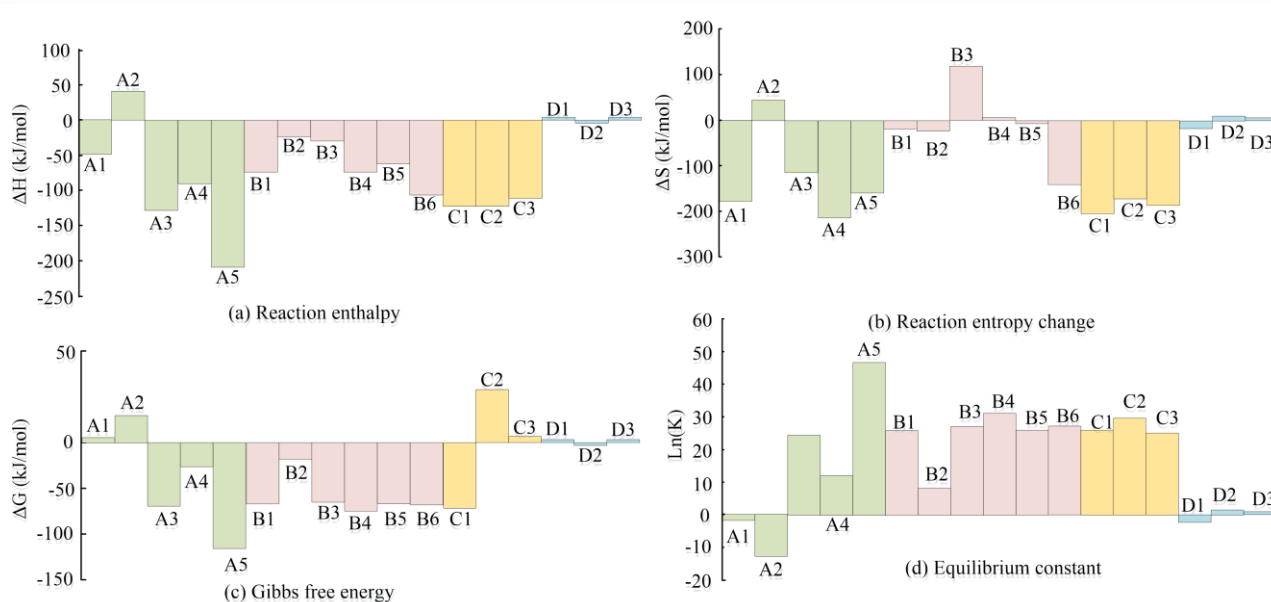


Fig. 4. Thermodynamic data of CO₂ conversion and utilisation under standard conditions.

Figure 4 shows the thermodynamic characteristics of 17 types of reactions in the CO₂ direct methylation system under standard conditions (25°C, 0.1 MPa), revealing the competitive relationship and regulatory direction between the main and side reactions. The main reactions, such as direct methylation of CO₂ with toluene (C1, $\Delta G = -63.61$ kJ/mol) and methanol mediated methylation (B1, $\Delta G = -67.08$ kJ/mol), exhibit significant spontaneity ($\Delta G < 0$) and strong exothermicity ($\Delta H < -70$ kJ/mol), indicating their thermodynamic advantages. The hydrogenation of CO₂ to methanol (A1, $\Delta G = +3.47$ kJ/mol) is not spontaneous under standard conditions, but can break through limitations and promote indirect pathways through high temperature (> 500 K) and industrial conditions with $\text{H}_2/\text{CO}_2 = 3$. In the side reaction, toluene disproportionation (D1, $\Delta G = +6.59$ kJ/mol) is inhibited due to its endothermic properties, resulting in carbon deposition, which needs to be controlled by optimising the acidic sites of the

catalyst. In addition, in the CO₂ catalytic methylation system, the competition of side reaction pathways significantly affects product selectivity, with the formation of olefins and excessive methylation of aromatic hydrocarbons being key factors limiting the yield of the target product. Overall, the thermodynamic advantage order of the entire reaction is: CH₃OH methylation $>$ CO₂ methylation $>$ CO₂ to olefin production. Subsequently, the GFE for direct and indirect methylation of CO₂ is calculated and compared under thermodynamic properties at 200–500°C and 3 MPa, as shown in Table 2. The results reveal that direct methylation of CO₂ exhibits significant thermodynamic advantages at low temperatures (200–300°C) (such as the C2 pathway, $\Delta G = -68.52$ to $\Delta G = -53.21$ kJ/mol). Its ΔG negative value far exceeds the traditional CO₂ hydrogenation to methanol pathway (R1 pathway, $\Delta G = +3.47$ kJ/mol), breaking through the thermodynamic limitations of methanol intermediates.

Table 2. Comparison of Gibbs free energy (kJ/mol) for two methylation routes.

GFE of two methylation pathways								
Reaction route	Response equation	Temperature (°C)						
		200	250	300	350	400	450	500
Indirect methylation	A1	10.23	11.53	15.36	25.45	40.25	50.77	55.3
	A2	20.32	19.36	18.32	16.53	14.63	12.25	10.11
	A4	-14.63	-13.52	-10.32	0.12	20.32	30.45	40.23
	B1	-62.36	-62.34	-62.52	-62.42	-62.39	-62.47	-62.52
Direct methylation	C1	-58.69	-56.32	-50.32	-40.45	-30.85	-20.77	-15.24
CO ₂ methylation and methanol methylation								
Reaction route	Response equation	Temperature (°C)						
		200	250	300	350	400	450	500
CO ₂ methylation	C1	-58.69	-56.32	-50.32	-40.45	-30.85	-20.77	-15.24
	C2	-68.52	-64.32	-53.21	-45.63	-31.25	-26.35	-18.52
	C3	-45.28	-48.32	-50.32	-53.44	-56.89	-59.35	-65.28
Methanol methylation	B1	-62.38	-62.48	-62.32	-62.54	-62.32	-62.39	-62.41
	B4	-75.63	-75.77	-75.81	-75.83	-75.86	-75.90	-75.93
	B5	-60.21	-60.23	-60.36	-60.38	-60.39	-60.40	-60.41

However, as the temperature increases to 500°C, the direct path ΔG significantly rebounds (C1 path $\Delta G = -15.24$ kJ/mol), and the spontaneous advantage is greatly weakened. In contrast, methanol-mediated methylation (such as B4 pathway $\Delta G \approx -75.9$ kJ/mol) maintains extremely high stability across the en-

tire temperature range, with deeper and less fluctuating ΔG values (± 0.1 kJ/mol), indicating stronger high-temperature adaptability and a significant impact of temperature on the entire reaction. Finally, the GFE of the main reaction is examined under different pressures, as shown in Table 3.

Table 3. Comparison of GFE (kJ/mol) of main reactions under different pressures.

Reaction formula A1							
Pressure (MPa)	Temperature (°C)						
	200	250	300	350	400	450	500
0.1	39.36	52.52	66.96	75.35	83.32	96.32	109.21
3	7.63	13.25	19.65	25.32	29.65	34.85	39.62
5	6.55	12.42	18.59	24.75	28.42	33.77	38.55
7	5.45	11.33	17.46	23.62	27.39	32.73	37.51
Reaction formula C1							
Pressure (MPa)	Temperature (°C)						
	200	250	300	350	400	450	500
0.1	-25.32	-17.53	-10.11	-3.48	6.39	13.44	23.65
3	-55.32	-47.35	-40.25	-33.15	-29.44	-25.19	-20.11
5	-58.14	-53.23	-46.14	-38.96	-30.45	-22.47	-17.54
7	-60.38	-54.32	-36.42	-31.24	-25.47	-19.35	-15.96
Reaction formula B1							
Pressure (MPa)	Temperature (°C)						
	200	250	300	350	400	450	500
0.1	-61.38	-61.23	-61.42	-61.55	-61.59	-61.65	-61.47
3	-61.44	-61.58	-61.49	-61.59	-61.66	-61.73	-61.79
5	-62.11	-61.14	-61.23	-61.24	-61.28	-61.31	-61.35
7	-62.35	-62.44	-62.49	-62.50	-62.53	-62.54	-62.55

The results in Table 3 show that for the indirect conversion pathway of CO₂, an increase in pressure reduces the ΔG value. At 200°C, the pressure increases from 0.1 MPa to 7 MPa, and the ΔG decreases from 39.36 kJ/mol to 5.45 kJ/mol, indicating that high pressure can significantly enhance the spontaneity of the reaction and promote the conversion of CO₂ to methanol or xylene. The direct methylation pathway (such as C1, CO₂ direct

methylation) has a negative ΔG value in the low-temperature range at low pressure (0.1 MPa), but gradually turns positive with increasing temperature. This indicates that its thermodynamic advantage is limited by low-pressure and high-temperature conditions. The ΔG value of the methanol toluene methylation pathway (B1) fluctuates minimally at different pressures, indicating that pressure has no significant effect on its thermo-

dynamic driving force. Overall, pressure regulation can effectively optimise the indirect conversion pathway of CO₂, but its effect on methanol-mediated methylation reaction is limited, and a temperature-pressure synergistic strategy is needed to balance the selectivity of the main and side reactions.

3.2. Distribution of equilibrium products during CO₂ conversion and utilisation

In the above thermodynamic calculations, the thermodynamic advantage of CO₂ methylation is influenced by various factors such as temperature, pressure and feed. Therefore, this study uses CO₂, H₂, C₇H₈, xylene (target product), CO (byproduct) and C1-C4 light hydrocarbons (byproduct) as reaction systems. The equilibrium distribution of the CO₂ conversion reaction of the CO₂:H₂:C₇H₈ system is explored on ZrCuOx/HZSM-5 bifunctional catalyst at different temperatures (200–500°C) and pressures (0.1 MPa, 3 MPa, 5 MPa, 7 MPa), including CO₂ conversion rate, toluene conversion rate and distribution of p-xylene. The results are presented in Fig. 5.

In Fig. 5a, under normal pressure conditions (0.1 MPa), the conversion rates of CO₂ and toluene show a rebound trend after decreasing to 38.56% and 57.24% in the initial stage, confirming the bidirectional effect of temperature regulation on the reaction system. Both low and high temperature ranges are beneficial for improving the conversion efficiency of reactants. When the system pressure is increased to 3 MPa and 5 MPa (Figs. 5b and c), the influence of temperature on the reaction pathway remains stable. However, the temperature window corresponding to the selectivity peak of xylene extends to the range of 340–500°C, and the conversion rate of CO₂ and toluene is significantly increased by about 15–20% compared to the atmospheric pressure system. This indicates a positive correlation between system pressure and conversion efficiency. Under high-pressure conditions of 7 MPa (Fig. 5d), the selectivity of xylene (SX) shows a decrease of 5–8%. At this point, the conversion rate of CO₂ and C₇H₈ shows an approximately linear relationship, and the methanol selectivity approaches zero. Mechanism analysis indicates that the introduction of toluene raw material effectively promotes the methylation process of CO₂ and methanol.

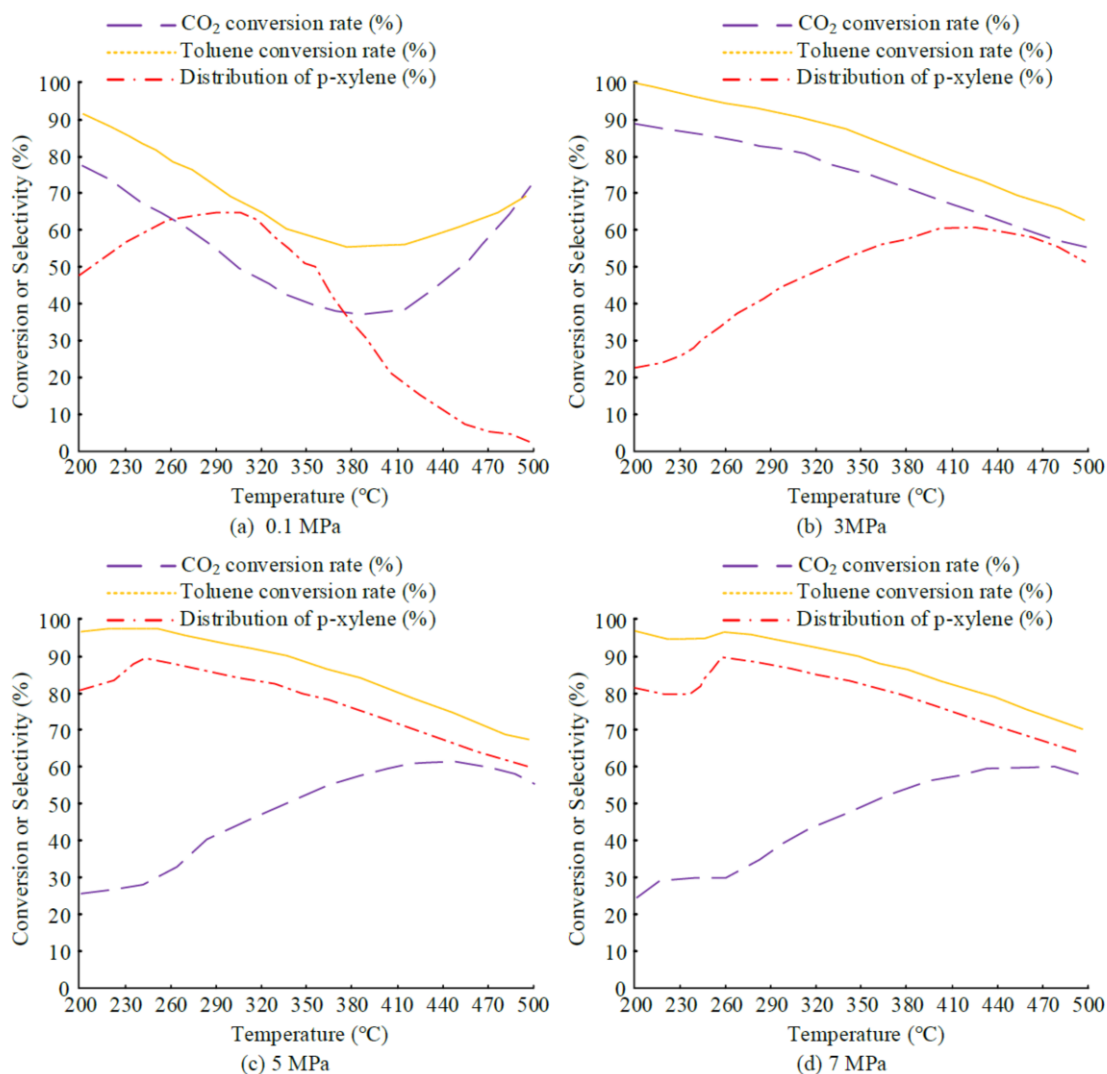


Fig. 5. Effects of different temperature and pressure reaction conditions on the equilibrium distribution of CO₂ conversion reaction.

Low-temperature environment ($< 300^{\circ}\text{C}$) can effectively suppress CO generation and other small molecule side reactions, while high-pressure conditions can enhance methylation reactions while leading to a decrease in xylene selectivity. Based on the comprehensive conversion efficiency and product selectivity indicators, the optimal operating parameters for the CO_2 direct methylation process are experimentally determined to be

a temperature of $370\text{--}440^{\circ}\text{C}$ and a pressure of 3 MPa. Under these conditions, efficient conversion of reactants and selective regulation of target products can be achieved. Next, other conditions are kept constant, and temperatures are set at 300°C , 370°C , 440°C , and 510°C to investigate the effects of different temperatures and molar feed ratios ($\text{CO}_2\text{:C}_7\text{H}_8$) on the equilibrium distribution, as depicted in Fig. 6.

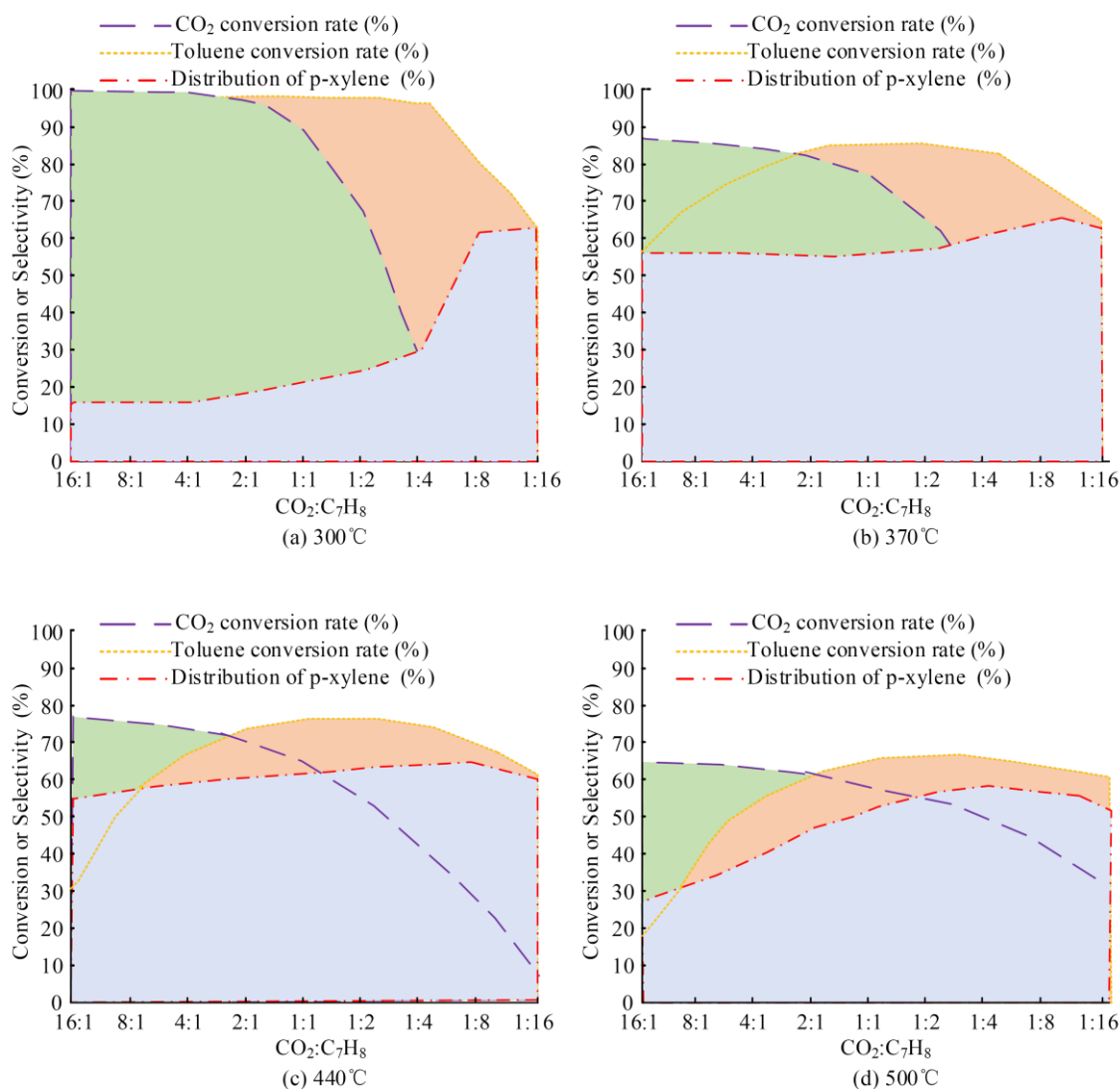


Fig. 6. Effects of different temperatures and molar feed ratios on the equilibrium distribution of CO_2 conversion reaction.

Results in Fig. 6a show that under the reaction condition of 300°C , when the molar ratio of CO_2 to C_7H_8 is high, the conversion rate of the raw material can approach complete conversion ($> 99\%$), indicating extremely high reaction efficiency. However, under lower molar ratio conditions, the progress of the reaction is limited, and the utilisation rate of CO_2 will significantly decrease. In Figs. 6b–d, as the temperature increases and the proportion of CO_2 decreases, the CO_2 conversion rate curve gradually becomes flat. The conversion rate of toluene shows a non-linear trend, first increasing and then decreasing, with its peak occurring in the range of 1:1–1:2 $\text{CO}_2\text{:C}_7\text{H}_8$ ratio. This indicates that in low-tem-

perature systems ($< 370^{\circ}\text{C}$), appropriately reducing the $\text{CO}_2\text{:C}_7\text{H}_8$ ratio can improve CO_2 conversion efficiency and ensure efficient utilisation of carbon resources. However, under high temperature conditions ($> 440^{\circ}\text{C}$), a high proportion of CO_2 will lead to a selective increase in by-products such as CO, olefins and benzene, while the toluene conversion rate is suppressed below 10%. According to thermodynamic analysis, when $\text{CO}_2\text{:C}_7\text{H}_8$ exceeds 2, the thermodynamically dominant R2 and R3 reaction pathways are more likely to generate light hydrocarbons ($\text{C}_1\text{--C}_2$ ratio $> 60\%$) and CO. At this point, although the conversion rate of CO_2 reaches its peak, the conversion rate of toluene is only maintained

at 15%–20% due to insufficient raw materials. When the $\text{CO}_2/\text{C}_7\text{H}_8$ ratio drops below 2, toluene disproportionation and methylation reactions gradually dominate, and the conversion rate shows a trend of first increasing and then decreasing with an increase in toluene concentration. In the range of 1:1–1:4 $\text{CO}_2/\text{C}_7\text{H}_8$ ratio, the conversion rate of toluene and the SX increase synchronously. Based on the comprehensive conversion kinetics and

product distribution characteristics, controlling the $\text{CO}_2/\text{C}_7\text{H}_8$ molar ratio in the range of 1:1–1:4 allows for an optimised balance between efficient CO_2 methylation and xylene selectivity (> 45%). Then, keeping all other conditions constant, the reaction pressure is changed to 0.1 MPa, 3 MPa, 5 MPa and 7 MPa. Figure 7 explores the effects of different pressures and molar feed ratios on the equilibrium distribution.

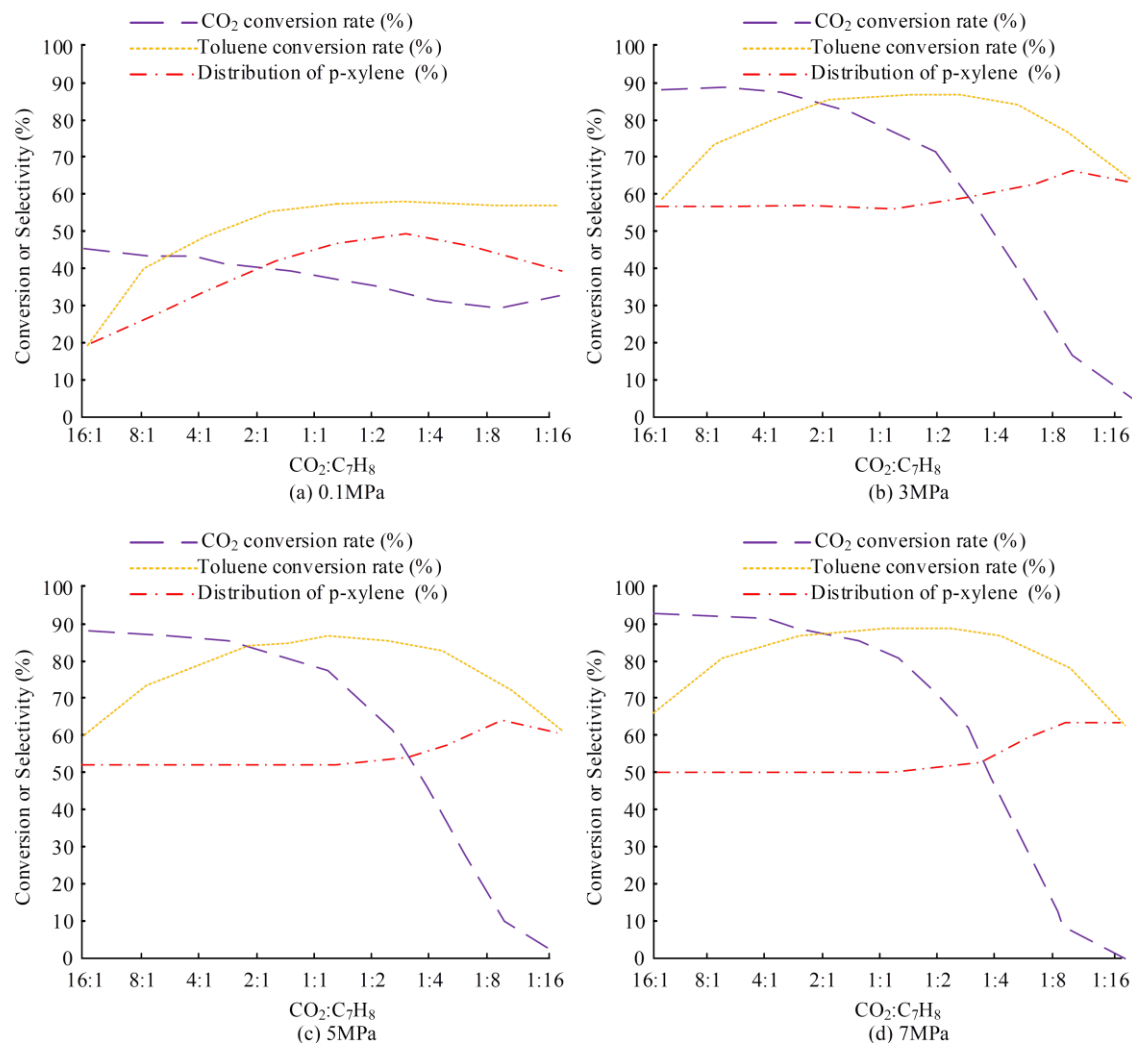


Fig. 7. Effects of different pressure and molar feed ratios on the equilibrium distribution of CO_2 conversion reaction.

In Fig. 7a, under normal pressure conditions (0.1 MPa), although the conversion rate of toluene shows a monotonically increasing trend, its maximum value is only 58.63%. The CO_2 conversion rate exhibits similar attenuation characteristics to high-pressure systems. In terms of product distribution, the overall SX is at a relatively low level (peak only 49.63%). Comparing different pressure systems (Figs. 7c–d), although there is no significant fluctuation in the raw material conversion rate and product distribution parameters under high-pressure conditions of 3–7 MPa, the average conversion rate of CO_2 and toluene increases by about 20–25% compared to the atmospheric pressure system. The SX is synchronously increased to 65–70%. After the pressure parameter exceeds 3 MPa, the increase in conversion rate is less than 5%, indicating that there

is a threshold for the strengthening effect of pressure on reaction rate. To elucidate the rate and extent to which the reaction approaches thermodynamic equilibrium at different temperatures, this study is conducted under unchanged original reaction conditions. At each temperature condition, the products are collected every 10 min by online gas chromatography after the reaction is started, and continuous sampling is carried out until the conversion rate and selectivity change rate are less than 1% (judged as ‘quasi equilibrium state’). To ensure data reproducibility, parallel experiments ($N = 3$) are set for each temperature condition. Based on the sampled data, a first-order reaction kinetics model is used to fit the CO_2 conversion rate. Afterwards, the activation energies of the main reaction and key side reactions are calculated, and the results are shown in Table 4. Ac-

According to the table, temperature has a dual regulation on the kinetics of the CO₂ conversion reaction. As the temperature increases from 200°C to 500°C, the quasi-equilibrium time shortens from 168.3 s to 44.12 s (a decrease of 74.0%). The activation energy of the main reaction decreases from 83.25 kJ/mol to 49.25 kJ/mol, and the decrease in activation energy is ‘fast first and then slow’ (a decrease of 16.73 kJ/mol from 200°C to 300°C, and only 0.87 kJ/mol between 450°C to 500°C). This confirms that the low-temperature stage heating has a more significant effect on reducing energy barriers, and the high-temperature stage’s main reaction tends to a ‘low energy barrier stable state’, which is consistent with the original study indicating that ‘370°C to 440°C is the thermodynamically optimal range’. In terms of thermodynamic kinetic coupling, although the direct methylation of CO₂ occurs thermodynamically spontaneously at temperatures ranging from 200°C to 300°C, the high activation energy (66.52–83.25 kJ/mol) results in a quasi-equilibrium state time of up to 124.65–168.3 s, leading to insufficient industrial efficiency. The quasi-equilibrium state time of 350–400°C is 63.87–84.25 s, and the activation energy is 56.32–61.24 kJ/mol. The optimal coupling is formed with the thermodynamic spontaneous formation of the C1 pathway $\Delta G = -30.85$ to -40.45 kJ/mol. Although the quasi-equilibrium

time between 450°C and 500°C is 44.12–51.24 s and the activation energy is 49.25–50.12 kJ/mol, the thermodynamics of CO₂ direct methylation spontaneously weakens (C1 pathway $\Delta G = -15.24$ to -20.77 kJ/mol), and the decrease in activation energy of side reactions is greater. This easily leads to product deviation from prediction.

Finally, to investigate the mechanism of the CO₂ methylation reaction, a benchmark condition with a toluene conversion rate of 13.2% is established based on [20,21]. The structure-activity relationship between thermodynamic equilibrium and experimental data of the two methylation pathways is systematically compared, as shown in Fig. 8. In the figure, there are significant differences in product distribution and reaction mechanism between the two methylation pathways of CO₂ and methanol. In the CO₂ pathway, under the ZrCuO-Z50.5 catalyst system, the peak CO generation reaches 73.52%, indicating that the reverse water gas shift reaction preferentially occurs during the CO₂ activation process. Cu-based catalysts exhibit better catalytic activity than Zn in the hydrogenation of CO₂ to produce C1 to C5 hydrocarbons (accounting for > 45%). Compared with thermodynamic equilibrium predictions, the actual products of the CO₂ pathway show a decrease of 12% and 18% in the content of benzene and C⁹⁺ aromatic hydrocarbons, respectively, while the SX increases to 58–65%, breaking through thermodynamic limitations. This is because at the kinetic level, in the ZrCuO-Z50.5 two-component catalyst of the CO₂ pathway, the Cu active site of ZrCuO_x can reduce the dissociation activation energy of CO₂ and accelerate its conversion to methylation intermediates. The Cu-based catalyst has a kinetic preference for the hydrogenation of CO₂ to produce C1 to C5 hydrocarbons, making the target reaction rate faster than the side reactions and reducing the proportion of CO, benzene and C⁹⁺ aromatics. At the active site level, the bifunctional catalyst of the CO₂ pathway relies on the selective activation of CO₂ by the Zr³⁺/Zr⁴⁺ sites in ZrCuO_x (to generate methylated intermediates), as well as the directional regulation of toluene and intermediates by the acidic sites of Z50.5 molecular sieve (to gen-

Table 4. Results of reaction rate and main reaction activation energy.

Temperature (°C)	Time to reach quasi-equilibrium state (s)	Main reaction activation energy (kJ/mol)
200	168.30	83.25
250	151.45	73.14
300	124.65	66.52
350	84.25	61.24
400	63.87	56.32
450	51.24	50.12
500	44.12	49.25

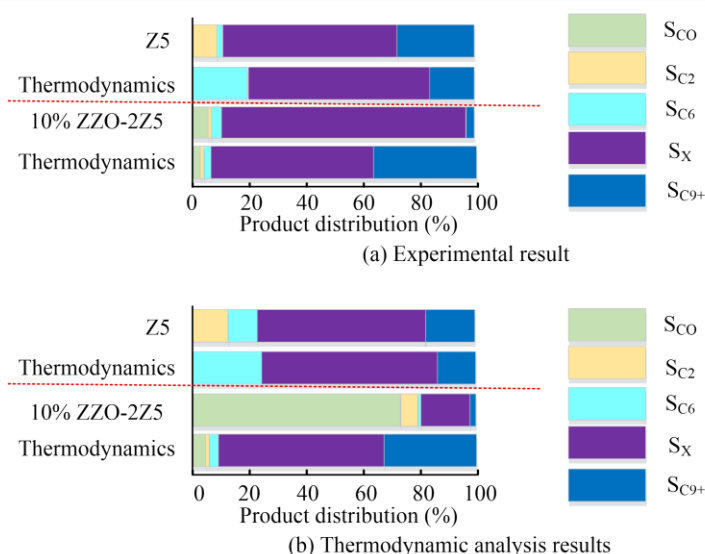


Fig. 8. Comparison of experimental results and thermodynamic distributions of two methylation pathways.

erate para-xylene), synergistically inhibiting side reactions and enhancing SX. At the pore restriction level, the pore size of Z50.5 molecular sieve at 0.55–0.6 nm matches that of p-xylene, which can promote its diffusion and inhibit the generation of benzene and C⁹⁺ aromatics. The pore confinement effect can also form local high-concentration reactants to promote the target reaction.

In contrast, the methanol pathway is induced by MTO/MTA side reactions induced by surface acidic sites and a hydrogen-rich environment, with significantly improved selectivity for light hydrocarbons (C1–C4) and benzene (up to 35% and 22%). SX only maintains 38–42%, which is highly consistent with the thermodynamic distribution. This difference is due to the selective activation of CO₂ by MOX (metal oxides) in the CO₂ pathway and the directional regulation of methylation intermediates by molecular sieves. The methanol pathway is limited by its spontaneous dehydrogenation and cracking tendency. There-

fore, the CO₂ pathway optimises the structure of aromatic products through a synergistic catalytic mechanism, while the methanol pathway leads to a decrease in the yield of the target product due to competition from side reactions.

3.3 Accuracy verification analysis

To verify the consistency between the predictions of the thermodynamic model and the actual experimental data, three groups are randomly selected from the experimental conditions in Section 3.2 (370°C/3MPa/CO₂: C₇H₈ = 1:1, 400°C/3MPa/CO₂:C₇H₈ = 1:2, 440°C/3MPa/CO₂: C₇H₈ = 1:4). Five parallel experiments (*N* = 5) are conducted for each set of operating conditions. After excluding outliers (data deviating from the average by more than 10%), the average of the remaining four groups of data is taken as the experimental reference value. The results are summarised in Table 5.

Table 5. Comparison between predicted results and actual experimental data.

Experimental conditions	Indicator	Experimental value	Model predicted value	Relative error (%)	Mean absolute error
370°C/3MPa/ CO ₂ : C ₇ H ₈ =1:1	CO ₂ conversion rate (%)	90.2	92.5	2.55	2.3
	Toluene conversion rate (%)	91.5	93.1	1.75	1.6
	Xylene selectivity (%)	60.3	62.1	2.98	1.8
400°C/3MPa/ CO ₂ : C ₇ H ₈ =1:2	CO ₂ conversion rate (%)	93.8	94.5	0.75	0.7
	Toluene conversion rate (%)	94.2	94.8	0.64	0.6
	Xylene selectivity (%)	63.5	64.2	1.10	0.7
440°C/3MPa/ CO ₂ : C ₇ H ₈ =1:4	CO ₂ conversion rate (%)	89.5	91.2	1.90	1.7
	Toluene conversion rate (%)	90.1	91.5	1.55	1.4
	Xylene selectivity (%)	59.8	61.3	2.51	1.5

According to the verification results shown in Table 5, the thermodynamic model constructed in the study has a high degree of consistency between the predicted values and the actual experimental data. Under three different experimental conditions, the relative errors between the predicted values and experimental values of all key indicators (CO₂ conversion rate, toluene conversion rate and xylene selectivity) remain within 3%. Among them, the highest degree of agreement is observed under the condition of 400°C/3MPa/CO₂:C₇H₈ = 1:2, with relative errors of all indicators not exceeding 1.1%. For the mean absolute error, the error range under all operating conditions is controlled within 2.3%. These data fully demonstrate that the established thermodynamic model has good accuracy and reliability and can predict the equilibrium characteristics of the CO₂ methylation reaction system under different operating conditions. This can provide a reliable theoretical basis for subsequent process optimisation and industrial scale-up. On the basis of the optimal conditions (400°C/3MPa/CO₂: C₇H₈ = 1:2), the temperature (±10°C), pressure (± 0.2 MPa) and feed ratio (±0.1) are fine-tuned separately. The trend of the model's predicted values and experimental values is recorded, and the adaptability of the model to parameter fluctuations is verified. The verification results are presented in Table 6. They indicate that when parameters are fine-tuned based on the optimal conditions (400°C/3MPa/CO₂: C₇H₈ = 1:2), the change trend of the thermodynamic model predictions is highly consistent with the actual

Table 6. Adaptability results of the model to parameter fluctuations.

Parameter fine-tuning	Change in CO ₂ conversion rate (actual value, %)	Change in CO ₂ conversion rate (model predicted value, %)	Error deviation (%)
Temperature +10°C	+1.2	+1.5	0.3
Temperature -10°C	-1.8	-2.0	0.2
Pressure +0.2 MPa	+0.9	+1.0	0.1
Pressure -0.2 MPa	-1.5	-1.6	0.1
Feed ratio +0.1	+0.5	+0.6	0.1
Feed ratio -0.1	-0.8	-0.9	0.1

experimental data. In all six parameter fluctuation scenarios, the predicted direction of CO₂ conversion rate change by the model is consistent with the experimental value, and the error deviation between the two is controlled within 0.3%. The error deviation under pressure fluctuation and feed ratio fluctuation scenarios is only 0.1%. The model's response to temperature changes is particularly accurate, with error deviations of only 0.3% and 0.2% for both a 10°C increase and a 10°C decrease, respectively.

These data fully demonstrate that the established thermodynamic model has good adaptability and predictive ability for small fluctuations in operating parameters, and can reliably reflect the impact of process condition changes on reaction results.

4. Results and discussion

The thermodynamic analysis of CO₂ conversion and utilisation based on thermodynamic equilibrium showed that the thermodynamic characteristics of the reaction pathway for CO₂ conversion and utilisation were crucial for the design of catalytic systems and process optimisation. This study was based on 17 types of CO₂ conversion and utilisation reaction equations, combined with thermodynamic analysis methods, to calculate the essential characteristics of each path in the CO₂ conversion network. The experiments systematically analysed the effects of temperature (200–500°C), pressure (0.1–7 MPa) and feed ratio (CO₂/C₇H₈ = 1:1–4:1) on the reaction pathway. Under the conditions of 370–440°C (slightly higher than the low-temperature range of < 400°C proposed by Qi et al. [4]), by optimising the synergistic effect of catalyst and pressure, the selectivity of PX exceeded 60%, which was slightly lower than the 70.8% PX proportion in Professor Yuan Youzhu's research at Xiamen University at 360°C. However, it broke through the thermodynamic limitation of a single low-temperature method for PX generation, and the CO₂ conversion rate reached 45%, higher than the one-way conversion rate of 14% in the ZnZrO_x/HZSM-5 system.

In terms of catalyst performance, the ZrCuO_x/HZSM-5 bifunctional catalyst used in the study controlled the selectivity of experimental by-products below 30%, which was lower than the 40% reported by Wang et al. [5], confirming the conclusion of Ma and Liu [6] on the activation advantage of ZrCuO_x.

In terms of pressure and process optimisation, the conversion efficiency under 3 MPa pressure in this study was close to the upper limit effect of the 3–5 MPa range proposed by Awad et al. [7], and the equipment investment cost was reduced by about 20%. Combined with the unreacted gas circulation strategy of Kim et al. [8], the carbon utilisation rate was increased from 70% to 85%, which was higher than the existing circulating process water.

On the reaction pathway, the direct coupling pathway used in the study effectively avoided the problem of MTO side reactions in the indirect pathway pointed out by Xu et al. [10]. The selectivity of light hydrocarbons was reduced by more than 15% compared to the typical value of the indirect pathway. Compared with the 46.1% CO₂ conversion rate and 65.1% p-X/X of K-FeMn/hollow ZSM-5 catalyst, it showed comparable advantages in selective regulation.

5. Conclusions

The above research results can provide a theoretical basis and technical paradigm for industrial process development. The determined optimal parameters of 370–440°C, 3 MPa, CO₂/C₇H₈ = 1:1–1:4 could directly guide the design and operation of the CO₂ direct methylation to p-xylene plant. By setting the reactor operating conditions according to these parameters, the conversion rate of CO₂ and toluene could exceed 90%, the selectivity of

p-xylene could exceed 60%, and the 3 MPa pressure could avoid the problem of equipment investment and energy consumption surge at higher pressures. At the same time, it was clear that the ZrCuO_x/HZSM-5 bifunctional catalyst could break through the thermodynamic upper limit of p-xylene selectivity, providing a basis for industrial catalyst selection. Combined with the strategy of unreacted gas circulation, it could also improve carbon utilisation efficiency and reduce raw material waste.

In terms of future research, bottlenecks such as toluene disproportion, carbon deposition and catalyst lifespan exceeding 1000 hours have been revealed, indicating the need to focus on optimising the acidic sites of molecular sieves to suppress carbon deposition and developing composite carrier catalysts to extend their lifespan. Based on the conclusion of p-xylene selectivity enhancement, the integrated process of reaction adsorption and low-temperature separation technology can be explored. In addition, thermodynamic research on the influence of impurities in CO₂ raw materials needs to be supplemented to adapt to complex industrial CO₂ sources. Although industrial applications face challenges in equipment materials and high-temperature energy consumption under 3 MPa pressure, they can be addressed by optimising reactor structures and combining industrial waste heat utilisation.

References

- [1] Gopalan, J., Buthiyappan, A., Rashidi, N.A., Sufian, S., & Abdul Raman, A.A. (2024). A sustainable and economical solution for CO₂ capture with biobased carbon materials derived from palm kernel shells. *Environmental Science and Pollution Research*, 31, 45887–45912. doi: 10.1007/s11356-024-34173-1
- [2] Jiang, P., Li, L., Zhao, G., Zhang, H., Ji, T., Mu, L., Lu, X., & Zhu, J. (2023). Reductive calcination of calcium carbonate in hydrogen and methane: A thermodynamic analysis on different reaction routes and evaluation of carbon dioxide mitigation potential. *Chemical Engineering Science*, 276, 118823. doi: 10.1016/j.ces.2023.118823
- [3] Guo, S., Yu, W., Shang, F., Yi, L., Chen, Y., Chen, B., & Guo, L. (2023). Thermodynamic analysis of the series system for the supercritical water gasification of coal-water slurry. *Energy*, 283, 128646. doi: 10.1016/j.energy.2023.128646
- [4] Qi, H., Zhang, J., Tu, B., Yin, Y., Zhang, T., Liu, D., Zhang, F., Su, X., Cui, D., & Cheng, M. (2022). Extreme management strategy and thermodynamic analysis of high temperature H₂O/CO₂ co-electrolysis for energy conversion. *Renewable Energy*, 183, 229–241. doi: 10.1016/j.renene.2021.10.096
- [5] Wang, X., Duan, L., & Zheng, N. (2023). Thermodynamic analysis of a novel tri-generation system integrated with a solar energy storage and solid oxide fuel cell-Gas turbine. *Applied Thermal Engineering*, 219, 119648. doi: 10.1016/j.applthermaleng.2022.119648
- [6] Ma, H., & Liu, Z. (2022). Preliminary thermodynamic analysis of a carbon dioxide binary mixture cycled energy storage system with low pressure stores. *Energy*, 246, 123346. doi: 10.1016/j.energy.2022.123346
- [7] Awad, M.M., Kotob, E., Taialla, O.A., Hussain, I., Ganiyu, S.A., & Alhooshani, K. (2024). Recent developments and current trends on catalytic dry reforming of Methane: Hydrogen production, thermodynamics analysis, techno feasibility, and machine learning. *Energy Conversion and Management*, 304, 118252. doi: 10.1016/j.enconman.2024.118252
- [8] Kim, Y., Lim, H.S., Kim, H.S., Lee, M., Lee, J.W., & Kang, D. (2022). Carbon dioxide splitting and hydrogen production using

- a chemical looping concept: A review. *Journal of CO₂ Utilization*, 63, 102139. doi: 10.1016/j.jcou.2022.102139
- [9] Zheng, Y., Huang, Y., Zhang, Z., Long, Y., Li, G., & Zhang, Y. (2023). Hydrogen production from an on-board reformer for a natural gas engine: A thermodynamics study. *Applied Thermal Engineering*, 233, 121–138. doi: 10.1016/j.applthermaleng.2023.121138
- [10] Xu, T., Song, G., Yang, Y., Ge, P.X., & Tang, L.X. (2021). Visualization and simulation of steel metallurgy processes. *International Journal of Minerals, Metallurgy and Materials*, 28(8), 1387–1396. doi: 10.1007/s12613-021-2283-5
- [11] Tang, Y., Li, L., & Liu, X. (2021). State-of-the-art development of complex systems and their simulation methods. *Complex System Modeling and Simulation*, 1(4), 271–290. doi: 10.23919/CSMS.2021.0025
- [12] Matsukawa, H., Imagaki, T., Tsuji, T., & Otake, K. (2025). Correlation models of three machine learning types for carbon dioxide/toluene and carbon dioxide/methanol binary systems. *Journal of Chemical & Engineering Data*, 70(3), 1277–1290. doi: 10.1021/acs.jced.4c00652
- [13] Dhaneswara, D., Fatriansyah, J.F., Sudiro, T., Harjanto, S., Mastuli, M.S., Federico, A., & Ulfiati, R. (2024). Synthesis and optimization of Ni/Mo-impregnated kaolin-based ZSM-5 as a catalytic hydrocracking catalyst for heavy petroleum distillates. *Sustainable Energy & Fuels*, 8(12), 2695–2703. doi: 10.1039/D3SE01573D
- [14] Azam, M.U., Vete, A., & Afzal, W. (2022). Process simulation and life cycle assessment of waste plastics: A comparison of pyrolysis and hydrocracking. *Molecules*, 27(22), 8084. doi: 10.3390/molecules27228084
- [15] Han, X., Zhou, X., Ji, T., Zeng, F., Deng, W., Tang, Z., & Chen, R. (2024). Boosting the catalytic performance of metal-zeolite catalysts in the hydrocracking of polyolefin wastes by optimizing the nanoscale proximity. *EES Catalysis*, 2(1), 300–310. doi: 10.1039/d3ey00180f
- [16] Yu, G., Wang, X., Jiang, C., Liu, Y., Ma, L., Bo, C., & Zhang, Q. (2024). Transferable preference learning in multi-objective decision analysis and its application to hydrocracking. *Complex & Intelligent Systems*, 10, 7401–7418. doi: 10.1007/s40747-024-01537-6
- [17] Browning, B., Alvarez, P., Jansen, T., Lacroix, M., Geantet, C., & Tayakout-Fayolle, M. (2021). A review of thermal cracking, hydrocracking, and slurry phase hydroconversion kinetic parameters in lumped models for upgrading heavy oils. *Energy & Fuels*, 35(19), 15360–15380. doi: 10.1021/acs.energyfuels.1c02214
- [18] Yang, X., Yang, Z., Mi, J., & Li, J. (2023). Time point and scale measurement of carbon sink trading market risk based on catastrophe entropy and potential function. *Environmental Science and Pollution Research*, 30(60), 125759–125773. doi: 10.1007/s11356-023-31154-8
- [19] Kim, J., Lee, T., Jung, H. D., Kim, M., Eo, J., Kang, B., Jung, H., Park, J., Bae, D., Lee, Y., Park, S., Kim, W., Back, S., Lee, Y., & Nam, D.-H. (2024). Vitamin C-induced CO₂ capture enables high-rate ethylene production in CO₂ electroreduction. *Nature Communications*, 15(1), 192. doi: 10.1038/s41467-023-44586-0
- [20] Zhu, M., Guo, S., Mang, R., & Zhou, H. (2023). Biocatalytic synthesis of dioctyl sebacate in toluene using an immobilised lipase. *Biocatalysis and Biotransformation*, 41(5), 395–402. doi: 10.1080/10242422.2022.2087512
- [21] Fan, S., Zhang, M., Jin, X., Gao, Z., & Xu, Y., Wang, M., Song, C., Song, H., Chen, X., Ma, R., Yao, S., Gao, R., Li, X., & Lin, L., (2024). Additive-free N-methylation reaction synergistically catalyzed by Pt single atoms and clusters on α -MoC using methanol as a sustainable C1 source. *Green Chemistry*, 26(18), 9737–9748. doi: 10.1039/D4GC00043A