

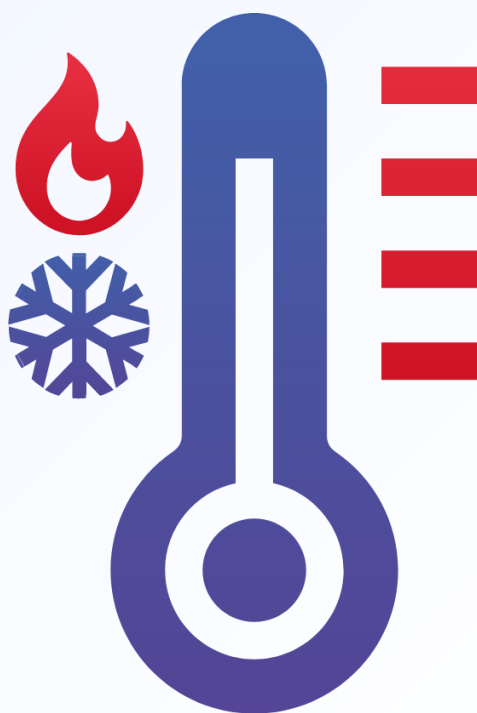
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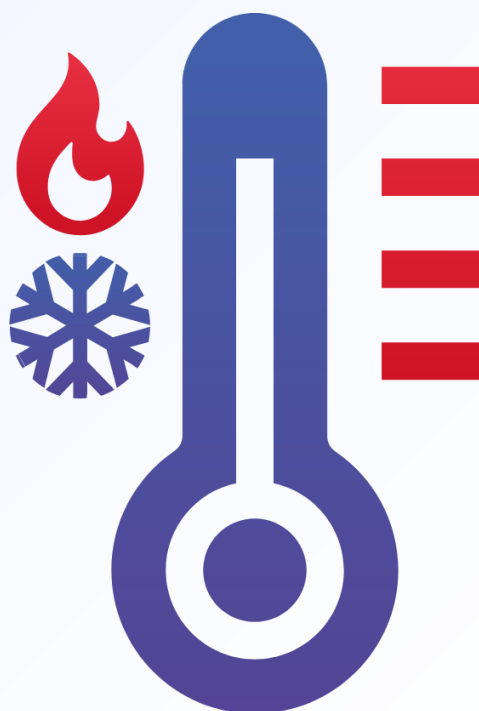
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Online monitoring of thermal loads of steam turbine components

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Abstract

The increasing share of renewable energy production in energy markets has resulted in the demand for more flexible operation of conventional power plants. Coal-fired units are forced to operate in a more cyclic duty with a higher number of starts, faster start-ups and increased load change rates. These unfavourable operating conditions result in additional thermal loading of steam turbine components and necessitate better monitoring of life-limiting components. The paper presents numerical investigations of thermal loading of a steam turbine valve at various operating conditions and identifies the requirements for an online monitoring method. A modified method based on Duhamel's integral is proposed and validated at different transient conditions, taking into account both convection and condensation heat transfer. It was shown that the calculation method provides reasonably accurate predictions of casing wall temperature. Relative differences between measured and calculated wall temperatures are within $(-2.6\%) \div (+1.5\%)$ for the investigated start-ups with condensation and convection heat transfer conditions. Advantages of this method and its suitability for online monitoring of thermal loads of steam turbine components are discussed in detail.

Keywords: Steam turbine; Thermal loads; Condensation; Duhamel's integral

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1. Introduction

The demand for more flexible operation of conventional power plants has significantly increased during the last years. The typical operation mode of coal fired plants has shifted from a more base load oriented regime towards cyclic operation. This is due to the increasing share of fluctuating renewable energy production [1].

Consequences of this trend are an increased number of starts, faster start-ups and increased load change rates. Additionally, the minimum load for stable plant operation has to be reduced to allow low load operation of conventional units. As a result of these changes in operation mode, thermal fatigue of power plant

equipment has become the main damage mechanism, which could potentially lead to component failures. This problem mainly affects the thick-walled components operating at high temperatures, like steam turbine rotors or valve casings.

In order to ensure safe and reliable operation of ageing power generation units, online monitoring of critical components has become necessary. In particular, for better understanding of the effect of changed operation mode on low cycle fatigue damage, accurate monitoring of thermal loads and stresses is essential.

The problem of thermal stress and load modelling for online monitoring of power plant equipment has been studied for many years, but has gained particular attention in recent years. Rusin et al. [2–4] proposed a new method of thermal stress modelling

Nomenclature

Bi – Biot number
 c_p – specific heat, J/(kg·K)
 d – hydraulic diameter, m
 g – heat source, J/(m³·s)
 h – enthalpy, J/kg
 H – Heaviside function,
 L – length, m
Nu – Nusselt number
 p – pressure, kPa
Pr – Prandtl number
 r – radial co-ordinate, m
Re – Reynolds number
 t – time, s
 T – temperature, K

Greek symbols

α – heat transfer coefficient, W/(m²·K)

λ – thermal conductivity, W/(m·K)
 ξ – friction factor
 ρ – density, kg/m³

Subscripts and Superscripts

a – air
 $cond$ – condensation
 $conv$ – convection
 f – fluid
 n – time step number
 sat – saturation
 st – steam
 $surf$ – surface
 w – wall

Abbreviations and Acronyms

FEM – finite element method
HS – hot start
WS – warm start

in turbine rotors employing Green's functions and Duhamel's integral, and steam temperature measurement at a critical location. Such an approach has also been adopted for monitoring of power boilers operation by Taler et al. [5], and as shown by Lee et al. [6], it can also be used for calculating stress intensity factors at transient thermal loads.

The major issue in using Green's function and Duhamel's integral method to model transient temperatures and thermal stresses in steam turbine components is the time-dependence of physical properties and heat transfer coefficients, affecting the proper evaluation of Green's functions. There are known approaches assuming determination of the influence functions at constant values of these quantities [7]. The inclusion of temperature dependent physical properties proposed by Koo et al. [8] relies on determining the weight functions for steady-state and transient operating conditions. A more important, in most cases, variation of the heat transfer coefficient can be taken into account by calculating the surface temperature using a reduced heat transfer model and employing Green's function to calculate stress response to a step change of metal surface temperature [9]. However, numerical solution of a multi-dimensional heat transfer model is complicated and time-consuming, and due to this, it cannot be used in online calculations. A full inclusion of time variability of the physical properties and heat transfer coefficients proposed by Zhang et al. [10] relies on the solution of a non-linear heat conduction problem by using the artificial parameter method and superposition rule, and replacing the time-dependent heat transfer coefficient with a constant value together with a modified fluid temperature. The effectiveness of the method has been proved by an example of a three-dimensional model of pressure vessel of a nuclear reactor.

Alternative methods of thermal stress modelling rely on using artificial neural networks [11–13] or inverse heat conduction methods [14–17]. Some researchers [18,19] investigated an approach combining direct heat conduction problems and Duhamel's integral.

This work presents numerical investigations of thermal loading of a steam turbine valve and proposes a modified method for

online monitoring of wall temperature employing Duhamel's integral. The main attention is given to appropriate consideration of different heat transfer mechanisms (condensation, convection) and determination of the temperature response function for a variable heat transfer coefficient. The proposed method is validated with the help of real measurements of casing wall temperature and finite element calculations.

2. Finite element modelling of turbine valve heating process

2.1. Problem formulation

The problem under consideration is transient heat conduction in a homogeneous isotropic solid described by the Fourier-Kirchhoff differential equation [20,21]:

$$\text{div}[\lambda \text{grad} T(\mathbf{r}, t)] + g(\mathbf{r}, t) = \rho c_p \frac{\partial T(\mathbf{r}, t)}{\partial t}, \quad (1)$$

where T is the metal temperature, g is the heat source, c_p is the specific heat, λ is the thermal conductivity and $\mathbf{r}$ is the spatial location.

Neglecting the heat source g and introducing the temperature Laplacian $\nabla^2 T$, Eq. (1) is rewritten in a rectangular co-ordinate system as follows:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T + \frac{\partial \lambda}{\partial T} \left(\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right). \quad (2)$$

In a rectangular co-ordinate system, the temperature Laplacian is defined as follows:

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}. \quad (3)$$

A uniform temperature distribution $T(\mathbf{r}) = T_0$ is assumed as an initial condition at $t = 0$.

For the turbine valve casings, the following boundary condition is applied:

$$\lambda \frac{\partial T}{\partial n} \Big|_s = -\alpha (T_{surf} - T_{st}) \text{ for } t \geq 0, \quad (4)$$

where n denotes the normal direction to the surface, T_{surf} is the surface temperature, T_{st} is the steam temperature and α is the heat transfer coefficient.

The Fourier-Kirchhoff Eq. (2) is solved by means of the finite element method [22].

2.2. Heat transfer coefficients

The variation of heat transfer coefficient $\alpha(t)$ is considered by the Nusselt number Nu changing in time as a function of the Reynolds number Re and Prandtl number Pr :

$$Nu = f(Re, Pr). \quad (5)$$

The Nusselt number is defined as follows:

$$Nu = \frac{\alpha d}{\lambda}, \quad (6)$$

where d denotes the hydraulic diameter.

The detailed form of Eq. (5) depends on the component geometry and flow character [23]. Practical methods of determining heat transfer coefficients are presented in [20]. The heat transfer model adopted in this study is based on the well-proven formulae for heat transfer coefficients in steam turbine components and can be employed in online calculations of temperatures and thermal stresses, which is not possible when more advanced thermal fluid-structure interaction (FSI) modelling is adopted [24–27]. Online calculations require fast and robust models which can be implemented and run on industrial computers. For typical start-up conditions and geometry of steam turbine components, the time constants of thermal processes require solving the model with new boundary conditions with a time step of around 1 minute. Start-up durations are in the range of 30 to 200 minutes, which are the required calculation times. Such times are achievable for the finite element method with convection or condensation boundary conditions, but for FSI calculations involving fluid flow and solid body simulations, these times can be several times longer. Moreover, iteration processes and time steps employed in FSI calculations adversely affect not only the calculation times but also stability and reliability of solutions. Such calculations require high performance computers while simple algorithms based for example on Duhamel's integral are easily implementable in programmable logic controllers (PLCs).

For the valve casing regions of circular cross-section, the Gnielinski correlation [28,29] is used:

$$Nu = \frac{\xi(Re-1000)Pr}{1+12.7\sqrt{\xi(Pr^{2/3}-1)}} \left[1 + \left(\frac{d}{L} \right)^{2/3} \right] \left(\frac{Pr_m}{Pr_w} \right)^{0.11}, \quad (7)$$

where L is the pipe length and symbol ξ denotes the friction factor, for smooth tubes calculated from Filonienko's formula [30]:

$$\xi = \frac{1}{(1.82 \log Re - 1.64)^2}. \quad (8)$$

The symbols Pr_m and Pr_w designate the Prandtl number at the bulk and wall, respectively.

For the valve main chamber, a simple power-law formula similar to those of Dittus-Boelter [31–33] and Chilton-Colburn [34] correlations is applied:

$$Nu = 0.018 Re^{0.8} Pr^{0.33}. \quad (9)$$

The factor 0.018 was established based on 3-dimensional thermal fluid-solid interaction calculations performed for this valve with time-dependent boundary conditions representing valve heating up from a cold, warm and hot state [35]. Direct simulation results using a 3-dimensional FSI method were available for design start-up conditions, assuming theoretical variations of steam temperature, pressure and mass flow rates. The primary goal of these calculations was to determine a relationship between the similarity numbers (i.e. Nusselt and Reynolds numbers) for the analysed valve geometry, which could be used in finite element calculations and online calculations for any start-up conditions (not only for the predefined design start-ups).

When the turbine starts from a cold state or during pre-warming, steam condensation can take place on valve casing inner surfaces, and condensation heat transfer should be taken into account. Steam condensation on casing surfaces can occur when steam is superheated or saturated. The film condensation mechanism is assumed. A schematic illustration of the condensation model adopted in this work is shown in Fig. 1. If the surface temperature is lower than the saturated steam temperature at the corresponding steam pressure, then steam condenses on the surface. The heat transfer to the surface is intensified as a result of water films appearing on the surface at the saturated steam temperature and the heat released due to condensation.

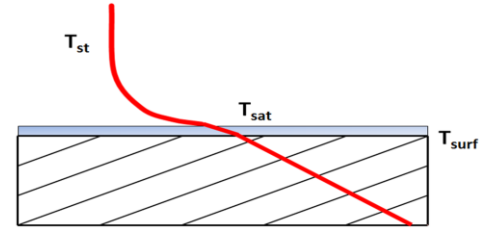


Fig. 1. Schematic illustration of film condensation model.

Condensation occurs if the following conditions are satisfied [36]:

- the metal temperature is below the saturation temperature T_{sat} at the local pressure p :

$$T_{sat} > T_{surf}, \quad (10)$$

- the heat flux from water film to metal is greater than the heat flux from steam to water film:

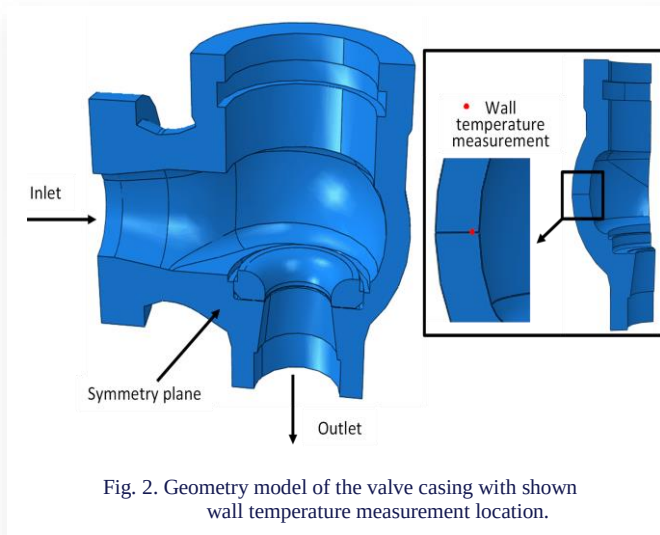
$$\alpha_{cond}(T_{sat} - T_{surf}) > \alpha_{conv}(T_{st} - T_{sat}). \quad (11)$$

As concluded in [23], available correlations for the Nusselt number for simple geometries, like a flat wall or pipe, can be used for estimating the condensation heat transfer coefficient usually with a large error. However, it is known that heat transfer coefficients during steam condensation assume large values reaching 120 000 W/(m²·K) in extreme cases during nucleate condensation of steam [37,38] and, consequently, the errors of their estimation will only slightly affect the variations of temperature and thermal stress fields in turbine components. It was shown that for Biot numbers $Bi > 20$, the variation of heat trans-

fer coefficient has little influence on the variation of temperature distribution [23]. Due to the lack of proper correlations for real turbine geometries and taking into account the above conclusions, the analyses of condensation heat transfer were performed with a constant heat transfer coefficient $\alpha = 11\,000\text{ W}/(\text{m}^2\cdot\text{K})$. This value is within the range of heat transfer coefficients predicted by Shah [39] for film condensation of water inside pipes.

2.3. Numerical results and model validation

For numerical calculations of valve heating-up, a three dimensional model has been prepared. The model includes half of the valve due to the symmetry of valve geometry and flow conditions (Fig. 2). Steam enters the valve through the inlet pipe and flows into the main chamber where the flow changes direction from horizontal to vertical. Further steam flows through a converging-diverging channel and leaves the valve through the outlet pipe. Wall temperature measurement is installed in the main chamber region in the wall section perpendicular to the inlet pipe axis and containing the outlet pipe axis. The measurement point is located at a 10 mm distance from the wall inner surface where the hot junction of the measuring thermocouple is located.



Numerical calculations have been performed using the Abaqus code [40]. Computational mesh size was approximately 94 000 second order diffusion elements with 89 546 tetrahedral and 4459 hexahedral elements. Casing material is heat resistant cast steel GS-17 CrMoV 5 11, and its physical properties as a function of temperature are given in Table 1 [35,41]. The cast steel density is constant and equal to $7850\text{ kg}/\text{m}^3$.

Table 1. Physical properties of GS-17 CrMoV 5 11 cast steel at different temperatures.

Temperature, T [°C]	20	100	200	300	400	500	550
Thermal conductivity, λ [W/(m·K)]	50	47	46	44	41	39	37
Specific heat, c_p [kJ/(kg·K)]	0.46	0.49	0.53	0.57	0.61	0.68	0.73

Fig. 3 presents inlet steam parameters for the warm start-up with pre-warming, together with the rotor rotational speed and

turbine load. Pre-warming is conducted by supplying steam to the valve chamber when the valve is in the closed position and evacuating condensate through a drain. Valve pre-warming takes place for 85 minutes until stop and control valves start to open. For the first 25 minutes, steam is at saturation and afterwards becomes superheated. During run-up and loading, the steam temperature and superheating increase, and reach maximum values at full load.

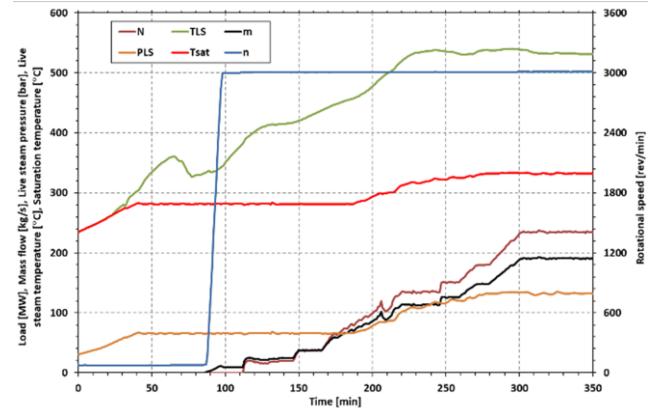


Fig. 3. Variation of inlet steam parameters, rotational speed and load during warm start-up with pre-warming.

A constant temperature of the valve casing equal to 240°C was assumed as an initial condition. Start-up simulations were carried out with time-dependent Fourier boundary conditions. Variation of fluid temperature and heat transfer coefficients for the inlet pipe and valve main chamber is presented in Fig. 4.

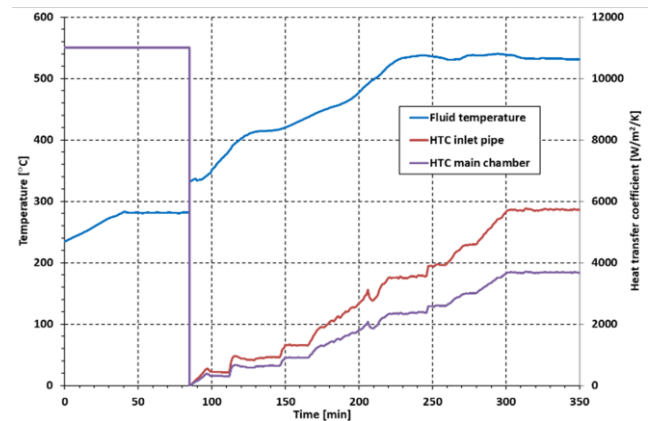


Fig. 4. Variation of temperature and heat transfer coefficients during warm start-up with pre-warming.

The calculated wall temperature at the measurement point is compared with the measured temperature in Fig. 5, which also presents steam and saturation temperatures. It is seen that the calculated wall temperature closely follows the measured value both during valve pre-warming and during start-up. The maximum deviation calculated as:

$$\text{Deviation} = \frac{\text{Predicted } T_{\text{wall}} - \text{Measured } T_{\text{wall}}}{\text{Measured } T_{\text{wall}}}, \quad (12)$$

does not exceed 1.5% during pre-warming and -2.6% during steam heating.

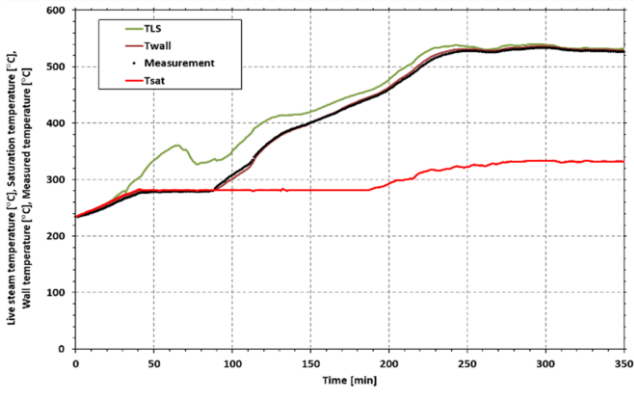


Fig. 5. Variation of steam and wall temperatures during warm start-up with pre-warming.

It is noted that condensation heat transfer prevails during the whole pre-warming phase until the beginning of run-up, and the wall temperature follows the saturation temperature. In early phases steam condensation on the valve inner surfaces takes place with saturated steam conditions and continues with superheated steam even for superheating reaching 80°C. When the valves start to open, the superheated steam flow through the valve brings about intensified heating of the valve and the wall temperature starts to increase above the saturation temperature. Since then, the casing temperatures continuously increase, slowly approaching the live steam temperature and departing the saturation temperature.

Temperature distributions in the valve casing at various phases of pre-warming and steam heating are shown in Figs. 6

and 7, respectively. Uniform temperature distribution was assumed as an initial condition for pre-warming (Fig. 6a). The first 20 minutes of pre-warming proceeds with saturated steam which condenses on the inner surfaces of the inlet pipe and main chamber. The wall surface temperature in these regions reaches 255°C (Fig. 6b), and the remaining regions are still at temperatures close to the initial. Further pre-warming takes place with superheated steam, and after 40 minutes, the wall surface temperature attains ~281°C (saturation temperature, Fig. 6c). After this time, considerable differences between surface temperature where condensation takes place and temperatures of remaining regions are still present. At the end of the pre-warming phase (Fig. 6d), temperature distribution becomes more uniform because the valve sections not being in contact with condensing steam are already heated-up via convection.

After 85 minutes of pre-warming, turbine run-up begins and steam starts to flow through the valve, bringing about intensive heating of the valve casing. The wall temperature reaches approximately 400°C after 60 minutes (Fig. 7a) and almost 500°C after 120 minutes (Fig. 7b) since the beginning of run-up. It should be noted that during the initial phases of steam heating, the temperatures of valve inlet and outlet regions tend to equalise as both regions are heated with comparable intensity by steam flow through the open valve. After 180 minutes, the nominal steam temperature is attained and the major part of the casing is at a temperature above 500°C (Fig. 7c). Since then, the temperature field is becoming more uniform while the inner surfaces being in contact with steam remain at almost constant temperature (Fig. 7d).

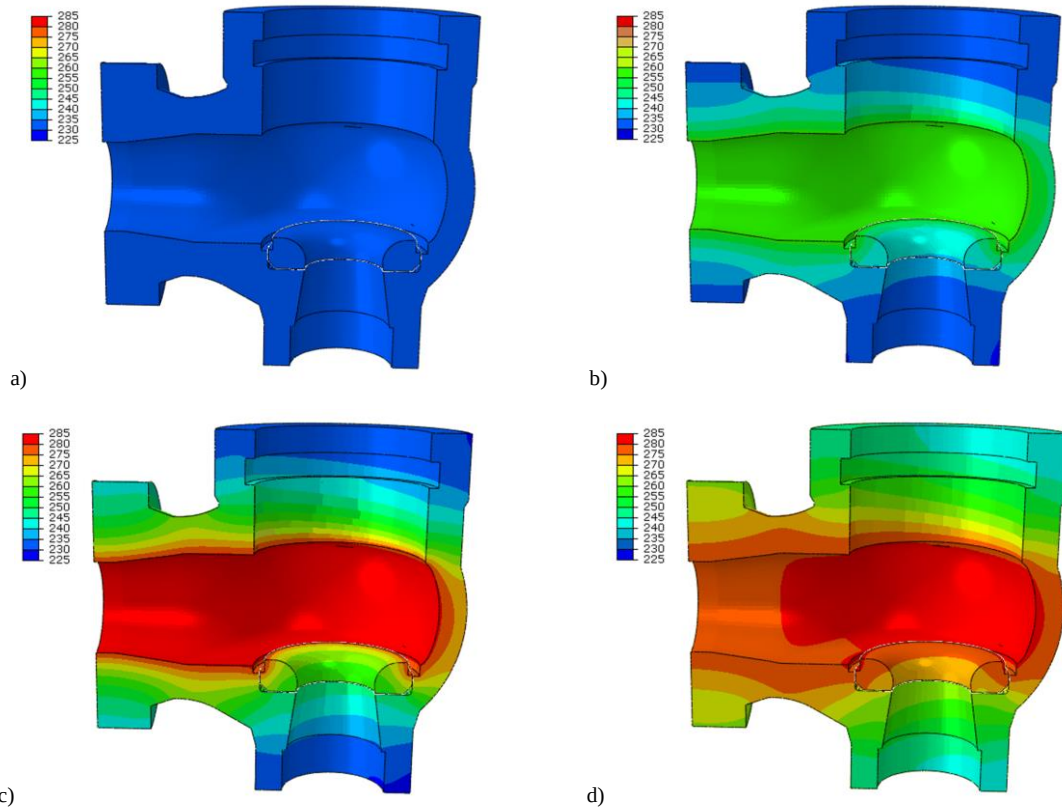


Fig. 6. Temperature distribution in the valve casing at various phases of pre-warming: a) $t = 0$ min, b) $t = 20$ min, c) $t = 40$ min, d) $t = 85$ min.

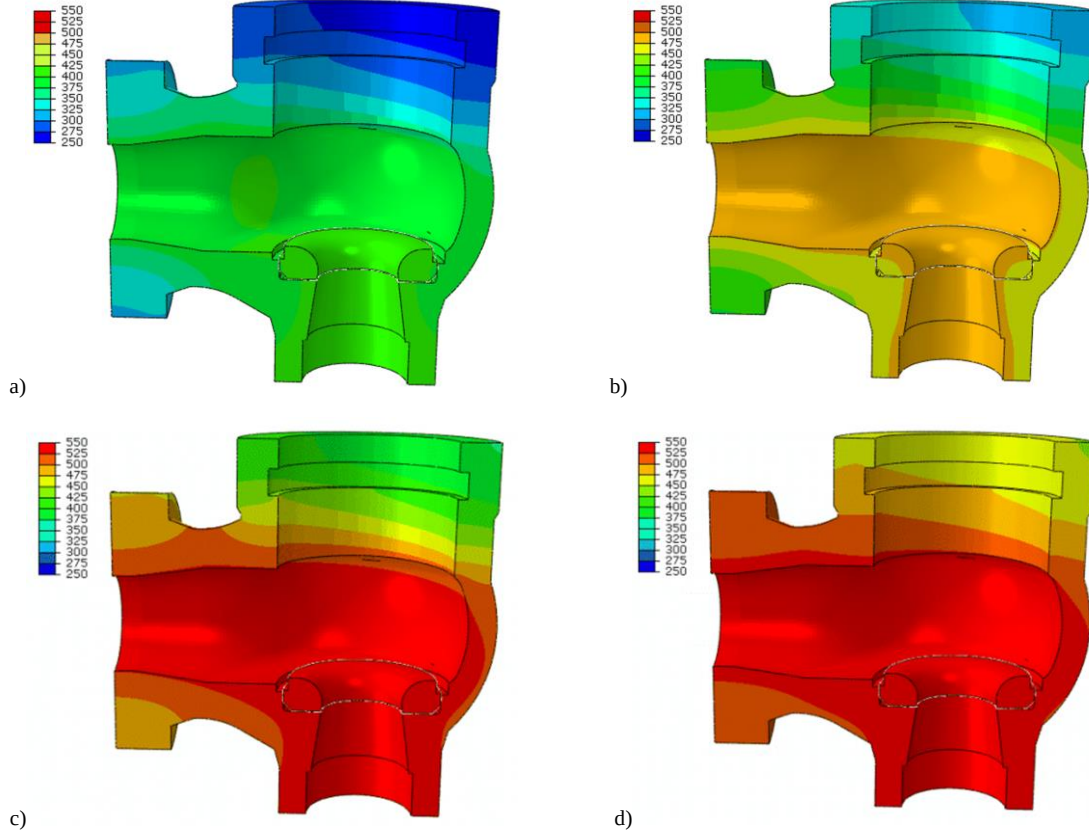


Fig. 7. Temperature distribution in the valve casing at various phases of steam heating: a) $t = 60$ min, b) $t = 120$ min, c) $t = 180$ min, d) $t = 240$ min.

3. Duhamel's integral method

3.1. Theoretical background

Duhamel's integral can be used for determining the temperature distribution in the body. For steam turbine components, a reliable and accurate measurement of surface temperature at the critical region is usually not possible, thus for on-line monitoring purposes, steam temperature measurement is used as a leading signal. Consequently, in the heat transfer model, Fourier's boundary condition (4) is applied.

Assuming that the steam temperature is described by Heaviside's function $H(t)$ [10], [42]

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \quad (13)$$

the boundary condition (4) can be written in the form [10]:

$$\lambda \frac{\partial x}{\partial n} = -\alpha(H(t) - X(\mathbf{r}, t)), \quad (14)$$

where $X(\mathbf{r}, t)$ is a solution of the direct heat conduction problem with a unit step change of steam temperature.

Using the solution $X(\mathbf{r}, t)$, we can calculate metal temperature for an arbitrary variation of steam temperature employing Duhamel's theorem [43]:

$$T(\mathbf{r}, t) = T_0 + \int_0^t X(\mathbf{r}, t - \tau) \frac{\partial T_{st}(\tau)}{\partial \tau} d\tau. \quad (15)$$

For simple geometries like a plate, cylinder or sphere, a solu-

tion of the one-dimensional transient heat conduction problem can be obtained analytically. In the case of more complex shapes, it is necessary to use numerical methods, e.g. the finite element method in order to obtain accurate solutions.

3.2. Numerical solution of Duhamel's integral

In order to solve Duhamel's integral given by Eq. (15), it is transformed from a continuous into discrete form:

$$T(\mathbf{r}, t) = T_0 + \sum_{t-t_d}^t X_t(\mathbf{r}, t - \tau) \Delta T_{st}(\tau) + X_s(\mathbf{r}, t), \quad (16)$$

where t_d is a cut-off time, $X_s(\mathbf{r}, t)$ is a value of the influence function in steady state, while $X_t(\mathbf{r}, t)$ is a transient part of the influence function.

In principle, Duhamel's integral, resulting from the superposition rule, can be applied to linear unsteady heat conduction problems. However, it has been shown in previous studies on thermal stress modelling using Duhamel's integral that it can be successfully applied to nonlinear thermoelasticity problems when the appropriately obtained solution of direct heat conduction problem $X(\mathbf{r}, t)$ is used [42]. This solution is obtained numerically with the assumption of time-dependent heat transfer coefficients $\alpha(t)$ and temperature-dependent specific heat $c_p(T)$ and thermal conductivity $\lambda(T)$.

It has been shown in Section 2 that for adequate modelling of thermal behaviour of turbine valves, both condensation and convection heat transfer have to be taken into account. Insofar as the condensation heat transfer coefficient can be assumed

constant and high, the convection heat transfer coefficient can vary by orders of magnitude during start-up. Equation (16) is then rewritten into the form:

$$T(\mathbf{r}, t) = T_0 + \sum_{t-t_d}^t X_{t,cond}(\mathbf{r}, t - \tau) \Delta T_{sat}(\tau) + k \sum_{t-t_d}^t X_{t,conv}(\mathbf{r}, t - \tau) \Delta T_{st}(\tau) + X_s(\mathbf{r}, t), \quad (17)$$

where $X_{t,cond}$ is a temperature response function for condensation determined with a constant heat transfer coefficient, $X_{t,conv}$ is a temperature response function for convection determined with a variable heat transfer coefficient and k is a factor responsible for compensating for nonlinearity of the problem. When necessary conditions for condensation are met, then $\Delta T_{st} = 0$, and when condensation vanishes, then $\Delta T_{sat} = 0$.

In steady state conditions, the two terms with summation over time tend to zero, and the steady state term $X_s(\mathbf{r}, t)$ is given by the following formula:

$$X(\mathbf{r}, t) = 0.9(T_{st}(t - t_d) - T(t - t_d)) \times \left(1 + 0.1 \exp\left(\frac{1}{t_d - t}\right)\right). \quad (18)$$

This term is assumed to be zero at transient states and is activated and contributes to the calculation of temperature when nearly steady state conditions are reached.

4. Validation of Duhamel's integral method

Validation of the proposed method based on Duhamel's integral and described by Eq. (17) was done by performing numerical calculations of wall temperature for three different start-ups using the proposed method. The results were then compared with real measurements of the valve casing temperature and predictions of the FEM model presented in Section 2.

4.1. Temperature response function for convection and condensation

The accuracy of the proposed method depends on the form of the temperature response function, which in turn is primarily determined by the variation of the heat transfer coefficient. As it has been previously shown, the condensation heat transfer coefficient can be assumed constant and good accuracy of temperature predictions is obtained with $\alpha_{cond} = 11\,000 \text{ W/(m}^2\cdot\text{K)}$. Figure 8 presents the temperature response functions for the wall surface and temperature measurement point located 10 mm below the surface. These functions were obtained using the finite element model of the valve presented in Section 2 and applying a step change of steam temperature from 125°C to 280°C. The temperature response was then related to the change of steam temperature (equal in this case to 155°C) to obtain a response function to a change of steam temperature by 1°C. In numerical simulations, material physical properties were temperature-dependent, and the heat transfer coefficient was kept constant.

Figure 8 also presents the temperature response functions for convection, which were obtained for variable heat transfer coefficients shown in Fig. 4. In this case, a step change of steam temperature from 280°C to 540°C was simulated, assuming temperature-dependent material properties. The shape of tempera-

ture response function reflects the variation in time of the heat transfer coefficient. It is seen from the figure that in condensation conditions, the temperature response is more rapid and regular than for convection. This behaviour results from the assumption of a constant and higher than for convection heat transfer coefficient during condensation.

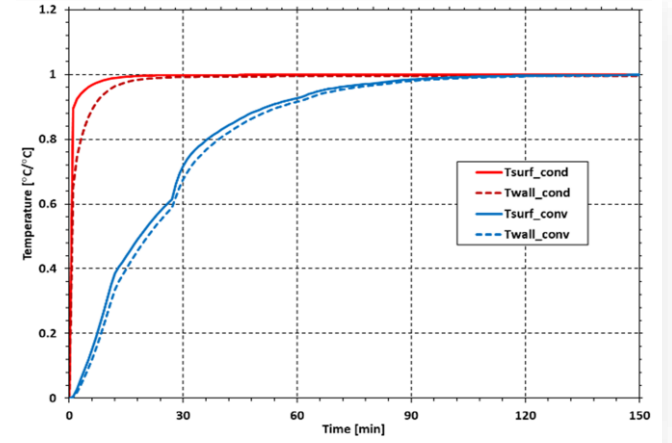


Fig. 8. Temperature response function for wall surface and measurement point location for condensation and convection.

4.2. Simulation of start-ups from various initial thermal states

The results of numerical calculations performed using Duhamel's integral were compared with wall temperature measurements and predictions of the finite element method for three different start-up conditions covering the whole range of turbine operation:

- i. pre-warming from a cold state (Fig. 9),
- ii. warm start with pre-warming (Fig. 10),
- iii. hot start (Fig. 11).

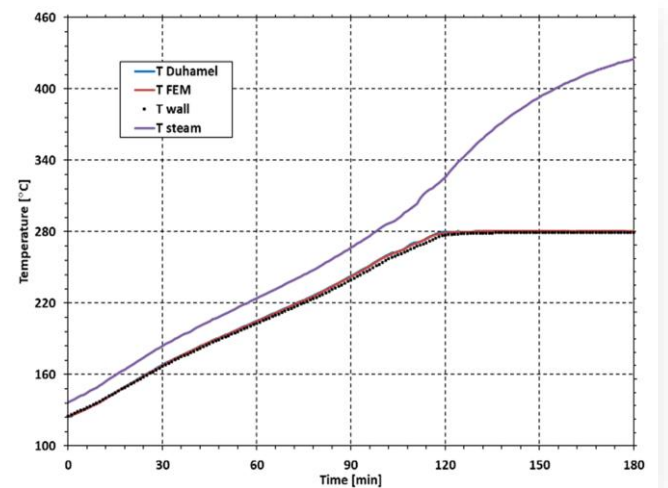


Fig. 9. Measured and calculated metal temperatures during pre-warming from a cold state.

In case i), steam condensation took place during the whole analysed period, while in case iii), only convection heat transfer was present. Case ii) included both condensation and convection. Steam condensation was modelled with the assumption of

a constant heat transfer coefficient $\alpha_{cond} = 11\,000\text{ W/(m}^2\cdot\text{K)}$, while for forced convection, variable heat transfer coefficients were used. Thermal boundary conditions for warm start with pre-warming were presented in Fig. 4, while those calculated for the hot start are shown in Fig. 12. A constant temperature of the valve casing equal to 479°C was assumed as an initial condition for the hot start. The factor $k = 1.12$ accounting for nonlinearity of thermal boundary conditions was obtained by tuning for the warm start conditions and further used in hot start calculations.

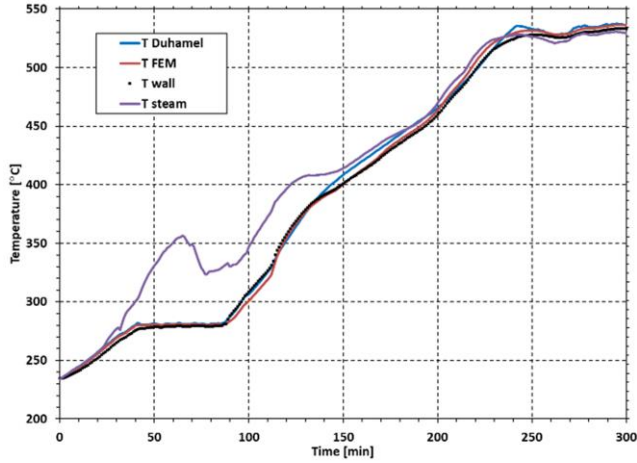


Fig. 10. Measured and calculated metal temperatures during warm start with pre-warming.

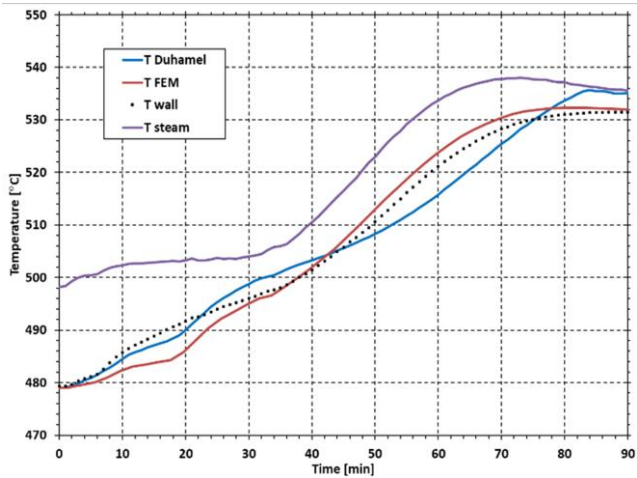


Fig. 11. Measured and calculated metal temperatures during hot start.

In all cases, very good agreement between calculations and measurements was found. At condensation conditions (Fig. 9 and Fig. 10), both methods (i.e. FEM and Duhamel's integral) predict nearly the same wall temperatures, which are very close to the measured temperature following steam saturation temperature. When the condensate film began to evaporate and convection heat transfer prevailed, larger deviations between calculated and measured temperatures were found. This is attributed mainly to inaccuracies in heat transfer coefficients determined with the help of simple correlations for pipe flows. However, the overall accuracy of the method based on Duhamel's integral can be assessed as high, taking into account the achieved deviations:

- i. $(-1.0\%)\div(+1.5\%)$,
- ii. $(-2.6\%)\div(+1.5\%)$,
- iii. $(-1.0\%)\div(+0.8\%)$.

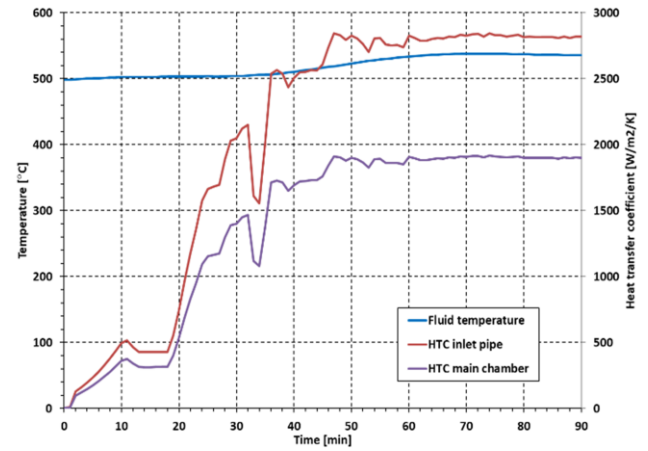


Fig. 12. Variation of temperature and heat transfer coefficients during hot start-up.

Temperature differences are also observed between the two calculation methods, in particular for forced convection. The two simulated start-ups show that Duhamel's integral tends to predict higher temperatures than FEM at lower heat transfer coefficients (lower steam velocities) and lower temperatures at higher heat transfer coefficients. Nevertheless, it should be noted that the degree of agreement between the detailed FEM calculations and predictions of the simple Duhamel's integral method is satisfactory. Finite element modelling of such complex components with nonlinear thermal boundary conditions is time consuming and results in a large amount of output data. Its use for online monitoring of power plant equipment is still impractical. However, the simple method described by Eq. (17) is fast, robust, accurate and easy to implement in industrial controllers. It considers in a simplified way the complex geometry of turbine and boiler components as well as the nonlinearity of thermal boundary conditions resulting from variation of steam temperature and heat transfer coefficient. The satisfactory accuracy of Duhamel's integral makes it very useful for online monitoring of thermal loading of turbine and boiler components.

For online checking of condensation conditions given by Eqs. (10) and (11), the wall surface temperature T_{surf} is calculated with the help of Eq. (17) and the temperature response function for the wall surface. Figure 13 presents the comparison of this temperature obtained with the help of Duhamel's integral and FEM for the warm and hot start-up. Thermal boundary conditions for the warm start are presented in Fig. 14, while for the hot start, the conditions shown in Fig. 12 were used. A constant temperature of the valve casing equal to 279°C was assumed as an initial condition for the warm start. Also for the surface temperature, reasonably good agreement between the two methods is found, in particular in the early phases of start-ups. These phases are very important because steam condensation on the valve surface can occur at the beginning of start-ups, and accurate prediction of the surface temperature is crucial in detecting steam condensation.

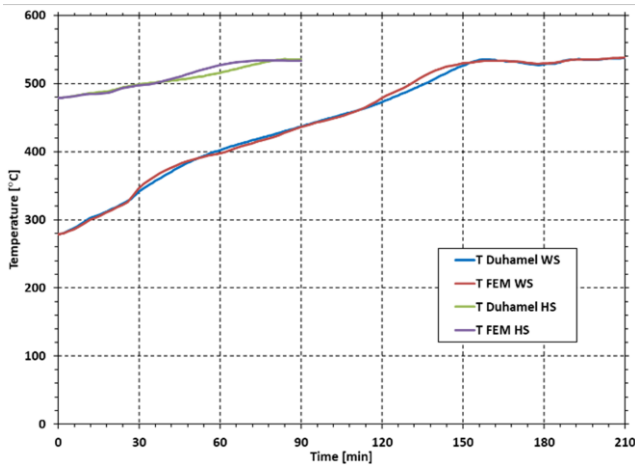


Fig. 13. Calculated wall surface temperatures during warm (WS) and hot start (HS).

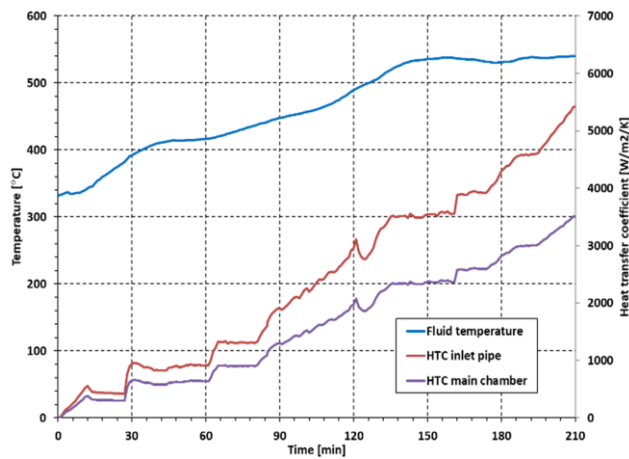


Fig. 14. Variation of temperature and heat transfer coefficients during warm start-up.

5. Summary

The paper presented numerical investigations of thermal loading of the turbine valve at various operating conditions. These included pre-warming from a cold state, warm start with pre-warming and hot start-up. Both condensation and convection heat transfer mechanisms were modelled. In the finite element modelling, it was done by applying condensation and forced convection heat transfer coefficients to the valve surfaces in contact with steam.

The proposed method for online calculation of metal temperature is based on Duhamel's integral. Different mechanisms of heat transfer are modelled in this algorithm by independent terms of Duhamel's integral. One term is responsible for condensation and the second one for convection, and each of them uses specific temperature response functions calculated with condensation and convection heat transfer coefficient, respectively.

It was shown that the calculation method provides reasonably accurate predictions of casing wall temperature. Comparison of the method predictions with the results of FEM simulations and measurements done at real operating conditions reveals the

same level of accuracy as the finite element method. Relative differences between measured and calculated wall temperatures are within $(-2.6\%) \div (+1.5\%)$ for the investigated start-ups with condensation and convection heat transfer conditions. This high level of accuracy was achieved by proper determination of temperature response functions which have to be calculated with a variable convection heat transfer coefficient. The quality of results is also improved by applying the constant factor k accounting for nonlinearity of thermal boundary conditions due to the variable heat transfer coefficient at convection.

The advantage of this method are fast and robust temperature calculations as well as its simple mathematical formulation, making it relatively easy for implementation in programmable logic controllers. It should also be emphasised that temperature calculations are performed using only steam temperature as an input without a need of installing metal temperature measurement in the area of interest. Turbine and valve casings are elements of complex geometry, and wall temperature measurements at critical locations are usually difficult in practical realisation. Moreover, in the case of rotating components, like rotors or blades, direct continuous measurements of metal temperature are still impractical and only short-term measurements with the help of optical probes are feasible [44]. Additional difficulty is also related to steam condensation, which reduces the accuracy of optical measurements.

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Steam and gas microturbines. Overview

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Abstract

In the contemporary energy landscape, governments are focusing on improving the efficiency of electricity generation and promoting distributed energy systems based on local and renewable sources. This has led to a substantial increase in the number of prosumer, polygeneration, and microgeneration power plants currently under development. In the case of micro-power plants, often turbine-based, particular emphasis is placed on the durability, reliability, cost-effectiveness, and efficiency of both individual components and the entire system. The construction of microturbine power plants involves several challenges, including component miniaturisation, achieving extremely high rotational speeds, efficient power transmission, maintaining adequate safety standards, reducing noise, minimising emissions of harmful compounds, and designing the electric generator. Additional difficulties concern the optimisation of thermodynamic and fluid-dynamic processes, the high-efficiency design of flow components, ensuring bearing and dynamic stability in high-speed rotating systems, miniaturising heat exchangers, and enhancing thermal processes. Despite the availability of commercial microturbine solutions, recent years have seen intensified research and development efforts. The evolution observed in this field suggests that efforts to improve efficiency, durability, reliability, and cost-effectiveness will continue, strongly supported by government involvement. Current progress and accumulated experience have enabled the development of engineering calculation methods for microturbine design, a crucial element of microgrids. This paper reviews the existing literature on microturbines, with particular attention to experimental results and examples of practical designs. It also outlines the main challenges faced by designers and engineers in developing such systems.

Keywords: Gas microturbines; Steam microturbines; Organic Rankine Cycle; Adhesion turbine

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1. Introduction

In the current 21st century, there is an increasing prevalence of trends towards miniature engineering. Their presence is evident in a variety of sectors, including the power industry, where they are employed in refrigeration, heat exchange, chemical processes, and power generation processes (microturbines, micro-pumps, micro-compressors, micro-valves, micro-heat exchangers, micro-motors, etc.) [1–3]. The popularity of microturbines

has increased significantly over the past two decades, due to their status as relatively small devices. The characteristics of the devices under discussion are as follows: they are characterised by relatively low electrical power (ranging from 25 to 500 kW), very high rotational speeds (ranging from 50 to 120 000 rpm, with some microturbines reaching speeds of up to 1 million rpm [4,5]), and relatively good efficiencies (ranging from 20% to 30% without regeneration). These devices allow for diversification of the energy source in the region [2,6].

Nomenclature

Abbreviations and Acronyms

ACM – air cycle machine
 CBC – closed Brayton cycle
 CHP – combined heat and power (system)
 EMG – electromagnetic generator
 FBG – fibre Bragg grating
 GMT – gas microturbine
 GWP – global warming potential
 HRSG – heat recovery steam generator

LSTM – long short-term memory
 MEMS – micro-electro-mechanical systems
 NZEB – net zero energy building
 ODP – ozone depletion potential
 ORC – organic Rankine cycle
 OTEC – ocean thermal energy conversion
 PLM – porous layer model
 STG – steam turbine generator
 TENG – triboelectric nanogenerator
 VOC – volatile organic compound

The advantage of microturbines is their versatility in terms of fuel selection. In addition to conventional fuels such as natural gas and diesel, microturbines have the capacity to utilise a wide range of alternative fuels, including landfill gas, sewage treatment plant gas, flue gas, coal gasification gas, biofuels, methanol, ethanol, and hydrogen, among others [7].

At present, a significant number of corporations are engaged in conducting extensive research and development projects with the objective of constructing microturbines that exhibit enhanced efficiency, reliability, durability and cost-effectiveness. The primary objective of the activity is to achieve efficiencies in excess of 40%, with a concomitant reduction in NO_x emissions and an increase in availability (continuous operation between overhauls of more than 50000 hours), reducing investment costs (< 500 USD/kW), improving fuel flexibility (the ability to utilise several types of fuel, including biofuels, etc.). It is evident that a significant number of globally renowned corporations are engaged in addressing these issues. While some entities have already commenced the provision of microturbines to the commercial sector, ongoing research in this domain persists, with continual advancements being made. The challenges posed by the miniaturisation of components are manifold. These include, but are not limited to, difficulties with bearing and transmission at very high speeds. In addition, there is a need to ensure a proper level of safety, to reduce noise and to reduce the emission of

harmful compounds into the atmosphere. A significant challenge confronting manufacturers pertains to the quality of electric generation, encompassing essential parameters such as required voltages, frequency, and power output, among others. This challenge persists during transient conditions. Concurrently, advanced research is being conducted into the use of new types of materials (synthetics, composites, ceramics, aluminium alloys, etc.) [8,9].

In the domain of micro systems, the primary concern pertains to the overall dimensions of the system. Miniaturisation facilitates the acquisition of information with greater efficiency, thereby reducing the material and energy requirements during the production process. Intelligent microsystems, equipped with microsensors and actuators, possess the capacity to interact with their environment and respond to changes therein. The utilisation of micro power supplies and microelectronics for signal processing is a prerequisite for such systems. The integration of these components enhances system efficiency, resulting in faster, more reliable, and cost-effective operation, while concurrently reducing power consumption and enabling the execution of more complex functions [10,11].

In the last two decades, there has been a marked increase in the number of articles dealing with microturbines, as evidenced by data from Elsevier's electronic journal database, which is available through the ScienceDirect platform (Fig.1).

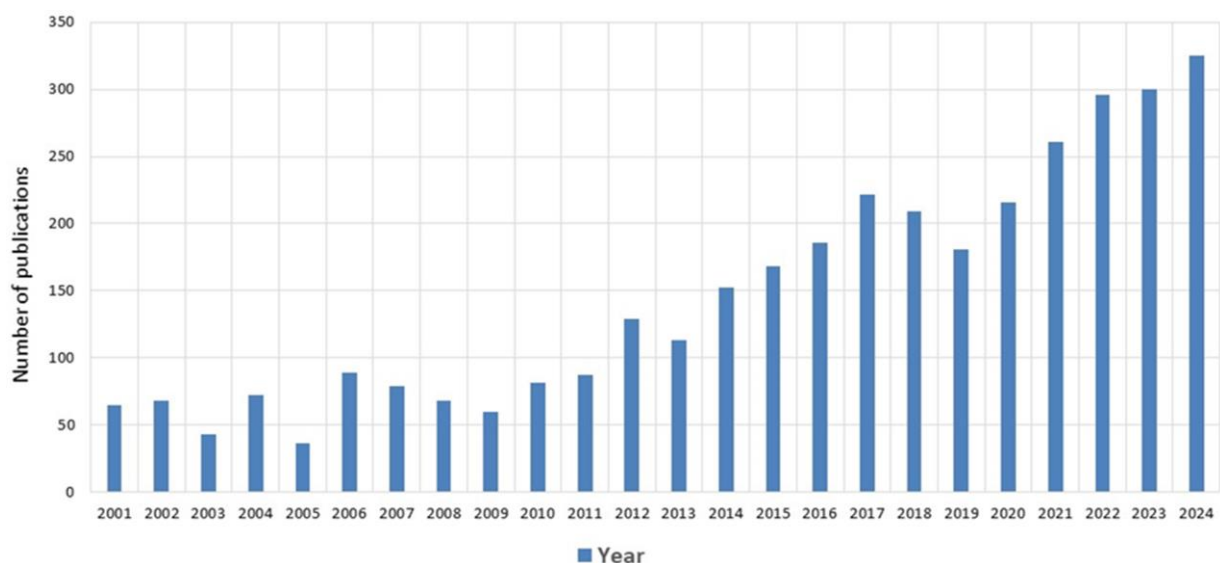


Fig. 1. Number of publications on microturbines from Elsevier's electronic journals database.

The objective of this paper is twofold: firstly, to present and systematise extant solutions for vapour and gas microturbines; and secondly, to explore the potential for future development in this area. The analyses were conducted using a range of academic databases, including Elsevier Open Access, Institute of Electrical and Electronics Engineers, and Springer, to name a few. In certain instances, the authors have encountered difficulties in accessing more detailed information due to the protection of intellectual property rights. The paper is organised into five main chapters, with each one focusing on specific aspects of these solutions. Vapour microturbines are categorised into two types: those using steam as the working medium (Rankine cycle) and those employing other working media in the Organic Rankine Cycle (ORC). The latter type of micro power plant, in particular, is undergoing significant development. The next chapter discusses gas microturbines, which are also experiencing rapid advancements. A separate chapter is dedicated to Tesla microturbines, given their ability to utilise various working media, including steam, organic vapour, air, or flue gases, which complicates their classification within a single category. Additionally, a chapter has been dedicated to microturbines (both steam and gas) that employ Microelectro-Mechanical System (MEMS) technology, in addition to the analyses presented in other chapters.

2. Steam microturbines

Steam turbines are among the most versatile and long-standing prime mover technologies still widely used today. The utilisation of steam turbines for power generation has a long history, having replaced reciprocating steam engines due to their superior efficiency and lower costs. In the extant literature, steam microturbines are referred to as devices that produce energy measured in kilowatts or even watts. Up to now, quite a few power stations have been equipped with steam microturbines. However, this knowledge and technology domain is still dynamically developing, in a similar manner to gas microturbines. The advantages of the steam microturbine are:

- a proven technology,
- broad power range,
- acceptable efficiencies (at least at nominal conditions),
- separation between combustion and power generation.

The main disadvantages are:

- low efficiency at partial load,
- relatively high costs for a small size [8].

The Rankine cycle is the most common thermodynamic cycle employed in steam turbines. This cycle constitutes the fundamental principle underlying conventional power-generating stations, Fig. 2, and it is realised in four ideal thermal processes [6]:

- heat supply at constant pressure (usually, in the boiler furnace, due to chemical reactions of fuel combustion, heat is generated and used for changing the water into steam, most frequently superheated),
- isentropic expansion (the steam of high pressure and tem-

perature generated in the boiler enters the turbine where it expands to the low pressure of the condenser),

- heat rejection at constant pressure (in the condenser, the steam rejects its heat of condensation to a cooling system),
- isentropic compression (in the pump, the isentropic process of compression takes place, transforming the water from a saturated liquid state to feed water at high pressure).

The steam microturbine has also been incorporated into hybrid cogeneration, trigeneration and combined steam-gas power systems [12].

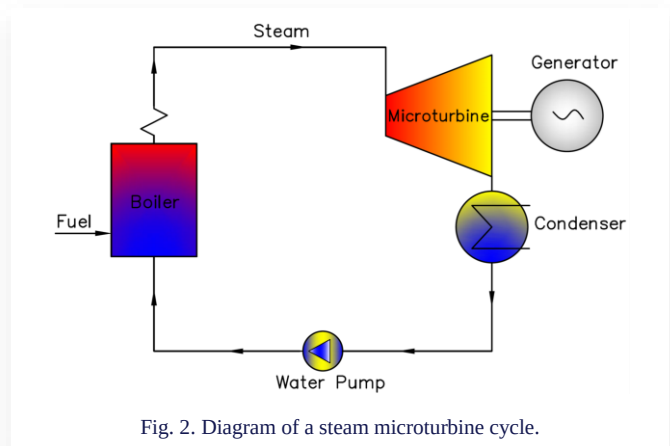


Fig. 2. Diagram of a steam microturbine cycle.

An example of a steam microturbine set is shown in Fig. 3 [13]. The Steam Turbine Generator (STG) system consists of a high-speed steam microturbine with an axial inlet and radial outlet with a high internal efficiency of 80%, a planetary gear-box reducing the speed in a ratio of 7.78:1 and a generator with efficiency above 95%. The efficiency depends on the parameters of the inlet steam. It has been demonstrated that the turbine set has the capacity to generate 275 kW of electrical power [14]. The centrifugal microturbine used in this solution allows large enthalpy drops to be realised with good efficiency at low mass flow rates.

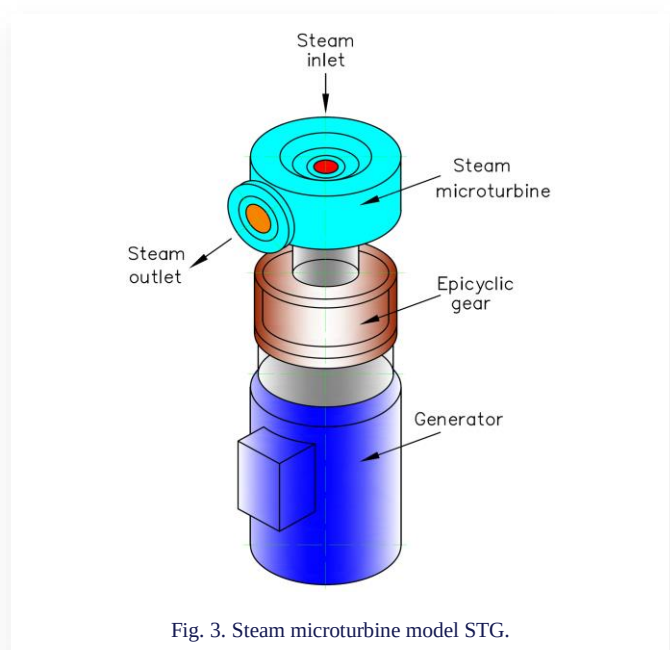
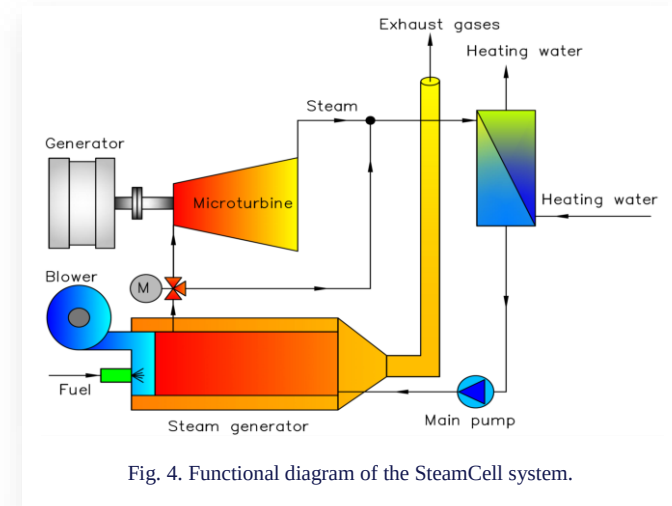


Fig. 3. Steam microturbine model STG.

In 2004, a prototype of a gas-powered domestic SteamCell unit for combined heat and power generation based on a steam unit was presented (Fig. 4) with an efficiency of 25%. The system has been demonstrated to deliver a range of electricity outputs between 0.5 kW and 4.6 kW, in conjunction with heat outputs ranging from 2 kW to 25 kW. In this particular instance, the conventional Rankine cycle is augmented by the incorporation of an additional domestic water heater [15].



The paper [16] presents a two-stream flow centripetal microturbine. The unit's maximum electrical power output is 50 kW at 12 000 rpm. The rotor is supported by rolling bearings. In experimental tests for an inlet steam pressure of 0.6 MPa, temperature of 159°C and mass flow rate of 0.21 kg/s, 30.37 kW of electrical power was obtained.

Another example of a steam turbine test rig is the system that was installed at the Maine Maritime Academy[17]. The facility has two steam turbine units. One is a single-stage Curtis turbine, and the other is a multi-stage axial turbine. Operation parameters: outlet pressure of 800 kPa, outlet temperature of 289°C, boiler inlet water temperature of 63°C, condenser pressure of 20 kPa, turbine outlet temperature of 110°C (Curtis stage) and 60.1°C (multi-stage turbine), rotational speed of 3 600 rpm, electric power of 8.21 kW (Curtis stage) and 13.32 kW (multi-stage turbine), with overall circuit efficiency of 11.4% (Curtis stage) and 18.55% (multi-stage turbine). There are some disadvantages to both solutions. The Curtis turbine stage is less efficient, and the multistage turbine is more complicated.

The New England Steam Turbine Corporation, on the other hand, makes NESTCO N17 H single-stage steam turbine units. These have a power output of 1260 hp (≈ 940 kW), an inlet steam temperature of 440°C, a pressure of 49 bar and a speed of 6 300 rpm. This is also a Curtis turbine stage device. The company also offers a vertical-flow version of the NESTCO N17 V microturbine. Both devices are used to drive pumps [18].

Another intriguing steam microturbine solution is a counter-rotation turbine with two separate rotors, without a stator between them, rotating in opposite directions, driving two separate high-speed permanent magnet generators (see Fig. 5). Figure 5a presents a microturbine set, while Fig. 5b shows a partially decomposed device with visible separate rotor blade cascades. It is a device that uses the summation of velocity vectors resulting

from the modification of the Ljungström turbine. The microturbine has been demonstrated to facilitate a power output of approximately 1.5 kW at an inlet steam pressure of around 5 bar, a temperature of approximately 220°C, and outlet parameters of 10 kPa and 40°C. These conditions are maintained while the microturbine operates at a rotational speed of 30 000 rpm and a mass flow rate of 5 g/s. This approach has been patented [19] and has undergone theoretical testing and analysis. The feasibility of a compact design that is capable of effective electricity production was demonstrated [20,21].

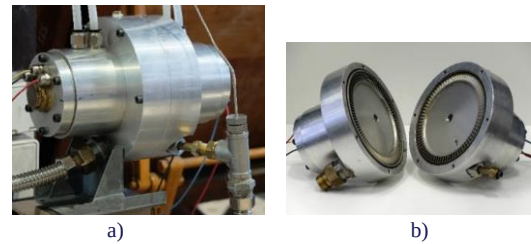


Fig. 5. View of a steam microturbine with two generators (a) and both counter-rotating rotors in separate housings (b).

For example, in Russian developments led by the Don Technologies consortium, single-stage, two-flow active centripetal steam microturbines have been applied, operating in the range of 30–50 kW electric power and up to 600 kW thermal power. Their design enables independent regulation of electricity and heat generation, which distinguishes them from gas turbine and piston units. Experimental investigations, however, revealed the need for further optimisation of blade profiles and reduction of efficiency losses during wet steam operation [22].

In studies on cogeneration systems, integrating gas and steam microturbines with ORC loops and solar collectors has been emphasised. Advanced concepts couple a microturbine with a parabolic dish collector and a Brayton cycle, while waste heat recovery is performed through dual-stage ORC modules. Such configurations significantly improve exergy efficiency and reduce fuel consumption, achieving overall efficiencies exceeding 70% [23].

Although steam microturbines were not the primary focus in research on building-integrated nanogrid systems, they are often discussed in the broader context of decentralised trigeneration. Integration of microturbines with thermal energy storage highlights their potential use in Net Zero Energy Buildings (NZEBs), where they can complement renewable sources and support local heating and cooling demands [24].

Another promising direction is the application of microturbines in mobile and transport energy systems. Research on sustainable tramway power supply has demonstrated the feasibility of integrating microturbines with biomass gasifiers. In such configurations, microturbines act as stabilising units for the entire energy system, working in conjunction with energy storage and renewable sources, thereby enhancing energy autonomy and reducing CO₂ emissions [25].

Utilising the microturbine configuration presented in Fig. 5, the authors have proposed a solution for a steam micro-turbine that operates in an emission-free mode. This solution involves the combustion of hydrogen in an oxygen, resulting in the generation of steam as the working medium. The concept of such

an installation is illustrated in Fig. 6, while the test stand is presented in Fig. 7. The proposed solution is straightforward and analogous to the Rankine system. The hydrogen burner generates high-temperature steam. The process of steam generation at a lower temperature, permissible for the turbine, is achieved by the supply of water to the burner. Steam expands in the turbine and is then condensed in a water-cooled condenser. The work of the turbine is used to produce electricity in the generator.

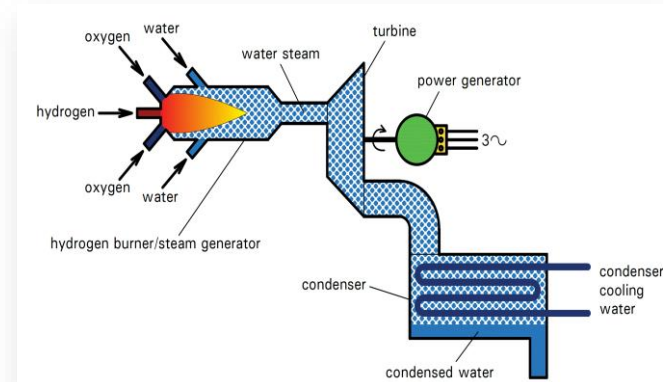


Fig. 6. Concept of operation of a hydrogen microturbine.

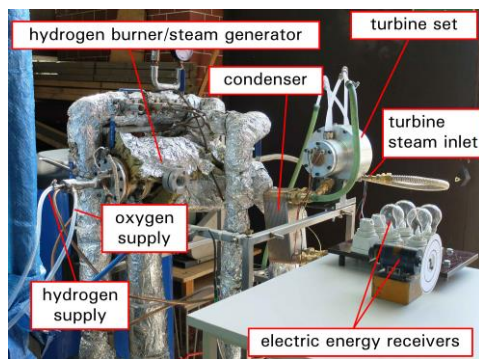


Fig. 7. View of the hydrogen microturbine test stand.

In addition to the commercial versions, laboratory-scale stations are also available. The steam parameters at the boiler outlet are a pressure of 827 kPa and a temperature of 250°C, and the resulting electrical power is approximately 15 W [26]. These are student stands designed to present the idea of the Rankine cycle.

In this chapter, the authors have conducted a review of extant design solutions for microturbines with steam as the working medium. Although few examples of steam-based microturbines have been cited, their development is ongoing. Steam microturbines have very low steam flow rates and high enthalpy drops, so special designs must be used to achieve good efficiency. A range of adaptations have been derived from high-power steam turbine solutions, including multi-stage axial micro-turbines, micro-turbines with Curtis stage, centripetal turbines, and turbines with contra-rotating rotors. Each of these has its own advantages and disadvantages, and ongoing development is focused on enhancing efficiency, durability, and reliability. However, recent trends in micro turbine development appear to be shifting towards the utilisation of working media other than water. These are ORC cycles, which will be discussed in more detail in the next chapter.

3. Microturbines in ORC cycles

The fundamental operating principle of ORC technology was first established in 1826 by Howard. He used ether as the working medium. It was not until more than a century later that D'Amelio constructed an ORC power plant in 1936, equipped with a single-stage impulse turbine and monochloroethane (C_2H_5Cl) as the working medium, heated by solar energy. Subsequently, during the 1960s, research into potential working fluids was initiated, which led to the identification of the advantages of chlorofluorocarbons (CFC) use and the establishment of the ORC cycle with regeneration. This cycle continues to be utilised extensively in the present day. This development subsequently prompted the conception of multiple prototypes by ORMAT in 1964 and Turboden in 1970. These enterprises have since emerged as the foremost entities within the ORC market [27].

The initial ORC systems, which were in operation between 1960 and 1980, are summarised in Table 1.

Table 1. First ORC systems built between 1960 and 1980.

Year	Power, kW	Working fluid	Temperature, °C	Pressure, bar	Number of systems	Country
1961	347	R11	-	-	1	USA
1966	190	R11	120	12,5	1	Japan
1967	750	R12	-	-	1	USSR
1968	3800	R11	121	12,8	1	Japan
1969	475	R11	116	11,6	1	Japan
1972	108	Fluorinol 85	288	48,3	1	USA
1974	500	R11	123	13,3	1	Japan
1975	600	Toluen	274	22	1	USA
1977	500	R113	90	12,4	1	Japan
1977	34	Fluorinol 50	321	55,2	1	USA
1978–1982	600	Toluen	251–274	16–22	5	USA
1979	200	Toluen	-	-	1	USA
1979	40	Tetrachloroethylen	110	0,7	1	Italy
1979	500	R11	88	6,2	1	USA
1979	21	R114	92	-	1	Germany
1979	100	Toluen	320	-	1	Germany
1979	170	Toluen	323	-	1	Germany
1979	500	R114	85	-	1	Germany
1979	30	R11	160	13	1	Germany
1980	45	C ₈ F ₁₆	280	11,6	1	Italy
1980–1983	50–600	R114	-	-	22	Israel
1981	14000	Fluorinol 85	260	34	1	Japan
1981–1984	1000–1500	R113	130	7,9	3	USA
1982	130	R11	-	24	1	Germany
1982	500	R114	105	15,3	1	Italy
1982	5000	R114	75	8,1	1	USA
1983	1000	Fluorinol 85	330	50	1	France
1983	3280	R11	90	6,7	1	Japan
1984	100	Dichlorobenzene	173	0,9	1	Italy
1984	100	C ₈ H ₁₀	130	0,8	1	Italy
1984	100	R114	170	22	1	Finland

The assembly of these power plants was primarily undertaken in the USA, Japan, Germany, Israel and Italy, with capacities ranging from a few tens of kilowatts to several megawatts. A solution with a capacity of up to 1 MW can be designated as a microturbine unit. At that time, the following refrigerants were typically utilised: R11, R12, R113, R114 and Fluorinol [7,28].

We can see from the table that the temperatures used in those years were quite low. They did not exceed 330°C. On the other hand, the pressures were already above 55 bar. It is generally accepted that steam power plants above 350°C have higher efficiencies than those using organic media [29]. At lower temperatures, thermodynamic cycles using organic fluids can achieve a higher level of efficiency than those using steam [30]. The construction of this type of power plant was later somewhat abandoned, but the last two decades have seen a rapid development of ORC systems and associated microturbine technology.

The ORC is similar to the cycle of a conventional steam turbine, except that the medium that drives the turbine is a high molecular weight organic fluid. The selected working fluids allow efficient use of low temperature heat sources to generate electricity over a wide power range (from a few kW up to 1 MW per unit). The organic working fluid is vaporised in the evaporator, then it expands in the turbine and is then condensed using cooling water in a heat exchanger (alternatively, ambient air can be used for cooling). The condensed medium is then pumped back to the evaporator, and the thermodynamic cycle is closed. Heating and cooling sources are not in direct contact with the working fluid or the turbine. For high temperature applications, e.g. combined heat and power biomass-powered plants, high temperature thermal oil is used as a heat carrier and a regenerator is added to further improve the cycle performance. The main technical advantages of ORC plants are as follows:

- relatively high cycle efficiency for a low value of upper cycle temperature,
- high turbine efficiency (up to 85%),
- no erosion of blades, due to the absence of moisture,
- long life time,
- no operator is required.

The system also has practical advantages, such as simple start-stop operation, quiet operation, minimum maintenance requirements, and good part load performance. Typical applications are:

- power plants with low temperature geothermal fluids, up to 1 MW electrical power per unit,
- biomass fueled plants,
- heat recovery applications (in the range of 400 to 1500 kW electrical power),
- solar systems application [31].

Many companies have developed and implemented their own technologies in recent years, e.g. radial flow turbines, axial microturbines, direct evaporation turbine units, centripetal radial flow microturbines or screw expanders [27,28]. By the end of 2016, 1754 ORC turbine units with a total installed capacity of more than 2.7 GW had been built, according to the study [27].

Often the main source of energy in ORC power plants is waste heat from various energy, industrial, agricultural or even domestic processes [32,33]. The utilisation of this particular

type of heat facilitates enhanced energy efficiency by means of its recovery and reuse, as opposed to its dissipation into the environment. To illustrate, in industrial facilities such as steel mills, cement plants and chemical plants, waste heat is frequently a by-product of technological processes [34,35]. Numerous examples in the extant literature illustrate the utilisation of waste heat from internal combustion engines [36] as well as from gas turbines [37] as a source of thermal energy for ORC systems. The implementation of these solutions has the potential to enhance the overall energy efficiency of these units, which, in practical terms, is evidenced by a reduction in energy losses and an increase in efficiency by several percentage points.

In addition to waste heat, geothermal energy is a frequently analysed and utilised energy source for ORC systems [38]. Geothermal heat sources, including hot springs and geothermal boreholes, are a stable and renewable energy source that can power ORC power plants throughout the year, irrespective of weather conditions. Another area of interest is the use of solar energy [39], which, when utilising suitable solar collector technology, can be efficiently converted to thermal energy to power the ORC cycle. In coastal or island regions, examples are also emerging of the use of ocean thermal energy, known as Ocean Thermal Energy Conversion (OTEC) energy, as a heat source for ORC power plants [40]. This type of solution is based on the temperature difference between the warmer surface of the ocean and the cooler layers of the deep sea, making it possible to generate electricity in a renewable and environmentally friendly manner. The above-mentioned heat sources vary considerably in temperature from around 30°C to 200°C, which has a significant impact on the selection of working medium and, consequently, the design of the microturbine.

The working fluids utilised in ORC power plants are required to adhere to a multitude of criteria, which poses a significant challenge in the design of such systems. Operational safety is paramount, necessitating the use of a working fluid that exhibits minimal toxicity and environmental impact. In practice, substances with high greenhouse gas potential (GWP) and high ozone depletion potential (ODP) are typically excluded or used only in exceptional cases. The environmental impact of a given substance is also associated with its capacity for biodegradation and the emission of volatile organic compounds (VOCs). In addition to the environmental aspects, the working fluid should be inexpensive and readily available on the market. The cost of the working fluid has a direct impact on the cost-effectiveness of the overall project, particularly in the case of larger installations, where the quantity of fluid required is substantial. Another important requirement is its chemical stability, both thermal and material. It is imperative that the working medium does not react chemically with the components of the flow system (e.g. heat exchangers, turbines or piping), as this could lead to their corrosion or degradation and, consequently, result in the premature failure of the plant. In practice, it is often challenging to identify a working medium that fulfils all these requirements while preserving optimal thermodynamic properties. Consequently, designers are compelled to make compromises, which may entail the implementation of more sophisticated and costly safety technologies, such as leak detection systems or fire protection systems, particularly in the context of flammable media, including

saturated hydrocarbons (methane, propane) and unsaturated hydrocarbons (ethane, ethylene). It is therefore evident that the selection of the operating medium constitutes a significant engineering challenge in the design of ORC systems, given the extensive range of potential substances that can be utilised.

The extant literature comprises analyses of diverse groups of compounds, including saturated hydrocarbons (e.g. n-pentane), unsaturated hydrocarbons (e.g. propylene), cyclic hydrocarbons (e.g. cyclohexane), heterocyclic hydrocarbons (e.g. furfural), aromatic hydrocarbons (e.g. toluene), as well as synthetic refrigerants (R134a, R1234yf), alcohols (ethanol, methanol), siloxanes (e.g. MM, MD2M) and other compounds [41]. A number of studies have been conducted on the selection of suitable operating agents for ORC systems, as it is crucial for process efficiency and stability. A range of studies have been conducted that consider both synthetic refrigerants and natural organic compounds with a view to determining whether they can meet the requirements of specific applications. The most commonly used working refrigerants include compounds such as R245fa, R123, n-butane, n-pentane and R1234yf [41]. These refrigerants possess relatively low boiling points, rendering them efficacious in ORC systems with low to medium heat source temperatures.

Ethanol, as an example of alcohol, is also used as a working fluid in ORC systems due to its thermodynamic properties and relatively low toxicity [40]. Other refrigerants, such as R134a, R152a, R236fa, R245fa, or mixtures such as R404a and SES36, have also been extensively studied in the context of ORC engine applications [37]. Many of these compounds have low ozone depletion potential (ODP) and moderate global warming potential (GWP), making them more environmentally friendly [42,43].

In the context of high-temperature ORC systems, compounds with higher boiling points, such as toluene, siloxanes (e.g. MDM, MM, MD2M), are often preferred [44], which show stability in higher temperature ranges. Furthermore, agents such as ammonia, cyclohexane or acetone are of interest in this context [45], which, due to their chemical and thermodynamic properties, can be used under more demanding conditions, e.g. using waste heat from higher temperature processes.

In the case of some ORC systems, mixtures of operating media are also considered. These mixtures may provide enhanced thermodynamic properties, but their composition and characteristics are often not commercially available and, in many cases, are protected by patents. Therefore, the selection of a suitable working medium should prioritise cycle efficiency and maximise the use of the available heat source, which directly affects the energy efficiency of the overall system.

A fundamental difference between ORC cycles and conventional Rankine cycles lies in the temperature of the working medium at the turbine inlet. This results from the specific properties of the organic fluids employed, such as isopentane, R245fa, or siloxanes. These fluids have a lower evaporation temperature than water, enabling the efficient utilisation of low- and medium-temperature heat sources. Consequently, conventional ORC power plants typically achieve efficiencies in the range of 10–12%, while modified ORC systems can reach efficiencies of up to 15–20% [46]. Such modifications may include an increase in the maximum temperature or a change in the structure of the

entire plant. Increasing the temperature of the working medium at the turbine inlet leads to a higher temperature difference over the cycle and, consequently, an increase in thermodynamic efficiency [47]. This is achieved by using stronger materials in heat exchangers and turbines, allowing for safe operation at higher temperatures. Another method of increasing efficiency is to modify the plant structure. The incorporation of an additional cycle is an illustrative example of the concept, as it results in the establishment of a cascade system. In such a system, the waste heat from the first cycle is utilised as a heat source for the second cycle, which results in a substantial increase in the overall efficiency of the plant [48].

In addition, the implementation of an auxiliary heater has been demonstrated to enhance the temperature of the working medium prior to its expansion within the turbine, thus leading to an enhancement in process efficiency [49].

As in conventional steam turbine systems, the efficiency of ORC cycles can also be increased by using an interstage superheater. Interstage superheating entails heating the medium after partial expansion in the turbine, which enhances the efficiency of subsequent expansion [50]. Moreover, the utilisation of regenerators facilitates the recovery of heat from the working medium following expansion. This process elevates the temperature of the medium prior to its entry into the evaporator, thereby diminishing the heat demand [51,52].

Another modification that has the potential to enhance efficiency is the implementation of a parallel evaporator. This apparatus facilitates the concurrent utilisation of diverse heat sources, each with distinct parameters. Consequently, it enables the optimised alignment of the cycle's properties with the available resources, thereby ensuring enhanced efficiency [53].

In addition to structural changes, efficiency improvements can also be achieved by optimising the operation of individual power plant components such as turbines, compressors, boilers, pumps and electrical generators. The optimisation of turbines and compressors through the utilisation of contemporary materials and precision machining technologies has been demonstrated to reduce mechanical losses and enhance efficiency. Improvements to heat exchangers, such as boilers and evaporators, can increase the efficiency of heat transfer. Moreover, modern electric generators with higher efficiency allow for better conversion of mechanical energy into electricity.

Nevertheless, despite these advances, a significant disparity in efficiency persists between ORC systems and conventional steam or gas cycles. Consequently, there is an ongoing focus on research endeavours aimed at enhancing the efficiency of ORC power plants. A pivotal advancement is the augmentation of working fluid parameters, including temperature and pressure. New working fluids with higher thermal stability can enable operation at higher temperatures, which would significantly increase cycle efficiency. Concepts such as Kalina cycles and supercritical ORCs represent promising research directions, as they offer the potential to achieve higher efficiencies by better matching the properties of the working fluid with the available heat sources [54,55].

The development of ORC microturbine design focuses both on enhancing flow efficiency and minimising losses, as well as

ensuring operational reliability. Of particular importance are sealing solutions aimed at reducing working fluid leakage. In addition to conventional labyrinth seals, honeycomb-type seals have been investigated, which effectively reduce leakage flows but require advanced manufacturing technology and may introduce additional frictional losses [56]. In parallel, experimental studies of microturbogenerators have demonstrated the existence of optimal heat source power and generator load values at which maximum electrical output can be achieved [57].

Another crucial direction of development is the implementation of advanced diagnostic and condition monitoring systems. Fibre Bragg Grating (FBG) optical sensors enable leakage detection and temperature monitoring of critical microturbine

components [58], while artificial intelligence algorithms, such as Bayesian-optimised Long Short-Term Memory (LSTM) networks, allow rapid identification of sensor malfunctions, thereby improving the safety and reliability of ORC systems [59].

As previously stated, the implementation of supercritical parameters has been demonstrated to enhance the efficiency of ORC cycles. However, it has been demonstrated that replacing adiabatic expansion with isothermal expansion can lead to an enhancement in efficiency of up to ten percentage points (see Fig. 8) [47,60].

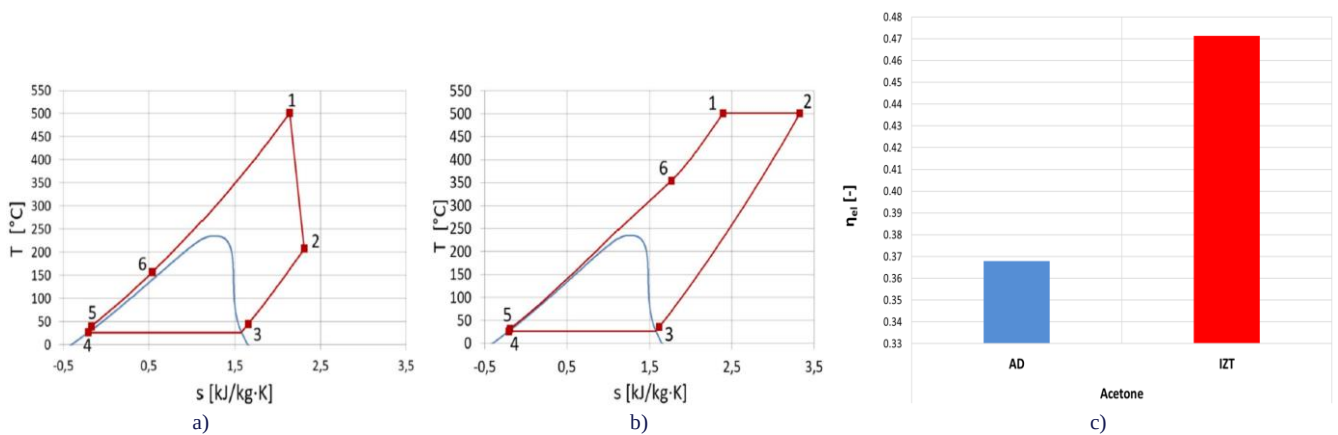


Fig. 8. Thermodynamic cycle of the ORC power plant with acetone as a working medium: (a) adiabatic expansion in the turbine, (b) isothermal expansion in the turbine, (c) comparison of electrical efficiency (AD – adiabatic expansion, IZT – isothermal expansion).

Moreover, endeavours are currently being undertaken to engineer more robust materials for heat exchangers and turbines that have the capacity to function at elevated pressures. This development has the potential to enhance efficiency.

This section of the paper provides a comprehensive overview of the current state of ORC technology. It places particular emphasis on the operating medium, temperature and pressure levels, as well as the specifics of each application, which are primarily influenced by the heat source. The investigation identified the type of working medium and the turbine as two key aspects of ORC technology. A comprehensive literature review has identified numerous studies addressing working fluids, underscoring their various limitations. The emphasis placed on design solutions by designers and constructors is notable for its simplicity, with axial or radial single-stage turbines being of particular interest. These turbines are characterised by their relatively high efficiency, durability and cost-effectiveness. However, it should be noted here that each case should be treated individually in the design of an ORC micro-turbine, depending on the operating parameters, heat source, intended use and other technological aspects. The primary technological characteristics of ORC solutions were enumerated, and a comparison with the conventional steam cycle demonstrated that ORC systems were more appropriate for low-power (microturbines) and low-temperature applications. The subsequent chapter will address gas microturbines, which are also undergoing development.

4. Gas microturbines

Various configurations of gas microturbine systems have been documented in the existing literature. In the majority of cases, two distinct microturbine power plant schemes are employed: an open cycle scheme (Fig. 9a) and a closed cycle scheme (Fig. 9b). The majority of systems incorporate a recuperator, the function of which is to recover a portion of the exhaust heat. This heat is then utilised to preheat the combustion air (see Figs. 9c and 9d). As demonstrated in Fig. 9c, the air is compressed in the compression section, subsequently mixed with fuel, and then burned to generate power for the turbine section [61].

Gas microturbines are typically fuelled with natural gas, with a generation capacity ranging from 25 kW to 500 kW of electricity. The rotational speed of these engines is notably high, ranging from 50 000 rpm to 120 000 rpm, and occasionally exceeding 100 000 rpm. Their thermal efficiency, ranging from 20% to 30% without regeneration, is considered acceptable. However, with regeneration, the efficiency can be further enhanced. The provision of a diverse energy source within a specific region is facilitated by this mechanism. Gas microturbines are advantageous due to their compact size and relatively low investment cost. Furthermore, the emission levels exhibited during operation are minimal. It is evident that they serve to minimise the ecological impact of substantial transmission and distribution systems. In the contemporary business environment,

a significant number of companies are engaged in extensive research and development initiatives related to the construction of gas microturbines. The primary objective of these endeavours is to enhance the efficiency, reliability and durability of these devices, while concurrently reducing their overall cost. The primary objective of these activities is to enhance the effective efficiency, whilst concomitantly reducing NO_x emissions, extending the time between successive overhauls beyond 45 000 hours, and minimising investment costs to below 500 USD/kW. Another issue that must be addressed is the necessity for greater flexibility with regard to the type of fuel utilised. This would entail the use of different fuels for the same turbine, with a particular emphasis on the use of biofuel.

operates up to a speed of 40 000 rpm. The compact and light-weight design of microturbines renders them a viable option for a wide range of light commercial and industrial applications [62].

In industrial practice, gas microturbines (GMTs) are increasingly employed as prime movers in combined heat and power (CHP) systems, simultaneously generating electricity and heat with overall efficiencies of 80–90%. Their advantages include fuel flexibility—from natural gas to biogas—and very low exhaust emissions. Feasibility studies, such as those conducted for industrial laundries, confirm the high economic viability of GMT-based installations as well as significant primary energy savings [63].

As previously stated, a salient benefit of gas microturbines is their capacity to operate on a range of fuels e.g. natural gas, biogas, sewage gas, waste gas, stack gas, gas produced as products of carbon gasification or pyrolysis processes, biofuel, diesel fuel, gasoline, heating oil, methanol, ethanol, propane, hydrogen, etc. Microturbines produce thermal output at a temperature range between 200°C to 350°C, suitable for supplying various thermal needs. Their life time is estimated in the range of 40 000 to 80 000 hours. To date, these units have exhibited adequate reliability. However, not enough time has passed for them to be included in commercial services. As a result, providing data on their entire lifetime is currently unfeasible. Microturbine sets can be connected in parallel to support larger loads and improve power reliability. Reducing the output power of microturbines leads to lower mass flow and combustion temperature; consequently, the efficiency at part load may drop below the design value. Single-shaft gas microturbines with air bearings do not require lubricating oil or water, resulting in lower maintenance costs compared to conventional gas turbines. Microturbines that use lubricating oil do not need frequent oil changes because the oil is kept separate from the combustion products [64].

A distributed power generation system covers a variety of generating technologies, and in addition to microturbines, it includes fuel cells, wind turbines, photovoltaic cells, Stirling engines and internal combustion engines. The main advantages of a distributed power generation system are reducing transmission losses, distributing costs, ensuring higher reliability (especially in smaller systems), and potentially achieving greater overall efficiency by utilising waste heat for cogeneration. While the potential market for microturbines remains uncertain, propane-fueled microturbines offer significant advantages. Propane is often the preferred fuel in several situations, such as when natural gas is not available, when propane is more affordable than natural gas, or when a backup fuel source is needed for greater reliability, especially in conjunction with natural gas or diesel [65,66].

Modern gas microturbines are expected to demonstrate a set of advanced features, including reduced emission levels achieved through water or steam injection as well as dry low-emission combustion technologies [67], along with high fuel flexibility enabled by new combustion and fuel systems supported by advanced coatings [68]. Their operation should also ensure improved reliability and availability through extended

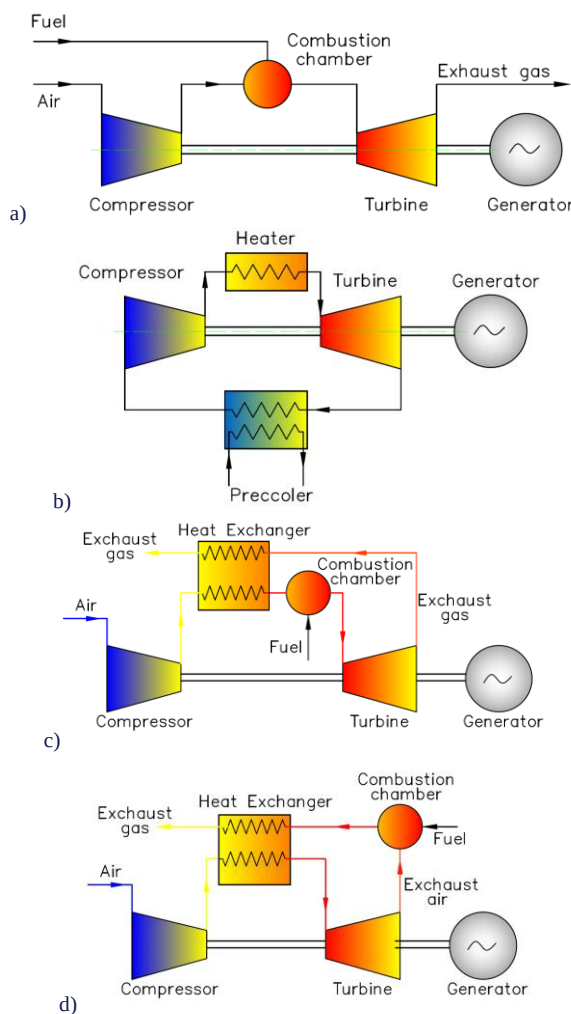


Fig. 9. Open cycle (a) and closed cycle (b) gas microturbine plant. Single shaft open cycle gas microturbines with a heat exchanger and a classical (c) or external (d) combustion chamber.

The majority of manufacturers are adopting a single-shaft design, whereby the compressor, turbine, and permanent-magnet generator are mounted on a single shaft supported by lubrication-free air bearings. These turbines are capable of operating at speeds of up to 120 000 rpm and are powered by natural gas, gasoline, diesel, and alcohol. The dual shaft design incorporates a power turbine and gear for mechanical drive applications and

component lifetimes, intelligent control, and condition monitoring systems, while maintaining higher overall efficiency and enhanced efficiencies of individual components resulting from tighter tolerances and advanced aerodynamic designs. Furthermore, progress is associated with higher firing temperatures, approaching the limits of material effectiveness [69], and the introduction of novel technological solutions such as recuperated cycles (Fig. 9c, [69]), intercooled recuperated cycles [70], integration with high-temperature fuel cells (solid oxide or molten carbonate), power plants equipped with Heat Recovery Steam Generators (HRSG) [71,72], wet cycles [6,70], combined heat and power systems, and photovoltaic integration [73]. At the same time, further developments are expected to reduce costs, increase power density through higher firing temperatures and innovative component designs based on advanced materials such as ceramics [74], and enable more compact turbomachinery with lower component costs resulting from more highly loaded elements.

The advanced cycle is a regenerative Closed Brayton Cycle (CBC) as presented in a paper [73]. The working fluid is a mixture of helium and xenon. The main components of the cycle are: the receiver, which has to collect solar energy, to store it in a phase-change eutectic salt (LiF-CaF_2) and to transfer it to the working fluid, the cooler, which refrigerates the working fluid and ejects the low temperature heat into space via the radiator device; the radial turbomachinery, consisting of the compressor, expander and electrical generator, all assembled on a single rotating shaft, and the recuperator, which recovers the exhaust heat from the expander outlet and transfers it to the compressed air.

Another example is the DynaJet 2.6 portable gas turbine generator, which has a rated output of 2.6 kVA and is shown in Fig. 10.

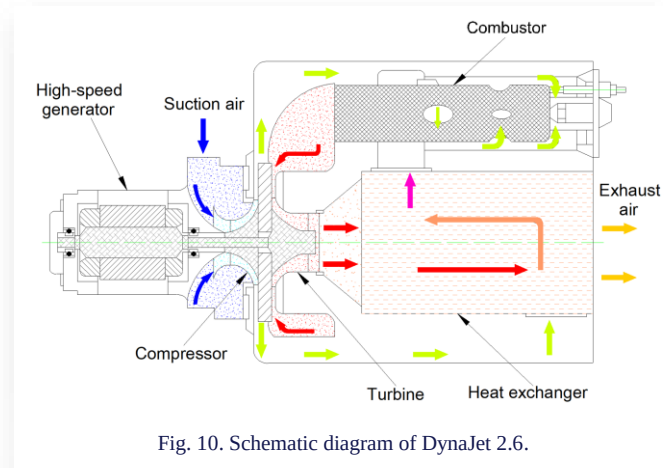


Fig. 10. Schematic diagram of DynaJet 2.6.

The unit is powered by a single-shaft regenerative gas turbine engine. It features a permanent magnet system for the generator, which requires effective cooling to ensure optimal performance at high speeds. The rated rotational speed is 100 000 rpm, and the rated power output is 100 V / 2,6 kVA. The device can operate at a frequency of either 50 Hz or 60 Hz, with the option to switch between these settings. It uses kerosene as fuel, with a consumption rate of less than 4.5 litres per hour at the rated output. The generator operates reliably within a temperature range of 20°C to 50°C. Its dimensions are 825 mm

(length), 420 mm (width), and 455 mm (height). Small gas turbines with a power rating of 100 kW or less are attractive not only as distributed power generation systems or for mobile power supply, but also as the core of a combined heat and power system due to the excellent quality of the exhaust gas [75].

Another type of gas turbine is the wave rotor system (Fig. 11). Wave rotors do not use mechanical components such as pistons or blade cascades to compress the fluid. Instead, the pressure increase is achieved by generating compression waves in appropriately designed geometries. It has been proven that, for the same inlet and outlet conditions, wave rotors can significantly enhance performance. The essential feature of wave rotors is an array of channels arranged around the axis of a cylindrical drum. The drum rotates between two end plates, each with several ports or manifolds that control the fluid flow through the channels [76].

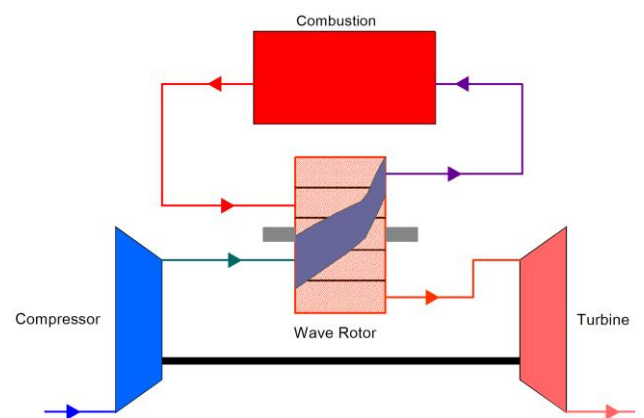


Fig. 11. Schematic diagram of a gas turbine topped by a four-port wave rotor.

The next example is a single-stage axial microturbine. It has been developed with a rotor diameter of 10 mm, see Fig. 12. The turbine is made from stainless steel using die-sinking electro-discharge machining. It has been tested for speeds up to 160 000 rpm and generates a maximum mechanical power of 28 W with an efficiency of 18% [77,78].

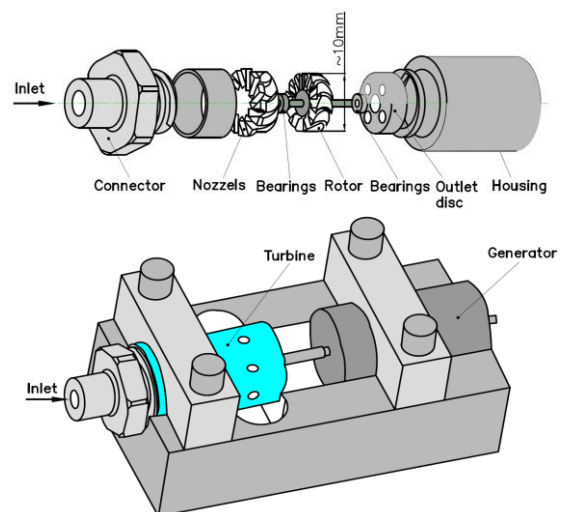


Fig. 12. Single-stage axial microturbine.

In contrast, a groundbreaking solution for a gas microturbine with isothermal expansion can be found in the paper [79] (see Fig. 13). In the proposed solution, the working medium, after isothermal expansion in the nozzles, is transferred to a guide

system that provides the correct flow angle to the rotor blades. The rotor of the turbine is shown in Fig. 13a, the compressor in Fig. 13b, and the whole stand is illustrated in Fig. 13c.

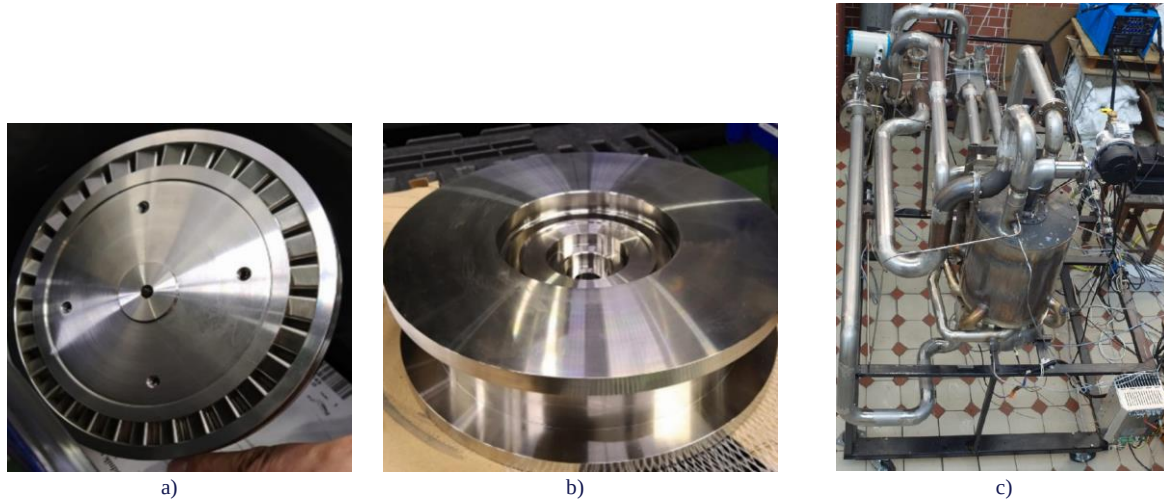


Fig. 13. Isothermal microturbine rotor (a), casing (b) and test rig (c).

The experimental rig consists of the combustion chamber, turbine, compressor, electric generator and regenerator. A large number of tests have been carried out on the operation with various changes and modifications. The test rig is equipped with sensors that measure the temperature and pressure of the working fluid at characteristic points of the unit (at the inlet and outlet of each component), the air flow rate, the rotor speed and the parameters of the electricity generated. The nominal power of the turbine was approximately 5.3 kW, and the rotational speed was 27 000 rpm. It is likely that there will be commercial offers for this type of microturbine set in the near future, as it is the solution with the highest achievable efficiency.

Gas turbines and turbojets are widely used in aviation. Micro jet turbines, on the other hand, have been used in modelling for many years. In these systems, they are small in size and the main parameters are: thrust of several hundred Newtons, temperatures behind the combustion chamber reaching up to 700°C, high revolutions exceeding 100 000 rpm. Table 2 shows examples of micro jet turbines and the operating parameters [80–83].

Microturbines are also used in air-conditioning systems in aircraft – Air Cycle Machine (ACM). ACM systems are the cooling units in the on-board atmosphere control system used in airtight aircraft cabins. Typically, aircraft have two or three ACM

Table 2. Parameters of exemplary airborne microturbines.

No.	Name	Thrust	Temperature, °C	Mass, kg	Diameter, mm	Length, mm	Mass flow, kg/s	Rotational speed, rpm	Compression
1	AMT Olympus	230 N	-	2.475	130	267	0.45	108000	4
2	KH-66	75 N	-	0.93	112	230	0.23	115000	2.2
3	Wren MW54	54 N	-	0.8	87	150	0.18	160000	2.3
4	JF-50 Bee Titanium	63 N	700	0.8	110	170	0.2	180000	2.3
5	JF-160 Rhino	35 lbs	650	1.5	110	300	0.52	116000	-
6	JF-120 SuperEagle	30 lbs	700	1.35	110	300	0.34	128000	2.37
7	Merlin JG-100	27.5 lbs	640	1.25	108	230	0.139	126000	2.37
8	Merlin KJ-66	92 N	590	-	108	230	0.154	128000	1.56
9	Merlin 160G	160 N	550–680	1.45	111	300	0.38	116000	3.2
10	GTM-70	80 N	600	1.2	115	300	-	120000	-
11	FD3 Schreckling	30 N	~700	0.75	-	-	0.16	-	1.5
12	Kamps	50 N	~650	1.14	-	-	0.166	-	2
13	T250P JPX	59 N	~740	1.55	-	-	0.177	-	2.15
14	JF-100 Falcon	22 lbs	650	1.4	98	300	0.27	122000	-
15	JF-50 Bee Tornado	25 N	565	0.4	58	132	0.2	200000	-

units. The cooling process uses air instead of freon in a gas cycle. There is no condensation or evaporation of the working medium in the cycle, the cooled air is used directly to cool the aircraft cabins. The microturbines in these systems run on gas bearings at speeds of 30 000–50 000 rpm [84].

Gas microturbines are increasingly recognised as a key component of modern energy systems, both in industrial applications and in specialised fields such as micro- and nanosatellites. Their design combines compact dimensions with the capability to operate in high-efficiency combined heat and power (CHP) systems as well as in applications requiring high precision and reliability. For space microengines, one of the fundamental design challenges is the modelling of flows in micronozzles and microchannels. The influence of the boundary layer, both laminar and turbulent, on velocity distribution and pressure losses is so significant that dedicated numerical and experimental models are required to design engines capable of delivering thrust in the order of milli-Newtons [85].

Small power turbines can also be found in various means of transport. Rolls-Royce manufactures turbines that have been used to power many small aircraft and helicopters for many years, such as the Bell helicopter. This turbine produces a power output of 533 kW. More than 29 000 units of this turbine have been installed since production began [86].

An older model of the Rolls-Royce 250 turbine was used to power Marine Turbine Technologies (MTT) motorbikes. Installing such a turbine allowed a top speed of around 400 km/h and acceleration from 0 to 320 km/h in around 15 seconds. The turbine produces 286 hp (≈ 213.4 kW) at 52 000 rpm [87].

Capstone, a company known primarily for its turbines for distributed heat and power generation, was awarded the contract in early April 2008 to manufacture 150 C30 turbines to power buses. The lightweight and robust buses, made mainly of aluminium, will be driven using the energy contained in the batteries; the turbine will only be used to recharge the batteries [88].

Volvo Aero, in turn, also developed a series of microturbines for use in cars, trucks and buses. Between 1988 and 1987, it developed a twin-shaft ceramic microturbine for cars of the KTT MK II/GT110 type with a power output of 115 kW. Later, it produced the new VT40 single-shaft microturbines for cars and the VT100 microturbines for trucks and buses [89].

Jaguar unveiled a prototype of the new Jaguar C-x75 car at the 2010 Paris Motor Show. It is a design that combines gas turbine engine technology with modern electric drive to create a hybrid car. The car has two gas microturbines with 97 hp (≈ 72.4 kW) each. These are used to drive the generators that charge the battery bank, which drives four electric motors with a total output of 800 hp (≈ 596.8 kW). The car is able to achieve acceleration in less than 14 s to 300 km/h. The range of this car on a full battery is 90 km, and with turbocharging 900 km can be driven on a full tank. The use of virtually any liquid fuel, gaseous fuel, biofuel, hydrogen, etc. opens a wide door to a new trend in motoring. The Jaguar C-X75 promised to be great, but never went into production [90].

Turbine drive of rotary tools is a relatively recent development and not widely used. In industry, microturbines can be found with powers from 1 to 5 kW, for driving precision tools.

They are used as power sources in high-speed grinding spindles, drills, in measuring, control and research apparatus (gyroscopes, mirrors and rotating prisms of laser optical systems, high-speed camera drives, etc.), as expansion turbines in air-conditioning and refrigeration systems and as medical tools (dental and surgical drills). These microturbines operate at relatively low power and high frequencies, so their development has undoubtedly been influenced by the fact that they have mastered the problems of gas bearing to achieve very high frequencies (up to 30 000 Hz and even more). These microturbines are usually powered by compressed air with a pressure of 0.4 to 0.8 MPa and a temperature of about 20°C, drawn from [64].

Medical drills, especially dental drills, were introduced into practice because of the discomfort experienced by patients when preparing hard dental tissue with low-speed drills. The first compressed-air-driven turbine had rolling bearings. Since the mid-1960s, air-bearing drills with a maximum speed of approximately 500 000 rpm have been introduced. These drills are much quieter, and the drill bit is not stimulated to vibrate by the bearing balls. They also do not require oil lubrication and are very durable. Due to these advantages, numerous companies have started to introduce air bearing in the dental drills they manufacture [91,92].

Nowadays, many companies offer, for example, micro turbine tools for precision engraving work based on dental-type turbine cartridges. They offer a low power output of 30–60 W at speeds of up to 400 000 rpm. Thanks to this high speed, the “floating” effect of the cutter tip (torque effect) is minimised [93,94].

Turbine grinders and polishers are another example of the use of microturbines. A turbine grinder offers double the power compared to motorised angle grinders. They are about half the size and weight of motorised tools, offering comparable power. The turbine motor is more efficient than a conventional grinder motor. As a result, it takes less time to do the same amount of work. The overall air consumption is also lower during a specific task. The GTG21 turbine grinder reaches a maximum speed of 12 000 rpm with a power output of 2.1 kW and an air consumption of 30 l/s. The GTG40 turbine grinder, on the other hand, reaches a maximum speed of 8 500 rpm at 4.5 kW and air consumption of 60 l/s. The turbine motor used in these grinders does not need oil in the air for lubrication [95].

Another example of a turbine-driven machine (single-stage axial turbine) with special gas bearings for up to 100 000 rpm and a cutting capacity of up to 100 kW is the TurboTool™ cutting tool [96].

Another way in which microturbines are used are air motors with outputs ranging from 0.1 kW to 35 kW. They have found wide use in industry as a powerful and trouble-free drive motor. They are smoothly controllable over a very wide speed range and provide high starting torque. They can operate at pressures from 2 to 7 bar. The speed of such a motor varies between 2 500 and 7 600 revolutions per minute [97].

A typical microturbine consists of a shaft, on which a generator, a compressor and a turbine are mounted. The turbine shaft rotates at 50 000–100 000 rpm, which corresponds to electrical frequencies ~ 833 –1667 Hz. This power has to be rectified and re-inverted to electrical energy at 50/60 Hz for interfacing with

grid power. Rectifiers and inverters are usually very energy efficient but are highly nonlinear, which can negatively affect the system [98–101].

The microturbine control system represents a key problem for stable operation at different working conditions, ensuring a flexible usage of the machine. The experience gained in large-size turbine control systems can also be profitably exploited in the microturbine field, but the time scales of the phenomena are extremely reduced due to the small mechanical inertia of the shafts, which makes microturbines difficult systems to control [102].

Another important development area involves the integration of microturbines with microgrids and hybrid systems. This requires advanced solutions for protection, synchronisation, and control, including adaptive algorithms and diagnostic systems supported by artificial intelligence [103]. In parallel, intensive research is being carried out on the utilisation of alternative fuels, with hydrogen emerging as a particularly promising clean energy carrier for cogeneration and trigeneration systems [104]. These advancements highlight the potential of gas microturbines as a flexible and low-emission technology supporting the transition towards decentralised and sustainable energy systems of the future.

During the last years, we have observed an increasing interest in small nuclear power stations in different countries around the world [105]. It concerns the construction of so-called minireactors (or microreactors), which are foreseen for energy supplying for houses or military uses. An example is a 200 kW microreactor built in Japan by Japan Atomic Energy Research Institute, which is cooled with liquid sodium. The most interesting thing is that the fuel should be changed every 10 years [106].

The literature analysis of micro gas turbines carried out in this chapter showed that there has been a tremendous development of this technology in the last two decades, and a large variety of proposed designs have emerged. This is due to the very different applications of this type of device. Designers' approaches are focused on improving efficiency, durability, reliability and, of course, low price. The next chapter presents a completely different type of microturbine design solution, namely the adhesion microturbine pioneered by the eminent inventor Nikola Tesla. Devices of this type have been classified in a separate chapter because they can operate as steam microturbines, including those for organic refrigerant vapours, and as gas microturbines.

5. Tesla microturbines

The first adhesion turbine, also known as the Tesla turbine, was designed, built and patented [107] by prominent inventor Nikola Tesla in 1913 (Fig. 14). It was an innovative type of bladeless turbine that operated on a completely different principle to traditional blade turbines.

The operating principle of this type of turbine is based on the phenomena of adhesion and friction, which primarily occur within the boundary layer of fluid between two closely spaced rotating discs. The fluid moves in a spiral pattern through the narrow gap between the discs, travelling from the outer diameter

towards the inner diameter. As it flows, the fluid transfers a portion of its kinetic energy to the discs, causing them to rotate. The turbine is powered by one or more inlet nozzles positioned at the outer edge of the discs, directing the working fluid into the gap. The fluid then exits through openings located near the central shaft of the turbine. The distance between the turbine discs is typically very small and is determined by the thickness of the boundary layer.

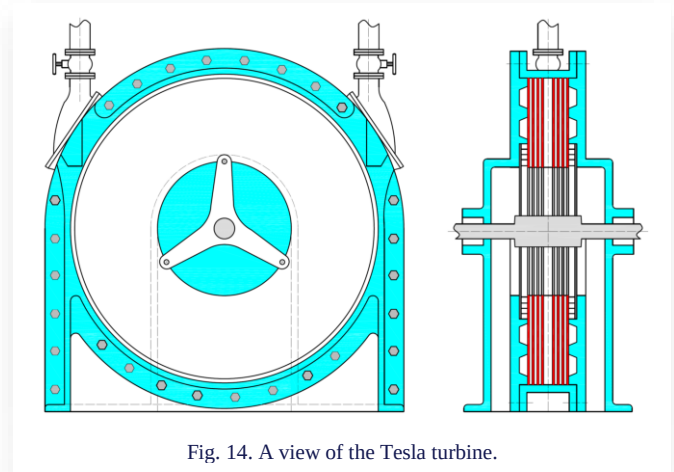


Fig. 14. A view of the Tesla turbine.

Tests conducted by Tesla on real adhesive turbines demonstrated that, as the diameter of the discs increased (and with it the power output, reaching up to 150 kW), structural damage could occur due to material deformation. It is important to note that during the time these experiments were performed, the technology for manufacturing and processing materials capable of withstanding high temperatures and high rotational speeds was limited. Despite the promise of his innovative approach, these material constraints hindered further progress. Tesla's financial situation had deteriorated significantly, and without sufficient resources, he was unable to continue developing and refining the turbine. Ultimately, the combination of material limitations and the absence of financial backing forced Tesla to abandon the project [108,109]. Although he had successfully patented the design and demonstrated its potential in laboratory tests, the adhesive turbine never reached commercial viability in Tesla's lifetime. Nevertheless, his work laid the foundation for future research into bladeless turbines and remains a source of inspiration for engineers and scientists exploring alternative methods of fluid dynamics and energy conversion.

For many years, Tesla's innovative turbine design was largely overlooked, as the scientific community and industry favoured more conventional technologies like traditional bladed turbines and reciprocating engines. However, by the mid-20th century, there was a renewed interest in alternative turbine designs as researchers sought more efficient and simpler mechanical solutions for energy conversion. It was not until the 1950s that scientists began to revisit Tesla's ideas, recognising the potential advantages of his bladeless turbine concept, particularly its simplified construction and reduced number of moving parts. One of the pioneers in this research was Armstrong, who in 1952 built his own turbine model. The device was steam-powered and

based on the Rankine cycle. Armstrong introduced an innovation by using discs of varying thickness, which increased the rotor's strength and reduced fluid turbulence. Thanks to these modifications, an efficiency of a few per cent was achieved at speeds between 4 000 and 6 000 revolutions per minute [110].

In 1965, Warren Rice, building on Tesla's design, constructed six models that he tested using air as the working fluid. The efficiencies he achieved ranged from 36% to 41%. Using the research results and mathematical analysis, he developed a design relationship between the disc radius and the spacing between them, which allowed the turbines to achieve maximum efficiency at different flow rates. In his conclusions, he noted that Tesla's turbine could be suitable for low-power applications, particularly in situations where low production costs are crucial or where the properties of the working fluid make the use of conventional turbines impractical [111].

In 1966, Beans developed a turbine powered by compressed air. He tested his system for various values of angular velocity, inlet pressure and disc spacing, obtaining efficiencies ranging from 7% to 25% [112]. Although the efficiency values were significantly lower compared to those achieved by Warren Rice, Beans' work provided valuable information on the effects of various operating parameters on turbine performance. By varying the angular velocity, Beans was able to determine the optimum operating speeds for different configurations, highlighting the importance of precise speed control to increase efficiency [113,114].

The reported experimental efficiency of Tesla microturbines in the literature varies considerably, for example:

- 32% efficiency for a turbine with a power output of approximately 3 kW and a rotor speed of 15 000 rpm [115],
- 23% efficiency for a turbine with a power output of approximately 1.5 kW and a rotor speed of 12 000 rpm [116],
- 21% efficiency for a turbine with a power output of approximately 50 W and a rotor speed of 1 000 rpm [33],
- 49% efficiency for a turbine with a power output of approximately 1.56 kW and a rotor speed of 12 000 rpm [112].

For example, the numerical calculations resulted in an efficiency of about 50% obtained for an 11-stage turbine with an output power of about 2 kW at a rotor speed of 18 000 rpm [117].

Tesla's turbine continues to attract significant interest, as evidenced by numerous publications analysing the turbine both theoretically and practically. As demonstrated by the work of Thomazoni et al. [118], the number of publications related to Tesla's turbine has increased significantly in the 21st century. Of these, approximately 36% are analytical studies, 31% are analyses using CFD, 31% are experimental studies, and the remaining 2% are review papers. For example, in the works of Guha and Smiley [119] and Sengupta and Guha [120], similar to Beans, Tesla's turbine was tested by varying selected design parameters. The best configuration solution generated 140 W with an efficiency of about 25%. Modifications to the inlet nozzles were also investigated, which reduced pressure losses and improved efficiency by about 1 percentage point.

In 2015, Krishnan [121], in his qualification work, presented a series of guidelines for the design and construction of Tesla

microturbines. The work contains a wealth of valuable data, analyses, and results of experimental studies related to Tesla's turbine and its potential applications. For example, the author determined the minimum radius-to-disc spacing ratio as 20.

Theoretical works employing CFD consider various aspects of designing Tesla-type turbine devices [122]. The efficiency of a Tesla turbine is influenced by both the working fluid parameters (pressure, temperature, inlet velocity) and its kinematics at the inlet and outlet, as well as geometric parameters (distance between rotor discs, diameter, thickness, and number of discs, roughness and micro-topography of their surfaces), rotor rotational speed, and mass flow rate. In the case of mass flow rate, there is a direct influence, namely, the greater the flow, the greater the output power [123]. Similarly, for the initial pressure, analyses have shown that an increase in pressure before the turbine increases efficiency [124]. For example, an increase in pressure by about 1 bar results in an increase in efficiency of about 5% [118].

Regarding the influence of temperature on the efficiency and power of a Tesla turbine, researchers' findings are divided. Publication [125] states that both power and efficiency increase with increasing inlet temperature, while publication [126] demonstrates the ambiguity of the influence of increasing temperature on output power. As in other works, including this one, there are no clear statements regarding the effect of the number and shape of inlet nozzles on turbine efficiency and power, and research in this area is still being developed [127].

In most published cases, the rotational speeds of Tesla turbine rotors are lower than those of bladed turbines [118]. Analyses can be found that define the output power and efficiency of a Tesla turbine as a non-linear function of rotational speed, approximately a second-order polynomial, for a fixed geometry and inlet conditions [128].

The outer diameter of the rotor affects both efficiency and output power. Most studies indicate an increase in turbine output power with an increase in the outer diameter of the rotor. According to some authors, there is an optimal value of the ratio of the outer to inner diameter of the rotor that maximises efficiency [129].

In the case of discs in a Tesla turbine, there is also no consensus among authors. Some argue that there is an optimal number of discs and their thickness that gives the maximum efficiency [130]. Other authors show that the fewer discs, the higher the efficiency [129], while still others argue that efficiency increases with the number of discs [131].

It is quite common to believe that bladed turbines are unrivalled compared to Tesla-type turbines in the range of higher powers, but at very low powers, one can expect a certain advantage of bladeless adhesive turbines [132].

For example, an analytical model incorporating wall roughness through the equivalent sand grain approach was proposed by Rusin et al. [133]. The model modifies dynamic viscosity in the governing equations, allowing the impact of roughness on velocity profiles and turbine efficiency to be assessed. Their results indicate that a surface roughness height of 10^{-5} m can noticeably improve both power output and isentropic efficiency.

In a complementary approach, Pahlavanzadeh et al. [134] introduced a porous layer model (PLM) for directly simulating roughness effects in the gap between rotating disks. This method eliminates the need for equivalent sand grain parameters and was validated against experimental data from minichannel flows. The study demonstrated that increasing surface roughness enhances the interaction between the working fluid and the rotor, thereby influencing overall turbine efficiency.

Similarly, there is no clear consensus on the influence of the number and shape of inlet nozzles on turbine performance. Research in this area is still ongoing [135].

Also worth noting is the growing interest in integrating Tesla turbines with low-temperature heat recovery technologies. A recent study by Teng et al. [136] presents a comprehensive experimental investigation of a Tesla microturbine integrated into an ORC system for low-grade heat recovery (90–130°C). The authors developed a prototype that achieved a maximum output power of 31.76 W and an isentropic efficiency of up to 62.28%, examining the effects of parameters such as heat source temperature, pump speed, and electrical load on system performance. The results highlight the potential of Tesla turbines as efficient and compact solutions for small-scale distributed energy systems and industrial waste heat recovery.

The concept of turbomachinery miniaturisation typically refers to the design of devices with reduced power output, intended to meet the demands of distributed, portable, or microscale systems. These scaled-down machines are used in applications such as portable electronics, IoT sensors, or microenergy systems. Energy micro-generators rely on piezoelectric, electromagnetic, electrostatic, or triboelectric transduction principles, effectively harvesting energy from low-frequency vibrations or human motion. Their power output is inherently limited and tailored to match their miniature form factor [138].

Huynh et al. [137] introduce a pioneering hybrid energy-harvesting system that combines a triboelectric nanogenerator (TENG) and an electromagnetic generator (EMG), both driven by a Tesla turbine. The optimised system achieves peak outputs of 312.5 V / 82 mA (TENG) and 4.2 V / 3.3 mA (EMG), with the hybrid configuration delivering 332 V / 3.5 mA. As demonstrated in the study, the device shows strong potential for use in wearable and low-power portable applications for biomechanical energy harvesting, as illustrated by practical demonstrations such as LED illumination and wireless sensor operation.

The miniaturisation of energy-conversion turbomachinery remains a significant challenge due to the nonlinear scaling of aerodynamic and mechanical phenomena. When reducing the physical size of rotating machines such as Tesla turbines, micro gas turbines, or centrifugal compressors, researchers must address issues including increased viscous effect and reduced flow uniformity. It is well documented that aerodynamic efficiency decreases at low Reynolds numbers due to boundary layer thickening and the dominant influence of viscosity [139].

Based on a literature review and the authors' own experience, a Tesla microturbine was built within a single enclosed hermetic housing (Fig. 15). The concept is based on a novel solution in which the magnets are placed in the rotating disc of the Tesla turbine. The stationary housing, on the other hand, houses the

poles where the electric current is induced. The diameter of the disc is 140 mm, the clearance between the moving and stationary disc is approximately 1 mm, and the inlet nozzle is set at an angle of 12° [140,141].

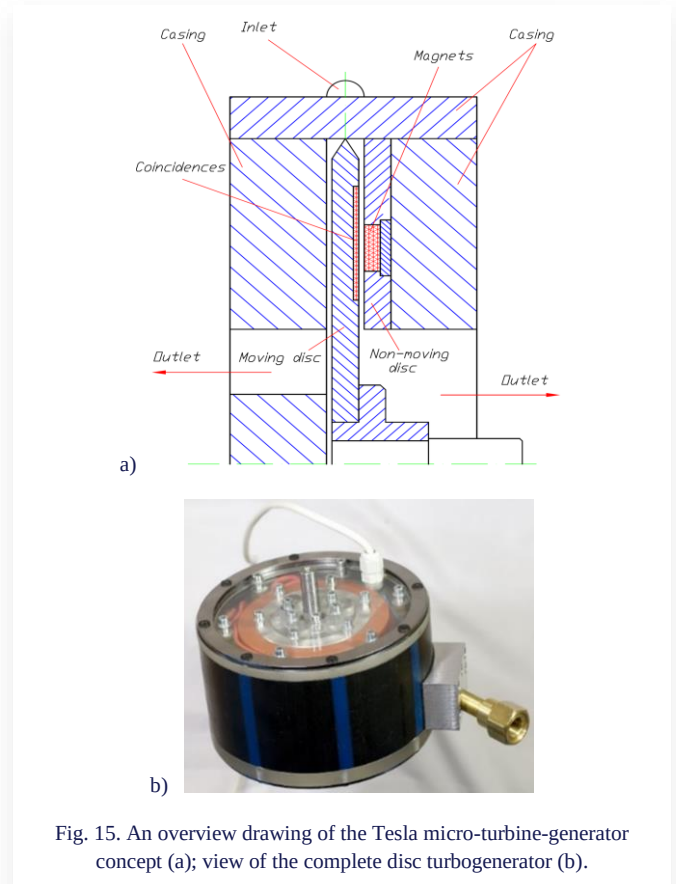


Fig. 15. An overview drawing of the Tesla micro-turbine-generator concept (a); view of the complete disc turbogenerator (b).

The conducted literature analysis has shown that despite the passage of over 100 years since Tesla's invention, work on its development is still ongoing and is certainly not complete. The next supplementary chapter presents examples of steam and gas microturbines made using MEMS technology.

6. Microturbines in MEMS technology

Micro-Electro-Mechanical Systems (MEMS) are miniaturised devices that integrate mechanical and electrical components on a single chip. These systems are fabricated using techniques derived from semiconductor manufacturing, such as photolithography, etching and deposition processes. This technology makes it possible to create devices that typically range in size from a few micrometres to a few millimetres. MEMS devices are extremely small, enabling the creation of compact systems with high functionality. They combine mechanical, electrical, thermal and sometimes optical components into a single system. MEMS devices are designed with nanometre precision and offer exceptional performance in precision applications [142].

Examples of steam microturbines made with MEMS technology are encountered in the literature [143,144]. More on MEMS can be found in publications [145–147], among others. An example of a steam microturbine, implementing the Rankine circuit, made with MEMS technology is presented in Fig. 16.

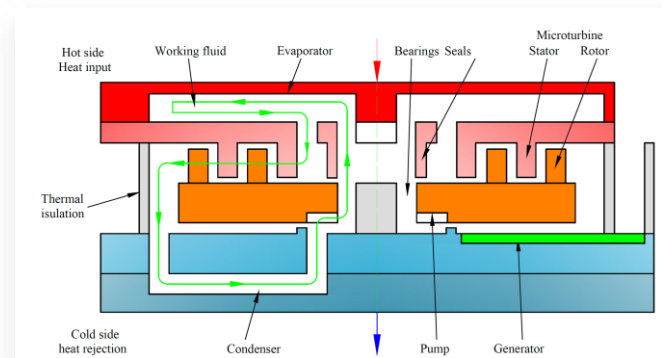


Fig. 16. Cross-section of a steam microturbine power plant on a chip.

The microturbine unit drives a micropump and a power generator and produces electricity with an efficiency of approximately 11%. The basic parameters of the microturbine unit are: steam pressures (inlet/outlet): 0,6/0,18 MPa, steam temperatures (inlet/outlet): 400/316°C, turbine diameters (inlet/outlet): 360/760 μm , blade height: 20–50 μm , steam mass flow rate: 24 mg/s, rotational speed: $\sim 1\,000\,000$ rpm, and the electrical power achieved 5 W.

MEMS technology has been effectively applied to micro gas turbines, resulting in the development of devices with dimensions measured in millimetres, power outputs ranging from 10 to 500 W, and compression ratios between 2:1 and 4:1. These microturbines have been developed alongside semiconductor technology and require exceptional precision during their manufacturing. Their power concentration factor of 33 kWh/kg is more than 20 times higher than that of lithium batteries, which have a density of 1.4 kWh/kg [148]. Microturbines represent a notable application of MEMS technology [149]. One specific example of a gas microturbine built using MEMS technology is the wafer genset, illustrated in Fig. 17.

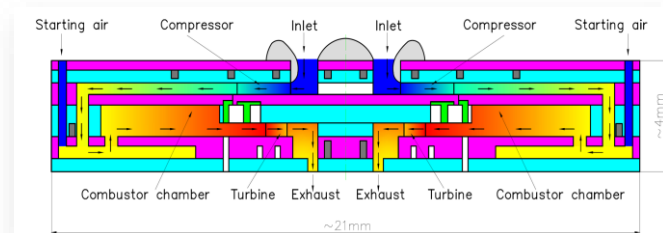


Fig. 17. General view of a wafer microturbine.

This device features a compressor with a diameter of 4 mm and a microturbine with a diameter of 21 mm, while the blade length measures 200 μm . The compressor achieves a tip velocity of 500 meters per second at a rotational speed of 1 million revolutions per minute, with an airflow rate of 0,3 milligrams per second. The blade angle, denoted as α_1 , varies between 15° and 20°. The temperature after the combustion chamber reaches 1600 K, and the power density of the combustor chamber is 3 GW/m³, significantly surpassing the typical value of approximately 2 GW/m³ for classical turbines. The radial clearance is around 2 μm , and the cost ranges from 1 000 to 5 000 USD per kilowatt [150]. These parameters underline the remarkable capabilities and efficiency of MEMS-based micro gas turbines.

The economic impact of these devices will be dependent on the performance levels and the manufacturing costs, both of which have yet to be proven. It is certainly possible, however, that MEMS gas turbines may one day be competitive with conventional machines in a cost per installed kilowatt. Even at much higher costs, they will be very useful as compact power sources for portable electronics, equipment, and small vehicles [148]. Figure 18 presents the concept of MEMS gas microturbine engine packaged as a standard military battery. The other advantages of miniaturisation is shortening the time needed for acquiring the information on system behaviour, as well as decreasing the amount of required materials and energy for the manufacturing process.

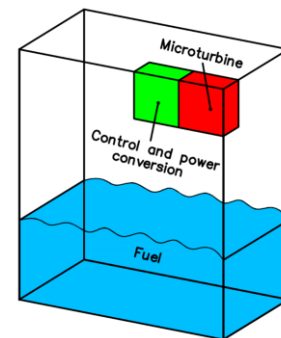


Fig. 18. Concept of MEMS gas microturbine engine packaged as a standard military battery.

Work on this type of device is also being developed, and it seems that in the future it could be used as an autonomous energy source for electronic equipment or small vehicles.

7. Summary

Modern power generation is increasingly focused on moving away from centralised power generation towards distributed systems. Microturbines, whether steam, gas or ORC, are the answer to the growing demand for local, flexible and renewable energy sources. They are a key component of prosumer micro-installations, cogeneration systems and small-scale energy networks (microgrids). Microturbines are extremely diverse in design, power range and application. They can be:

- steam turbines ranging from a dozen watts to hundreds of kilowatts,
- gas-fired microturbines with or without heat recovery,
- turbines in ORC systems to recover waste heat,
- Tesla bladeless turbines, simplified in design,
- MEMS turbines, which function as miniature energy sources, e.g. for portable electronics.

This differentiation allows the solution to be tailored to the specific application, from powering a laptop to recovering heat from a cement plant to driving a turbocharger or micro-generator.

These devices allow energy to be produced close to the point of consumption, minimising transmission losses and increasing supply reliability. Steam and gas microturbine design issues began to develop at the beginning of this century. Technological solutions, although using well-established knowledge of high-

power turbines, are at an early stage of development. This requires the integration of knowledge from a number of disciplines, such as:

- thermodynamics (optimisation of thermal cycles),
- fluid mechanics (design of flow systems),
- strength of materials (use of ceramics, aluminium alloys, composites),
- mechatronics and electronics (control systems, energy conversion and storage),
- microengineering (manufacture of MEMS turbines).

The advances concern not only the turbine itself, but entire energy systems including generators, heat exchangers, bearings, gearboxes and cooling systems. Microturbines are part of the green energy transition because they can be:

- powered by biomass, biogas, hydrogen, alcohol, synthesis gas,
- used for waste heat recovery (in ORC, cogeneration, trigeneration),
- coupled to fuel cells or solar collectors,
- used in zero-emission projects (e.g. hydrogen-oxygen combustion).

They are thus part of the drive to decarbonise energy and improve energy efficiency in various sectors of the economy.

Despite tremendous progress, microturbine development still faces a number of technical barriers, including:

- difficulties in miniaturisation and high temperature resistant materials,
- dynamic stability problems at very high speeds (up to 1 million rpm in MEMS),
- challenges in lubrication and cooling (e.g. oil-free bearings),
- complications in designing very efficient, small heat exchangers,
- the need for precision control systems and power electronics (e.g. for high-frequency generators).

Key research directions in the coming years will be:

- increasing efficiency to > 40%,
- lowering the investment cost below 500 USD/kW,
- development of multi-fuel and zero-emission drives,
- integrated hybrid systems (e.g. microturbine + photovoltaics (PV) + energy storage),
- implementation of advanced materials (ceramics, superalloys).

Microturbines are increasingly used in means of transport (cars, motorbikes, buses, drones, aircraft) as:

- power sources,
- energy recovery systems (e.g. turbochargers),
- direct drives (in hybrids or research models).

In industry, they are used for:

- powering high-speed instruments (e.g. dental turbines),
- powering mobile diagnostic equipment,
- air conditioning and refrigeration (e.g. in aviation).

This shows that microturbines not only produce energy but also increase the efficiency of other technical systems.

The review of existing steam and gas microturbine designs carried out in this article does not provide clear answers as to

what direction their development will take. According to the authors, the observed trends in the development of these devices allow the conclusion that they will move towards the construction of more efficient, reliable, and sustainable microturbines, both steam and gas. Analyses have also shown that there is now also a very strong emphasis on emissions, so the development of these devices will also move towards the use of low- or zero-emission fuels, such as hydrogen.

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Measurements of electrical capacitance in the flow of reservoir fluid with gas bubbles

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Abstract

The paper discusses tests of an electric capacitance sensor designed and built to measure gas fraction in a reservoir fluid flow. These tests were conducted under conditions typical for oil field testing and productivity monitoring. In such an application, the electrical permittivity of the liquid and gas components is of the same order of magnitude, which poses a significant challenge in capacitance metering of the phase content. The studied two-electrode capacitance sensor is a novel design. It uses a commercial, high-resolution $\Delta\Sigma$ capacitance-to-digital converter. The sensor was experimentally tested on the flow of reservoir fluid (from Lubiatów-Międzychód-Grotów field in Poland) heated to 40–50°C to reproduce field conditions during actual gas and oil extraction. The gas fraction in the reservoir fluid was generated by the injection of air or methane bubbles. Fluid velocities during the testing ranged from 1 to 3.1 m/s, and the void fraction reached 4.4%. After calibration, the examined capacitance sensor was able to detect a void fraction as low as 1%. The measured capacitance differences due to gas content were dependent on the spatial distribution of voids in the inter-electrode space. This effect was confirmed by photographs of the flow patterns and numerical simulations of the electric field distributions.

Keywords: Capacitance sensor; Reservoir fluid; Oil-gas flow; Oil and gas extraction; $\Delta\Sigma$ capacitance-to-digital converter;

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1. Introduction

Electrical capacitance measurement has a wide range of applications, both in industry and laboratory practice. In particular, it enables the measurement of the void fraction, i.e. the gas content in a liquid. Measuring gas content, both quantitatively and qualitatively, is crucial in many technological processes. One of the most important potential applications is two-phase liquid-gas flows, which are common, for example, in the refrigeration industry [1,2]. Detecting the presence of liquid at the steam turbine inlets is equally important. In the drilling industry, void fraction measurement is used during the drilling process to determine the

amount of drilling fluid flowing from the well. This is a crucial parameter for blowout prevention. Using void fraction measurement during oil field sampling provides additional information about separator performance, which positively affects the control of operating parameters and separator efficiency. In the later stages of the oil field development, void fraction measurements are important for determining the phase composition of the reservoir fluid during well testing and productivity monitoring.

The most common method for measuring the phase composition of the reservoir fluid involves using a separator to measure each phase separately. This method is expensive and requires ongoing maintenance by qualified personnel. Due to the

Nomenclature

C – capacitance, F
 $\mathbf{E}$ – electric field vector, V/m
 $\mathbf{n}$ – unit normal vector
 S – surface area, m²
 U – electric potential, V
 V – voltage, V

Greek symbols

ε – absolute permittivity, F/m
 ε_r – relative permittivity, –
 ε_0 – vacuum permittivity, $\varepsilon_0 = 8.8541878188(14) \times 10^{-12}$ F/m
 φ – void fraction, –

Abbreviations and Acronyms

CDC – capacitance-to-digital converter
 LMG – Lubiatów-Międzychód-Grotów

separator size, it also requires considerable space, which can be problematic on offshore drilling platforms. Due to the high inertia of the separator, this method has low time resolution but offers high measurement accuracy for quasi-steady flows. Therefore, implementing a system that allows for direct and continuous measurement of the void fraction without prior separation offers many advantages [3]. In this context, non-invasive and continuous capacitance measurement can be applied [4,5].

Over the years, many capacitance sensor designs based on analogue electronic circuits have been described in the literature. Salehi et al. [6], for instance, investigated a capacitance sensor aimed at measurements in oil–gas two-phase flow. The focus was on finding the optimal exciting frequency of the voltage applied to the electrodes. Based on the sensor impedance modelling and static experiments representing annular flow (with a void in the core) were made. They found that a frequency of 0.8–1.0 MHz gives the best sensitivity for the oil–gas medium. Continuing their research, Salehi et al. [7] investigated the performance of different electrode configurations for oil–gas flows in bubbly, plug, stratified and slug regimes. They concluded that an interlocking electrode arrangement (termed TRFLC) or a concave shape is best for identifying flow regimes. For measuring gas fraction, a helical shape is preferable. Ahmed and Ismail [8] also reviewed the performance of capacitance sensors with different electrode shapes.

In addition to previous studies, a system with a capacitance sensor specifically designed to measure water content in crude oil was constructed by Aslam and Tang [9]. Its electronics operated based on capacitance to phase angle conversion [10,11] and achieved a resolution of ± 50 ppm of water content during laboratory tests on crude oil samples. A new design of the capacitance sensor also dedicated to oil–water flows was proposed by Demori et al. [12]. It was tested on an experimental setup with controlled two-phase annular and slug flows in a horizontal tube for oil fractions ranging from 20% to 65%.

Similarly, capacitance sensors with two concave, two helical, or double ring electrodes were experimentally investigated by dos Reis and da Silva Cunha [13] in water–air horizontal flow. Their study covered the full range of void fractions, from 0% to 100%. Their measurements showed that the highest sensitivity was observed for the concave electrodes, while the lowest for the double ring ones. However, measurements with concave electrodes were most dependent on the air–water flow patterns, and the double ring was least affected in this respect. The characteristics of the helical electrodes were similar to those of the double ring.

The problem of calibration of capacitance sensor was studied, among others, by Abou-Arkoub et al. [14]. They postulated

that simple capacitance-sensing techniques can be used for online validation of multiphase flow meters. It was demonstrated that capacitance measurement combined with statistical and fuzzy-logic-type methods can provide the basis for a simple and robust validation of multiphase flow meters' performance by identifying the carrier, dispersed phase and flow structures. Furthermore, the proposed system could be retrofitted to existing multiphase flow meters.

Recent research has explored the use of digital signal processing in capacitance sensor electronics. For example, Wu et al. [15] used digital control in their combined conductance and capacitance sensor designed to measure water holdup in oil–water flow. Maurya and Parasuraman [16] built two- and four-electrode sensors for micro-droplet detection, which were based on the AD7150 capacitance-to-digital converter and were intended for use in drug delivery systems. Their sensor was mounted on an 8 mm outer diameter tube. It had a high sensitivity of 0.7409 fF/ μ L with repeatability of $\pm 0.032\%$. Dos Santos et al. [5] developed an oil and gas pipeline monitoring system with a fast capacitance sensor designed for operation in hazardous areas. The sensor operated based on a resonant circuit with a 40 MHz reference oscillator with a frequency tolerance of ± 100 ppm/K. The frequency output was digitised with a 15-bit resolution. Data was transmitted in real time to a microcomputer for analysis. The system achieved absolute uncertainty for a gas fraction content of 1.41% in slug flow.

The research presented below aimed to experimentally verify a two-electrode capacitance sensor using a commercially available high-resolution $\Delta\Sigma$ capacitance-to-digital converter (CDC).

The examined system is intended to be developed for use in real-time reservoir fluid characterisation, supporting well testing and productivity monitoring in the oil (petroleum) and gas industry. In this application, the electrical permittivity of liquid and gas components can be of the same order of magnitude, which constitutes a significant difficulty in capacitance metering of the phase content. Despite this, laboratory tests clearly demonstrate that the $\Delta\Sigma$ CDC enables the accurate detection of even small amounts of gas in reservoir fluid, successfully overcoming this difficulty. This finding highlights the capability and sensitivity of the system. Moreover, the measurements were carried out using the fluid at a temperature of 40–55°C, which is typical for oil and gas field production.

2. Materials and methods

The electrical capacitance in the pipe flow of reservoir fluid with gas bubbles was investigated experimentally using a capacitance

sensor of our design and construction. The measurements were made on a custom-built laboratory test stand with a horizontal test section.

2.1. Experimental setup

The test stand used during the present research is schematically shown in Fig. 1.

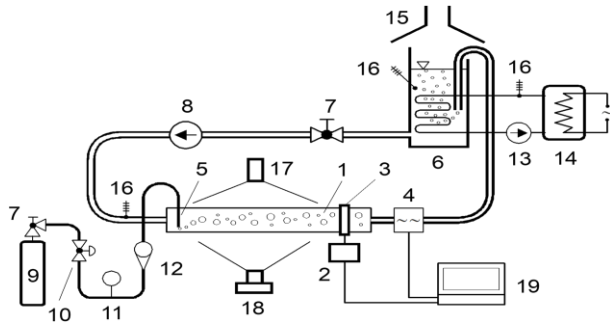


Fig. 1. Experimental setup: 1 – test section, 2 – capacitance sensor, 3 – measuring electrodes, 4 – ultrasonic flow meter, 5 – gas injection, 6 – degasser (flash tank), 7 – cut-off valve, 8 – reservoir fluid circulation pump, 9 – compressed gas cylinder (air or methane), 10 – gas flow control valve, 11 – gas pressure gauge, 12 – rotameter, 13 – heating water circulation pump, 14 – water heater, 15 – gas extraction chimney, 16 – temperature sensor, 17 – electronic flash, 18 – digital still camera, 19 – data acquisition.

It essentially consists of a loop in which the reservoir fluid circulates, including the test section 1 made of a horizontal, transparent polycarbonate tube with an internal diameter of 45 mm and a length of 400 mm. The electrodes 3 of the capacitance sensor 2 were precisely wrapped around the outer surface of this tube at a position near the downstream end. Upstream, a clean segment was used to observe and photograph gas bubbles injected by the needle 5, which was mounted at the beginning of the test section. Downstream of the test section, a polyamide tube segment served as the mounting point for the ultrasonic flow meter 4, with the instrument secured directly onto this segment. The rest of the loop was made of a rubber hose with an internal diameter of 50 mm. Between the flow meter and the circulating pump was the degasser 6, an open tank with

a coil used for heating the fluid to the required temperature. The heating medium was water, which circulated between the degasser 6 and the electric boiler 14 via the pump 13. Gases (especially methane) separated from the reservoir fluid were removed from the laboratory through the extraction chimney 15. Downstream of the degasser tank 6, the cut-off valve 7 was installed. It was closed when the circulation pump 8 was turned off. Gas injected into the reservoir fluid was drawn from cylinder 9. The valve 10 adjusted its pressure to achieve the required flow rate. These parameters were monitored using the manometer 11 and the rotameter 12. To control the temperature, sensors 16 were placed at three specific locations: upstream of the test section, within the degasser and in the heating water loop. The flow pattern upstream of the capacitance sensor was photographed using the still camera 18, which was synchronised with the electronic flash 17. The readings from the capacitance sensor 2 and the ultrasonic flow meter 4, both fixed at their described positions, were recorded on-line and processed by laptop computer 19.

Main pump 8 of the centrifugal type was driven by a 2.2 kW electric motor. While the normal rotating speed of this pump was 1450 rpm, it was reduced by an inverter to achieve the required flow rate of the reservoir fluid. To determine this value, the flow velocity was measured with a Flexim Flexus F608 portable ultrasonic meter with an accuracy of ± 0.005 m/s. The rotameter 12 (Brooks Instruments Sho-Rate glass tube) used to control gas flow had an accuracy of $\pm 5\%$. The temperature was measured with industrial PT100 sensors with an accuracy of $\pm 0.5^\circ\text{C}$.

2.2. Capacitance sensor

The capacitance sensor used in this research was developed in-house based on the AD7746 microchip, an integrated $\Delta\Sigma$ capacitance-to-digital converter from Analog Devices [17]. The converter can measure capacitances up to 21 pF with an accuracy of up to 4 fF ($4 \cdot 10^{-15}$ F), a resolution of 4 aF ($4 \cdot 10^{-18}$ F), and a frequency of up to 90 Hz, depending on the configuration. It operates based on a $\Delta\Sigma$ modulator with a maximum resolution of 24 bits and a square-wave excitation signal with a frequency of 32 kHz.

The measurement system was monitored by software running on a PC, see Fig. 2.

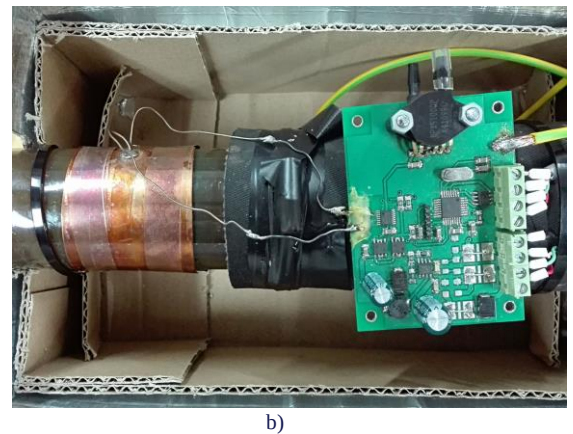
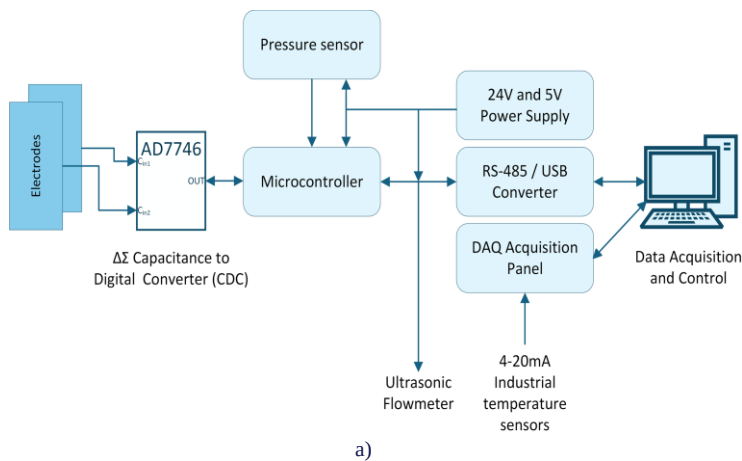


Fig. 2. Capacitance sensor with $\Delta\Sigma$ CDC: a) block diagram of the sensor components; b) photograph of the sensor board end electrodes mounted in the test section.

The RS-485 interface allowed for configuration and readout of the AD 7746, as well as readings from the ultrasonic flowmeter. Additionally, the DAQ panel connected to the Ethernet interface enabled readings from the temperature sensors. All read data was saved to a file every second.

2.3. Material properties

The relative permittivity of the materials used in the present study is listed in Table 1. The reservoir fluid came from the Lubiatów-Międzychód-Grotów (LMG) gas and oil field in western Poland. The air and methane were of industrial grade and were drawn from pressurised cylinders. The test section was made of a polycarbonate tube with inner and outer diameters of 45 mm and 50 mm.

Table 1. Relative permittivity.

Material	Relative permittivity ϵ_r (-)
Reservoir fluid	2.5
Air	1.00054
Methane	1.00081
Polycarbonate	2.9

The permittivity of the reservoir fluid was measured by the authors using the air capacitor method. It should be noted that it is only 2.5 times larger than that of the gases.

2.4. Test procedure

Measurements were performed at a constant flow rate and temperature. Capacitance values were sampled at a rate of 1 Hz. Each individual measurement for a specified flow parameter lasted 2 minutes, and the average capacitance during this time interval was recorded for further analysis. For each specified value of liquid flow rate, the liquid-only capacitance was first recorded. Next, two-phase flow capacitance measurements were taken for six values of injected gas flow rate. Before each series of measurements, the reservoir fluid temperature was stabilised at the desired level with an accuracy of $\pm 2^\circ\text{C}$ or better. Flash tank in the flow circuit ensured that the injected gas was removed and did not circulate in the test loop, its content did not increase during the two-minute data acquisition. Several photographs of the flow structure were also taken.

2.5. Supplementary numerical simulations

To gain insight into the phenomena occurring in the inter-electrode space of the capacitive sensor, numerical simulations were performed using the finite element method implemented in the FlexPDE 5 software [18,19]. The electrodes form a capacitor in which the distribution of the electric field in static conditions can be described by the Laplace equation [20]:

$$\nabla \cdot (-\epsilon \nabla U) = 0, \quad (1)$$

where U denotes electric potential and $\epsilon = \epsilon_0 \epsilon_r$ is absolute permittivity. Equation (1) was solved numerically in a cylindrical domain comprising a segment of the polycarbonate tube with two sensor electrodes, as shown in Fig. 3. The value of U at one

of the electrodes was set to 0 and at the other to $U_e = 0.625$ V. The boundary condition on the outer surface of the solution domain was imposed in the form:

$$\nabla U \cdot \mathbf{n} = 0, \quad (2)$$

where $\mathbf{n}$ is a unit vector normal to this surface.

The condition (2) means that the electric field vector:

$$\mathbf{E} = -\nabla U, \quad (3)$$

is perpendicular to the outer surface of the domain, and the electrical charge induced on this surface is zero.

When the solution of Eq. (1) is known, the capacitance C can be calculated from the integral:

$$C = \frac{1}{V_e} \iint_S \epsilon (\mathbf{E} \cdot \mathbf{n}) dS, \quad (4)$$

where S is a surface enclosing the excited electrode at the potential U_e , and $V_e = U_e$ is the voltage between electrodes.

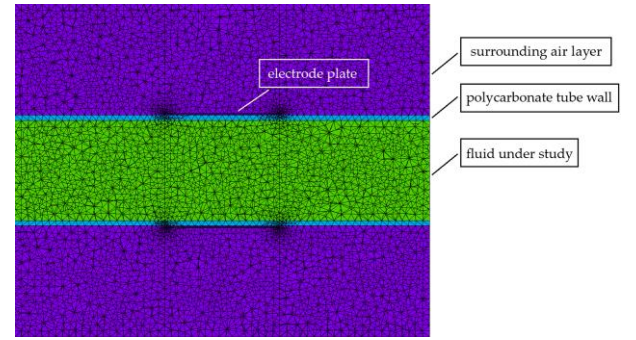


Fig. 3. Axial cross-section through the computational mesh (949 802 cells in total).

Details of the computational mesh are shown in Fig. 3. It forms a cylinder with a diameter of 150 mm and a length of 180 mm, which means that a 50 mm thick air layer was added around the outer walls of the polycarbonate tube and the electrode plates. The 50 mm wide plates were placed halfway along the tube. The mesh cells were generated automatically, but their size could be controlled. Mesh independence tests confirmed that meshes with approximately 1 million cells produced solutions with sufficient accuracy. Capacitance calculated for a fully filled tube with 1 million cells differs by around 0.1% from capacitance calculated with a minimum possible 300 thousand cells, and differs by around 0.01% from capacitance calculated with 2 million cells.

3. Results

The experiments began by assessing how the angular position and width of the electrode gap affected the capacitance readings. Next, with the electrodes set in their final configuration, tests were performed to evaluate the effect of liquid temperature. These measurements were done over a preselected range of fluid temperature, velocity and air void fraction. A similar set of experiments was later performed using methane instead of air. Finally, tests on the long-term stability and repeatability of the sensor concluded the study.

3.1. Effect of electrodes gap position and width

The performance of the capacitance sensor was investigated using three sets of electrodes with different gap widths: 3 mm, 26 mm and 40 mm, corresponding to 1/60, 1/6 and 1/4 of the pipe circumference, respectively. The width of each electrode was 50 mm. The effect of both gap size and angular position was investigated in the following sequence. First, the gaps were placed on the sides of the pipe, in the horizontal plane (0°). Next, the electrodes were rotated so that the centre of the gap was at a 45° angle. Finally, the gaps were placed at the top and bottom of the pipe, i.e. at a 90° angle. These gap positions and sizes are illustrated in Fig. 4.

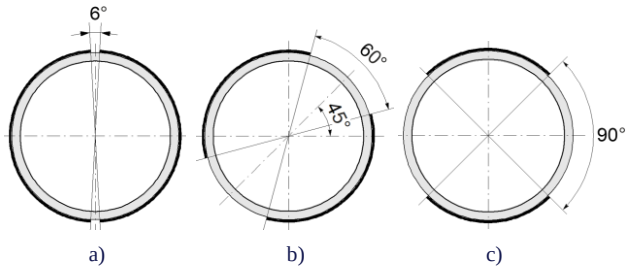


Fig. 4. Tested electrodes differing in gap angular position and width (note: each electrode size was tested at the three angular positions illustrated): a) large electrode at 90° angle (gap width 3 mm), b) medium electrode at 45° angle (gap width 26 mm), c) small electrode at 0° angle (gap width 40 mm)

Simultaneously with the measurement of electrical capacitance, photographs of the flow structure were taken on the pipe section upstream of the capacitance sensor.

During each test run, the flow rate was maintained at 5.8, 9.8, 13.9 or 17.8 m^3/h , corresponding to flow velocities in the test section ranging from 1 to 3.1 m/s. For each flow rate value, the air content in the reservoir fluid was increased stepwise from 0 to 4.2 L/min, which resulted in a maximum void fraction φ ranging from 1.4% in the fastest flow to 4.4% in the slowest one.

A single test series uses one electrode configuration and includes 28 individual measurements (4 liquid flow rates times 7 gas fraction values). For each test series, capacitance measured with only the reservoir fluid flowing (no gas added, void fraction equal to 0) served as the reference value, and was subtracted from the subsequent capacitances measured in two-phase gas-liquid flow. The resulting capacitance differences are shown in Figs. 5–7 for the three electrode sizes investigated, i.e. large, medium and small electrodes with the gaps of 3 mm, 26 mm and 40 mm, respectively. The fluid temperature during the tests ranged from 45°C to 55°C , with the variation of no more than $\pm 2^\circ\text{C}$ during a single measurement series.

The average capacitances measured for each electrode size and angular position are shown in Fig. 8 for liquid-only flow. The average value was calculated from 4 measurements corresponding to the number of flow rates tested. The standard deviation for these averages ranges from 0.001 pF to 0.004 pF, except for the large electrode at the 0° position, for which it is 0.008 pF.

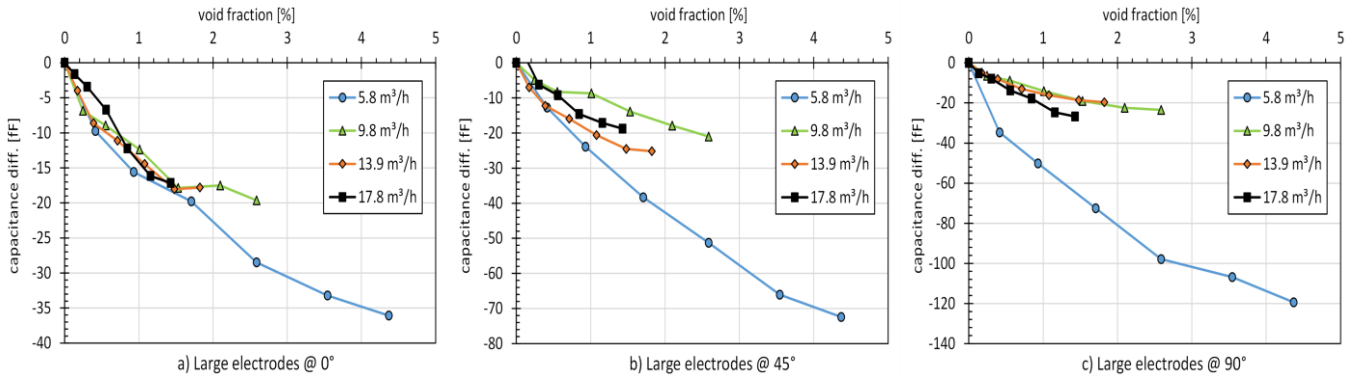


Fig. 5. Capacitance difference (in fF) measured for large electrodes (gap width 3 mm) in three gap angular positions.

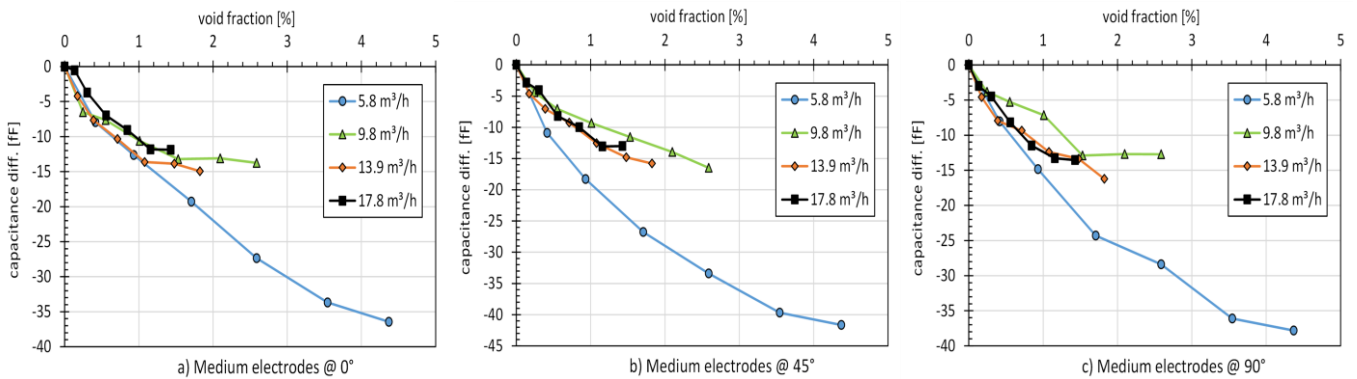


Fig. 6. Capacitance difference (in fF) measured for medium electrodes (gap width 26 mm) in three gap angular positions.

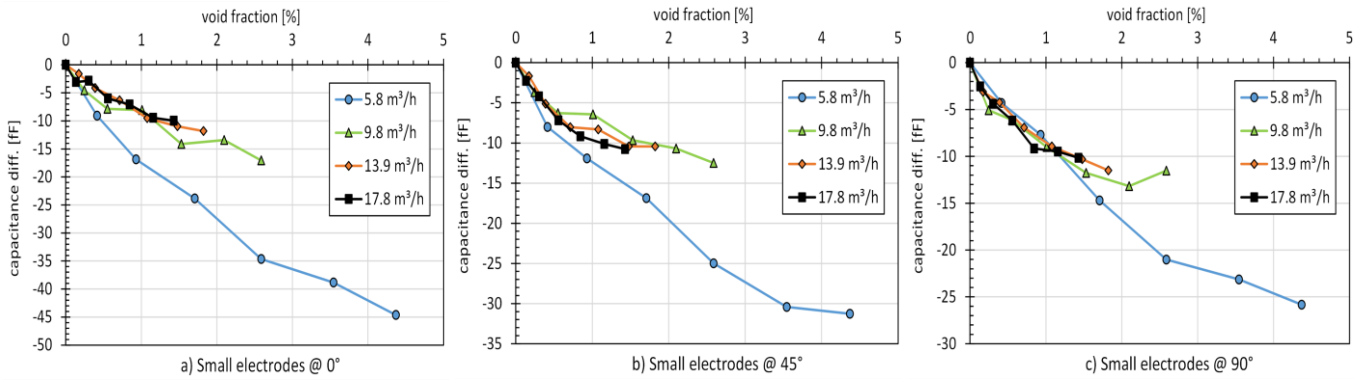


Fig. 7. Capacitance difference (in fF) measured for small electrodes (gap width 40 mm) in three gap angular positions.

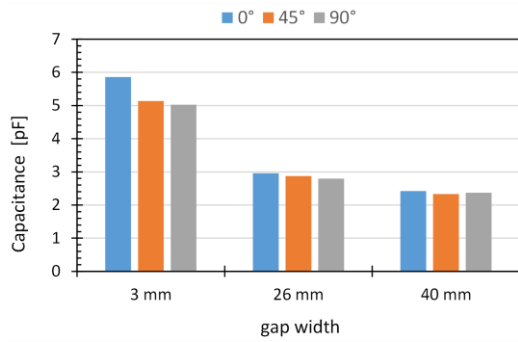


Fig. 8. Average capacitance measured for each configuration of the electrodes and liquid-only flow.

3.2. Effect of reservoir fluid temperature

The effect of fluid temperature on the capacitance sensor operation was checked using the largest electrodes with a 3 mm gap positioned at a 90° angle. For each measurement series, the temperature was constant and the set value was selected from 23°C to 50°C. For each temperature, the air content in the fluid was increased stepwise from 0 to 4.2%.

The test results for varying flow temperature are shown in Fig. 9, illustrating how the capacitance difference changes with air fraction. For each temperature tested, this relationship was measured sequentially for three liquid flow rates: 5.8, 9.8 and 13.9 m³/h. As before, the capacitance difference was calculated using the value of liquid-only flow as a reference.

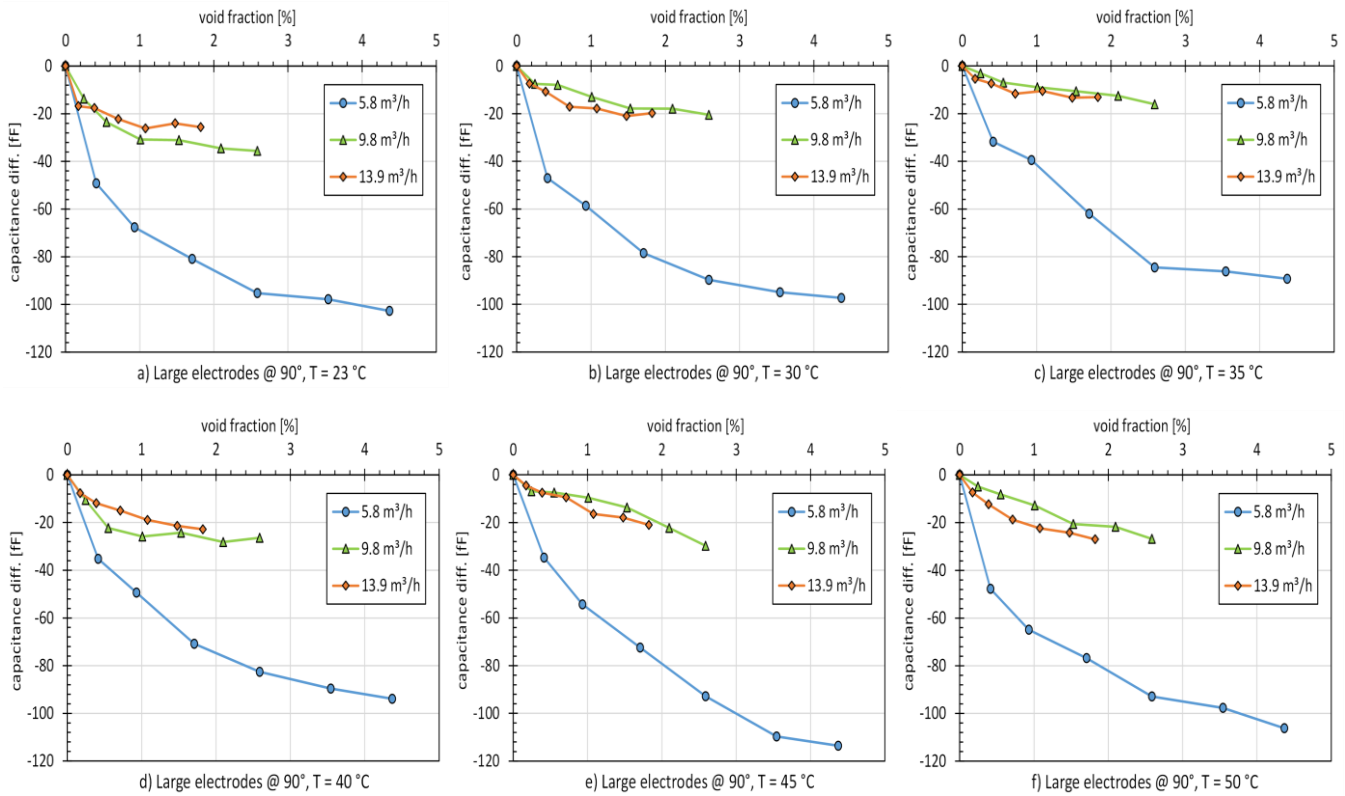


Fig. 9. Capacitance difference measured for large electrodes (gap width 3 mm at 90° angle) for selected reservoir fluid temperature values between 23°C and 50°C.

3.3. Test with methane

The tests with methane injection were carried out for fluid temperatures of 45°C and 50°C. Figure 10 shows the test results for methane. The format is the same as for the measurements with air injection. The capacitance difference relative to the value of liquid-only flow was calculated for three liquid flow rates of 5.8, 9.8, and 13.9 m³/h. The volume flow rate of methane covers the same range as for the air case.

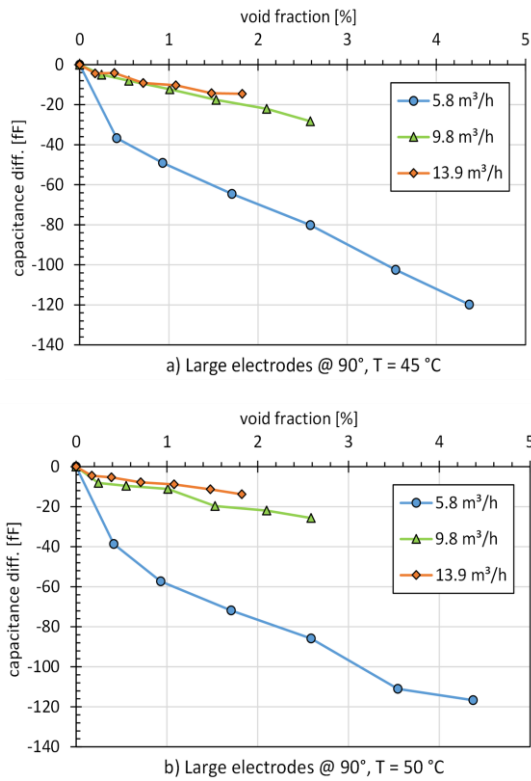


Fig. 10. Capacitance difference measured for large electrodes (gap width 3 mm at 90° angle) for methane injected into the reservoir fluid at temperature 45°C and 50°C.

3.4. Sensor repeatability tests

Continuous measurements of capacitance were taken under steady flow conditions to assess the stability of the sensor readings.

One of the stability tests examined the effect of reservoir fluid temperature on capacitance during a liquid-only flow, as shown in Fig. 11a. Continuous measurements were recorded for approximately 2 hours at 45°C. The temperature was then increased to 50°C, and recording continued for another 2 hours. The next day, the test at 50°C was repeated. All recorded capacitance values were within ± 15 fF of the value recorded at the beginning of the measurement session on day one. Another stability test involved a gas injection period, as shown in Fig. 11b. The purpose of gas injection was to check whether the sensor readings returned to their initial level after the gas injection ended. It turned out that the capacitance in the second period of liquid-only flow was close to the initial level, although lower on average by about 4 fF.

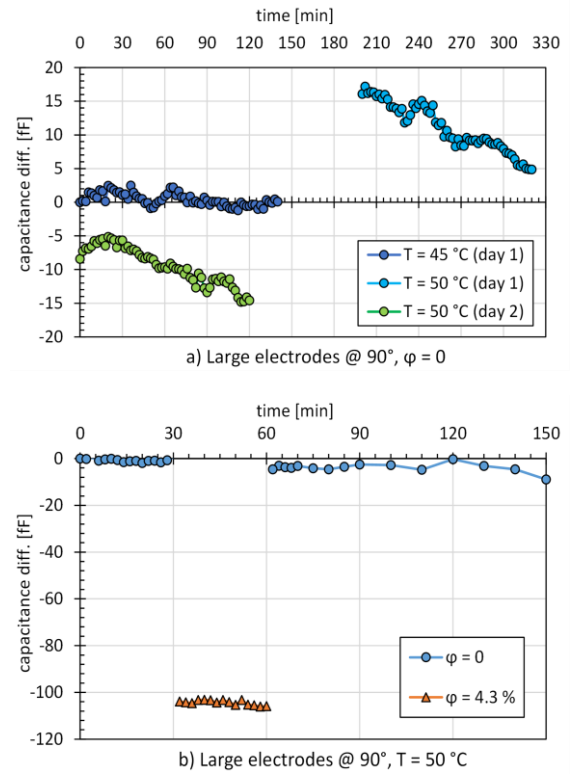


Fig. 11. Capacitance stability tests: a) repeatability of liquid-only capacitance, b) return of the sensor readings after a period of gas injection.

3.5. Flow structure photographs

Photographs of gas bubbles in the tube segment between the gas injection point and capacitance sensor were taken for each individual capacitance measurement. Figures 12 and 13 show example images that are typical for the used values of gas flow rate.

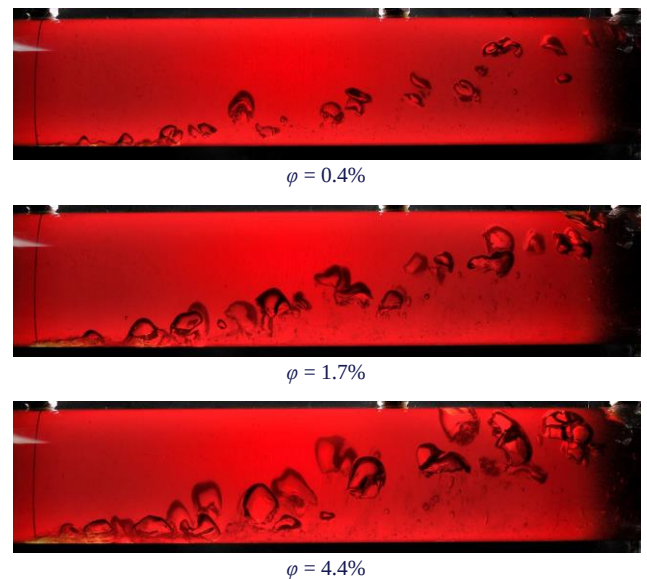


Fig. 12. Backlit photographs of flow patterns. Gas bubbles emerged from the needle outlet located at the bottom of the test tube, on the left. The capacitance sensor was located downstream (on the right), just outside the image. Liquid flow rate – 5.8 m³/h, temperature – 52°C, void fraction $\phi = 0.4\%$, 1.7% or 4.4% .

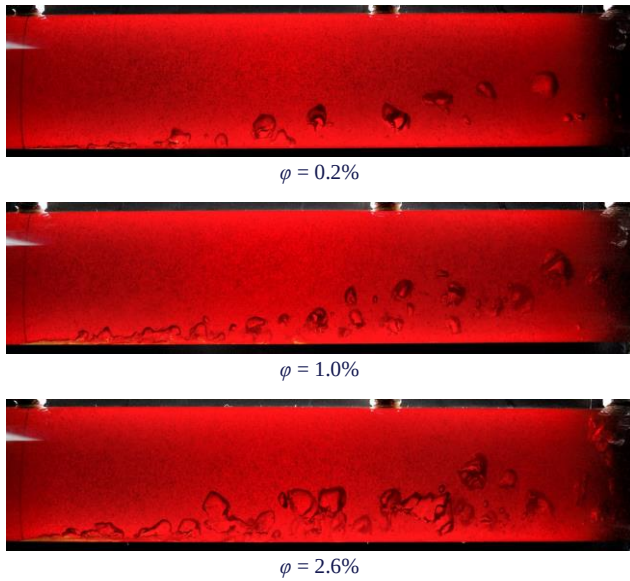


Fig. 13. Backlit photographs of flow patterns. Gas bubbles emerged from the needle outlet located at the bottom of the test tube, on the left. The capacitance sensor was located downstream (on the right), just outside the image. Liquid flow rate – 9.8 m³/h, temperature – 51°C, void fraction $\varphi = 0.2\%$, 1.0% or 2.6% .

The photographs show the flow inside the polycarbonate tube viewed from the side, illuminated by light from the flash coming from behind the tube (backlight). Under this illumination, the colour of the reservoir fluid appeared red. The camera and flash were positioned in the horizontal plane. The gas was injected in the upstream part of the photographed segment, on the left in the images. The injection needle outlet was located near the bottom part of the tube inner wall, so the gas bubbles originate in the lower part of the images and move to the right, rising and expanding.

4. Discussion

The experiments discussed here aimed at reproducing reservoir fluid flow conditions similar to those found in a production well. Therefore, the temperature was elevated to 40–50°C in most tests. Due to limitations encountered (only one injection needle could be used), the maximum gas fraction in the flow did not exceed 4.4%. Nevertheless, the presence of gas was reliably detected by the capacitance sensor.

Most experiments have shown that capacitance differences due to increasing void fraction were greater at the lowest flow rate (5.8 m³/h) than at higher rates. This difference decreased as the gap between the sensor electrodes increased, and was also more pronounced when the gaps were arranged vertically at the top and bottom of the test tube wall (position angle 90°, Fig. 4a). Similar trends appear in tests using methane instead of air.

Numerical simulations and photographs of actual flow explain the above-described effects. It turns out that the capacitance in the inter-electrode space depends not only on the amount of gas in the liquid, but also on its spatial distribution. As can be seen in Fig. 14, gas voids concentrating near the gap between the electrodes cause the largest decrease in capacitance.

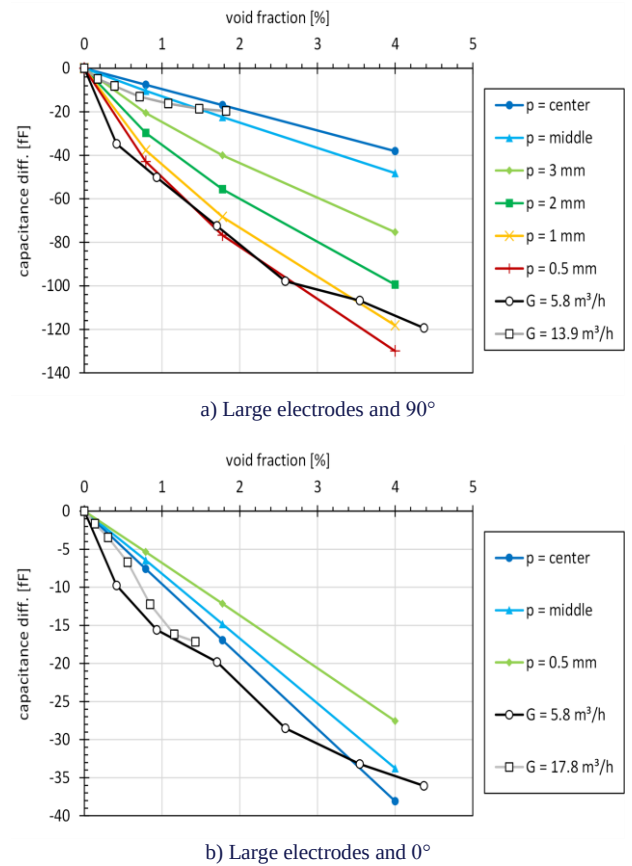


Fig. 14. Comparison of measurements with numerical simulations of capacitance changes depending on the void fraction and the void position “p” relative to the electrode plates. The effect of the void position is different when it approaches (a) the gap between the electrodes or (b) their centre.

The same amount of gas, when concentrated near the centre of the flow tube cross-section has a much smaller effect, as seen in Fig. 14a. The same can be said for gas concentration near the centre of the electrode plate, Fig. 14b. Comparison of the measurements with simulations leads to the hypothesis that for the lowest flow rate (and velocity), the injected gas created a bubble train that concentrated near the top of the tube wall, where the electrode gap at a 90° angular position was also located. For higher velocities and flow rates, the simulation results suggest that the gas bubbles concentrated and flowed along the centre of the tube. This is confirmed by the images of the flow patterns shown in Figs. 12 and 13. With this assumption about gas phase position in the tube cross-section, quantitative agreement is achieved between the measured and calculated capacitance difference values, Fig. 14a. Similarly, good agreement is also observed when the angular position of the gap was 0°, Fig. 14b. In this electrode configuration, bubbles flowing near the top of the tube cross-section approached the middle of the electrode plate and did not cause as large a capacitance drop as that near the gap. In summary, the concentration of gas voids in different areas of the tube cross-section causes different changes in electrical capacitance. Other researchers have reported this effect as well [7,8,21].

The calculated distributions of electric potential in the inter-electrode space, shown in Figs. 15 and 16, provide more details

on the influence of the location of gas voids. To explain these details, Figure 15 illustrates the potential distributions when the flow tube is completely filled with the reservoir fluid. The appearance of gas voids causes a disturbance of this potential, which affects the capacitance. Examples of potential distribu-

tions with voids representing gas bubble trains at a void fraction of 4% are shown in Fig. 16 (with the detailed views of parts of Figs. 16c and 16d presented in Fig. 17). The capacitance differences resulting from introduction of the voids can be found in Fig. 14 (position p = center, middle or 0.5 mm).

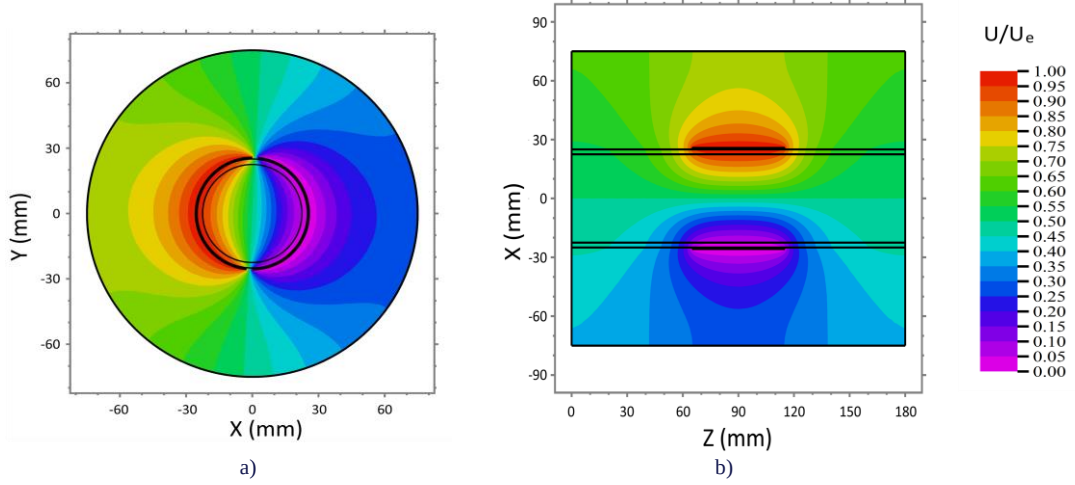


Fig. 15. Electric potential distributions in the inter-electrode space filled with reservoir fluid: a) transverse section in the middle of the electrodes, b) axial section.

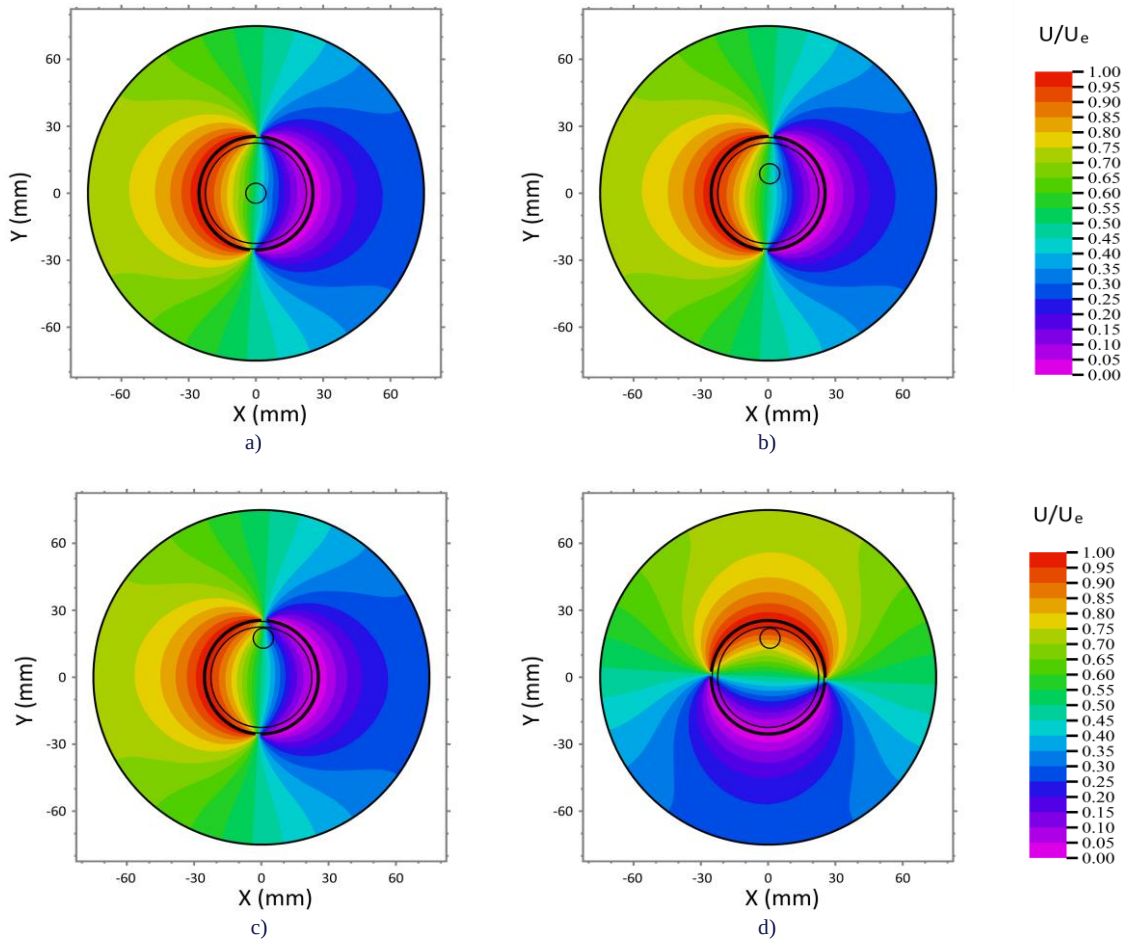


Fig. 16. Electric potential distributions in inter-electrode space for simulated two-phase flow with a 4% void fraction: a) gas phase concentrated in the tube centre, b) gas phase in the "middle" position shifted towards the gap, c) gas phase in the position 0.5 mm from the gap, d) gas phase in the position 0.5 mm from the plate centre.

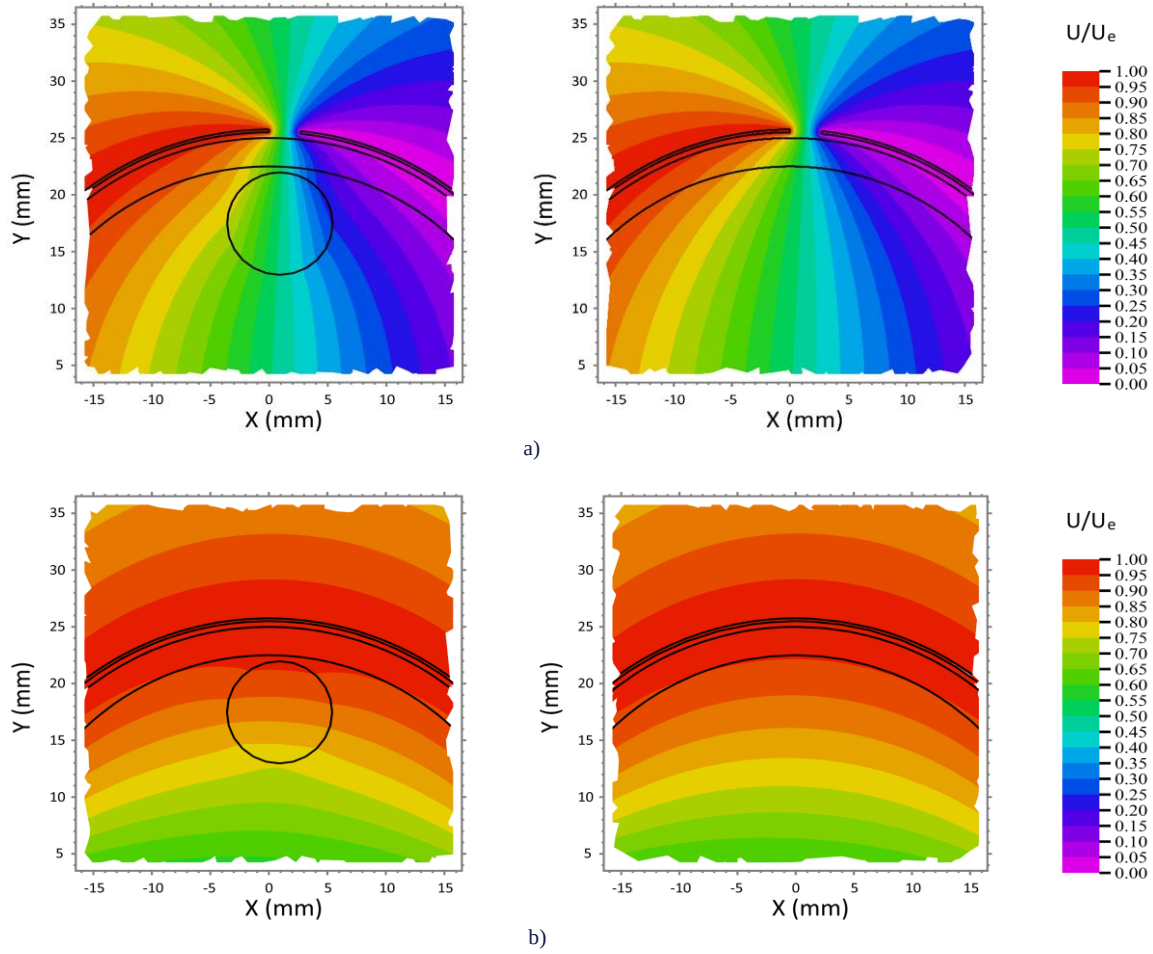


Fig. 17. Detail view of electric potential distributions near the simulated gas void with a 4% void fraction: a) gas phase in the position 0.5 mm from the gap, b) gas phase in the position 0.5 mm from the plate centre. Details at left are parts of Fig. 16c and 16d. Undisturbed liquid-only potentials are also shown at right for comparison.

The dependence of the measured capacitance difference on the electrode gaps angular position is weakest for large electrodes at 0° (when the gaps are located on the sides of the tube, Fig. 4c) or the gap itself is large, as in the tests with small electrodes (Fig. 7). However, as the gap increases (and therefore the plate becomes smaller), the measured capacitances decrease, as shown in Fig. 8. As a result, accurately measuring gas content becomes difficult due to the decrease in the signal-to-noise ratio. For example, the capacitance measured in liquid-only flow at temperatures of 45°C and 50°C averaged 5.23 ± 0.05 pF, which is close to the value of 5.11 pF calculated from numerical simulations. The capacitance of gas-only flow, determined from the simulation, is 3.99 pF. Therefore, the full range of the sensor readings would be 1.12 pF.

The permittivity of the reservoir fluid varies with temperature [22], as verified in the tests described in Section 3.2. These measurements, however, did not demonstrate a clear relationship between capacitance and temperature. This is probably due to the fact that the change in the electrical capacity of the reservoir fluid in the tested temperature range from 23°C to 50°C was smaller than the measurement uncertainty of the sensor.

When methane instead of air filled the voids in the reservoir fluid, there were no significant changes in the measured capaci-

tance difference. This conclusion can be supported by comparing the results in Fig. 5c or Fig. 9e,f and Fig. 10, where data for large electrodes with the gap at a 90° angle and flow temperatures of 45°C and 50°C are presented. In all these cases, the largest measured capacitance difference reached 120 fF for a flow rate of $5.8 \text{ m}^3/\text{h}$ and a void fraction of 4.4%. At higher flow rates, this difference was much lower and amounted to approximately 30 fF at most.

For most liquid-only measurements, repeatability of 5 fF or better was achieved. Only for some tests of the reservoir fluid at 50°C , the repeatability was worse, reaching 15 fF. This means that the tested meter can measure with satisfactory accuracy the gas content above about 1% at low flow rates (corresponding to a velocity of about 1 m/s) and gas content above 2% at higher flow rates. Improving repeatability is only possible through periodic calibration of the device.

5. Conclusions

The experimental research described above concerned capacitance measurements of two-phase liquid-gas flow under conditions encountered during well testing and productivity monitoring in the oil and gas industry. Therefore, the liquid fraction used was actual reservoir fluid at elevated temperatures of 40 – 50°C .

To further increase the realism of the research, methane was intentionally used as the gas phase in some measurements to better simulate real-world conditions relevant to the study.

The following general conclusions can be drawn after analysing the experimental results discussed above:

- The commercial $\Delta\Sigma$ CDC can reliably detect small changes in capacitance, but quantitative measurements of small gas fractions require calibration of the sensor at the beginning of the measurement session.
- The measured capacitance value depends on both the gas fraction and the spatial distribution of voids in the reservoir fluid.
- Voids concentrated near the electrode gaps have the most significant impact on the capacitance drop in the inter-electrode space.

The capacitance sensor used in this research can be easily developed into a multi-electrode version that allows the reconstruction of the two-phase flow patterns.

Acknowledgements

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Heat Transfer Analysis of a Longitudinal Fin Attached to a Beta-Type Stirling Engine

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Abstract

Our proposed model is a highly non-linear parabolic heat transfer equation for the longitudinal fin with an insulated fin tip, temperature dependent thermal conductivity, and internal heat generation in the transient condition. The research comprises several components, namely, a discussion of the longitudinal fin heat transfer model, its solution, the validation of the numerical scheme and solution analysis, and finally, the influence of fin attachment on the system. Mathematical formulations were developed to represent identified fin parameters. This enables us to investigate further their variation patterns, therefore making future modelling easy as well as providing a better understanding of their applicability in developing energy storage systems.

Keywords: Stirling engine; Longitudinal fin; FERK (4,5) scheme; Heat transfer; Thermoelectric generator

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1. Introduction

Stirling engines were developed in 1816 by Rev. Robert Stirling. They work in a closed regenerative cycle in which a working fluid is externally heated. The cyclic compression and expansion of fluid is utilised by the piston to produce power. As a result of its multi-fuel functionality, it has a wide range of applications, Yang et al. [1]. Based on the location of the piston and cylinder configuration, these engines are classified into three types: Alpha, Beta and Gamma type Stirling Engines. Other classifications include free-piston Stirling engines, double-acting Stirling engines, thermal lag Stirling engines, multi-cylinder liquid-piston Stirling engines, pulse tube Stirling engines, and thermo-acoustic Stirling engines, etc. [2]. Stirling engines have been de-

signed in a wide range from large marine engines to the nanoscale [3,4]. Its applications are prophesied from outer space to the human heart [3,5].

It has been found that although the Stirling cycle claims to have an efficiency on the same scale as the Carnot cycle, its practical application does not have that high efficiency. Simultaneous heating and cooling come as the biggest challenge, in maintaining the desired temperature difference.

The first analytical model for the Stirling engine was developed by Schmidt in 1871 [6]. Building on this, researchers like Finkelstein et al. [7], Walker [8], and Kirkley [9] explored optimal phase angles and swept volume ratios for alpha, beta, and gamma types of Stirling engines and refrigerators. In their findings, it appears that the β -type Stirling engine produces the high-

Nomenclature

A_c	– cross-sectional area of fin, m^2
A_m	– dimensionless amplitude of base temperature
c	– specific heat of fin material, $J/(kg \cdot K)$
Fo	– Fourier number
f	– frequency related to engine operating speed
G	– internal heat generation coefficient or generation number
h	– Newton's cooling constant, $W/(m^2 \cdot K)$
$k(T)$	– thermal conductivity, $W/(m \cdot K)$
L	– length of the fin, m
N	– thermogeometric fin parameter
P	– fin perimeter, m
q	– internal heat generation, W/m^2
Re	– Reynolds number,
T	– temperature, K
t	– time, s
x	– space co-ordinate, m

Greek symbols

ε	– internal heat generation parameter, $1/K$
θ	– local fin temperature
X	– coordinate space
ε_G	– associate coefficient of internal heat generation
ρ	– density of fin material, kg/m^3
ω	– periodicity, s^{-1}
Ω	– non-dimensional periodicity
Φ	– phase shift

Subscripts and Superscripts

b	– base
∞	– ambient
m	– mean

Abbreviations and Acronyms

FERK	– finite element Runge-Kutta
ERS	– energy recovery systems
TEG	– thermoelectric generator

est indicated work, while the gamma-type yielded the lowest. Further studies confirmed that β -types are more efficient than alpha or gamma types under similar conditions [10].

In addition to thermodynamic models, a three-dimensional CFD approach can be used to analyse the Stirling engine. Mahkamov [11] conducted a CFD simulation for a solar powered Stirling engine, while Hung and Cheng [12] used CFD methods to predict flow, pressure, and thermal fields in a piston-cylinder assembly. Although CFD methods provide detailed information, they are more time-consuming and computationally intensive than thermodynamic models [13].

The performance of the Stirling engine depends on the temperature difference between the hot and cold regions. Once the engine starts to work, the heated working fluid is forced to expand and moves towards the colder region. This exchange of working fluid between the cold region and hot region, if remains unchecked, will generate a uniform temperature profile throughout the cylinder and will seize the engine operation. Fins are attached to perform this task efficiently. Kumaravelu et al. [14] studied the Stirling engine performance without and with different fin configuration models. They found that engine efficiency without fin attachment is 10.26%; with circular fins attached, its efficiency is 14.41%, with pin fins attached, its efficiency is 16.64%, and when rectangular fins are attached, its efficiency is 19.03%. They concluded that rectangular fins are more efficient due to their larger surface area. Since fin properties and geometry vary, mathematical studies are needed to assess their impact on cooling efficiency. In Stirling engines, since there are no valves or ports for induction or exhaust, energy is transferred through cylinder walls or heat exchangers. Thus, fins play a crucial role in indirect heat rejection to the surroundings. Numerous studies have modelled heat transfer in fins [14–18].

Energy recovery systems (ERS) such as a thermoelectric generator (TEG) work on the principle of the Seebeck effect. When combined with Stirling engines, TEG provides an opportunity to convert waste heat into additional electric power. It has been found by Faraj et al. [19] that attaching fins to a $40 \times 40 \text{ cm}^2$ flat plate TEG system generates 3.8 W of electrical

power in combination with solar radiation of 2000 W/m^2 and reduces one ton of carbon dioxide emission yearly. Energy recovery systems are strategies used to recover system energy that would otherwise be squandered. In recent years, ERS has been regarded as a cost-effective choice for increasing efficiency, and as a result, the potential advantages of those systems are being fully utilised [20]. Ideally, when applying ERS, energy must be utilised continuously, and the efficiency gain must be significant in all conditions. Waste energy from the Stirling engine can be harvested in developing ERS utilising the fins attached to its colder section. For a waste heat recovery system for power generation, in a lower temperature range, Chen et al. [21] suggested adding fins within a range of 0–27 items, which significantly enhances heat transport and the performance of thermoelectric modules (TEMs). At $Re = 10$ and 100 , a perfect number of fins is 21, whereas it is 27 at $Re = 1000$. At $Re = 1000$, the highest overall power generated is 0.411 W , and the average efficiency of conversion is 0.95% . When compared to TEMs without fins, these figures represent 105.5% and 43.94% enhancements, respectively.

1.1. Present contribution

Our proposed model is a highly non-linear parabolic heat transfer equation for the longitudinal fin with an insulated fin tip, temperature dependent thermal conductivity and internal heat generation. It is a realistic model of the longitudinal fin compared to the existing model. This type of model is for the first time used in the transient and highly non-linear conditions in the fin material. FERK(4,5) scheme is used for the solution of our non-linear parabolic heat transfer equation. The research comprises six components. The first component is to discuss longitudinal fin heat transfer modelling. The second section contains a system description; the third section concentrates on solving the problem. In the fourth section, the validation analysis of the numerical scheme has been presented. The fifth section discusses the influence of parameters on the system. The sixth section is devoted to final conclusions.

A key gap identified in the available research literature is the absence of transient heat transfer modelling of a fin attached to the cold region of a β -type Stirling engine. The given model represents a realistic situation as:

1. the mathematical formulation was developed to represent identified fin parameters; this enables us to investigate further their variation patterns, therefore making future modelling easy as well as providing a better understanding of their applicability in developing energy storage systems;
2. the model provides a mathematical backup to the study of heat transfer through the fin.

2. System description

We consider a system consisting of a β -type Stirling heat engine operating between a heat source and a heat sink at the temperatures T_H and T_L , respectively (Fig. 1). Near the wall area, hydrodynamically non-slip walls satisfying the temperature continuity and heat flux continuity at both sides of the wall are applied to accurately simulate the thermo-hydraulic output [14]. The cold region includes longitudinal fins (Fig. 2).

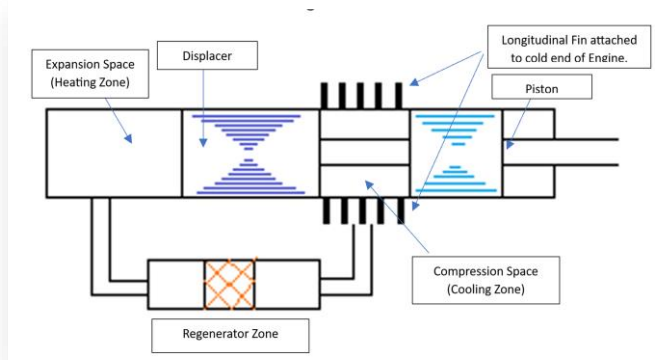


Fig. 1. Fin attachment to "cold region" of β -type Stirling engine.

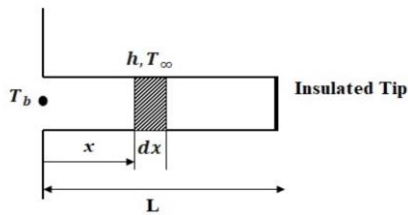


Fig. 2. Schematic geometry of the longitudinal fin.

For boundary conditions, a conjugated heat transfer was assumed for the fin fluid/solid wall [14]. The temperature at the base of the longitudinal fin varies cyclically due to cyclic changes in the temperature of the working gas, within the cylinder. The area of the cross-section of the fin is A_c , thickness b , length L , and perimeter P . Width is dominated by length, i.e. $b/L \ll 1$. The amount of heat dissipated from the tip can be discounted to be zero as the tip area is negligible in comparison to the total fin area [22,23]. When heat is generated inside the fin by utilising an external energy source, an opportunity for dynamic thermal management, hybrid thermal circuit and energy recovery is observed. The internal heat generation term, q , has

been added in the analysis to cover such scenarios. The physical properties of the longitudinal fin and system assumptions are considered as follows:

1. A transient state is assumed.
2. The fin material has a temperature dependent thermal conductivity.
3. The density of longitudinal fin material is ρ , and the specific heat is c .
4. Fin is assumed to have a uniform cross-sectional area.
5. Conduction is assumed to be in one dimension.
6. We assume a constant convective heat transfer coefficient h . The convection is assumed to happen at the fin surface exposed to the surrounding only.
7. A local fin temperature-dependent internal heat generation is denoted by q .
8. Temperature variation at the fin base is assumed to be a function of engine dynamic characteristic, phase angle Φ .

2.1. Mathematical Formulation

Under the above assumptions, the differential equations governing heat transfer in the longitudinal fin come out as [23–25]:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left\{ k(T) \frac{\partial T}{\partial x} \right\} - \frac{hP}{A_c} (T - T_\infty) + q(T), \quad (1)$$

where:

$$q(T) = q_\infty \{1 + \varepsilon(T - T_\infty)\},$$

$$k(T) = k_\infty \{1 + \alpha(T - T_\infty)\}.$$

The initial conditions are assumed as:

$$T(x, 0) = T_\infty, \quad (2)$$

and the temperature at the base of the longitudinal fin is given by [24–27]:

$$T(0, t) = T_{b,m} + A_m (T_{b,m} - T_\infty) \cos(\omega t + \Phi), \quad (3)$$

where $T_{b,m}$ is the mean base temperature of the cold side during the cycle. A_m is the dimensionless amplitude of the base temperature and $\omega = 2\pi ft$, where frequency f is directly related to the engine operating speed. Time in seconds is represented by t . $\Phi \in [0, 2\pi]$ represents the phase shift, covering the one complete cycle of the cosine wave.

In the actual phenomenon, the temperature profile decreases with respect to x , see Fig. 2. We consider the insulated tip [22,23] in our assumption, so at $x = L$, we use the condition:

$$\frac{\partial T}{\partial x}(L, t) = 0. \quad (4)$$

By introducing non-dimensional variables and similarity criteria as follows:

$$\theta = \frac{T - T_\infty}{T_{b,m} - T_\infty}, \quad X = \frac{x}{L}, \quad \text{Fo} = \frac{k_\infty t}{\rho c L^2}, \quad N^2 = \frac{h P L^2}{k_\infty A_c}, \quad \Omega = \frac{\rho c L^2 \omega}{k_\infty} \quad (5)$$

$$G = \frac{q_\infty A_c}{h P (T_{b,m} - T_\infty)}, \quad \varepsilon_G = \varepsilon(T_{b,m} - T_\infty), \quad \beta = \alpha(T_{b,m} - T_\infty),$$

the system of Eqs. (1)–(4), in a non-dimensional form reduces to:

$$\frac{\partial \theta(X, Fo)}{\partial Fo} = (1 + \beta \theta) \frac{\partial^2 \theta(X, Fo)}{\partial X^2} + \beta \left(\frac{\partial \theta}{\partial X} \right)^2 - N^2(1 - G \varepsilon_G) \theta(X, Fo) + N^2 G, \quad (6)$$

$$\theta(X, 0) = 0, \quad (7)$$

$$\theta(0, Fo) = 1 + A_m \cos(\Omega Fo + \Phi), \quad (8)$$

$$\frac{\partial \theta(1, Fo)}{\partial X} = 0. \quad (9)$$

3. Solution of the problem

The hybrid numerical scheme combining two techniques has been developed. The first one is discretization in space coordinates, and the other one is the Runge-Kutta (4,5) scheme. We discretised the space coordinate $[0, 1]$ into $n+1$ equal size subintervals of length h . Using finite differences, the boundary value problem (Eqs. (6)–(9)) reduces to an initial value problem of the system of first-order non-linear ordinary differential equations, as follows:

$$\frac{d\theta_1}{dFo} = - \left[\frac{2}{h^2} + N^2(1 - \varepsilon_G G) \right] \theta_1 + \frac{\theta_2}{h^2} + \frac{\beta}{h^2} \left(-2\theta_1^2 + \theta_1 \theta_2 + \frac{1}{4} \theta_2^2 \right) + \frac{\beta}{h^2} \left(\theta_1 - \frac{1}{2} \theta_2 \right) (1 + A_m \cos(\Omega Fo + \Phi)) + \frac{(1 + A_m \cos(\Omega Fo + \Phi))}{h^2} + \frac{\beta}{4h^2} (1 + A_m \cos(\Omega Fo + \Phi))^2 + N^2 G, \quad (10)$$

$$\frac{d\theta_i}{dFo} = \frac{\theta_{i-1}}{h^2} - \left[\frac{2}{h^2} + N^2(1 - \varepsilon_G G) \right] \theta_i + \frac{\theta_{i+1}}{h^2} + \frac{\beta}{h^2} (\theta_i \theta_{i+1} - 2\theta_i^2 + \theta_i \theta_{i-1}) + \frac{\beta}{4h^2} (\theta_{i+1}^2 - 2\theta_{i+1} \theta_{i-1} + \theta_{i-1}^2) + N^2 G, \quad (11)$$

$$i = 2, \dots, n-1,$$

$$\frac{d\theta_n}{dFo} = \frac{\theta_{n-1}}{h^2} - \left[\frac{2}{h^2} + N^2(1 - \varepsilon_G G) \right] \theta_n + \frac{\beta}{h^2} [-2\theta_n^2 + \theta_n \theta_{n-1}] + \frac{\beta}{4h^2} \theta_{n-1}^2 + N^2 G + \frac{1}{h^2} \left[1 + \beta \theta_n - \frac{\alpha}{2} \theta_{n-1} \right] \theta_{n+1} + \frac{\beta}{4h^2} \theta_{n+1}^2, \quad (12)$$

under the initial condition:

$$\theta_i(0) = 0. \quad (13)$$

To calculate the tip temperature of the fin, using a three-point formula, we write:

$$\theta_{n+1} = \frac{1}{21} (13\theta_n + 17\theta_{n-1} - 9\theta_{n-2}). \quad (14)$$

The highly non-linear system of first-order non-linear ordinary differential equations (10)–(12) under the initial condition (13) is solved using the Runge-Kutta (4,5) technique. For all computational work, MATLAB-R2024 software is utilised.

3.1. Exact solution

For the validation of the proposed numerical scheme of the prob-

lem, an analytical solution is required. When we assume the associated coefficient of thermal conductivity $\beta=0$, and the boundary condition at the base of the fin as constant in Eqs. (6) and (8), then the proposed model of heat transfer is changed into the following form:

$$\frac{\partial \theta(X, Fo)}{\partial Fo} = \frac{\partial^2 \theta(X, Fo)}{\partial X^2} - N^2(1 - G \varepsilon_G) \theta(X, Fo) + N^2 G, \quad (15)$$

$$\theta(X, 0) = 0, \quad (16)$$

$$\theta(0, Fo) = 1, \quad (17)$$

$$\frac{\partial \theta(1, Fo)}{\partial X} = 0. \quad (18)$$

Applying the Laplace transform technique, the exact solution of the system of Eqs. (15)–(18) comes out as follows:

$$\theta(X, Fo) = \frac{\cosh(\sqrt{N^2(1-G\varepsilon_G)}(X-1))}{\cosh(\sqrt{N^2(1-G\varepsilon_G)})} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{s_n Fo} (2n-1) \pi \cos(2n-1) \frac{\pi}{2} (X-1))}{s_n} - \frac{G \cosh(\sqrt{N^2(1-G\varepsilon_G)}(X-1))}{(1-G\varepsilon_G) \cosh(\sqrt{N^2(1-G\varepsilon_G)})} + \frac{G e^{-N^2(1-G\varepsilon_G) Fo}}{(1-G\varepsilon_G)} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4N^2 G e^{s_n Fo} \cos((2n-1) \frac{\pi}{2} (X-1))}{s_n (2n-1) \pi} + \frac{G(1-e^{-N^2(1-G\varepsilon_G) Fo})}{(1-G\varepsilon_G)}, \quad (19)$$

where: $s_n = -(2\pi - 1)^2 \left(\frac{\pi^2}{4} \right) - N^2(1 - G \varepsilon_G)$; $n \in N$.

4. Numerical simulation and analysis

In the proposed work, the temperature in the longitudinal fin is studied under the Neumann boundary condition. We consider a temperature dependent thermal conductivity and internal heat generation term, which are functions of local temperature. The FERK (4,5) scheme is applied for the solution of this non-linear parabolic equation to calculate the temperature in the longitudinal fin. The obtained solution is presented in the form of two- and three-dimensional figures. Only the parameters whose values differ from the reference values have been indicated. The selected reference values of dimensionless parameters used to calculate the non-dimensional temperature profile in the longitudinal fin in the finite domain are given in Table 1.

Table 1. The selected reference value of dimensionless parameters.

Parameter	Symbol	Numerical value	Reference
Associated coefficient of thermal conductivity	β	0.2	[28]
Fin parameter	N	1	[28]
Internal heat generation coefficient	G	0.2	[1]
Associated coefficient of internal heat generation	ε_G	0.2	[1]
Dimensionless amplitude of the base temperature	A_m	0	[18]
Dimensionless periodicity	Ω	0	[18]

4.1. Validation of proposed numerical scheme

The FERK (4,5) scheme has been used to obtain a numerical solution of the highly non-linear parabolic equation of heat transfer in the longitudinal fin. By unifying the two techniques, we achieve a greater accuracy with a negligible local error [29,30]. Figure 3 shows a comparative analysis between the FERK (4,5) scheme and the exact analytical solution obtained using the Laplace transform technique. It is found that the developed FERK (4,5) scheme is in excellent agreement with the exact solution. By analysing Fig. 3, we observe a good accuracy with minimum computational complexity, highlighting the advantage of the proposed method [28].

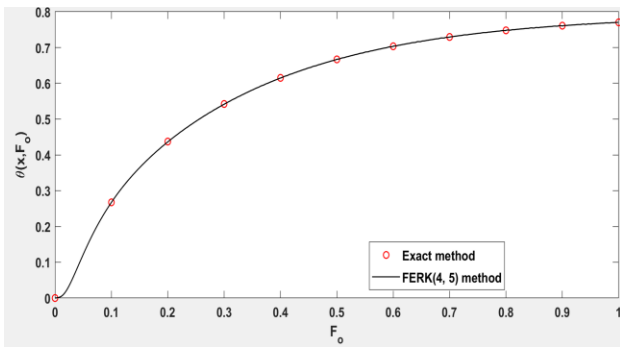


Fig. 3. Comparison between the exact method and FERK (4,5) method.

5. Results and discussion

We discuss the effects of several non-dimensional parameters, affecting heat transfer analysis in the longitudinal fin.

5.1. Effect of Ω and A_m at $\Phi = 0, \pi/2, \pi$

In order to include cyclic temperature variation in the cold wall side, we have introduced three parameters, namely, A_m , dimensionless amplitude of base temperature, Ω , non-dimensional periodicity, and Φ , representing the phase shift, which are included in Eq. (8) [18].

Here A_m represents thermal inertia, reflecting temperature fluctuations, in reference to the mean temperature difference in the cold cylinder wall. In other words, by using A_m in the model, the intensity of thermal oscillations can be quantified. From previous studies [31,32], it was evident that a suitable value of A_m for stable heat transfer in Stirling engines lies between 0.1 and 0.3. Higher values beyond 0.7–1.0 provide a non-ideal behaviour with a thermal lag leading to lower efficiency. Very high values of A_m may cause entropy generation, reducing the network output of the engine. It is evident from Fig. 4 that the non-dimensional temperature increases with the non-dimensional space coordinate as A_m increases, for $\Phi = 0$; the non-dimensional temperature decreases with the non-dimensional space coordinate as A_m increases, for $\Phi = \pi$, and remains constant for $\Phi = \pi/2$. A similar result is obtained by Singh et al. [18] for $\Phi = 0$, as they have studied for $\Phi = 0$, only. Real Stirling engines have thermal inertia: there is a delay between heating the wall and gas temperature rising. Φ can model this delay, accounting for thermal diffusion time. ωt tracks the cyclical evolution of the

engine over time. It helps predict when in time the system reaches peak or trough temperatures for optimal heat exchange.

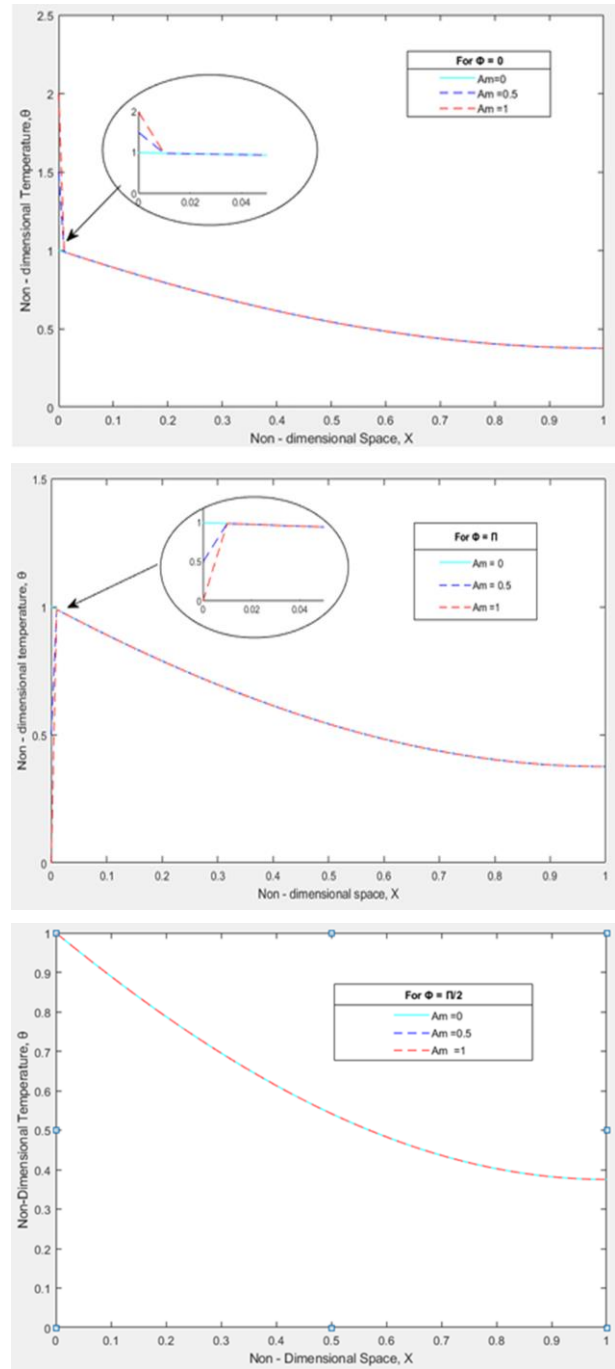


Fig. 4. Variation in dimensionless amplitude of base temperature A_m for fixed values of $\alpha = 1, G = 0.2, \Omega = 0, N = 1, \epsilon_G = 0.2, Fo = 0.3$.

Figure 5 shows the variation of non-dimensional temperature with respect to non-dimensional space coordinates for different values of Ω . It is evident from these figures that θ is the maximum for $\Omega = 0$, and the lowest for $\Omega = 10$, while it lies in between for $\Omega = 100$ when $\Phi = 0$. For $\Phi = \pi$, the non-dimensional temperature θ with respect to non-dimensional space coordinate is the highest when $\Omega = 10$, and the lowest when $\Omega = 0$, and in between for $\Omega = 100$. Further, the non-dimensional temperature, θ , is the highest for $\Omega = 100$ and the lowest for $\Omega = 0$, but in

between when $\Omega = 10$, for $\Phi = \pi/2$. A similar result is obtained by Singh et al. [18] for $\Phi = 0$.

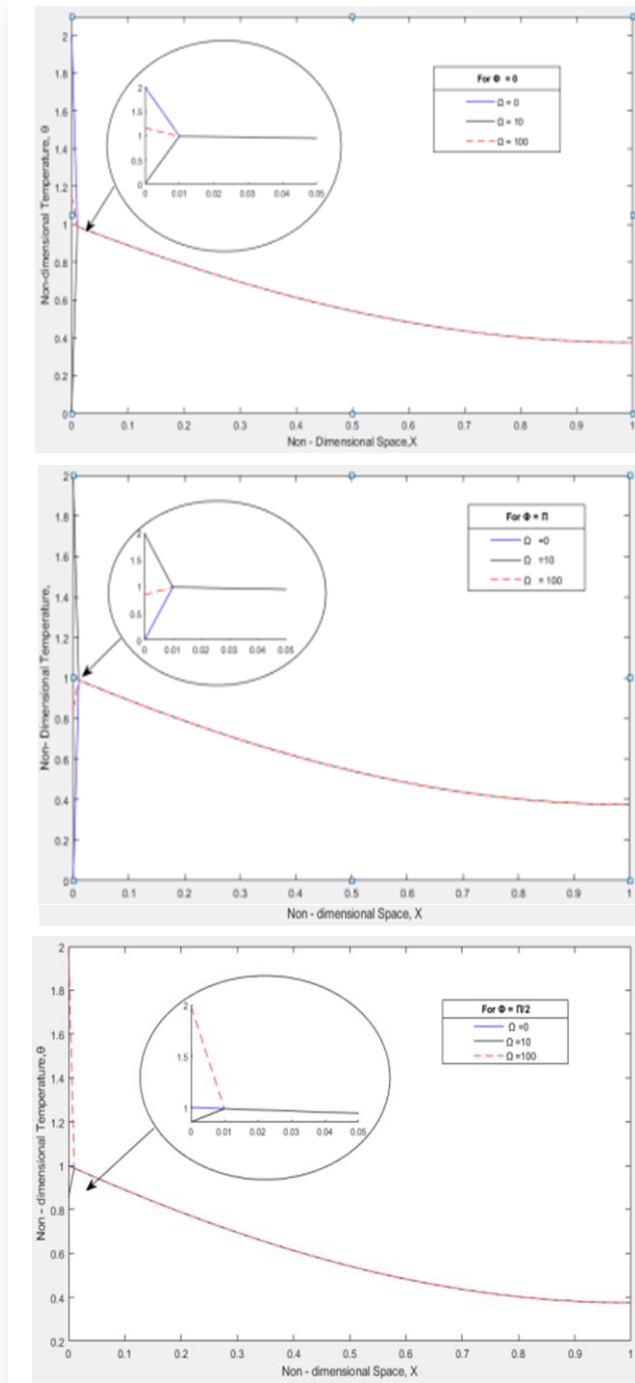


Fig. 5. Variation in non-dimensional periodicity, Ω , for fixed values of $\alpha = 1$, $G = 0.2$, $A_m = 1$, $N = 1$, $\varepsilon_G = 0.2$, $Fo = 0.3$.

5.2. Effect of Fo and X

In Figure 6, we analyse the behaviour of temperature distribution in the longitudinal fin with respect to non-dimensional time, Fo , and non-dimensional space coordinate X . This figure has two components – Figs. 6(a) and 6(b). Figure 6(a) demonstrates the variation of non-dimensional temperature with Fo for different values of X , when $\beta = G = \varepsilon_G = 0.2$ and $N = 1$. We observe from this figure that the non-dimensional temperature increases

with Fo but decreases as X increases. Figure 6(b) presents the variation of non-dimensional temperature vs. X for varying Fo , for the same values of fin parameters. It shows that the tip temperature increases as Fo grows, and decreases with time.

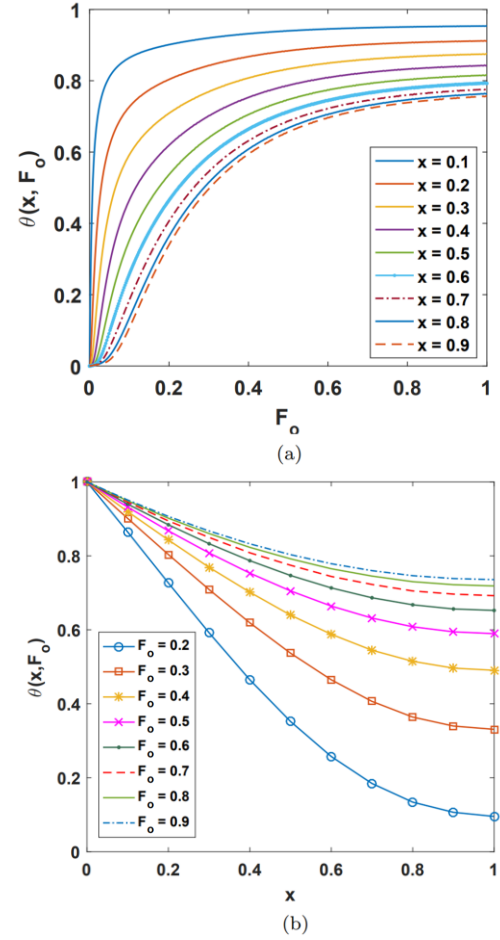


Fig. 6. Temperature distribution in fin vs.: (a) dimensionless time coordinate (Fo), (b) dimensionless space coordinate (x) when $\beta = 0.2$, $N = 1$, $G = 0.2$, and $\varepsilon_G = 0.2$.

5.3. Effect of β

Figure 7 shows the effect of β in two- and three-dimensional coordinate space. It is observed that as β increases, the temperature in the fin increases with Fo and X . This confirms the improved fin conductivity and a more uniform temperature distribution in the fin.

5.4. Effect of N

Figure 8 demonstrates the effect of fin parameter N in two- and three-dimensional coordinate systems. We draw Fig. 8(a) as non-dimensional temperature vs. non-dimensional time, Fo , and Fig. 8(b) as non-dimensional temperature vs. non-dimensional space coordinate. It is observed that the temperature in the longitudinal fin increases along Fo and X as the values of N decrease. The fin parameter N increases as convective heat transfer coefficient h , fin perimeter p , and fin length L increase, while the conductivity of fin material, k , and its cross-sectional area A_c decrease. Therefore, it can be concluded that a higher value of N promotes convection, resulting in a better cooling effect.

Thus, a long thin fin will have a prominent temperature difference between both ends. Therefore, it can be concluded that, as N increases, the conductivity pattern deteriorates. In other

words, a lower N number denotes thick and short fins, whereas a higher value of N shows a better cooling effect.

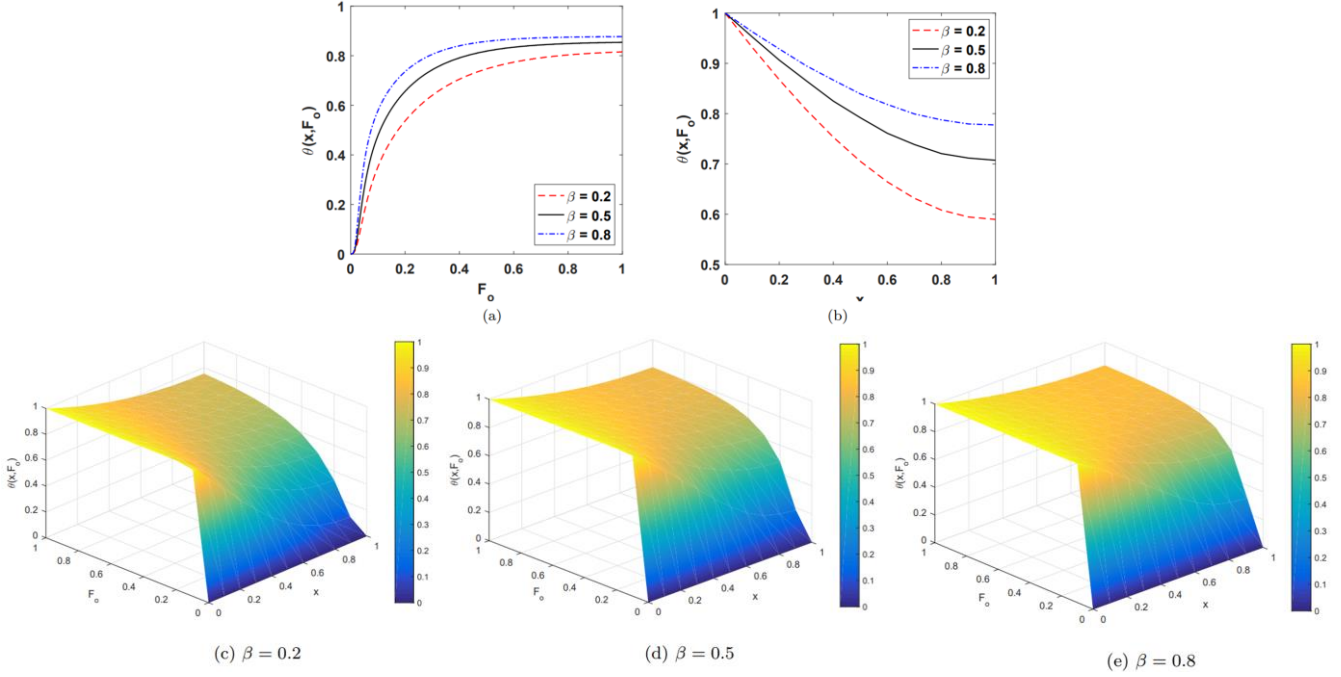


Fig. 7. Temperature distribution in fin vs.: (a) dimensionless time coordinate (F_o), when $\beta = 0.2, 0.5, 0.8$ and $X = 0.5$; (b) dimensionless space coordinate (X), when $\beta = 0.2, 0.5, 0.8$ and $F_o = 0.5$; (c) F_o and X , when $\beta = 0.2$; (d) F_o and X , when $\beta = 0.5$; (e) F_o and X , when $\beta = 0.8$ for fixed numerical values of $\beta = 0.2, N = 1, G = 0.2$, and $\varepsilon_G = 0.2$.

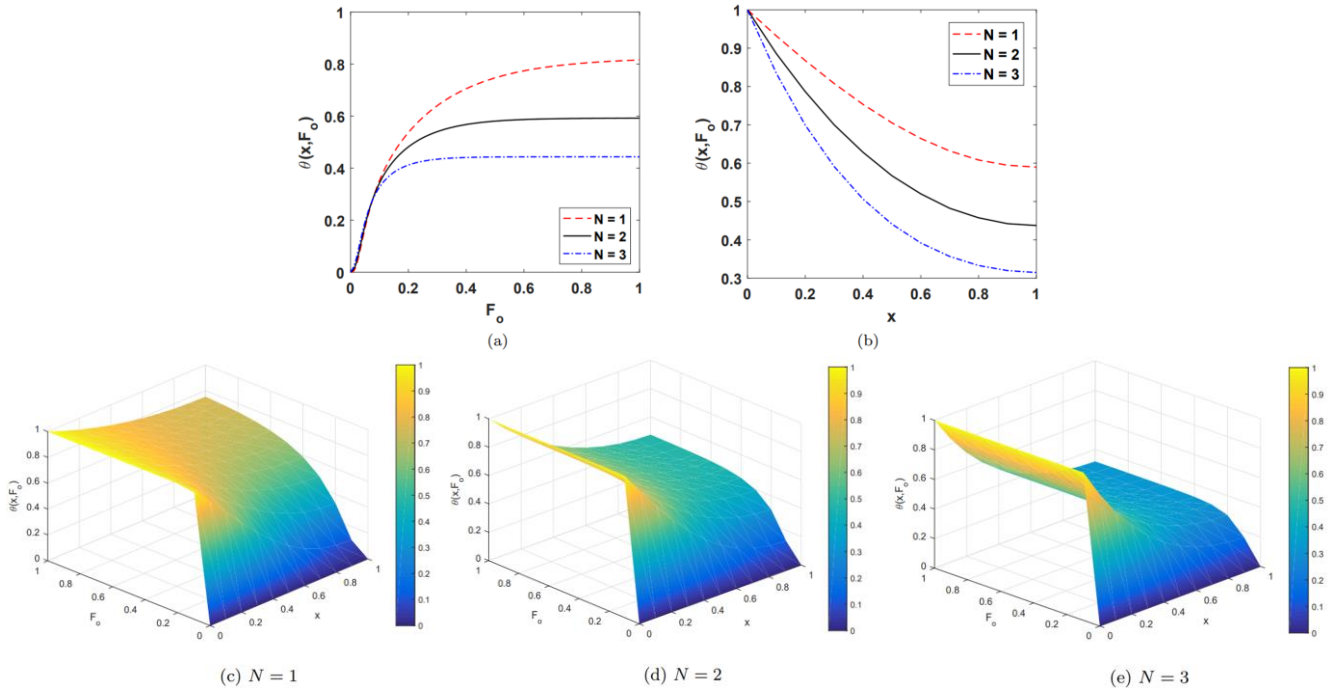


Fig. 8. Temperature distribution in fin vs.: (a) dimensionless time coordinate (F_o), when $\beta = 0.2, 0.5, 0.8$ and $X = 0.5$; (b) dimensionless space coordinate (X), when $\beta = 0.2, 0.5, 0.8$ and $F_o = 0.5$; (c) F_o and X , when $\beta = 0.2$; (d) F_o and X , when $\beta = 0.5$; (e) F_o and X , when $\beta = 0.8$ for fixed numerical values of $\beta = 0.2, N = 1, G = 0.2$, and $\varepsilon_G = 0.2$.

5.5. Effect of G

The effect of the internal heat generation coefficient in two- and three-dimensional coordinate systems is shown in Fig. 9. We

draw Fig. 9(a) for the non-dimensional temperature vs. non-dimensional time coordinate, and Fig. 9(b) for the non-dimensional temperature vs. non-dimensional space coordinate. The temperature in the longitudinal fin increases along F_o and X ,

with an increase in G . This can be achieved by fin geometry where the cross-sectional area A_c has a smaller value and the perimeter p of the fin is large. This again supports a long fin for better cooling performance.

5.6. Effect of ε_G

Internal heat generation represents the amount of heat generated

internally in the fin by consistent heating of the cooling section in the Stirling engine. Figure 10 depicts the effect of the associated coefficient of internal heat generation ε_G in two- and three-dimensional coordinate systems. We draw Fig. 10(a) for the non-dimensional temperature vs. non-dimensional space coordinate. The temperature in the longitudinal fin increases along both F_o and X as the values ε_G increase.

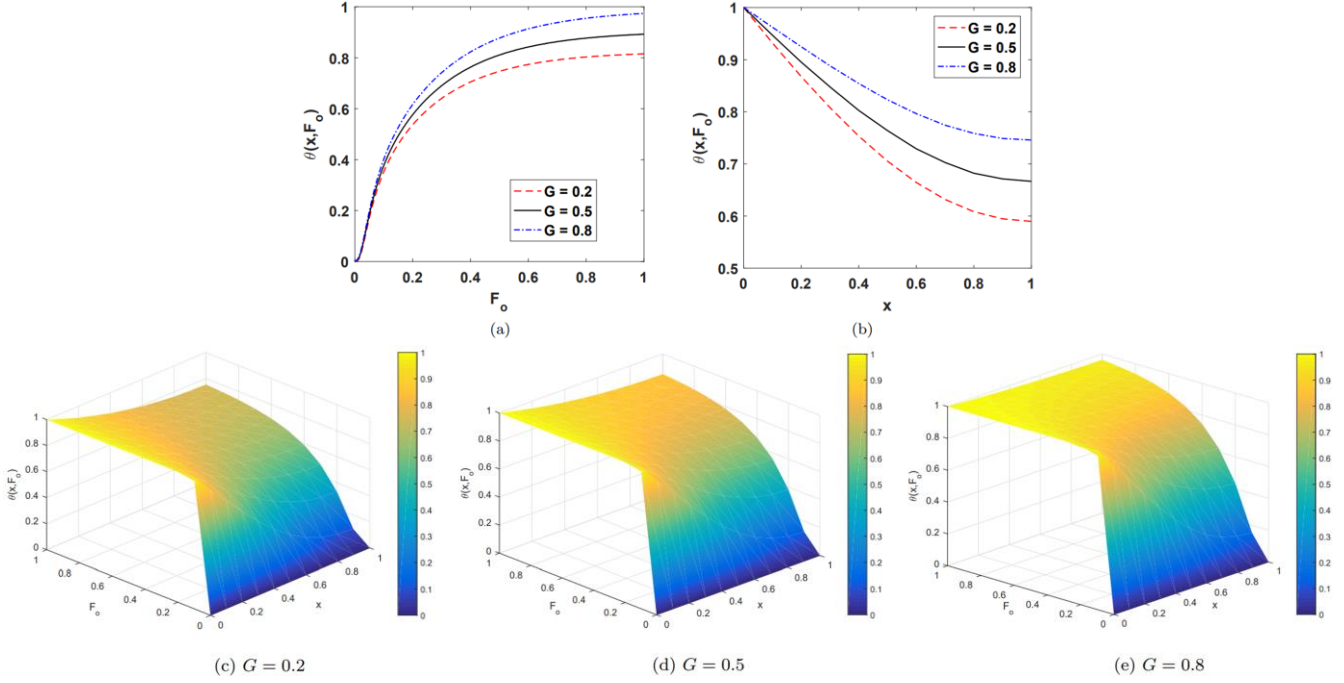


Fig. 9. Plot the temperature distribution in fin vs.: (a) dimensionless time coordinate (F_o), when $G = 0.2, 0.5, 0.8$; (b) dimensionless space coordinate (x), when $G = 0.2, 0.5, 0.8$; (c) F_o and x , when $G = 0.2$; (d) F_o and x , when $G = 0.5$; (e) F_o and x , when $G = 0.8$ for fixed numerical values of $\beta = 0.2$, $N = 1$ and $\varepsilon_G = 0.2$.

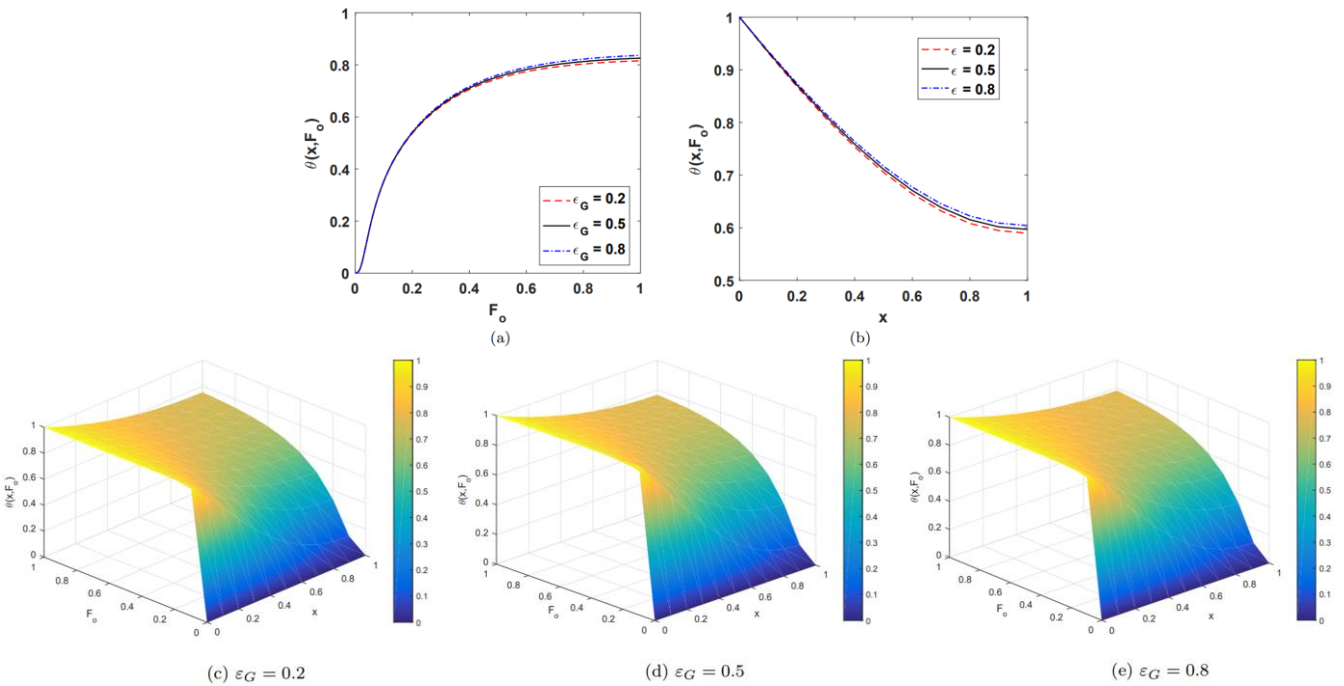


Fig. 10. Plot of the temperature distribution in fin vs.: (a) dimensionless time coordinate (F_o), when $\varepsilon_G = 0.2, 0.5, 0.8$; (b) dimensionless space coordinate (X), when $\varepsilon_G = 0.2, 0.5, 0.8$; (c) F_o and X , when $\varepsilon_G = 0.2$; (d) F_o and X , when $\varepsilon_G = 0.5$; (e) F_o and X , when $\varepsilon_G = 0.8$ for fixed numerical values of $\beta = 0.2$, $N = 1$ and $G = 0.2$.

6. Conclusions

The goal of this study is to provide validation of the FERK (4,5) method and use this technique for evaluation of the thermal profile in a longitudinal fin attached to a cooler region of a β -type Stirling engine. The wasted heat through the fin is proposed to be utilised by an ERS in generating electricity. The obtained results were analysed for the variability of thermal conductivity and internal heat generation with temperature, thermogeometric fin parameters, and generation number. The important findings are summarised as follows:

1. FERK (4,5) method is justified with exact results in a specific case, and good accuracy is observed.
2. A_m and Ω do not significantly affect temperature distribution in the non-dimensional space over a wide range. For different values of A_m , graphs representing the non-dimensional temperature distribution in non-dimensional space quickly overlap.
3. A suitable value of phase angle Φ was shown by various researchers to be in the range of 60° – 80° ; As a function of cosine, the range in which Φ varies is from -1 to 1 . It has been observed that by varying Φ , a delay can be introduced in the temperature curve for different values of A_m and Ω . The variation only affects the boundary condition at the base of the fin, and the nature of the curve soon follows similar trends for different values.
4. The temperature profile in the longitudinal fin increases along Fo and X both as the value of β increases. A higher value of β represents conduction as a prominent phenomenon, resulting in a uniform temperature profile in the fin.
5. The temperature profile in the longitudinal fin decreases when the fin parameter N increases, i.e. the temperature profile is inversely proportional to N . For fast cooling, we require a higher value of the N parameter.
6. An increase in the value of G will result in an increase in ambient heat generation as well as internal heat generation. This will elevate the fin temperature.
7. ε_G increases as the internal heat generation parameter ε increases. This will reduce the heat flux through the fin. As a result, the temperature of the fin will increase, providing an opportunity for hybrid energy system utilisation.

The above study concludes that thermodynamic parameters of the fin, temperature dependent conductivity of fin material, generation number G , and coefficient of internal heat generation play a significant role in releasing internally generated heat of machines into the environment. This research may be expanded to create a mathematical framework combining thermoelectric production in plate fins using simulation to calculate a low-temperature waste energy harvesting system to produce electricity. The developed system will consist of several thermoelectric modules (TEMs) attached to the cooling zone of the Stirling engine, which will provide an opportunity for waste heat utilisation in combination with the Stirling engine power. As a result, the system will incorporate a waste energy channel, a cooling zone, and thermoelectric modules. TEM is placed amid the cooling fins and the waste heat pipes. The temperature differential between the thermoelectric materials hot and cold surfaces will be utilised to transform wasted energy into electrical energy

[32,33]. The heat transfer rate provides the researchers with sufficient opportunity to generate electricity utilising ERS. This model can be utilised to design the cold section of the Stirling engine to maintain the desired temperature difference for proper operation.

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Sensitivity Analysis of Magnetohydrodynamic Mixed Convective Trapezoidal Heat Exchanger Containing Hybrid Nanofluid: Numerical and Statistical Approach

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Abstract

The consequences of magnetohydrodynamic mixed convection in a trapezoidal heat exchanger are investigated through numerical analysis. Due to the extensive applications of both mono and hybrid nanofluids in manufacturing and thermal engineering, the Ag-MgO-H₂O hybrid nanofluid is selected as the working material for the entire domain. Additionally, a horizontal magnetic field is applied to the cavity. The finite element method is involved to solve the corresponding mathematical equations. The physical implications of the results are examined over a range of Reynolds numbers ($10 \leq Re \leq 200$), Hartmann numbers ($0 \leq Ha \leq 100$), and nanoparticle volume fractions ($0 \leq \phi \leq 0.08$) using streamlines, isotherms, and line graphs. The impact of key factors on the response function is illustrated using the response surface methodology with 2D and 3D visualizations. Sensitivity rates are analysed by developing a best-fit correlation. It is concluded that the thermal enhancement of the hybrid nanofluid is achieved up to 11.4% by incorporating hybrid nanoparticles, and due to the upsurge of the Reynolds number. Conversely, the influence of the magnetic field leads to a decline in this rate to 10.02%. The use of Ag-MgO-H₂O hybrid nanofluid improves the heat transfer efficiency of water by 6.62%. Finally, the results of this study may offer valuable insights for designing an efficient mixed convective mechanical device.

Keywords: Hybrid nanofluid; Mixed convection; Heat exchanger; Response surface methodology; Sensitivity analysis

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1. Introduction

Current researchers are also captivated by mixed convective heat transfer involving fluid movement in enclosed spaces, where the combination of forced and natural convection results in mixed convection. This phenomenon has gained increasing attention in metalworking and metallurgical sciences due to its substantial influence on the heat transfer of electrically conduc-

tive fluids. The investigation, understanding, and prediction of fluid flow dynamics have been of great interest. Leonardo da Vinci was the first to observe and document fluid flow phenomena in the early fifteenth century, sparking a substantial amount of research in fluid mechanics. Mixed convective heat transfer in various enclosures is a crucial topic in engineering sectors because of its extensive applications in heat exchangers, nuclear reactors, drying technologies, solar panels, building in-

Nomenclature

c_p – specific heat at constant pressure, J/(kg·K)
 Ha – Hartman number
 Nu – Nusselt number
 p – pressure, kPa
 P – dimensionless pressure
 Pr – Prandtl number
 Re – Reynolds number
 Ri – Richardson number
 T – temperature, K
 u, v – dimensional velocity, m/s
 U, V – dimensionless velocity

Greek symbols

α – thermal diffusivity, m²/s²
 β – coefficient of thermal expansion, K⁻¹

θ – dimensionless temperature
 μ – dynamic viscosity, kg/(m·s)
 ν – kinematic viscosity, m²/s
 ρ – density, kg/m³
 σ – electric conductivity, $\Omega^{-1}\cdot\text{m}^{-1}$
 ϕ – particle concentration
 ψ – stream function
 Ω – vorticity vector

Subscripts and Superscripts

HS – hot surface
 hnf – hybrid nanofluid

Abbreviations and Acronyms

FEM – finite element method
RSM – response surface methodology

sulation, lubrication technologies, and chemical industries. Among the most studied areas are mixed convective flows in enclosed spaces, aiming for a quantitative understanding and expanding their practical applications [1–5]. Nanofluids, which consist of nanoparticles with diameters less than 100 nm, enhance the thermodynamic properties and thermal efficiency of base fluids even at low concentrations when properly dispersed and stabilised [6–10]. Numerous researchers have explored methods to improve heat transfer and cooling effects by considering various geometries and configurations of different nanofluids [11–15]. In the context of a variety of engineering and industrial applications for nanofluids, it is essential to significantly modify the attributes of mono nanofluids significantly to enhance their thermophysical and rheological properties. To meet these needs, a new type of fluids known as hybrid nanofluids (HNFs) were developed. These fluids allow for the proper combination of two or more nanoparticles within a single base fluid. Essentially, composite or hybrid nanofluids are a novel category of nanofluids created by mixing metal oxide, metal particles, or both into a primary fluid. Significant research has already been conducted on hybrid nanofluids. For instance, Kaushik et al. [16] conducted both numerical and experimental studies to compare the flow behaviour between CuO-ZnO-H₂O and the base fluid in a small channel. They concluded that due to the addition of solid nanoparticles, the outcomes improved up to 18–21%. Hussain et al. [17] used numerical methods to study an open enclosure with a square adiabatic barrier, using Al₂O₃-Cu-H₂O hybrid nanofluid. Their findings indicated that the rate of heat transfer was enriched by increasing the Richardson number, Reynolds number, and nanoparticle size. Zaboli et al. [18] reported that hybrid nanofluids could enhance heat exchange in solar systems, including trough collectors. Mahalakshmi [19] conducted a numerical study on a lid-driven mixed convection with heat sources containing hybrid nanofluids. The study concluded that the Ag-CuO-water hybrid nanofluid transfers heat more efficiently compared to Ag-MgO-water and Ag-TiO₂-water. Mandal et al. [20] numerically explored the effects of several geometric factors on a porous W-shaped cavity experiencing mixed convection with a Cu-Al₂O₃-H₂O hybrid nanofluid. They found that increasing the bottom undulation amplitude im-

proved thermal energy transmission despite the reduced fluid volume. Anee et al. [21] applied the Lattice Boltzmann method (LBM) to examine the heat transfer behaviour of hybrid nanofluid in an enclosed shape with multiple heaters, revealing that the size of the nanoparticles and the Hartmann number (Ha) significantly affected heat transfer. Thumma et al. [22] studied the heat transfer phenomena of a magnetised hybrid nanofluid (Cu-Ag-H₂O) in a radiative flow across a rotating disc, taking into account the Hall current and heat source. Their investigation showed that the Hall current parameter not only regulates cross-radial velocity and energy but also increases radial motion. The hybrid nanofluid exhibited superior heat transfer performance compared to a single fluid with respect to thermal radiation and ESHS coefficients.

The study of the influence of magnetic forces on electrically conducting fluids is known as magnetohydrodynamics (MHD). MHD encompasses mechanisms such as the Earth's magnetic field, nuclear fusion, cooling of fission reactors, X-ray radiation, and solar wind cooling. Various researchers have examined MHD heat transfer in different enclosures over time due to these practical applications [23–27]. Owing to its numerous uses, MHD mixed convection involving various geometries, temperatures, and boundary conditions has garnered significant attention in scientific research. For instance, Tayebi et al. [28] explored heat transfer behaviour of a hybrid nanofluid in a square cavity including a wavy cylinder. Gibanov et al. [29] investigated MHD of ferrofluid in a chamber with moving upper edges and a porous layer. Mebarek-Oudina et al. [30] studied the magnetohydrodynamic transport of a hybrid nanofluid within a porous chamber, finding that the upsurge of magnetic field intensity inhibits the convective heat transfer rate. Selimefendigil and Chamkha [31] completed a numerical study on MHD flow in a square cavity with a partial triangular porous layer, filled with Ag-MgO-water hybrid nanofluid. They observed that increasing the porosity of the container significantly boosts heat transfer, while the presence of a magnetic field substantially decreases it. Ma et al. [32] statistically examined the MHD effects using a Shamse Knot-shaped cavity filled with Ag-TiO₂-water hybrid nanofluid, discovering that MHD influences heat transfer and that increases in the Rayleigh number (Ra) and cavity side

length (D) enhance heat transport. Munawar et al. [33] numerically studied MHD mixed convection using Ag-MgO-water in an inclined cavity with a circular heater. They determined that increasing the Hartmann number (Ha) reduces the average Nusselt number (Nu_{av}), and greater heat transfer rates were achieved by using central heaters with smaller radii for free convection and larger radii for forced convection. Mourad et al. [34] explored the impact of MHD on a hybrid nanofluid within a wavy cavity using the finite element method (FEM). Their findings indicated that Nu_{av} increases with the Darcy number (Da) and Ra but decreases with Ha .

An apparatus that moves heat from an energy source to a working fluid is called a heat exchanger. It is utilised in systems that heat and cool, where the fluids may be near one another or kept apart by a solid barrier to avoid direct contact. Heat exchangers are essential in different applications, such as ventilation, freezing, sewage treatment, chemical and pharmaceutical plants, power plants, natural gas processing, and space heating [35–39]. Chen et al. [40] developed a heat exchanger model to explore the relationship between fin pitches and tube diameters in a fin-tube heat exchanger, using both numerical and experimental methods to solve it. Lee et al. [41] created a 3D model of unsteady free convection for a circular fin-tube heat exchanger. Additionally, Pakalka et al. [42] developed a theoretical and experimental framework for a heat exchanger based on fin-tube design.

The review of publications mentioned above makes it evident that the MHD mixed convective heat exchanger is a subject of great attraction to investigators on account of its creative uses across a wide range of engineering sectors. While numerous studies have been conducted on a variety of cavities to examine mixed convective fluid with MHD and heat transfer mechanisms, some recent studies have focused on schematic cavities that can hold a wide variety of nanoparticles [43–46]. Similarly, comparatively little research was done on heat exchangers to construct a fast heat transmission structure [47–49]. Yet, in the current research, a sensitivity analysis of a trapezoidal heat exchanger that contains an Ag-MgO- H_2O hybrid nanofluid using RSM is performed after integrating these two phenomena, heat exchanger and mixed convection. Moreover, in real life applications, there are numerous trapezoidal-shaped industrial architecture where mono or hybrid nanofluids are used for distinct purposes. To the greatest of the writer's understanding, no investigation has been done on this topic yet. The FEM is a popular computational technique that is used to simulate the governing formulas [50, 51]. Another reason to implement the RSM is to investigate the geometric and statistical impact of involved parameters with a best-fitted correlation with Nu_{av} and involved parameters. Investigating the heat transfer mechanisms for this hybrid nanofluid-based mixed convective trapezoidal heat exchanger mathematically and statistically using FEM and RSM is the primary goal of this study.

2. Physical description and mathematical model

Using water (H_2O) as the primary fluid, a schematic-shaped cavity is examined numerically as a fluid region containing Ag and MgO nanoparticles. This fluid is an incompressible, steady and

Newtonian fluid that takes magnetic field impacts into account. The height and length of this schematic-shaped cavity, which is mass and heat isolated, are H and L , accordingly. With a lid motion of u_0 , the top horizontal wall is moving. On the right and left sides, accordingly, two cylinder pipes with a $0.1L$ radius can be utilized as a heater and cooler. Figure 1 shows the arrangement of this fluid framework with a schematic-shaped heat exchanger. It is proposed that the left cylindrical pipe indicates a cool surface T_c while the right pipe indicates a hot surface T_h .

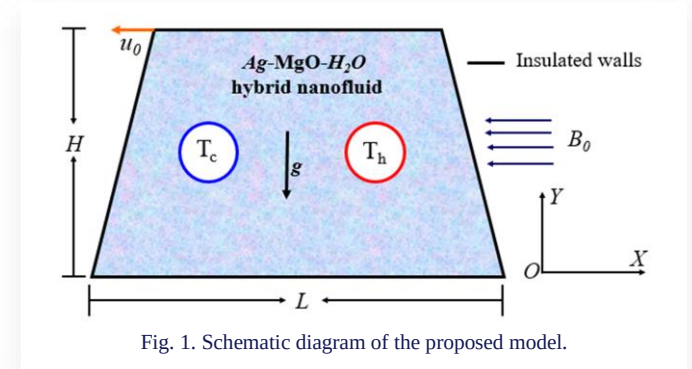


Fig. 1. Schematic diagram of the proposed model.

The outer boundaries of the fluid area are totally intact and adiabatic. Additionally, the gravitational acceleration g works in the exact reverse position of the Y -dimension. Further, the frame is enveloped by a B_0 form of Ag-MgO magnetic field that is constant and flows from right to left. The nearby medium is considered non-slip because of the assumed equality of size and nanoparticles. In this situation, Table 1 lists the thermophysical properties of the considered nanofluid.

Table 1. Characteristics of nanoparticles and base fluid [52].

Nanoparticle and base fluid	c_p , $J \cdot kg^{-1} \cdot K^{-1}$	ρ , $kg \cdot m^{-3}$	κ , $W \cdot m^{-1} \cdot K^{-1}$	β , K^{-1}	σ , $S \cdot m^{-1}$	μ , $kg \cdot m^{-1} \cdot s^{-1}$
Ag	235	10500	429	5.4×10^{-5}	8.1×10^{-4}	-
MgO	879	3970	30	3.36×10^{-5}	8×10^{-4}	-
H_2O	4179	997.1	0.613	21×10^{-5}	5.5×10^{-6}	8.91×10^{-4}

Some assumptions served as the foundation for this study and the creation of the model. The following assumptions were considered:

- this hybrid nanofluid has a two-dimensional laminar, incompressible flow,
- magnetohydrodynamic mixed convection is considered,
- the governing equations are simulated using the Galerkin weighted residual finite element method,
- the flow problem is considered under the Boussinesq approximation.

To see the entire fluid flow area, a two-dimensional (2D) Cartesian structure is set up, with the left sidewall marked by the Y -axis and the bottom wall by the X -axis. The basic governing equations to make a mathematical form for this mixed convective two dimensional model using hybrid nanofluid are the continuity, momentum, and energy equations as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\rho_{hnf} \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_{hnf} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F, \quad (2a)$$

$$\rho_{hnf} \left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_{hnf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + F, \quad (2b)$$

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_p)_{hnf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \quad (3)$$

Here, in the u -momentum equation, there is no external effect; as a result, Eq. (2a) does not include any additional body force ($F=0$); Eq. (2b) adds $F = g(\rho\beta)_{hnf}(T - T_c) - \mu_{hnf}B_0^2v$ because of an outside magnetic field and gravitational force. Also, the dimensionless initial and boundary conditions are:

$$\left. \begin{array}{l} \text{At right circular surface: } T = T_h, \quad u = v = 0 \\ \text{At left circular surface: } T = T_c, \quad u = v = 0 \\ \text{Rest of the walls: } \frac{\partial T}{\partial n} = 0, \quad u = v = 0 \end{array} \right\}. \quad (4)$$

Additionally, Table 2 explains the correlations of hybrid nanofluids which have been taken into account between base fluid (H_2O) and nanoparticles (Ag and MgO). Here, n is the orthogonal unit vector on the XY -plane. To put it practically, base fluid and nanoparticles are needed to obtain the properties of nanofluid. The correlations that are taken into consideration between primary fluid (H_2O) and nanoparticles (Ag and MgO) are explained in this part of the paper. The characteristics of nanofluid are truly dependent upon the primary fluid and nanomaterials. In order to estimate the characteristics of nanofluids, correlations gathered in Table 2 are used.

Table 2. Used correlations of hybrid nanofluid [53].

Properties	Applied Correlations
Concentration of nanoparticles	$\phi = \phi_{Ag} + \phi_{MgO}$
Density of nanofluid	$\rho_{hnf} = (1 - \phi)\rho_{bf} + \phi\rho_{sp}$ where: $\phi\rho_{sp} = \phi_{Ag}\rho_{Ag} + \phi_{MgO}\rho_{MgO}$
Specific heat capacity	$(\rho c_p)_{hnf} = (1 - \phi)(\rho c_p)_{bf} + \phi(\rho c_p)_{sp}$ where: $\phi(\rho c_p)_{sp} = \phi_{Ag}(\rho c_p)_{Ag} + \phi_{MgO}(\rho c_p)_{MgO}$
Thermal conductivity	$\kappa_{hnf} = \kappa_{bf} \left\{ \frac{\kappa_{sp} + 2\kappa_{bf} - 2\phi(\kappa_{bf} - \kappa_{sp})}{\kappa_{sp} + 2\kappa_{bf} + \phi(\kappa_{bf} - \kappa_{sp})} \right\}$ where: $\phi\kappa_{sp} = \phi_{Ag}\kappa_{Ag} + \phi_{MgO}\kappa_{MgO}$
Thermal diffusivity	$\alpha_{hnf} = \frac{k_{hnf}}{(\rho c_p)_{hnf}}$
Dynamic viscosity	$\mu_{hnf} = \mu_{hnf}(1 + 2.5\phi + 6.2\phi^2)$
Thermal expansion coefficient	$(\rho\beta)_{hnf} = (1 - \phi)(\rho\beta)_{bf} + \phi(\rho\beta)_{sp}$ where: $\phi(\rho\beta)_{sp} = \phi_{Ag}(\rho\beta)_{Ag} + \phi_{MgO}(\rho\beta)_{MgO}$
Electrical conductivity	$\sigma_{hnf} = \sigma_{bf} \left[1 + \frac{3\phi \left(\frac{\sigma_{sp}}{\sigma_{bf}} - 1 \right)}{\left(\frac{\sigma_{sp}}{\sigma_{bf}} + 2 \right) - \phi \left(\frac{\sigma_{sp}}{\sigma_{bf}} - 1 \right)} \right]$ where: $\phi\sigma_{sp} = \phi_{Ag}\sigma_{Ag} + \phi_{MgO}\sigma_{MgO}$

By incorporating the dimensionless quantities in Eq. (5) into Eqs. (1–3), the subsequent dimensionless governing equations (6–9) are generated:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad P = \frac{p}{\rho_{nf}u_0^2} \quad \text{and} \quad \theta = \frac{T - T_c}{T_h - T_c}. \quad (5)$$

The adjusted set of dimensionless equations is as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (6)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \left(\frac{\nu_{hnf}}{\nu_{bf}} \right) \frac{1}{Re} (\nabla^2 U), \quad (7)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{\nu_{hnf}}{\nu_{bf}} \right) \frac{1}{Re} (\nabla^2 V) + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_{bf}} Ri \theta - \left(\frac{\rho_{bf}\sigma_{hnf}}{\rho_{hnf}\mu_{bf}} \right) \frac{Ha^2}{Re} V, \quad (8)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left(\frac{\alpha_{hnf}}{\alpha_{bf}} \right) \frac{1}{Re Pr} (\nabla^2 \theta), \quad (9)$$

where $Re = \frac{u_0 L}{\mu_{bf}}$, $Pr = \frac{\nu_{bf}}{\alpha_{bf}}$ and $Ha = \frac{LB_0\sqrt{\sigma_{bf}}}{\sqrt{\mu_{bf}}}$ represent the Reynolds number, Prandtl number and Hartmann number, respectively. Also, again, $\frac{Gr}{Re^2} = Ri$ is known as the Richardson number, where $Gr = \frac{g\beta_{bf}(T_h - T_c)L^3}{\nu_{bf}^2}$ is the Grashoff number. Additionally, the reduced boundary conditions are:

$$\left. \begin{array}{l} \text{At right circular surface: } \theta = 1, \quad U = V = 0 \\ \text{At left circular surface: } \theta = 0, \quad U = V = 0 \\ \text{Rest of the walls: } \frac{\partial \theta}{\partial N} = 0, \quad U = V = 0 \end{array} \right\}. \quad (10)$$

Furthermore, from the heated left circular surface, the average Nusselt number (Nu_{av}), which is employed to quantify the rate of heat transfer, is obtained by employing:

$$Nu_{av} = - \left(\frac{k_{hnf}}{k_{bf}} \right)_S \int \frac{\partial \theta}{\partial N} ds, \quad (11)$$

where S stands for the surface that is heated, and N is the perpendicular unit vector on the XY -plane. Moreover, ψ signifies the stream function that is associated by $U = \frac{\partial \psi}{\partial X}$ and $V = \frac{\partial \psi}{\partial Y}$.

Furthermore, $\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = - \left(\frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \right) = -\Omega$, where Ω expresses the vorticity vector, and U, V stand for the velocity vector along the X and Y axis, respectively.

3. Numerical methodology

3.1. Solution technique

The dimension-free governing expressions (6) to (9) are mathematically solved utilising the Galerkin weighted residual finite element method (FEM) with boundary settings (10). The entire region is distributed into separate triangular elements, requiring six nodes and accounting for quadratic interpolation functions, in order to monitor the thermal performance and fluid motion. Furthermore, the linear interpolation approach is used to calcu-

late the gradient of pressure. Moreover, the participation of interpolation functions roughly represents the dependent variables of each element as local element coordinates. These equations can be simulated by using the Newton-Raphson iteration method by MATLAB, which generates a set of global nonlinear algebraic equations. In this process, the convergence condition is defined as follows: $|\Gamma^{m+1} - \Gamma^m| < 10^{-5}$, where $m+1$ and m stand for two successive repetitions, and $\Gamma(U, V, \theta)$ stands for the iterative value. The complete set of FEM was detailed in [54,55]. The whole flowchart of this computation process is shown in Fig. 2.

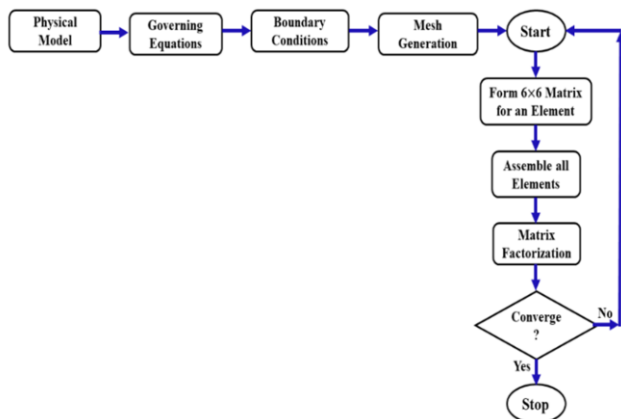


Fig. 2. A complete flow chart of the computational procedure.

This finite element scheme can have the optimal number of elements by taking the following factors into account, which can be determined via a grid sensitivity evaluation. $Ri = 1$, $Ha = 20$, $Pr = 6.8377$, $Re = 100$, $\phi_{Ag} = 0.02$ and $\phi_{MgO} = 0.02$ are the magnitudes. Again, the magnitude of Nu_{av} is designated in order to continue this independence test for meshing. For applying FEM, the entire geometry is subdivided into five distinct triangular elements. These are 2369, 3400, 9570, 27305, and 35665. The computed results of Nu_{av} using FEM for different numbers of triangle members are demonstrated in Table 3 and Fig. 3.

Table 3. Grid independence analysis for the present study.

Elements	2369	3400	9570	27305	35665
Nu_{av}	10.975	11.046	11.207	11.277	11.276

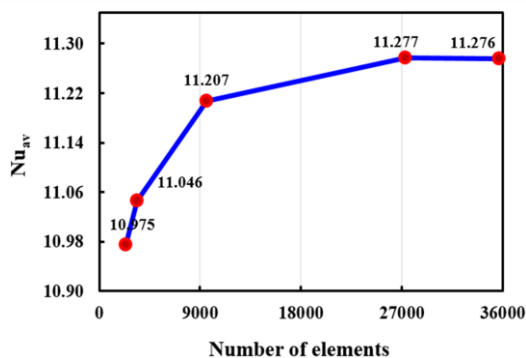


Fig. 3. Grid test by using Nu_{av} and number of elements.

Though for the first three meshing types, the differences are clearly noticeable, it is clear that the magnitude of Nu_{av} for 27305 triangular elements is nearly identical to the cases ensuing a greater number of elements. As a result, 27305 triangular elements are selected for discretisation and solving this proposed heat exchanger model.

3.2. Code validation

The purpose of this section is to validate the findings of this study by comparing streamlines and isotherms with those obtained from a lid-driven mixed convective investigation conducted by Sivakumar et al. [56]. To ensure accuracy, the results of Sivakumar's proposed mixed convective heat exchanger model are compared with the present outcomes. The simulation was performed with parameters $Re = 100$, $Pr = 0.71$, and $Ri = 0.01$, within a partially heated square cavity on the left wall, with the right wall maintained at a relatively low temperature and a moving lid on the upper wall. Additionally, we have replicated the work of Sivakumar et al. [56] in this study and compared the streamlines and isotherms (as shown in Fig. 4).

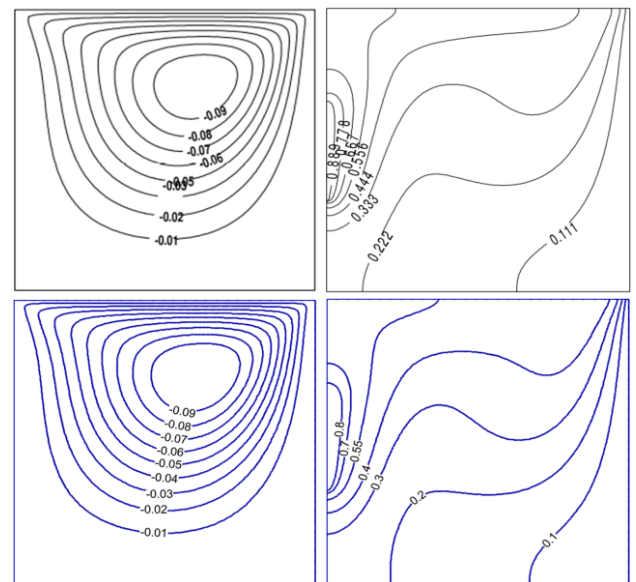


Fig. 4. Comparison of results of Sivakumar et al. [56] (top row) with the current study (bottom row): streamlines (left), isotherms (right).

The top two figures are from Sivakumar's research, while the bottom two are from the current study. The present results closely match the streamline and isotherm patterns, reinforcing our confidence in the accuracy of this mixed convective analysis in near cavities, as it demonstrates strong consistency with the current numerical analysis.

4. Results and discussion

This section explores the effects of physical variables with an engineering focus on trapezoidal mixed convective flow of hybrid nanofluid, including the influence of a magnetic field. The results obtained within this cavity are presented through velocity profiles, streamlines, isotherms, and 2D and 3D response surfaces. The study involves a hybrid nanofluid composed of Ag,

MgO, and H₂O, with spherical solid nanoparticles confined within a trapezoidal chamber. Key parameters such as the Reynolds number (Re), nanoparticle volume fraction (ϕ), Richardson number (Ri), and Hartmann number (Ha) are analysed to show their impact on the heat exchanger model, as in Figs. 5–18. Additionally, the average Nusselt number (Nu_{av}) is used to evaluate the performance of the heat exchanger for water, mono-nano-

fluid, and hybrid nanofluid. For this analysis, standard values for the above-mentioned key parameters are $Ri = 1$, $Ha = 20$, $Pr = 6.8377$, $Re = 100$, $\phi_{Ag} = 0.02$ and $\phi_{MgO} = 0.02$.

4.1. Influence of Reynolds number

Figures 5–6 illustrate how the contours of streamlines and isotherms change with varying Reynolds numbers (Re).

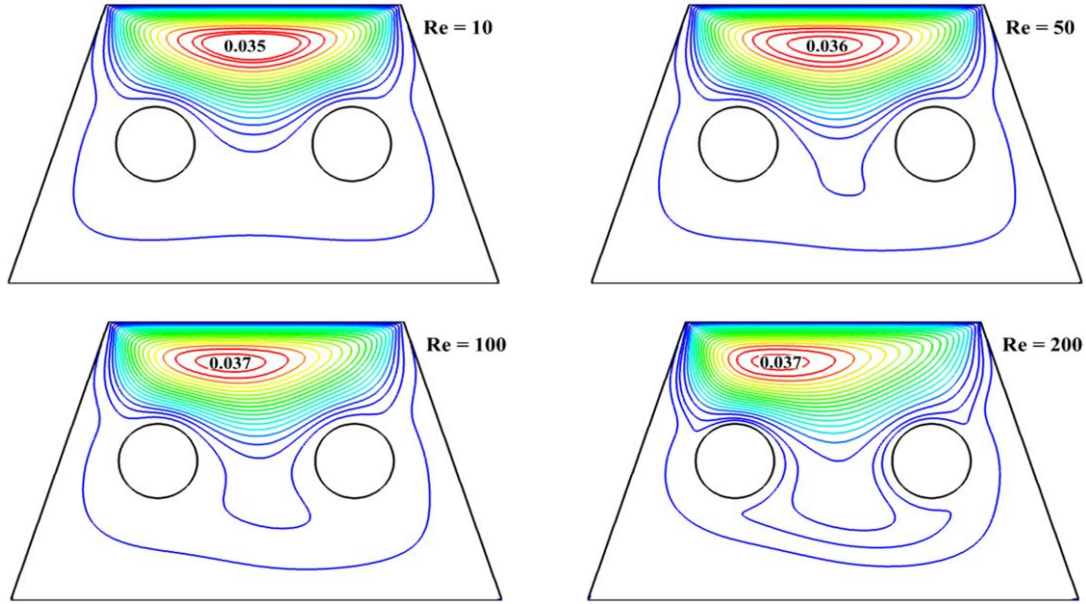


Fig. 5. Effect of Re on streamlines.

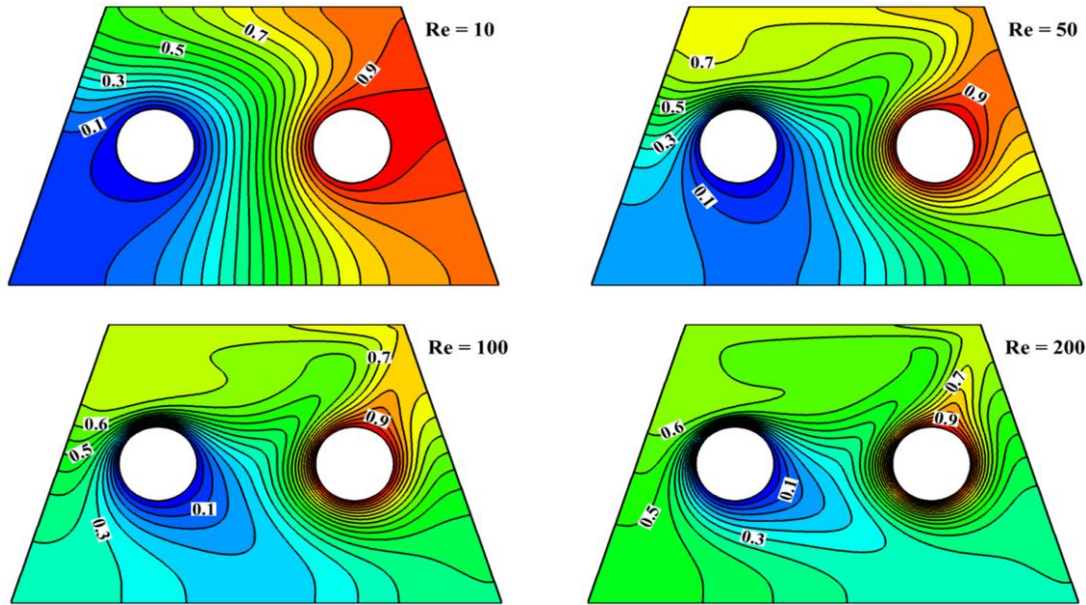


Fig. 6. Effect of Re on isotherms.

Figure 5 uses streamline contours to demonstrate how Re (10–200) may be controlled for a fluid velocity field when the other parameters are $Ri = 1$, $Ha = 20$, $Pr = 6.8377$ and $\phi_{Ag} = 0.02$, $\phi_{MgO} = 0.02$. By contrasting the effects of viscous forces and inertia, Re provides insight into the pattern of fluid motion under various circumstances. The streamline concentration and flow

circulation are uniformly circulating throughout the uppermost part of the wall when $Re = 10$. Physically, this is caused by the combined effects of shear force and buoyant force, both of which are controlled at low Re. After that, the streamline vortex and lid velocity both noticeably alter as the Re spikes from 50 to 100. Higher Re values physically result in increased fluid in-

ertia, which raises the flow circulation and magnitude with streamlines concentration. Furthermore, the strengthening of Re signifies the lid velocity of the trapezoidal cavity's top wall; fluid flow follows the lid velocity along the upper wall from the right to the left. The fluid's movement over the upper surface, from right to left, is growing as Re rises. This movement reaches its peak when $Re = 200$. This is because the velocity of the top lid increases with increasing Re , and as a result, forced convection becomes more prominent than sheer force and buoyant force. It is evident that the entire domain experiences an increase in fluid velocity along a line $(0.5, 0)$ to $(0.5, 0.6)$ as Re develops, which can be seen in Fig. 7.

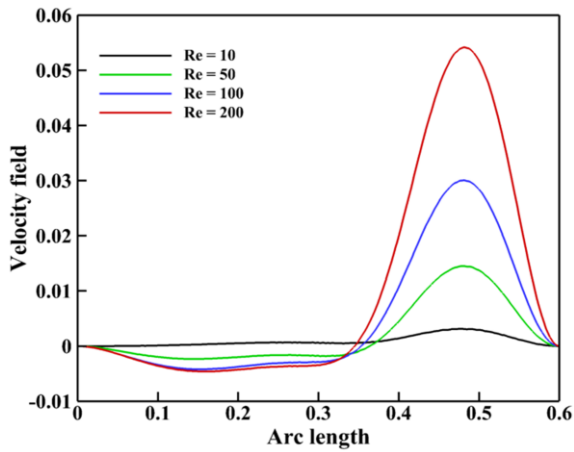


Fig. 7. Velocity profile for Re .

Moreover, Fig. 6 shows how Re affects the interior isotherms of the cavity. Natural convection is dominant in fluid flow when $Re = 10$. As a result, from the right hot cylinder to the left cooler one, the isotherm lines change uniformly. Force convection occurred, as evidenced by the rising value of Re (50), brought about by the upper wall's rising lid velocity. As can be seen, the red contour lines in the isothermal patterns indicate how fluid movement carries heat from the heated surface to the colder surface. Consequently, there is a 97.07% intensification of heat transmission rate compared to that before. It is apparent, therefore, that when Re rises, the isotherm contour lines distort.

4.2. Influence of Richardson number

The relative significance of forced convection caused by a lid vs. thermal natural convection forces is measured by the Richardson number (Ri) where $Ri = Gr/Re^2$. Here, Figs. 8 and 9 are applied to explain the effect of fluid flow and heat transmission using streamlines and isotherms at various Ri values when $Re = 100$, $Ha = 20$, $Pr = 6.8377$, and $\phi_{Ag} = 0.02$, $\phi_{MgO} = 0.02$. In this investigation, the parameter that has the largest influence on describing the mixed convection is Ri . It is evident from Fig. 8 that the fluid flow pattern is also affected by changes in Ri . Firstly, when $Ri = 0.1$, a forced convection is evident, with the streamlines moving more along the enclosure's upper wall and to the left. Furthermore, the bulk motion inside the cavity is circulating as a result of the top moving lid's motion, which is caused by the dominance of inertial forces over buoyancy forces. Furthermore, when the forces of buoyancy and inertia are balanced (for $Ri = 1$), examine the mixed convection mode.

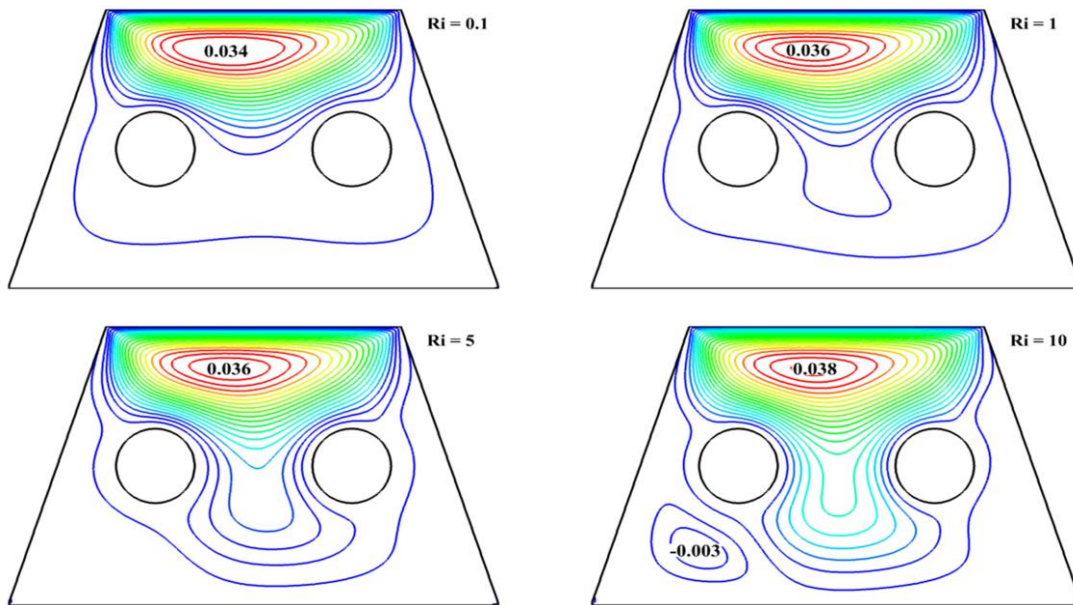


Fig. 8. Effect of Ri on streamlines.

Consequently, the top lid wall experiences a movement effect from the core vortex. Moreover, the streamlines' vortex widens as the value of Ri rises. According to the physical explanation, an escalation of Ri signifies the predominance of natural convection, which reduces shear stress. This created shear stress

provides hurdles for fluid motion. The fluid velocity decreases as the value of Ri rises from 1 to 5, indicating that natural convection is now more significant in this condition.

As a result, the streamlines are shifted somewhat towards the lid wall. A clockwise rotating vortex can be noticed in the cavi-

ty's lower left corner at the highest value of $Ri = 10$. Also, Fig. 9 illustrates how isotherms are impacted by Ri in the entire trapezoidal cavity. A significant reduction of heat transmission is designated by Nu_{av} of 10.549 at $Ri = 0.1$. The heat transfer rate rises 6.9% over the lower value prior to the expansion of Ri from 0.1 to 1. Moreover, for $Ri = 1$, the mixed convection occurs

in the entire cavity and at that time Nu_{av} is 11.277. An additional 15.46% enhancement in heat transmission may be seen by increasing Ri from 1 to 5. This analysis shows an increase in Nu_{av} with the enlargement of Ri values. And, there is an 11.16% increase in Nu_{av} as Ri increases from 5 to 10.

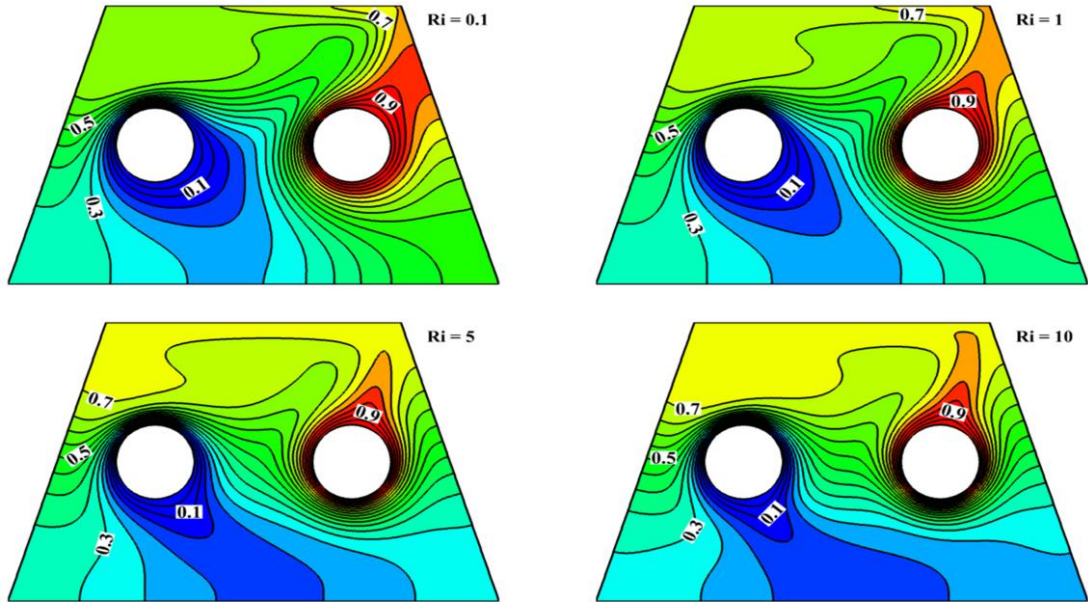


Fig. 9. Effect of Ri on isotherms.

4.3. Influence of Hartmann number

The streamline and isotherm contours in Figs. 10 and 11 demonstrate the consequences of Hartmann number (Ha) on fluid velocity and thermal transportation, keeping $Ri = 1$, $Re = 100$, $Pr = 6.8377$, $\phi_{Ag} = 0.02$ and $\phi_{MgO} = 0.02$. In reality, Ha illustrates how the magnetic force influenced this trapezoidal heat exchanger.

From Fig. 10, the streamline's concentrations are significantly denser without the existence of an external magnetic field ($Ha = 0$). It appears that the flow pattern is consistent across the hollows. However, as the magnitude of fluid vorticity inside the insertion becomes slow, the flow circulation becomes slightly blocked following the intensification of the magnetic field ($Ha = 20$).

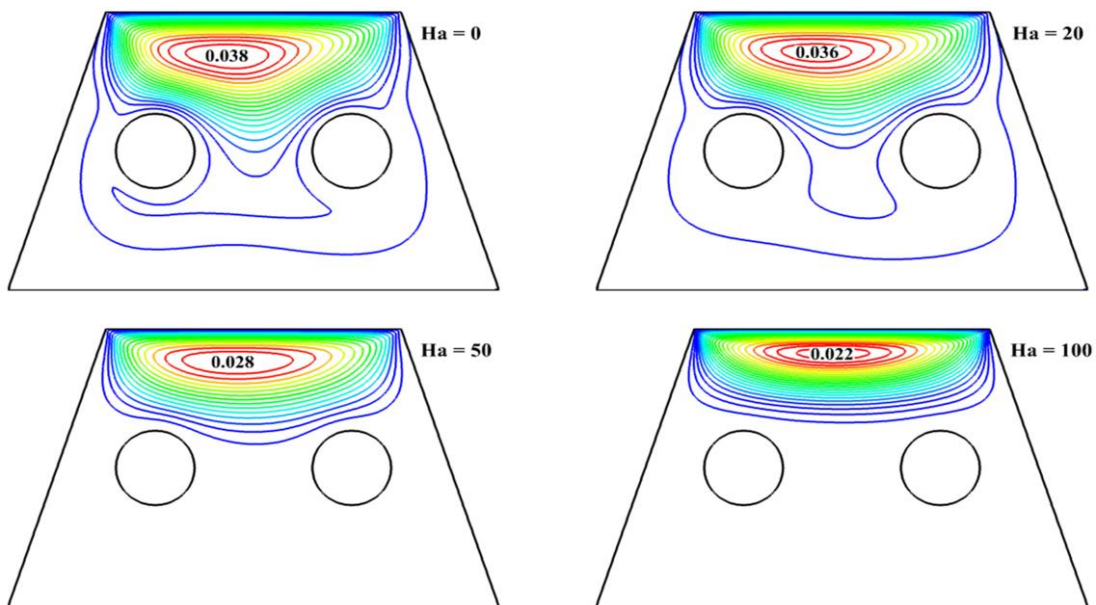


Fig. 10. Effect of Ha on streamlines.

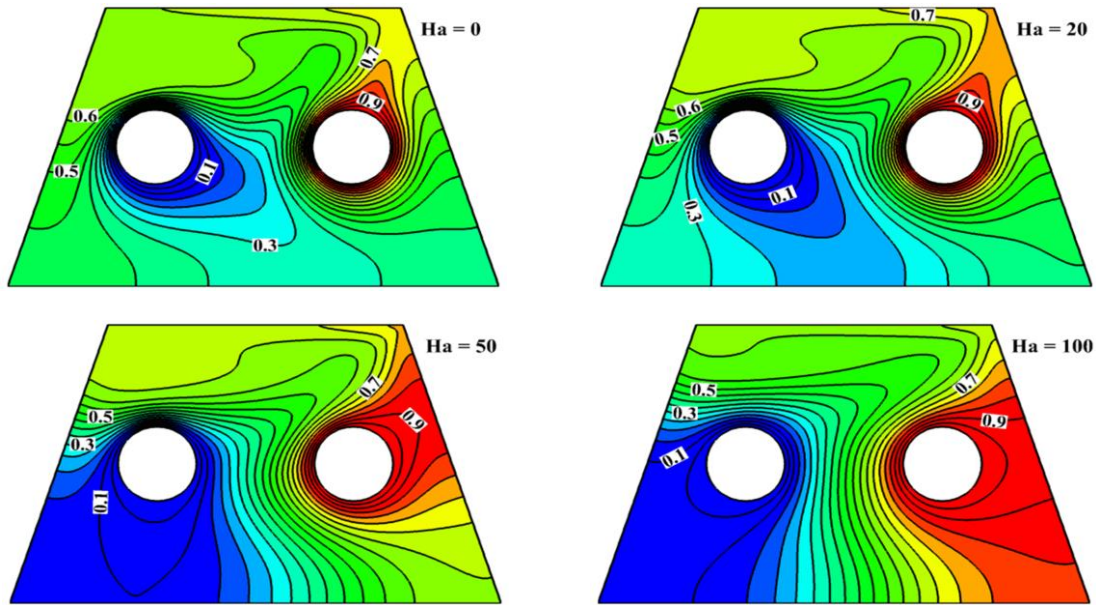
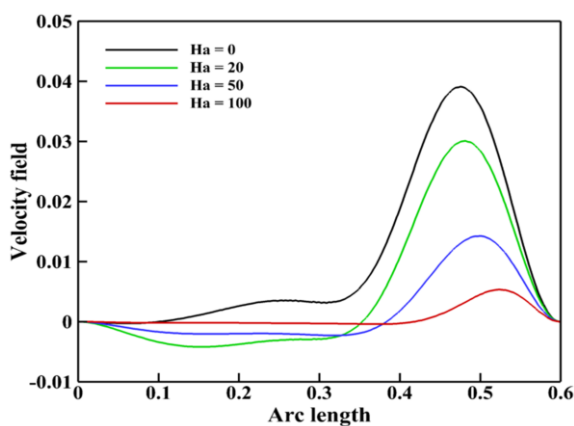


Fig. 11. Effect of Ha on isotherms.

It is evident that, in comparison to greater magnetic field effects ($Ha = 20, 50, 100$), lower magnetic fields ($Ha = 0$) boost flow behaviour. After increasing the value of Ha from 20 to 50 and from 50 to 100, there is a perceptible effect on the flow circulation. Because of the magnetic force effect, there is an active resistive force inside the enclosure known as the Lorentz force, which has the potential to slow down the mobility of the nanofluid. Precisely, the isotherm lines have a consistent shape in the vertical direction as the convection type of heat transmission progressively gives way to the conduction mode. This physical phenomenon causes the fluid flow to diminish more in the direction of the top lid surface. More importantly, the velocity profile falls with increasing Ha , as Fig. 12 illustrates the velocity profile of Ha . Furthermore, for greater Ha values ($Ha = 20, 50, 100$), the isothermal lines in Fig. 11 show that the variation of isothermal lines is rather mild (not significant).

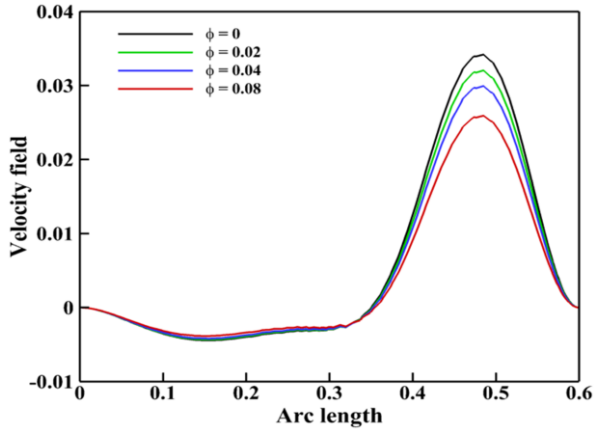
Fig. 12. Velocity profile for Ha .

When there is no active magnetic field, convective heat transfer (Nu_{av}) occurs at a high rate. This indicates that due to the addition of magnetic field, the fluid flow meets impedance.

When the value of Ha is enforced from 0 to 20, there is a 7.85% decrease in heat transfer. After the value reached 50, there was a significant decline in heat transfer, around 33.04%. Also, this decreased rate is 31.43% at the maximum magnetic impact value ($Ha = 100$). The investigation shows that the rate of heat transmission declines at 58.7% from the lowest active magnetic field to the greatest active magnetic field.

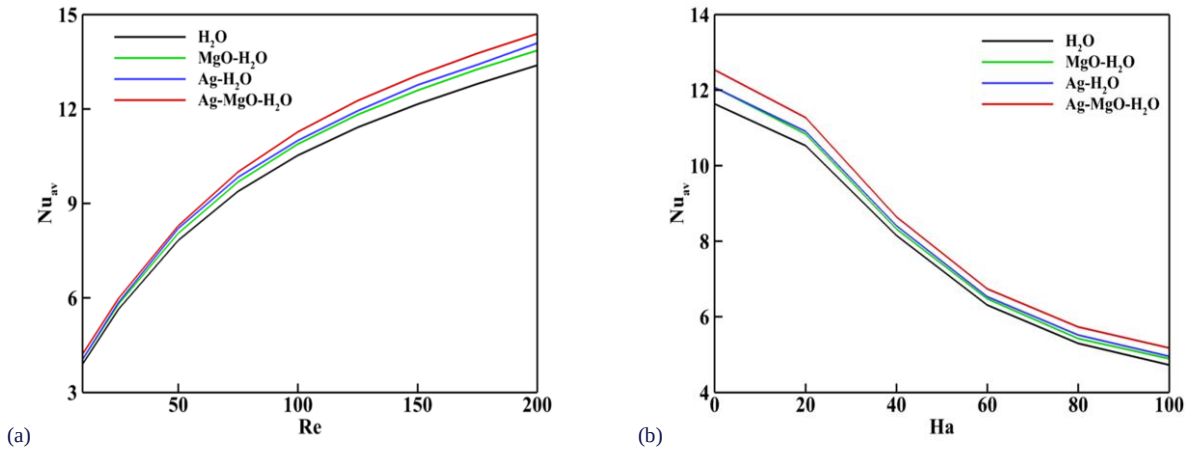
4.4. Importance of nanofluids

This section uses Figs. 13 and 14 to expound the nature of heat transfer and fluid motion on the present model for different magnitudes of the ϕ . Since the velocity profile is therefore controlled by the inherent characteristics of the base fluid, such as density and viscosity, the fluid is a pure base fluid devoid of nanoparticles ($\phi = 0$). The introduction of a modest percentage of nanoparticles ($\phi = 0.02$) really starts to affect the fluid dynamics and causes resistance to the particles' mobility in this fluid area. The fluid's inertia force is increased by the cavity's total mass of the fluid. The fluid velocity is significantly reduced as a result of the significant rise in viscosity, which increases flow resistance. An additional factor is that the solution becomes more viscous when more nanoparticles are added. From Fig. 13, it is evident that when ϕ rises, the fluid velocity along the line (0.5, 0) to (0.5, 0.6) decreases significantly. Conversely, the heat transfer rate (Nu_{av}) is rising due to intensification of ϕ , as seen in Fig. 14. The parameters of the base fluid alone dictate Nu_{av} , which is 10.648 in the absence of nanoparticles ($\phi = 0$), as shown by this line graph. Nanoparticle addition begins to marginally increase the heat transfer rate to 3% (10.968) at modest $\phi = 0.02$. Because of the nanoparticles' increased thermal conductivity, and therefore improved heat transmission within the fluid, there has been an improvement. By increasing the mass of the nanoparticles in the fluid up to 0.04, Nu_{av} is increased by 2.82%. On top of that, compared to $\phi = 0$, Nu_{av} is greatly increased by 11.4% when ϕ is raised by 8% ($\phi = 0.08$).

Fig. 13. Velocity profile of ϕ .

Superior heat transfer rates arise from the fluid's increased thermal conductivity, which is further enhanced by the high concentration of nanoparticles. Moreover, let's take a look at the significance of accumulation of solid nanoparticles in water more closely (Fig. 14). Here, utilising the properties of Re and Ha , Nu_{av} is used to clarify and illustrate the rate of heat transfer for several fluid types. For pure fluid (H_2O), $MgO-H_2O$ nano-

fluid, $Ag-H_2O$ nanofluid and $Ag-MgO-H_2O$ hybrid nanofluid, Fig. 14(a) shows that Nu_{av} grows monotonically with the increasing Re . Without the existence of any nanoparticles in water, Nu_{av} is equal to 10.53 by taking all other parameters in a standard form. When $\phi_{MgO} = 0.02$ and $\phi_{Ag} = 0$, adding MgO nanoparticles quickly improves Nu_{av} (10.885) relative to the pure base fluid, which becomes 3.37% greater than that of the base fluid. A similar tendency can be found while adding Ag nanoparticles ($\phi_{MgO} = 0$ and $\phi_{Ag} = 0.02$), which increases the Nu_{av} value relative to the pure base fluid by 3.5% (10.908). But both solid nanoparticles ($\phi_{MgO} = 0.02$ and $\phi_{Ag} = 0.02$) increased Nu_{av} significantly, which is 11.277. This time, the improvement rate is 7.09%. In addition, Fig. 14(b) illustrates how adding mono or hybrid nanoparticles enhances Nu_{av} for distinct values of Ha . The findings indicate that the $Ag-MgO-H_2O$ hybrid nanofluid has a larger Nu_{av} than $Ag-H_2O$ nanofluids, yet for all fluid combinations, the rate of Nu_{av} decreases as the Ha factor rises. This indicates that because of the hybrid nanofluid's exceptional thermal properties, the $Ag-MgO-H_2O$ hybrid nanofluid performs better in terms of heat transport than either $MgO-H_2O$ or $Ag-H_2O$ nanofluid. This is the primary reason for utilising hybrid nanofluid as opposed to the base fluid or nanofluid containing a particular nanoparticle.

Fig. 14. Deviation of Nu_{av} for base fluid and nanofluid using (a) Re and (b) Ha .

4.5. Response surface methodology

A well-known statistical analytic technique that explains how the included parameters (Re , ϕ and Ha) impact the response function (Nu_{av}) for this fluid model is called the response surface methodology, or RSM, as described by [57]. This is a useful method for simulating multivariate scenarios in which the input components simultaneously influence the responses that generate interest. Generally, the second-order model produces an acceptable approximation of the response, even in cases where alternative RSM models exist. As per certain accounts, the quadratic RSM model is:

$$y = s_0 + \sum_{i=1}^3 s_i x_i + \sum_{i=1}^3 s_{ij} x_i x_j + \sum_{i=1}^3 s_{ii} x_i^2. \quad (12)$$

Here, y is the response function, s_0 , s_i , s_{ij} and s_{ii} are the corresponding terms' coefficients. Here, Nu_{av} is considered the response function (y), while the significant parameters Re , Ha and

the ϕ function are input parameters. Finding the response function that most closely matches the interaction between independent parts is the primary goal. Here, a second-order RSM model based on central composite design (CCD) is used [58]. This model has 20 runs in total for 3 independent factors: 6 centres, 8 cubes, and 6 axial points per factor. The coded level for CCD-based RSM is shown in Table 4. In addition, Table 5 expresses the real values, response function values, and 20 runs of coded values. The results of this statistical investigation of this model utilising RSM are also displayed in Table 6. The greatest number of autonomous terms is what we refer to as degrees of freedom, or DOF. Moreover, a significant piece of evidence from this analysis is p-value, which reflects the probability that the null hypothesis is accurate for a definite statistical technique. When the p-value is low, it indicates that the null hypothesis is rejected, which indicates that the model is significant. Typically, this is less than 5%.

Table 4. Original and coded levels for CCD-based RSM.

Factors	Coded & original level		
	Lowest value (-1)	Mid value (0)	Lowest value (-1)
Re	10	105	200
ϕ	0	0.04	0.08
Ha	0	50	100

Table 5. Magnitude of the response function for distinct cases.

Run	Coded values			Real values			Response Nu_{av}
	Re	ϕ	Ha	Re	ϕ	Ha	
1	1	0	0	200	0.04	50	10.569
2	-1	-1	1	10	0	100	3.1986
3	1	1	-1	200	0.08	0	16.28
4	0	0	1	105	0.04	100	5.2279
5	0	1	0	105	0.08	50	8.1926
6	0	0	0	105	0.04	50	7.7234
7	1	-1	1	200	0	100	6.0078
8	-1	1	-1	10	0.08	0	4.8334
9	0	0	0	105	0.04	50	7.7234
10	0	0	0	105	0.04	0	12.722
11	1	1	1	200	0.08	100	6.8295
12	-1	0	0	10	0.04	50	3.8024
13	0	0	0	105	0.04	50	7.7234
14	0	-1	0	105	0	50	7.2798
15	1	-1	-1	200	0	0	14.227
16	0	0	0	105	0.04	50	7.7234
17	-1	1	1	10	0.08	100	3.9276
18	-1	0	1	10	0.04	100	7.7234
19	-1	-1	-1	10	0	0	4.2158
20	0	0	1	105	0.04	100	7.7234

Table 6. Analysis of variance for Nu_{av} .

Source	DOF	F-value	p-value	Comment
Model	9	38.18	< 0.0001	Significant
Re	1	210.44	< 0.0001	
ϕ	1	69.67	0.019	
Ha	1	9.65	< 0.0001	
Re^2	1	7.59	0.0273	
ϕ^2	1	0.0059	0.3358	
Ha^2	1	0.0317	0.0302	
$Re \cdot \phi$	1	37.64	0.3743	
$Re \cdot Ha$	1	0.3335	< 0.0001	
$\phi \cdot Ha$	1	0.5012	0.511	
Lack-of-Fit	5	-	-	Insignificant

Here, $R^2 = 98.53\%$, Adjusted $R^2 = 97.21\%$

The model's statistical analysis and the procedures for testing indicate that the R^2 value (97.85%) for Nu_{av} is superior, indicating that this model is apposite for calculating the response function Nu_{av} , despite having a lower adjusted R^2 value (95.92%). Another crucial statistic that must be exceptionally small for a model to be deemed suitable is lack-of-fit. In order

to investigate the relation between Nu_{av} and the factors Re, Ha and ϕ , a general RSM model is developed as indicated below:

$$y = s_0 + s_1 Re + s_2 \phi + s_3 Ha + s_{12} Re \cdot \phi + s_{13} Re \cdot Ha + s_{23} \phi \cdot Ha + s_{11} Re^2 + s_{22} \phi^2 + s_{33} Ha^2, \quad (13)$$

where $s_0, s_1, s_2, s_3, s_{12}, s_{13}, s_{23}, s_{11}, s_{22}$ and s_{33} are taken as the coefficients of the regression line concerning Re, Ha and ϕ . Furthermore, Nu_{av} 's anticipated coefficients of Eq. (13) are provided in Table 7 and are calculated as coded units.

Table 7. Coefficients of the regression equation with corresponding p-values based on RSM.

Coefficients	S_0	S_1	S_2	S_3	S_{11}
Values	7.87	3.39	0.5134	-2.71	-0.905
p-values	-	<0.0001	0.019	<0.0001	0.0273
Coefficients	S_{22}	S_{33}	S_{12}	S_{13}	S_{23}
Values	-0.3545	0.8842	0.1910	-1.97	-0.14
p-values	0.3358	0.0302	0.3743	< 0.0001	0.511

A suitable regression equation has only been developed utilising the significant model terms with lower p-values, owing to their importance. On the contrary, the terms that are not essential have been neglected (bold emphasised). Therefore, the terms ϕ^2 , $Re \cdot \phi$ and $\phi \cdot Ha$ have no influence whatsoever on Nu_{av} 's regression equation (13). Consequently, the subsequent mathematical correlation can be applied to relate Nu_{av} with the parameters Re, ϕ and Ha:

$$Nu_{av} = 7.87 + 3.39Re + 0.5134\phi - 2.71Ha - 0.905Re^2 + 0.8842Ha^2 - 1.97Re \cdot Ha. \quad (14)$$

4.6. Response surface analysis

With a view to analysing the influence of independent components on the response function, in this segment, Figs. 15–17 provide 2D and 3D contour plots regarding the response surface developed using RSM (Nu_{av}). Figure 15(a) demonstrates Nu_{av} 's reaction to Re and ϕ . This 2D contour map clearly illustrates that the response function rises with both Re and ϕ . For example, the rate of heat transmission proliferations increases by about 103.1% when the value of Re goes up from 10 (coded value 1) to 105 (coded value 0). At this point, Nu_{av} grows by around 36.84% when the magnitude of Re is raised from 105 (coded value 0) to 200 (coded value 1). At the greatest magnitude of Re and ϕ (coded value 1), the fluctuating rate of Nu_{av} is maximal, as seen in the 2D contours in Fig. 15(a). Moreover, Fig. 15(b) displays a 3D surface plot of the effects of Re and ϕ on Nu_{av} . Moreover, several 2D and 3D graphical representations are also shown in Fig. 16(a) to show how Re and Ha influence Nu_{av} in this hybrid nanofluid model. Furthermore, although ϕ remains constant, a rise in Ha diminishes the rate of Nu_{av} . Additionally, Fig. 16(b) displays a 3D surface depiction of Re and Ha's effects on Nu_{av} . Likewise, Fig. 17 illustrates Nu_{av} 's impact from Ha and ϕ . Here, the altering rate of Nu_{av} is advanced by increasing the size of ϕ while decreasing the magnetic field. Nu_{av} 's fluctuating rate is lower than it was in the prior two cases, though.

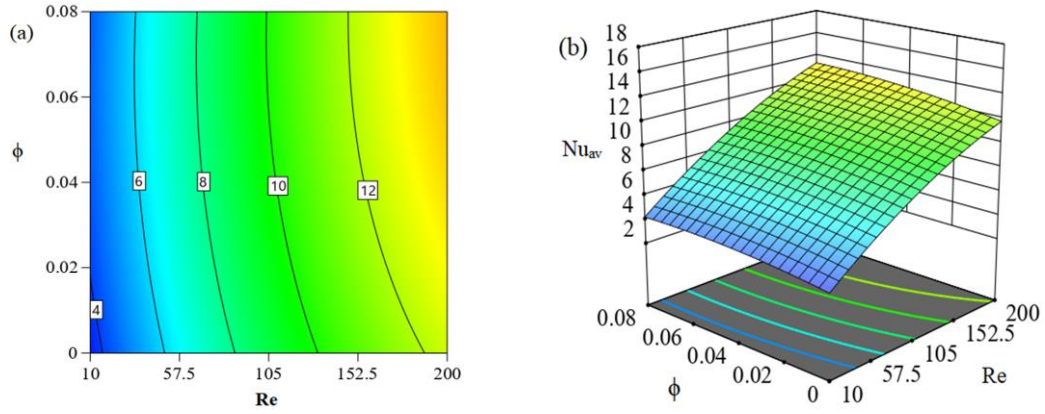


Fig. 15. Effect of Re and ϕ on Nu_{av} : (a) 2D sight; (b) 3D sight.

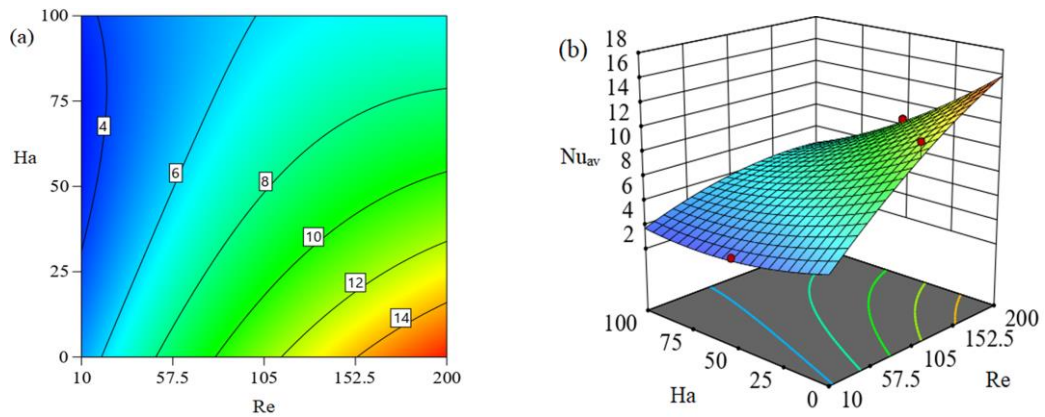


Fig. 16. Effect on Nu_{av} for Re and Ha: (a) 2D sight; (b) 3D sight.

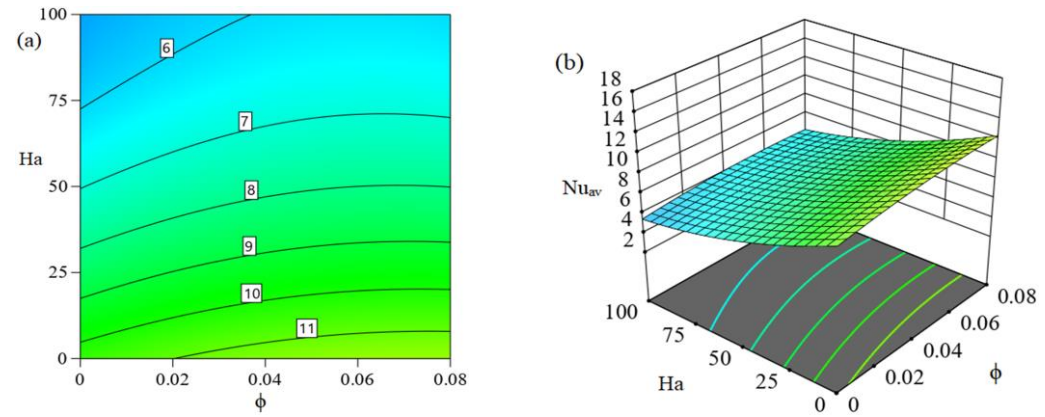


Fig. 17. Effect on Nu_{av} for ϕ and Ha: (a) 2D sight; (b) 3D sight.

4.7. Sensitivity analysis of parameters

Sensitivity analysis, a process for the determination of how input uncertainty affects the model's response, is crucial to numerical simulation. By doing a "sensitivity analysis", it is possible to determine the extent to which the model's parameter distresses the generated variables [59]. Using the results of the sensitivity analysis, the important variables can be ranked in order of influence to determine which parameter is the most beneficial. By

differentiating (partially) the response function with respect to autonomous factors (Re, ϕ and Ha), one can analytically evaluate the sensitivity of the output. Consequently, the following computation is made for Nu_{av} with respect to the input parameters as in Eq. (14):

$$\frac{\partial Nu_{av}}{\partial Re} = 3.39 - 1.81Re - 1.97Ha, \quad (15)$$

$$\frac{\partial \text{Nu}_{av}}{\partial \phi} = 0.5134, \quad (16)$$

$$\frac{\partial \text{Nu}_{av}}{\partial \text{Ha}} = -2.71 + 1.7684\text{Ha} - 1.97\text{Re}. \quad (17)$$

Eqs. (15) to (17) can now be used to calculate the rate of sensitivity of Nu_{av} to Re , ϕ and Ha . Table 8 presents the findings.

Table 8. Sensitivity analysis for Nu_{av} .

Re	φ	Ha	$\frac{\partial \text{Nu}_{av}}{\partial \text{Re}}$	$\frac{\partial \text{Nu}_{av}}{\partial \phi}$	$\frac{\partial \text{Nu}_{av}}{\partial \text{Ha}}$
-1	0	-1	7.17	0.5134	-0.226
		0	5.2	0.5134	-0.74
0		-1	5.36	0.5134	-4.478
		0	3.39	0.5134	-2.71
1		-1	3.55	0.5134	-6.448
		0	1.58	0.5134	-4.68
-1	1	-1	7.17	0.5134	-0.226
		0	5.2	0.5134	-0.74
0		-1	5.36	0.5134	-4.478
		0	3.39	0.5134	-2.71
1		-1	3.55	0.5134	-6.448
		0	1.58	0.5134	-4.68

The values obtained by applying this model are as follows: Ha at -1 and 0 (0 and 50), ϕ at 0 and 1 (0.04 and 0.08), and Re at -1, 0 and 1 (10, 105 and 200). Furthermore, it's critical to remember that a positive sensitivity indicates that the input factors are intensifying this response. This demonstrates how the Re and ϕ stimulate Nu_{av} in a positive way. Conversely, a negative sensitivity signifies the exact opposite trend, where raising input factors causes the response to drop. This signal suggests that the input factor Ha has a negative effect on Nu_{av} . Moreover, Fig. 18 illustrates negative sensitivity with respect to Nu_{av} by the flipped bar and positive sensitivity with respect to Nu_{av} by the upright bar. The sensitivity level is also displayed along the entire length of the bar.

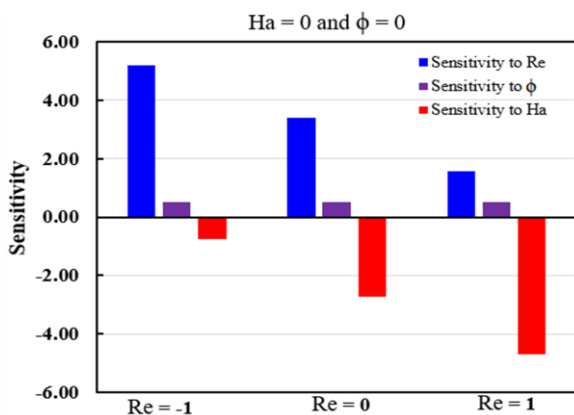


Fig. 18. Rate of sensitivity of Nu_{av} for independent factors at $\text{Ha} = 0$ and $\phi = 0$.

6. Conclusions

This study focuses on the interaction between heat transport processes and the effects of magnetic fields, examining the behaviour of mixed convective heat fluxes in a confined cavity using a hybrid (Ag-MgO- H_2O) nanofluid. A numerical solution is developed using the Galerkin finite element method to capture the complex behaviour. The key findings are outlined below:

- Sensitivity analysis of the response function is performed using RSM to gain a comprehensive understanding of the heat transportation mechanism.
- The impact of the hybrid nanofluid on streamlines, isotherms, and Nu_{av} is investigated.
- Surface plots in two and three dimensions provide a clear visual representation of the heat transmission process for significant components involved.
- The hybrid nanofluid (Ag-MgO- H_2O) shows superior thermal properties, resulting in better heat transfer performance than the base fluid or mono nanofluid, with heat transfer improvements of 3.37% and 7.09%, respectively.
- The Hartmann number (Ha) has a negative impact on the velocity profile.
- Sensitivity analysis results show that Reynolds number (Re) and nanoparticle volume fraction (ϕ) positively influence heat transfer from hot surfaces, while Ha has a negative impact on it.

This study can be extended in the future for non-Newtonian fluids and temperature-dependent thermophysical properties with a three-dimensional approach.

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Numerical analysis of the Soret and Dufour effects on boundary layer flow past a curved stretching surface in porous media with thermal slip and variable magnetic field

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Abstract

This study presents a novel investigation into the boundary layer flow, heat, and mass transfer of a Newtonian, incompressible and electrically conducting viscous fluid flow past a permeable curved stretching surface embedded in porous media with Soret and Dufour effects subject to varying magnetic fields and thermal slip. The research uniquely combines these complex phenomena, addressing a significant gap in the literature. This configuration has potential applications in electronic cooling systems, polymer manufacturing and compact heat exchangers in various industries. The main objectives are to understand how various parameters influence fluid flow, heat and mass transfer behaviour to optimize the design for enhanced performance. To attain the numerical results the governing equations are transformed into a set of nonlinear ordinary differential equations, which are then solved numerically by using MATLAB's built-in solver bvp4c. The parameters of engineering interest are tabulated and validated with previous results. The outcomes of the study suggest that the presence of a larger magnetic parameter, Dufour number, Joule parameter, thermal slip and curvature parameter contribute to reducing the cooling rate of the system by 2.73%, 10.53%, 12.39%, 9.29% and 6.28%, respectively. Also, the curvature of the surface can be enlarged from 5 to 10 to obtain 6.87% reduction in surface drag force.

Keywords: Curved stretching surface; Variable magnetic field; Soret; Dufour; Porous media

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1. Introduction

In boundary layer flow, heat and mass transfer across a curved stretching surface is an important topic among researchers due to its numerous applications in polymer production, metallurgy and chemical engineering. Designing and creating new machinery and equipment with a high rate of heating or cooling draws the curiosity of many engineers and scientists. In industries, the intended final products are mostly determined by the rate of cooling. Additionally, a magnetic field has the potential to change heat transfer properties by rearranging fluid concentration. Furthermore, the direction and intensity of the applied magnetic field can be used to

alter the fluid's behaviour. In industry, pure crystal semiconductor growth may be regulated by varying the magnetic field. Variable magnetic fields have been used to cure peptic ulcers, as well as for medical diagnostic and therapeutic purposes. Thus, studying the interactions between flow, heat, and mass transfer across curved stretching surfaces provides insights into complex physical phenomena and facilitates the development of innovative technologies and solutions across a wide range of industries.

Sajid et al. [1] were the first to explore the viscous flow caused by a curved stretching surface, discovering that dimensionless curvature increases boundary layer thickness while decreasing skin friction coefficients. Abbas et al. [2] investigated viscous fluid flow

Nomenclature

a – stretching rate
 B – strength of magnetic field, T
 B_0 – constant magnetic field, T
 C – species concentration, kg m⁻³
 c_p – specific heat at constant pressure, J kg⁻¹ K⁻¹
 C_{fs} – local skin friction coefficient
 C_∞ – ambient fluid concentration, kg m⁻³
 C_w – surface concentration, kg m⁻³
 c_s – concentration susceptibility
 Da – Darcy parameter, $= \frac{\nu}{k_p a}$
 D_m – mass diffusion coefficient, m² s⁻¹
 Du – Dufour number, $= \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}$
 Ec – Eckert number, $= \frac{u_w^2}{c_p (T_w - T_\infty)}$
 f – dimensionless stream function
 J – Joule heating parameter, $= M^2 Pr Ec$
 K – curvature parameter, $= R \sqrt{\frac{a}{\nu}}$
 k – thermal conductivity, W m⁻¹ K⁻¹
 k_p – coefficient of permeability
 k_T – ratio of thermal diffusion
 L – thermal slip parameter, $= L_1 \sqrt{\frac{a}{\nu}}$
 L_1 – thermal slip constant
 M – magnetic parameter, $\sqrt{\frac{\sigma B_0^2}{\rho a}}$

Nu_s – local Nusselt number
 P – dimensionless pressure
 p – pressure, N m⁻²
 Pr – Prandtl number, $= \frac{\mu c_p}{k}$
 R – radius of curved surface, m
 Re_s – local Reynolds number, $= \frac{u_w s}{\nu}$
 r, s – curvilinear coordinates
 Sc – Schmidt number, $= \frac{\nu}{D_m}$
 Sh_s – local Sherwood number
 Sr – Soret number, $= \frac{D_m k_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}$
 T – fluid temperature, K
 T_∞ – ambient fluid temperature, K
 T_w – surface temperature, K
 T_m – mean fluid temperature, K
 u, v – velocity components in r - and s - directions, m s⁻¹
 $(\cdot)', (\cdot)'', (\cdot)'''$ – differentiation with respect to variable η

Greek symbols

ν – kinematic viscosity of the fluid, m² s⁻¹
 η – similarity variable
 μ – dynamic viscosity
 σ – electrical conductivity of the fluid, S m⁻¹
 ρ – fluid density, kg m⁻³
 θ – dimensionless fluid temperature
 ϕ – dimensionless fluid concentration
 λ – suction/injection parameter

and heat transfer in the presence of a constant applied magnetic field over a curved stretched surface. Rosca and Pop [3] used mass suction to study the problem of unstable viscous flow across a curved stretching or shrinking surface and found that, in the case of a curved stretching surface, the pressure inside the boundary layer cannot be disregarded. Sanni et al. [4,5] studied the transfer of heat from a non-linear curved stretched surface in the absence and presence of a varying magnetic field. Hayat et al. [6] investigated the flow of viscous fluid across a nonlinear curved stretching surface under convective heat and mass circumstances. Saranya et al. [7] analysed bio-convective nanofluid flow over a curved surface employing the shifted Legendre collocation method, concluding that the concentration of motile bacteria significantly decreases with an increase in the curvature parameter. Abbas and Shatanawi [8] employed the bvp4c numerical technique to examine micropolar fluid on a curved surface. Majeed et al. [9] observed that the presence of a magnetic field reduces the heat transfer rate and shear stress in the analysis of bio-convective nanoparticle fluid flow across a stretched surface. Abbas et al. [10] examined cilia flow in a curved channel subjected to a radial magnetic field, revealing a substantial influence on the thermal characteristics of the system. Mahesha et al. [11] investigated nanofluid behaviour within an annular space formed by two concentric cylinders, indicating that quadratic resistance significantly affects both flow dynamics and heat transfer characteristics.

A medium with holes or blank spaces is said to be porous. Some examples of materials with pores include sand, wood, boulders, and soil. Porosity and permeability are two of a porous material's most crucial characteristics. Porosity is the quality of being porous, whereas permeability measures how easily gases, liquids, or certain compounds may transport through a material. Understanding porous media may help explain a lot of elements of transport processes. The porous material can regulate the heating or cooling process. Because of their importance in geophysics, petroleum technology, cooling of nuclear reactors, biochemical processes and other fields, transport mechanisms in porous media have received a lot of attention in recent years. Ahmed et al. [12] discussed boundary layer flow over a curved surface embedded in a porous medium and it was observed that increasing values of the porosity parameter decreases the fluid velocity. Using the homotopy analysis method, Riaz et al. [13] examined the motion of nanoparticles in a non-Darcy porous curved channel. Adeyemo et al. [14] emphasized understanding the impact of changing viscosity on nonlinear convective fluid flow in a porous medium. Sonam and Yadav [15] examined steady, laminar Casson fluid flow generated by a semi-infinite vertical plate, incorporating the effects of the Darcy-Forchheimer model and employing the bvp4c numerical method. Similarly, Jagadha et al. [16] developed a model for the steady flow of Jeffrey nanofluid over a nonlinearly stretching flat sheet of variable thickness, also governed by the Darcy-Forchheimer relation, using the Runge-Kutta numerical approach.

Due to its numerous applications, including the solidification of binary alloys, oil reservoirs, isotope separation, chemical reactors, and mixtures of gases, Soret and Dufour effects are significant fields of study in heat and mass transfer processes. Energy flux due to concentration gradient and mass flux due to temperature gradient in a medium are known as the Dufour and Soret effects, respectively. They are important mechanisms in transport phenomena and are often ignored because of their low order of magnitude. Imtiaz et al. [17] identified the impacts of Soret and Dufour in the flow of viscous fluid by a curved stretching surface. Magnetohydrodynamic (MHD) boundary layer mixed convection flow past an exponentially extending permeable sheet in porous media was presented by Konwar et al. [18] together with the Soret and Dufour effects. In another numerical investigation conducted by Jamir et al. [19], the significance of Soret and Dufour effects on the fluid flow system about a stretching sheet was presented. Soret and Dufour effects were found to impact the concentration and temperature profile for a system involving ferromagnetic hybrid nanofluid flow over a permeable surface [20].

In engineering and industries, the study of viscous dissipation is a crucial factor in understanding the energy balance within a system. The dissipation of mechanical energy into heat through viscous effects can have implications for the efficiency and heat generation in various processes, such as fluid transport in pipelines, blood flow in arteries, or any situation involving the flow of viscous fluids. Heat and mass transfer through a porous medium was discussed by Haile and Shankar [21] on MHD flow of nanofluids with the effects of viscous dissipation, thermal radiation and chemical reaction. A study conducted by Oladapo et al. [22], who reported that the presence of slip effect significantly influences the flow and heat transfer of the system, referred to the case of a non-Newtonian fluid for enhanced heat transfer devices. Sademaki [23] elucidated the impact of viscous dissipation on sustained heat transfer in chemically reactive MHD nanofluid flow, incorporating mass and heat diffusion characteristics within a vertical cone filled with a saturated porous medium. Another study indicated that the Soret and Dufour effects impact the heat transfer rate in the flow of viscoelastic fluid caused by a stretching surface with viscous dissipation [24].

In the above literature survey, it is evident that MHD flow over curved surfaces is of great interest to researchers and is applicable in various fields of engineering. Taking into account the significance of the flow on curved surface, the present problem aims to provide insight into the problem of boundary layer flow, heat, and mass transfer over a permeable curved stretching sheet in porous media with Soret and Dufour effects subject to variable magnetic fields and thermal slip, which has not been accounted for in any of the literature cited above. The study aims to analyse the outcomes as well as find answers to how these parameters may possibly contribute to improving or reducing the boundary layers of the system under consideration. The study also looks into the parameters of engineering interest, which will significantly contribute towards finding new innovations to enhance cooling rates in curved devices and equipment,

such as cooling electronics, heat exchanger design, polymer manufacturing and metal forming.

2. Mathematical formulation

This issue pertains to examining the flow of a two-dimensional, steady, Newtonian, incompressible, and electrically conducting viscous fluid flow past a permeable curved stretching sheet curled

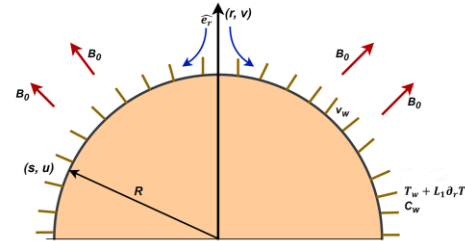


Fig. 1. Geometric configuration of the flow.

in a circle of radius R and the sheet is being stretched in s -direction with a velocity $u = as$, where $a > 0$ (see Fig. 1). Also, a variable magnetic field of strength $B(r) = RB_0(R + r)^{-1}\hat{e}_r$ is applied in the radial direction of the curved sheet and takes a constant value, when $R \rightarrow \infty$, where $\hat{e}_r$ is the unit vector in the radial direction. Assuming the magnetic Reynolds number is negligible, the influence of the induced magnetic field can be disregarded. Additionally, the work presents thermal slip conditions at the surface-fluid interface. This enhancement enriches our analysis and enables a more detailed examination of fluid flow.

The present analysis includes a thorough array of assumptions, such as:

1. Integration of porous media effects, recognizing the impact of permeability on flow dynamics.
2. Assessment of suction-injection effects, essential for evaluating fluid transport phenomena.
3. Incorporation of viscous dissipation, Soret and Dufour effects to improve heat and mass transfer, particularly relevant to energy systems, augmentation of heat transfer, polymer processing, and advanced thermal management technologies.
4. Utilization of boundary layer approximation methods to analyse fluid flow near walls.
5. Consideration of thermal slip effect to reflect the realistic scenarios.

The flow in curvilinear coordinates is modelled by the governing equations, which are expressed as [5,7]:

$$\partial_r[(R + r)v] + R\partial_s u = 0, \quad (1)$$

$$\frac{u^2}{R+r} = \frac{1}{\rho}\partial_r p, \quad (2)$$

$$v\partial_r u + \left(\frac{Ru}{R+r}\right)\partial_s u + \frac{uv}{R+r} = -\frac{1}{\rho}\left(\frac{R}{R+r}\right)\partial_s p + v\left[\partial_r^2 u + \left(\frac{1}{R+r}\right)\partial_r u - \frac{u}{(R+r)^2}\right] - \frac{\sigma B_0^2 R^2 u}{\rho(R+r)^2} - \frac{vu}{k_p}, \quad (3)$$

$$\left[v \partial_r T + \left(\frac{Ru}{R+r} \right) \partial_s T \right] = \frac{k}{\rho c_p} \left[\partial_r^2 T + \left(\frac{1}{R+r} \right) \partial_r T \right] + \frac{\mu}{\rho c_p} \left(\partial_r u - \frac{u}{R+r} \right)^2 + \frac{D_m k T}{c_s c_p} \left[\partial_r^2 C + \left(\frac{1}{R+r} \right) \partial_r C \right] + \frac{\sigma B_0^2 R^2 u^2}{\rho c_p (R+r)^2}, \quad (4)$$

$$v \partial_r C + \left(\frac{Ru}{R+r} \right) \partial_s C = D_m \left(\partial_r^2 C + \frac{1}{R+r} \partial_r C \right) + \frac{D_m k T}{T_m} \left(\partial_r^2 T + \frac{1}{R+r} \partial_r T \right). \quad (5)$$

The appropriate boundary conditions are [8]:

$$u = u_w = as, v = -v_w, T = T_w + L_1 \partial_r T, C = C_w \quad \text{at } r = 0, \quad (6)$$

$$u \rightarrow 0, \partial_r u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } r \rightarrow \infty, \quad (7)$$

where $v_w = \frac{Rv_0}{R+r}$ and v_0 is the initial value and variable velocity at the wall, respectively.

The following similarity transformations [1]:

$$u = asf'(\eta), v = \frac{-R}{R+r} \sqrt{av} f(\eta), \eta = \sqrt{a/v} r, \\ p = \rho a^2 s^2 P(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \quad (8)$$

satisfy Eq. (1), and Eqs. (2) and Eq. (3) transform to:

$$P' = \frac{f'^2}{\eta+K}, \quad (9)$$

$$\frac{2K}{\eta+K} P = f'''' + \frac{f''}{\eta+K} - \frac{f'}{(\eta+K)^2} - \frac{Kf'^2}{\eta+K} + \frac{Kff''}{\eta+K} + \frac{Kff'}{(\eta+K)^2} - \frac{M^2 K^2 f'}{(\eta+K)^2} - Da f'. \quad (10)$$

Eliminating pressure P from Eqs. (9) and (10), the following dimensionless momentum equation is obtained:

$$f''v + \frac{2f'''' + K(ff'''' - f'f''')}{\eta+K} - \frac{f'' - K(ff'' - f'^2)}{(\eta+K)^2} + \frac{f' - Kff'}{(\eta+K)^3} + \frac{M^2 K^2}{(\eta+K)^2} \left[f'' - \frac{f'}{(\eta+K)} \right] - Da \left(f'' + \frac{f'}{\eta+K} \right) = 0. \quad (11)$$

Prime in this case represents derivatives with regard to the similarity variable η .

With the transformations (8), the energy and species concentration equations in dimensionless form become:

$$\theta'' + \frac{\theta'}{\eta+K} + \frac{PrK}{\eta+K} f \theta' + PrEc \left(f'' - \frac{f'}{\eta+K} \right)^2 + PrDu \left(\phi'' + \frac{\phi'}{\eta+K} \right) + \frac{JK^2 f'^2}{(\eta+K)^2} = 0, \quad (12)$$

$$\phi'' + \frac{\phi'}{\eta+K} + \frac{ScK}{\eta+K} f \phi' + ScSr \left(\theta'' + \frac{\theta'}{\eta+K} \right) = 0 \quad (13)$$

together with transformed boundary conditions:

$$f(\eta) = \lambda, f'(\eta) = 1, \theta(\eta) = 1 + L\theta'(\eta), \phi(\eta) = 1 \text{ at } \eta = 0, \quad (14)$$

$$f'(\eta) \rightarrow 0, f''(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty, \quad (15)$$

were suction is represented by $\lambda = \frac{v_0}{\sqrt{av}} > 0$ and injection by $\lambda < 0$ [18].

The skin friction coefficient $C_{fs} = \tau_{rs}/(\rho u_w^2)$, Nusselt number $Nu_s = sq_w/\{k(T_w - T_\infty)\}$, and Sherwood number $Sh_s = sq_m/\{D_m(C_w - C_\infty)\}$ are engineering characteristics that contribute significantly to understanding heat and mass transfer.

Here, $\tau_{rs} = \mu \left(\partial_r u - \frac{u}{R+r} \right)_{r=0}$ is the wall shear stress, $q_w = -k(\partial_r T)_{r=0}$ is the wall heat flux and $q_m = -D_m(\partial_r C)_{r=0}$ represents the wall mass flux.

Upon using Eqs. (8), the following are determined:

– the skin friction coefficient:

$$Re_s^{-\frac{1}{2}} C_{fs} = f''(0) - \frac{f'(0)}{K}, \quad (16)$$

– the Nusselt number:

$$Re_s^{-\frac{1}{2}} Nu_s = -\theta'(0), \quad (17)$$

and the Sherwood number:

$$Re_s^{-\frac{1}{2}} Sh_s = -\phi'(0). \quad (18)$$

3. Numerical procedure

The bvp4c method (a finite difference method that provides C^1 -continuous solution) [25] is utilized to derive the computational solution of Eqs. (11)–(13), subject to the limitations defined by Eqs. (14) and (15). Current flow difficulties include dimensionless ordinary differential equations that exhibit a particularly high nonlinear nature. The principal research objective is to precisely tackle challenges pertaining to non-linear flow phenomena. The bvp4c numerical approach within the MATLAB software is the most suitable for acquiring the quantitative solution of the present model [25]. It is important to supply three distinct categories of data: the first-order equations to be solved, the corresponding boundary conditions, and the first guess employed to ascertain the solution to the problem. The integration interval is defined from zero to five and from zero to ten, with a relative tolerance set at 10^{-7} . The mesh is automatically changed to enhance accuracy based on this tolerance.

We reformulate the provided equations for the bvp4c code as a system of first-order ordinary differential equations (ODEs):

$$f = y_1, f' = y_2, f'' = y_3, f''' = y_4, \quad (19) \\ \theta = y_5, \theta' = y_6, \phi = y_7, \phi' = y_8.$$

By substituting Eqs. (19) into equations Eqs. (11)–(13), the resultant system of 1st-order ODE can be derived:

$$y_1' = y_2, \quad (20)$$

$$y_2' = y_3, \quad (21)$$

$$y_3' = y_4, \quad (22)$$

$$y_4' = -\frac{2y_4 + K(y_1 y_4 - y_2 y_3)}{\eta+K} + \frac{y_3 - K(y_1 y_3 - y_2^2)}{(\eta+K)^2} - \frac{y_2 - K y_1 y_2}{(\eta+K)^3} + \frac{M^2 K^2}{(\eta+K)^2} \left(y_3 - \frac{y_2}{\eta+K} \right) + Da \left(y_3 + \frac{y_2}{\eta+K} \right), \quad (23)$$

$$y_5' = y_6, \quad (24)$$

$$y_6' = \frac{1}{1 - \text{PrDuSrSc}} \left[-\frac{y_6}{\eta + K} - \frac{\text{PrKy}_1y_6}{(\eta + K)} - \text{PrEc} \left(y_3 - \frac{y_2}{(\eta + K)} \right)^2 + \right. \\ \left. - \frac{JK^2y_2^2}{(\eta + K)^2} - \text{PrDu} \left(-\frac{K\text{Scy}_1y_8}{\eta + K} - \frac{\text{ScSry}_6}{\eta + K} \right) \right], \quad (25)$$

$$y_7' = y_8, \quad (26)$$

$$y_8' = -\frac{y_8}{\eta + K} - \frac{K\text{Scy}_1y_8}{\eta + K} - \text{ScSr} \left\{ \frac{1}{(1 - \text{PrDuSrSc})} \left[-\frac{y_6}{\eta + K} + \right. \right. \\ \left. - \frac{\text{PrKy}_1y_6}{\eta + K} - \text{PrEc} \left(y_3 - \frac{y_2}{\eta + K} \right)^2 - \frac{JK^2y_2^2}{(\eta + K)^2} + \right. \\ \left. \left. - \text{PrDu} \left(-\frac{K\text{Scy}_1y_8}{\eta + K} - \frac{\text{ScSry}_6}{\eta + K} \right) \right] + \frac{y_6}{\eta + K} \right\} \quad (27)$$

with the transformed boundary conditions as follows:

$$y_1(0) - \lambda, \quad y_2(0) - 1, \quad y_5(0) - 1 - Ly_6(0), \quad y_7(0) - 1, \quad (28)$$

4. Validation of results

To validate the methodology used in this study, numerical values that have been obtained in the study have been compared to those obtained by Abbas et al. [2] and Sanni et al. [5], using the shooting method with the Runge-Kutta algorithm in MATLAB. This is shown in Table 1, which tabulates the values of the surface drag force and the heat transfer rate upon varying the curvature parameter and the magnetic parameter. The data recorded in Table 1 are seen to be in good convergence, which shows the accuracy of the present numerical scheme.

Table 1. Comparison of $-\text{Re}_s^{\frac{1}{2}}C_{fs}$ and $\text{Re}_s^{\frac{1}{2}}\text{Nu}_s$ for different values of K and M , when $\text{Pr} = 0.7$, $\text{Ec} = 0.2$, in absence of other parameters.

K	M	Abbas et al. [2]		Sanni et al. [5]		Present result	
		$-\text{Re}_s^{\frac{1}{2}}C_{fs}$	$\text{Re}_s^{\frac{1}{2}}\text{Nu}_s$	$-\text{Re}_s^{\frac{1}{2}}C_{fs}$	$\text{Re}_s^{\frac{1}{2}}\text{Nu}_s$	$-\text{Re}_s^{\frac{1}{2}}C_{fs}$	$\text{Re}_s^{\frac{1}{2}}\text{Nu}_s$
5	0.2	1.22881	0.43268	1.20372	0.42418	1.20442	0.42716
10	0.2	1.12311	0.41896	1.10709	0.41132	1.10485	0.41137
20	0.2	1.07541	0.41094	1.06389	0.40365	1.06045	0.40009
50	0.2	1.04849	0.40571	1.03958	0.39864	1.02779	0.39370
100	0.2	1.03982	0.40390	1.03175	0.39691	1.02389	0.39113
200	0.2	1.03553	0.40298	1.02788	0.39604	1.02080	0.38981
10	0.2	1.12311	0.41896	1.10709	0.41132	1.11909	0.41137
10	0.4	1.18306	0.40717	1.16408	0.39975	1.17579	0.39754
10	0.6	1.27633	0.38927	1.25344	0.38190	1.26228	0.38109
10	0.8	1.39562	0.36727	1.36870	0.35953	1.37096	0.35223

5. Results and discussion

This paper examines the steady, two-dimensional boundary layer flow, heat and mass transfer, thermal slip, and variable magnetic field of a Newtonian, incompressible, viscous, and electrically conducting fluid past a permeable curved stretching surface in porous media. The distributions of fluid temperature, concentration, and velocity are represented graphically with the numerical findings, and the behaviour of different parameters is examined. Different parameter values are taken into consideration, with Prandtl number $\text{Pr} = 0.7$ taken for air and Schmidt number $\text{Sc} = 0.6$ taken for water vapour, $K = 5$, $\text{Da} = 0.5$, $M = 0.2$, $J = 0.2$, $\text{Ec} = 0.2$, $\text{Sr} = 0.4$, $\text{Du} = 0.2$, $\lambda = 0.2$, $L = 0.2$, unless stated otherwise.

Figures 2–7 illustrate the outcome of the curvature parameter (K) and the magnetic parameter (M) on the boundary layers of the fluid. It has been observed that $f'(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ exhibit an increase as K becomes greater. As K grows, the surface shape transforms, hence modifying the fluid flow over it. Enhanced curvature results in elevated gradients within the velocity field owing to the more significant bending of the surface. This improves velocity distributions. The extended surface will improve the thermal and species dispersion of the fluid due to its significant capacity for fluid movement and contact with the surface. Consequently, the thickness of the thermal and concentration boundary layers will increase.

A reverse effect is noticed for the magnetic parameter. The resistive force known as the Lorentz force, produced due to the effect of the magnetic field on an electrically conducting fluid, suppresses the bulk motion of the fluid and, as a result, both the components of fluid velocity decrease. Moreover, because of the friction caused by this resistive force, more heat is generated, which eventually increases the temperature and concentration distribution in the flow. According to tabulated data in Table 3, upon increasing curvature of the surface ($5 \leq K \leq 10$) and magnetic field intensity ($0.2 \leq M \leq 0.5$), both the heat transfer rate and mass transfer rate reduce, by 6.28% and 2.73%, and by 7.3% and 1.15%, respectively. Thus, one can increase the cooling rate in conducting fluid flows by considering lesser curvature parameter and magnetic field. Moreover, for a larger curved surface, the

Table 2. Numerical values of $-\text{Re}_s^{\frac{1}{2}}C_{fs}$ with $\text{Pr} = 0.7$, $\text{Ec} = 0.2$, $\text{Du} = 0.2$, $J = 0.2$, $\text{Sc} = 0.6$, $\text{Sr} = 0.4$ and $L = 0.2$.

K	M	Da	λ	$-\text{Re}_s^{\frac{1}{2}}C_{fs}$
5	0.2	0.5	0.2	1.54627
10	0.2	0.5	0.2	1.44009
5	0.5	0.5	0.2	1.61541
5	0.2	0.7	0.2	1.62783
5	0.2	0.5	0.4	1.63839

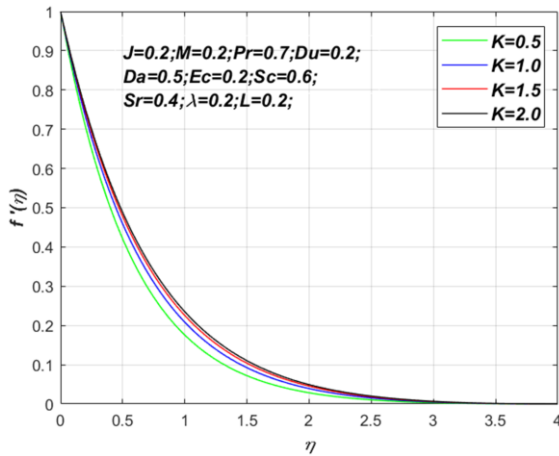


Fig. 2. Velocity distribution over varying K .

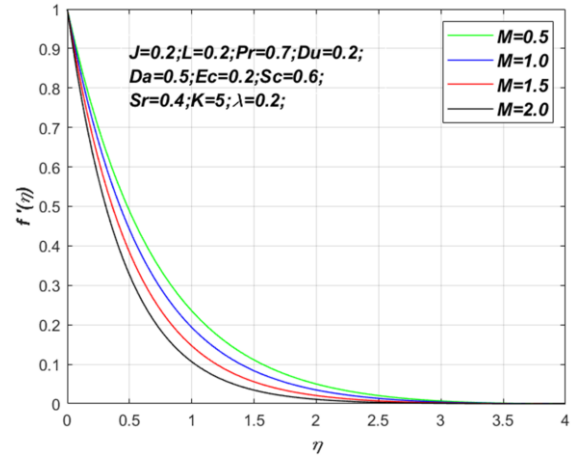


Fig. 5. Velocity distribution over varying M .

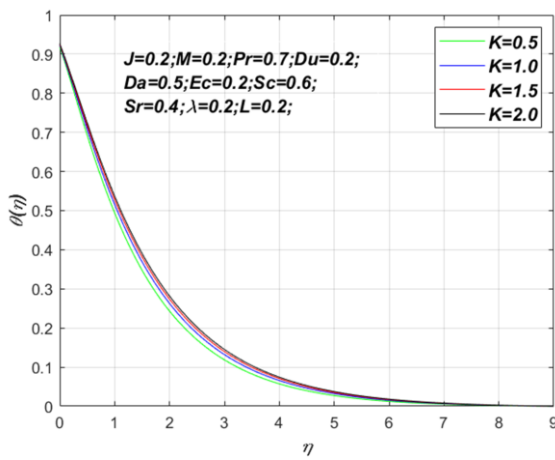


Fig. 3. Temperature distribution over varying K .

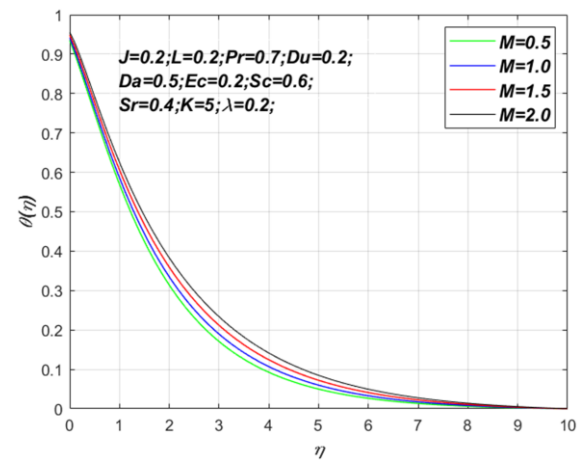


Fig. 6. Temperature distribution over varying M .

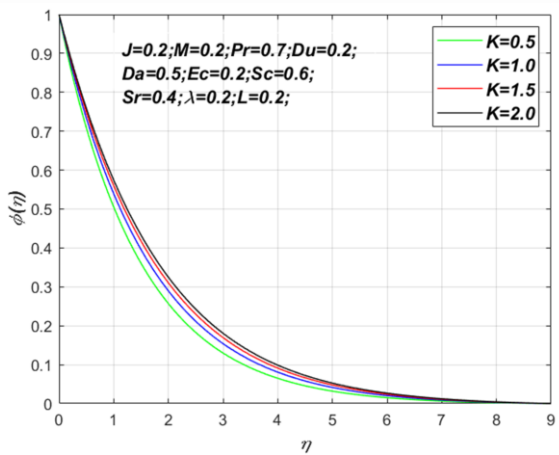


Fig. 4. Concentration distribution over varying K .

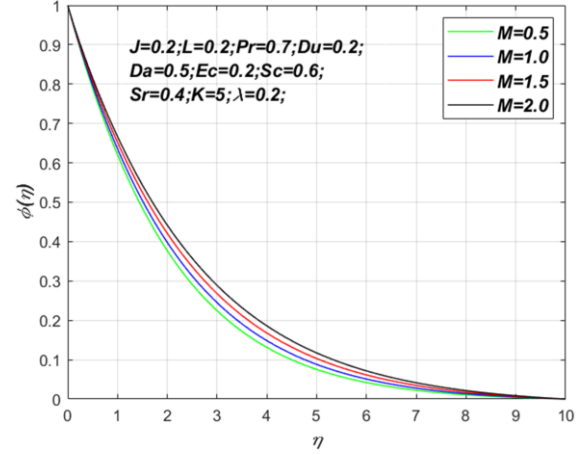


Fig. 7. Concentration distribution over varying M .

surface drag force decreases by about 6.87%, while as the magnetic field intensifies, the surface drag also improves by around 4.47% (see Table 2).

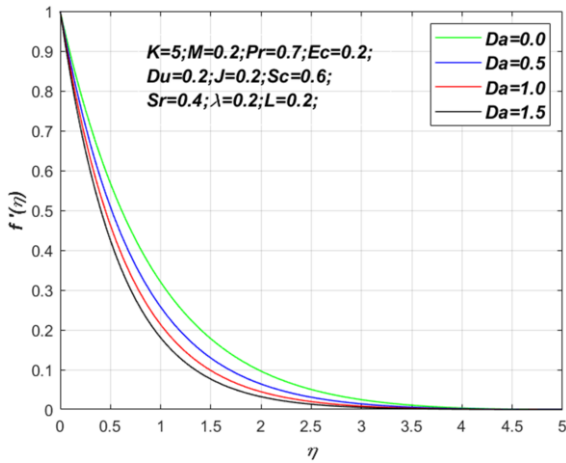
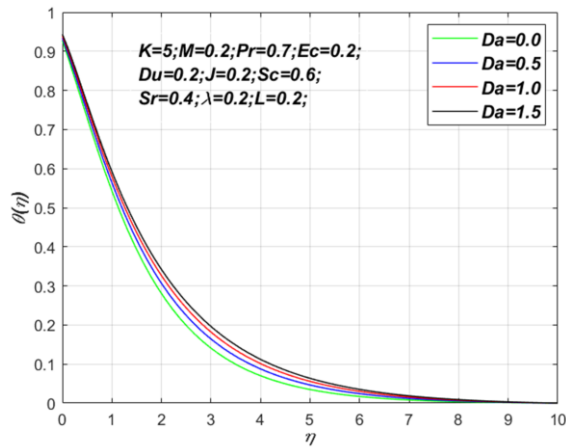
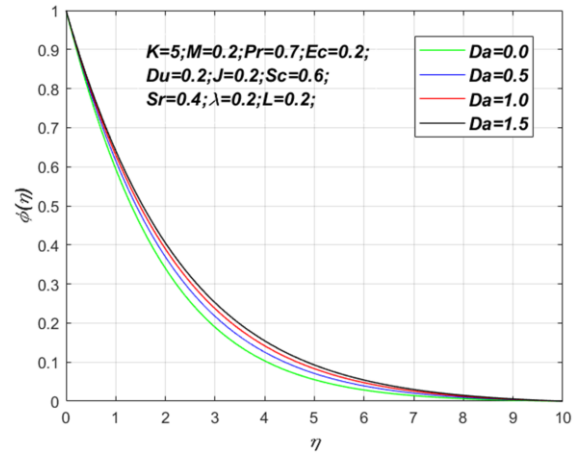
Figures 8–10 represent the effects of Darcy parameter (Da) on velocity component ($f'(\eta)$), temperature ($\theta(\eta)$) and concentration ($\phi(\eta)$) distributions. The porous medium physically exemplifies

a type of external resistance that restricts fluid movement, resulting in a reduction of fluid velocity. It is established that an increase in Darcy number alters the permeability rate of the fluid through the porous medium, consequently diminishing the velocity distribution. However, the temperature of the fluid increases due to the rising values of Da . This is because increased porosity facilitates greater

Table 3. Numerical values of $Re_s^{-\frac{1}{2}}Nu_s$ and $Re_s^{-\frac{1}{2}}Sh_s$ when $\lambda = 0.2$.

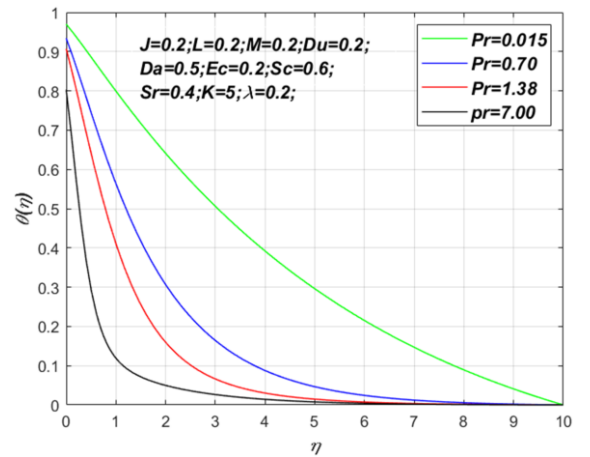
K	M	Da	Pr	Ec	Du	J	Sc	Sr	L	$Re_s^{-\frac{1}{2}}Nu_{fs}$	$Re_s^{-\frac{1}{2}}Sh_{fs}$
5	0.2	0.5	0.7	0.2	0.2	0.2	0.6	0.4	0.2	0.33151	0.47911
10	0.2									0.31066	0.44414
5	0.5									0.32246	0.47359
	0.2	0.7								0.32085	0.47264
		0.5	1.38							0.46581	0.44967
			0.7	0.4						0.22965	0.50344
				0.2	0.4					0.29661	0.48721
					0.2	0.4				0.29044	0.48892
						0.2	0.94			0.31418	0.62965
							0.6	0.8		0.33513	0.44537
								0.4	0.4	0.30072	0.48362

fluid movement across the medium, hence enhancing convective heat transfer, which effectively transports heat and causes a temperature increase within the porous medium. Also, an increase in the concentration distribution is noted when the values of Da increase. This is due to the rate of permeability of the porous medium, which makes the thickness of the boundary layer continuously increasing (see Fig. 10). From Tables 2 and 3, it has


 Fig. 8. Velocity distribution over varying Da .

 Fig. 9. Temperature distribution over varying Da .

 Fig. 10. Concentration distribution over varying Da .

been observed that surface drag increases by about 5.27%, while the heat and mass transfer rates both reduce by 3.22% and 1.35%, respectively, within $0.5 \leq Da \leq 0.7$.

Figures 11 and 12 depict the impact of the Prandtl number (Pr) on the temperature distribution ($\theta(\eta)$) and the concentration distribution ($\phi(\eta)$), correspondingly. The parametric values evaluated for Pr are 0.015, 0.7, 1.38 and 7.0, which correspond


 Fig. 11. Temperature distribution over varying Pr .

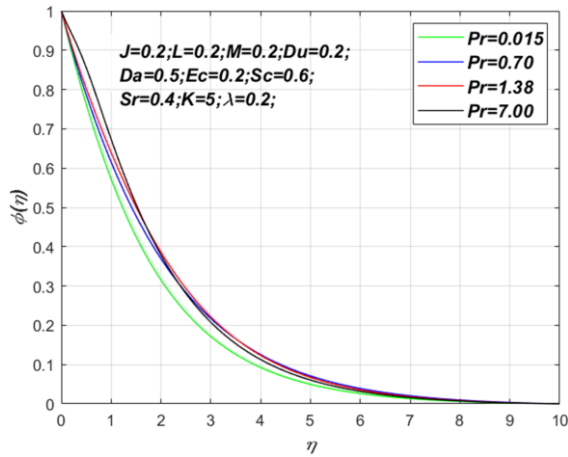


Fig. 12. Concentration distribution over varying Pr.

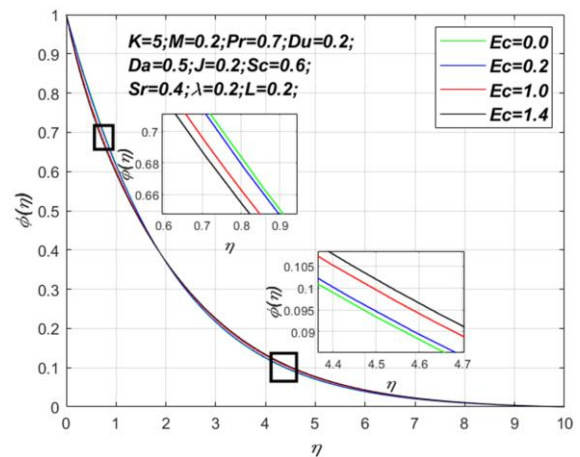


Fig. 14. Concentration distribution over varying Ec.

to liquid metal, air, ammonia, and water, respectively. The results indicate a significant decrease in temperature for large Prandtl numbers, resulting in the weakening of the thermal boundary layer. The concentration of fluid along the wall increases and falls away from the sheet when the value of η is greater than 2.3 for materials with high Pr values. Physically, a higher Pr value indicates a lower thermal diffusivity, which means that heat is dispersed more slowly from the surface, resulting in a drop in the temperature of the fluid. Table 3 demonstrates that when the values of Pr grow ($0.7 \leq Pr \leq 1.38$), the heat transfer rate experiences a substantial increase of 40.51%; however, the mass transfer rate decreases by approximately 6.14%. Fluids with lower Prandtl numbers have stronger thermal conductivities, facilitating faster heat dissipation from the surface compared to fluids with higher Pr values. Prandtl number can therefore be used to increase the cooling rate in conducting fluid flows.

The outcome of the rising values of the Eckert number (Ec) on the temperature ($\theta(\eta)$) and concentration ($\phi(\eta)$) distributions is depicted in Figs. 13 and 14. It is known that temperature is considered the average of kinetic energy, while on the other

hand, Ec exhibits a relation with kinetic energy. Thus, keeping these two conditions, it can be concluded that upon improving the energy dissipation, the kinetic energy increases, which in turn significantly improves the temperature distribution. It is also important to note that because of this factor, the concentration of the fluid near the vicinity of the surface declines, while upon entering the region $\eta > 1.9$, a reverse trend is noticed in the concentration distribution. Also, from Table 3, it can be concluded that Ec is an important parameter in the heat transport phenomenon and can be incorporated to significantly reduce, by 30.73%, the rate of cooling and enhance by 5.08% the mass transfer rate.

In Figs. 15 and 16, the influence of the Joule heating parameter (J) on the distributions of temperature and concentration is presented. Physically, as the Joule heating parameter enhances the impact of the energy dissipation, as well as the magnetic field, intensifies, thus the resulting Lorentz force aids in promoting the heating of the curved surface; also, due to inside friction of molecules, the mechanical energy, which gets converted to thermal energy aids in improving the temperature distribution significantly (mainly near the vicinity of the curved surface).

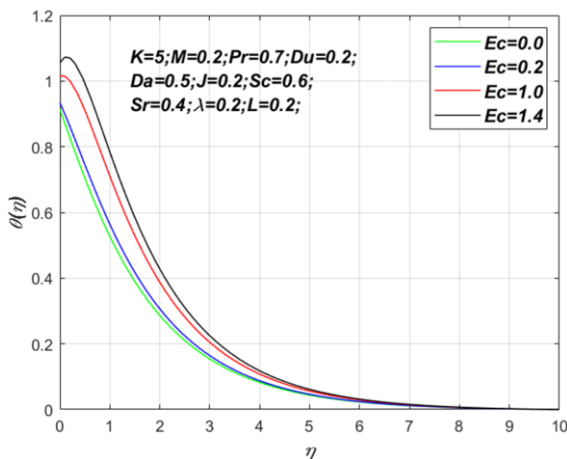


Fig. 13. Temperature distribution over varying Ec.

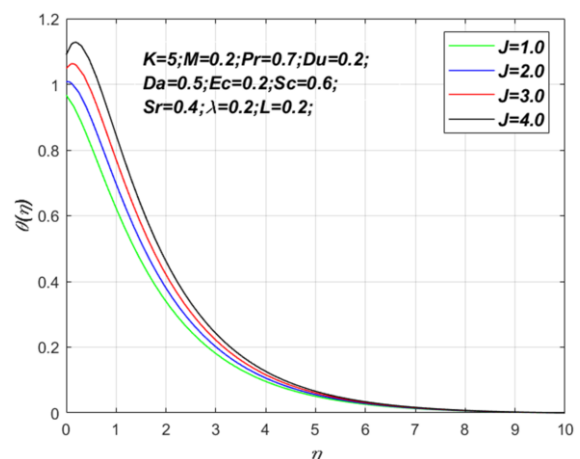
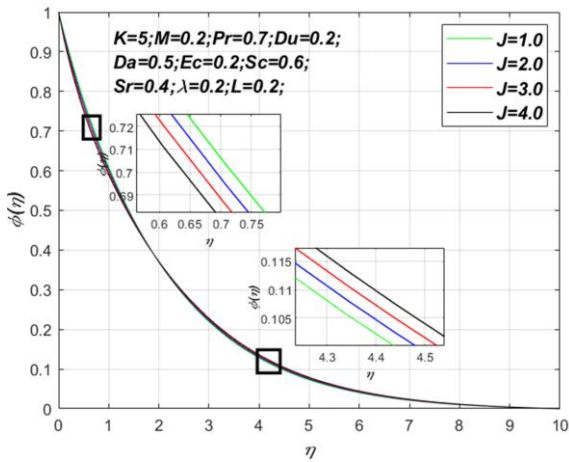
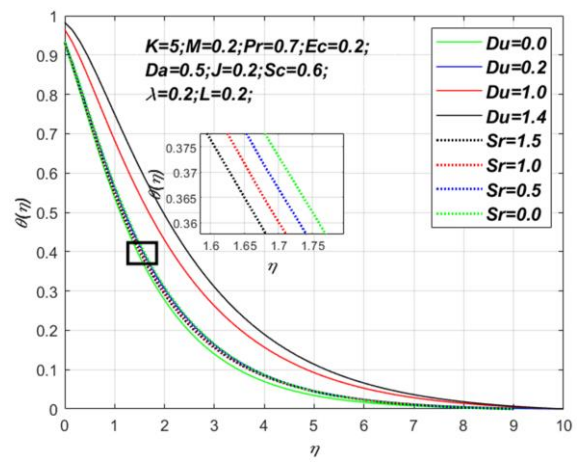


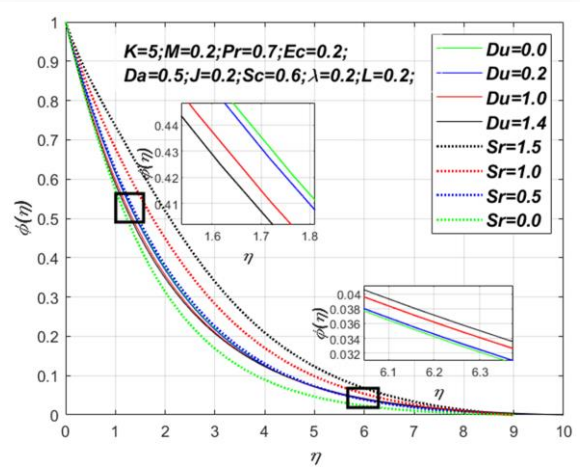
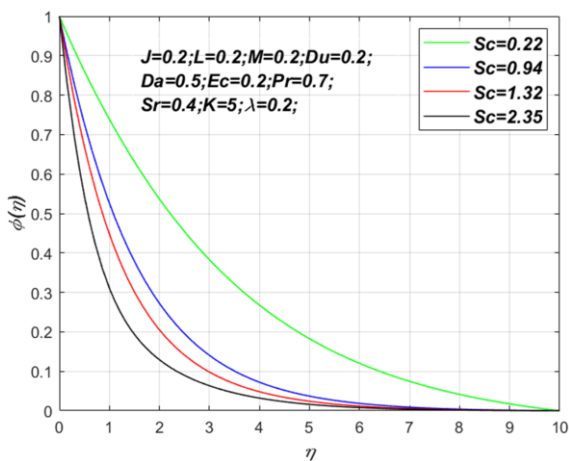
Fig. 15. Temperature distribution over varying J.


 Fig. 16. Concentration distribution over varying J .

 Fig. 18. Temperature distribution over varying Du and Sr .

Because of these factors, although the concentration distribution is found to be declining near the vicinity of the surface in a region beyond $\eta > 1.9$, the species concentration distribution slowly rises. From Table 3, it can be concluded that improving J can significantly reduce, by 12.39%, the cooling rate and enhance by 2.05% the mass transfer rate.

The effects of Schmidt number (Sc) on concentration ($\phi(\eta)$) distribution of the fluid are shown in Fig. 17. Parametric values considered for Sc are 2.35, 1.32, 0.94, and 0.22 corresponding to naphthalene, ethanol, hydrogen sulphide, and helium, respectively. Data plotted show that as the Sc concentration increases, the fluid concentration within the boundary layer drops. The reason for this is that a greater value of Sc indicates a higher kinematic viscosity, which hinders the movement of mass and, consequently, results in a reduction of the fluid concentration. A significant hike in the mass transfer rate (by 31.42%) is also recorded in Table 3 for $0.6 \leq Sc \leq 0.94$.

Figures 18 and 19 depict the effects of Dufour (Du) and Soret (Sr) on the temperature ($\theta(\eta)$) and species concentration ($\phi(\eta)$) distributions, respectively. It has been observed that an increase in the Dufour number causes a rise in the temperature of the


 Fig. 19. Concentration distribution over varying Du and Sr .

 Fig. 17. Concentration distribution over varying Sc .

fluid, while under the same influence, a varying outcome is observed on the concentration distribution. As reflected in Fig. 19, the concentration of species declines near the vicinity of the curved surface; however, beyond $\eta > 4.5$, the species concentration begins to rise. When the Dufour number is relatively elevated, thermal diffusion prevails over mass diffusion. This may lead to fluctuations in temperature gradients, which can subsequently generate buoyancy effects, altering the density distribution and perhaps affecting flow patterns through natural convection. The Dufour phenomenon results in a broader temperature distribution compared to its absence. From Figs. 18 and 19, it is also clear that Sr significantly improves species concentration, while reducing the temperature profile to a certain extent. The distribution of concentrations in the Soret phenomenon is influenced by the temperature gradient. In fact, higher Soret numbers cause more convective flow since they are linked to larger temperature gradients. The dispersion of the concentration rises as a result. Analysis from Table 3 shows that a higher $0.2 \leq Du \leq 0.4$ aids in reducing the heat transfer rate significantly (by 10.53%), while enhancing the mass transfer rate (by 1.69%). On the contrary, higher $0.4 \leq Sr \leq 0.8$ aids in enhancing the heat

transfer rate (by 1.09%) and significantly reduces the mass transfer rate (by 7.04%).

Figure 20 depicts the graphical data of the temperature ($\theta(\eta)$) distribution upon enhancing the thermal slip parameter (L). It is clear from Fig. 20 that increasing L results in a substantial decrease in the temperature distribution in the region closer to the curved wall. Physically, this decrease in surface temperature happens as a result of thermal slip, which facilitates heat transfer from the fluid surface to its interior. The temperature inside the thermal boundary layer drops as a result of the improved heat transfer brought on by thermal slip. The data presented in Table 3 indicates a notable decrease of around 9.29% in heat transfer rates for $0.2 \leq L \leq 0.4$.

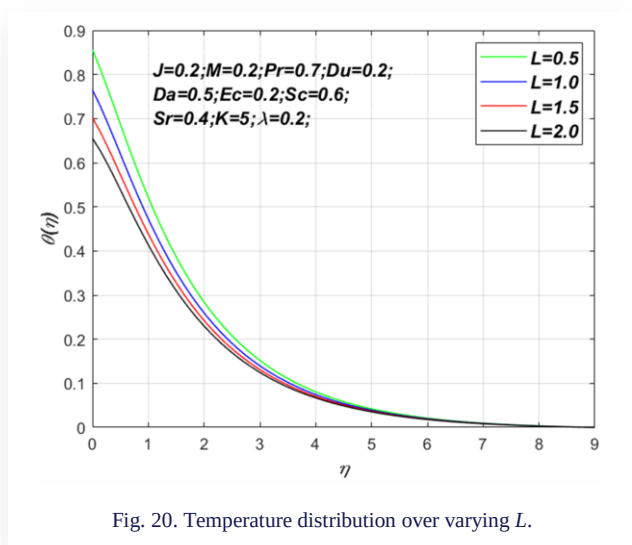


Fig. 20. Temperature distribution over varying L .

6. Conclusions

Based on the preceding discussions, the following conclusions can be drawn:

- Velocity of fluid increases for higher curvature parameter and decreases with increasing Darcy parameter and magnetic parameter.
- The curvature parameter, magnetic parameter and Darcy parameter can be increased for the reduction of the heat transfer rate (by around 6.28%, 2.73%, and 3.22%, respectively), as well as the mass transfer rates (by around 7.30%, 1.15%, and 1.35%, respectively).
- For large Prandtl numbers, there is a significant decrease in temperature. Therefore, the Prandtl number can be utilized to enhance the cooling rate (by around 40.51%) in the flow of conducting fluids.
- The Eckert number, Dufour number and the Joule heating parameter tend to reduce the species concentration in the vicinity of the curved region, while at a distance away from the surface, they also aid in gradually boosting the species concentration.

- The Soret number increases the concentration of the fluid. The heat transfer rate increases (by 1.09%), but the mass transfer rate decreases (by 7.04%) for larger values of the Soret number.
- The Dufour number significantly contributes towards raising the temperature of the fluid and aids in reducing the cooling rate by 10.53%.
- The mass transfer rate increases for higher values of the Dufour number (by 1.69%), the Schmidt number (by 31.42%), the Joule heating parameter (by 2.05%) and the Eckert number (by 5.08%).

The physics of flow on curved surfaces with variable magnetic field and heat slip can be employed in various physical domains, including industries, aeronautics, medicinal research, and engineering sciences.

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Shape effects on steady flow and heat transfer of Cu-nanofluid over a nonlinear stretching surface with Joule heating

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Abstract

Nanofluids have garnered significant interest in various fields due to their numerous advantages and potential applications. The appeal of ethylene glycol (EG)-based Cu-nanofluid lies in its excellent thermal conductivity, stability, and ability to enhance heat transfer properties, making it a promising candidate for diverse industrial applications. In this study, we analyse the flow behaviour of EG-based copper (Cu) nanofluid over a nonlinear stretching surface under the influence of Joule heating effects. The research involves the development of a mathematical model and formulating of governing equations represented as system of partial differential equations, which are subsequently transformed into nonlinear ordinary differential equations through suitable transformations. A numerical framework based on MATLAB bvp4c solver technique is employed to obtain approximate solutions. The study examines the influence of dimensionless parameters on velocity and thermal distributions. The findings for the local Nusselt number and skin friction are presented in tabular form, highlighting the effects of key parameters. The results are benchmarked against three different sources from existing literature, showing strong agreement in the case of reduced Nusselt number.

Keywords: Joule heating; MHD flow; Cu-nanofluid; Stretching surface

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1. Introduction

Nanofluids are stable dispersions of nanoparticles in a base fluid, in which the thermophysical properties, such as thermal conductivity, are enhanced several times compared to those of the pure liquid. On the other hand, these nanofluids are used in various sectors such as heat transfer, lubrication, biomedicine,

and solar panels to have better optical characteristics and absorption ability of CO₂. Although nanofluids have many advantages over other types of fluids, their stability is still a major hurdle when using them in diverse engineering fields. To overcome the issue, different stabilization strategies combined with numerous performance evaluation approaches have been introduced. It is

Nomenclature

A – proportionality constant
 B – magnetic field, T
 B_0 – constant magnetic field, T
 c – dimensional constant
 C_p – specific heat capacity, (J/kg K)
 Ec – Eckert number
 $f'(\eta)$ – dimensional velocity distribution
 J – current density vector, A/m²
 k – thermal conductivity, W/(m K)
 K – slip parameter
 L – velocity gradient tensor, 1/s
 m – shape factor
 n – stretching parameter
 M – magnetic parameter
 Nu – Nusselt number
 Pr – Prandtl number
 q – electric charge, C
 Re – Reynolds number
 T – temperature parameter, K
 u, v – velocity components in the x- and y-axis directions, m/s
 U – surface velocity, m/s
 V – velocity vector, m/s
 x, y – Cartesian coordinates, m

Greek symbols

α – thermal diffusivity, m²/s
 ε_i – constants ($i = 1, 2, 3, 4, 5$)
 η – similarity variable
 $\theta(\eta)$ – dimensionless temperature distribution
 μ – dynamic viscosity, Pa-s
 ν – kinematic viscosity, m²/s
 ρ – density, kg/m³
 σ – electrical conductivity, S/m
 ϕ – volume fraction

Subscripts and Superscripts

f – fluid
 nf – nanofluid
 s – solid particles
 w – wall
 ∞ – surrounding fluid
 $(\cdot)', (\cdot)'', (\cdot)'''$ – differentiation with respect to η

Abbreviations and Acronyms

EG – ethylene glycol
MHD – magnetohydrodynamics
ODE – ordinary differential equation
PDE – partial differential equation

generally accepted that the nanoparticle and base fluid characteristics, preparation methods, external factors and stabilizer content play critical roles in assessing nanofluid stability over time.

Literature review reveals that nanofluids are a rapidly growing field of research that is currently applied in many different fields. Several attempts have been made simultaneously to create stable nanofluids.

Copper (Cu) nanofluid can be defined as a liquid that comprises copper nanoparticles suspended in the base fluid, which can be, among others, water, ethylene glycol, or oil. The following properties of copper are some of the reasons why copper nanofluids are applied in heat exchangers, solar collectors, cooling of electronic devices, and some other loads where the process of heat transfer needs improvement, high thermal conductivity and electrical conductivity. This is because the conduction of the copper within the nanofluids is high, hence improving the energy and overall performance of the nanofluids. Copper is also less expensive to incorporate into other nanomaterials and other related compounding materials to which it can be added. Copper nanoparticles are also known to have an antimicrobial property that makes these fluids useful in reducing biofouling in systems where water is used as the heat transfer medium. Hence, copper is employed in nanofluids due to its thermal/electrical conductivity, reasonable cost, and bacteriostatic properties. Eastman et al. [1] investigated that the nanofluid of copper nanoparticles in ethylene glycol has significantly higher effective thermal conductivity than pure or oxide nanoparticles. This increase can be up to 40%, contradicting previous predictions of particle shape and size having no effect on thermal conductivity. Chakraborty and Panigrahi [2] discussed the stability of nanofluid and its significance in industrial applications, and offer recommendations for its wider practical adoption. Patel et al. [3] explored thermal

conductivities of nanoparticles in water and toluene media, revealing variations based on particle type and stabilization, with additional factors influencing the thermal conductivity mechanism in nanofluids. Liu et al. [4] examined the flow and heat transfer near a rotating disk containing nanofluids, showing variations in boundary layer thicknesses and heat transfer rates with different nanoparticle volume fractions and materials. The flow and heat transfer properties of nanofluids in a rotating frame were studied by Das [5], who considered two types of nanofluids and analysed the effects of different parameters. Nanofluid flow and heat transfer are extensively studied for various industrial and engineering applications.

Utilizing the Buongiorno model, the work [6] investigates mass and heat transfer in nanofluid-connected flow and discovers that heat transmission diminishes as thermal radiation and convection parameters grow. Research [7] explores the impact of external forces like natural convection, thermal radiation, and chemical reactions on heat and mass transfer. Nanofluid combinations can minimize the skin friction coefficient and maximize heat transfer rates. Magnetic fields and thermal radiation influence hybrid nanofluids, enhancing thermal conductivity. The study of Sakiadis [8] offers valuable insights into optimizing nanofluid flow and heat transfer processes. Ferdows et al. [9] analysed the effects of MHD flow and Hall currents on boundary layer flow over a three-dimensional extending surface to demonstrate improvements in velocity, profile and heat flux, and transfer rates.

Ahmad et al. [10] examined the mixed convection flow of hybrid nanofluid over a nonlinear stretching sheet, showing that the magnetic field velocity decreases and thermal radiation temperature increases. The influence of a magnetic field and nano-

particles on Carreau fluid flow and heat, and mass transfer over a porous nonlinear stretching surface was studied by Eid et al. [11]. Hayat et al. [12] present a numerical simulation study on boundary layer flow over a nonlinear curved stretchable surface, examining various parameters. Zaimi et al. [13] present a mathematical study on boundary layer flow and heat transfer in a nanofluid, examining the effects of parameters and identifying dual solutions. When dust particles and polymers combine in a viscous liquid, Athar et al. [14] observed that the polymers undergo deformation inside the boundary layer but not outside of it. Shah et al. [15] used graphs and the homotopy analysis approach to analyse the heat transfer and boundary layer flow of couple stress fluid at the stagnation point over an exponentially extending surface.

Venkateswarlu et al. [16] and Vidya et al. [17] conducted a study on the incompressible stagnation point flow of a conductive hyperbolic radiative nanofluid past a stretching surface. They found that increasing Weissenberg's number reduced fluid velocity. Hashim et al. [18] extended the mathematical formulation for the aligned magnetic field on Williamson fluid over a stretching sheet with Newtonian heating, analysing the effects of various parameters on temperature and velocity profiles.

Chen [19] examined both Joule heating and viscous dissipation on MHD (magnetohydrodynamic) flow over a stretching sheet, considering the influence of free convection, thermal radiation, and surface suction/blowing. The study of Khashi'ie et al. [20] reveal the variation of flow and heat transfer characteristics of a hybrid nanofluid of a shrinking cylinder with Joule heating with distinct types of nanofluids. Joule heating effects on fluid movement and heat transmission over an exponentially extending sheet are explored by Srinivasacharya and Jagadeeshwar [21] mainly in view of its relevance to industrial applications and magnetohydrodynamic flows. Ganesh Kumar et al. [22] discuss the effect of viscous dissipation and Joule heating on Jeffrey nanofluid and how the temperature profile and the thickness of the thermal boundary layer are affected by it. Ghaffar pasand [23] investigates MHD natural convection in a cavity filled with Fe_3O_4 -water nanofluid, revealing improved heat transfer with ferrite nanoparticles and increased entropy generation under Joule heating and Lorenz force. Mutuku et al. [24] analyse the impact of Joule heating and viscous dissipation on nanofluid flow past an inclined plate, highlighting the significant influence of controlling parameters on flow fields. Ali et al. [25] conducted a study on the impact of various forces and parameters on ethylene glycol-based nanofluids suspended above a thin vertical needle.

Shankar Goud et al. [26] investigated the effects of Joule heating on surfaces that stretch exponentially, and Khashi'ie et al. [27] communicated about the Joule heating effect of hybrid nanofluid. According to the research of Rana et al. [28] on an exponentially stretched surface, the thermal boundary layer structure is improved by both Joule and viscous heating mechanisms, which lowers nano-liquid velocity. Viscoelastic nanofluid has greater momentum, according to research by Tarakaramu et al. [29] on the effects of different physical parameters on MHD nanofluid flow to a stretched surface.

The research community in engineering and modelling has

extensively investigated the boundary layer flow and heat transfer of nanofluids under Joule heating, focusing on factors like hydromagnetic forces, mixed convection, thermal stratification, chemical reactions, and viscous dissipation. Saleem et al. [30] investigated the impact of various copper nanoparticle shapes on the flow and heat transfer characteristics of a water-based nanofluid over a flat surface. Fakour et al. [31] studied the flow and heat transfer characteristics of nanofluid thin films over an unsteady stretching elastic sheet using the lattice Boltzmann method (LBM). Using aluminum and copper with water, Mathews and Tallab [32] discussed the nonlinearity of the stretching sheet. Triveni et al. [33] analyse the heat transfer of MHD Casson nanofluid flow over a nonlinear stretching sheet in the presence of a nonuniform heat source. Hayat et al. [34] analysed MHD thin film flow with viscous dissipation and slip effects for nanofluids of various nanoparticle shapes on an unsteady radially stretching surface.

A numerical study using a vertical cone and Williamson nanofluid flow has been discussed by Sathyanarayana and Goud [35]. Joachimia et al. [36] investigated the effects of different physical criteria on thermochemical attitudes while conducting a numerical study of an aluminum alloy. In his discussion of the various nanoparticle shape parameters, Hayat and Shahzad [37] included a numerical study and analysis of the impacts of heat transmission and thin film flow for various gold nanoparticle shapes on a vertically stretched sheet. In another study [38], authors considered gold nanoparticles and the effects of different shape factors on a radial stretching sheet. Ali et al. [39] proposed a model for the unsteady flow of silica nanofluid over a stretching cylinder and considered the shape effects of the nanoparticles. Gangadhar et al. [40] investigated the flow performance of silver-engine oil-based nanofluids with different non-Newtonian models through a Riga plate. In [41], Gangadhar and co-authors presented a numerical study on the performance of an HVAC (heating, ventilation, and air conditioning) system utilizing Sutterby flow ternary nanofluids, with a focus on a Cu-Ag-AA7075/ $\text{C}_6\text{H}_9\text{NaO}_7$ nanofluid. In other works [42–44], Gangadhar et al. provided the PDEs (partial differential equations) model-based Williamson fluid over a heated cylinder with convective heating. Furthermore, they considered thermal radiation and Cattaneo-Christov energy diffusion. They incorporated graphene and magnetic oxide nanoparticles to improve thermal conductivity. Their studies showed that enhanced thermal radiation increases the temperature of the fluid, while suction reduces skin friction coefficients but enhances heat and mass transfer rates. Their model was validated by comparing results with existing literature. Furthermore, Gangadhar et al. [45] explored the self-propelled movement of gyrotactic swimming microorganisms in a Casson nanofluid slip flow over a stretching cylinder with a strong magnetic field, using a non-dimensional numerical model and bvp4c technique. In another work [46], Gangadhar et al. examined the effect of a strong magnetic field and thermal radiation on the free convective flow of a ternary nanofluid over a moving wedge surface.

The PDE model provided in the manuscript has potential applications in heat transfer enhancement. It is evident that the copper-based nanofluid has extraordinary thermal conductivity.

This peculiar property makes them an ideal candidate for enhancing heat transfer for real-life applications, including cooling microelectronics and solar collectors. The proposed model, which includes PDEs with complex boundary conditions and the study of different particle shapes, is the first of its kind. Nanofluidic models with nonlinear stretching surfaces are highly applicable in industrial processes like sheet drawing, sustainable polymer production. The literature suggests studying efficient cooling and uniform heat distribution for high-quality engineering products. Nanofluid models are used for controlled heating processes in aerospace, automation and renewable energy. A better PDE model with realistic boundary conditions will enable the research community to design efficient devices.

This article investigates the steady heat transfer and flow in Cu nanoparticles with the added effects of thermal radiation, Joule heating, magnetohydrodynamics, and velocity slip over time-independent nonlinear stretching surfaces. It is the first study to investigate steady-state heat transfer and flow over a time-independent stretching surface with ethylene glycol (EG) as the base liquid. Here, we utilize a Cartesian coordinate system for the mathematical description of the problem and solve the obtained system of simplified nonlinear ordinary differential equations (ODEs) by utilizing an effective and convergent technique bvp4c. All the necessary calculations are performed in MATLAB. Additionally, a graph is included to explain the impact of physical parameters on temperature, velocity, skin friction number and Nusselt number.

2. Materials and methods

Heat and mass transfer analysis of a steady, viscous, incompressible nanofluid under laminar flow through a stretching and two-dimensional, regulated, nonlinear and stretching surface is depicted in Fig. 1. Parameter T_w represents the surface temperature and T_∞ denotes the temperature of the surrounding fluid. The x -axis is taken along the surface and the y -axis is taken perpendicular to it.

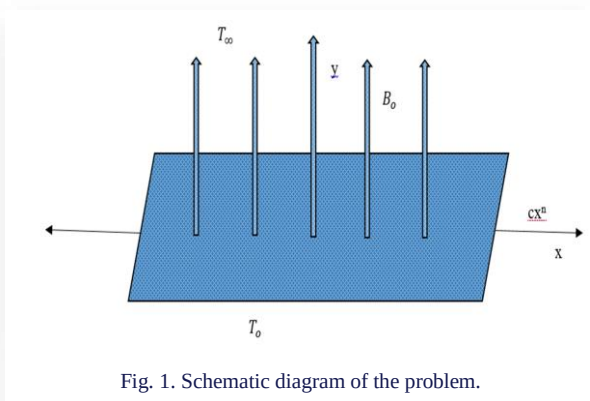


Fig. 1. Schematic diagram of the problem.

It is further assumed that the stretching velocity of the surface is given by

$$U = cx^n, \quad (1)$$

where n is the stretching parameter, and the magnetic field applied perpendicular to the surface is equal to

$$B = B_0 x^{\frac{n-1}{2}}, \quad (2)$$

where B_0 is a constant magnetic field. It was also assumed that any generated magnetic field component is much smaller than the external magnetic field and the base fluid is ethylene glycol containing copper nanoparticles. These nanoparticles exist in various shapes and have a uniform size without any distortions, remaining in thermodynamic equilibrium with the fluid in which they are suspended.

In light of the above assumptions, the governing equations of continuity, momentum and energy for the current study are expressed as follows:

$$\nabla \cdot \mathbf{V} = 0, \quad (3)$$

$$\rho_{nf}(\mathbf{V} \cdot \nabla)\mathbf{V} = \nabla \cdot \boldsymbol{\tau} + \mathbf{J} \times \mathbf{B}, \quad (4)$$

$$\rho_{nf} \frac{dT}{dt} = \boldsymbol{\tau} \cdot \mathbf{L} - \nabla \cdot \mathbf{q} + \frac{J^2}{\sigma_{nf}}, \quad (5)$$

where $\mathbf{V}$ is the velocity vector, $\boldsymbol{\tau}$ is the stress tensor, $\mathbf{J}$ is the current density vector, T is the temperature, t is the time, ρ_{nf} is the density of the nanofluid, $\mathbf{L}$ is the velocity gradient tensor, $\mathbf{q}$ is the electric charge of particles. In the case of a two-dimensional nanofluid model, the vector form reduces to the following system of equations [39,40]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (6)$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma_{nf} B^2}{\rho_{nf}} u, \quad (7)$$

$$\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf} B^2}{(\rho C_p)_{nf}} u^2, \quad (8)$$

where μ_{nf} is the dynamic viscosity of the nanofluid and α_{nf} is the thermal diffusivity of the nanofluid, B is the strength of the magnetic field and C_p is the specific heat.

The boundary conditions for the problem under consideration at $y = 0$ are:

$$\begin{cases} u = U + A v_f \frac{\partial u}{\partial y}, \\ v = 0, \\ T = T_w, \end{cases} \quad (9)$$

as $y \rightarrow \infty$, u approaches 0 and T approaches T_∞ .

The pertinent physical parameters from literature are defined as [31]:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$$

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s,$$

$$\mu_{nf} = \mu_f(1 + A_1\phi + A_2\phi^2), \quad (10)$$

$$\sigma_{nf} = \sigma_f(1 - \phi)\sigma_f + \phi\sigma_s,$$

$$\begin{aligned}
 (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi (\rho C_p)_s, \\
 \frac{K_{nf}}{K_f} &= \left[\frac{K_s + (m - 1)K_f + (m - 1)(K_s - K_f)\phi}{K_s + (m - 1)K_f - (K_s - K_f)\phi} \right]
 \end{aligned}
 \quad \begin{cases} f(0) = 0, \\ f'(0) = 1 + Kf''(0), \\ f'(\infty) = 0, \\ \theta(0) = 1, \\ \theta(\infty) = 0. \end{cases} \quad (17)$$

where ϕ is the volume fraction, m is the shape factor, K_{nf} , K_f , and K_s are slip parameters of nanofluid, base fluid and solid particles, respectively.

Using the similarity transformation [30] mentioned below, we will obtain the dimensionless form of the governing equations:

$$\begin{aligned}
 \psi &= e^{-\frac{1}{2}\eta} f(\eta), \\
 \eta &= \frac{y}{x} \text{Re}^{\frac{1}{2}}, \\
 \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty},
 \end{aligned} \quad (11)$$

where ψ is the stream function, η is the similarity variable, and θ is the dimensionless temperature. Function f represents a dimensionless similarity function that describes the velocity distribution within the boundary layer. The pattern of flow is defined by the stream function, and the components of velocity can be obtained as follows:

$$u = \frac{\partial \psi}{\partial y} \quad (12)$$

$$v = -\frac{\partial \psi}{\partial x}$$

Here

$$\text{Re} = \frac{xU}{\nu_f} \quad (13)$$

is the Reynolds number. From the above transformation, we have

$$u = Uf(\eta), \quad (14)$$

$$v = -U\text{Re}^{\frac{1}{2}} \left[\frac{n+1}{2} f(\eta) + \frac{n-1}{2} \eta f'(\eta) \right].$$

By using the above transformation, the continuity Eq. (6) holds automatically, and Eqs. (7) and (8) take the following non-dimensional forms, respectively:

$$\begin{aligned}
 \varepsilon_1 f'''' - n f'^2 - \frac{n+1}{2} f f'' - \varepsilon_3 M f' \\
 - (n-1) \eta f' f'' = 0,
 \end{aligned} \quad (15)$$

$$\begin{aligned}
 \theta''(\eta) + \frac{\text{Pr}}{\varepsilon_2} \left(\frac{n+1}{2} f \theta'(\eta) + \varepsilon_4 \text{Ec} f''^2 \right. \\
 \left. + \varepsilon_5 \text{Ec} M f'^2 \right) = 0.
 \end{aligned} \quad (16)$$

Now, the boundary conditions in terms of the transformed variable become:

The dimensionless constants are defined as follows:

$$\begin{aligned}
 \text{Prandtl number} \quad \text{Pr} &= \frac{(\rho C_p \nu)_f}{k_f}, \\
 \text{slip parameter} \quad K &= A \sqrt{\frac{\nu U_w}{x}}, \\
 \text{Eckert number} \quad \text{Ec} &= \frac{U^2}{c_p (T_w - T_\infty)}, \\
 \text{magnetic parameter} \quad M &= \frac{B_0^2 \sigma_f}{c \rho_{nf}}.
 \end{aligned}$$

Since the Nusselt number (Nu) and skin friction coefficient (C_f) are significant in engineering, they can be defined as:

$$\begin{aligned}
 C_f &= \frac{\tau_w}{\rho_f U^2}, \\
 \text{Nu} &= \frac{x q_w}{k_f (T_w - T_\infty)}.
 \end{aligned} \quad (18)$$

Further the constants ε_i , for $i = 1, \dots, 5$ are defined as:

$$\begin{aligned}
 \varepsilon_1 &= \frac{1 + A_1 \phi + A_2 \phi^2}{1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)}, \\
 \varepsilon_2 &= \frac{\frac{K_{nf}}{K_f}}{1 - \phi + \phi \left[\frac{(\rho C_p)_s}{(\rho C_p)_f} \right]}, \\
 \varepsilon_3 &= \frac{1 - \phi + \phi \left(\frac{\sigma_s}{\sigma_f} \right)}{1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)}, \\
 \varepsilon_4 &= \frac{1 + A_1 \phi + A_2 \phi^2}{1 - \phi + \phi \left[\frac{(\rho C_p)_s}{(\rho C_p)_f} \right]}, \\
 \varepsilon_5 &= \frac{1 - \phi + \phi \left(\frac{\sigma_s}{\sigma_f} \right)}{1 - \phi + \phi \left[\frac{(\rho C_p)_s}{(\rho C_p)_f} \right]}.
 \end{aligned} \quad (19)$$

3. Numerical solution

This article uses the bvp4c [31] in MATLAB to get the mathematical solution of the reduced Eqs. (15) and (16) along with the boundary conditions Eq. (17). The main characteristics of the suggested bvp4c technique, which include handling single boundary value problems (BVPs), direct acceptance of both

two-point and multipoint BVPs with improved accuracy, and faster convergence with reduced error, are well recognized in the research community. The basic numerical method implemented in bvp4c is a form of Simpson's method that is rather generic and can be found in numerous programs. To apply this strategy, the third-order ODE (15) and the second-order ODE (16) are transformed into the first-order differential equations as follows:

$$f = y_{(1)}. \quad (20)$$

Then we can write

$$f' = y'_{(1)} = y_{(2)},$$

$$f'' = y'_{(2)} = y_{(3)},$$

$$f''' = y'_{(3)} = \frac{1}{\varepsilon_1} \left(ny_{(2)}^2 + \frac{n+1}{2} y_{(1)} y_{(3)} + \varepsilon_3 M y_{(2)} + n\eta y_{(2)} y_{(3)} \right), \quad (21)$$

$$\theta = y_{(4)},$$

$$\theta' = y'_{(4)} = y_{(5)},$$

$$\theta'' = y'_{(5)} = \frac{\text{Pr}}{\varepsilon_2} \left[-\left(\frac{n+1}{2}\right) y_{(1)} y_{(5)} - \text{Ec} \varepsilon_4 y_{(3)}^2 - \varepsilon_5 \text{Ec} M y_{(2)}^2 \right].$$

Boundary conditions are:

$$\begin{cases} y_{(2)}(0) = 1 + K y_{(3)}(0), \\ y_{(1)}(0) = 0, \\ y_{(2)}(\infty) = 0, \\ y_{(4)}(\infty) = 0. \end{cases} \quad (22)$$

4. Results and discussion

In this research, we explore a model that investigates how nanofluids, containing copper nanoparticles of various shapes, behave in two-dimensional boundary layer flow. By employing similarity transformations, we transform the given differential equations into simpler ordinary differential equations. Afterward, we analyse how factors like the magnetic parameter, slip parameter, volume fraction, stretching parameter and Eckert number impact the velocity and temperature profiles within the layer, considering various shapes of copper nanoparticles. These influences are illustrated through graphs shown in Figs. 2 to 12.

The pertinent impact of the magnetic parameter, which occurs through the modelling of Maxwell's equations on the velocity profile within the boundary layer, is comprehensively analysed and illustrated in Fig. 2 for EG-Cu nanofluid for various nanoparticle shapes. The results are presented for blade, brick, cylinder, platelets, and sphere, highlighting the influence of the shape factor on the velocity profile. Increasing the magnetic parameter leads to a noticeable reduction in the velocity of copper

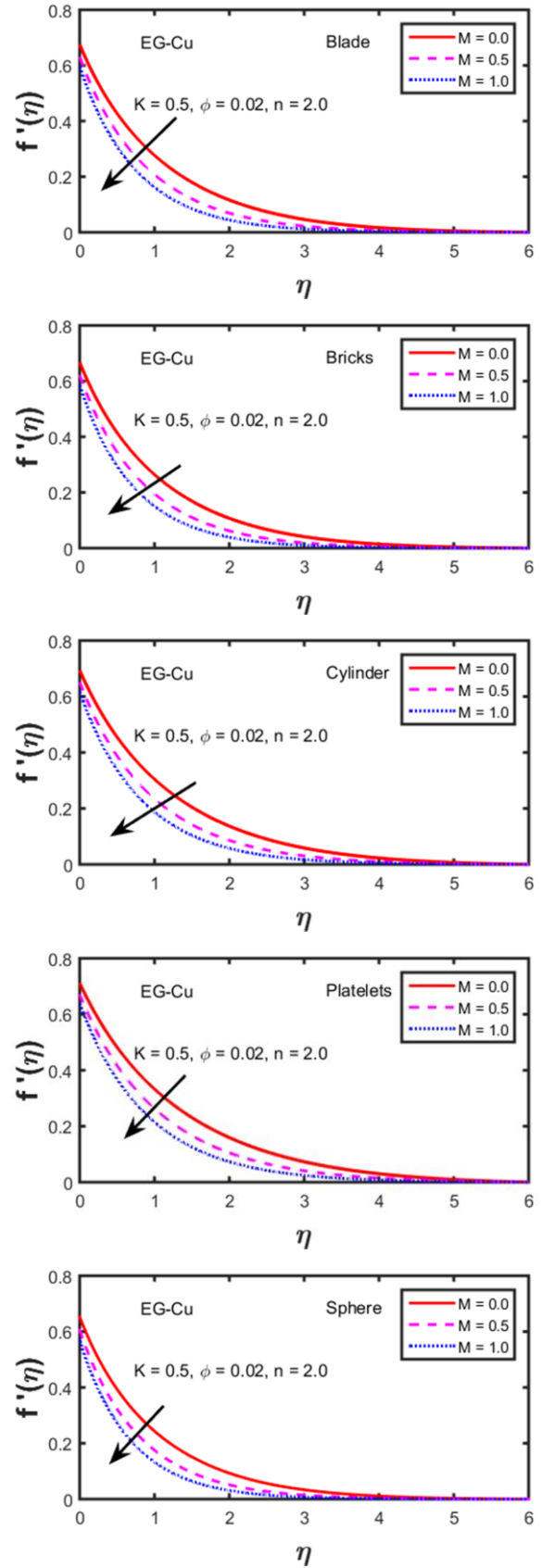


Fig. 2. Impact of the magnetic parameter (M) on the velocity profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

nanofluids across various nanoparticle shapes. This effect is attributed to the enhancement of Lorentz forces, which act as resistive forces opposing the fluid's motion, thereby decreasing the velocity profile.

From the graphical representation of the velocity profile, it is clear that an increase in the magnetic parameter (M) leads to a noticeable reduction in the velocity of the copper nanofluid across all nanoparticle shapes. This peculiar behaviour can be attributed to the presence of the Lorentz force, which arises from the interaction between the externally applied magnetic field and the induced electrical currents within the nanofluid. The Lorentz force acts as a resistive drag force, opposing the motion of the fluid and thereby reducing its velocity profile.

This effect is particularly significant within the boundary layer, where the interplay between magnetic forces and viscous forces becomes more pronounced. The reduction in velocity due to the magnetic parameter highlights the potential for controlling fluid dynamics in magnetohydrodynamic applications, where the magnetic field is utilized to manipulate the flow properties of conducting fluids, such as nanofluids, for engineering and industrial purposes.

Figure 3 presents the relationship between the magnetic parameter M and the fluid temperature of EG–Cu nanofluid for different nanoparticle shapes, including blade, brick, cylinder, platelets and sphere. The results clearly demonstrate that fluid temperature increases as M rises, emphasizing the critical role of nanoparticle geometry in dictating thermal behaviour. This trend is explained by the Lorentz force, which is induced when a magnetic field interacts with an electrically conducting nanofluid. The Lorentz force resists the motion of nanofluid particles, thereby dissipating their kinetic energy and converting it into thermal energy. This process enhances the overall heat transfer rate of the fluid. Importantly, the results highlight a dual effect: while the Lorentz force restricts particle movement and slows fluid motion, it simultaneously boosts thermal energy generation through dissipation. Consequently, higher values of M lead to a substantial rise in fluid temperature, as depicted in Fig. 3. These observations underscore not only the impact of magnetic fields on thermal transport but also the significance of nanoparticle shape in modifying velocity and temperature distributions within nanofluids.

Figure 4 illustrates the influence of the slip parameter K on the velocity profile of EG–Cu nanofluid for various nanoparticle shapes. As K increases, the velocity decreases for all particle geometries due to reduced momentum transfer at the fluid–wall interface under partial slip conditions. The results also highlight that nanoparticle shape strongly influences this behaviour: particles with larger surface areas, such as blades or platelets, exhibit more pronounced velocity reductions, while smoother geometries, like spheres, show smaller variations. These observations emphasize the importance of nanoparticle shape in governing flow characteristics under slip conditions.

Figure 5 depicts the influence of the partial slip condition (K) on the thermal field in EG–Cu nanofluid for five distinct nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

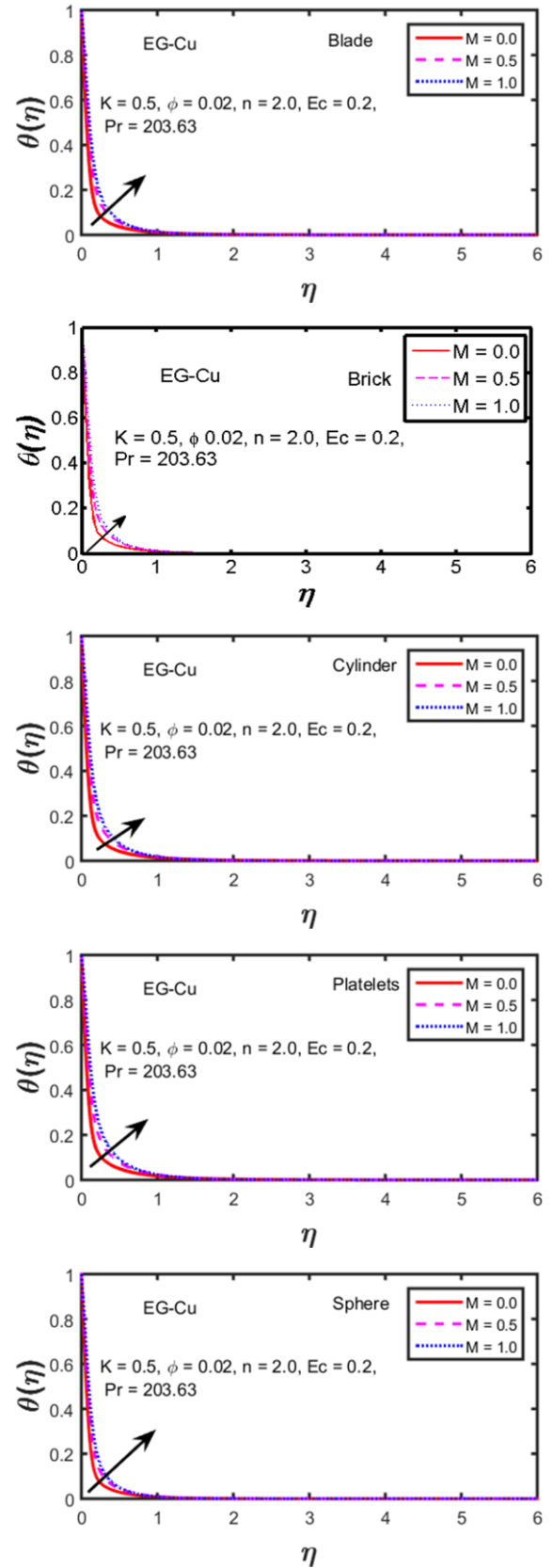


Fig. 3. Influence of magnetic parameter (M) on temperature profile of EG–Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

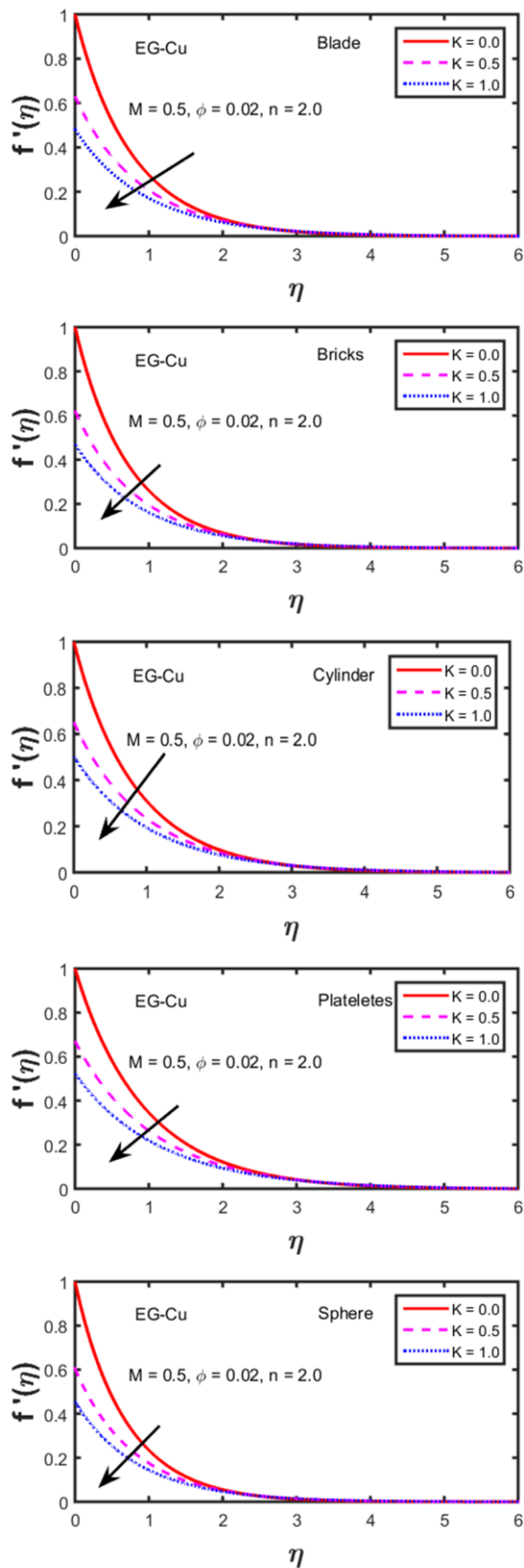


Fig. 4. Impact of slip parameter (K) on the velocity profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

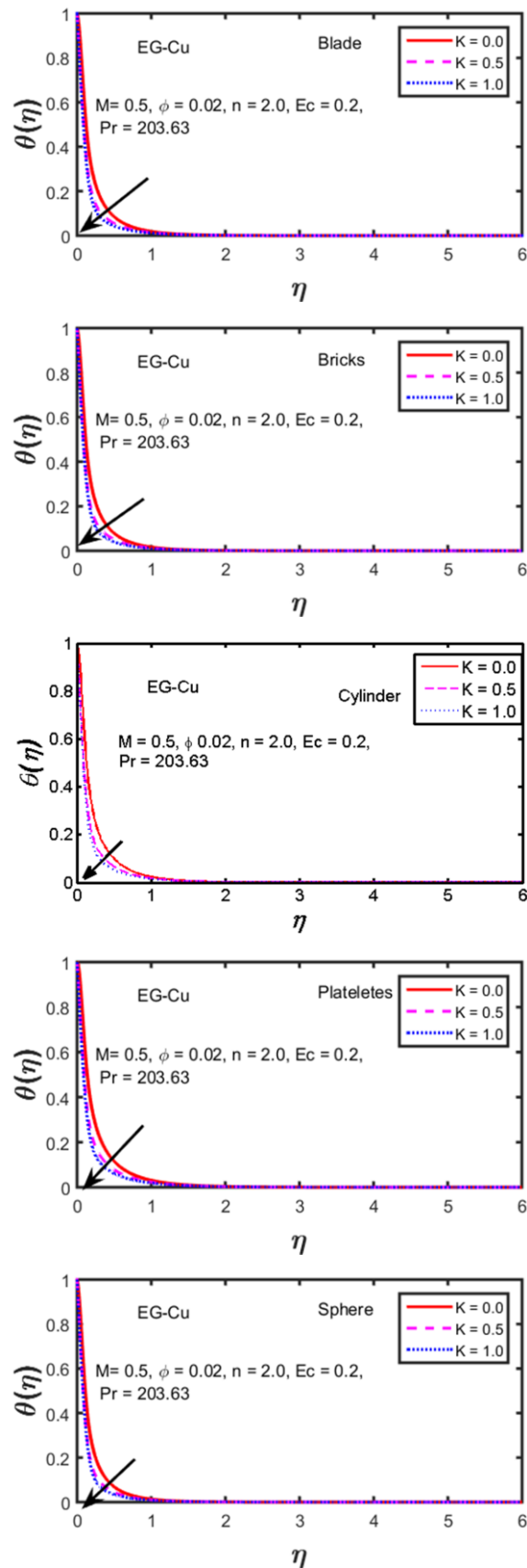


Fig. 5. Impact of the slip parameter (K) on the temperature profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

Increasing the slip parameter K reduces the friction at the boundary, which in turn reduces the rate of convective heat transfer. This reduction in heat transfer efficiency leads to a decrease in the temperature distribution within the EG-Cu nanofluid, regardless of the nanoparticle shape (blade, brick, cylinder, platelets, or sphere). Different shapes will exhibit different magnitudes of this effect due to their varying thermal conductivity and interaction with the base fluid, but the overall trend with respect to K will be the same.

Figure 6 demonstrates the influence of the nanoparticle volume fraction (ϕ) on the velocity profile for EG-Cu nanofluid for five nanoparticle shapes: blade, brick, cylinder, platelets, and sphere. The results demonstrate how the concentration of nanoparticles affects fluid flow, with each shape exhibiting unique flow characteristics. For instance, blade-shaped nanoparticles, with their sharp and elongated design, tend to increase resistance and turbulence, causing noticeable changes in the velocity profile. In contrast, spherical particles, due to their smooth and symmetrical shape, enable more uniform and stable flow. Platelet-shaped particles, with their flat and wide structure, enhance the velocity profile, while cylindrical and brick-shaped particles show intermediate flow patterns. Notably, increasing ϕ enhances the velocity for most copper nanoparticle shapes, including cylinders, platelets, blades, and bricks. However, a unique exception is observed for spherical nanoparticles, where increasing ϕ leads to a velocity decrease. This contrasting behaviour can be attributed to the interplay between various factors such as particle shape, interparticle interactions, and heat transfer mechanisms. Figure 7 delves into the influence of nanoparticle volume fraction (ϕ) on the temperature for EG-Cu nanofluid for various nanoparticle shapes. The results are presented for blade, brick, cylinder, platelets, and sphere, highlighting the influence of shape factor on the temperature profile. Interestingly, increasing ϕ generally leads to a temperature increase for most copper nanoparticle shapes, including cylinders, platelets, blades, and bricks. This can be attributed to enhanced heat generation within the nanofluid due to higher particle concentration.

The impact of the stretching parameter (n) on the velocity profile with different shapes of copper nanoparticles is shown in Fig. 8 for the EG-Cu nanofluid for five nanoparticle shapes: blade, brick, cylinder, platelets, and sphere. The results demonstrate how the rate of surface stretching influences fluid flow, with each nanoparticle shape showing unique flow characteristics. These findings highlight the significant role of nanoparticle geometry in determining fluid dynamics during stretching, providing critical insights for applications, such as industrial coating, polymer processing, or flexible electronics, where controlling flow behaviour under deformation is essential for optimizing performance. The velocity profile also tends to reduce with a rise in n values, as mentioned earlier; a rise in the values of n reduces the boundary layer thickness.

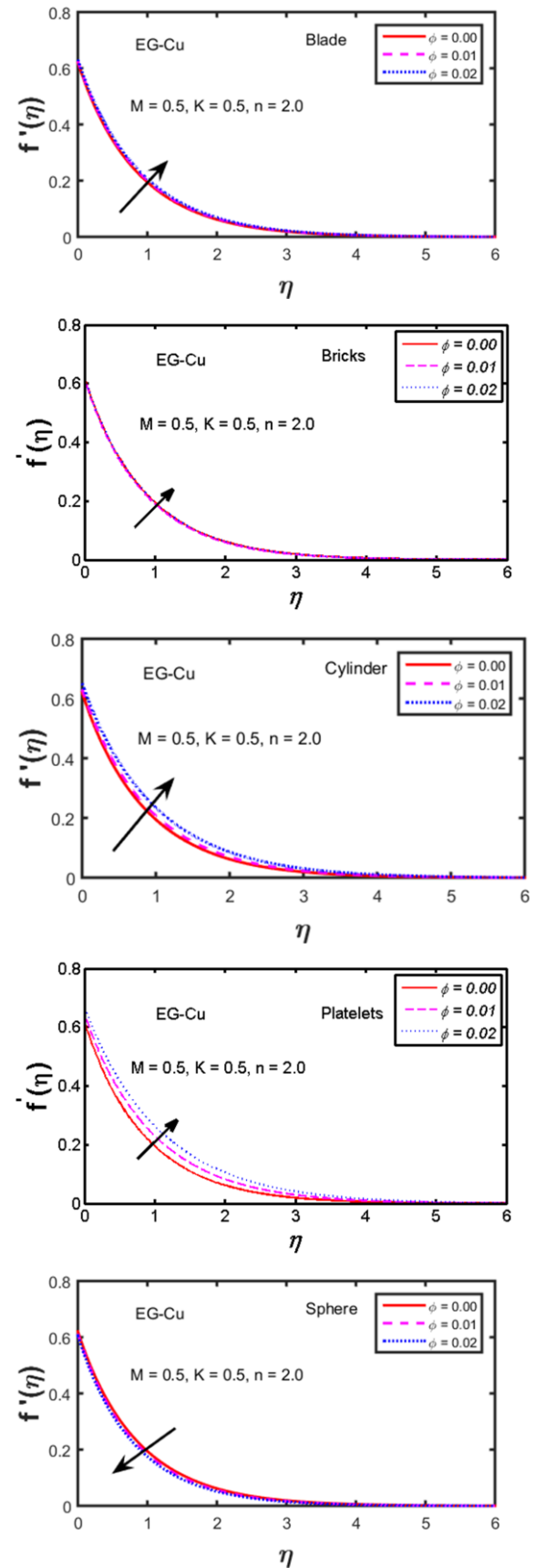


Fig. 6. Impact of volume fraction (ϕ) on the velocity profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

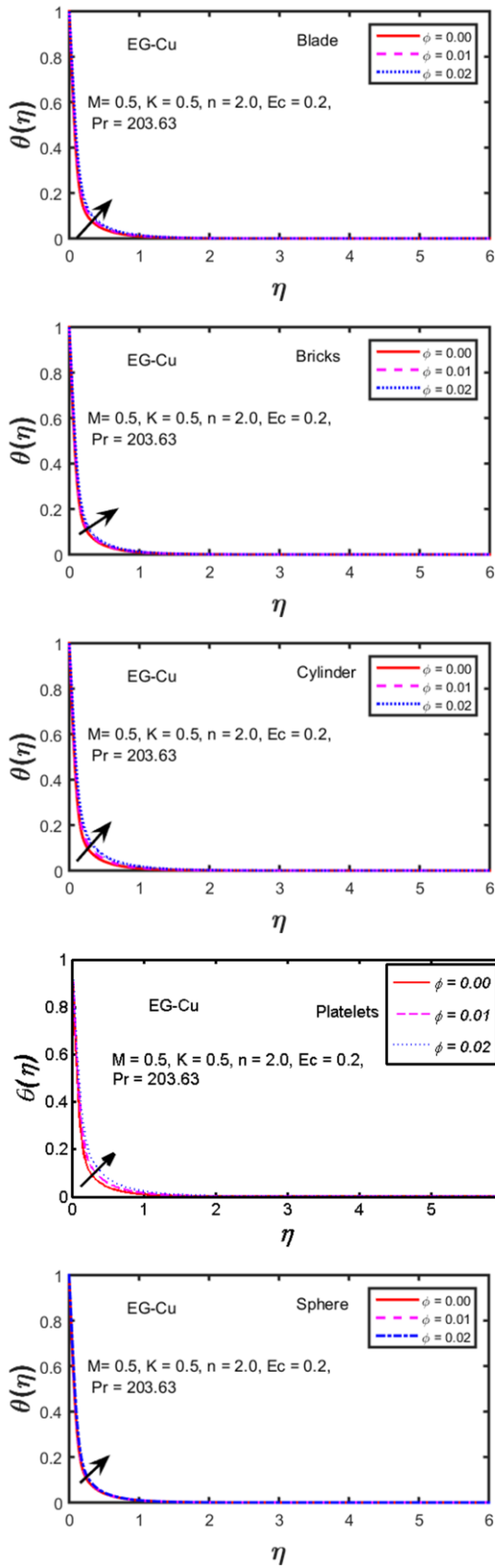


Fig. 7. Impact of volume friction (ϕ) on the temperature profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

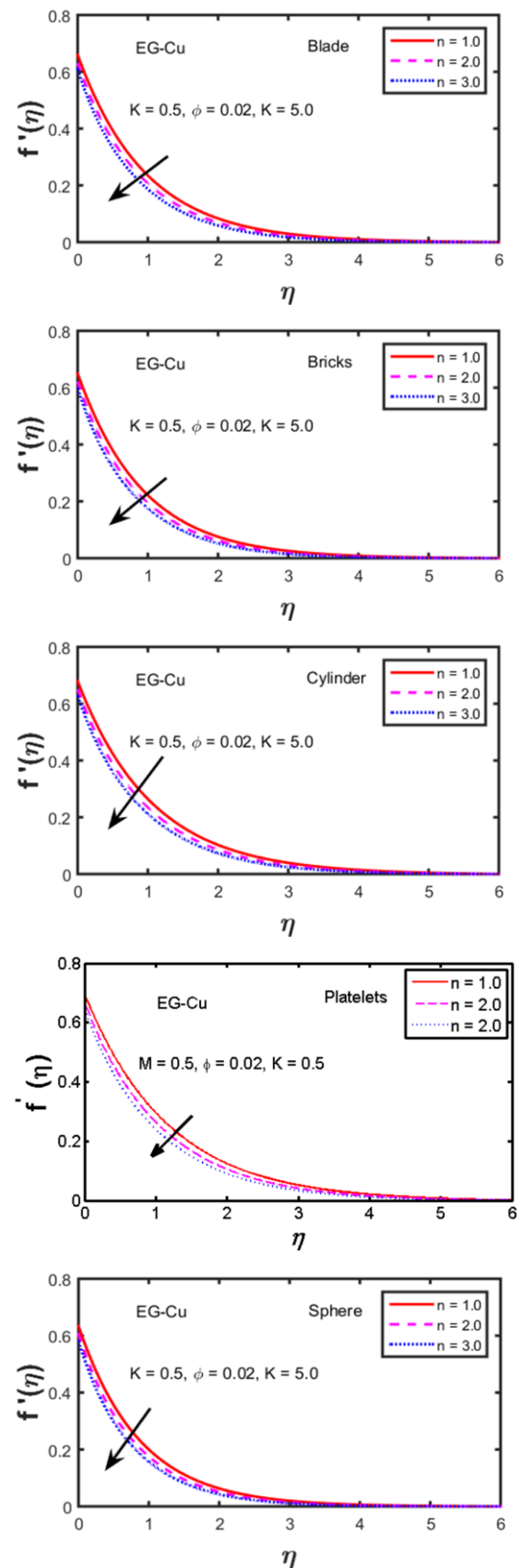


Fig. 8. Effect of the stretching parameter (n) on the velocity profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

Figure 9 portrays the impact of stretching parameter (n) on the temperature profile for EG-Cu nanofluid for five nanoparticle shapes: blade, brick, cylinder, platelets, and sphere. The results illustrate how the rate of surface stretching influences thermal behaviour, with each nanoparticle shape displaying distinct heat transfer characteristics.

The findings emphasize the importance of optimizing nanoparticle shape to achieve desired thermal performance in systems, where stretching and heat transfer are interconnected. It is noted that the temperature also reduces with a rise in the stretching parameter.

Figure 10 sheds light on the role of the Eckert number (Ec) in shaping the temperature profile. As Ec increases, we observe a further enhancement in temperature. This signifies the growing importance of viscous dissipation with increasing fluid velocity.

Figure 11 provides an intriguing comparison of the velocity profiles for different copper nanoparticle shapes. We observe that the velocity ranking, from highest to lowest, is as follows: platelets, cylinders, blades, bricks and spheres. This suggests that platelet-shaped nanoparticles offer the strongest enhancement in nanofluid velocity, while spherical nanoparticles exhibit the least impact.

Finally, Fig. 12 offers a scientifically compelling results for comparison of temperature profiles for different copper nanoparticle shapes. We can discern that the temperature ranking, from highest to lowest, follows the order: platelets, cylinders, blades, bricks and spheres. This suggests that platelet-shaped nanoparticles have the most significant impact on heat generation, while spherical nanoparticles contribute the least. These findings unveil the intricate relationships between various parameters and their impact on the thermal behaviour of copper nanofluids. Such insights can guide the design and optimization of nanofluid-based systems for diverse applications in thermal management, energy harvesting, and beyond.

Table 1 shows the physical and thermal characteristics of base fluids and copper nanoparticles [32] and Table 2 exhibits the varying values of the shape factor and viscosity of fluid under consideration [31]. In Table 3, the current result is contrasted with the findings of Khan and Pop [47], Wang [48], and Reddy Gorla and Sidawi [49] across a range of Prandtl numbers. We noticed that our results are in great agreement with existing literature for specific values. In Table 4, numerical values of skin friction are listed for different shapes of copper nanoparticles under various physical conditions. The data show that an increase in volume fraction (ϕ), magnetic field strength (M) and stretching parameter (n) leads to an increase in the skin friction coefficient for multi-shaped copper nanoparticles. However, the trend is reversed for the slip parameter (K). In Table 5, the heat transfer coefficient (Nusselt number, Nu) is computed for parameters such as magnetic field strength (M), volume fraction (ϕ) stretching parameter (n) and slip parameter (K). The Nusselt number reduces with increasing values of magnetic field strength (M), volume fraction (ϕ) and Eckert number (Ec), for multi-shaped copper nanoparticles. Conversely, a different pattern is observed for the slip parameter (K) and stretching parameter (n).

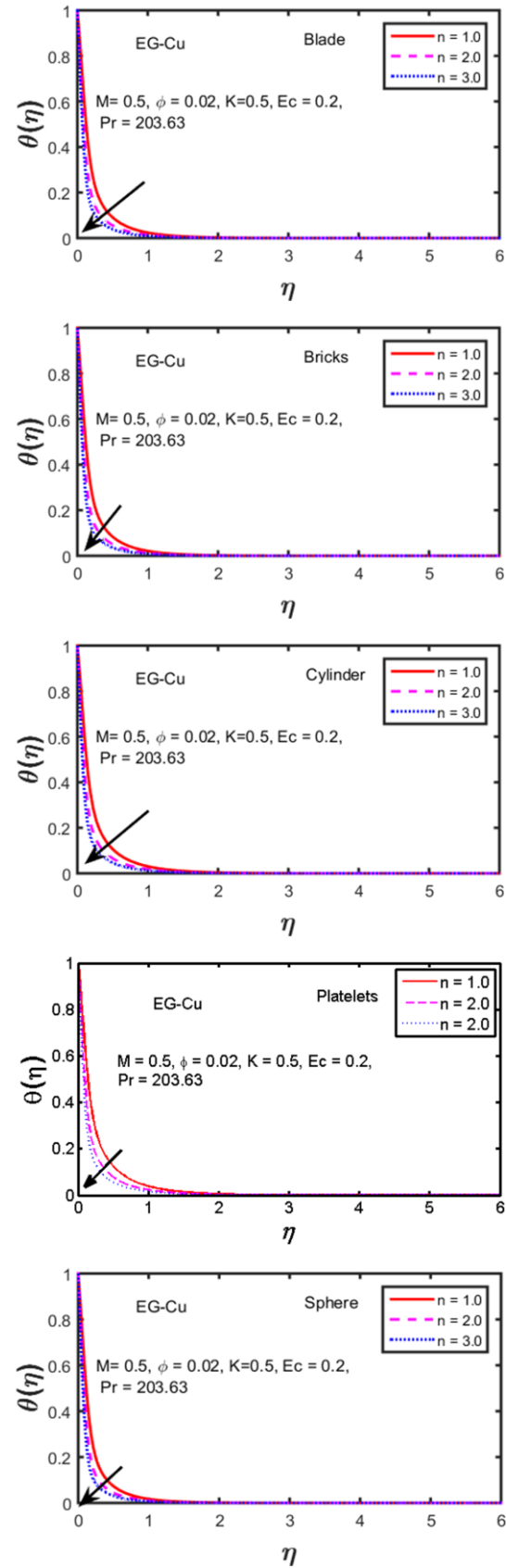


Fig. 9. Effect of the stretching parameter (n) on the temperature profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, and sphere.

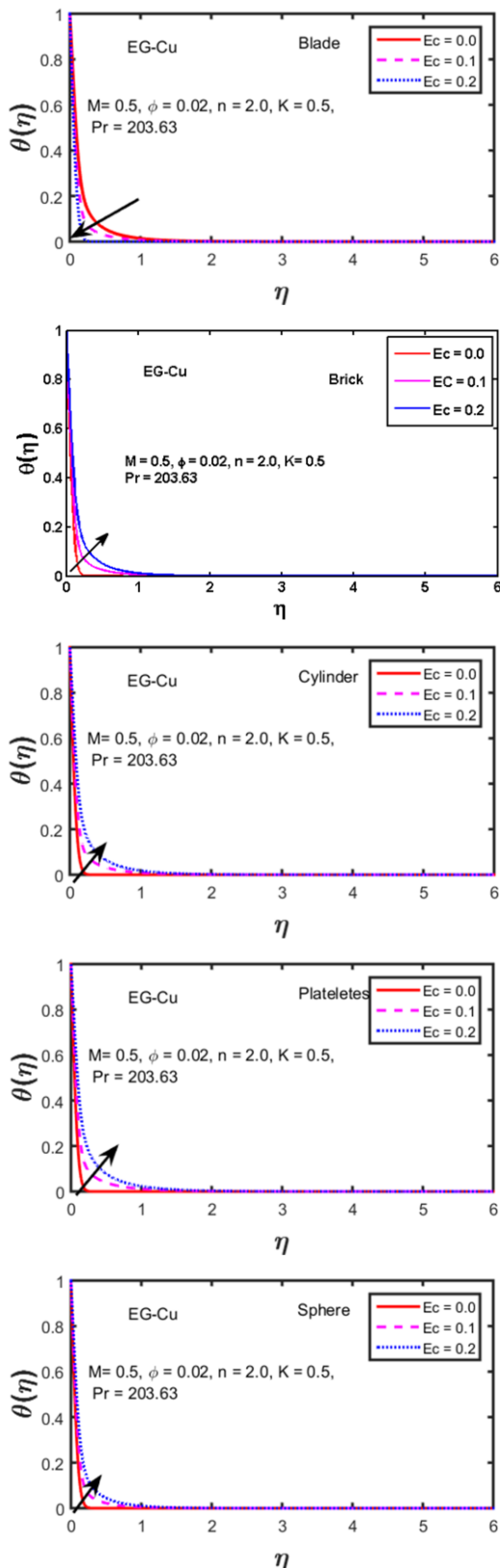


Fig. 10. Impact of the Eckert number (Ec) on the temperature profile of EG-Cu nanofluid for various nanoparticle shapes: blade, brick, cylinder, platelets, sphere.

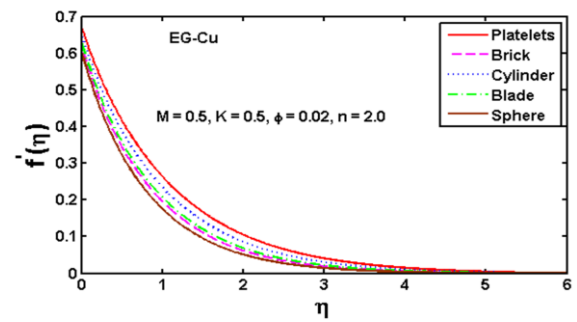


Fig 11. Impact of nanoparticle shape on the velocity profile.

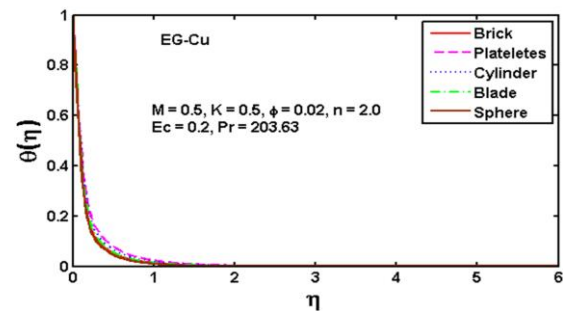


Fig. 12: Impact of nanoparticle shape on the temperature profile.

Table 1. Physical and thermal characteristics of base fluids and copper nanoparticles [32].

Nanoparticles / Base fluid	Specific heat, C_p (J/kgK)	Thermal conductivity, k (W/m K)	Density, ρ (kg/m ³)
Copper	385	401.0	8933
Ethylene glycol	2430	0.253	1115

Table 2. Varying values of the shape factor and viscosity [31].

Parameters / Nanoparticles	A_1	A_2	m
Blade	14.6	123.3	8.26
Brick	1.0	471.4	3.72
Cylinder	13.5	904.4	4.82
Platelets	37.1	612.6	5.72
Sphere	2.5	6.5	3.00

Table 3. Comparison of results for the reduced Nusselt number $\theta'(0)$ considering: $K = 0$, $M = 0$, $\phi = 0$, $Ec = 0$, $n = 1$.

Pr	Khan [1]	Wang [2]	Sidawi [50]	Present
0.7	0.4539	0.4539	0.53	0.4544
2.0	0.9113	0.9114	0.9114	0.9114
7.0	1.8954	1.8954	1.8905	1.8954
20.0	3.3539	3.3539	3.3539	3.3539
70.0	6.4621	6.4622	6.4622	6.4622

Table 4. Values of skin-friction coefficient for multi-shape nanoparticles.

M	K	ϕ	n	$C_f Re^{\frac{1}{2}}$				
				Cylinder	Platelets	Sphere	Blade	Brick
0.5	0.5	0.02	2	-1.138160	-1.306940	-0.830585	-0.989867	-0.928154
0.0	–	–	–	-0.998848	-1.145640	-0.731201	-0.869857	-0.816152
1.0	–	–	–	-1.246380	-1.433560	-0.906007	-1.082130	-1.013850
.5	0.0	–	–	-2.023270	-2.232800	-1.624970	-1.834350	-1.754120
–	1.0	–	–	-0.811870	-0.945852	-0.573870	-0.696044	-0.648416
–	0.5	0.00	–	-0.758920	-0.758920	-0.758920	-0.758920	-0.758920
–	–	0.01	–	-0.903964	-1.010440	-0.794841	-0.868059	-0.817518
–	–	0.02	1	-1.046630	-1.197250	-0.770443	-0.913785	-0.858335
–	–	–	3	-1.210200	-1.393640	-0.877531	-1.049520	-0.982801

Table 5: Comparison of the numerical values of Nusselt number for multi-shape nanoparticles.

M	K	Ec	ϕ	n	$Nu Re^{-\frac{1}{2}}$				
					Cylinder	Platelets	Sphere	Blade	Brick
0.5	0.5	0.2	0.02	2	5.797680	5.45369	6.40553	6.41026	6.22636
0.0	–	–	–	–	8.654940	8.54567	8.79801	9.11713	8.77341
1.0	–	–	–	–	3.635770	3.07073	4.66863	4.40239	4.3504
0.5	0.0	–	–	–	0.340441	-0.13803	1.35575	1.03829	1.0155
–	1.0	–	–	–	6.716180	6.57152	6.89114	7.12693	6.8602
–	0.5	0.0	–	–	11.574800	11.8706	10.91440	11.7129	11.1567
–	–	0.1	–	–	8.686250	8.66215	8.65996	9.06153	8.69152
–	–	0.2	0.00	–	6.892040	6.89204	6.89204	6.89204	6.89204
–	–	–	0.01	–	6.472940	6.26860	6.64252	6.68574	6.62141
–	–	–	0.02	1	3.334780	2.91268	4.12472	3.94395	3.87874
–	–	–		3	7.679450	7.39147	8.15487	8.29984	8.02463

5. Conclusions

In this article, heat transfer enhancement due to variation in copper nanoparticle shape factor for blades, spheres, bricks, cylinders and platelets in a nanofluid over stretching surfaces has been studied theoretically. Additionally, the effect of thermophysical quantities like volume fraction on heat transfer rates has been investigated. It is observed in this study that the surface area of nanoparticles greatly influences the thermal conductivity and all other properties of the nanofluid, and the increase in shape factor values enhances the heat transfer rate. Hence, the heat transfer rate for bricks is the highest in comparison to all other shapes of nanoparticles due to the large interfacial contact area. Regarding the role of the shape factors in heat transfer, it has been concluded that brick-shaped nanoparticles can be considered a good choice for future nanofluid applications in heat exchangers.

Based on the obtained results, the new findings are as follows:

- Nanoparticles in the shape of platelets offer excellent flow and bricks offer excellent heat transfer properties.
- Spherical nanoparticles have poor flow and heat transfer rates as compared to other shapes.
- An increase in the magnetic parameter M reduces velocity and hence decreases the boundary layer thickness.
- The interaction of the magnetic field and the conducting fluid causes Joule heating, which occurs when electrical energy is transformed into heat by flow-induced currents. This increased heat generation influences the temperature distribution and results in an increase in temperature.
- Increasing the slip parameter reduces both velocity and temperature profiles.
- Increasing the stretching parameter value reduces both the velocity distribution and the temperature of the copper nanofluid.
- An increase in the volume fraction enhances velocity and temperature profiles.
- Nusselt number enhances with increasing values of all parameters, such as the slip parameter and stretching parameter.
- Comparing the impact of the Eckert number, volume fraction and magnetic parameter, it was found that the Nusselt number decreases as these parameters increase.
- Skin friction magnitude increases for increasing values of volume fraction, stretching parameter, and magnetic parameter, and decreases for increasing values of slip parameter.

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Exploring potential of paraffin wax: A phase change material for improving yield of solar still

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Abstract

The necessity of providing fresh water to the existing population has remained a significant challenge, highlighting the utmost necessity of a sustainable water purification technique. Therefore, the present review article investigates the recent developments in paraffin wax-based enhancements for solar stills for improving freshwater yield. Paraffin wax, a widely used phase change material (PCM), offers high latent heat storage and chemical stability; however, its low thermal conductivity and long-term reliability present significant challenges. Therefore, this article aims to provide sufficient knowledge on the most practical designs of solar still with paraffin wax. Also, the review highlights design modifications, placement strategies, nanoparticle incorporation, and external attachments like heat exchangers and collectors that have been employed to enhance distillate yield. Economic analyses indicate that paraffin-enhanced systems offer lower water production costs compared to conventional solar stills. Challenges such as phase segregation, volumetric expansion, and material degradation over repeated cycles are discussed. This review aims to provide a comprehensive understanding of optimizing solar still performance using paraffin wax, supporting future research and practical implementation for sustainable water desalination. The key findings indicate that pyramid-shaped solar stills are the most suitable design and their integration with paraffin wax can significantly improve the distillate yield by 100 %.

Keywords: Paraffin wax; Latent heat storage; Solar still; Yield improvement; Placement of PCM; Economic analysis

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1. Introduction

Solar distillation has emerged as an efficient technique for desalinating ocean water. Optimal yield is generally achieved when the operating temperature is maintained within the range

of 40°C to 80°C [1]. Other techniques, such as reverse osmosis (RO) [2], multi-effect desalination [3], nanofiltration [4], humidification, and dehumidification [5,6], can be employed for ocean water treatment, but these techniques cannot be scaled down for remote regions, which is unviable for the end user.

Nomenclature

AC – annual cost, \$
 AFC – annual fixed cost, \$
 AMC – annual maintenance cost, \$
 ASV – annual salvage value, \$
 CPL – cost per litre, \$
 CRF – capital recovery factor
 NPBP – net payback period
 SFF – sinking fund factor

Abbreviations and Acronyms

CSS – conventional solar still
 DSSS – double slope solar still
 PCM – phase change material
 SSSS – single slope solar still
 SBSS – single basin solar still
 TESM – thermal energy storage system
 TPSS – triangular pyramid solar still
 TSS – triangular solar still

Also, the energy supplied to operate these techniques is mainly supplied through thermal power plants that emit a significant amount of CO₂, which is harmful to the environment.

Among the various existing techniques, solar distillation, as shown in Fig. 1, is a promising solution to bridge the gap between accessibility and sustainability, offering a cost-effective, low-maintenance alternative, while significantly mitigating CO₂ through its operational lifespan. The solar distillation technique employs solar still devices to desalinate water, relying on the fundamental principles of evaporation and condensation [7,8]. In this process, initially, impure water is introduced into a basin constructed from high-absorptivity materials such as GI sheet, SS sheet, aluminium, copper, or mild steel [9]. The basin's temperature is elevated as solar radiation passes through a transparent surface, causing an increase in the temperature of the basin's impure water. Subsequently, the impure water undergoes evaporation, transforming into vapour that adheres to the inner surface of the transparent cover. During the condensation process, the evaporated water releases its latent heat by virtue of the temperature gradient between the impure water and the transparent surface (glass). The condensed vapour then drips down and is collected in a container. Later, the water can be used for domestic purposes without compromising taste at an economical cost. Negi et al. [10] reported a cost of \$0.01621 per litre of fresh water produced by integrating black painted Khes wick in a single slope solar still (SSSS). Based on the results, it is evident that the water obtained using a solar still is more economical than the packaged water. Additionally, the study claims yearly carbon mitigation of 2.01 tons and 0.25 tons based on energy and exergy studies.

Solar stills are classified as passive solar stills and active solar stills. Passive solar stills are the primary design, also termed

as conventional solar still (CSS), reaching a maximum efficiency of 34.3% with a total yield of 3.22kg/m²/day [11]. Despite their several advantages, literature quoted the low distillate yield as the biggest disadvantage of the solar still. Hence, literature [12] has recommended solar stills for small families. According to [13], the yield of the solar still depends on the temperature difference between the basin water and the inner glass surface. Therefore, several studies have been conducted to enhance the yield of the CSS through internal and external modifications. These modifications include external glass cooling [14], the application of glass coatings [15], the use of wicks [16], thermal energy storage materials [17], encapsulated PCM [7], concentrator [18], as well as the integration of fins [19], reflectors [20], and design alterations [20,52]. Additionally, it has been observed that further enhancements, such as coupling the system with flat plate collectors (FPC) [16], evacuated tube collectors (ETC) [22] and heat pipes (HP) [23], are highly recommended.

Thermal energy storage materials (TESMs), especially phase change materials (PCMs), are considered due to adequate phase transition temperature, high latent storage capacity, adequate thermal conductivity and specific heat [24,25]. Their thermal energy is stored in the form of sensible heat and latent heat, during storage and release, as can be seen in Fig. 2. High latent heat enthalpy results in a significant energy storage capacity, making it ideal for a wide range of applications. The high latent heat mechanism allows paraffin wax to store a significant amount of energy in a relatively small volume, making it an attractive modification to be incorporated in solar stills. PCMs are frequently placed beneath the basin water to absorb thermal energy during peak load hours and maintain the water temperature suitable for evaporation by liberating heat. PCMs have demonstrated their

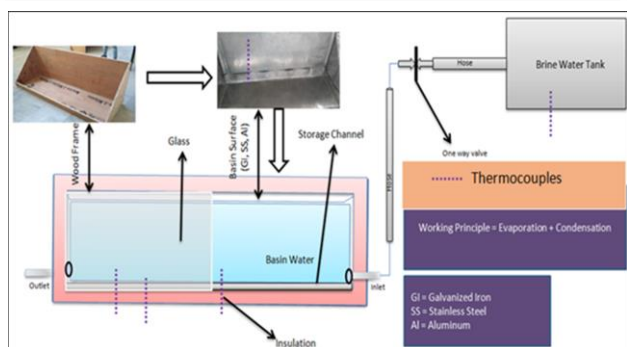


Fig.1. Schematic diagram of solar distillation.

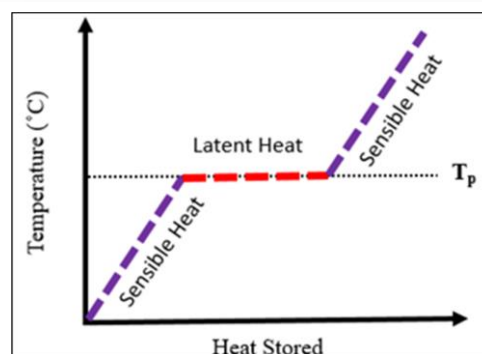
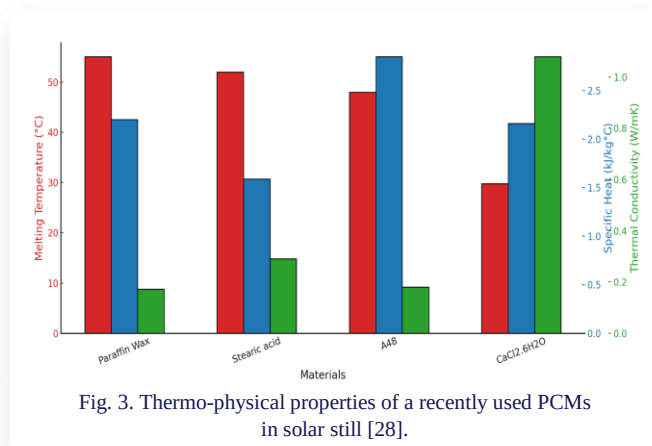


Fig. 2. Energy stored by latent heat storage system (phase change materials).

potential in desalination. According to the literature, myristic acid, palmitic acid, stearic acid, lauric acid, paraffin wax, A39, A42, A48, A55, A58, A62, petroleum jelly, soy wax, bees wax and sheep fat are among the various PCMs, which are already being used in solar stills for improving yield. The corresponding values of the mentioned properties for various PCMs have been discussed by Negi et al. [26]. Moreover, the study reports paraffin wax PCM as an ideal candidate due to the heat storage capacity of approximately 350 kJ/kg, the highest among the reviewed PCMs.

Furthermore, literature [27] has recommended paraffin wax and bees wax for solar distillation application by utilizing a proper MCDM (Multi-Criteria Decision Making) model. In the study, 18 PCMs were investigated based on 7 criteria, including thermo-physical properties, cost, toxicity and compatibility with the container material. The melting temperature, specific heat and thermal conductivity of the few PCMs are illustrated in Fig. 3. The figure suggests that paraffin wax has the highest melting point, followed by stearic acid, A48 PCM and $\text{CaCl}_2 \cdot 6\text{H}_2\text{O}$ PCM. However, the least thermal conductivity has been observed for paraffin and A48 PCM, while the highest was observed for inorganic PCM, i.e. $\text{CaCl}_2 \cdot 6\text{H}_2\text{O}$. This comparative study enlightens the drawbacks of paraffin wax, which can be improved by incorporating nanoparticles [28].

Several reviews are available on the application of PCMs in solar distillation, especially paraffin wax. But, its practicality for real-world application is still lacking, and it is an important aspect to evaluate before application in solar stills. Therefore, this study aims to provide the ongoing recent advancement in paraffin wax-based solar distillation. As a result of this review, the best design, climatic and operational parameters related to paraffin wax use will be considered by summarizing recent literature. The article is divided into five sections: Section 1 introduces the background of the study. Section 2 explores the potential of paraffin wax in solar stills. Section 3 examines the effect of paraffin wax placement on solar still performance, and Section 4 discusses the economic feasibility of a paraffin wax-enhanced solar still. Section 5 addresses the challenges and limitations associated with its application, and Section 6 summarizes the conclusions and proposes future research directions.



2. Paraffin wax incorporated solar still

The effect of a design modification is crucial for improving solar still yield. For instance, an investigation on a triangular solar still (TSS) with paraffin wax was evaluated and the reported results were later compared with conventional solar still (CSS) to observe the feasibility of the modified solar still (MSS).

Figure 4 shows the improvement reported by the various authors using paraffin wax and other modifications in the solar still. Ravishankara et al. [29] reported a noticeable improvement in freshwater yield by 20%. Additionally, the authors claimed that incorporating paraffin wax led to a higher yield at night as compared to day. Moreover, Sathyamurthy et al. [30] tested paraffin wax on a triangular pyramid solar still (TPSS) under the ambient conditions of South India. They reported a 35% improvement compared to CSS with 20 kg water mass and paraffin wax, suggesting water mass as an important parameter to be considered while designing a solar still.

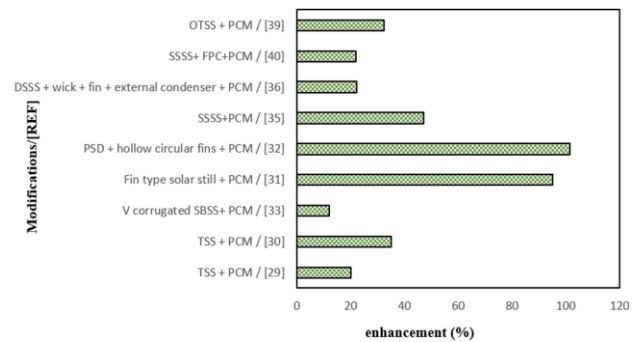


Fig. 4. Distillate yield enhancement for PCM-incorporated solar stills reported by various authors.

The heat transfer has a significant importance when considering a solar still. Therefore, fins are utilized to improve heat transfer through increased surface area, resulting in improved evaporation rate and, consequently, higher yield. Work [55] reported a significant reduction in charging and discharging rate of PCM by 22% and 42%, respectively, for orthogonal fin orientation compared to circumferential fin and no fin configurations. However, the selection of the design of the fins may be a deciding factor and yet needs thorough attention for upcoming studies. In the recent study [31], square hollow fins and paraffin wax improved distillate yield by 95%. Also, the night productivity could be improved by twice as much as a conventional one. Additionally, the effect of a circular hollow fin on a pyramid solar still (PSS) with paraffin wax was investigated by Kabeel et al. [32]. The modified solar still (MSS) significantly improved the yield by 101.5% compared to conventional systems. It was also concluded from the study that the PCM acts as a thermal barrier, which helps to minimize loss from the sides and bottom.

It is obvious to optimize the PCM mass for better yield. So, with a similar perspective, in [33] authors employed paraffin wax in different quantities (18 – 25kg) on the V-shaped corrugated basin surface of the single basin solar still (SBSS). It can be observed from the study that a higher quantity of paraffin wax

increases the yield. Furthermore, the wick placed inside the basin increases the capillary action and reduces evaporation time, leading to higher distillate yield. Additionally, a theoretical study was performed with varying masses of paraffin wax. The results revealed that the mass of PCM is the critical factor in the improvement of the distillate yield [34]. During the study, PCM pouches were heated outside under sunlight and afterwards placed inside the distillation unit. Moreover, an improvement of 47.1% was observed with paraffin wax pouches heated outside and later placed inside the basin. Also, frequent opening of the system while placing the PCM pouches inside the system reduced the water temperature due to heat losses [35]. This can be reduced to a certain level by placing the paraffin wax within the solar still.

As mentioned earlier, the yield of the solar still depends on the basin water and glass temperature. To achieve the maximum water temperature, external modifications such as collectors, heat exchangers, condensers, wick or fin are employed. Tuly et al. [36] earlier reported a 22.33% improvement in the yield by integrating fins, condenser, cotton wick and paraffin with a double slope solar still (DSSS). Meanwhile, they observed a trend in the heat transfer shift of the PCM. Kateker et al. [37] reviewed various PCMs, including organic and inorganic PCMs, in a solar still. They recommended paraffin wax and copper oxide nanoparticles for improvement in distillate yield. However, their study fails to determine the optimal placement of the PCM within the solar still. Furthermore, literature [38] has mentioned the significance of reducing the glass cover temperature. Aly et al. [39] reduced glass temperature by employing a sprinkler attachment, while paraffin wax was used for increasing the water temperature of the MSS. The water depth during the experiments varied at different water depths from 0.5 – 2 cm. The best output was reported for the lowest water depth, with an improvement of 32.42% compared to CSS. Apart from this, Table 1 tabulates various parameters that were considered during experimental research on the solar stills.

3. Placement of paraffin wax on the solar still

Paraffin wax PCM is frequently used by researchers, owing to its adequate thermo-physical properties, chemical stability, low cost and low toxicity. Despite this, placement of a PCM in solar distillation is a crucial step that needs to be optimized according to design, weather, and operational conditions, before its implication. For instance, Banoqitah et al. [41] placed paraffin wax beneath the stepped solar still with and without nanoparticles. Additionally, an adaptive neuro-fuzzy inference system (AN-FIS) and a hybrid adaptive neuro-fuzzy inference system-particle swarm optimizer (PSO) model were utilized to predict the yield of the solar still. The distillate yield was significantly improved by 66.8% with PCM, while 101.2 % improvement was reported with nPCM compared to CSS.

Furthermore, Tan et al. [42] studied the effect of paraffin wax and petroleum jelly on the yield of the solar still. The PCM was placed beneath the basin water in a separate container. The paraffin wax improved the distillate yield by certain values compared to CSS, resulting in achieving 12.17% efficiency, while 6.36% was reported for CSS. Apart from this, utilizing paraffin wax with external attachments is a fruitful modification. Dawood et al. [43] performed an experimental study by integrating paraffin wax with the SSSS. In their study, they modified SSSS with evacuated tube collectors (ETC) filled with paraffin wax, a parabolic trough collector (PTC), and a serpentine heat exchanger. The system was capable of purifying 11.1 L/m²/day of potable water while achieving the highest system efficiency of 34% at the flow rate of 0.5 L/min.

It is evident that paraffin wax has low thermal conductivity. Its utility can be improved by incorporating nanoparticles or using paraffin wax with an active solar still. For instance, Alqsair et al. [44] investigated the effect of a parabolic solar concentrator, paraffin wax and silver (Ag) nanoparticle-based paraffin wax on an active drum solar still. The PCM was placed inside the aluminium drum of the solar still. The study optimized the speed of the rotating drum, with 0.3 rpm reported as the optimal

Table 1. Summary of the relevant studies for solar still with incorporated paraffin wax.

Type of solar still	Location	Angle of inclination	Basin material	Basin area	PCM	Modification	Enhancement, %
Triangular [29]	Chennai	-	GI	-	Paraffin	TSS + PCM	20
Triangular [30]	Chennai	-	GI	-	Paraffin	TSS + PCM	35
Single basin [33]	Egypt	30	Copper	-	Paraffin wax	V-corrugated SBSS+ PCM	12
Single slope [31]	India	12	Galvanized Sheet	0.5 m ³	Paraffin wax	Fin type solar still + PCM	95
Pyramid [32]	Egypt	-	Galvanized Steel	0.64 m ²	Paraffin wax	PSD + hollow circular fins + PCM	101.4
Single slope [35]	Poland	30	Stainless-steel	0.5 m ²	Paraffin wax	SSSS+PCM	47.1
Double slope [36]	Bangladesh	25	GI	0.5 m ²	Paraffin	DSSS + wick + fin + external condenser + PCM	22.33
Single slope [40]	India	17	GI sheet	1.5 m ²	Paraffin wax	SSSS+ FPC+PCM	22
Oval tubular [39]	-	-	Stainless-steel	0.24 m ²	Paraffin wax	OTSS + PCM	32.42

Table 2. Parameters evaluated for economic analysis of paraffin wax incorporated solar still [47–51].

Equations	Economic parameter	Formula
(1)	Annual fixed cost (AFC)	$AFC = CRF \times \text{Initial Investment}$
(2)	Capital recovery factor (CRF)	$CRF = \frac{i(1+i)^n}{((1+i)^n - 1)}$
(3)	Annual maintenance cost (AMC)	$AMC = 0.15 \times AFC$
(4)	Annual salvage value (ASV)	$ASV = S \times SFF$
(5)	Sinking fund factor (SFF)	$SFF = \frac{i}{(1+i)^n - 1}$
(6)	Annual cost (AC)	$AC = AFC + AMC - ASV$
(7)	Cost per L of distillate yield (CPL)	$CPL = \frac{\text{Annual Cost}}{\text{Total Annual Yield}}$
(8)	Net payback period (NPBP)	$NPBP = \frac{\ln \left[\frac{CF}{CF - (AFC \times i)} \right]}{\ln(1+i)}$
Note: i = interest rate, S = salvage value, N = useful life span, CF = (selling price of 1 liter water * annual yield of the solar still)		

speed. The nPCM-based solar still showed a 310% improvement in the yield, while achieving a thermal efficiency of 70.5%. Moreover, the authors observed a significant improvement in the evaporation rate with the use of a parabolic solar collector, which was identified as the driving force behind the enhanced yield. Additionally, the authors reported a significant reduction in losses due to the placement of PCM.

4. Economic, energy and exergy analysis of paraffin wax enhanced solar still

An economic study is an important criterion in assessing the viability of implementing a solar still system. The relevant parameters for evaluation are summarized in Table 2. The annual fixed cost (AFC) of the system can be determined using Eq. (1), where the capital recovery factor (CRF) is calculated as per Eq. (2). In this context, ‘i’ represents the annual interest rate, while ‘n’ denotes the expected useful lifespan of the solar still.

Furthermore, initial investment and annual maintenance are essential to ensure the efficient and reliable operation of the system. The maintenance cost is typically estimated at 15% of the fixed cost, as calculated using Eq. (3). Upon completion of the system's useful life, a salvage value equivalent to 20% of the initial investment is considered, which can be evaluated using Eq. (4). The sinking fund factor (SFF), necessary for long-term financial planning, is computed using Eq. (5). The total annual cost (AC) of operating the system is a key parameter in the economic assessment and can be estimated from Eq. (6). At last, the cost per litre (CPL) of the purified water produced is determined using Eq. (7), providing a direct measure of the system's economic performance. Finally, the net payback period, an essential indicator of investment recovery, is calculated using Eq. (8).

In this context, Suraparaju et al. [45] reported a cost of \$0.0246 per litre for a single basin solar still (SBSS) modified with a pin-finned absorber and paraffin wax, compared to \$0.026 per litre for a conventional solar still (CSS). Concur-

rently, Kabeel et al. [46] investigated the impact of incorporating composite materials and paraffin wax, which resulted in a substantial reduction in the cost of distillate yield, reporting a CPL as low as \$0.00189 per litre.

The feasibility of the modified system can be ensured by calculating the energy and exergy efficiency. In this context, Suraparaju et al. [52] reported \$0.0084 CPL for CSS incorporated paraffin wax. Apart from this, the modified system achieves 26.51% and 2.58% of average energy and exergy efficiency. Moreover, Methre et al. [53] obtained 30.42%, 4.93% and 4.17 L/day of daily average energy efficiency, exergy efficiency and overall yield, respectively. In their study, the thermo-physical properties of the PCM were optimized by varying the Al_2O_3 nanoparticles concentration. Apparently, external attachments are much more beneficial when considering exergy and energy efficiency of the system. For instance, the energy and exergy efficiency in heat pump-assisted solar stills were improved by 65% and 82.3%, respectively, compared to CSS [54].

5. Challenges and limitation associated with paraffin wax for solar distillation application

The past discussions are notably significant, highlighting paraffin wax as a commonly used PCM in the solar still due to its adequate heat storage capacity. However, primary challenges, such as low thermal conductivity, limit its effectiveness during phase transition, resulting in reduced heat transfer rate. Slow heat transfer reduces the availability of stored energy during off-sunshine hours, which ultimately affects the yield of the solar still. Despite the heat transfer limitations, paraffin wax also faces challenges related to its long-term reliability. It has been stated in a past study that its performance deteriorates over repeated thermal cycles. Apparently, phase segregation and volumetric expansion during the melting and solidification process are also a significant concerns to be discussed as they can lead to leakage, encapsulation failures, and reduced latent heat stor-

age capacity over time. These limitations necessitate further research into thermal conductivity enhancement techniques, such as the incorporation of nanoparticles to ensure the long-term stability and viability of paraffin wax in solar distillation applications. Meanwhile, eutectic PCMs are yet to be fully explored as they may offer a solution to the low thermal conductivity of PCM without compromising its other thermo-physical properties.

6. Conclusions and recommendation

Solar distillation systems have attracted significant research interest due to their versatility and ease of use. The current review study focuses on: (1) paraffin wax (PCM) and its role in solar distillation (2) placement of PCM in solar still, (3) the cost analysis.

- a) Paraffin wax is preferred for solar applications due to its high latent heat storage capacity and chemical stability. Paraffin wax remains the most widely used PCM owing to its availability, wide operating temperature range, and non-corrosive behaviour.
- b) The use of paraffin wax inside the basin of the solar still is often mentioned in the literature. An improvement of 47.1% was observed for the case with paraffin wax pouches heated outside and later placed inside the basin. However, losses associated with the time of opening of the solar still are to be considered.
- c) One key challenge observed in traditional solar stills is the frequent opening of the system to add or adjust paraffin wax, leading to temperature losses and reduced efficiency. This issue can be mitigated by placing paraffin wax within the solar still, effectively reducing heat loss and maintaining a more stable water temperature throughout the distillation process.
- d) To optimize solar still performance, the choice of storage container material plays a significant role. Copper, stainless steel, and aluminium are all viable options for storing paraffin wax, with each offering advantages in terms of thermal conductivity and durability.
- e) Copper and aluminium have excellent thermal conductivity and could enhance the heat retention capacity of paraffin wax, improving the overall distillation rate.
- f) Future studies could focus on optimizing the placement of paraffin wax within the still to improve heat transfer efficiency and overall water yield.

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Exergoeconomic Optimisation of a Novel Tri-Evaporator Solar-Biomass Multigeneration System Coupled with Fuel Cell and Electrolyser

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Abstract

This study presents a novel, cost-effective multigeneration system configuration that utilises locally available solar irradiance and biomass resources to produce hydrogen, electricity, heating, hot water, and cooling. The system integrates a range of energy conversion technologies, including non-conventional parabolic trough collectors, conventional biomass gasification, the Kalina cycle, organic Rankine cycle, vapour absorption system, an electrolyser, and a fuel cell. A thermoeconomic analysis was conducted to evaluate system performance, with exergy-based costing applied to determine the operating costs while optimising overall net output. The study also assessed the exergetic sustainability of the system by analysing thermodynamic inefficiencies. It was found that an optimal ambient temperature of approximately 297.4 K maximises the system's exergetic sustainability index, reaching a value of 1.00. The leveled cost of electricity from the Kalina and organic Rankine cycle subsystems was calculated as 0.04308 USD/kWh and 0.0245 USD/kWh, respectively, corresponding to an exergoeconomic factor of 44.51%. When converted to Nigerian currency, these values equate to 69.12 NGN/kWh and 39.31 NGN/kWh – significantly lower than the prevailing electricity tariff in Nigeria, which stands at approximately 209.50 NGN/kWh (0.1306 USD/kWh). Under specified operating conditions, the optimal work outputs of the organic Rankine cycle and Kalina cycle turbines were 47.97 kJ/kg and 435.3 kJ/kg, respectively. The overall energy and exergy efficiencies of the integrated plant were recorded at 52.2% and 16.14%. This multigeneration system demonstrates strong potential as an alternative to fossil-fuel-based power generation, particularly in applications and sectors with low energy demand.

Keywords: Multigeneration; Exergy efficiency; Exergoeconomics; Optimisation; Exergetic sustainability

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1. Introduction

The likelihood of exhausting conventional energy sources becomes increasingly certain in light of the recent correlation between wealth and energy usage, as well as the sharp rise in energy consumption. In addition, sudden changes in the environment brought on by the release of heat and waste materials have

had a severe and unsettling impact on human life as well as endangered animal species. In this regard, global warming represents a challenge that humanity has yet to conquer. It is therefore more important than ever to discover an environmentally acceptable substitute for conventional fuels that performs as well. Renewable energy sources, such as waste heat and waste material sources, may be preferable options in this regard.

Nomenclature

c	– specific cost of product
c_p	– specific heat, J/(kg·K)
$\dot{C}$	– cost rate
$E_{D,Ratio}$	– exergy destruction ratio
$\dot{E}_{D_k}$	– exergy destruction rate, kJ/s
$\dot{E}_{Q_k}$	– exergy associated with heat, kJ
$\dot{E}_{W_k}$	– exergy associated with work, kJ
f	– exergoeconomic factor
h	– specific enthalpy, kJ/kg
H	– higher operating limit
L	– lower operating limit
$\dot{m}_i$	– mass flow rate, kg/s
n	– estimated plant life in years
N	– annual operational hours
P	– pressure
$\dot{Q}_k$	– heat rate to the k^{th} component, kJ/s
s	– specific entropy, kJ/(kg·K)
T	– temperature
$\dot{W}_k$	– rate of work output from the k^{th} component, kJ/s
$\dot{Z}_k$	– levelised cost rate

Greek symbols

ψ_{plant}	– exergy efficiency
Γ_{eff}	– environmental effect factor
θ_{est}	– exergetic sustainability index

Subscripts and Superscripts

F	– fuel
i	– inlet

j	– outlet stream
k	– element, component
o	– outlet
P	– product
q	– heat transfer
turb	– turbine
w	– work

Abbreviations and Acronyms

Al ₂ O ₃	– aluminium dioxide
CMP	– compressor
CON	– condenser
COP	– coefficient of performance
CRF	– capital recovery factor
EES	– engineering equation solver
ESI	– exergetic sustainability index
EVP	– evaporator
HEX	– heat exchanger
KC	– Kalina cycle
LiBr	– lithium bromide water
MWCNT	– multi-walled carbon nanotube
NH ₃	– ammonia
ORC	– organic Rankine cycle
PEC	– purchase and equipment cost
PECF	– purchase of equipment cost fund
PEM	– proton exchange membrane
PTC	– parabolic trough collector
PUM	– pump
S	– separator
VAS	– vapour absorption system
VG	– vapour generator

However, the idea of integrating systems with multiple production offers a big possibility [1]. In this regard, multiple attempts have been made to integrate diverse energy technologies in order to reduce carbon emissions while also providing a more efficient and reliable energy supply [2]. Integration, which involves connecting two or more energy technologies inside a single structure, provides advantages over their stand-alone equivalents due to the mutual utilisation of substructures and feedstock [3]. However, unique engineering methodologies, such as thermodynamic studies, should be used to analyse the efficacy of such novel integrated systems from an environmental and budgetary standpoint [4].

A multigeneration system generates electricity, heating, cooling, hot water, freshwater, hydrogen, and other energy from a single or multiple sources. The multi-generation system produces electricity using one or more prime movers, such as a gas turbine, Rankine cycle, or organic Rankine cycle (ORC). Because of its suitability for recovering energy from low-grade heat sources, the ORC is a process that might be incorporated into a multigeneration system [5]. Multi-generation systems can achieve better efficiency than single-generation systems [1]. Furthermore, it has been demonstrated that in these integrated systems, fuel consumption may be lowered by using waste heat and renewable energy. As a result, these sources have the potential to reduce emissions while also diversifying commodities [6]. The complexity of energy flows in integrated systems, such as

multigeneration systems with various outputs, makes cost allocation to each product a big difficulty [7]. Researchers have used a variety of cost allocation approaches, but the thermoeconomic or exergoeconomic method is suggested due to its ability to properly calculate the costs of various items [1]. Exergoeconomic analysis is a useful method for improving the performance of multigeneration systems [8] by integrating exergy and economic analysis [9]. Several research works utilised the exergoeconomic technique to improve their systems. A thermoeconomic multi-objective optimisation was performed by Ahmadi et al. [10] on a novel integrated multigeneration system consisting of a biomass combustor, proton exchange membrane (PEM) electrolyser, organic Rankine cycle, double effect absorption chiller, domestic water heater, reverse osmosis desalination, and a heat exchanger. They observed that increasing design parameters within their maximum values enhances both the multi-objective function of the total cost rate and the system's exergy efficiency. Calise et al. [11] simulated a novel solar-geothermal-based multigeneration system producing electricity, heating, cooling and fresh water. The system is built on a combination of thermal recovery system (TRS), ORC, parabolic trough collectors (PTC) and multi-effect distillation. Their exergetic and exergoeconomic analysis results were presented on a daily, weekly and annual basis. The system cost led to a high price of electricity in the energy market. Baghernejad et al. [12] conducted an exergoeconomic optimisation and environmental analysis of

a new solar trigeneration system based on an integrated solar combined cycle system (ISCCS) (power generating unit), a cooling unit, a heating unit and an electrical generator. Optimisation results obtained show a 26.34% increase in exergy efficiency and 11.5% decrease in unit cost of products, a 24.17% decrease in fuel exergy, a 21.63% reduction of total exergy destruction and a 24.17% reduction in CO₂ emission cost. Moharamian et al. [13] performed an exergoeconomic and thermodynamic analysis of the influences of injecting hydrogen into the combustion chamber of a biomass-based externally fired combined cycle hydrogen injection (EFCCHI) and compared it with hydrogen production with no injection (EFCCH). They found that hydrogen injection into the combustion chamber reduces fuel consumption by 27%, exergy and energy efficiencies by 45% and CO₂ emissions by 32%. Also, exergy loss and destruction cost rates were reduced by 78% and 10% respectively. Khalid et al. [14] proposed a thermoeconomic analysis of a hybrid solar-biomass multigeneration system for a community comprising an absorption chiller, a solar cycle, two organic Rankine cycles and a Brayton cycle to produce electricity, hot water, cooling and heating. Their results show energy, exergy efficiencies, electricity system cost and levelized cost of electricity as 91.0%, 34.9%, \$2 700 496 and \$0.117/kWh, respectively. Khanmohammadi et al. [15] conducted a thermoeconomic modelling and optimisation of a multigeneration system comprising a solar flat plate collector, an organic Rankine cycle, a PEM electrolyser and a single effect absorption chiller. Their results indicate that exergy efficiency and cost of the system can simultaneously increase from 1.72% to 3.2% and 19.59 \$/h to 22.28 \$/h, respectively. Also, raising the evaporator temperature of the ORC unit to the upper limit increases electric power generation and increases the production rate of hydrogen from 0.23 to 0.35 kg/day but decreases the cooling capacity from 44.29 kW to 22.6 kW. A novel solar power-based polygeneration system producing electricity, fresh water, cooling, domestic hot water, and hydrogen was analysed by El-Emam and Dincer [16]. The results of the thermoeconomic and multi-objective optimisation of the system show a maximum exergy efficiency of 39% obtained at a cost rate of 309 \$/h and a lowest exergy efficiency of 21.7% at a rate of 241 \$/h. Khanmohammadi et al. [17] proposed and studied the thermoeconomic and multi-objective optimisation of a multigeneration structure with a thermoelectric waste heat recovery system (WHRS). The system is composed of a gas turbine cycle, an organic Rankine cycle and a domestic water heater. They obtained that the efficiencies of the system can be raised from 16.76% to 17.93% by attaching a WHRS on the ORC condenser. Behzadi et al. [18] proposed an integrated energy system composed of a biomass-based solid oxide fuel cell (SOFC) with a gas turbine, a reverse osmosis desalination unit and a double effect absorption chiller. Thermoeconomic and optimisation study results show the gasifier as the primary source of irreversibility with an exergy destruction rate of 179.8 kW; the pump and SOFC stack have the highest exergoeconomic factor at optimal operating conditions; the exergy efficiency is 38.16% and the total product unit is 69.47 \$/GJ. Anvari et al. [19] proposed and conducted a thermodynamic, exergoeconomic and environmental analysis of a gas turbine cycle-based

multigeneration system with absorption refrigeration cycle, ORC and desalination unit subsystems. The results of their assessment show that the overall cost and CO₂ emissions are 1943.5 \$/h and 0.163 kg/kWh, respectively. For every 83.56 MW energy input into the cycle, electric power of 30.5 MW, cooling of 1MW, heat of 40.8MW and freshwater of 0.364 kg/s were generated. Also, raising temperatures of the pre-heater outlet air (800 K–1000 K) and gas turbine inlet (1320–1570 K) reduces CO₂ emissions of the cycle by 26.6% and 53%, respectively. Abdelhay et al. [20] conducted a thermodynamic analysis of a solar-based multigeneration system powered by a natural gas heater. The system layout includes a solar power cycle, a multi-effect desalination unit, and an absorption refrigeration unit, which generates electricity, fresh water, and cooling for around 1000 households. The modelling findings suggest that the maximum total system exergy efficiency is 23.95%, compared to 23.14% or less for any subsystem that is stand-alone or dual-purpose. The system also offers the lowest unit prices of water and cooling, at 1.247 \$/m³ and 0.003 \$/kWh, respectively.

Alirahmi et al. [21] conducted a thermo-economic analysis and multi-criteria design optimisation of a geothermal and solar-driven multigeneration system, using the metrological data of three cities. The system layout consists of a geothermal cycle, solar cycle, Rankine cycle, thermoelectric generator, ORC, absorption chiller, desalination unit and an electrolyser, which they aimed to produce space heating, space cooling, fresh water, hot water, electric power and hydrogen. Results of their study show exergy efficiency and cost rate as 21.63% and 63.89 \$/h, respectively. Also, on a typical sample day, the system produced 373.8 m³ of fresh water and 26.38 kg of hydrogen in one city. A thermodynamic and exergoeconomic analysis of a novel solar-assisted multigenerational system utilising high-temperature phase change material and hybrid nanofluid was performed by Khan et al. [22]. Their system integrated a parabolic dish collector with hybrid nanofluids, double effect lithium-bromide/water absorption cycle, PEM electrolyser, recompression sCO₂ Brayton cycle, and a desalination unit, producing electricity, cooling, hydrogen and fresh water. Results show that fresh water and cooling productions are 1.564 kg/s and 196.1 kW, respectively, and the total energy and exergy efficiencies of the analyzed system are 31.59% and 30.02%, respectively, while the exergoeconomic results show the total cost rate of exergy destruction and levelised cost of electricity are 530 \$/h and 0.1387 \$/kWh, respectively.

The following evaluation objectives were taken into consideration while conducting the above thorough literature review: the architecture of the multigeneration system, the system's primary energy resources and how they are converted, the effective utilisation of energy sources for each subsystem, the method of producing hydrogen, the decarbonization of power production, the system and its primary end products. It is found that a multigeneration energy system powered by a combination of biomass and solar energy sources with PTC system running on a hybrid nanofluid (MWCNT-Al₂O₃ / therminol VP1), incorporating and utilising the tri-evaporator system has not been investigated by integrating a unique ORC system combined with

a Kalina cycle (KC), vapour absorption system, proton exchange membrane, and a fuel cell, using minimum energy of the configured system to generate electricity, heating, cooling, hot water, and hydrogen. Furthermore, the performance of linked subsystems in such a multigeneration system must be investigated under various operational conditions. It is anticipated that these systems will open up a number of new possibilities for using biomass and solar energy sources.

Inspired by these gaps in the literature, this study proposes an integrated energy system that generates clean energy from renewable sources. A key innovation lies in the use of a single working fluid to drive three interconnected subsystems. By strategically bleeding the fluid during turbine expansion, the same medium is repurposed for cooling production. This integrated design shortens the thermodynamic process path, reduces system complexity, and minimises the number of components required. As a result, it enhances overall performance, boosts energy utilisation, and achieves a higher cooling-to-power ratio – an especially noteworthy aspect of this study. Furthermore, the paper develops models for an innovative cycle featuring a tri-evaporator system. This includes a conventional evaporator in the vapour absorption system (VAS) and unconventional evaporators in both the ORC and KC subsystems. The tri-evaporator configuration provides numerous benefits, such as increased cooling capacity, improved heat recovery, enhanced flexibility, reduced temperature differentials, greater system reliability, better heat transfer, and improved overall energy efficiency. By enabling heat recovery from multiple sources, mini-

misizing thermal stress, and offering operational redundancy, this configuration represents a significant advancement in integrated energy systems.

The objectives of this research include the development of a new renewable energy-based integrated system capable of producing various utilities, economic assessment of the system in terms of the levelized cost of electricity and net present cost, and the integrated system exergetic sustainability analysis and optimisation. To achieve this, the study involves a thorough investigation using: exergoeconomic balance equations for each system component analysing how state properties and varying operating conditions impact individual subunit performance as well as the system's overall performance; sustainability analysis with regards to total system output and the gross exergy destruction in the plant; and optimization, with respect to optimum selection of operating parameters for the ORC and Kalina turbines.

2. System description

Figure 1 depicts the suggested multigeneration design. According to the system overview, the integrated energy system's primary energy source is the solar-powered parabolic trough collector (PTC). Other energy sources include the KC unit, the ORC unit, the vapour absorption system (VAS), a proton exchange membrane (PEM) electrolyser that produces hydrogen through a biomass combustor, and a fuel cell that runs on the hydrogen produced by the PEM electrolyser. The workings of these subsystems are vividly described below.

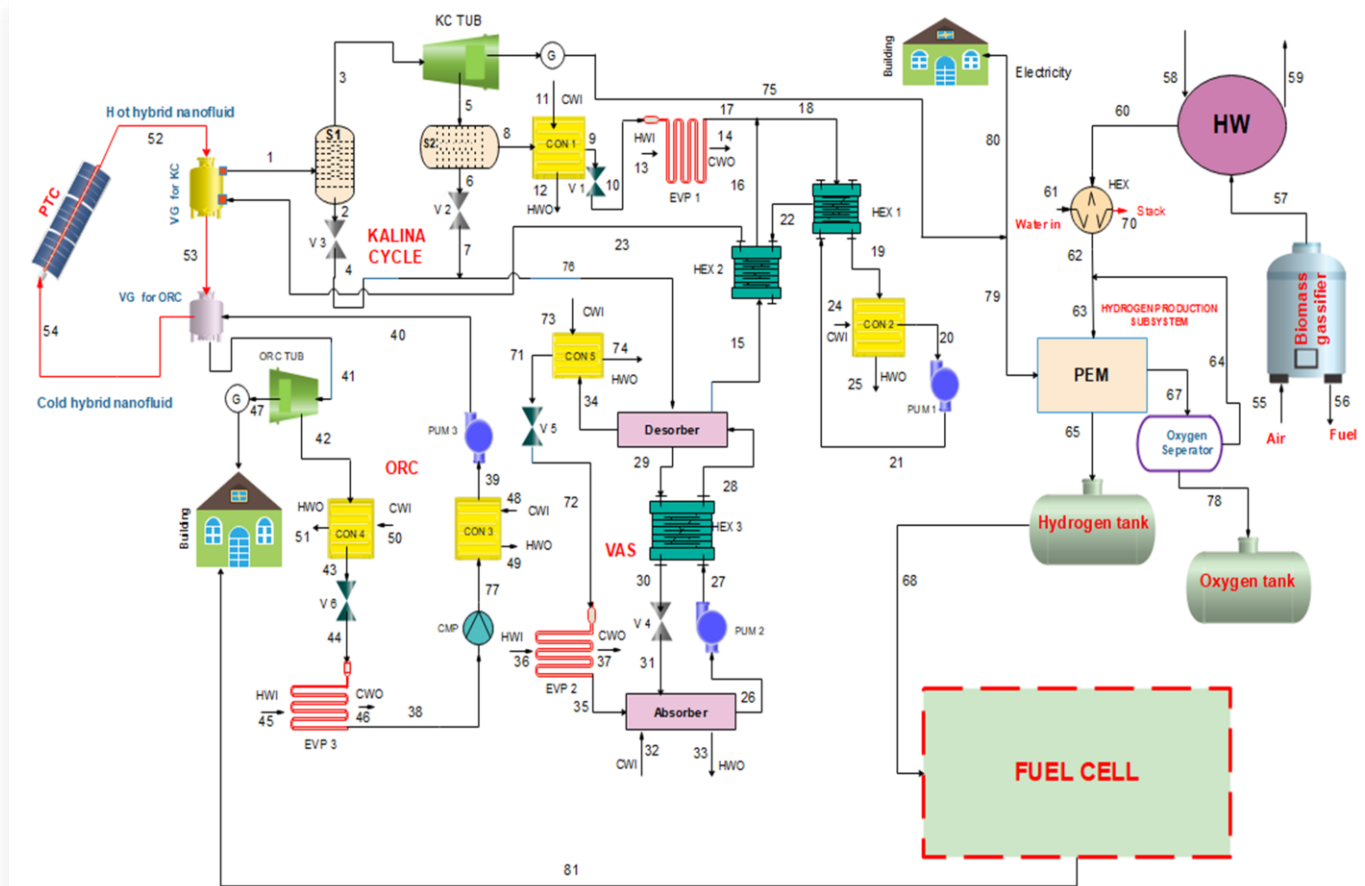


Fig. 2. Schematic diagram of the proposed solar-biomass-based multigeneration system.

2.1. Parabolic solar trough collector

The process begins with PTC, where a hybrid nanofluid, consisting of multi-walled carbon nanotubes and aluminium dioxide (MWCNT- Al_2O_3) mixed with therminol-VPI, serves as the heat transfer fluid. This fluid enters the solar collector as stream 54, where its energy is increased. The superheated fluid, now exiting as stream 52, first flows into the vapour generator associated with the Kalina cycle (VG for KC), transferring a portion of its thermal energy to the Kalina cycle's working fluid. After this, the fluid, now as stream 53, enters the vapour generator for the organic Rankine cycle (VG for ORC) and transfers additional heat energy to the ORC working fluid. Finally, the hybrid nanofluid returns to its initial state as stream 54, continuing the cycle uninterrupted.

2.2. Kalina cycle

The superheated steam-ammonia ($\text{NH}_3\text{-H}_2\text{O}$), which serves as the working fluid (stream 1) for the Kalina cycle, exits the vapour generator (VG for KC) and enters a separator (S1), where it splits into stream 2 and stream 3. Stream 2 is a high-pressure, high-temperature condensate that passes through a throttling valve, resulting in low-pressure stream 4. Stream 3, consisting of high-temperature and high-pressure steam-ammonia, expands through the Kalina turbine, converting some of its thermal energy into mechanical shaft work that drives an alternator to produce electricity. The expanded vapour (stream 5) then enters a second separator (S2), splitting into a lower-pressure condensate (stream 6) and another lower-pressure lower-temperature vapour (stream 8). Stream 6 is throttled into low-pressure stream 7, which then mixes uniformly with stream 4 to form stream 76. Stream 8 is condensed into a lower-temperature stream 9 and throttled into a lower-pressure stream 10, where it gains heat through the cooling process in the evaporator (EVP1). It exits as stream 17, mixing uniformly with stream 16 to form stream 18. This stream transfers its heat energy in the heat exchanger (HEX1), exits as stream 19, and further loses heat in the low-pressure condenser (CON2), emerging as stream 20. The pressure of stream 20 is increased by the pump (PUM1), resulting in stream 21, which gains heat in HEX1 to become stream 22 and then gains additional heat in heat exchanger (HEX2), exiting as stream 23, which returns to VG for KC, continuing the cycle in this manner.

2.3. Organic Rankine cycle

The working fluid (R245fa) in the ORC subsystem absorbs heat energy from the vapour generator (VG for ORC) and flows as stream 41 to the turbine, where it expands and converts part of its heat energy into mechanical shaft work, driving an alternator to generate electricity. The fluid then exits as stream 42 and enters the high-pressure condenser (CON4), where it loses some energy before exiting as stream 43. The pressure of stream 43 is further reduced through throttling, resulting in stream 44. It then gains heat energy through the cooling process in the evaporator (EVP3) and exits as stream 38, which is compressed by the compressor (CMP) into stream 77. Stream 77 flows into the low-pressure condenser (CON3), releases some heat energy, and exits as a low-temperature low-pressure stream 39. This stream is

then pumped by pump (PUM3) back to the VG for ORC as stream 40, continuing the cycle in this sequence.

2.4. Vapour absorption system

The ammonia-water fluid (stream 76) from the Kalina cycle separators (S1 & S2) transfers heat to the lithium bromide water ($\text{LiBr-H}_2\text{O}$) solution in the desorber and exits as stream 15. When the weak $\text{LiBr-H}_2\text{O}$ solution (stream 28) is heated, some of the absorbed water is released as vapour (stream 34). This process makes the remaining solution more concentrated (stream 29). The vapour from the generator/desorber is then condensed into a saturated liquid in a condenser (stream 34 to stream 71), where the rejected heat is used for space heating or hot water production. The condensed liquid is then throttled to a lower pressure, determined by setting the evaporator temperature to the saturation temperature and finding the corresponding saturation pressure. After throttling, the refrigerant is in a saturated liquid-vapour state (stream 72). It then passes through the evaporator, absorbing heat from the cooled or air-conditioned space. After the evaporator, the refrigerant goes to the absorber (stream 35). The strong solution from the desorber passes through a heat exchanger (HEX3), recovering some heat (stream 29 to stream 30) that was added in the desorber. This heat is then transferred to the incoming weak solution (stream 27 to stream 28). After the heat exchanger, the strong solution is throttled to the absorber's lower pressure (stream 31). The strong solution (stream 31) entering the absorber is cooled by releasing some heat into the atmosphere. The lower temperature increases its ability to absorb water vapour, so the vapour from the evaporator is absorbed to form a weak $\text{LiBr-H}_2\text{O}$ solution (stream 26). This weak solution is then pumped to the desorber's higher pressure (stream 27) and passes through the heat exchanger (HEX3) to gain some heat (stream 28) before returning to the desorber. The cycle then repeats.

2.5. Biomass combustor, proton exchange membrane electrolyser and fuel cell

The heat released as flue gases (stream 57) from burning fuel derived from biomass gasification is directed to the hot water generating unit, where it transfers its heat to produce domestic hot water. The flue gases then exit (stream 60) into a heat exchanger (HEX), transferring energy to the incoming water (stream 61). The spent flue gases (stream 70) are expelled through the stack, while the heated water (stream 62) and hot water by-product (stream 64) from the proton exchange membrane (PEM) electrolyser mix uniformly as stream 63. Part of the electric power generated from the Kalina cycle (stream 79) is used in the electrolysis reaction to decompose the heated water (stream 63) in the PEM electrolyser, producing hydrogen (stream 65) for the H_2 storage tank, and a mixture of oxygen and hot water (stream 67). This mixture is separated into oxygen (stream 78) for the O_2 storage tank and hot water (stream 64), which is partially fed back into the PEM electrolyser. The chemical energy of the fuel, hydrogen (stream 68), and oxygen from the air is converted into electricity (stream 80) in the fuel cell unit through redox reactions.

3. Exergoeconomic Modelling and Analysis

The system's exergy-based costing is performed to determine the cost of the systems' products with regard to the components' cost, the lifetime of the system, and the number of hours of operation of the system. It also assists in quantifying the payback period and the economic losses accruable due to exergy destruction. This will assist in proper equipment sizing for reduction in economic losses during plant life. The components' purchase and equipment cost (PEC) are written as functions of their operating parameters, and are required to calculate the levelised cost rates during plant life operations in consonance with the cost equation balance. The PEC for the plant components can be found in related literature [23,24].

The general cost balance for a control volume is [25]:

$$\dot{C}_{q,k} + \sum_i \dot{C}_{i,k} + \dot{Z}_k = \sum_e \dot{C}_{e,k} + \dot{C}_{w,k}. \quad (1)$$

The terms in Eq. (1) represent the cost of heat transfer to the system's k^{th} component $\dot{C}_{q,k}$, the cost associated with the j^{th} sum of exergy streams to the system's k^{th} component $\sum_i \dot{C}_{i,k}$, and the levelised cost rate for the k^{th} component, $\dot{Z}_k$. The other terms are the cost associated with the j^{th} sum of exergy streams from the system's k^{th} component $\sum_e \dot{C}_{e,k}$, while the work associated with the k^{th} component in the system is denoted with the term $\dot{C}_{w,k}$.

The cost of the j^{th} stream is related to the cost of specific cost and the exergy, work or heat with the relationships:

$$\dot{C}_j = c_j \dot{E} x_j, \quad (2)$$

$$\dot{C}_{w,k} = c_w \dot{W}, \quad (3)$$

$$\dot{C}_{q,k} = c_q \dot{Q}. \quad (4)$$

The cost rate $\dot{Z}_k$ for the components is expressed as:

$$\dot{Z}_k = \frac{PECF \times CRF \times \phi}{N \times 3600}, \quad (5)$$

where PECF, CRF, and ϕ represent the purchase of equipment cost function, capital recovery factor, and maintenance factor, respectively.

The system's annual operational hours are denoted with N . Accordingly, the capital recovery factor is obtained with the relationship:

$$CRF = \frac{i|1+i|^n}{|1+i|^n - 1}, \quad (6)$$

where n is the estimated plant life in years.

Another exergy-related index is the cost of exergy of the product and fuel for the component. Detailed description of component cost of fuel and product is found in [26], but the principle is applied to this multigeneration system, and the balance is shown in Table 1.

The general relationships for evaluating the specific cost of product for the k^{th} component $c_{p,k}$ (\$/kJ), and that of fuel $c_{F,k}$ (\$/kJ), as well as the exergoeconomic factor, f are listed in Eqs. (7)–(9), respectively:

$$c_{p,k} (\$/kJ) = \frac{\dot{C}_{p,k}}{\dot{E}_{p,k}}, \quad (7)$$

$$c_{F,k} (\$/kJ) = \frac{\dot{C}_{F,k}}{\dot{E}_{F,k}}, \quad (8)$$

$$f = \frac{\dot{Z}_k}{\dot{Z}_k + \dot{C}_{D,k}}. \quad (9)$$

The cost of exergy destruction, which quantifies the economic loss attributed to inefficiencies, is expressed with the relationship:

$$\dot{C}_{D,k} = c_{p,k} E_{D,k}. \quad (10)$$

From Eq. (1), the cost balances and auxiliary relations are formulated for each of the components of the multigeneration system in Table 1.

3.1. Exergy based sustainability analysis

Exergy based sustainability analysis is used to analyse the availability of energy resources at low cost, with little or no effect on the environment [27]. The environmental effect is accounted for based on the magnitude of exergy destruction in the system, while the availability is based on the total net products from the system. The exergetic sustainability indicators considered in this study are exergy efficiency, exergy waste ratio, environmental effect factor and exergetic sustainability index were obtained from [28] and [29].

3.2. Sustainability indicators

The sustainability indicators are listed as a function of exergy efficiency, ψ_{plant} , exergy destruction ratio, $E_{D, Ratio}$, waste exergy ratio, Γ_w , environmental effect factor, Γ_{eff} , and exergetic sustainability index, θ_{esi} . The exergetic efficiency of the multigeneration plant is expressed as the ratio of exergy of product to fuel exergy [30]; the exergy waste ratio as the ratio of exergy waste to the chemical exergy of fuel. The ratio of total exergy destruction in the plant to the exergy of fuel is termed the exergy destruction ratio; while the environmental effect factor is the ratio of the waste exergy ratio to the exergy efficiency. Lastly, the exergetic sustainability index is the reciprocal of the environmental effect factor. The range of this index is between 0 and ∞ [28]. Mathematically, the indicators are expressed mathematically as:

$$\psi_{plant} = \frac{E_{x,product}}{E_{x,fuel}}, \quad (11)$$

$$\Gamma_w = \frac{E_{waste}}{E_{x,fuel}}, \quad (12)$$

$$E_{D,Ratio} = \frac{\sum E_D}{E_{x,fuel}}, \quad (13)$$

$$\Gamma_{eff} = \frac{\Gamma_{w,e}}{\psi_{plant}}, \quad (14)$$

$$\theta_{esi} = 1/\Gamma_{eff}. \quad (15)$$

3.3. System optimisation

The optimisation of the system is performed to maximise the turbine work of the ORC and Kalina systems. With regard to the

Table 1. Summary of component costs and auxiliary equations.

Component	Component exergoeconomic balance	Auxiliary equation
Parabolic trough collector	$\dot{c}_{54} + \dot{c}_{Q_{solar}} + \dot{Z}_{trough} = \dot{c}_{52}$	Not applicable
Kalina VG	$\dot{c}_{52} + \dot{c}_{23} + \dot{Z}_{VG_{KC}} = \dot{c}_1 + \dot{c}_{53}$	$E_{53}\dot{c}_{52} - E_{52}\dot{c}_{53} = 0$
ORC VG	$\dot{c}_{53} + \dot{c}_{40} + \dot{Z}_{VG_{ORC}} = \dot{c}_{54} + \dot{c}_{41}$	$E_{54}\dot{c}_{53} - E_{53}\dot{c}_{54} = 0$
ORC turbine	$\dot{c}_{41} + \dot{Z}_{ORC_{TURB}} = \dot{c}_{42} + W_{TURB} + W_{P_3} + W_{ORC_{CMP}}$	$E_{42}\dot{c}_{41} - E_{41}\dot{c}_{42} = 0;$ $W_{P_3}\dot{c}_{W_{turb}} - W_{turb}\dot{c}_{W_{P_3}} = 0;$ $W_{ORC_{CMP}}\dot{c}_{W_{turb}} - W_{turb}\dot{c}_{W_{ORC_{CMP}}} = 0.$
ORC CON 4	$\dot{c}_{42} + \dot{c}_{50} + \dot{Z}_{CON4} = \dot{c}_{51} + \dot{c}_{43}$	$E_{43}\dot{c}_{42} - E_{42}\dot{c}_{43} = 0$
ORC valve (43, 44)	$\dot{c}_{43} + \dot{Z}_{VALVE} = \dot{c}_{44}$	Not applicable
ORC EVP 3	$\dot{c}_{44} + \dot{Z}_{EVP3} + \dot{c}_{45} = \dot{c}_{38} + \dot{c}_{46}$	$E_{46}\dot{c}_{45} - E_{45}\dot{c}_{46} = 0$
ORC Compressor	$\dot{c}_{38} + \dot{Z}_{ORC_{CMP}} + \dot{c}_{ORC_{CMP}} = \dot{c}_{77}$	Not applicable
ORC CON 3	$\dot{c}_{77} + \dot{c}_{48} + \dot{Z}_{CON3} = \dot{c}_{49} + \dot{c}_{39}$	$E_{39}\dot{c}_{77} - E_{77}\dot{c}_{39} = 0$
ORC pump 3	$\dot{c}_{39} + \dot{Z}_{ORC_{P_3}} + \dot{c}_{W_{P_3}} = \dot{c}_{40}$	Not applicable
Kalina separator 1	$\dot{c}_1 + \dot{Z}_{S_1} = \dot{c}_2 + \dot{c}_3$	$E_3\dot{c}_2 - E_2\dot{c}_3 = 0$
Kalina separator 2	$\dot{c}_5 + \dot{Z}_{S_2} = \dot{c}_6 + \dot{c}_8$	$E_8\dot{c}_6 - E_6\dot{c}_8 = 0$
Kalina valve V3 (2,4)	$\dot{c}_2 + \dot{Z}_{V_3} = \dot{c}_4$	Not applicable
Kalina turbine	$\dot{c}_3 + \dot{Z}_{KC_{TURB}} = \dot{c}_5 + \dot{c}_{W_{KC_{TURB}}} + \dot{c}_{W_{P_1}} + \dot{c}_{W_{P_2}}$	$E_5\dot{c}_3 - E_3\dot{c}_5 = 0;$ $W_{P_1}\dot{c}_{W_{KC,turb}} - W_{KC,turb}\dot{c}_{W_{P_1}} = 0;$ $W_{P_2}\dot{c}_{W_{P_1}} - W_{P_1}\dot{c}_{W_{P_2}} = 0.$
Kalina valve, V2(6/7)	$\dot{c}_6 + \dot{Z}_{V_2} = \dot{c}_7$	Not applicable
Kalina CON 1	$\dot{c}_8 + \dot{c}_{11} + \dot{Z}_{CON1} = \dot{c}_9 + \dot{c}_{12}$	$E_9\dot{c}_8 - E_8\dot{c}_9 = 0$
Kalina EVP 1	$\dot{c}_{10} + \dot{c}_{13} + \dot{Z}_{EVP1} = \dot{c}_{17} + \dot{c}_{14}$	$E_{14}\dot{c}_{13} - E_{13}\dot{c}_{14} = 0$
Kalina CON 2	$\dot{c}_{19} + \dot{c}_{24} + \dot{Z}_{CON2} = \dot{c}_{25} + \dot{c}_{20}$	$E_{20}\dot{c}_{19} - E_{19}\dot{c}_{20} = 0$
Kalina pump 1	$\dot{c}_{20} + \dot{c}_{W_{P_1}} + \dot{Z}_{P_1} = \dot{c}_{21}$	Not applicable
Kalina HEX 1	$\dot{c}_{18} + \dot{c}_{21} + \dot{Z}_{HEX1} = \dot{c}_{22} + \dot{c}_{19}$	$E_{19}\dot{c}_{18} - E_{18}\dot{c}_{19} = 0$
Kalina HEX 2	$\dot{c}_{22} + \dot{c}_{15} + \dot{Z}_{HEX2} = \dot{c}_{16} + \dot{c}_{23}$	$E_{23}\dot{c}_{22} - E_{22}\dot{c}_{23} = 0$
VAS desorber	$\dot{c}_{76} + \dot{c}_{28} + \dot{Z}_{Desorber} = \dot{c}_{34} + \dot{c}_{29} + \dot{c}_{15}$	$E_{15}\dot{c}_{76} - E_{76}\dot{c}_{15} = 0; \quad E_{34}\dot{c}_{29} - E_{29}\dot{c}_{34} = 0.$
VAS CON. 5	$\dot{c}_{34} + \dot{c}_{73} + \dot{Z}_{CON5} = \dot{c}_{71} + \dot{c}_{74}$	$E_{71}\dot{c}_{34} - E_{34}\dot{c}_{71} = 0$
VAS valve, V5 (71,72)	$\dot{c}_{71} + \dot{Z}_{V_5} = \dot{c}_{72}$	Not applicable
VAS EVP 2	$\dot{c}_{76} + \dot{c}_{36} + \dot{Z}_{EVP2} = \dot{c}_{35} + \dot{c}_{37}$	$E_{37}\dot{c}_{36} - E_{36}\dot{c}_{37} = 0$
VAS absorber	$\dot{c}_{35} + \dot{c}_{31} + \dot{c}_{32} + \dot{Z}_{Abs} = \dot{c}_{26} + \dot{c}_{23}$	$(E_{33} - E_{32})\dot{c}_{26} - E_{26}(\dot{c}_{33} - \dot{c}_{32}) = 0$
VAS pump 2	$\dot{c}_{30} + \dot{c}_{W_{P_2}} + \dot{Z}_{P_2} = \dot{c}_{27}$	Not applicable
VAS valve 4	$\dot{c}_{30} + \dot{Z}_{V_4} = \dot{c}_{31}$	Not applicable
VAS HEX 3	$\dot{c}_{27} + \dot{c}_{29} + \dot{Z}_{HEX3} = \dot{c}_{28} + \dot{c}_{30}$	$E_{30}\dot{c}_{29} - E_{29}\dot{c}_{30} = 0$
Biomass combustor	$\dot{c}_{55} + \dot{c}_{56} + \dot{Z}_{BMC} = \dot{c}_{57}$	Not applicable
Water heater (HW)	$\dot{c}_{57} + \dot{c}_{58} + \dot{Z}_{HW} = \dot{c}_{60} + \dot{c}_{59}$	$E_{60}\dot{c}_{57} - E_{57}\dot{c}_{60} = 0$
HEX	$\dot{c}_{60} + \dot{c}_{61} + \dot{Z}_{HEX} = \dot{c}_{70} + \dot{c}_{62}$	$E_{62}\dot{c}_{60} - E_{60}\dot{c}_{62} = 0$
PEM electrolyser	$\dot{c}_{76} + \dot{c}_{63} + \dot{Z}_{PEM} = \dot{c}_{65} + \dot{c}_{67}$	$E_{67}\dot{c}_{65} - E_{65}\dot{c}_{67} = 0$

schematic of the energy system, the ORC turbine output is maximised with the objective function:

– maximise:

$$\dot{W}_{ORCT} = \dot{m}h|T_i, P_i| - h|s_i, P_o| = f|T_i, P_i, P_o|. \quad (16)$$

This is subject to the variables with constraints as:

$$T_{i,L} < T_i < T_{i,H}, \quad (17)$$

$$P_{i,L} < P_i < P_{i,H}, \quad (18)$$

$$P_{i,o} < P_o < P_{o,H}. \quad (19)$$

Equation (16) expresses the work output as a function of the enthalpy at the turbine inlet specified with turbine inlet tempera-

ature and pressure, and the exit enthalpy is defined with the entropy at the turbine inlet and the back pressure after expansion. The subscripts “i” and “o” respectively represent the inlet and outlet from the turbine, while “L” and “H” are for lower and higher operating limits.

Similarly, the turbine output from the Kalina system is maximised with the expression:

– maximise:

$$\dot{W}_{Kal.T} = \dot{m}h|T_i, P_i, q_i| - h|s_i, P_o| = f|T_i, P_i, P_o|, \quad (20)$$

where the quality of ammonium vapour at turbine inlet is denoted with q_i , and the objective function is subject to the variables with limits as:

$$T_{i,L} < T_i < T_{i,H}, \quad (21)$$

$$P_{i,L} < P_i < P_{i,H}, \quad (22)$$

$$P_{i,o} < P_o < P_{o,H}. \quad (23)$$

4. Results validation

To validate the thermodynamic simulation results for the proposed Multigeneration system, a suitable code was written in Engineering Equation Solver (EES) software [31] to compare the obtained findings to those in the literature. In this study, four case studies from the literature are chosen, and each is simulated in a constant situation. These case studies include the KC, ORC, VAS subsystem, and PEM electrolyser system. Validation for each case study is performed below.

4.1. Validation of Kalina cycle subsystem

In the first case study, a KC is selected from Parikhani et al.[32], and simulated under constant input parameters. In this simulation, some of the KC design input parameters of [32] were plugged into KC of the present work model. It can be expressed that the results of the current investigation properly corroborate the results of [32], with some improvements in performance indices. According to Table 2, a performance parameter (i.e. net output power) is selected, and the calculated value is compared with the literature. There is a good agreement between the obtained results in the present model and those reported in the literature.

Table 2. Model validation of the present work with Parikhani et al. [32] for the Kalina cycle subsystem.

Parameter	Reference [32]	Present work
System working fluid	Ammonia-water	Ammonia-water
Mass flow rate (kg/s)	2.917	2.917
Ammonia concentration at turbine outlet (%)	99.97	83.06
Turbine inlet temperature (°C)	160.2	160.2
Turbine inlet pressure (bar)	30	30
Turbine outlet pressure (bar)	2.986	2.986
Net power output (kW)	925.26	941.3

4.2. Validation of organic Rankine cycle subsystem

For the second case study, the results of the model for the organic Rankine cycle system are presented in Table 3. Assuming these input design parameters through the analysis of ORC, it can be stated that the results of the current investigation properly verify the results of Ozturk and Dincer [33], with some enhancements in the performance indices. From Table 3, a performance parameter (i.e. turbine power output) is selected, and the calculated value is compared with the literature. This comparison, of the different parameter, validates well with the literature.

Table 3. Model validation of the present work with Ozturk & Dincer [33] for the ORC generation subsystem.

Parameter	Reference [33]	Present work
System working fluid	Isobutane	Isobutane
Mass flow rate (kg/s)	34.24	34.24
Turbine inlet temperature (°C)	146.8	146.8
Turbine inlet pressure (bar)	32.5	32.5
Turbine outlet pressure (bar)	4.10	4.10
Turbine power output (kW)	2436	2437

4.3. Validation of vapour absorption subsystem

Presented in Table 4 is the third case study, which is the vapour absorption system (VAS). In the simulation, some of the VAS design input parameters of Karaca and Dincer [34] were substituted into VAS of the present work model. From Table 4, two performance parameters (i.e. evaporator cooling rate and energetic COP) are selected, and the calculated values are compared with the literature. This comparison, of the different parameters, agrees well with the literature.

Table 4. Model validation of the present work with Karaca & Dincer [34] for the VAS subsystem.

Parameter	Reference [34]	Present work
System working fluid	Ammonia-water	Ammonia-water
Mass flow rate (kg/s)	0.0089	0.0089
Pressure across the evaporator (bar)	4.62	4.62
VAS cooling rate (kW)	5.08	4.946
Energetic COP	0.96	0.9783

4.4. Validation of proton exchange membrane electrolyser subsystem

The fourth validation step is using a PEM electrolyser. In the mathematical modelling of the PEM electrolyser, the operating temperature is set at 90°C, the power supplied is 7.5 kW, and the oxygen and hydrogen pressures are fixed at 1 bar. Based on Table 5, a performance indicator (i.e., hydrogen production rate) is selected, and the calculated value is compared with the litera-

ture. It can be expressed that the results of the current investigation substantiate the results of Khanmohammadi et al. [35] as seen in Table 5, with some improvements in performance indices. A minor discrepancy is attributed to the different design operating conditions, like the unavailable or varying mass flow rate of water. Since the mass flow rate of water affects hydrogen production if it is insufficient, but once sufficient, the hydrogen flow rate increases with an increase in power supplied, as long as efficiency losses at high power are managed.

Table 5. Model validation of the present work with Khanmohammadi et al. [35] for PEM electrolyser.

Parameter	Reference [35]	Present work
Electrolyzer hot water mass flow rate (kg/s)	-	1.99
Electrolyzer temperature (°C)	90	90
Electrolyzer pressure (bar)	1	1
Hydrogen production rate (kg/h)	0.245	0.255

4.5. Validation of exergoeconomic performance

The final but not least significant step in validation is to use the comprehensive exergoeconomic and environmental results of a comparative system (Table 6).

Table 6. Exergoeconomic model validation of present work with Hashemian & Noorpoor [36] for the multigeneration system.

Parameter	Value	Present work
Total product cost rate (\$/s)	0.83	0.798
Electricity cost rate (\$/s)	0.0001	0.00016
Hydrogen cost rate (\$/s)	0.0002	0.00021
Energy efficiency (%)	82.4	82.43

In the simulation, some of the system design input parameters of Hashemian & Noorpoor [36] were substituted into the present work model. From Table 6, six performance parameters (i.e. total product cost rate, electricity cost rate, cooling power cost rate and hydrogen cost rate) are selected, and the calculated values are compared with the literature. This comparison, of the different parameters, agrees well with the literature.

5. Results and discussion

5.1. Exergoeconomic results

The results of the exergoeconomic considerations of the plant as the main outcome of this study are presented in this section. It was conducted to determine the exergoeconomic factor of the plant, the cost related to exergy destruction, the total cost rate per second for the plant components, as well as the unit cost of products per energy. The unit cost of products was estimated separately for the electricity generation systems (ORC and Kalina cycle) only. This was based on the solution of the cost balance and auxiliary equations that yielded monetary quantification of the exergy streams at the state points. The cost parameters are summarised in Table 7. From the costs associated with the exergy stream and that of turbine work generated, the system's electricity cost from the Kalina cycle and ORC were computed as 0.04308 and 0.0245 USD/kWh, respectively. The calculated unit cost of electricity values translates to 69.12 ₦/kWh and 39.31 ₦/kWh based on the current exchange rate. These values are much desired when compared to the current electricity tariff of about 209.50 ₦/kWh (0.1306 USD/kWh) in Nigeria. The system's total cost of exergy destruction of 5.18 \$/hr and equipment cost rate of 4.2 \$/hr results in about 44.51% of the total exergoeconomic factor. Therefore, up to 55% of economic loss in the plant is related to the component's thermodynamic inefficiencies. The degree of each component contribution to economic loss is highlighted by the corresponding exergoeconomic factors in Table 7.

Table 7. Exergoeconomic results of the novel system.

Component	Equipment cost (USD)	Levelised equipment cost rate (USD/h)	Specific exergy cost of product (USD/GJ)	Cost of destruction (USD/h)	Exergoeconomic factor (%)
Biomass combustor	58124.00	1.368	0.000436	0.0009464	99.93
Kalina condenser 1	2444.00	0.05755	0.006077	0.0309744	65.01
Kalina condenser 2	1953.00	0.04598	0.003294	0.0207000	68.96
Kalina evaporator 1	1694.00	0.03988	0.001944	0.1502640	20.97
Kalina HEX 1	1533.00	0.0361	0.01521	0.0638640	36.11
Kalina HEX 2	955.60	0.0225	0.009738	0.1139400	16.49
Kalina pump 1	25.19	0.000593	0.01203	0.0000581	91.08
Kalina separator 1	805.80	0.01897	0.001355	0.0228060	45.41
Kalina separator 2	805.80	0.01897	0.001562	0.0002475	98.71
Kalina turbine	30940.00	0.7285	0.01197	0.0000000	100.00
Kalina valve 1	161.60	0.003805	0.001944	0.0131328	22.46
Kalina valve 2	161.60	0.003805	0.01112	0.0005753	86.87
Kalina valve 3	330.20	0.007775	0.003261	0.0336276	18.78
Kalina vapour generator	5635.00	0.1327	0.000567	0.0434160	75.35
ORC compressor	1374.00	0.03235	0.007699	0.0521640	38.28
ORC condenser 3	1137.00	0.02678	0.000603	1.0483200	2.49

Table 7 continued. Exergoeconomic results of the novel system.

Component	Equipment cost (USD)	Levelised equipment cost rate (USD/h)	Specific exergy cost of product (USD/GJ)	Cost of destruction (USD/h)	Exergoeconomic factor (%)
ORC condenser 4	1311.00	0.03087	0.01424	0.2283840	11.91
ORC evaporator 3	1311.00	0.03087	0.002451	0.1681920	15.51
ORC pump	29.79	0.000701	0.006892	0.0001922	78.49
ORC turbine	41103.00	0.9677	0.006822	0.0924120	91.28
ORC vapour generator	1423.00	0.03351	0.000567	0.0434160	43.56
ORC valve	140.80	0.003315	0.02226	0.2611080	1.25
PEM electrolyser	2843.00	0.06694	0.00261	0.0305388	68.67
PEM HEX	1137.00	0.02678	0.002187	0.3209040	7.70
Trough collector	10026.00	0.2361	0.000113	0.0000000	100.00
VAS absorber	1368.00	0.0322	0.0124	0.5958000	5.13
VAS condenser 5	1137.00	0.02678	0.02249	0.0085500	75.80
VAS desorber	1694.00	0.03988	0.003571	0.2546280	13.54
VAS evaporator 2	1254.00	0.02952	0.002666	0.0021600	93.18
VAS HEX3	1137.00	0.02678	0.006502	0.3054960	8.06
VAS pump	3.63	8.53E-05	0.01119	0.0101664	0.83
VAS valve 4	166.80	0.003926	0.01498	0.0808200	4.63
VAS valve 5	166.80	0.003926	0.002666	0.7822800	0.50
Water heater	2202.00	0.05185	0.002815	0.4010400	11.45
Total	176533.61	4.155991	0.226233	5.1811234	44.51

5.2. Sensitivity analysis

The effect of some key thermodynamic parameters on the key performance criteria of the system is investigated in this section. Figure 2 reveals a significant relationship between the turbine inlet temperature, pressure, and total cost rate. Specifically, a 20°C increase in turbine inlet temperature from 140°C to 160°C yields a 3.8% rise in total cost rate, while a 66.7% increase in turbine inlet pressure from 15 bar to 25 bar results in a 3.5% cost rate escalation. These findings underscore the intricate balance between thermodynamic efficiency, heat transfer rates, and component costs.

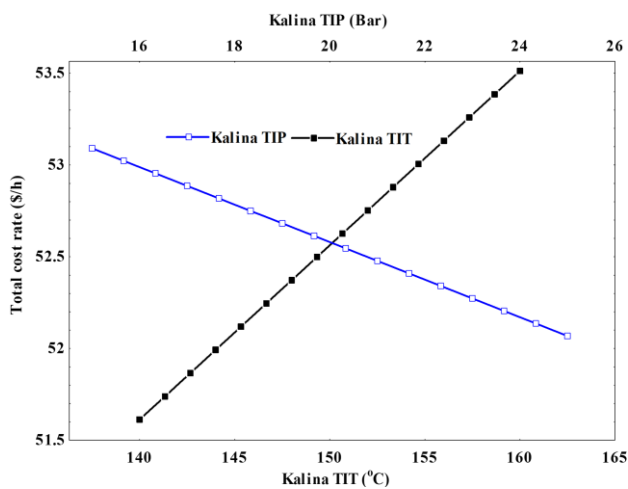


Fig. 2. Variation of the Kalina turbine inlet temperature and pressure with the total cost rate.

The system's configuration, comprising the Kalina cycle, organic Rankine cycle, and vapour absorption system, exhibits

sensitivity to turbine inlet conditions, emphasising the importance of optimising operating parameters to mitigate increased costs.

Figure 3 shows the relationship between the ORC turbine inlet temperature and pressure and the multigeneration energy system's cost rate. As the inlet temperature increases from 125°C to 160°C, the total cost rate remains relatively stable, fluctuating between \$52.45 and \$53.65. Conversely, the inlet pressure exhibits a more pronounced impact, with the cost rate escalating from \$53.35 at 22 bar to \$52.25 at 27 bar. This disparity suggests that optimising turbine inlet pressure may yield greater cost savings. Notably, the system demonstrates thermal efficiency resilience within the 125°C–155°C temperature range, but experiences a slight cost increase beyond 155°C.

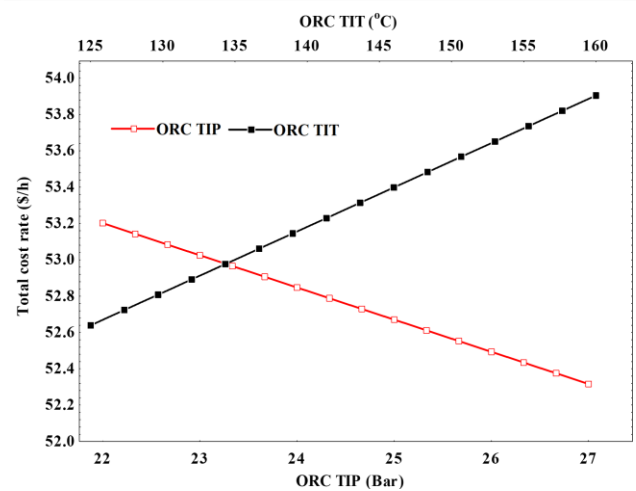


Fig. 3. Variation of the ORC turbine inlet temperature and pressure with the total cost rate.

Figure 4 reveals a positive linear relationship between the parabolic trough collector (PTC) area and total cost rate, with a minimal increase of approximately 0.4% per 5 m² increase in the PTC area. As the PTC area expands from 20 m² to 45 m², the total cost rate rises from \$52.61/h to \$52.83/h, demonstrating efficient scaling and economies of scale. This trend is attributed to reduced costs per unit area, efficient heat collection, linear scaling, and fixed costs associated with system components. The system's configuration benefits from increased heat input and maintains relatively consistent thermal efficiency.

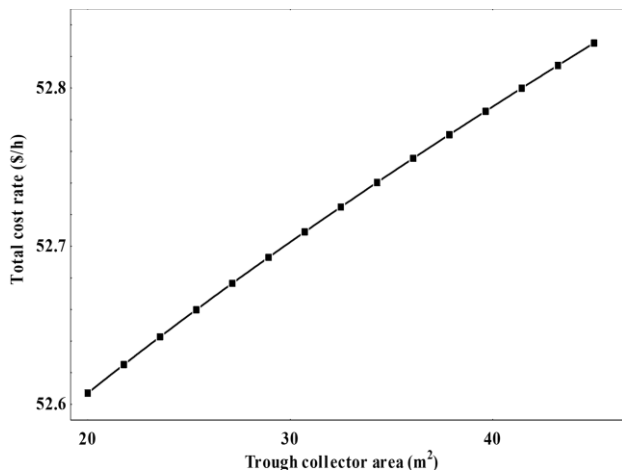


Fig. 4. The effect of trough collector area on the total cost rate.

Figure 5 reveals a trade-off between the total cost rate and total plant exergy efficiency as the parabolic trough collector (PTC) mass flow rate increases from 2 kg/s to 3.5 kg/s. The total cost rate rises linearly by 4.7% from \$50.67/h to \$53.15/h, while the total plant exergy efficiency decreases non-linearly by 14.6% from 80.52% to 65.6%. The efficiency drop accelerates beyond 2.75 kg/s, highlighting the importance of balancing cost and efficiency. This trend is attributed to increased heat transfer and thermal losses, pumping power requirements, and system design limitations.

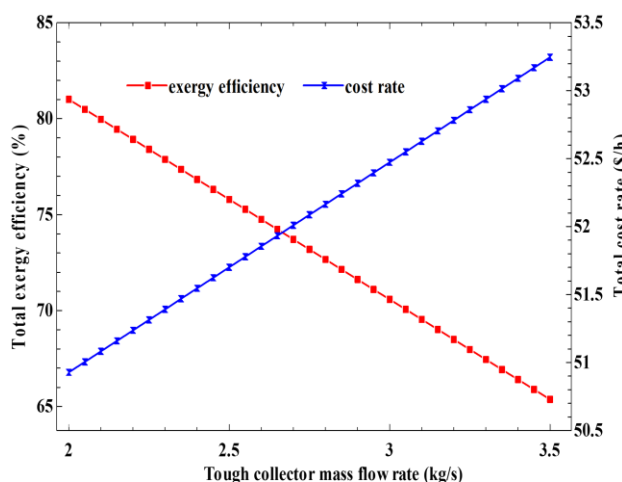


Fig. 5. Variation of the collector mass flow rate with total exergy efficiency and total cost rate.

Figure 6 shows the trade-off between the total cost rate and total efficiency as the parabolic trough collector (PTC) mass flow rate increases from 20 m² to 45 m². The total cost rate rises linearly by 4.8% from \$50.67/h to \$53.15/h, while the total efficiency decreases non-linearly by 18.5% from 80.52% to 65.6%. This decline is attributed to increased thermal losses, reduced heat recovery, and decreased organic Rankine cycle and vapour absorption system efficiencies.

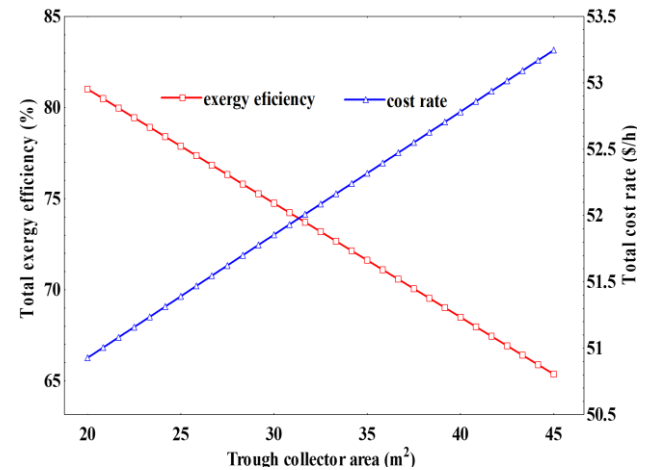


Fig. 6. Variation of the collector area with total exergy efficiency and total cost rate.

The efficiency drop accelerates beyond 30 kg/s, highlighting a critical mass flow rate threshold. Optimising the PTC mass flow rate, component selection, and system design can mitigate this efficiency decrease, achieving a more favourable cost-efficiency balance in the multigeneration energy system.

5.3. Optimisation results

The optimisation was performed with regards to the optimum selection of operating parameters for the ORC and Kalina turbines. The work output from these turbines was optimised subject to the ideal temperature and pressure mix within the thermodynamic operation domain. The ORC work output optimisation results are shown in Fig. 7 after 2152 iterations performed with 64 genetic algorithm generations. The optimum work produced from the ORC turbine was obtained as 47.97 kJ/kg at a turbine inlet temperature and pressure of 140°C and 25.7 bar, respectively. The turbine back pressure (TBP) was optimised to 2.003 bar. The optimum parameters for the ORC are summarised in Table 8. The surface plot of the optimum turbine work with regards to TIP and TBP in the optimisation search space is also shown in Fig. 8.

Table 8. ORC optimum operating parameters.

Parameter	Limits	Optimum value	Unit
Turbine work	–	47.97	kJ/kg
TIT	110 – 140	140	°C
TIP	15 – 30	25.7	bar
TBP	2 – 6	2.003	bar

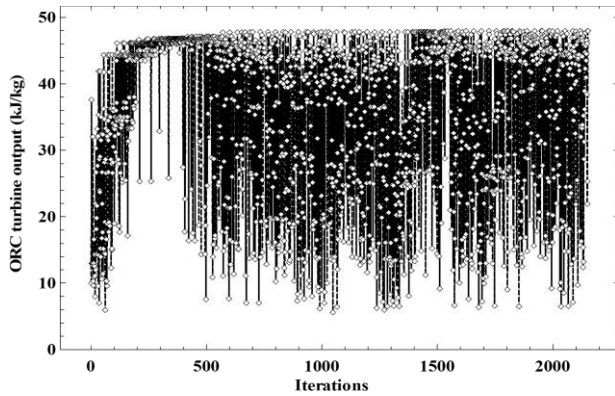


Fig. 7. ORC optimum work output.

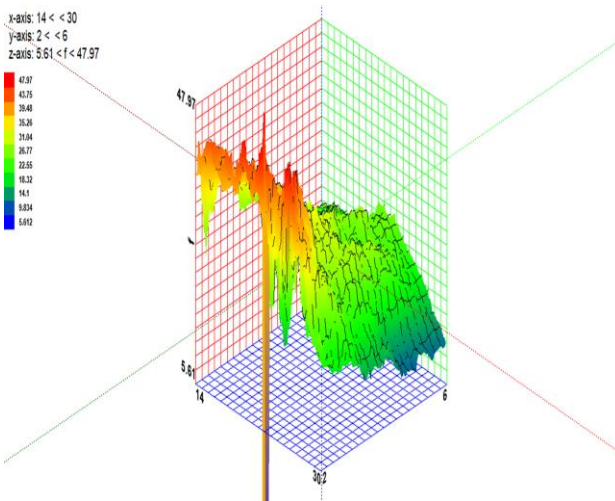


Fig. 8. Surface plot of the optimum turbine work with regards to TIP and TBP.

Similar optimisation results for the Kalina system are shown in Fig. 9. Several optimization runs yielded the maximum turbine work outputs of approximately 435 kJ/kg. However, the operating turbine inlet temperature and pressure tended towards the upper constraints, while the pressure after expansion tended towards the lower bound. The optimum parameters are shown in Table 4, while all iterations involving the objective function and decision variables are well documented.

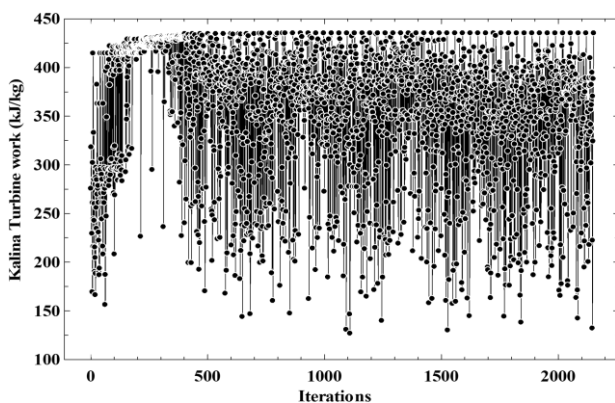


Fig. 9. KC optimum work output.

Table 6. Kalina system optimum operating parameters.

Parameter	Limits	Optimum value	Unit
Turbine work	—	435.3	kJ/kg
TIT	120 – 160	159.1	°C
TIP	15 – 30	30	bar
TBP	2 – 8	2.006	bar

5.4. Exergetic sustainability analysis of the system

The results of the sustainability index of the system were found with regards to the total system output and gross exergy destruction in the plant. At design condition, the system has an exergy efficiency of 52.2%, total exergy destruction of 963.3 kW, and total product of 964.9 kW. Accordingly, the system's exergy sustainability index was 1.002. This shows that the system's output is slightly higher than the total exergy destruction in the system, although there were massive gross exergy destructions, especially in the parabolic trough collector, due to the high temperature gradient between the PTC surface and the surroundings. An improvement on the exergetic sustainability index can be achieved with a reduction in the net exergy destruction through variation in operating parameters. One such parameter is the ambient temperature. The results in Fig. 10 show that high ambient temperatures will result in ESI. Interestingly, an optimum ambient temperature exists at about 297.4 K where the system's ESI can reach 1.00.

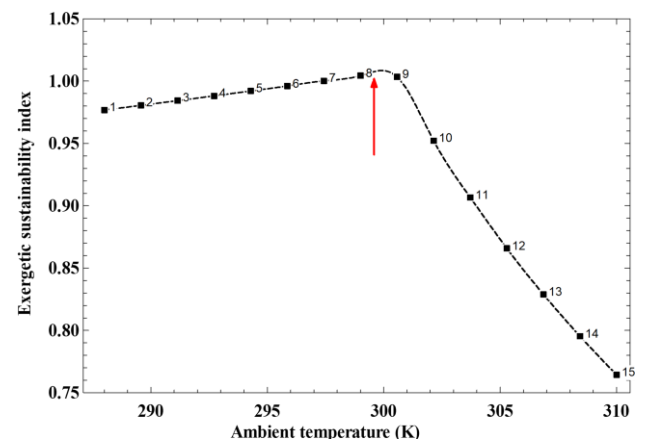


Fig. 10. Effect of ambient temperature on the system's ESI.

6. Conclusions

The present study performed an exergoeconomic investigation and optimisation of a solar-biomass tri-evaporator multi-generation plant equipped with a fuel cell and a proton exchange membrane. The system includes a parabolic trough collector powered by the sun, which is the primary energy source for the integrated energy system; a Kalina system that provides power and cooling; an organic Rankine cycle (ORC) for power and cooling; a vapour absorption system for refrigeration; and a proton exchange membrane (PEM) electrolyzer for hydrogen production supported by a biomass combustor. A sensitivity analy-

sis is carried out to examine the effect of various design factors. The main conclusions are that:

- The system's electricity cost from the Kalina and ORC were computed as 0.04308 USD/kWh and 0.0245 USD/kWh, respectively, with a total system cost of 176533 USD. The system's total cost of exergy destruction of 5.18 \$/h and equipment cost rate of 4.2 \$/h results in about 44.51% of the total exergoeconomic factor.
- The calculated unit cost of electricity values translates to 69.12 ₦/kWh and 39.31 ₦/kWh based on the current exchange rate. These values are much desired when compared to the current electricity tariff of about 209.50 ₦/kWh (0.1306 USD/kWh) in Nigeria.
- The optimum power generation from the ORC turbine was obtained as 47.97 kW at a turbine inlet temperature and pressure of 140°C and 25.7 bar, respectively. The turbine back pressure (TBP) was optimised to 2.003 bar.
- The optimum power generation from the Kalina turbine was obtained as 435 kW at a turbine inlet temperature and pressure of 159.1°C and 30 bar, respectively, and TBP of 2.006 bar.
- An optimum ambient temperature exists at about 297.4 K where the system's ESI can reach 1.00.

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Forecasting the energy efficiency of an earth-air heat exchanger with integrated twisted tabs using machine learning techniques

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Abstract

This study investigates the potential of machine learning models as efficient alternatives to traditional computational methods for evaluating the performance of earth-air heat exchanger systems. A validated numerical model was used to simulate system behaviour under varying parameters, including soil type, pipe material, number of internal twisted tabs and outlet temperature. Based on these simulations, a dataset comprising 216 entries was generated to train three machine learning models: support vector regression, gradient boosting and decision trees. The gradient boosting model achieved the highest predictive accuracy, with a root mean square error of 0.0188 and a mean absolute error of 0.0138. Support vector regression and decision trees also demonstrated strong performance, with prediction accuracies of 97% and 96%, respectively. Additionally, the proposed earth-air heat exchanger design with twisted tabs showed superior thermal performance compared to the conventional configuration. Over extended operation, the temperature difference between inlet and outlet exceeded 7°C, with the new configuration system incorporating 10 tabs yielding optimal performance. This configuration led to more than a 10% improvement in thermal efficiency and an increase of approximately 25% in the heat transfer coefficient. These results confirm that integrating machine learning with advanced earth-air heat exchanger designs offers a reliable and computationally efficient approach for enhancing system performance.

Keywords: Machine learning; Earth-air heat exchanger; Twisted tabs; Heat transfer; Dataset

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1. Introduction

Heat transfer has always played a crucial role in industry and mechanical systems. It enables the transfer of thermal energy between substances without direct contact or mixing [1]. Applications of heat exchangers span various sectors, including the chemical industry, food processing, space heating, heat recovery systems, vegetable drying, paper treatment and air conditioning

[2]. The design of heat exchangers can vary significantly depending on their type and intended function. As a result, numerous configurations have been developed by modifying shapes, sizes, materials, and incorporating various types of flow disruptors or mixers into the heat exchange regions.

Methods for enhancing the thermal performance of heat exchangers are generally classified into three categories: active, passive and combined enhancement techniques. Active methods

Nomenclature

C_p – pipe thermal specific heat, kJ/(kg K)
 C_{e1}, C_{e2}, C_μ – constants of the k – ε model
 D – pipe diameter, m
 h – heat transfer coefficient, W/(m²·K)
 k – turbulent kinetic energy, (J/kg)
 L – length of the pipe, m
 Pr – Prandtl number
 Q_c – cooling capacity, W
 T – temperature, K
 u_i – velocity component in the i -th direction, m/s
 V – flow velocity of the fluid, m/s
 Z – pipe buried depth, m

Greek symbols

ρ – density of the fluid, kg/m³
 λ – thermal conductivity, W/(m K)
 ε – dissipation rate of k , W/kg
 μ – viscosity, kg/(m s)
 $\sigma_k, \sigma_\varepsilon$ – turbulent Prandtl numbers for k – ε

Subscripts and Superscripts

in – inlet
 out – outlet
 s – soil
 p – pipe
 t – turbulent

Abbreviations and Acronyms

AL – aluminium
 ANN – artificial neural networks
 CFD – computational fluid dynamics
 EAHE – earth-to-air heat exchanger
 EAHET – earth air heat exchanger with twisted tab
 HDPE – high-density polyethylene
 MAE – mean absolute error
 NN – neural network
 NSE – Nash-Sutcliffe efficiency
 PVC – polyvinyl chloride
 RANS – Reynolds-averaged Navier-Stokes equations
 RMSE – root mean square error
 RSM – response surface methodology
 SD – standardised value
 SVR – support vector regression

rely on external power sources and employ techniques such as magnetic fields, auxiliary mechanical systems, fluid injection or surface vibrations. In contrast, passive methods are more commonly studied and applied, as they require no additional energy input and are relatively easy to implement [3,4]. Aziz and Rehman [5] numerically simulated a small heat exchanger equipped with six baffles. By sequentially removing each baffle plate, they found that removing the inlet and outlet baffles had minimal impact on pressure drop, while removing the central baffles caused significant pressure fluctuations. Muszyński and Dorao [6] conducted an experimental study to evaluate the performance of a heat exchanger module with an enhanced surface. Their results indicated a 60% improvement in heat flux. The enhancement of heat transfer in various innovative heat exchanger designs using computational fluid dynamics (CFD) has also been widely documented in the literature. A comparative study conducted by Serrao et al. [7] using ANSYS Fluent evaluated the performance of smooth and corrugated tubes. Their CFD analysis showed that the corrugated tube exhibited a 54.7% increase in the overall heat transfer coefficient compared to the smooth tube. Vignesh et al. [8] and Wang et al. [9] used computational methods to investigate the impact of dimples on the thermal performance of heat exchangers. They found that dimpled tubes led to a higher temperature rise and pressure drop compared to smooth tubes. Kumar et al. [10] analysed the heat transfer and flow characteristics of double helically coiled tube heat exchangers. Extensive research has also been conducted on the performance and efficiency of tubes operating in turbulent flow regimes [11]. Kishan et al. [12] performed a numerical study on different flow patterns inside the tubes of shell-and-tube heat exchangers. Sharma et al. [13] examined flow patterns, pressure drops, and heat transfer coefficients in both staggered and inline configurations of shell-and-tube heat exchangers. Larwa and Kupiec [14] proposed analytical relationships based on Green's

function theory to model heat transfer in spiral ground heat exchangers. Rahimi et al. [15] carried out both numerical and experimental investigations on four types of inserts: classic, perforated, notched and jagged twisted tapes, and found that the jagged insert yielded the best heat transfer performance. Molcrette and Autier [16] introduced a new formula to estimate earth energy for earth-air pipe heat exchangers during winter heating. Their study emphasised how various parameters affect the proper sizing of the system. In an experimental study, Nawaz and Prakach [17] tested three horizontal PVC pipes of different diameters to evaluate the thermal performance of an earth-to-air heat exchanger, concluding that the smallest pipe diameter provided the highest efficiency.

In recent years, artificial intelligence (AI) has attracted considerable attention due to its exceptional ability to deliver positive outcomes across various fields. Among AI techniques, machine learning (ML) has been widely adopted by researchers for identifying optimal process parameters [18]. This approach offers significant advantages, including reduced computational time, enhanced productivity and minimal human intervention. Of the many ML techniques available, supervised and unsupervised learning models are the most commonly employed in thermal system research [19,20]. In the context of heat exchangers, the heating or cooling capacity, typically quantified by the heat transfer rate, is a critical performance parameter.

Numerous studies have applied machine learning methods to predict heat exchanger performance. For instance, Karami et al. [21] assessed the effectiveness of an artificial neural network (ANN) in predicting heat transfer. Their experimental and numerical investigations on tubes with butterfly inserts considered the influence of insert inclination angle and Reynolds number variation. The ANN, developed in MATLAB, achieved a training error of 0.109% and a testing error of 0.509%, indicating high agreement between predicted and experimental results.

Li et al. [22] and Shojaeefard et al. [23] focused on predicting cooling capacity. Li et al. implemented a neural network based on response surface methodology (RSM), while Shojaeefard et al. explored different ANN architectures.

Deb et al. [24] employed a regression-based classification ML model to predict output parameters from a given set of inputs. The model's accuracy was evaluated using the residual sum of squares method, with results showing strong agreement between predicted and simulated values for fluid outlet temperature and pressure drop. Liao et al. [25] combined multilayer perceptron (MLP) networks with Bayesian optimisation to design an optimal near-field thermal radiative modulator, considering variables such as rotation angle, layer thickness, and gap distance between layers.

Ren et al. [26] explored advanced techniques by integrating reinforcement learning with proximal policy optimisation (PPO) and the non-Oberbeck–Boussinesq (NOB) approximation. They achieved a 76% improvement in heat transfer by developing a smart active flow control system for laminar flow enhancement. Wang and Vafai [27] applied support vector regression (SVR) algorithms in conjunction with thermal simulations to accurately predict temperature variations in hotspots within multi-layer 3D electronic chips.

Çolak et al. [28] employed the Levenberg–Marquardt training algorithm to develop two predictive models aimed at estimating key performance parameters, including the overall heat transfer coefficient, pressure drops on both the tube and annulus sides, and the total cost. Each model was trained using a dataset comprising 438 samples and incorporated a hidden layer with 15 neurons. Model 1 achieved estimation deviations of 0.16% for the overall heat transfer coefficient, -0.23% for the tube-side pressure drop, -0.02% for the annulus-side pressure drop, and -0.003% for the overall cost. Model 2 yielded deviations of 0.02%, -0.18% , -0.16% , and -0.15% , respectively, for the same parameters. Due to the limited availability of experimental and parametric studies in nanofluid applications, Çolak et al. [29] evaluated the use of an AI-based approach to estimate key performance parameters. Utilising six input variables, the model achieved deviation rates of -0.66% , 0.001% , and 0.12% for the respective outputs, demonstrating the method's effectiveness in compensating for the lack of empirical data.

Shell and helically coiled tube heat exchangers are increasingly utilised in industrial applications due to their compact design and enhanced heat transfer surface area compared to conventional models. In the study conducted by Çolak et al. [30], two distinct ANN architectures were developed using a dataset of 105 samples. These models were designed to predict key output parameters, including the heat transfer coefficient, pressure drop, Nusselt number, and performance evaluation criteria. In recent study by Çolak et al. [31], an ANN was employed to predict the Nusselt number, friction factor and performance evaluation criteria for heat transfer in straight corrugated tubes, based on flow rate and corrugation parameters. The model was specifically developed to estimate the flow and thermal behaviour of corrugated tubes operating at low flow rates. This proposed approach offers a valuable tool for thermal system applications,

providing accurate predictions that can assist designers in improving system efficiency.

The purpose of this study is to enhance the energy efficiency of climate control systems through the optimisation of earth-to-air heat exchanger (EAHE) design and operation using twisted tabs. While EAHEs are well-established for utilising stable subsoil temperatures to moderate indoor climates, their thermal performance can still be significantly improved. However, several critical research challenges persist in the literature. First, the integration of passive flow control devices, such as twisted tabs, into EAHE systems remains underexplored, with limited numerical or experimental investigations of their effect on flow structure and heat transfer. Second, traditional CFD-only approaches, while accurate, are often computationally expensive and impractical for real-time design or operational optimisation. Third, there is a lack of scalable, predictive frameworks that combine physical modelling with data-driven techniques to address performance variability across diverse environmental and geometric conditions.

To address these gaps, this study introduces a dual-strategy approach that combines passive geometric enhancement with advanced data-driven modelling. Specifically, twisted tabs are employed within the EAHE ducts to enhance airflow turbulence and improve thermal exchange between ambient air and surrounding soil. In parallel, a hybrid framework combining (CFD) simulations with (ML) algorithms is developed to predict and optimise thermodynamic performance. This integrated methodology offers a scalable and practical solution for real-time performance assessment and efficient HVAC system design in modern sustainable buildings.

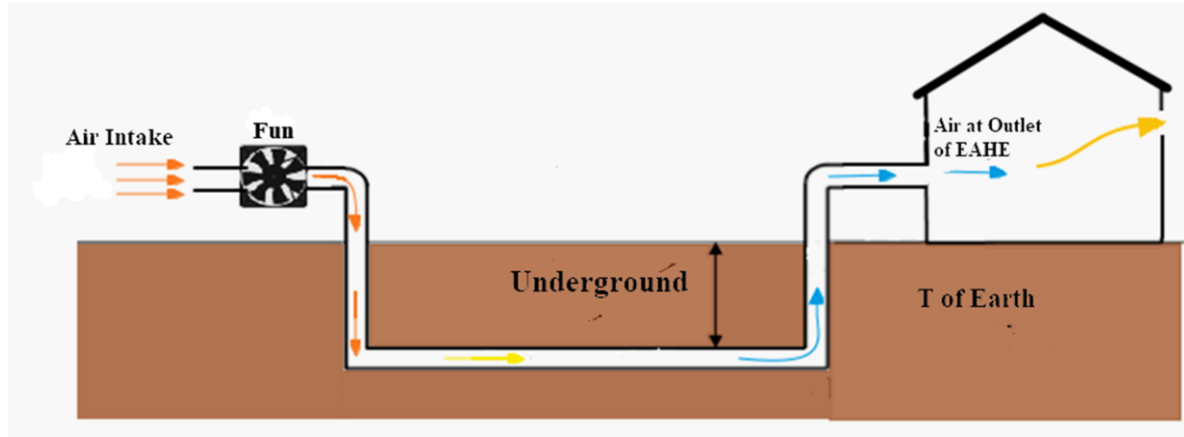
2. Materials and methods

2.1. Geometry and input description

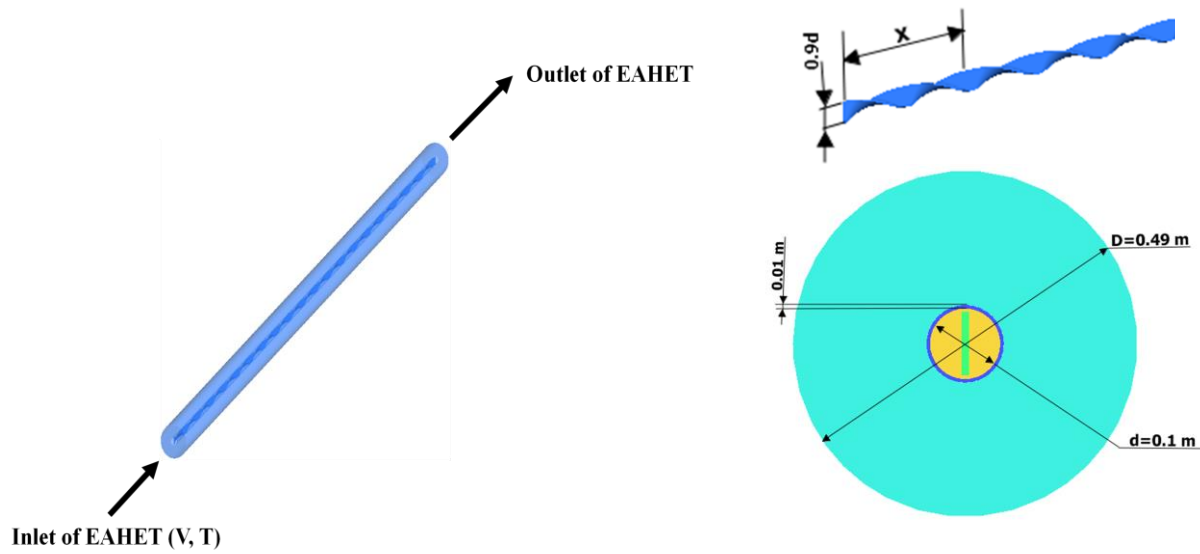
2.1.1. Computational setup

To analyse the complex airflow and heat transfer phenomena within an earth–air heat exchanger (EAHE) system, this study adopts the geometric configuration proposed by Bansal et al. [32] as a baseline reference. A schematic of the investigated EAHE setup is provided in Fig. 1. The system comprises a polyvinyl chloride (PVC) pipe horizontally buried at a depth of 2.7 meters below the ground surface, with an inner diameter of 0.15 meters and a total length of 23.42 meters.

At the inlet, ambient air is introduced into the pipe with a uniform velocity of 5 m/s, a temperature of 42°C , and a turbulence intensity of 5%, consistent with typical hot and arid climatic conditions. The outlet boundary is treated as a pressure outlet with zero gauge pressure, allowing for fully developed flow. The pipe wall is assumed to be thermally conductive, allowing heat exchange between the airflow and the surrounding soil. The outer soil boundary is modelled as a fixed temperature boundary, based on the annual average subsoil temperature at the burial depth. No-slip conditions are applied to all solid surfaces.



(a)



(b)

Fig 1. Schematic of the integrated EAHE system (a) and computational domain of EAHE system with twisted tabs (b).

The simulation employs the standard $k-\varepsilon$ turbulence model, selected for its enhanced performance in capturing swirling and recirculating flows, as commonly observed in EAHE applications. The residual convergence criteria for all transport equations are set to 10^{-6} to ensure numerical stability and accuracy. Furthermore, the working fluid (air) is treated as an incompressible ideal gas, and the surrounding soil is assumed homogeneous with constant thermal properties. This study investigates a novel configuration of an (EAHET), wherein twisted tabs are inserted within the inner tube to enhance thermal performance. As depicted in Fig. 1, the twisted tabs are strategically placed along the inner wall of the tube to promote turbulence and improve heat transfer between the air and the surrounding soil. The analysis explores the impact of the number of twisted tabs, specifically 2, 5, 10, 15 and 20 tabs, on the system's thermal behaviour. Each configuration is labelled accordingly as EAHET-02, EAHET-05, EAHET-10, EAHET-15 and EAHET-20. In addition to tab quantity, the study examines the influence of different

inner tube materials, including polyvinyl chloride (PVC), aluminium (AL) and high-density polyethylene (HDPE), as well as the effect of soil thermal conductivity. These parameters are evaluated through detailed CFD simulations to assess their roles in enhancing the heat transfer efficiency of the EAHET system. The details of thermo-physical properties of air, soil and inner tube materials are presented in Table 1. The temperature selected at the pipe's entrance in these suggested designs is based on the location of our nation; on average, it is around 42°C .

2.1.2. Turbulence model validation

To perform this study, multiple simulations were conducted using ANSYS CFX software [33]. In this solver package, the Reynolds-averaged Navier–Stokes (RANS) equations are solved using the finite volume method to discretise the continuity, momentum, and energy equations. A second-order upwind scheme is applied to solve the momentum, energy, and turbulence model equations. For pressure–velocity coupling, the

Table 1. Properties of materials.

Material	Density [kg/m ³]	Specific heat capacity [J/(kg K)]	Thermal conductivity [W/(m K)]
Air	0.0242	1.225	1006
PVC	1380	900	0.16
HDPE	940	2000	0.4
AL	2702	903	237
Soil 1	2050	1840	0.52
Soil 2	2215	1260	1.26
Soil 3	1700	906	2.1

SIMPLEC algorithm [34] is employed. The governing equations, which represent the conservation of mass, momentum, and energy, are expressed as follows [35]:

$$\frac{\partial(\rho u_i)}{\partial x_i} = 0, \quad (1)$$

$$\rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \overline{\rho u'_i u'_j} \right], \quad (2)$$

$$\frac{\partial}{\partial x_i} (\rho u_i T) = \frac{\partial}{\partial x_j} \left[\left(\frac{\lambda}{c_p} + \frac{\mu_t}{Pr_t} \right) \frac{\partial T}{\partial x_j} \right], \quad (3)$$

where p is the pressure, T is the temperature and u_i represents the velocity component in the i -th direction. Parameter c_p represents the specific heat capacity, ρ is the density, λ is the thermal conductivity, μ_t and Pr_t are the turbulent viscosity and turbulent Prandtl number, respectively.

Modelling of the Reynolds stresses ($-\overline{\rho u'_i u'_j}$) in Eq. (2) is done using the Boussinesq hypothesis [36], which relates the Reynolds stresses to the mean velocity gradients via the following equation:

$$-\overline{\rho u'_i u'_j} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \left(\rho k + \mu_t \frac{\partial u_i}{\partial x_i} \right) \delta_{ij}. \quad (4)$$

Turbulent quantities in the Navier–Stokes equations are treated using the turbulent viscosity μ_t , which is given by:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}, \quad C_\mu = 0.085.$$

According to the literature, this choice aligns with several recent CFD-based EAHE studies. For instance, Rahimi et al. [14] and Bansal et al. [32] utilised k - ε -based turbulence models and obtained reliable predictions of temperature distribution and energy efficiency. In the present study, the standard k - ε model effectively captured the axial temperature decay and overall heat exchange behaviour, while maintaining reasonable computational costs for long-duration simulations. Additionally, the k - ε model exhibited good agreement with experimental data by accurately representing turbulent flow characteristics and predicting heat transfer rates, making it well-suited for assessing the thermal performance of EAHE systems under complex flow conditions.

The governing transport equations for the turbulent kinetic energy (k) and its dissipation rate (ε) are solved using the RNG

k - ε turbulence model, which provides enhanced accuracy for flows with strong streamline curvature and high strain rates, such as in film-cooling jets. The general form of these equations can be expressed as follows:

$$\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_j} (\rho k u_j) = \frac{\partial}{\partial x_j} \left[\left(\mu \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k + P_{kb} - \rho \varepsilon, \quad (5)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial x_j} (\rho \varepsilon u_j) &= \frac{\partial}{\partial x_j} \left[\left(\mu \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \\ &+ \frac{\varepsilon}{k} (C_{\varepsilon 1} P_k - \rho C_{\varepsilon 2} \varepsilon + C_{\varepsilon 1} P_{\varepsilon b}), \end{aligned} \quad (6)$$

$$P_k = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \left(3 \mu_t \frac{\partial u_k}{\partial x_k} + \rho k \right), \quad (7)$$

where P_{kb} and $P_{\varepsilon b}$ stand for the buoyancy forces' influence. The turbulence caused by viscous forces, or P_k , is represented by the following model:

$$P_k = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \left(3 \mu_t \frac{\partial u_k}{\partial x_k} + \rho k \right). \quad (8)$$

The empirical constants used in the RNG k - ε model are summarised in Table 2. These constants are derived from the model calibration and govern the production, dissipation and diffusion of turbulence quantities within the computational domain.

Table 2. The RNG k - ε turbulence model constants.

Symbol	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_μ	σ_k	σ_ε
Value	1.44	1.92	0.09	1.0	1.3

The heat transfer coefficient (h) is expressed as:

$$h = \frac{Q_C}{\pi d L (T_{in} - T_{out})}, \quad (9)$$

where Q_C is the total thermal energy extracted from the air, d and L is the inner diameter and length of EAHE, respectively; the fluid temperatures at the inlet and outlet are T_{in} and T_{out} , respectively.

2.1.3. Grid independence study

The transient temperature field surrounding the horizontally buried pipe of the EAHE system was analysed using an unstructured grid and CFD-based modelling. As seen in Fig. 2, temperature gradients are more noticeable close to the pipe wall, thus a denser mesh was used there and a coarser one further away. To predict the system's thermal performance and evaluate the cooling capacity of the earth-air heat exchanger, a transient and implicit numerical model was used. This model is based on the coupled simulation of heat transfer and turbulent airflow.

To accurately replicate real conditions and ensure high precision, element type and mesh density were varied according to the temperature sensitivity of each region. As illustrated in Fig. 2a, the number of mesh elements significantly affects the outcome of the CFD analysis. Increasing the element count improves solution quality up to a certain threshold. A mesh with

1.4 million elements was selected for the 54 simulation cases examined in this study. Figure 2b presents the simulated temperature distribution along the length of the PVC pipe. With the exception of the inlet region, the CFD predictions exhibit consistently lower temperature values compared to the corresponding experimental measurements throughout the pipe.

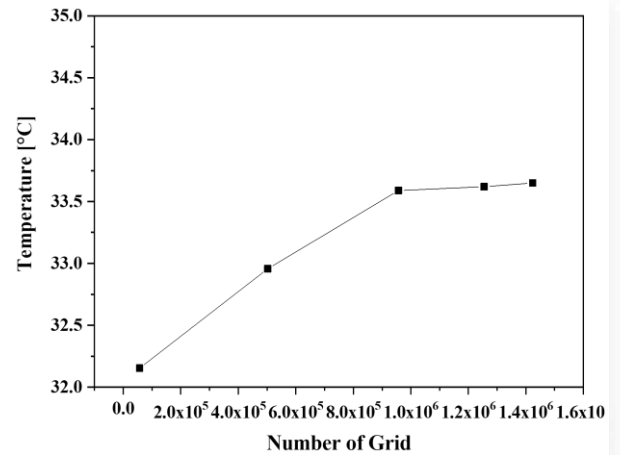
3. Artificial neural network modelling and validation

The goal of this stage is to collect sufficient data to build a comprehensive dataset for training and testing artificial intelligence models. The selected input parameters include the number of twisted tabs, the thermal conductivity (λ) of the soil, and the material of the tube. The outlet air temperature is designated as the output variable. Current machine learning models rely on small datasets for training, enabling more effective extraction of patterns and relationships within the data. This approach enhances the model's generalization ability, simplifies data collection and reduces the time required for numerical simulations.

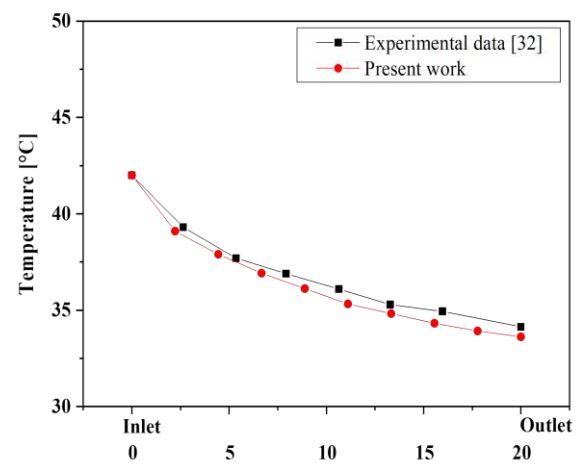
3.1. Methodology

Boundary conditions obtained from numerical simulations are commonly used as input parameters for machine learning prediction models (Vu et al. [37], Chen et al. [38]). Traditional models typically require large datasets generated through extensive simulations to effectively capture the underlying patterns and relationships in the system. In contrast, the current study demonstrates that high predictive performance can be achieved using only 216 data points. Despite the limited dataset, the machine learning models showed excellent accuracy when tested against the observed data.

The goal of this stage is to collect sufficient data to build a comprehensive dataset for training and testing artificial intelligence models. The selected input parameters include the number of twisted tabs, the thermal conductivity (λ) of the soil and the material of the tube, while the outlet air temperature is des-



(a)



(b)

Fig. 2. Grid independence (a) and model validation (b).

ignated as the output variable. As illustrated in Fig. 3, the machine learning workflow involves data pre-processing, model training using algorithms, such as decision tree (DT), SVR and

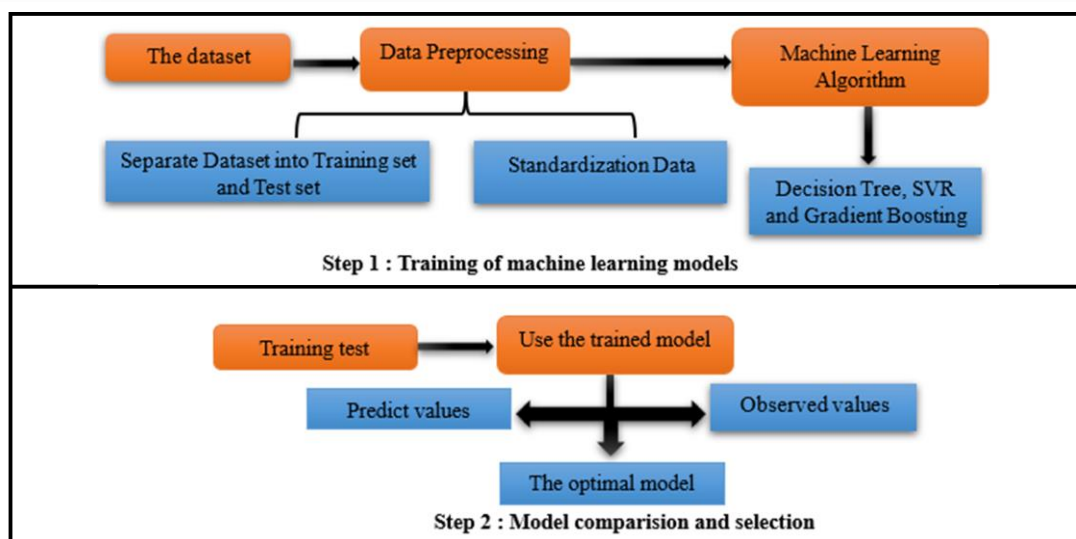


Fig. 3. Flowchart explaining the methodology.

gradient boosting (GB), and subsequent model evaluation and selection. Current machine learning models rely on small datasets for training, enabling more effective extraction of patterns and relationships within the data. This approach enhances the model's generalisation ability, simplifies data collection, and reduces the time required for numerical simulations.

3.2. Data preparation for training

Before training a machine learning model, it is essential to preprocess the dataset to ensure optimal performance. In this study, standardisation was applied, which is a common feature scaling method. Standardisation transforms the data so that it has a mean of 0 and a standard deviation of 1, ensuring that all features contribute equally to the model, regardless of their original scales [39,40]. The data were transformed using the following standardisation equation:

$$\text{Standardised value} = \frac{\text{original value} - \text{mean value}}{\text{standard deviation}}. \quad (10)$$

For this research, multiple predictive models were evaluated, and only three models, namely DT, SVR and GB, were chosen

based on their accuracy. The model that is selected for this study is the gradient boosting model. Prior studies have suggested that gradient boosting is effective in working with smaller datasets (Jiang et al. [41], Lubke et al. [42], Cha et al. [43]).

Gradient boosting is a unique example of ensemble learning in machine learning that is well known for its accuracy in performing regression tasks is gradient boosting. It uses the predictions of several weak learners, typically decision trees, to create a strong predictive model. The strategy is to build models iteratively, where every new model attempts to minimise the error left by its previous counterpart by fitting it to the residuals (errors) of a pre-existing ensemble, thereby minimising a chosen metric, for instance, mean squared error. Gradient boosting is popular because of its effectiveness with many types of data and also because it reveals the most important features to be considered, making the task of finding key factors in the data quite easy and straightforward.

Figure 4 presents a comparison between actual and predicted values across six machine learning models. Ensemble methods such as GB, CatBoost, and XGBoost exhibit strong agreement

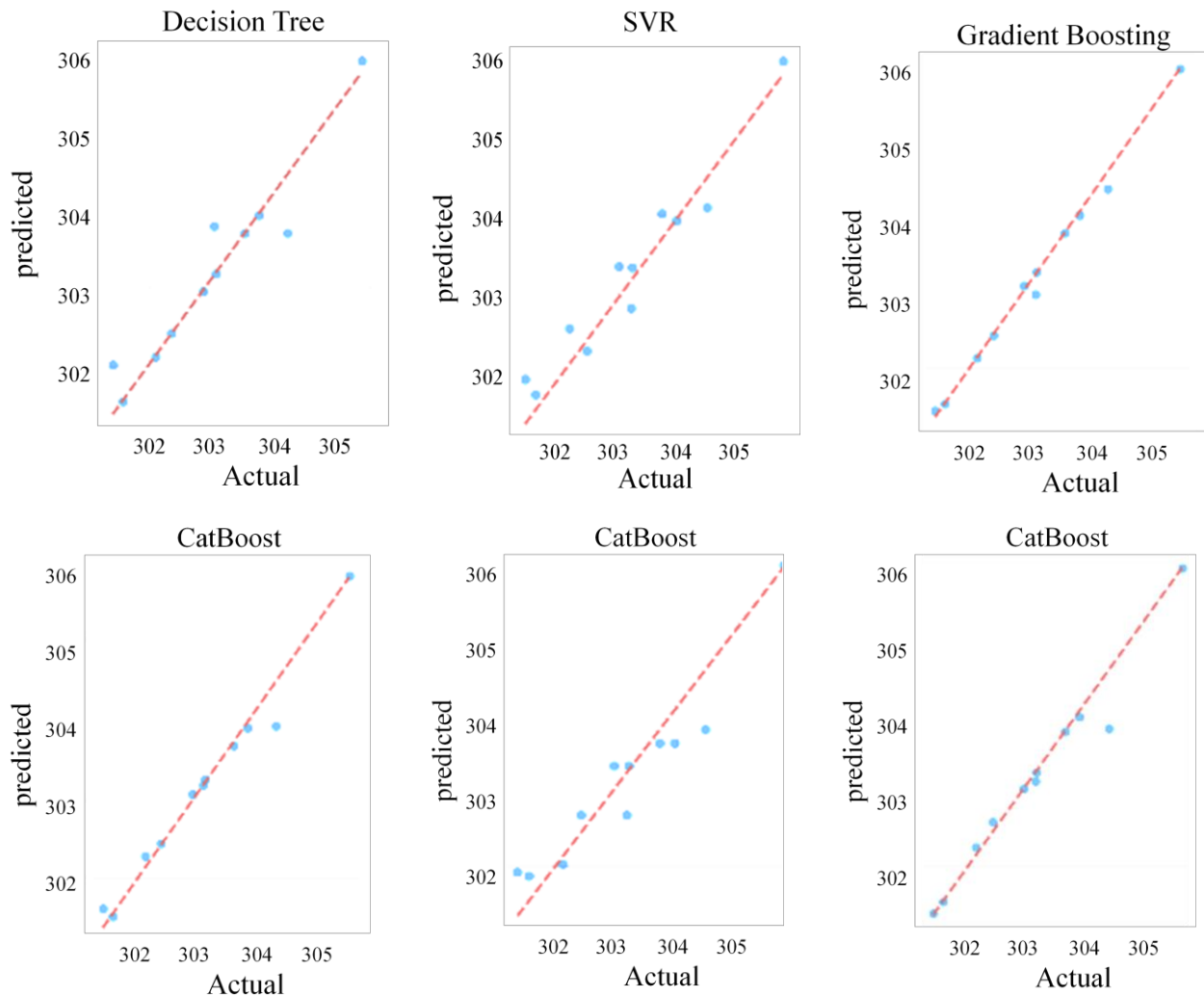


Fig. 4. Comparison of observed and predicted values from the trained models.

Table 3. Statistical evaluation of the performance of the three models.

MODEL	RMSE		MAE		NSE / R ²	
	Train Dataset	Test Dataset	Train Dataset	Test Dataset	Train Dataset	Test Dataset
SVR	0.2629	0.3175	0.1790	0.2768	0.9776	0.9333
SVR, Gong et al. [48]	0.3647	0.30	0.2169	0.2331	0.9579	0.9611
Decision tree	0.0126	0.3531	0.102	0.2011	0.9661	0.9175
RF, Gong et al. [48]	0.2868	0.3151	0.178	0.2335	0.9740	0.9572
Gradient boosting	0.0188	0.01032	0.0138	0.0612	0.9999	0.9930
LGBM, Gong et al. [48]	0.1929	0.2856	0.134	0.2242	0.9882	0.9648

with the ideal prediction line, validating their credibility in capturing the data trends accurately. In contrast, although SVR and DT exhibit some deviation from the reference line, they still demonstrate satisfactory performance in this study, proving to be effective modelling tools. Despite their limitations, they contribute valuable insights and remain relevant within the scope of the analysis.

The training set comprises 80% of the total data, while the remaining 20% is allocated for testing the model's performance. This split is done prior to any pre-processing, such as standardisation, to prevent data leakage and ensure that the model's evaluation is unbiased and reflects its true generalisation capability [44]. The model's performance is checked using a line graph of the data fit and three key evaluation measures. These measures are root mean squared error (RMSE) [45], mean absolute error (MAE) [46], and the Nash-Sutcliffe efficiency (NSE) [47]. These methods are well-known and often used to assess how well machine learning models make predictions. This quantitative comparison shows that the GB model developed in this work not only outperforms other models used in this study (e.g. SVR, DT), but also clearly exceeds the performance of existing traditional or physics-based models reported in the literature. This affirms the robustness and accuracy of the GB model for modelling the thermal behaviour of earth-air heat exchangers (EAHEs), especially when enhanced with integrated twisted tabs.

In terms of goodness of fit, the GB model achieved an NSE/R² value of 0.9930 on the test set, which is higher than the 0.9648 reported for light gradient boosting machine (LGBM) in and 0.9572 for random forest (RF) in [48], and 0.9611 for SVR, highlighting its superior ability to capture the variance in the data.

The results presented in Table 3 clearly demonstrate the superior predictive performance of the GB model compared to both the other machine learning models evaluated in this study and the traditional/physics-based models reported in the literature [48]. During the testing phase, the GB model achieved the lowest RMSE of 0.01032 and MAE of 0.0612, outperforming Gong et al.'s LGBM model, which yielded an RMSE of 0.2856 and an MAE of 0.2242. Similarly, in the training phase, the GB model achieved an RMSE of 0.0188 and an MAE of 0.0138, which are significantly better than those reported for the LGBM (0.1929 and 0.134, respectively), and much lower than the SVR (0.3647, 0.2169) and RF (0.2868, 0.178) models in [48].

4. Results and discussion

Twisted tabs are incorporated into (EAHEs) to improve their thermal performance. As depicted in Fig. 5, these internal elements alter the heat transfer characteristics by increasing the effective surface area for thermal exchange and enhancing convective flow. The inclusion of twisted tabs leads to a noticeable improvement in the heat transfer coefficient when compared to the baseline configuration without inserts. However, this enhancement does not scale linearly with the number of tabs. Beyond a certain threshold, the increasing number of twisted tabs induces excessive turbulence within the airflow, which, while initially beneficial, ultimately results in reduced residence time of the air in contact with the heat transfer surface. Consequently, the thermal exchange efficiency begins to deteriorate despite higher turbulence levels. Furthermore, the system may reach a state of excessive mixing, wherein the intensification of turbulence ceases to contribute positively to thermal performance and may even become counterproductive.

The graphs in Fig. 6 illustrate the temperature variation along the normalised length of the tube for various tube materials (PVC, HDPE and AL) and six different design

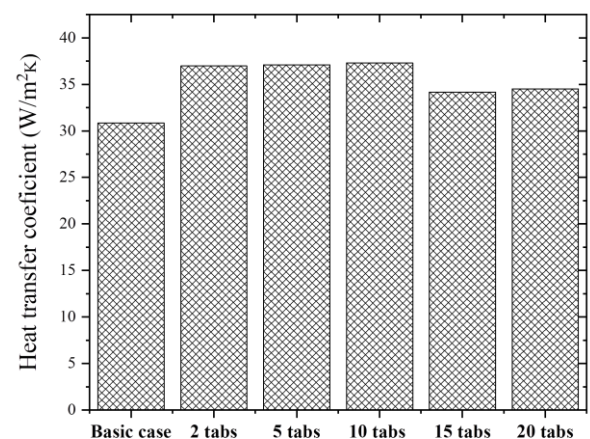


Fig.5. Heat transfer coefficient for various configurations of EAHE.

configurations, all evaluated under the same soil condition (soil 3). The temperature profiles consistently show a decline with increasing x/L , indicating progressive cooling of the airflow along the tube. Notably, all five modified configurations

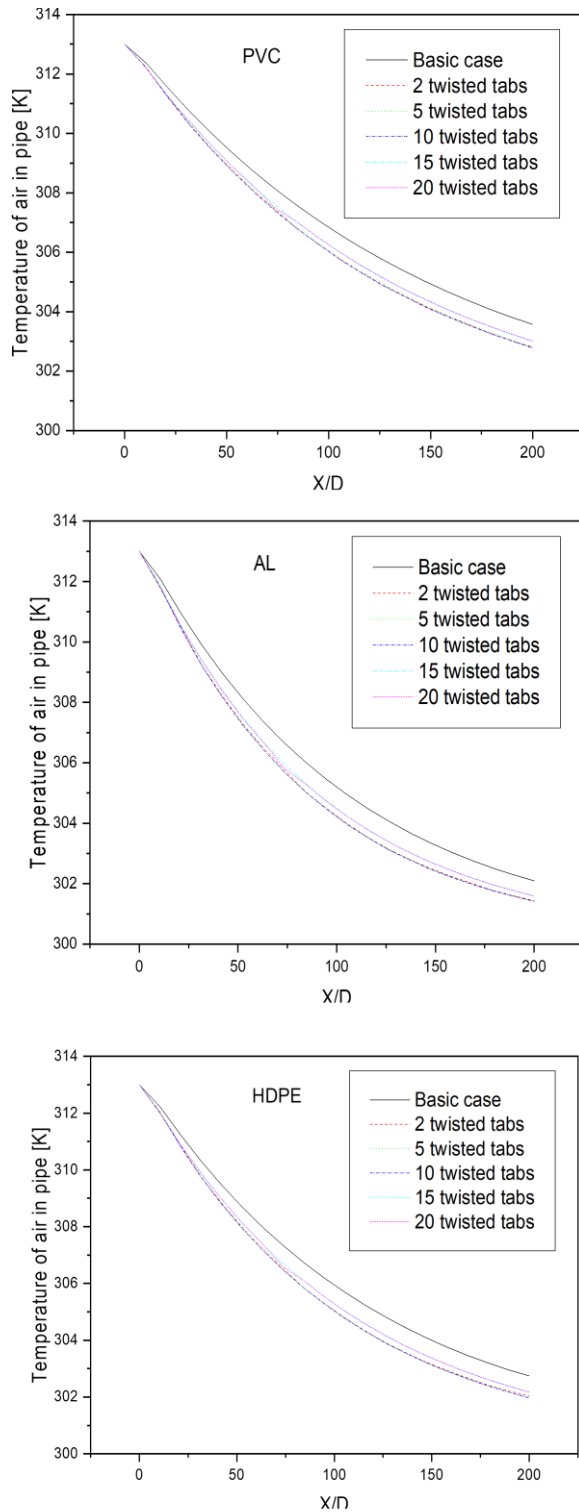


Fig.6. Temperature in the pipe centerline for various tube materials.

yield lower outlet temperatures compared to the basic case, which can be attributed to the enhanced turbulence and improved convective heat transfer resulting from the incorporation of twisted tabs. As only soil 3, characterised by a relatively high thermal conductivity (see Table 1), was considered in this analysis, its pronounced effect on the thermal performance of the EAHE system is evident. Higher conduc-

tivity facilitates more efficient heat exchange between the subsurface soil and the air flowing through the tube, thereby improving the overall cooling effectiveness of the system.

Figure 7 illustrates the temperature contours at the outlet of a heat exchanger, emphasising the impact of incorporating a twisted tab within the inner tube. The presence of twisted tabs generates vortices along their surfaces, which enhances turbulence in the airflow, disrupts the thermal boundary layer,

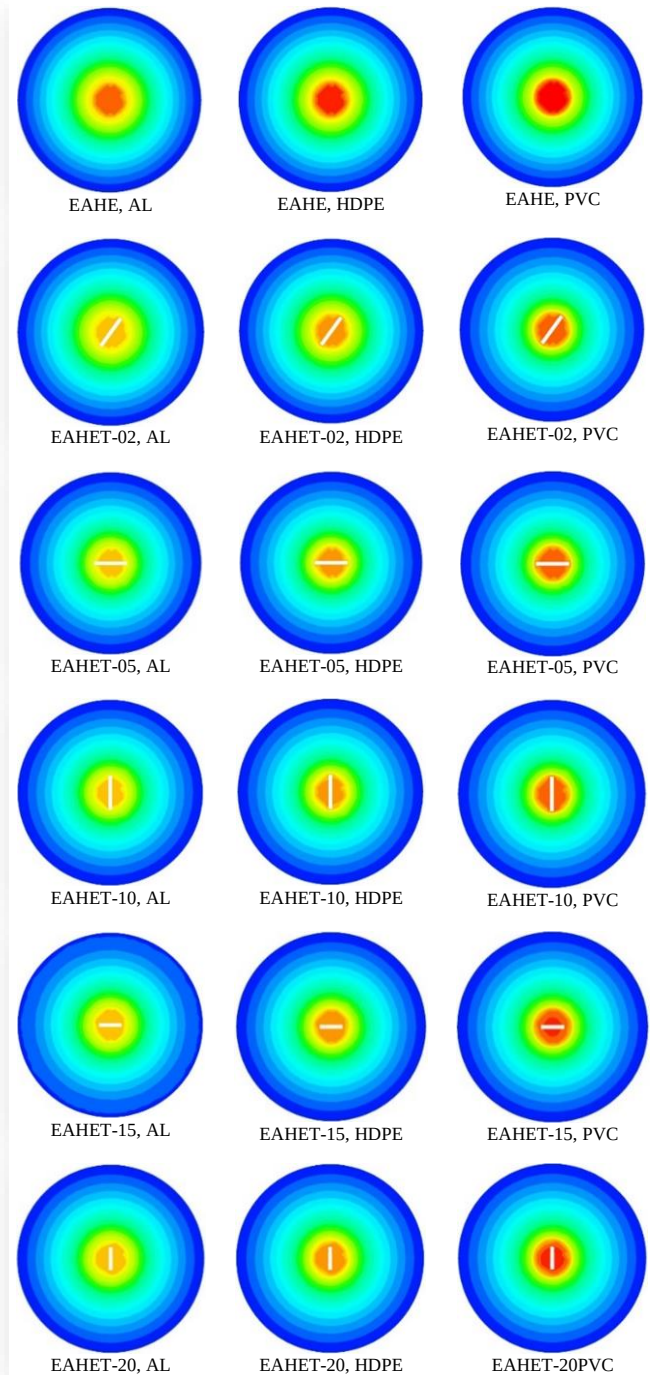


Fig.7. Temperature contours at the outlet of a heat exchanger.

and promotes better mixing between hotter and cooler regions. As depicted in the figure, the flow contours in the basic case without twisted tabs are more irregular and display cooler regions at the outlet. This indicates less effective heat transfer, primarily due to the presence of laminar flow near the tube walls. This behaviour is consistently observed across the three tube materials studied: PVC, HDPE and AL.

The effectiveness of incorporating twisted tabs into the EAHE system (referred to as EAHETs) is evident in the temperature contours, where the temperature difference between the inlet and outlet can reach up to 7°C. This cooling effect can be further enhanced with extended system operation. Additionally, Fig. 7 highlights the influence of tube materials on the performance of both EAHE and EAHET systems. Among the tested materials, the aluminium tube yielded the lowest outlet temperature. This indicates that the combination of high thermal conductivity and the presence of twisted tabs significantly improves heat transfer. The twisting of the wall promotes enhanced fluid mixing, maintaining a strong temperature gradient near the tube surface and thereby increasing the overall heat exchange efficiency.

Figure 8 illustrates the findings of the sensitivity analysis, which quantifies the relative impact of various input parameters on the outlet air temperature of the (EAHE) system. Among the examined factors, the thermal conductivity (λ) of the surrounding soil emerges as the most influential, contributing approximately 45.5% to the variation in outlet temperature. This substantial influence underscores the critical role of soil thermal properties in governing heat exchange between the buried pipe and its environment. In contrast, the effects of the pipe material and the number of internal twisted tabs are markedly less signif-

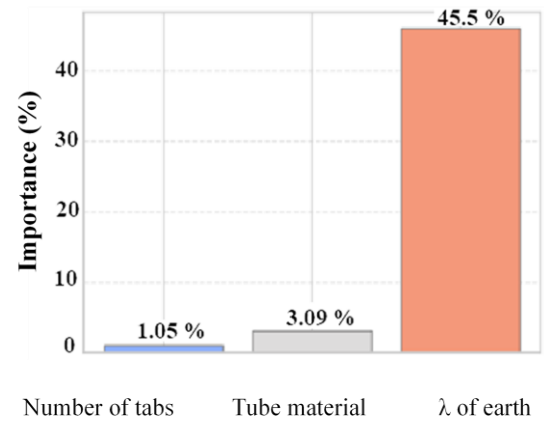


Fig. 8. Relative importance of input variables on the outcome temperature.

icant, accounting for only 3.09% and 1.05%, respectively. The comparable and minimal contributions of these two variables highlight their secondary roles relative to soil conductivity. These findings suggest that while geometric and material modifications within the EAHE system can offer marginal performance improvements, optimising the thermal characteristics of the soil environment should be prioritised for maximising system efficiency.

Figure 9 presents a comparative analysis between the actual outlet temperatures (T_{out}) obtained through numerical simulations of the EAHE system and those predicted by the GB machine learning model. Each subfigure corresponds to distinct combinations of system parameters, including the type of pipe material, the thermal conductivity (λ) of the surrounding soil,

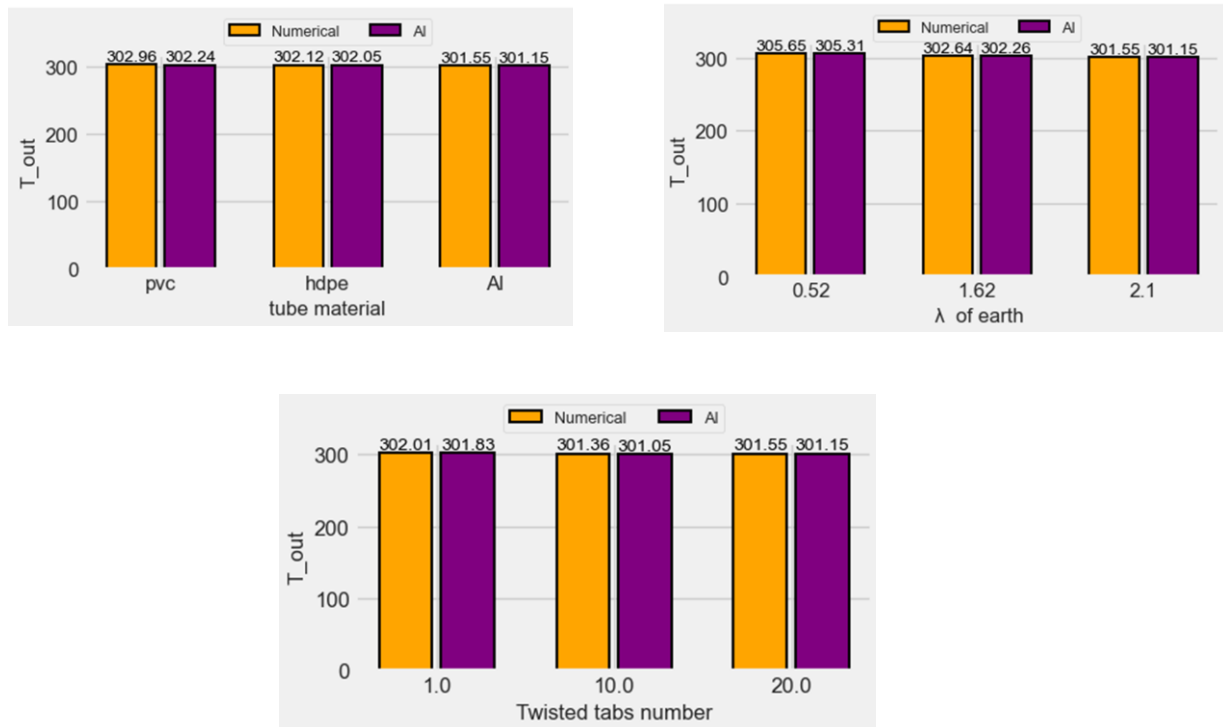


Fig. 9. Comparison between the numerical results and predicted outlet temperature for all tested parameters.

and the number of twisted tab inserts within the pipe. The predicted temperature values exhibit a strong correlation with the simulated results across all parameter configurations. This high level of agreement underscores the GB model's robustness, accuracy, and reliability in capturing complex nonlinear interactions within the system, thereby validating its potential as a powerful predictive tool for evaluating EAHE performance.

4.1. Industrial applicability and scalability

The present study proposes a hybrid approach combining computational fluid dynamics (CFD) and machine learning (ML) to evaluate and optimise twisted tab-based earth-air heat exchangers (EAHEs).

The configuration is particularly important in industrial and commercial buildings needing sustainable and passive cooling strategies. The design results have direct implications for actual HVAC and green building systems, where sub-surface temperature can be used to pre-condition ventilation air. The twisted tabs added improve air-side turbulence and augment heat exchange with the adjacent soil, rendering the system viable in high-density city centres with limited areas of land for installation of conventional cooling devices. At a scalability level, the datasets generated by CFD that are employed to train the ML models can be scaled across a broader span of operating conditions, geometries and soil properties. The trained models, especially ensemble learners XGBoost and CatBoost, produced excellent prediction precision with minimal computational overhead and can therefore be employed for integration into industrial control systems, real-time monitoring platforms or digital twins. Furthermore, these ML models are platform-agnostic and lightweight, making them deployable on embedded systems or cloud platforms for large-scale monitoring and optimisation. Subsequent research can examine integrating these models with SCADA systems or building energy management systems (BEMS) to support automated, real-time decision-making for energy-efficient operation.

5. Conclusions

This study presented a numerical and data-driven hybrid approach to examine and optimise the thermal performance of an earth-air heat exchanger (EAHE) system equipped with twisted tab inserts. A dataset of 216 entries from CFD simulations was used to train and test various machine learning (ML) models. The originality of this study lies in the integration of turbulence-inducing geometrical modifications with predictive ML models for the enablement of rapid performance analysis of EAHE configurations.

The key findings are summarised as follows:

- CFD analysis, including temperature contour plots and profiles, centreline revealed that twisted tabs significantly enhance convective heat transfer. This is attributed to the generation of localised turbulence and thermal boundary layer disruption, thereby leading to better thermal mixing and increased energy exchange. These enhancements are beneficial to the performance of the EAHE in both heating and cooling modes;

- Among the ML algorithms tried (DT, SVR and GB), GB worked better than the others in predictive accuracy. It learned nonlinear interactions between input parameters nicely and offered a good surrogate model for the rapid and accurate prediction of EAHE performance parameters.
- Sensitivity analysis revealed soil thermal conductivity (λ) as the most influential parameter on system efficiency. Though the tube material and the number of twisted tabs both played a secondary individual effect (4.14%), they remain important design parameters for optimising system performance by internal geometry and material selection.

Overall, this study demonstrates that the combination of CFD simulations and machine learning provides an effective framework for EAHE system analysis and optimisation. While the results are promising, limitations include the use of steady-state simulations and a static geometric range. Transient flow regimes, longer datasets with a wider variety of geometries, and experimental validation should be the focus of future work to enhance model generalizability even further. In addition, the deployment of trained ML models within real-time building control systems can open doors towards adaptive and energy-saving climate control.

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Investigating the impact of heat treatment on mechanical behaviour of A36 low-carbon steel

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Abstract

Quenching and tempering processes are commonly employed to enhance the physical and mechanical properties of multiphase steels. This study investigated the effects of heat treatment on ASTM A36 low-carbon steel, a material well-suited for structural applications. The novelty of this study lies in the detailed investigation of the quenching and tempering processes applied to ASTM A36 low-carbon steel, focusing on specific tempering temperatures and varying soaking times. The specimens were heated to 900°C and soaked for 2 hours, followed by quenching in an oil bath. Subsequently, they were tempered at low (200°C), medium (300°C), and high (400°C) temperatures for 60, 90, and 120 minutes, respectively. The mechanical properties of the processed specimens, including hardness, tensile strength, and impact strength, were evaluated. The Rockwell hardness showed a significant improvement of 22.75% after treatment. Oil bath quenching followed by tempering increased the ultimate tensile strength by 31.51% and 29.36%, respectively, compared to steel without any heat treatment. However, elongation at break and impact strength decreased by 11.55% and 27.27%, respectively, during quenching. Low, medium, and high temperature tempering at various soaking times released the internal stresses, refined the grain structure and exhibited the effect on tensile strength, as well as improved the elongation at break and impact strength by 35.08% and 125%, respectively, compared to quenched steel.

Keywords: Heat treatment; Quenching; Tempering; Tensile strength; Low-carbon steel; A36

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1. Introduction

Steel is one of the most critical materials in modern industry, essential in construction, automotive, manufacturing, and infrastructure applications due to its versatility and strength. The heat treatment of low-carbon steel plays a vital role in optimising its mechanical properties for various industrial uses. Low-carbon steel A36, with a carbon content of up to 0.3%, is widely used due to its balanced combination of strength, ductility, and cost-effectiveness [1–3]. However, untreated low-carbon steel often lacks the hardness and wear resistance required for demanding applications. Heat treatment processes such as quenching, normalising, annealing and tempering are applied to alter the mi-

crostructure of the steel, significantly enhancing its strength, hardness and durability. By controlling parameters such as temperature and cooling rate during these treatments, the material properties can be customised to meet specific requirements [4,5].

The heat-treatment process is a combination of operations involving the heating and cooling of metals in different media such as water, oil, atmosphere, and brine solutions. It softens the metal, refines grain size, improves strength, and relieves internal stress [6]. The duration of heating at a particular temperature and the rate of cooling in different media play an essential role in achieving the desired mechanical properties of any steel [7].

Nomenclature

Greek symbols

Ø – diameter, mm

Abbreviations and Acronyms

ASTM – American Society for Testing and Materials

HTT – high temperature tempering
LTT – low temperature tempering
MTT – medium temperature tempering
QCH – quenching
UTS – ultimate tensile strength
WHT – without heat treatment
YS – yield strength

Multi-step heat treatment can enhance the toughness of low-carbon, precipitation-strengthened steel with minimal reduction in yield strength. The high strength of steel is achieved by carefully creating internal barriers that hinder dislocation movement [8,9]. The improvement in toughness is partially attributed to reduced copper (Cu) segregation. However, tempered martensite may exhibit a slight decrease in strength due to a reduction in dislocation density, coarsening of precipitates, and a decrease in solute content [10–14]. Heat treatment of bimetallic additively manufactured structures (BAMS) at elevated temperatures and specific durations significantly improved ultimate tensile strength and elongation compared to untreated BAMS [15]. As a result of heat treatment, the failure shifted from the low-carbon steel side to the stainless-steel side [16,17].

The heat treatment process is an effective method for alleviating post-weld effects. Numerous studies have examined the impact of heating temperature, cooling medium temperature, heating rates, duration and cooling rate. Findings show that soaking temperature plays a major role in reducing residual stresses, while heating rate has minimal effect [18–20]. The addition of the niob (Nb) element improves the tensile strength and elongation of break of the weld metal without the need for heat treatment after welding, while having no impact on toughness. The Nb-free weld metal's strength was reduced by stress relief annealing; however, its elongation and impact toughness were improved [21]. The duration of austenitization significantly influenced the austenitic grain size, resulting in enhanced toughness of the alloy [22].

Nagar et al. [23,24] explored the critical role of heat-treatment in enhancing the durability and performance of manganese steel cone crusher liners. Rawat et al. [25] also investigated the effects of heat treatment on the microstructure and mechanical behaviour of A356/SiC metal matrix composites (MMCs), specifically focusing on tensile properties, hardness, and impact behaviour. They reported that adopting the heat-treatment processes reduced internal stresses in the metal matrix, contributing to improved mechanical performance. Li et al. [26] studied the cryogenic treatment of low-carbon steel and observed that applying cryogenic treatment combined with quenching and tempering to low-carbon martensitic stainless bearing steel enhances its mechanical properties by increasing hardness and tensile strength [27]. Cryogenic treatment reduces retained austenite, though some remains despite high-temperature tempering. Repeated cryogenic and tempering cycles further refine the martensite laths, leading to improved strength and wear resistance, but also result in a slight reduction in toughness [28,29]. These findings indicate that a well-optimised sequence of cryogenic and heat treatments can significantly enhance the durability and stability of this steel type for demanding applications [30]. The retention of austenite is enabled by austenitization, followed by

quenching and partitioning, whereas carbon partitioning from martensite to austenite has a limited role in the creation of multiphase low-carbon steel [31,32].

Aminah et al. [33] examined how cryorolling, a low-temperature processing method, affects low-carbon steel with varying pre-annealing temperatures. The optimal results were achieved with pre-annealing at 550°C and 50% thickness reduction, leading to the highest grain aspect ratio (4.19), smallest crystallite size (28.8 nm), maximum microhardness (162.3 HV) and tensile strength (567 MPa). These enhancements were due to nano/micro-scale ferrite grains, dispersed Fe₃C particles, and increased dislocation density. Other research [34,35] highlights cryorolling as a simple, efficient method for producing ultrafine-grained materials, with initial grain size playing a key role in final properties. Similarly, Kumar et al. [36] focused on the fabrication and analysis of zinc sulfide (ZnS) thin films deposited on glass substrates using the spin-coating technique at a high temperature of 500°C. They observed that post-deposition heat treatment process and precise control of film thickness played a crucial role in achieving desired material performance by improving the crystalline structure of the ZnS films.

Literature studies show the importance of heat treatment, which plays a vital role in achieving the desired mechanical properties of low-carbon steel. This study lies in its detailed investigation of the quenching and tempering processes applied to ASTM (American Society for Testing and Materials) A36 low-carbon steel, focusing on specific tempering temperatures (200°C, 300°C and 400°C) and varying soaking times (60, 90 and 120 min). A comprehensive evaluation of mechanical properties, such as hardness, tensile strength, elongation at break, and impact strength, demonstrates the systematic influence of these parameters on the steel's performance.

2. Experimental study

An experimental investigation was conducted on low-carbon steel ASTM A36 following heat treatment. Mechanical tests were performed to evaluate changes in tensile strength, hardness, and impact strength of the heat-treated samples. ASTM A36 was selected because of its suitability for structural applications. Table 1 shows the chemical composition of ASTM A36 low-carbon steel derived from spectrographic analysis.

Table 1. Composition of ASTM A36 by wt.%.

C	Mn	Cu	P	Si	S	Fe
0.25	1.03	0.2	0.04	0.28	0.05	98

2.1. Preparation of specimen

For the specimen preparation of low-carbon steel A36, a 10 m long steel rod of diameter 12 mm was considered along with a 10 mm × 10 mm bar. The three sets of dimensions: set I (15 × Ø 12 mm), set II (100 × Ø 12 mm), and set III (55 × 10 × 10 mm) were utilised for hardness, tensile, and Charpy impact tests, respectively. Three replicates of each sample were prepared for each experiment and tested. The average result has been presented for the accuracy of the results. For the specimen preparation, the ASTM E10, ASTM E8, and ASTM E32 standards were followed for the hardness, tensile strength, and impact strength tests, respectively. The dimensions and prepared specimens are shown in Figs. 1 and 2, respectively. The specimens were precisely machined using a BatliBoi 16TC CNC lathe and a vertical milling machine to achieve the required dimensional accuracy. This machining process ensured uniformity in specimen geometry and minimised stress concentration, thereby improving the reliability of the mechanical test results.

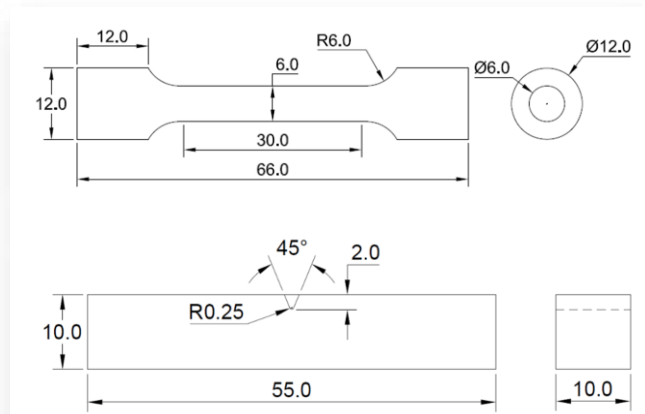


Fig. 1. Dimensions of tensile and impact strength test specimen [37,38].

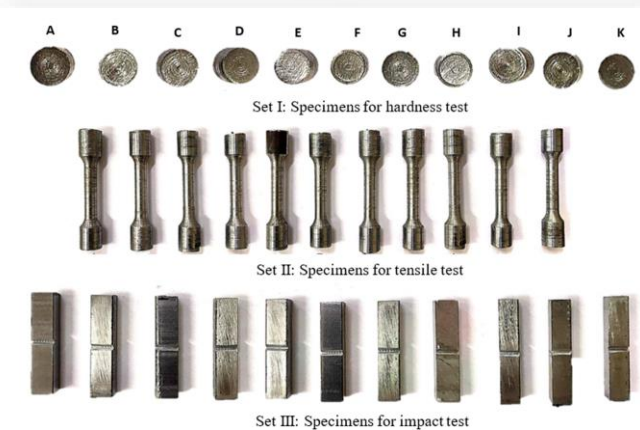


Fig. 2. Prepared test specimens based on ASTM standards.

2.2. Heat treatment of the specimen

For heating the specimens, a muffle furnace was utilised. The specimens were heated inside the furnace above the critical tem-

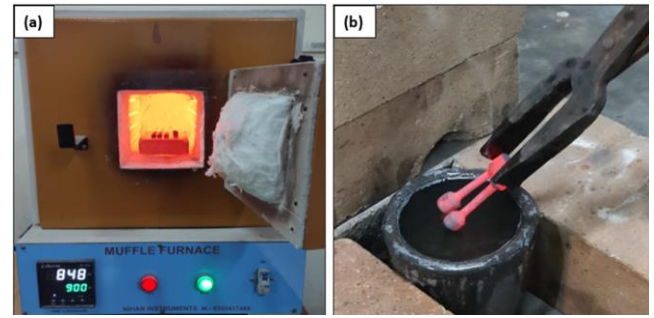


Fig. 3. Heat treatment process: (a) heating in a furnace, (b) quenching.

perature, i.e., 900°C for 2 hours. In this study, samples underwent quenching at 900°C using both oil and water as quenching media. The heated specimens were left to cool for 30 min in both quenching media. Quenching is a critical process that helps to improve steel's hardness, strength, and wear resistance. The taken-out, cooled and cleaned specimens were dipped into fresh water at ambient temperature for 10 min. The process of heating and cooling is shown in Fig. 3.

Following the quenching process, tempering was performed to reduce brittleness and relieve internal stresses in the specimens. This involved reheating the specimens below the recrystallisation temperature. During the tempering process, the specimens were heated to temperatures of 200°C (low), 300°C (medium) and 400°C (high). The tempering process was conducted for 60, 90, and 120 minutes. After completing the process, the specimens were left to cool in the air at room temperature.

3. Mechanical characterisation

The analysis of the mechanical properties of low-carbon steel A36 was performed. The Rockwell hardness, tensile, and impact strengths were tested with and without heat-treated specimens.

3.1. Hardness testing

The Rockwell hardness of the A36 low-carbon steel was measured using a hardness tester following the ASTM E10 standard. A diamond indenter with a 120° cone angle was used on the C scale. First, a minor load of 10 kgf was applied to remove any surface irregularities. Then, a major load of 150 kgf was applied through the indenter. The indentation depth was recorded from the dial gauge as arbitrary hardness numbers, which were then converted to standard hardness values using a conversion chart.

3.2. Tensile strength testing

Tensile strength testing was conducted on ASTM E8 standard specimens, using an electronic tensometer (PC-2000, KIPL) equipped with a 20 kN load cell and operating at room temperature, shown in Fig. 4. The tests were conducted at the Noida Institute of Engineering and Technology (NIET), Greater Noida. The ultimate tensile strength (UTS), yield strength (YS), and percentage elongation were measured. A round specimen with a gauge length of 30 mm and a gauge diameter of 6 mm was used. The test was performed at a constant strain rate of 1 mm/s,

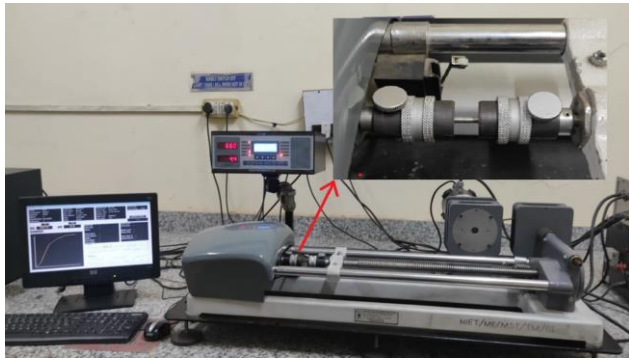


Fig. 4. Tensile strength test instrument, tensometer.

recording load vs. displacement and stress vs. strain plots. UTS, YS, and percentage elongation were calculated from the recorded data.

3.3. Impact strength testing

The impact strength of the specimens was measured using the Charpy impact test, following ASTM E23 standards. This test measures the impact energy or toughness of the specimens by quantifying the energy absorbed before fracture.

4. Results and discussion

The Rockwell hardness, tensile strength, and impact strength tests were conducted on standard specimens to evaluate the effect of the heat-treatment process.

Primarily, in the present study, samples underwent quenching at 900°C using both oil and water as quenching media. The results indicate that water quenching significantly reduces the material's tensile strength, elongation, and impact energy absorption by 5.83%, 10.24%, and 14.28%, respectively, compared to oil quenching of A36 low-carbon steel. This decline in tensile strength, ductility and impact strength is attributed to the

rapid cooling rate, which promotes the formation of brittle martensite, a phase that absorbs less energy before fracturing. However, hardness increased by 8.06% from 95.5 HRC to 103.2 HRC due to martensitic transformation, which enhances the material's resistance to deformation. Both oil and water quenching significantly impact material properties. While water quenching results in higher hardness but greater brittleness, oil quenching offers a more balanced approach, reducing the risk of cracking and distortion. Water quenching has the most significant effect on material properties due to its rapid cooling rate, which drastically alters hardness and microstructure. However, for applications requiring a combination of strength and dimensional stability, oil quenching is often preferred.

4.1. Rockwell hardness

The Rockwell hardness test results of the specimens are shown in Table 2, comparing conditions without heat treatment, after quenching, and at different tempering temperatures and soaking times. Three samples were tested for each condition, and the mean hardness value was calculated from the three trials.

The variation in hardness due to quenching and tempering the A36 steel at different conditions is shown in Figs. 5–7. It is observed that the Rockwell hardness of the quenched sample (B) is 22.7% higher than that of the sample without heat-treatment (A). The variation due to different tempering temperatures and soaking times was also studied. The variation due to tempering temperatures, viz. low (200°C), medium (300°C) and high (400°C), along with soaking times of 60, 90, and 120 min among the samples, is not significant. Overall, the hardness of the A36 material increases due to the heat treatment process. It is due to the steel structure changing into austenite with a considerable amount of cementite. The rapid rate of cooling changes developed austenite into martensite. Due to structural changes in the steel, the hardness of the A36 improved. The quenching of steel improved its hardness and improved strength of steel. It has also been noticed that tempering and soaking time decreased the quenched samples' hardness.

The effect of tempering temperature and the soaking time has also been analysed and shown in Fig. 8. It has been found

Table 2. Rockwell hardness of A36 low-carbon steel.

Sample ID	Heat treatment process#	Heating condition (°C/min)	Rockwell hardness (HRC)			Mean hardness (HRC)
			Trial 1	Trial 2	Trial 3	
A	WHT	–	76.2	78.5	78.8	77.8
B	QCH	900 / 120	97.2	95.4	93.8	95.5
C	LTT	200 / 60	87.9	86.3	88.1	87.4
D	LTT	200 / 90	86.8	85.5	89.0	87.1
E	LTT	200 / 120	86.9	85.9	86.2	86.3
F	MTT	300 / 60	83.9	81.8	82.9	82.9
G	MTT	300 / 90	85.2	80.2	82.8	82.7
H	MTT	300 / 120	79.3	82.6	83.8	81.9
I	HTT	400 / 60	77.5	81.2	76.3	78.3
J	HTT	400 / 90	76.6	79.4	78.4	78.1
K	HTT	400 / 120	79.6	77.7	76.3	77.9

#WHT – without heat treatment, QCH – quenching, LTT – low temperature tempering, MTT – medium temperature tempering, HTT – high temperature tempering.

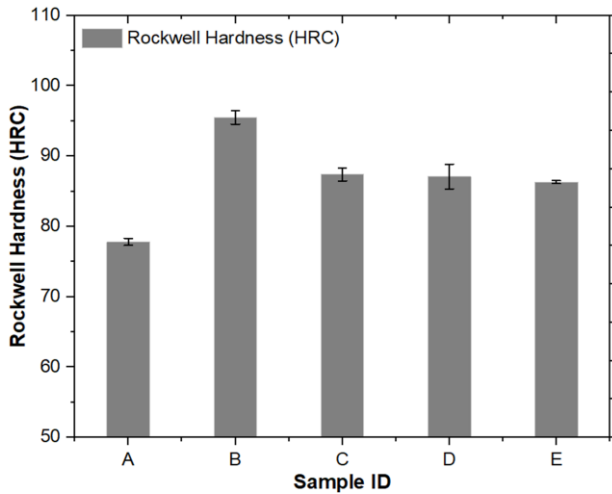


Fig. 5. Effect on the Rockwell hardness for WHT (A), QCH (B) and LTT (C, D and E).

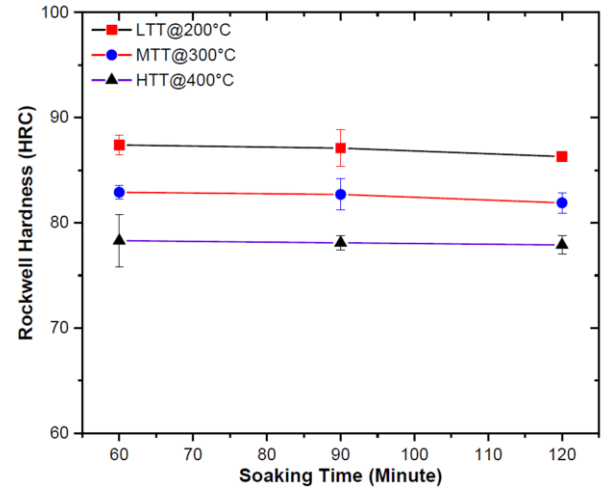


Fig. 8. Rockwell hardness behaviour of heat-treated (LTT, MTT and HTT) samples.

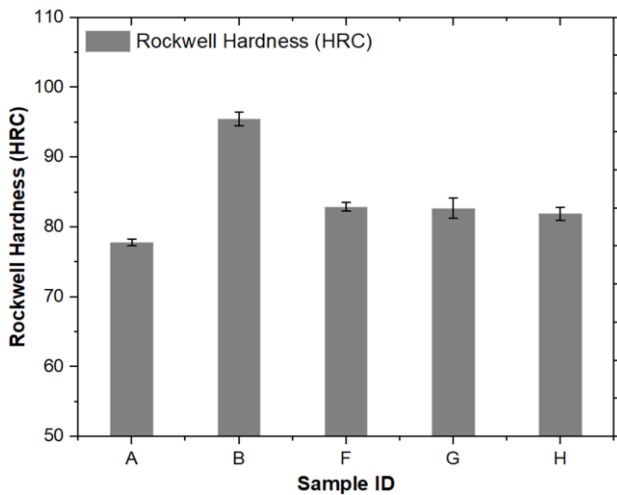


Fig. 6. Effect on the Rockwell hardness for WHT (A), QCH (B) and MTT (F, G and H).

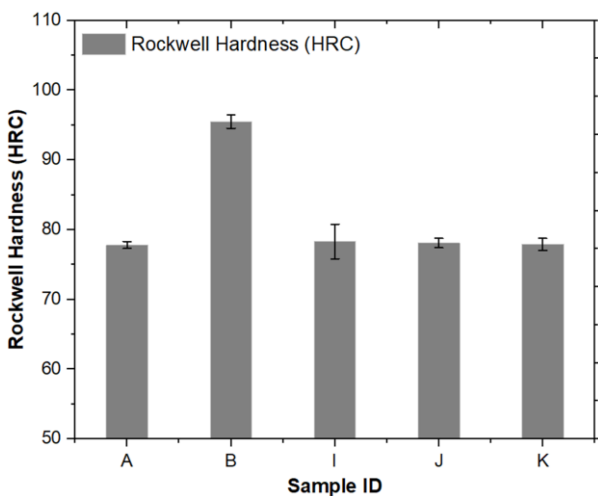


Fig. 7. Effect on the Rockwell hardness for WHT (A), QCH (B) and HTT (I, J and K).

that tempered steel's hardness decreases because of grain re-arrangement in the subsequent tempering heat-treatment process. With an increase in tempering temperatures, the developed martensite during quenching breaks faster, resulting in a decrement in hardness.

4.2. Tensile strength

The tensile test experimental results of A36 ASTM standard samples are shown in Table 3. The ultimate tensile strength and yield strength were measured. The experimental results of tensile tests of heat-treated specimens are shown in Figs. 9–11 for LTT, MTT, and HTT conditions, respectively.

The results showed that the quenching process had improved both the ultimate tensile strength and yield strength by 31.51% and 32%, respectively, compared to the material samples without quenching. At the same time, the elongation has decreased from 17.04% to 15.39% compared to the samples without quenching. It has also been observed that the tempering with different soaking times has an insignificant effect on ultimate tensile strength, yield strength, and percentage of elongation. The strength of the samples dropped significantly due to the tempering process. This is possibly due to relieving the developed internal stresses and breaking down the martensite. Whereas the percentage of elongation increases due to improvement in ductility.

From Fig. 12, it can be clearly observed that the tensile strength decreases with an increase in the tempering temperature. Although the soaking time has an insignificant effect on the material's tensile strength.

4.3. Impact strength

The impact test experimental results of ASTM standard samples are shown in Table 4. The samples' impact strength at different conditions is shown in Figs. 13–15.

It has been observed that the impact strength of the quenched sample (B) dropped by 27.27% in comparison to the non-

Table 3. Tensile strength of A36 low-carbon steel.

Sample ID	Heat treatment process [#]	Heating condition (°C/min)	Ultimate tensile strength (MPa)	Yield strength (MPa)	Elongation (%)
A	WHT	–	76.2	78.5	78.8
B	QCH	900 / 120	97.2	95.4	93.8
C	LTT	200 / 60	87.9	86.3	88.1
D	LTT	200 / 90	86.8	85.5	89.0
E	LTT	200 / 120	86.9	85.9	86.2
F	MTT	300 / 60	83.9	81.8	82.9
G	MTT	300 / 90	85.2	80.2	82.8
H	MTT	300 / 120	79.3	82.6	83.8
I	HTT	400 / 60	77.5	81.2	76.3
J	HTT	400 / 90	76.6	79.4	78.4
K	HTT	400 / 120	79.6	77.7	76.3

[#]WHT – without heat treatment, QCH – quenching, LTT – low temperature tempering, MTT – medium temperature tempering, HTT – high temperature tempering.

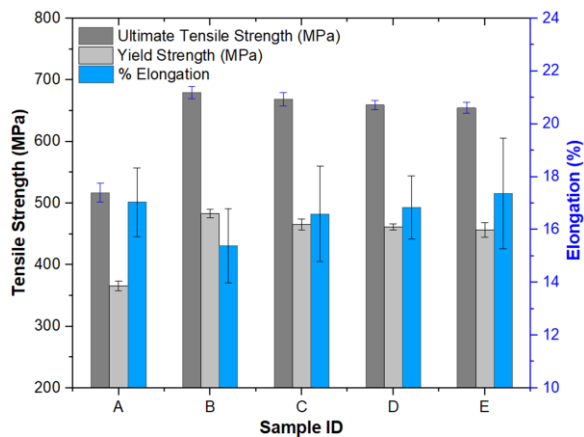


Fig. 9. Effect on the tensile strength for WHT (A), QCH (B) and LTT (C, D and E).

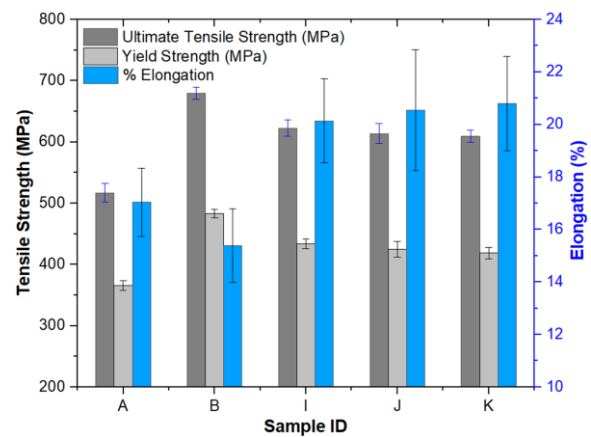


Fig. 11. Effect on the tensile strength for WHT (A), QCH (B) and HTT (I, J and K).

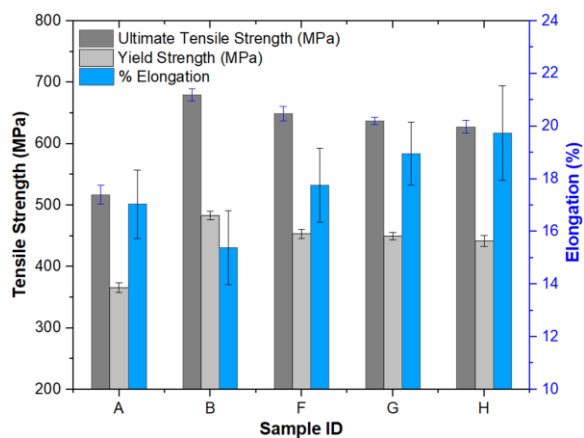


Fig. 10. Effect on the tensile strength for WHT (A), QCH (B) and MTT (F, G and H).

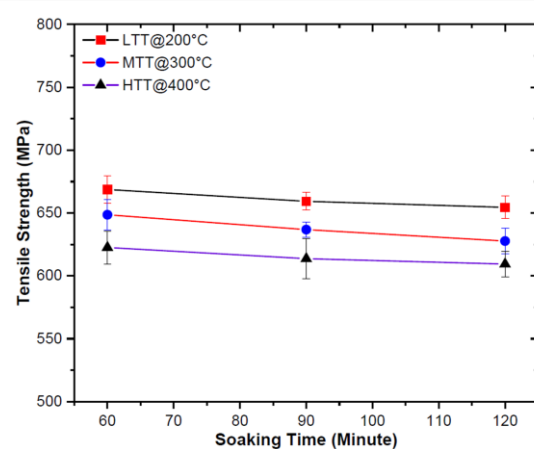


Fig. 12. Tensile strength behaviour of heat-treated (LTT, MTT and HTT) samples.

quenched sample (A). Due to the quenching process, residual stresses developed, resulting in increased material brittleness and decreased toughness. The subsequent tempering process of the sample material developed martensite and relieved the inter-

nal stresses, resulting in a significant improvement in the material's impact strength. The maximum improvement in impact strength of the material is 63.6% and 125% for HTT in comparison to samples without heat-treatment (A) and quenched samples (B), respectively (refer to Fig. 15).

Table 4. Impact strength of A36 low-carbon steel.

Sample ID	Heat treatment process [#]	Heating condition (°C/min)	Impact energy (J)	Impact strength (kJ/m ²)
A	WHT	–	88	1100.0
B	QCH	900 / 120	64	800.0
C	LTT	200 / 60	72	900.0
D	LTT	200 / 90	79	987.5
E	LTT	200 / 120	88	1100.0
F	MTT	300 / 60	103	1287.5
G	MTT	300 / 90	115	1437.5
H	MTT	300 / 12.	121	1512.5
I	HTT	400 / 60	133	1662.5
J	HTT	400 / 90	140	1750.0
K	HTT	400 / 120	144	1800.0

[#]WHT – without heat treatment, QCH – quenching, LTT – low temperature tempering, MTT – medium temperature tempering, HTT – high temperature tempering.

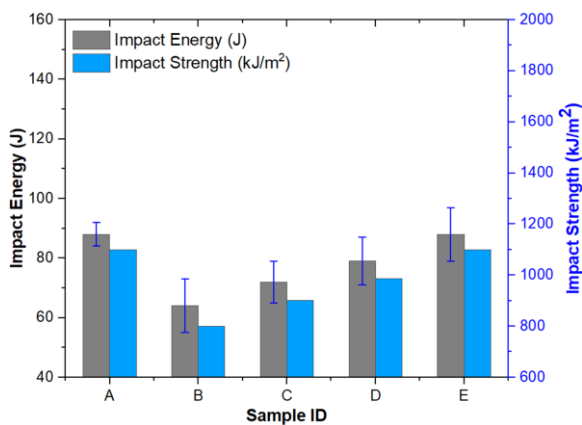


Fig. 13. Effect on the impact strength for WHT (A), QCH (B) and LTT (C, D and E).

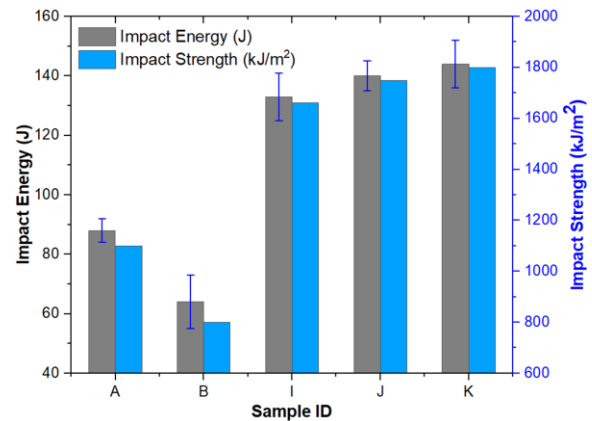


Fig. 15. Effect on the impact strength for WHT (A), QCH (B) and HTT (I, J and K).

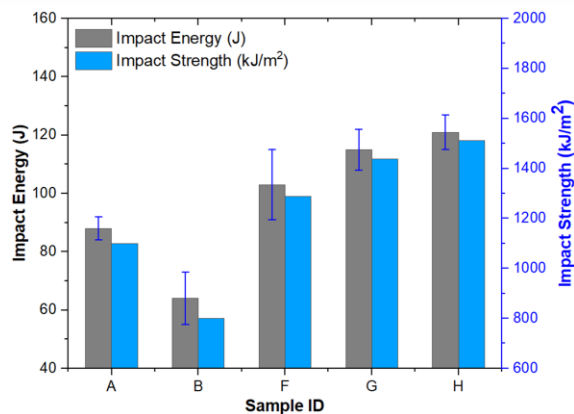


Fig. 14. Effect on the impact strength for WHT (A), QCH (B), and MTT (F, G and H).

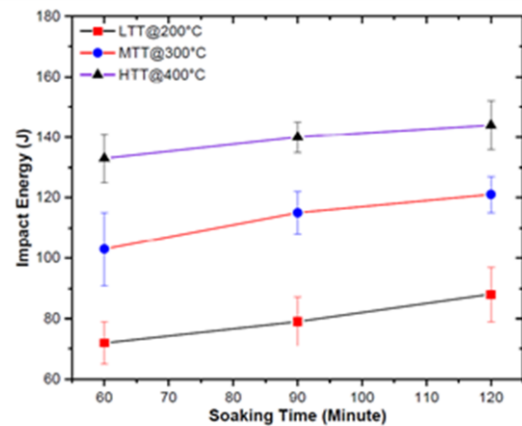


Fig. 16. Impact strength behaviour of heat-treated (LTT, MTT and HTT) samples.

The effect of tempering temperature and soaking time can be seen in Fig.16. It is also observed that with the increase in tempering temperature and soaking time, the impact strength of the material increases. The maximum impact strength of the test material is 144 J for HTT with a soaking time of 120 min. The material had sufficient time to eliminate the internal stresses.

5. Conclusions

The experimental investigation of the effect of heat treatment on low-carbon steel A36 has been carried out. The tests of Rockwell hardness, tensile strength, and impact strength of the ASTM A36 specimens were performed. The results state that the mechanical properties of low-carbon steel depend on heat treatment operations, heating temperatures above or below the critical temperature, and soaking time at a specific temperature. The following significant conclusions are drawn from this study.

- The Rockwell hardness of the samples is improved by 22.75% due to the heat treatment of the material.
- The quenching process improved the ultimate tensile strength and yield strengths by 31.51% and 32%, respectively, compared to those obtained without quenching. At the same time, the elongation decreased from 17.04% to 15.39%.
- Low, medium, and high temperature tempering at various soaking times released the internal stresses, refined the grain structure and exhibited the effects on tensile strength, and improved the elongation at break by 35.08% compared to quenched steel.
- The impact strength of the samples dropped by 27.27% due to quenching compared to the non-quenched sample. However, the subsequent tempering process of the samples improved the material's impact strength by 63.6% and 125% for HTT compared to samples without heat-treatment and quenched samples, respectively.

Therefore, the improved mechanical properties of A36 low-carbon steel through quenching and tempering make it highly suitable for structural applications, including construction, automotive, and machinery components. The increased hardness and tensile strength enhance wear resistance, while tempering at different temperatures optimises toughness and ductility, reducing the risk of failure under dynamic loads. These findings provide valuable insights for industries aiming to balance strength, toughness, and manufacturability in low-carbon steel applications.

The potential areas for further study may include investigating the effect of varying quenching temperatures on the microstructure and mechanical properties of A36 low-carbon steel. Additionally, incorporating different alloying elements such as chromium, nickel, or vanadium could enhance specific properties such as wear resistance, corrosion resistance, and toughness. Future studies could also explore the influence of alternative quenching media and advanced heat-treatment techniques to optimise the performance of A36 steel for diverse industrial applications.

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Experimental Investigation of the Performance and Emission Characteristics of Cotton Seed Biodiesel / Diesel Blends with Titanium Dioxide Nanoparticles

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Abstract

This study uniquely investigates cotton seed oil as a biodiesel source, a less-explored feedstock, and evaluates the role of TiO₂ nanoparticles in enhancing performance and emission characteristics. The use of a variable compression ratio engine under diverse load conditions further distinguishes this work from existing studies on nanoparticle-blended biofuels. The biodiesel was produced via transesterification and blended at various proportions, with and without TiO₂ additives. Experimental tests were conducted on a variable compression ratio diesel engine under varying load conditions: one-fourth, half, three-fourths, and full load. The results were compared against conventional diesel fuel. Among the tested blends, B40 (40% biodiesel, 60% diesel) without TiO₂ exhibited the highest brake thermal efficiency, exceeding that of diesel by 5.4%, and achieved the highest mechanical efficiency (58.31%) at full load condition. Fuel consumption increased proportionally with load across all fuel types. Notably, B40 without TiO₂ also recorded the lowest hydrocarbon emissions, with a reduction of 34 ppm at full load, which corresponds to a 45% decrease compared to conventional diesel. Carbon monoxide emissions were effectively eliminated in TiO₂-blended fuels due to improved combustion. Additionally, nitrogen oxides emissions were reduced by 1.1% in B40 when TiO₂ was incorporated. Overall, the findings highlight that cotton seed biodiesel, particularly when enhanced with TiO₂ nanoparticles, can offer substantial improvements in engine performance and emission control compared to conventional diesel.

Keywords: Biodiesel; Cotton seed biodiesel; Titanium dioxide; Transesterification; VCR engine; Nanoparticles

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1. Introduction

Researchers dedicate their time and efforts to discover alternative fuels to replace petrochemical fuels [1–4]. Certainly, the researchers across the world also work in various fields to meet the environmental sustainability goals [5–10]. It is crucial, however, to develop alternative fuels with properties comparable to

petroleum-based fuels. Biodiesel has emerged as a promising substitute, attracting significant attention due to its environmentally friendly characteristics. Coating diesel engine pistons with a combination of titanium aluminide and yttria-stabilised zirconia significantly improved brake thermal efficiency and reduced emissions when using gulmohar biodiesel blends, with optimal results observed at a 200 µm coating thickness [11]. The addi-

Nomenclature

M – mass, kg
 t – time, s
 T – temperature, °C

Greek symbols

ν – kinematic viscosity, cSt
 ρ – density, g/cm³

Subscripts and Superscripts

m – mean

Abbreviations and Acronyms

BTE – brake thermal efficiency
CO – carbon monoxide
CV – calorific value, kJ/kg
FC – fuel consumption, kg/h
HC – hydrocarbon
ME – mechanical efficiency
NaOH– sodium hydroxide
NO_x – nitrogen oxide
TiO₂ – titanium dioxide
VCR – variable compression ratio

tion of titanium oxide (TiO₂) nanoparticles to *Chlorella vulgaris* biodiesel blends significantly enhanced engine performance, reduced emissions (Carbon monoxide (CO), Hydrocarbon (HC), smoke), and improved tribological properties, with optimal results observed at 100 ppm concentration [12].

The study demonstrated that blending water hyacinth biodiesel with n-pentanol and diesel (WHB20D75P5) enhances engine performance and reduces HC and CO emissions, indicating its potential as a viable diesel alternative [13]. The cotton seed oil stands out as a viable option among potential biodiesel sources. Alabi et al. [14] examined the combustion properties of diesel-vegetable oil blends using an industrial fuel burner, providing insight into critical parameters influencing the feasibility of these alternative fuels in industrial applications. The effects of viscosity on vegetable oil-based fuels can be efficiently minimized by mixing with diesel. This drop in viscosity improves fuel atomization, increases combustion efficiency, and reduces nozzle obstructions. They observed that greater vegetable oil concentration reduces flame duration and stability, which is mostly due to the higher density and viscosity of vegetable oil. The effective use of *Mimusops elangi* methyl ester (MEME) blended with TiO₂ nanoparticles as a biodiesel alternative, highlighting improved engine performance and reduced emissions at an optimized injection pressure of 220 bar [15].

Jain et al. [16] investigated the potential of Nahar biodiesel blended with TiO₂ nanoparticles (NBD-TNP) as a sustainable diesel alternative, demonstrating improved performance, combustion efficiency, and reduced emissions at an optimized injection pressure of 260 bar. Musthafa et al. [17] conducted an experimental study to minimize NO_x emissions from palm oil methyl ester mixed with diesel fuel (B20) utilizing a cetane number improver – di-tert-butyl peroxide (DTBP), and to improve the ignition performance of the test fuel. B20 with additives improves thermal efficiency by 2–3.5% and reduces specific energy consumption by 10–15% when compared to diesel and B20 fuels. In exchange for a little rise in HC, the B20 gasoline without additives showed a significant reduction in CO and nitrogen oxide (NO_x). Machine learning models, particularly random forest regression, have been effectively applied to predict combustion characteristics such as cylinder pressure and heat release rate in dual-fuel diesel engines using sustainable fuel blends, demonstrating high predictive accuracy and potential for emission reduction [18].

Radhakrishnan Lawrence et al. [19] evaluated the performance and emission magnitude of a single cylinder compression

ignition (CI) engine fuelled with M. Elangi methyl ester (MEME) blends including TiO₂ nano-additions. MEME20 + 25TiO₂ and MEME20 + 25TiO₂ @20%EGR fuels had higher brake thermal efficiency (3.6% and 2.8%, respectively) than MEME20 fuel at full load. The addition of TiO₂ nanoparticles to MEME20 gasoline reduced HC and CO emissions, but the EGR approach increased both emissions when compared to MEME20 fuel. Their findings further suggest that the MEME20 + 25TiO₂ @20%EGR with 35% urea injection with Titanium dioxide (TiO₂) coated catalyst (SCR) technology improves engine performance and reduces exhaust emissions.

Keskin et al. [20] investigated the effect of biodiesel-diesel fuel blends, including acetylferrocene and palladium-based additives, on diesel engine performance and emissions. They also confirmed that CO and PM emissions were decreased by up to 60.07% and 51.33%, respectively. Jiaqiang et al. [21] evaluated the combined use of water additives and metal-based additives in biodiesel-diesel mix fuels. They conducted comparative studies on various combinations of biodiesel-diesel (B5), water (2%, 4%, and 6% wt.) and cerium oxide (CeO₂) nanoparticles (90 ppm) with pure diesel in terms of combustion emissions and performance. They have proven that the correct amount of water addition improves performance while decreasing CO, PM, NO_x, and HC emissions. Mehregan and Moghiman [22] evaluated the effects of nano-additives on the performance and emission characteristics of a mixed biodiesel. Their testing results revealed an increase in brake specific fuel consumption and brake thermal efficiency for manganese oxide and cobalt oxide nanoparticles mixed gasoline, while NO_x and CO emissions were significantly reduced. Soudagar et al. [23] conducted a comprehensive evaluation of the influence of nano-additives in diesel-biodiesel fuel blends on engine performance and emission parameters. They confirmed that the majority of the outcomes improved thermo-physical parameters, increased heat transfer rate, and stabilized fuel blends. Furthermore, scientists discovered that the amount of nanofluid additives increased engine performance characteristics while decreasing exhaust emissions.

Figure 1 represents a comprehensive flow diagram illustrating the life cycle and energy cycle of biodiesel production and its investigation process. The primary source is, i.e. cotton crop cultivation, where cotton seeds are collected as a by-product for the purpose of biodiesel preparation. These seeds undergo an oil extraction process to produce raw oil. The extracted oil is subjected to transesterification, a chemical process that removes fatty acids and glycerine, converting the oil into biodiesel. To

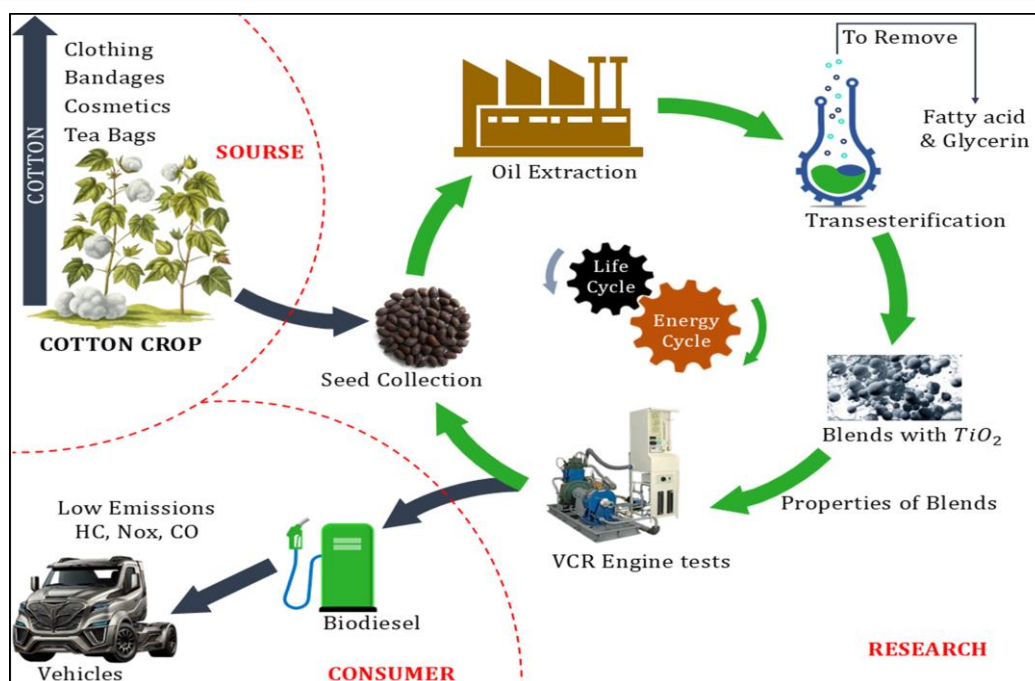


Fig. 1. Flow diagram illustrating the preparation and experimentation with blends – life cycle & energy flow cycle.



Fig. 2. Flow diagram of blends preparation.

enhance fuel properties, the biodiesel is blended with TiO_2 nanoparticles, forming experimental blends. Figure 2 also highlights the subsequent steps of evaluating the properties of these blends and testing their performance using a VCR engine.

The process contributes to the energy cycle by assessing emissions such as HC, NO_x and CO when used in vehicles, demonstrating low-emission outcomes. Additionally, the life cycle aspect integrates the biodiesel production process into practical applications, emphasizing sustainability and its potential to serve as an alternative fuel source. The overall figure connects source, research, and consumer applications, presenting a holistic approach to biodiesel production, experimentation, and utilization. This study investigates the performance and emission characteristics of cotton seed biodiesel-diesel emulsion blends with the addition of TiO_2 nanoparticles. The results are compared to those obtained using conventional diesel fuel. Cotton seed biodiesel was produced and tested under various loading conditions, utilizing a factorial design of experiments. The article examines the impact of different blends of this environment-

tally friendly alternative fuel on diesel engine performance and emission parameters.

2. Methodology

2.1. Preparation of biodiesel

The biodiesel was produced using cotton seeds obtained from local cotton farmers. Initially, the collected cotton seeds were sent to an oil extraction mill. The extracted oil was then subjected to a series of steps to produce biodiesel. A 1000 ml flat-bottom flask was selected as the reactor, which was placed on a magnetic stirrer set to maintain a temperature between 55°C and 60°C . The stirrer was positioned centrally to ensure uniform mixing of the reaction mixture. Before transesterification, the cotton seed oil underwent pre-treatment. The oil was first filtered to remove any solid impurities and then preheated to 100°C . It was maintained at this temperature for 25 minutes to eliminate moisture content, ensuring the oil was suitable for the biodiesel production process.

Subsequently, transesterification was carried out using a base-catalysed reaction to remove fatty acids and glycerine from the oil. A mixture of alcohol and catalyst was combined with 1000 ml of preheated cotton seed oil in the reactor, maintaining a temperature of 60–70°C. In a separate beaker, 4 grams of NaOH were dissolved (calculated as 3.8 grams per litre of oil: 3.5 grams stoichiometric equivalent plus 0.3 grams for neutralizing free fatty acids) in 2.4 litres of methanol, with a ratio of 200 ml methanol per litre of oil. NaOH was added gradually to ensure proper dissolution, forming sodium methoxide. The sodium methoxide solution was then added to the cotton seed oil and vigorously stirred to facilitate the reaction. During the process, the cloudy free fatty acids and glycerine separated and settled at the bottom of the reactor, while the fatty acid methyl ester (biodiesel), a clear translucent liquid, remained on the top. Stirring was stopped once the separation process began to plateau, and the mixture was allowed to settle overnight to ensure complete separation. The upper layer containing the fatty acid methyl ester (biodiesel) was carefully collected, but any remaining soaps or salts needed to be removed through further purification before use. The glycerine, which settled at the bottom, can be repurposed for applications such as cosmetic production. While the reactor was resting during the settling process, a new batch could be initiated to optimize the production efficiency.

The fatty acid methyl ester (biodiesel) was transferred into a clean container and thoroughly washed to remove any residual soaps and salts. Warm water was added to the methyl ester, and the mixture was gently swirled to allow separation. After settling, warm water was carefully drained from the bottom. This rinsing process was repeated until the discharged rinse water reached a neutral pH of 6–7 and was free of soap bubbles. The resulting liquid appeared hazy, indicating the presence of trapped water, which needed to be evaporated by gently heating the biodiesel. Any white solids settling at the bottom or bubbles forming on the surface were signs of residual soap; in such cases, further re-washing or removal was performed. Dehydration of the methyl ester was carried out at 100°C to ensure the final biodiesel product was free from moisture.

Blends of biodiesel with diesel fuel were prepared in varying proportions, analysed, and compared to pure diesel fuel. The blends included 15% flax methyl ester with 85% diesel, 10% flax methyl ester with 90% diesel, 5% flax methyl ester with 95% diesel, and 100% pure flax methyl ester.

To ensure uniform dispersion of nanoparticles in the biodiesel blend, a magnetic stirrer was initially used to mix the TiO₂ nanoparticles thoroughly with the biodiesel. This was followed by ultrasonication (ultrasonic agitation) to break up nanoparticle clusters, resulting in a stable and homogenous dispersion of TiO₂ within the blend. To minimize the risk of agglomeration during engine operation, fresh blends were prepared immediately before testing to reduce the chances of sedimentation. Additionally, the TiO₂ concentration was carefully optimized (≤ 100 ppm), as higher concentrations are more prone to agglomeration. During engine operation, continuous fuel injection pressure and in-cylinder turbulence further aid in maintaining a uniform nanoparticle dispersion throughout the combustion process.

2.2. Estimation of thermophysical properties

Kinematic viscosity

The kinematic viscosity of several cotton seed biodiesel blends is evaluated using a Redwood viscometer as shown in Fig. 3a. This device does not provide a direct measurement of viscosity in absolute units, but it does allow for the comparison of oil viscosities by measuring the efflux of 50 ml of oil through the instrument's standard orifice under standard circumstances.



Fig. 3. Redwood viscometer set-up (a), Cleveland set-up (b), bomb calorimeter set-up (c).

Flash and fire point

The flash and fire points of each mix of biodiesel samples were evaluated as per IS1448 by using the Cleveland set-up. The oil sample under test was poured into an oil cup until it reached the mark. Figure 3b shows the equipment used to determine the flash and pour points. The gasoline sample was swirled at a modest continuous pace of around 60 rpm. The oil sample was heated at a rate of 5 to 6 degrees of Celsius per minute. A shudder was used to introduce flame for a brief period of time with each 1°C increase in temperature. The flash point was defined as the lowest temperature at which a clear flash was seen. The fire point was defined as the temperature at which fuel vapour ignites and remains in flames for at least five seconds.

Calorific value

The heat of combustion of fuel samples was evaluated using an IS1448-compliant Wilson Scientific Works isothermal bomb calorimeter, as illustrated in Fig. 3c. A 1ml fuel sample was burned in a calorimeter bomb with pure oxygen. The sample was ignited electrically. Temperature increase was measured as heat was generated. The water equivalent (effective heat capacity of the calorimeter) was also calculated using pure and dry benzoic acid as the test fuel. Every sample was repeated three times. The gross heat of combustion for the fuel samples was determined using the equation shown in Table 1.

Table 1. Kinematic viscosity of cotton seed biodiesel blends.

Kinematic viscosity	Calorific value
If $t_m < 34 \Rightarrow \nu = 0.24 \times t_m - 50/t_m$, If $t_m < 34 \Rightarrow \nu = 0.26 \times t_m - 179/t_m$,	$H_c = W_c \times \Delta T/M_s$,
where:	
t_m – mean time, s,	
ν – kinematic viscosity, cSt,	
H_c = heat combustion of the fuel sample, cal/g,	
W_c = water equivalent of the calorimeter, cal/°C,	
ΔT = rise in temperature, °C,	
M_s = mass of sample burnt, kg.	

The properties of various cottonseed biodiesel blends are presented in Table 2.

Table 2. Properties of different blends of cotton seed biodiesel.

Blends	Kinematic viscosity, cSt	Calorific value, kJ/kg	Flash point, °C	Fire point, °C	Density, g/cm ³
Diesel	4.18	45270	60	64	0.81
Biodiesel	4.89	41760	172	184.4	0.874
B10	4.15	40890	71	77.2	0.82
B20	3.87	39640	82.4	89.6	0.81
B30	2.62	3710	91.7	97.5	0.79
B40	2.26	35110	103.8	110.2	0.792
B10+TiO ₂	2.68	39750	77	83	0.841
B20+TiO ₂	2.8	38170	87	93.5	0.849
B30+TiO ₂	4.3	36190	96.4	101.9	0.857
B40+TiO ₂	4.4	34260	106.2	112.3	0.861

3. Experimental set-up and procedure

The experimental setup consists of an engine, a dynamometer, and an exhaust gas analyser. The engine is a single cylinder, four stroke, vertical, water-cooled, direct injection computerised VCR CI engine, as shown in Fig. 4. The eddy current dynamometer comprises of a stator with electromagnets and a rotor disc that is connected to the engine's output shaft. When the rotor rotates, eddy currents form in the stator owing to the magnetic flux created by the passage of field current via the electromagnets. These eddy currents impede rotor rotation, therefore loading the engine.

These eddy currents enhance heat production. As a result, a cooling system is used to disperse the heat. A moment arm measures torque. The load is controlled by regulating the current using electromagnets. First, tighten the nut bolts on the engine, dynamometer, propeller shaft, and base frame. Ensure that the engine sump tank contains enough lubricating oil. This may be verified by marking the level stick. Ensure that there is enough fuel in the tank. Remove any air that may be in the fuel line. Turn on the electric power supply to the piezo powering unit (PPU)/AX-409, the dynamometer loading unit (DLU)/AX-155, the load indicator, and the voltmeter. Configure the engine model for configuration variables and system constants using the "Enginesoft" program on your computer. Start water circulation by using a water pump and adjust the flow rate of 250–350 l/h for the engine and 75–100 l/h for the calorimeter by regulating the valves provided at the rotameter inlet. Ensure that water is flowing through a dynamometer at a pressure of ~0.5–1 bar.

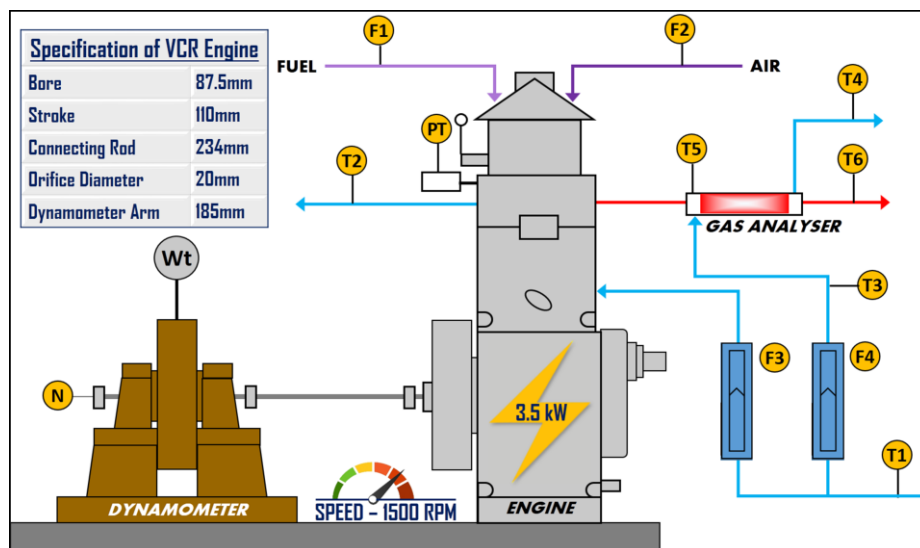


Fig. 4. Experimental set-up with instrumentation.

3.1. Factorial design of experimentation

Initially bypass the measuring tube as shown in Fig. 5 and allow the fuel to engine and adjust the knob on the dynamometer loading unit (DLU) for zero kg load and wait for 3 minutes to ensure the load is constant. To estimate the fuel consumption, regulate the valve to direct the fuel to the engine through the measuring

tank. Measure the time for the consumption of 5cm³ fuel under zero load. Again, bypass the measuring tube and change the load to 3 kg by using DLU. After ensuring the constant load reading, operate the valve to direct the fuel to the engine through the measuring tube. Measure the time for the consumption of 5 cm³ fuel under 3 kg load. Repeat the above procedure for the different set of loads as shown in Table 3 and record data. After re-

ording all the required data, remove the load on the engine by using DLU, and Click "Scan Stop" on a PC. Stop the engine and allow the water to circulate for 5 minutes. All the blends and blends with TiO_2 have been tested, and data have been collected by repeating the above experimental procedure.

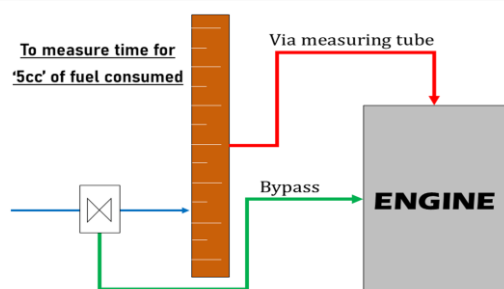


Fig. 5. Fuel flow configuration (to regulate the fuel flow direction) to conduct the experiments.

Table 3. Factorial design of experiments of diesel, B10, B20, B30, B40, B10+ TiO_2 , B10+ TiO_2 , B20+ TiO_2 , B30+ TiO_2 , B40+ TiO_2 .

Experiment number	Fuel flow direction	Regulate the DLU knob for load, kg	Record the time for fuel consumption, cm^3
1	Bypass measuring tube	0	5
	Through measuring tube	0	5
2	Bypass measuring tube	3	5
	Through measuring tube	3	5
3	Bypass measuring tube	6	5
	Through measuring tube	6	5
4	Bypass measuring tube	9	5
	Through measuring tube	9	5
5	Bypass measuring tube	12	5
	Through measuring tube	12	5

The exhaust emission parameters were determined using the AVL flue gas analyser. The AVL analyser was used to examine HC emissions. Furthermore, emission characteristics such as CO and nitrogen oxide (NO_x) were assessed. The sensors were installed into the exhaust gas outlet under each load state. The exhaust gases were fed through the analyser using sensors. At least three readings of the analyser digital screen were noted down, and the mean was be considered.

4. Results and discussion

The blends of varying proportions of flax methyl ester and diesel were prepared, analysed, and compared with diesel fuel. The different blends in the proportions of 10% alternative oil with 90% diesel, 20% alternative oil with 80% diesel, 30% alternative oil with 70% diesel, and 40% alternative oil with 60% diesel were tested in the present investigation. These blends were also tested by incorporating TiO_2 nanoparticles at a concentration of 30 ppm.

All the experiments were conducted under a 200 bar pressure with a compression ratio of 16:1. The experiments were conducted for the performance and emission characteristics of a cotton seed biodiesel prepared through the transesterification method, followed by volume-based blending with the addition of TiO_2 . The parameters viz. brake thermal efficiency (η_{BTE}), mechanical efficiency (η_{ME}), fuel consumption (FC) and brake specific fuel consumption (BSFC) were estimated to examine the performance characteristics. Further, the fraction of CO, the fraction of HC, and the fraction of NO_x were estimated to examine the emission characteristics.

Figure 6 shows the variation of brake thermal efficiency (BTE) and the mechanical efficiency with respect to load. Figure 6a shows the increase in BTE for various cotton seed biodiesel blends, both with and without TiO_2 , under different load condi-

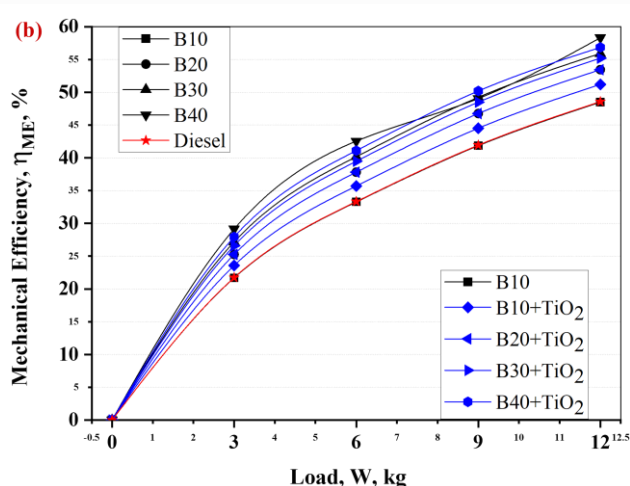
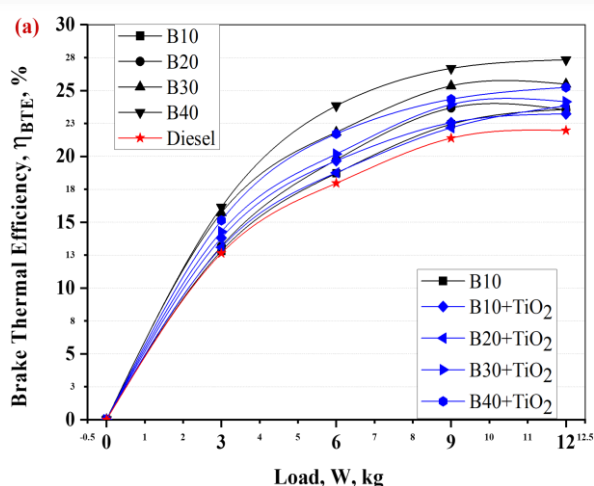


Fig. 6. Variation of efficiencies with load for different blend proportions (a) BTE (b) ME.

tions (3 kg, 6 kg, 9 kg, and 12 kg). The zero line denotes pure diesel, serving as a reference point. The biodiesel blends (B10, B20, B30, B40) show a noticeable increase in BTE compared to pure diesel under all loads. The biodiesel blends with TiO_2 ex-

hibit higher BTE improvements than their corresponding biodiesel blends without TiO_2 , suggesting that TiO_2 improves efficiency. The trend generally shows that higher biodiesel blends tend to yield greater efficiency improvements, with B40 + TiO_2

achieving the highest increase in BTE across most load conditions. The increase in BTE tends to be greater at higher loads for all fuel blends. The improved combustion behaviour with TiO_2 inclusion can be attributed to its catalytic properties, which promote surface oxidation reactions and enhance fuel-air mixing. Additionally, TiO_2 acts as an oxygen buffer, aiding in the oxidation of unburnt hydrocarbons and thereby improving overall combustion efficiency. However, the B40 blend without TiO_2 outperforms the TiO_2 -enhanced blends in terms of brake thermal efficiency. This may be due to more complete combustion occurring at the higher pressures and temperatures present under full-load conditions, even without the aid of the catalyst.

The graph shown in Fig. 6b represents the variation of mechanical efficiency with respect to load for different fuel blends and diesel, with and without the addition of TiO_2 nanoparticles. The fuel blends containing TiO_2 exhibit reduced mechanical efficiency compared to their counterparts without TiO_2 . However, this trade-off is considered worthwhile because the addition of TiO_2 helps reduce emissions. Diesel shows the lowest mechanical efficiency across all load values compared to the biodiesel blends, indicating that biodiesel blends (both with and without TiO_2) offer better efficiency. The blends B40 and B40 + TiO_2 show the highest mechanical efficiency values, especially at higher load levels. Figure 6b indicates that biodiesel blends, particularly those enhanced with TiO_2 , can achieve better mechanical efficiency than conventional diesel fuel. However, the B40 blend without TiO_2 outperformed the TiO_2 -enhanced blends in terms of mechanical efficiency. This can be attributed to the inherently higher oxygen content, better volatility, and improved atomization characteristics of the B40 blend, which facilitate more complete combustion. Although TiO_2 is known to promote oxidation, under full load conditions, its excessive thermal activity may lead to premature combustion, resulting in engine knocking and a subsequent reduction in mechanical efficiency.

Figure 7 depicts the relationship between fuel consumption and load for various fuel types, including biodiesel blends (B10, B20, B30, B40) and diesel, with and without the addition of TiO_2 nanoparticles. All fuel types exhibit the increased fuel

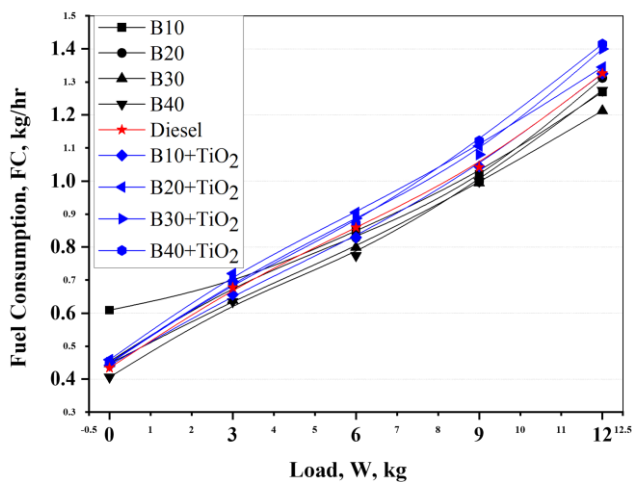


Fig. 7. Fuel consumption vs. load for different blend proportions.

consumption as the load on the engine increases, ranging from approximately 0.5 kg/h at zero load to over 1.2 kg/h at 12 kg load. The fuel blends with TiO_2 show slightly higher fuel consumption compared to their non- TiO_2 counterparts at the cost of decrease of exhaust gas emissions with the addition of TiO_2 particles. The fuel consumption of cotton seed biodiesel blends has been decreased when compared with pure diesel by 1.3 % to 9.8%. While in the case of cotton seed biodiesel blends with addition of TiO_2 particles, the fuel consumption has been raised by 1.8% to 7.6% when compared to diesel.

Figure 8 depicts the fluctuation in brake specific fuel consumption (BSFC) with respect to the engine load with a compression ratio of 16:1. BSFC increases with all blends and diesel and gradually falls to full load for all blends and fuel. This is because when the load grows, the temperature within the cylinder rises, reducing the delay period. At full load, B10 has the lowest BSFC of any mix of diesel. At maximum load, the minimum BSFC for B10 is 0.373 kg/kWh.

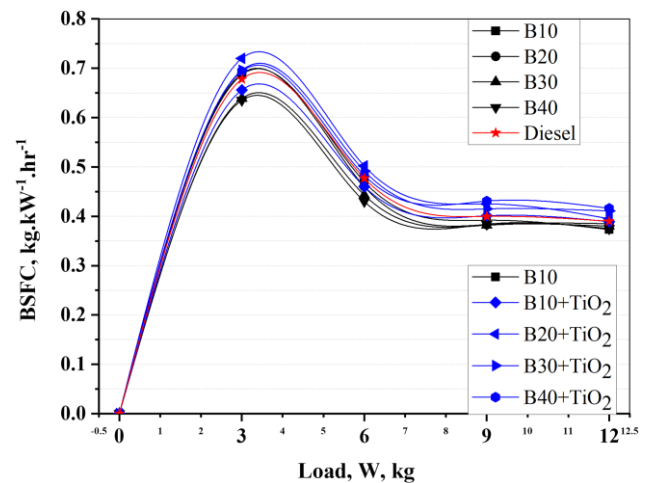


Fig. 8. BSFC vs. load for different blend proportions.

The graph shown in Fig. 8 illustrates the relationship between brake specific fuel consumption (BSFC) and load (kg) for different fuel blends, including biodiesel blends (B10, B20, B30, B40) and diesel, with and without the addition of titanium dioxide (TiO_2) nanoparticles. The addition of TiO_2 results in a slight improvement in fuel efficiency, as seen by the marginally lower BSFC values compared to their corresponding biodiesel blends without TiO_2 . As the engine load increases from 3 kg to 12 kg, BSFC consistently decreases, suggesting that the engine operates more efficiently at higher loads. B10 shows the highest BSFC values among biodiesel blends, while higher biodiesel blends like B40 exhibit slightly lower BSFC values at the higher load ranges.

Figure 9a depicts the variation of HC emissions in parts per million (ppm) with respect to load (in kg) for various biodiesel blends and diesel, both with and without the addition of TiO_2 nanoparticles. The fuel blends are B10, B20, B30, and B40, while B10 + TiO_2 , B20 + TiO_2 , B30 + TiO_2 , and B40 + TiO_2 represent the respective blends with TiO_2 . For all fuel types, HC emissions increase as the load increases, especially beyond 6 kg.

The trend is more pronounced in conventional diesel and biodiesel blends without TiO_2 . Diesel exhibits the highest HC emissions across all load levels, with a sharp increase as load increases. Biodiesel blends without TiO_2 (B10, B20, B30, B40) show moderate HC emissions. Among these, B10 has the highest emissions, while B40 has the lowest. The addition of TiO_2 nanoparticles significantly reduces HC emissions for all biodiesel blends (B10 + TiO_2 , B20 + TiO_2 , B30 + TiO_2 , B40 + TiO_2) across the entire load range. B40 + TiO_2 shows the lowest HC emissions, highlighting the efficiency of the higher concentration biodiesel blend with TiO_2 in reducing emissions. B40 + TiO_2 achieves the minimum HC emissions at all load levels, indicating the effectiveness of using high biodiesel blends with TiO_2 nanoparticles for reducing HC emissions.

The graph shown in Fig. 9b shows the variation of CO emission (in ppm) with respect to load for various fuel types, including blends of biodiesel (B10, B20, B30, B40) and standard diesel. As the load increases, CO emissions also increase for all fuels, but the rate of increase varies across the fuels. The increase in CO emission becomes more pronounced at loads above

6 kg, with biodiesel blends generally producing higher emissions than pure diesel. Higher biodiesel content in the blend (e.g. B20, B40) appears to result in greater CO emissions as the load increases, suggesting a trade-off in CO emissions with biodiesel use. Diesel has the lowest CO emissions across all loads compared to biodiesel blends.

The graph shown in Fig. 9c represents the relationship between NO_x emissions (in ppm) and load (in kg) for various fuel types and biodiesel blends, including the effect of adding TiO_2 to the blends. B10 has the highest NO_x emissions as the load increases, reaching above 600 ppm at the highest load. B40 and B30 also exhibit high NO_x emissions as the load increases, but lower than B10, B10 + TiO_2 and other biodiesel blends with TiO_2 additives, which show reduced NO_x emissions compared to their non-additive counterparts, indicating the effectiveness of TiO_2 in reducing NO_x emissions. Adding TiO_2 to biodiesel blends appears to reduce NO_x emissions across all load conditions, with a significant drop observed when comparing the emissions of blends like B20, B30, and B40 with and without TiO_2 .

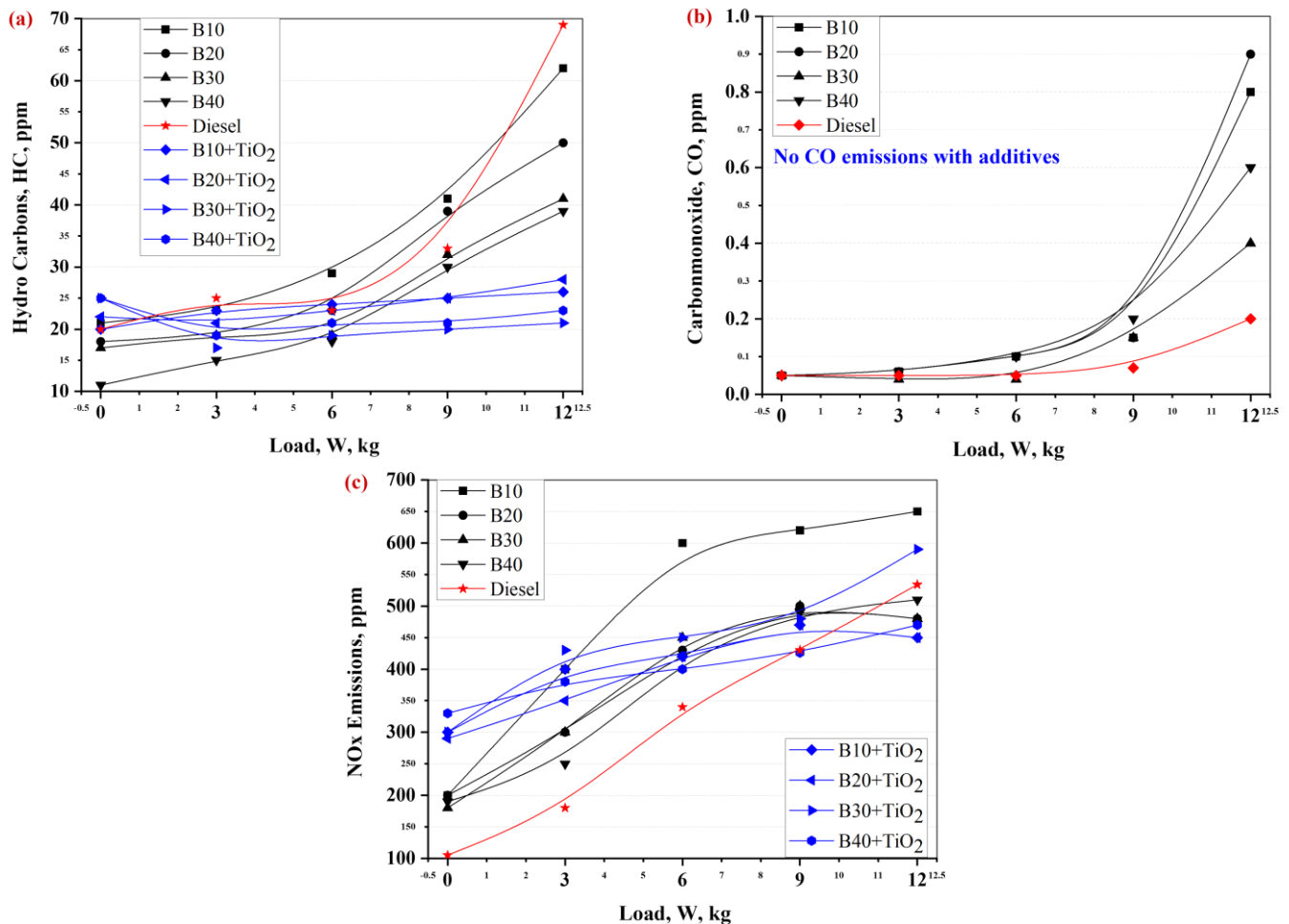


Fig. 9. Emission characteristics: (a) HC, (b) CO, (c) NO_x .

The increase of brake thermal efficiency (BTE) of different blends of cotton seed biodiesel with respect to diesel is quantified and shown in Fig. 10. In case of B10, the brake thermal efficiency has been increased by 1.3%, 4.1%, 4.9% and 7.2% for 3 kg, 6 kg, 9 kg and 12 kg loads, respectively. In the case of B20,

the brake thermal efficiency has been increased by 16%, 13%, 11% and 10% for 3 kg, 6 kg, 9 kg and 12 kg loads, respectively. The fuel consumption has been decreased for B10, B20, B30 and B40, respectively. However, the fuel consumption is increased with the addition of TiO_2 particles.

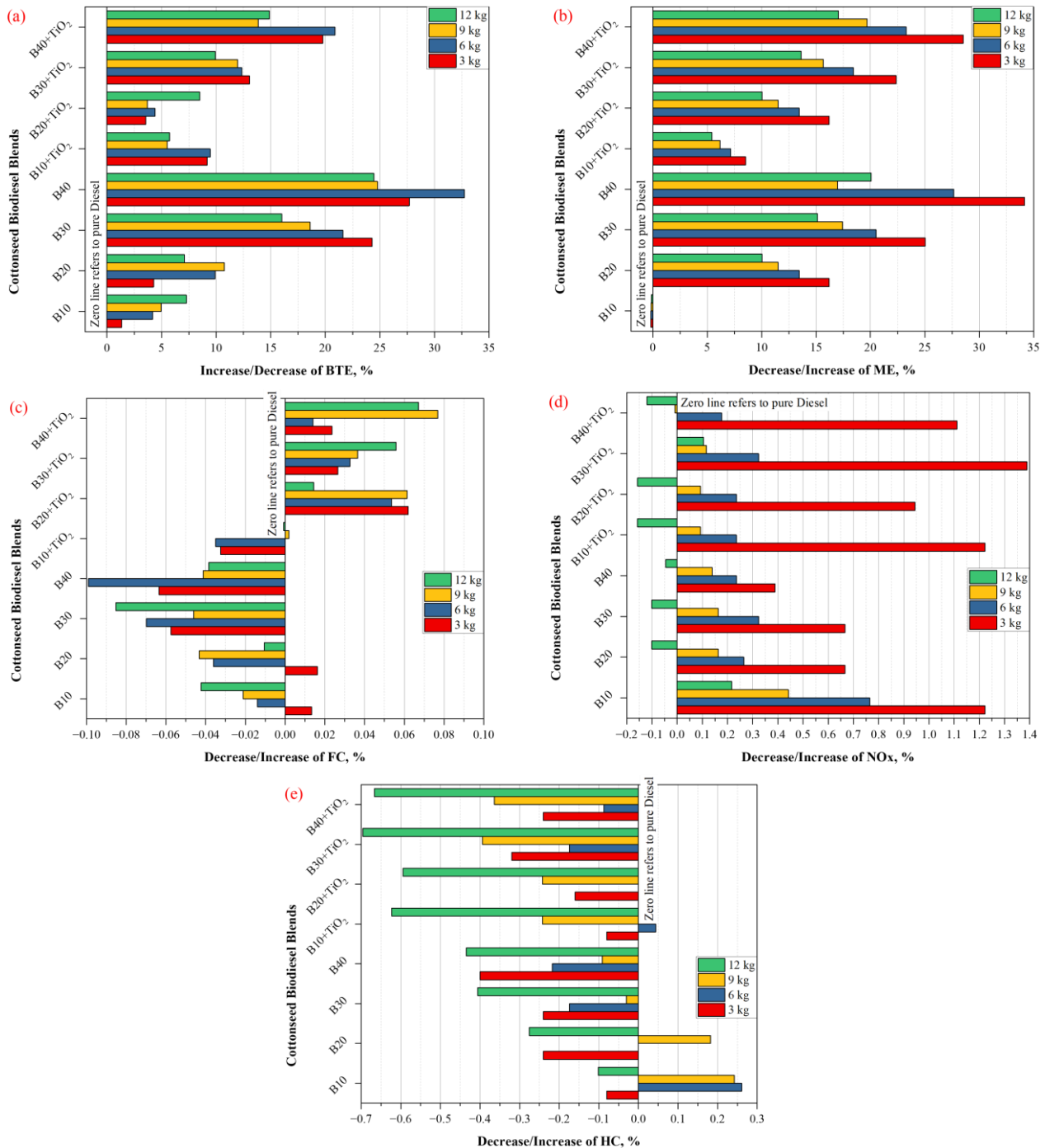


Fig. 10. Variation of performance and emission parameters of different blends of cotton seed biodiesel with respect to diesel: (a) BTE, (b) ME, (c) FC, (d) NO_x, (e) HC.

5. Conclusions

An experimental investigation was conducted to study the performance and emission characteristics of a variable compression ratio engine using various blend proportions, with and without additives, and compared to diesel. The key findings are as follows:

- Across all diesel blends, BTE improves as the temperature increases. Among the blends, B40 without additive achieved the highest BTE, surpassing diesel by 5.4%.

- Mechanical efficiency increases gradually with load for all blends. At full load, B40 without additive exhibited the highest mechanical efficiency at 58.31%, outperforming all other blends and diesel.
- Fuel consumption rises with engine load for all blends and diesel. B40 without additive demonstrated the lowest fuel economy at full load but still reduced fuel usage by 0.058 kg/h compared to diesel.
- Initially, BSFC increases for all blends and diesel, then decreases at full load. B10 without additive had the lowest

BSFC at full load, reducing BSFC by 0.017 kg/kWh compared to diesel.

- ➔ B10 without additive produced higher HC emissions than diesel up to 34% load but lower emissions at full load. The B40 blend without additive emitted fewer hydrocarbons than all other blends and diesel, reducing emissions by 34 ppm at full load, which corresponds to a 45% decrease compared to conventional diesel.
- ➔ CO emissions increased steadily from no load to full load for all blends without additives. The B30 blend without additive emitted less CO than diesel at half load. CO emissions were zero for all blends with additives, as TiO₂ promotes combustion, enhancing CO oxidation and reducing HC emissions.
- ➔ NO_x emissions increased with load for all blends and diesel. The addition of TiO₂ is beneficial in lowering NO_x emissions in biodiesel blends, especially for higher percentages of biodiesel content. NO_x emissions were reduced by 1.1% with the addition of TiO₂ in biodiesel blends (B40).

Beyond performance and emission improvements, the practical viability of TiO₂-enhanced biodiesel blends depends on factors such as cost, engine compatibility, and nanoparticle stability. Although TiO₂ is relatively low-cost and widely available, the long-term effects on engine wear, injector clogging, and catalyst degradation require further study. Moreover, ensuring uniform dispersion and preventing nanoparticle agglomeration during storage are essential for maintaining consistent combustion benefits. These considerations are vital for translating laboratory results into real-world applications.

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CFD based design of blast furnace gas cleaning cyclone separator

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Abstract

The paper focuses on the design of the geometry and operational parameters of a cyclone separator. Its main objective is to identify a combination of dimensional and flow parameters that enables the efficient capture of large particles containing carbon and iron, while allowing particles rich in zinc and lead to pass through. The cyclone separator is intended to separate dust with a high content of valuable materials so that it can be reprocessed in a blast furnace. A computational fluid dynamics model, previously validated against similar datasets, is employed to support the design process. Three different geometries and inlet velocities ranging from 2 m/s to 12 m/s were tested to determine the most effective configuration. Depending on the selected parameters, the overall separation efficiency varied between 46% and 93%. Specifically, the system achieved separation efficiencies of 60–95% for carbon, 38–90% for iron, and 14–67% for zinc. The final design reached an overall efficiency of 86.5%, successfully separating over 90% of carbon, more than 80% of iron, as well as 45% of zinc and 55% of lead.

Keywords: Computational fluid dynamics; Cyclone; Dust; Blast furnace; Zinc

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1. Introduction

This study aims to determine the basic geometric parameters of a cyclone used for the preliminary purification of blast furnace (BF) gas from coarse dust fractions. This cyclone represents the first stage in the BF gas cleaning process. The blast furnace produces pig iron from iron ore, coke, and fluxes. The process involves the reduction of iron ore by carbon monoxide, which is

generated through the incomplete combustion of coke. A byproduct of BF operation is blast furnace gas, characterised by high temperature, elevated pressure, a significant concentration of carbon monoxide, and a high dust content. This dust contains substantial amounts of carbon, iron, zinc, and lead.

During gas purification, a dust containing carbon and iron, valuable components for the BF process, is returned to the furnace. However, dust containing zinc and lead, which are harmful

Nomenclature

D – diameter, m
 d_{50} – cut-off point
 g – mass fraction
 H – height, m
 m – mass, kg
 r – mass ratio

Greek symbols

α – under-relaxation factor
 η – efficiency

Subscripts and Superscripts

A, B, C – simulation variant
 d – dust catcher
 el – element

f – fraction
 gc – cleaned gas
 raw – raw dust
 ws – wet scrubber
 Zn – zinc

Abbreviations and Acronyms

BF – blast furnace
 CFD – computational fluid dynamics
 CFL – flow Courant number
 CS – cyclone separator
 DC – dust catcher
 DPM – discrete phase model
 LES – large eddy simulation
 RSM – Reynolds stress model
 WS – wet scrubber

to the process [1], must be separated and subjected to metallurgical treatment for the recovery of these metals. Once the dust is removed, the BF gas can be expanded in turbines and used to preheat the air supplied to the tuyeres at the base of the furnace.

The various dust fractions differ in composition: coarser and medium-sized fractions tend to be richer in organic carbon (originating from coke) and iron, while finer fractions typically contain higher concentrations of zinc and lead [2]. Therefore, the cyclone should be designed to ensure efficient separation of coarse and medium-sized particles with high separation efficiency. The fine fractions are captured in a wet scrubber and removed as sludge, which is then processed externally from the ironworks. The criterion for the separation of coarse and fine fractions (i.e., the cut-off point) is based on the concentration of zinc and lead in the separated dust.

The upgrade of an obsolete BF gas cleaning system necessitated the development of a new cyclone separator. The current setup utilised a dust catcher (DC), a sizeable chamber where dust particles were separated using gravity and inertia. Despite its usage, this approach failed to offer the necessary selectivity and separation effectiveness. The enhanced system aimed to replace the dust catcher with a cyclone separator (CS). Figure 1 illustrates the former configuration of the installation.

In industrial settings, the separation of solid particles from gas or liquid is frequently required. The selection of a specific separation technique is influenced by factors like particle size and composition, along with operational conditions such as gas temperature, velocity and pressure. A common method involves using centrifugal force to extract solid particles from a rotating fluid [3], a process executed within CSs. These separators are advantageous due to their low cost, straightforward design, and minimal upkeep needs. Moreover, cyclone separators are capable of functioning under high temperatures and pressures, and they manage a broad spectrum of flow rates [4].

Cyclone separators tend to have lower effectiveness in capturing smaller particles, particularly those of sizes smaller than a micrometre. Since CSs are designed for specific dust types, any variation in dust characteristics can greatly influence their

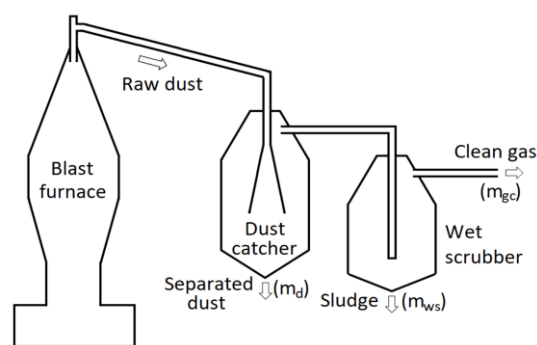


Fig. 1. Schematic of the dusting installation to be modernized with marked measurement locations.

efficiency. Cyclone efficiency is often described by the fractional efficiency curve and a cut-off point, enabling evaluation under different dust scenarios. Cyclone separators are primarily categorised into two types: axial and tangential, differentiated by their induction of swirling motion. In axial CSs, internal vanes or blades direct the gas, whereas in tangential CSs, the gas enters from one or more inlets tangentially at the top of the side wall [5]. Modifications in separation efficiency are possible for both types, with axial systems utilising adjustable vane or blade angles, and tangential systems generally employing adaptable shutters in the inlet duct. Such modifications usually necessitate a temporary halt of the system. Research on cyclones focuses on their performance and dust removal efficiency under varying conditions, using experimental and numerical methods. The effect of swirl vanes on cyclone flow is discussed in [6]. Hsu et al. [7] examined how different geometries affect cyclone efficiency, proposing a quality factor that relates to cyclone dimensions. Li et al. [8] analysed heat recovery possibilities from blast furnace slag by employing the CS. Yoshida [9] and Zhang et al. [10] studied how an apex cone influences separation efficiency. Yang et al. [11] investigated performance variations under different operational settings.

Computational fluid dynamics (CFD) is frequently employed to simulate and assess cyclone behaviour. It facilitates the examination of both established and novel cyclone designs across a variety of operational scenarios [12,13]. Such simulations typically involve optimising geometry and assessing enhancements in performance [14–17]. The potential of CFD can be augmented by incorporating deep learning techniques [18] or artificial neural networks [19], and optimisation algorithms can identify optimal operational conditions or designs. Furthermore, CFD is highly effective in determining the cut-off point for cyclone separators [3,20]. The choice of an appropriate turbulence model is crucial for precise Reynolds-averaged simulations. While simpler models like $k-\epsilon$ or $k-\omega$ have traditionally been used, there is a shift towards utilising the more sophisticated Reynolds stress model (RSM) [3,19–22]. The RSM turbulence model was validated for a variety of cases against experimental results and is relatively resource-efficient. An alternative technique for simulating turbulence in cyclone separators is the large eddy simulation (LES), which, although demands considerably more computational resources and a finer numerical grid, is particularly effective for studying transient turbulence fluctuations [23–25]. Cyclone separators typically operate under quasi-steady state conditions. For this reason, numerical simulations are usually performed in the steady state [26,3,27]. This approach is acceptable as long as the study aims to determine the cut-off point or the separation efficiency. Transient simulations are performed when the LES approach is used [28] or when quasi-steady flow fields are obtained by time averaging of the obtained transient fields in time [15]. Numerous papers dealing with CFD simulation of CSs, most focus on analysing a single aspect or feature of CSs. It can be a geometrical feature like apex cone [9], entrance design [22], etc., or a physical phenomenon like particle motion [25]. Furthermore, the analyses often use assumed dust parameters (size, composition, etc.). There is no complete analysis of a CS working under real conditions with the dust collected at an existing facility. Therefore, this research focuses on a robust steady-state numerical model incorporating the RSM turbulence model that is employed to determine the geometric setup and flow parameters of a full-scale industrial cyclone separator that efficiently separates valuable particles from BF gas. Dust and sludge samples were collected from the blast

furnace plant and used for comparison with the calculation results to ensure the accuracy of the obtained results.

This article is an extension of our previous study [20], in which a CFD model for a laboratory-scale axial cyclone was developed. The current work uses this model to simulate a full-scale industrial cyclone separator (CS) and tackle issues related to its dimensions and flow parameters. The primary objective is to configure the CS to effectively separate valuable materials such as iron and carbon while allowing zinc and lead, which are detrimental to the production of pig iron, to pass through [1]. Since carbon and iron are typically found in larger or denser dust particles, separating them helps conserve the natural resources and energy used in the blast furnace (BF). These larger particles generally contain minimal lead and zinc, mostly found in smaller and lighter particles. Particles not captured by the CS are removed in the second cleaning stage, often involving a wet scrubber. Another important aspect is the control of the cyclone separator, particularly due to variations in the ore composition entering the BF, which can change the dust composition. Here, the CFD model is crucial as it can forecast the cut-off point of CS.

2. Dust analysis

To design a CS, it is necessary to use the actual dust composition of the BF gas as input data. This task is complex due to the duct of the BF outlet gas configuration, high pressure, toxic carbon monoxide levels, and the duct's substantial size. To circumvent these challenges, the dust composition at the inlet was deduced from results obtained at two outputs of the existing dust removal system: DC and the wet scrubber (WS) (see Fig. 1). By gathering and examining the dust separated by the former and the sludge produced by the latter, the original dust composition was determined. At the BF plant, dust was weighed and categorised using laser diffraction according to the ISO 13320:2009 standard with the Fritsch Analysette 22 particle sizer, into size ranges between 56 μm and 1.6 mm. Smaller fractions, specifically those under 56 μm , were further divided using a counter-flow Bahco high-speed centrifuge. A total of 18 fractions were identified, with the smallest having a diameter of less than 1.9 μm . Essential elements of the dust, such as iron, carbon, zinc and lead, were determined and analysed. The dust density was measured using a pycnometer.

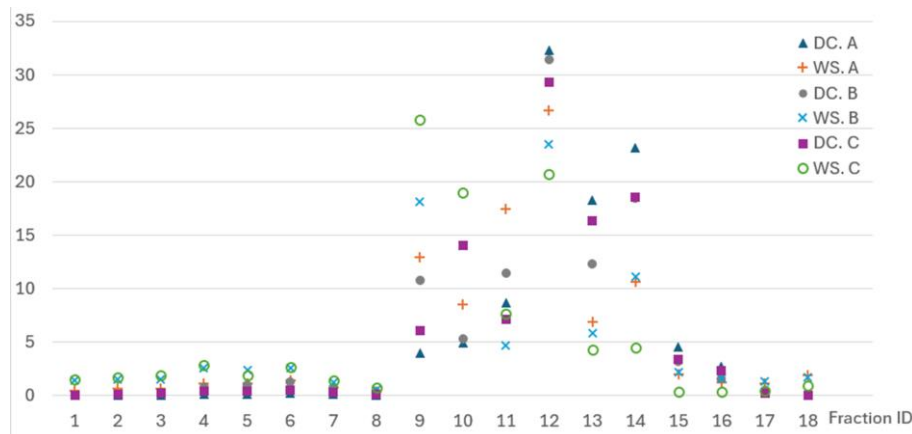


Fig. 2. Composition of dust collected in dust catcher and wet scrubber, %.

Figure 2 illustrates the detailed composition of the dust, showing three dust samples collected from DC and WS. While some variation in the dust composition is apparent, the bulk of the mass in all samples is found in medium-sized fractions, with both the smallest and largest fractions containing less than 22% of the total mass.

These measurements, combined with the known overall separation efficiency of the dust catcher ($\eta_d = 31\%$), allow to calculate the raw dust composition, that is, the dust that enters the dust removal installation. The overall separation efficiency of the dust catcher is given by

$$\eta_d = \frac{m_d}{m_{raw}}, \quad (1)$$

where m_d is the mass of dust collected in the dust catcher and m_{raw} is the mass of raw dust. The same can be written separately for each fraction to obtain the fractional efficiency:

$$\eta_{d,f} = \frac{m_{d,f}}{m_{raw,f}}, \quad (2)$$

where $m_{d,f}$ is the mass of fraction f of dust collected in the dust catcher, and $m_{raw,f}$ is the mass of fraction f of raw dust. The fractional efficiencies of the dust catcher are not known. The fractional mass balance can be written as

$$m_{raw,f} = m_{d,f} + m_{ws,f} + m_{gc,f}, \quad (3)$$

where $m_{ws,f}$ and $m_{gc,f}$ is the mass of dust separated in the wet scrubber and the mass of dust that remained in clean gas, respectively. Since the dust content after the wet scrubber $m_{gc,f}$ is approximately 1000 times less than the raw dust, it is omitted in the subsequent calculations. To find the composition of raw dust and dust catcher fractional efficiencies, a set of equations was solved:

$$\begin{cases} \sum_{f=0}^{18} g_{raw,f} = 1, \\ \eta_d = \sum_{f=0}^{18} \eta_{d,f} g_{raw,f}, \\ m_{d,f} = \eta_{d,f} m_{raw,f} & \text{for } f = 1, \dots, 18, \\ m_{ws,f} = (1 - \eta_{d,f}) m_{raw,f} & \text{for } f = 1, \dots, 18, \end{cases} \quad (4)$$

where $g_{raw,f}$ represents the mass fraction of the component f in the raw dust. This set features four primary equations. The first equation ensures that the sum of all mass fractions in the raw dust equals one. The second equation pertains to the overall efficiency of the dust collection system. The third equation details the fractional mass captured by the dust collector, derived from the second equation. The fourth equation calculates the fractional mass accumulated in the scrubber, based on the third equation. Altogether, the set includes 38 equations and 36 variables. The nonlinear gradient method was employed to solve this system. The goal was to minimise the sum of the squared residuals from the equations.

For each measurement, an independent calculation of the raw dust composition was performed and the results were presented in Fig. 3. The dust composition measurements were conducted over a three-month period. During this timeframe, both the blast furnace's operation and the iron ore's composition were considered constant. As depicted in Fig. 3, most of the dust is found in medium-sized fractions, with approximately 25% of it accumulating in the 100–160 μm range. For the simulations, the mean of the three measured compositions was utilised, and the same methodology was applied to the chemical compound content. These findings are detailed in Table 1. The measurements indicate that the bulk of the dust mass concentrates in particles sized between 71 μm and 200 μm . A mere 15% of the mass is attributed to the smallest and largest particles. Zinc, iron, and lead concentrations in the dust were analysed using an ICP-OES Agilent 5100 SVDV spectrometer. Irrespective of the fraction size, the iron content ranges from 10% to 30%, but a slight decrease is noted as particle size increases. Due to the measurement method, which involves burning the sample at 1000 K and analysing carbon dioxide in flue gas, determining organic carbon content for each fraction is unfeasible due to insufficient sample mass. Carbon content exceeds 50% in larger fractions, but is approximately 30% in smaller ones. The zinc mass fraction peaks in the smallest particles, diminishing as particle size grows. Overall, each gram of dust contains about 0.01 g of zinc, with over 70% of the zinc mass being associated with particles smaller than 71 μm . As seen in Table 1, zinc was present mainly

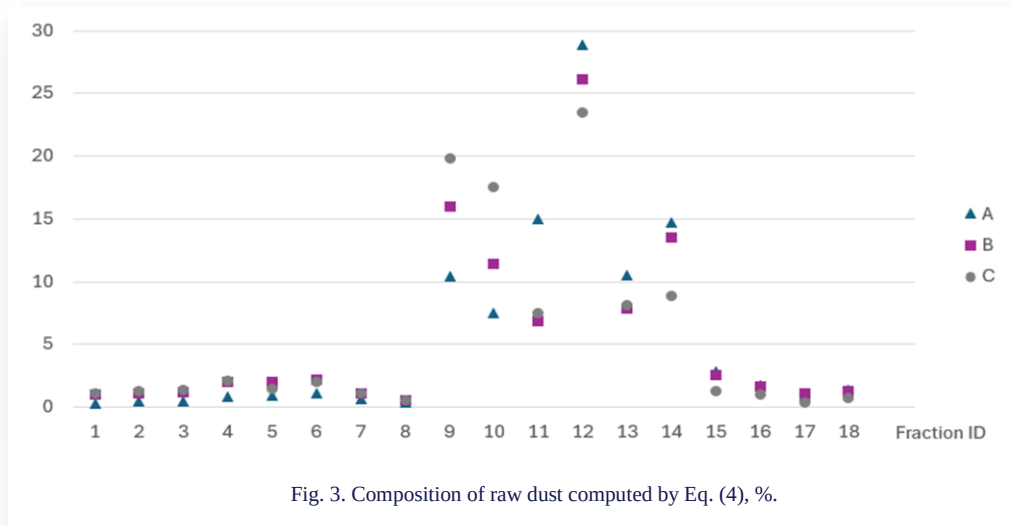


Fig. 3. Composition of raw dust computed by Eq. (4), %.

Table 1. Raw dust fractional composition and elements content.

Fraction ID	Fraction, μm	Mass, %	Iron, %	Carbon, %	Zinc, %	Lead, %
1	<1.9	0.797	27.2	*	7.113	0.1496
2	1.9–3.9	0.951	24.4	*	5.443	0.2500
3	3.9–6.4	1.015	16.4	*	3.387	0.1644
4	6.4–8.8	1.664	14.6	*	2.913	0.0586
5	8.8–12.4	1.446	14.6	*	1.716	0.1001
6	12.4–18.0	1.777	*	49.2	1.833	0.0681
7	18.0–23.2	0.946	21.1	*	1.614	0.2484
8	23.2–27.7	0.479	17.9	*	1.449	0.0594
9	27.7–56.0	15.385	32.4	15.2	1.298	0.0903
10	56.0–71	12.162	24.5	35.5	0.768	0.0455
11	71–100	9.771	24.6	31.6	0.492	0.0520
12	100–160	26.129	16.2	48.0	0.420	0.0787
13	160–200	8.835	8.1	64.0	0.305	0.0765
14	200–315	12.361	8.2	62.2	0.270	0.0860
15	315–400	2.238	11.9	51.2	0.241	0.1415
16	400–630	1.458	6.6	61.0	0.241	0.0659
17	630–800	0.785	19.6	30.0	0.340	0.0639
18	800–1600	1.118	20.3	*	0.390	0.0584

*denotes sample too small to analyse.

in the smallest fractions, with its mass fraction almost 7% for the smallest particles. With the increasing particle diameter, the zinc mass fraction shrank to less than 0.5%. The lead content does not vary with the diameter, and the overall lead content is an order of magnitude lower than that of zinc.

The dust contains a total lead content of 0.8 mg/g. A significant portion of this lead (65%) is found in medium-sized dust fractions in the diameter interval of 56–315 μm . The primary objective of the cyclone separator is to filter out larger particles rich in iron and carbon. These particles are then recycled back

into the BF. It is crucial to minimise the quantities of zinc and lead returning to the BF due to their detrimental effects on the BF process, with zinc posing a greater risk as its amount in the dust is ten times larger than lead. The operational parameters of BF are given in Table 2. During BF work, the operating parameters may vary by approximately 20%. The dust content may also vary in terms of the fractional and zinc content. Therefore, the fractional separation efficiency of the CS needs to be evaluated.

Table 2. Measured operating parameters of blast furnace gas duct.

Parameter	Value	Unit
Volumetric flow rate	396–756	Nm^3/h
Temperature	243	$^{\circ}\text{C}$
Gauge pressure	155	kPa
Dust content	15.2	g/Nm^3

The examination of the gathered dust and sludge (refer to Fig. 4) indicates that the sludge contains significantly higher zinc levels compared to the dust. This occurs because zinc-containing particles are lighter and, despite being larger, they tend to traverse the DC or CS due to their inertia. Conversely, smaller particles that encompass carbon and iron are denser, thus more likely to be separated than particles of the same size that contain zinc or lead. The evaluation of experimental results (refer to Figs. 2 and 4) reveals that the separation ratio of zinc in the sludge (WS) and dust (DC), as computed by the formula:

$$r_{f,Zn} = \frac{g_{Zn,ws,f}}{g_{Zn,dc,f}}, \quad (5)$$

ranges from 4 to 6.

The ratio of size is typically greater for smaller and medium-sized particles, while it tends to decrease for those that are larger. This ratio helps to estimate the amount of zinc recirculated back to the BF when collecting the dust. Based on these analyses and on the chemistry of the BF process, it was determined to separate approximately 85% of the dust mass with the cyclone separator.

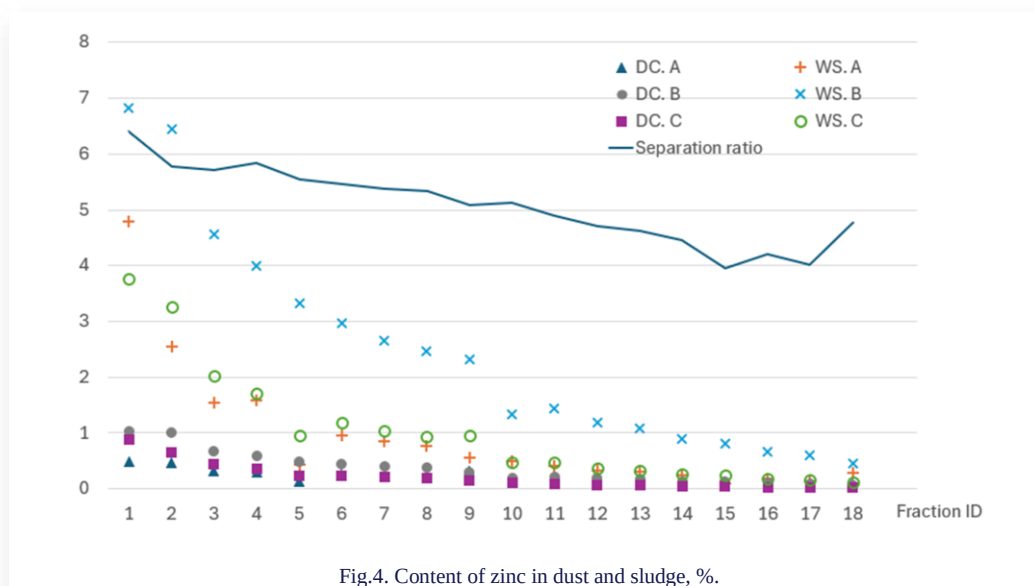


Fig.4. Content of zinc in dust and sludge, %.

It is presumed that in the CS, all larger particle fractions are eliminated, whereas the smaller ones pass through, resulting in about 40% of zinc returning to the BF. In real-world operations, the separation efficiency for the largest fractions reaches 100%, albeit it is never zero for the smallest fractions. Hence, the actual amount of zinc going back to the BF is slightly higher. The separation ratio for other elements analysed was calculated as an average across all fractions employing a similar approach:

$$r_{el} = \frac{g_{el,ws}}{g_{el,dc}}, \quad (6)$$

where *el* denotes a specific element. For instance, the separation ratios for iron, carbon, and lead are 1.4, 0.49, and 5.2, respectively.

Notably, lead demonstrates behaviour quite similar to zinc. Iron is almost evenly distributed between the dust and the sludge, whereas the amount of carbon in the dust is double that found in the sludge. These ratios will be used to assess the resources and energy savings resulting from the return of dust to the BF.

3. CFD model outline and modelling strategy

The CFD model for the CS was developed using Ansys/Fluent 19.2 and largely corresponds to the setup of a previously verified model of a lab-scale device, fully detailed in our earlier study [20]. For clarity, this text will briefly outline the parameters both models share. Conducted under steady-state conditions, the analysis yielded results akin to transient calculations but at a much lower computational expense. For 3D turbulence modelling, we employed an RSM with linear pressure strain and standard model constants, suitable for cyclonic applications. The coupled scheme was utilised for pressure-velocity coupling, while the PRESTO! scheme was applied for pressure discretisation. Convective terms of Reynolds stresses, momentum, turbulent kinetic dissipation rate, and energy were spatially discretised using the QUICK scheme. Convergence was deemed satisfactory when all residuals levelled off at 10^{-4} for continuity and 10^{-5} for velocity and turbulence. Additionally, changes in outlet velocity and pressure were monitored as further convergence indicators. To track the particles in the gas, the discrete phase model (DPM) was implemented. Reflection coefficients for the discrete phase were taken from [20], considering the particulate matter is identical in both instances and experimentally verified, with coefficients at 0.7 and 0.8 for tangential and normal components, respectively.

3.1. Geometry of the cyclone

The geometrical model of the CS consists of various elements contained in the main body, presented in Fig. 5. The particle-laden gas (1) is introduced axially at the top of the cyclone via the 3.2 m in diameter inlet and then flows around the inlet cone (6). The cone distributes the gas on the flow guides equipped with vanes (7), causing the flow to swirl (2) and thus initiating particle separation. Dust particles (4) descend due to gravity and are trapped upon reaching the inlet of the dust bin (surface 10). As the gas travels down the barrel of the cyclone, it encounters

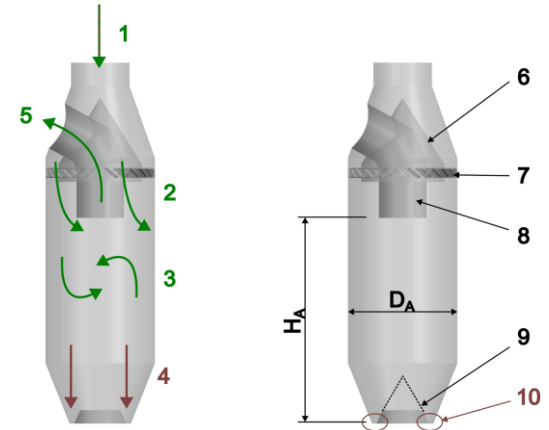


Fig.5. Cyclone – basic geometry (case A): 1 – dust-laden air inlet, 2 – air swirled by vanes, 3 – dust separation process in the cylindrical section, 4 – separated dust, 5 – purified gas outflow, 6 – inlet cone, 7 – flow guide with vanes, 8 – vortex finder, 9 – vortex cone, 10 – particle trap; D_A – cyclone internal diameter, 6 m; H_A – separation section height, 11.5 m.

a 3.37 m high conically narrowing section with a centrally located truncated vortex cone (9). This cone serves to stabilise the flow pattern within the cyclone separator. The narrowing of the barrel causes the flow to constrict, facilitating mass transfer from the outer downward vortex to the inner upward vortex. This mechanism directs the purified gas to the vortex finder (8), which eventually leaves the apparatus via the outflow pipe (5), which is 2.6 m in diameter. The vortex cone (9) is truncated, as this design choice minimally affects the velocity field while reducing the mesh size, improving its quality, and enhancing the convergence of simulations.

The main goal of this work was to numerically investigate the impact of different ratios of separation section height to barrel diameter (H/D) on the fractional separation efficiency of the axial cyclone. Hence, three cases (geometries) were considered, all presented in Fig. 6. Case A represents the baseline geometry of the height of 11.5 m in the separation section (H_A) and 6 m in the diameter of the barrel (D_A), resulting in a H/D ratio equal to approximately 1.92. The alternative scenarios, namely B and C,

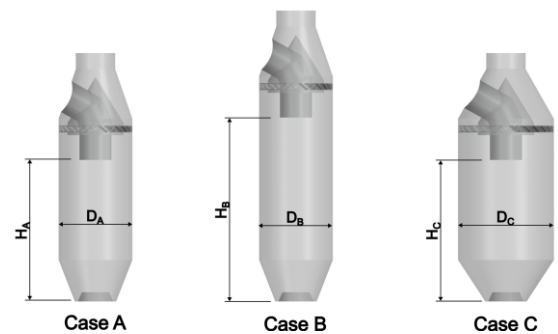


Fig. 6. Cyclone - considered geometrical variants:
Case A – basic geometry, Case B – extended separation section H_B ,
Case C – increased diameter D_C ; $H_A = 11.5$ m, $D_A = 6$ m,
 $H_B = 14.95$ m, $D_B = 6$ m, $H_C = 11.5$ m, $D_C = 7.8$ m.

demonstrate modifications of the reference case, where the separation section height (H_B) was enhanced by 30%, changing from 11.5 m to 14.95 m (Case B), or the barrel diameter (D_C) was augmented from 6 m to 7.8 m (Case C). In Case B, the H/D ratio is 2.42, whereas in Case C, it stands at 1.47.

3.2. Boundary conditions

Figure 7 illustrates the three primary boundary conditions in the computational domain: (1) velocity inlet, (2) pressure outlet, and (3) particle trap (wall). Inlet (1) features a uniform velocity profile and particles evenly distributed in space. Each dust fraction is modelled as a distinct DPM injection. At the pressure outlet (2), 0 Pa gauge pressure was assumed, and the wall condition (3) acts as a particle trap. The gas used in the model is the BF gas, which is treated as an ideal gas. The heat capacity was averaged over the composition, dynamic viscosity was computed using the Wilke equation, and thermal conductivity was calculated using the Chapman-Enskog model. The composition of BF gas (see Table 3) was measured by the industrial partner. The analyses were not isothermal, so the physical properties of the BF gas were temperature-dependent, and a convective boundary condition was applied to the external walls of the cyclone, assuming a heat transfer coefficient of 16 W/(m² K) and a free stream temperature of 0 °C.

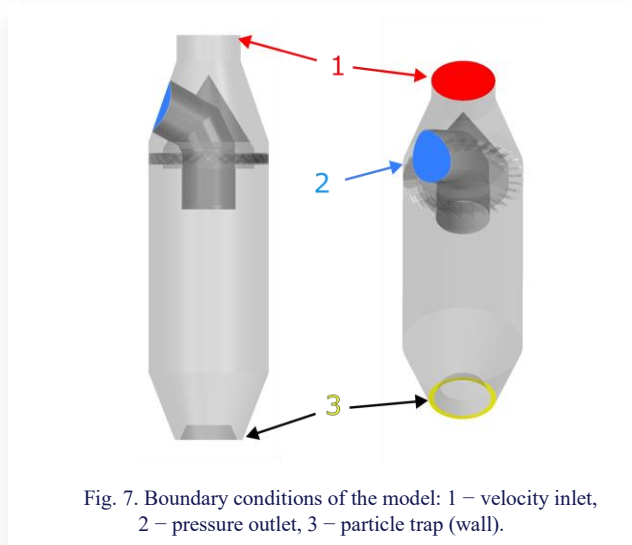


Fig. 7. Boundary conditions of the model: 1 – velocity inlet, 2 – pressure outlet, 3 – particle trap (wall).

The temperature of the BF gas was 150 °C and an operating pressure was set at 2 bar. However, due to the very low temperature drop (magnitude of several °C only), the changes in the BF gas properties were insignificant. Hence, only the average values of specific heat (1085.8 J/(kg K)), thermal conductivity (0.038 W/(m K)) and dynamic viscosity (2.14×10^{-5} kg/(m s)) are provided here.

3.3. Numerical mesh

Because the pressure drop in the cyclone, defined as the difference between the static pressure at the inlet and outlet, was a significant factor for evaluating the function of the designed apparatus, it was meticulously tracked during the mesh sensitivity analysis. This study included five mesh sizes, varying from

0.37 to slightly over 2.25 million elements. As indicated in Fig. 8, where the pressure drop computed for each mesh is displayed, increasing the number of elements to approximately 1.2 million exerts a minimal impact on the pressure drop value, as it stabilises at around 670 Pa. Relative pressure difference concerning mesh 5 is shown as labels in Fig. 8.

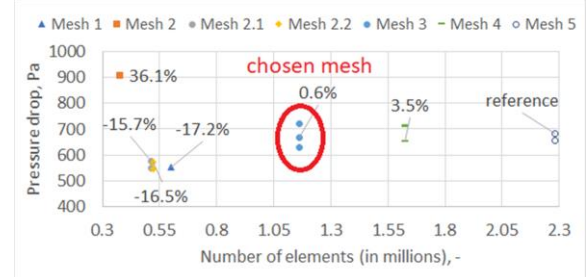


Fig. 8. Mesh sensitivity study. Mesh 3 was tested for various CFL values.

The same behaviour of results for various meshes can be observed on the profiles of tangential velocities across the cyclone separator (see Fig. 9). Smaller meshes (mesh 1–2.2) underpredict maximum and minimum gas velocities and do not properly resolve the stagnation at the CS axis. On the contrary, meshes 3–5 produce very similar velocity profiles.

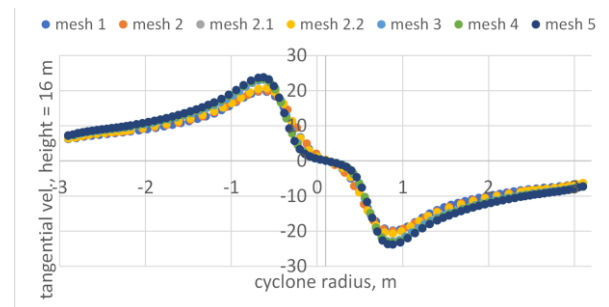


Fig. 9. Tangential velocity (m/s) profile at height 16 m.

The final mesh used in further research is presented in Fig. 10. It consists of 1.16 million mostly well-structured hexahedral elements. The only exception is the region of the inlet cone, where polyhedral elements are used instead. As previously discussed, reducing the size of the vortex cone significantly enhanced the mesh quality at the cyclone's base. Additionally, improving the mesh structure could be achieved by splitting the

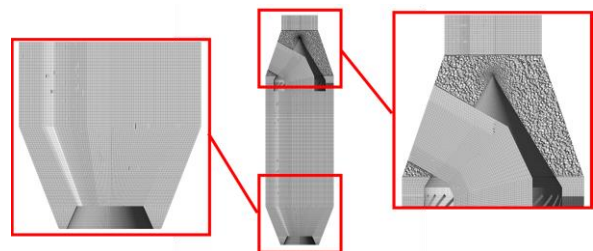


Fig. 10. Numerical mesh: 1.16 million elements, max. skewness of 0.2, min. orthogonal quality of 0.19.

computational domain into bodies of rotational symmetry, such as the vortex finder and the area near the cyclone's axis.

A maximum skewness of 0.25 and a minimum orthogonal quality of 0.19 ensure adequate stability and convergence for the solution. The computations were carried out on Intel(R) Core(TM) i7-5820K CPU @ 3.30GHz and 16,0 GB RAM. Solving a single case using mesh 3 took approximately 13 hours.

3.4. Courant flow number

Given the utilisation of the coupled scheme for pressure-velocity coupling, it became essential to modify the flow Courant number (CFL). This dimensionless metric indicates the number of mesh cells traversed over a specific time increment. In the context of steady-state conditions and the coupled pressure-velocity method, the CFL acts to stabilise the convergence process. The CFL is characterised as

$$CFL = \frac{\alpha}{1-\alpha}, \quad (7)$$

where α represents the under-relaxation factor. Four CFL values were examined: 10, 25, 40, and 60. To select an appropriate CFL value, the velocity distribution was investigated for each scenario. Each case shared the same setup, incorporating a uniform velocity profile at the inlet, with the only variable being the distinct CFL values. The asymmetry arises from the curved outlet duct, which extends from the vortex finder, resulting in an unbalanced velocity profile with a low-velocity zone along the vertical axis. The velocity fields for each case are illustrated in Fig. 11. Notable differences in velocity distributions are evident at first glance, especially when comparing CFL = 60 and the other cases, where the low-velocity region (< 6 m/s) near the axis is heavily blurred. In the CFL = 40 scenario, the region in question is noticeable but heavily blurred in the bottom-to-centre sector. In contrast, scenarios with CFL = 10 and CFL = 25 reveal a clearly defined narrow, deep blue band, indicating velocities below 6 m/s, surrounded by a high-velocity yellow/orange zone ranging from 23 to 33 m/s, with the peak velocity at the vortex finder entry. Across all profiles, there is a consistent velocity area near the walls within the separation sections (barrel and bottom cone), showing velocities between 6 and 13 m/s. The results for the CFL = 10 and CFL = 25 cases were quite similar;

however, some instabilities, observed as wavy yellow/orange regions, appeared in the CFL = 10 scenario, while the flow structure for the CFL = 25 case was more stable. Thus, CFL = 25 was chosen for further analysis.

3.5. Discrete phase model setup

The dispersion of particulate matter was simulated using the DPM, where the paths of particles were traced with a maximum of 50 000 steps and a step length factor of 5. The particle momentum exchange was accounted for by the spherical drag law combined with a discrete random walk model that included 5 iterations and a time-scale constant set at 0.15. Since the particle concentration was low, specifically 15.2 g/Nm³ interactions between particles were disregarded, and a one-way interaction between gas and particles was used. The dust was divided into 18 distinct injections, each corresponding to a specific fraction. Each injection was characterised by a constant particle diameter matching the maximum grain size of that fraction (e.g. for the 56 µm fraction, the diameter was set to 56 µm). These particles were non-reactive, with the density determined by earlier measurements. The wall reflection coefficients, based on the validation outlined in our prior research [20], were assumed to be 0.7 for tangent components and 0.8 for normal components. The dust density was assumed to be 2310 kg/m³. This value is based on measurements and is the average of all fractions.

4. Results and discussion

Once the appropriate CFL value was identified, numerical simulations were performed. The results are compiled for the three scenarios outlined earlier in Section 4.1, denoted as A, B, and C, which represent diverse geometric configurations of the cyclone. Case A serves as the reference geometry, while Cases B and C embody modifications featuring a 30% augmentation in the height of the separation section and the barrel diameter, respectively. For each configuration, the impact of gas velocity on fractional separation efficiency was examined, resulting in ten different cases with variant geometries and flow environments. In CS, higher gas velocities are desired as they enhance separation efficiencies. Consequently, Case A was tested with five distinct inlet gas velocities: 2, 4, 6, 9, and 12 m/s, to assess its influence on the flow conditions within the apparatus. The observed velocity fields are depicted in Fig. 12. The region exhibiting low velocity near the

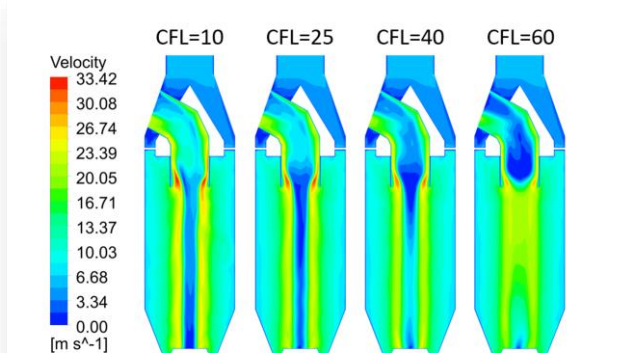


Fig. 11. CFL analysis - case A: mesh – 1.16 million elements, inlet gas velocity – 6 m/s.

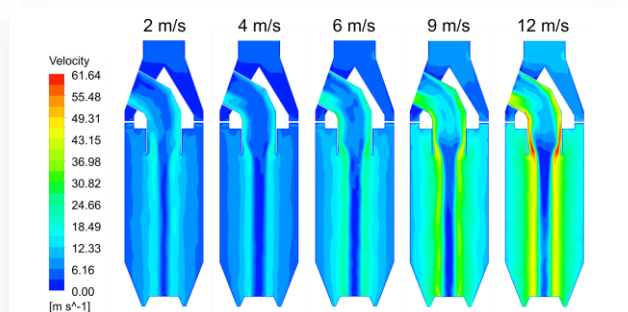


Fig.12. Case A. Velocity field in regard to different gas velocities at the inlet: 2, 4, 6, 9 and 12 m/s; CFL = 25.

central axis remains consistent and visibly apparent across all scenarios. Similarly, regions exhibiting medium to high velocity maintain stability, with no notable anomalies in flow. An increase in gas velocity substantially impacts the peak velocity at the vortex finder entrance, reaching over 60 m/s when the inlet gas velocity is set at 12 m/s. Such high velocity levels could result in excessive local erosion or vibrations. This consideration is relevant for the 9 m/s scenario, where the local velocity surpasses 40 m/s. The velocity fields recorded in case C, which features an expanded barrel diameter, are shown in Fig. 13 for four different inlet gas velocity values: 6, 9, 10, and 12 m/s. The low-velocity region along the central axis is observed, albeit smaller compared to the 6 m/s scenario. The local maximum velocity is located at the vortex finder entrance, reaching nearly 44 m/s for an inlet gas velocity of 12 m/s. This represents almost a 30% reduction compared to the related case A, also with a 12 m/s inlet gas velocity. Generally, as expected, extending the barrel diameter lowers the velocity within the cyclone globally, including the local peak velocity at the vortex finder.

Figure 14 illustrates the velocity fields determined for a 9 m/s inlet gas velocity in the three geometric configurations examined.

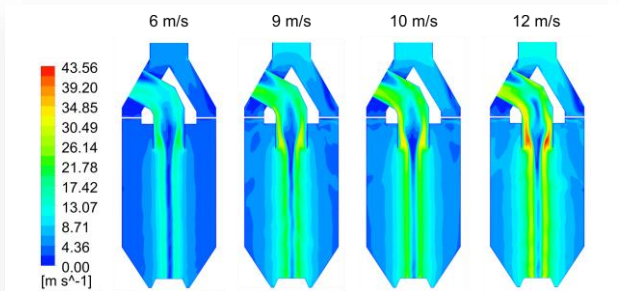


Fig. 13. Case C. Velocity field corresponding to different gas velocities at the inlet: 6, 9, 10 and 12 m/s; CFL = 25.

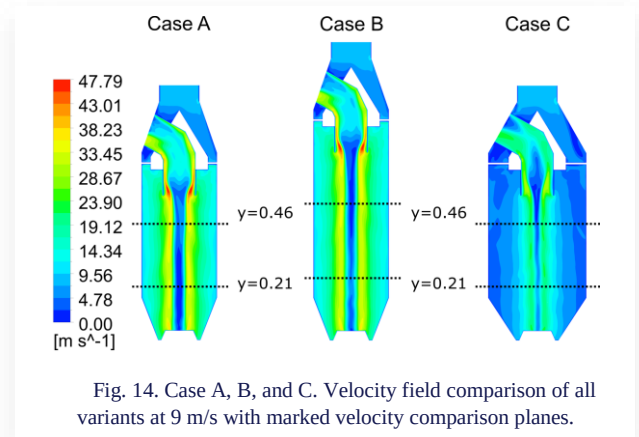


Fig. 14. Case A, B, and C. Velocity field comparison of all variants at 9 m/s with marked velocity comparison planes.

The findings indicate that altering the height of the separation section does not influence the velocity field, as observed in similar profiles of case B (with increased height) and case A (the baseline). In both instances, the local peak velocity is around 48 m/s. However, case C (increased barrel diameter) differs, showing a reduced overall velocity with a peak of approximately 34 m/s. In Fig. 15, axial and tangential velocity profiles for all three designs at two height levels are shown. For comparison purposes, the radial coordinate was normalised. The first height was close to the outlet and was located in 22% of the overall height of CS. The second height was closer to the middle – 46% of the overall height. This analysis reveals that while the separation section's height does not notably affect the velocity field (see Fig. 14 and Figs. 15a, b, and d), the cyclone barrel's diameter significantly influences the velocity profile; specifically, enlarging the diameter markedly lowers the peak velocity and decreases the overall velocity, particularly close to the walls. In Fig. 15c, there is a visible difference between cases A and B. This is due to the larger distance from the CS outlet – the velocity field is more uniform there. As dust particles enter the cyclone, they

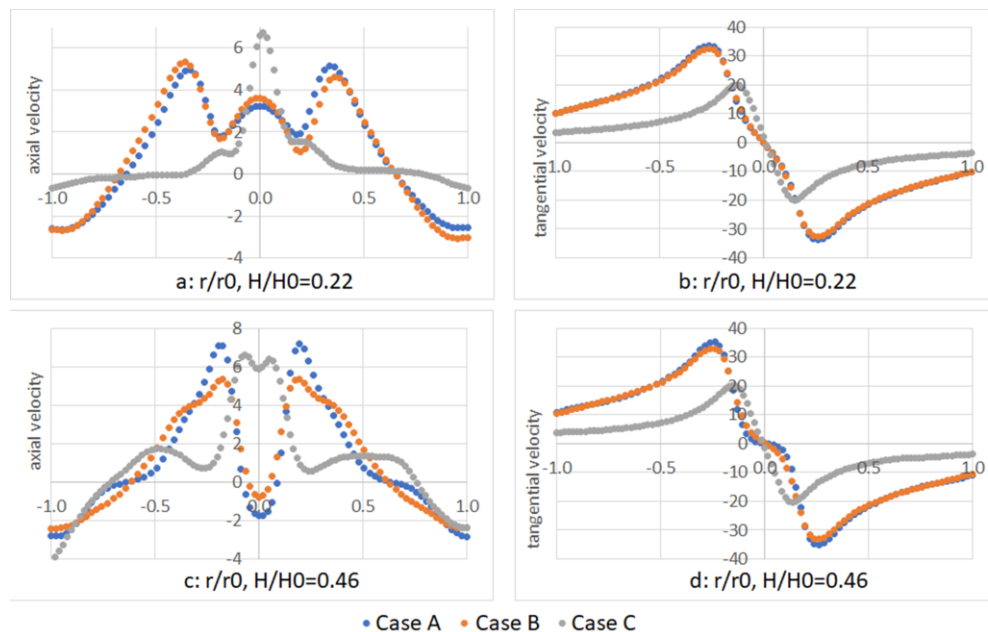


Fig.15. Velocity profiles: a, c – axial and b, d – tangential, in %.

flow around the inlet cone and through the flow guides towards the side walls of the barrel (separation section). If the near-wall velocity is reduced, the separation efficiency should also decrease, which results from the decrease in centrifugal force. Since for Cases A and B, tangential velocities are very similar, the separation efficiency should also be at the same level. Third analysed geometry – Case C is characterised by a lower tangential velocity both near the wall and in the CS volume, thus its separation efficiency is expected to be lower. Analysing axial velocities (Figs. 15 a and c), it is visible that Cases A and B are similar, while the profile for Case C differs. For the latter, an axial velocity is much higher in the axis, while smaller near the wall. This leads to the conclusion that increasing the CS diameter lowers the separation efficiency.

To investigate the separation behaviour in terms of geometry and inlet gas velocity, the fractional separation efficiencies are gathered in Table 4, which contains the results of all the cases analysed. The table is organised column-wise, where each column corresponds to a single inlet velocity. The columns are grouped into three categories, each corresponding to a case: A, B, or C, respectively. In the very first column on the left, dust fractions are listed. The columns are filled with separation efficiencies given in percentage, according to the fraction listed in the first left column. The bottom, green-filled cells store the complete separation of a given fraction. Orange cells represent a fraction range, where the cut-off point is expected. It is defined as a fraction, half of which is separated successfully, often denoted as d_{50} [24].

Table 4 reveals that the cut-off range tends towards lower velocities, indicating a decrease in fractional separation efficiency and a shift of the cut-off point towards larger particle sizes. In Case A, particles up to 56 μm are almost fully separated with over 90% efficiency, though efficiency drops sharply for the nearest smaller particles (highlighted in orange). The cut-off particle sizes range from 23.2 to 27.7 μm at 12 m/s and from 56.0 to 71.1 μm at 2 m/s. In Case B, both the separation efficiencies and the cut-off sizes are virtually the same as in Case A when the inlet velocity is 9 m/s. Similarly, in Case C, there is

a steady decline in separation efficiency, with the cut-off point moving to larger sizes. However, there is a global shift to larger particles in the 100–200 μm range, resulting in lower overall separation efficiency. The requirement to separate 85% of dust is achievable in the CS configuration A with an inlet velocity of 4 m/s.

The overall separation efficiency is computed as

$$\eta_d = \sum_{f=0}^{18} \eta_{CS,f} g_{raw,f}, \quad (8)$$

while the zinc separated is given by

$$\eta_{Zn} = \frac{\sum_{f=0}^{18} \eta_{CS,f,Zn} g_{CS,f,Zn}}{m_{raw,Zn}}, \quad (9)$$

where

$$g_{CS,f,Zn} = \frac{m_{raw,f,Zn}}{\eta_{CS,f} + (1 - \eta_{CS,f})r_f}, \quad (10)$$

where r_f is a ratio of zinc, Eq. (5), according to which it is distributed between sludge and separated dust. It is assumed that the zinc distribution between dust collected in CS and that which passes through will be the same as in DC. This cyclone separator can recover roughly 45% of the zinc (Table 4). Concurrently, about 55% of the lead is expected to return to the BF (see Fig. 16). More than 80% of iron and 90% of carbon from the dust are also anticipated to be returned to the BF. It is important to note that the volumetric flow rate of BF gas can vary significantly, potentially by $\pm 30\%$. Fluctuations in BF gas flow impact the gas velocity, thereby affecting its separation efficiency. To minimize this effect, the system includes adjustable vanes. Changing the vane angle influences the flow swirl, affecting the separation efficiency [18]. Such adjustments can, however, be made only during a pause in operations. Configuration B showcases a slightly higher efficiency at the same inlet velocity. The last geometric configuration tested exhibits a significant drop in efficiency by around 30%. Consequently, for this setup, the CS

Table 4. Fractional separation efficiency for case A, B, C at different inlet gas velocities (green – full separation; yellow expected cut-off point).

Fraction, μm	Case A					Case B		Case C			
	12 m/s	9 m/s	6 m/s	4 m/s	2 m/s	9 m/s	12 m/s	10 m/s	9 m/s	6 m/s	
1.9	15	3	9	8	7	13	7	6	5	7	
3.9	20	9	12	9	11	25	9	6	8	7	
6.4	24	19	17	15	8	27	10	5	11	7	
8.8	25	25	26	27	7	30	10	5	7	7	
12.4	28	34	31	31	17	35	10	4	13	8	
18.0	35	39	37	38	21	35	20	5	10	8	
23.2	47	45	41	38	29	43	21	7	12	7	
27.7	52	51	45	38	36	49	22	7	12	9	
56.0	97	96	88	68	41	99	31	16	17	18	
71.1	100	100	96	90	55	100	36	23	24	23	
100.0	100	100	100	100	100	100	44	37	34	30	
160.0	100	100	100	100	100	100	87	78	72	44	
200.0	100	100	100	100	100	100	100	98	99	84	
315.0	100	100	100	100	100	100	100	100	100	100	
400.0	100	100	100	100	100	100	100	100	100	100	
630.0	100	100	100	100	100	100	100	100	100	100	
800.0	100	100	100	100	100	100	100	100	100	100	
$\eta_{CS}, \%$	92.5	92.2	90.4	86.5	77.1	93.0	64.2	56.4	55.3	46.1	
Zn separated, %	64.7	63.4	55.5	45.8	35.5	67.0	24.4	19.9	19.1	14.3	

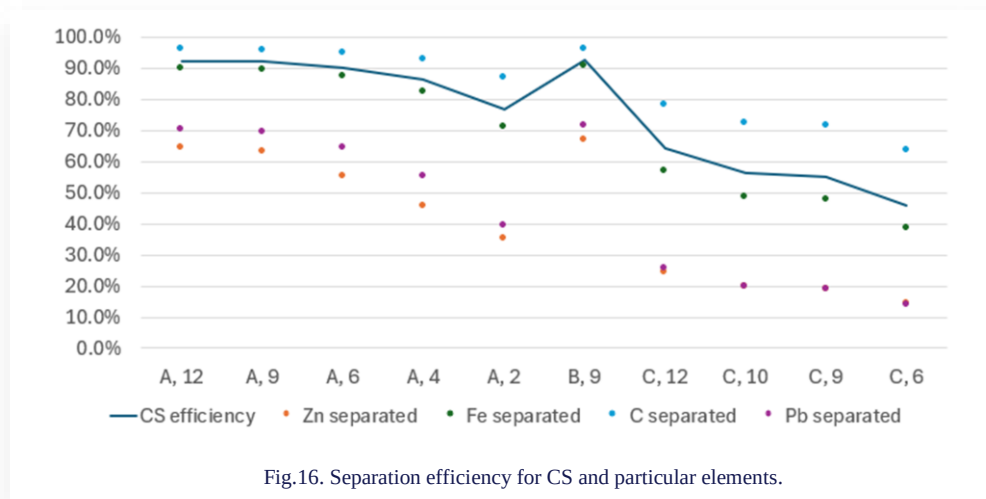


Fig.16. Separation efficiency for CS and particulate elements.

should keep an H/D ratio ranging from 1.5 to 2. Increasing the ratio beyond this markedly reduces efficiency and necessitates higher velocity, which is undesirable due to heightened erosion of the CS walls. In the case of Configuration A, a 20% fluctuation in gas flow alters separation efficiency by around 24% and changes zinc collection by 5%, which is an acceptable value. Considering the annual average operation parameters displayed in Table 2, the adoption of the proposed cyclone separator could conserve approximately 14 500 tons of iron and 13 000 tons of coal. This translates to a substantial conservation of natural resources and energy.

5. Conclusions

The main objective of this study was to create a comprehensive model of a cyclone separator designed to purify the blast furnace gas. This model was based on an already validated and published CFD model of a smaller-scale cyclone separator. The input to the model was derived from analysing measurements taken from a working BF gas purification system. The research was based on measurement data to determine the composition of raw dust and assess the zinc content. Using the CFD model, the influence of three factors on the separation efficiency and the amount of zinc extracted has been investigated: the inlet gas velocity, the height of the separation section and the diameter of the barrel. Gas velocity was varied between 2 and 12 m/s; two different heights for the separation section were tested: 11.5 and 14.95 m; and two barrel diameters: 6 and 7.8 meters were evaluated. Alterations in the dimensions of the cyclone separator yielded three unique geometries with different height-to-diameter ratios. $A = 1.92$, $B = 2.42$ and $C = 1.47$. Analysis revealed that configuration A achieved the highest dust separation efficiency (approximately 93%) at an inlet velocity of 12 m/s. Configuration B matched the dust separation efficiency of configuration A at a velocity of 9 m/s, suggesting that the separation section height minimally affects the separation efficiency. Configuration C consistently demonstrated a dust separation efficiency roughly 30% lower than A and B, at all input velocities, resulting in a reduction in zinc separation by nearly 40%. With a flow and geometric configuration of A at 4 m/s, 85% of dust and 46% of zinc are separated, resulting in minimal zinc separation (with the

peak zinc separation noted at 65% for A at 12 m/s), while still achieving a notably high dust separation. This result is beneficial since the dust mainly comprises iron and carbon, which are to be recycled back into the blast furnace. The considerable quantity of dust rerouted to the BF carries only a minor amount of zinc, whose presence in the gas is detrimental to the blast furnace process.

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Online monitoring of thermal stress and heat transfer coefficient in thick-walled cylindrical elements

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Abstract

The article presents a method for online monitoring of thermal stress and heat transfer coefficient on the inner surface of a thick-walled cylindrical element using two separate applications. Both applications are based on the inverse heat conduction problem and use the control volume method. The first one allows the determination of thermal stresses based on measuring the wall temperature at a single point near the inner surface. This software is suitable for one-dimensional heat transfer, i.e. in a radial direction. The second application allows the heat transfer coefficient on the inner surface to be determined based on temperature measurements at six spatially distributed points. Knowledge of the heat transfer coefficient on the inner surface allows the stress concentration factor to be determined for elements weakened by an opening. This can be used to determine the optimum heating or cooling rates for pressure elements or to determine the thermal stresses in elements weakened by an opening. This method is suitable for heat transfer cases in the radial, longitudinal and circumferential directions. The correct operation of both original applications has been tested on a laboratory stand, where there was a sudden change in the working fluid temperature in a steam header from 16.8°C to 142.5°C. At the beginning of the temperature change, the thermal stresses on the header inner surface reached a maximum value of -195.8 MPa, and the heat transfer coefficient was approximately 5000 W/(m²K). Then, the thermal stresses began to decrease, and the heat transfer coefficient began to increase.

Keywords: Inverse heat conduction problem; Thermal stress; Heat transfer coefficient; C++ Builder; Visualisation

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1. Introduction

Monitoring thermal stresses is essential in many industries, primarily in the energy sector (conventional, nuclear and renewable energy), but also in the metallurgical industry, semiconductor industry, automotive and aviation industries, transport, civil engineering and even medicine. The occurrence of excessive thermal stresses in structural components is one of the reasons for their failure due to material fatigue.

Non-destructive stress measurement methods include the magnetic property, strain gauge, and ultrasonic wave methods.

However, these techniques are often problematic in the case of thermal stresses, although attempts are being made to adapt the measurement methods. For example, in [1], an improvement to the ultrasonic wave method was proposed by using piezoelectric zero-order shear horizontal (SH0) wave transducers bonded to the sample with adhesive. Work on this method was related to monitoring thermal stresses, which can cause cracking or bulging in railway rails.

The calculation method most commonly used in recent years for the analysis of thermal stresses is the finite element method (FEM).

Nomenclature

a – distance between nodes in the circumferential direction, m
 A – surface area, m²
 b – distance between nodes in the longitudinal direction, m
 c – specific heat capacity, J/(kg K)
 E – Young's modulus, MPa
 Fo – Fourier number
 h – heat transfer coefficient, W/(m² K)
 k – thermal conductivity, W/(m K)
 p – pressure, MPa
 $\dot{q}$ – heat flux, W/m²
 $\dot{q}_v$ – energy generation rate per unit volume, W/m³
 $\dot{Q}_{i,j}$ – heat flux rate transferred from node j to node i , W
 r – radius or radial variable, m
 s – thickness of the wall, m
 t – time or time variable, s
 T – temperature, K
 $\bar{T}$ – mean temperature across wall thickness, K
 v_T – rate of temperature change, K/s

Greek symbols

α – stress concentration factor
 β – coefficient of linear thermal expansion, 1/K
 δ – distance between the measuring point and the inner surface, m
 Δr – spatial step in the radial direction, m
 Δt – time step, s
 ΔV – volume of the cell, m³
 $\Delta\phi$ – spatial step in the circumferential direction, °

ε – tolerance, K
 κ – thermal diffusivity, m²/s
 ν – Poisson's ratio
 ρ – density, kg/m³
 σ – circumferential stress, MPa
 ϕ – shape factor

Subscripts and Superscripts

al – allowable
 b – spigot
 $direct$ – direct area
 f – fluid
 i – general node number index
 in – inner
 inv – inverse area
 m – mean
 out – outer
 p – caused by pressure
 P – on the edge of the hole at point P
 s – shell
 T – thermal

Abbreviations and Acronyms

FEM – finite element method
FVM – finite volume method
GUI – graphical user interface
IDE – integrated development environment
RAD – rapid application development
SH0 – zero-order shear horizontal (wave)
VCL – visual component library

The need to monitor thermal stresses in conventional power plants is related to the development of renewable energy. Renewable energy sources, primarily wind and solar energy, are unpredictable, necessitating more frequent start-ups and shut-downs of power units. This results in faster wear and tear due to fatigue of installation components, especially thick-walled components with complex shapes in steam boilers or steam turbines. Monitoring thermal stresses is also important because power plants often exceed their design life. Analysing the stresses that arise in the plant can help ensure safe operation and assess the residual strength of critical components. Finally, accurately determining thermal stresses makes it possible to determine the optimal start-up time [2] or shutdown time [3] for a conventional power plant. This time is determined based on the condition that the allowable stresses are not exceeded.

In energy installations, thick-walled components are used in high-pressure working fluid cases. In significant changes in fluid temperature, the greatest thermal stresses will occur on surfaces in contact with the fluid. The procedures for determining thermal stresses in these components are described in standards TRD 301 [4] and EN 12952-3 [5]. Monitoring thermal stresses using computational methods is strictly related to determining the temperature distribution in the analysed element and determining the heat transfer coefficient on the surface washed by the working fluid. If the component is not weakened by an opening, it is sufficient to know the temperature distribution in the wall. However, in practice, thick-walled elements have stubs that increase thermal stresses on the inner surface of the shell opening.

Thermal stresses should then be calculated using the stress concentration factor, the value of which depends on the heat transfer coefficient.

Many studies on determining thermal stresses concern steam turbines [6–10]. In [6], a concept for an advanced turbine life monitoring system was proposed based on analysing damage caused by material fatigue. In [7], a modification of the algorithm based on Green's functions and the Duhamel integral was proposed, allowing the determination of thermal stresses in valves and turbine casings during start-up, considering the variable heat transfer coefficient. This method can be used for online stress monitoring. Online stress monitoring is also discussed in the article [8]. It presents a programme based on measurement data and calculations performed using FEM, which estimates stresses in critical turbine components (rotor, shut-off and control valves) and allows for optimisation of the device start-up. In [9], the thermal stresses occurring in the rotor of combined medium- and high-pressure turbines (with a common casing) were compared with the stresses in the rotors of turbines operating separately (in separate casings). In [10], thermal stresses occurring, among others, in a high-pressure steam turbine during start-up in the case of a deviation of the live steam temperature from that specified in the instructions were analysed.

Correct evaluation of thermal stresses is crucial in nuclear power plants, as it is related to the facility's safe operation. The main component of a pressurised water reactor is the pressure vessel, which must withstand the highest possible loads without any damage. Research on stresses and stress intensity factors in

reactor pressure vessel cracks was made in [11]. The work is based on the results of an experiment in which samples were exposed to thermal shock. Stress measurements were performed on samples with and without cracks using synchrotron X-ray diffraction as a function of time. At the same time, it was found that it is not possible to use FEM to simulate stresses during thermal shock under industrial conditions. The corners of the nozzles are critical points for pressure vessels in reactors. Research on methods for determining stresses in these areas, replacing the computationally expensive FEM method in this case, is presented, for example, in [12,13]. In [12], a simple method was developed for determining temperature distributions in the corner along the cross-section, taking into account heat conduction in two directions. Then, thermal stresses are calculated based on temperature gradients. In [13], on the other hand, it was proposed to determine the stress intensity factor based on simple predictive equations as a function of thermal load under cooling and heating conditions.

Thermal stresses naturally occur in other components of power plants where temperatures can change rapidly over time. For example, in [14], an analysis was performed of unsteady thermal stresses occurring in a safety condenser. The condenser is a shell-and-tube heat exchanger used in the event of an emergency shutdown of a steam turbine when hot steam needs to be cooled. The authors of the study used FEM to determine the temperature distribution in the shell and tubes of the condenser and assessed the thermal stresses during rapid start-up of the device. The calculations were performed for cases where a thermal expansion compensator was used in the condenser and without it.

The temperature field in a pressure element, knowledge of which allows thermal stresses to be determined, may result from a direct solution of the heat conduction problem. For example, [15] presents a method for predicting the behaviour of thick-walled elements with complex shapes, using the example of a Y-shaped junction, under unsteady conditions. The proposed method uses measurements obtained on the object with calculations performed using FEM. The paper also points to the significant influence of the heat transfer coefficient on the internal surface of the element on thermal stresses. Another example of determining the unsteady distribution of temperature and stresses in a cylindrical element using FEM is presented in [16]. The model developed for the calculations uses measurements of the ambient temperature, the external surface temperature and the temperature of the working fluid washing the internal surface. Also in [17], a cylindrical element was analysed using FEM for calculations. The model assumed that the pipeline was thermally insulated, and the temperature was measured using a new thermometer. The newly designed thermometer was used to obtain more accurate measurements of the working fluid temperature.

However, to solve the direct problem, it is necessary to know the internal surface temperature or the exact temperature of the working fluid and the heat transfer coefficient. In both cases, accurate measurements are difficult to achieve. Measuring the temperature of a fluid at high pressure and temperature requires the use of thermometers in heavy housings, which causes a significant dynamic measurement error. Another approach to cal-

culating the temperature distribution in a pressure element, which then allows the determination of thermal stresses, is to solve the inverse heat conduction problem [18]. In this case, it is sufficient to know the temperature at one or more points inside the wall of the element. Examples of articles in which this method is developed are [19–22]. In [19], the temperature distribution in a thermally insulated cylindrical wall is determined based on the measurement of the external surface temperature, and in [20,21] based on the measurement of the temperature near the internal surface. In these cases, heat was assumed to be transferred only in the radial direction. In [22], on the other hand, the inverse problem is solved by measuring the wall temperature at six points, which allows the internal surface temperature and the heat transfer coefficient to be determined for a three-dimensional temperature distribution. The inverse methods can also be used to assess the influence of wall material heterogeneity on temperature distribution and stress. For example, in [23], the temperature distribution and thermal stresses in a spherical, hollow element made of functionally graded material were analysed based on the known temperature and circumferential strain on the outer surface. The problem was reduced to an inverse thermoelasticity problem and solved using FEM.

In inverse methods, the problems of direct methods are eliminated, but they are sensitive to random errors in the measurement data used as input. The impact of random errors can be reduced by using regularisation methods, such as the Tikhonova method and its modifications [24,25] or the method based on the notation of the thermal balance [26]. Random errors can also be eliminated by smoothing the measurement data using digital filters [27], which is a simple and effective solution.

This article presents simple and effective methods for monitoring thermal stresses in an element without an opening and determining the heat transfer coefficient on the inner surface of a pressure element in unsteady states. Both methods apply to thick-walled cylindrical elements and are based on solving the inverse heat conduction problem. The first method is based on measuring the temperature inside the element wall at a single point and is suitable for heat conduction in the radial direction. Thermal stresses are determined based on the standard [5]. The second method is based on measuring the temperature at six points inside the wall of the element. It is suitable for heat transfer in three directions (longitudinal, circumferential and radial). The operation of both applications has been verified on a laboratory stand equipped with a thick-walled steam header.

2. Determination of circumferential stress in thick-walled cylindrical elements

Pressure components exposed to variable loads, e.g. due to cyclic start-up and shutdown of units, should be subjected to fatigue analysis. Operating and environmental influences that cause variable loads include, among others, changing pressure and temperature of the working fluid, unsteady temperature conditions, restrictions on expansion or contraction of the material during normal temperature changes, forced vibrations, or changing external loads. Circumferential stresses in cylindrical elements should meet condition [5]:

$$\sigma \leq \sigma_{al}. \quad (1)$$

The allowable stresses σ_{al} for the designed number of cycles are determined for a given material and specific tensile strength R_m using, for example, the Wohler diagram [5].

If the circumferential stresses are caused only by changes in the pressure and temperature of the working medium, the maximum circumferential stresses on the inner surface of the hole at point P (Fig. 1) can be determined using the equation [5]:

$$\sigma_P = \alpha_p \sigma_p + \alpha_T \sigma_T. \quad (2)$$

Figure 1 shows a section of a thick-walled pipe with thermal stress and stress caused by pressure at point P on the inner surface of the hole marked.

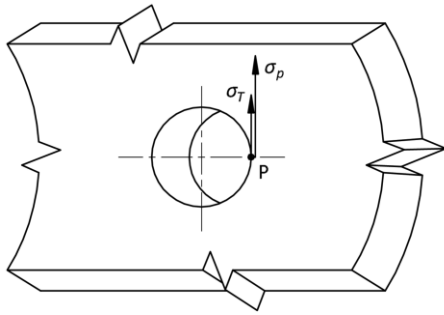


Fig. 1. View of a fragment of a thick-walled element weakened by a hole with marked circumferential stresses at point P on the edge of the hole.

Circumferential stresses caused by pressure are calculated using the following equation:

$$\sigma_p = \frac{pr_{ms}}{s}. \quad (3)$$

The mean radius of a cylindrical element r_m is calculated as the arithmetic mean of the inner r_{in} and outer r_{out} radii.

Thermal stresses reach their highest values on the inner surface and are determined using the formula:

$$\sigma_T = \frac{E\beta}{1-\nu} (\bar{T} - T|_{r=r_{in}}). \quad (4)$$

The mean temperature function across the cylindrical wall thickness $\bar{T}$ in Eq. (4) is determined by the following equation:

$$\bar{T} = \frac{2}{r_{out}^2 - r_{in}^2} \int_{r_{in}}^{r_{out}} rT(r, t) dr. \quad (5)$$

Standard [5] presents a procedure for calculating thermal stresses for a quasi-steady state when the temperature of the pipeline wall changes over time at a constant rate and the pipeline is thermally insulated. The formula contained in standard [5] is derived from the solution of the heat conduction equation, which for isotropic bodies has the general form [28]:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = \nabla \cdot [k(T)\nabla T] + \dot{q}_v, \quad (6)$$

and which for heat transfer in the radial direction, constant pipeline material properties and zero energy generation per unit volume has the form:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = \frac{v_T}{\kappa}. \quad (7)$$

From solving Eq. (7) with boundary conditions:

$$T|_{r=r_{in}} = v_T t, \quad (8)$$

$$-k \frac{dT}{dr} \Big|_{r=r_{out}} = 0, \quad (9)$$

we obtain:

$$T = v_T t + \frac{1}{4} \frac{v_T}{\kappa} \left(r^2 - r_{in}^2 - 2r_{out}^2 \ln \frac{r}{r_{in}} \right). \quad (10)$$

After substituting Eq. (10) into Eqs. (5) and (4), we obtain:

$$\sigma_T = \phi \frac{E\beta}{1-\nu} \frac{v_T s^2}{\kappa}, \quad (11)$$

where

$$\phi = \frac{1}{8} \frac{(u^2-1)(3u^2-1)-4u^4 \ln u}{(u^2-1)(u-1)^2}, \quad (12)$$

and $u = r_{out} / r_{in}$.

For an unsteady state, the integral in Eq. (5) can be calculated using, for example, the trapezoidal method. This requires calculating the temperature T_i at N equidistant discrete points (at a distance Δr) on the radii r_i :

$$\bar{T} = \frac{1}{r_{out}^2 - r_{in}^2} \sum_{i=2}^N (r_{i-1} T_{i-1} + r_i T_i) \Delta r. \quad (13)$$

The temperature at N points can be obtained using numerical methods, e.g. the finite volume method (FVM) or finite element method (FEM).

The stress concentration factor due to pressure is determined by Eq. [5]:

$$\alpha_p = 2.2 + e^A \zeta^B, \quad (14)$$

where

$$A = -1.14 \left(\frac{s_b}{s_s} \right)^2 - 0.89 \frac{s_b}{s_s} + 1.43, \quad (15)$$

$$B = 0.326 \left(\frac{s_b}{s_s} \right)^2 - 0.59 \frac{s_b}{s_s} + 1.08, \quad (16)$$

$$\zeta = \frac{r_{mb}}{r_{ms}} \sqrt{\frac{r_{ms}}{s_s}}. \quad (17)$$

The thermal stress concentration factor is calculated as follows [5]:

$$\alpha_T = \left\{ \left[2 - \frac{h+2700}{h+1700} z + \frac{h}{h+1700} (e^{-7z} - 1) \right]^2 + 0.81z^2 \right\}^{\frac{1}{2}}, \quad (18)$$

where $z = r_{mb} / r_{ms}$.

Standard [5] proposes to adopt constant values for the heat transfer coefficient h : 3000 W/(m² K) for water and 1000 W/(m² K) for steam. Determining the exact value of the heat transfer coefficient, especially in the case of significant deviations from the value given in the standard, increases the accuracy of thermal stress calculations.

3. Method for determining a one-dimensional temperature field by measuring the temperature at a single point on the wall

The one-dimensional temperature field in the wall of a cylindrical element, knowledge of which is necessary to determine the mean wall temperature from Eq. (13) and thermal stresses from Eq. (4), can be determined using the inverse method. The proposed method uses a wall temperature measurement at a single point and the fact that the pipeline is thermally insulated. The temperature measurement point N divides the cross-sectional area of the pipe into a direct ($r_N \leq r \leq r_{out}$) and inverse ($r_{in} \leq r \leq r_N$) area (Fig. 2).

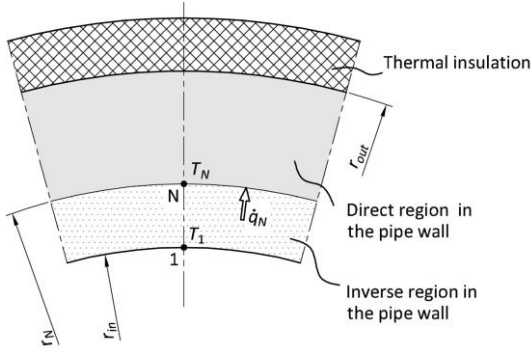


Fig. 2. Cross-section of the pipe with a marked temperature measurement point N and division into inverse ($r_{in} \leq r \leq r_N$) and direct ($r_N \leq r \leq r_{out}$) areas.

To determine the temperature field, the heat conduction Eq. (6) was used, which for heat transferred in the radial direction, the lack of energy generation rate per unit volume and the temperature-dependent physical properties of the pipeline material takes the form [20,27]

$$c(T)\rho(T)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[k(T)r\frac{\partial T}{\partial r}\right]. \quad (19)$$

The equation is first solved for the direct area and boundary conditions

$$T|_{r=r_N} = T_N, \quad (20)$$

$$-k(T)\frac{\partial T}{\partial r}\Big|_{r=r_{out}} = 0. \quad (21)$$

From the solution of the direct problem, Eqs. (19)–(21), we obtain the temperature distribution over the wall thickness in the direct region and the heat flux at point N. Thus, the second boundary condition on the isothermal surface on which point N is located is known and has the form of Eq. (23). The boundary conditions for solving the inverse problem are as follows:

$$T|_{r=r_N} = T_N, \quad (22)$$

$$-k(T)\frac{\partial T}{\partial r}\Big|_{r=r_N} = \dot{q}_N. \quad (23)$$

From the solution of the obtained inverse problem, Eqs. (19) and (22)–(23), we get the temperature distribution over the wall

thickness in the inverse region, including the temperature at the inner surface of the pipeline.

3.1. Solution of the inverse problem

FVM will be used to solve the inverse problem. The heat balance equation for a single control volume, which is obtained by transforming Eq. (6), is of the form [28]:

$$\Delta V_i c(T_i) \rho(T_i) \frac{\partial T_i}{\partial t} = \sum_{j=1}^{n_c} \dot{Q}_{i,j} + \Delta V_i \dot{q}_v(T_i). \quad (24)$$

The inverse area in the proposed algorithm was divided into two control volumes, as shown in Fig. 3. The division of the inverse area into only two finite volumes was dictated by the small distance of the temperature measuring point from the inner surface. If the measuring point is at a greater distance from the inner surface, the algorithm can be modified, and the number of control volumes can be increased.

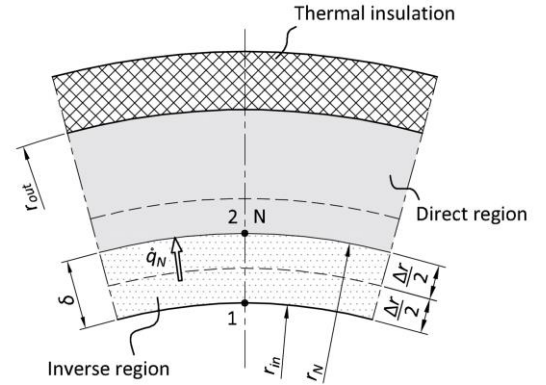


Fig. 3. Division of the inverse region into two finite volumes.

In this case, the internal surface temperature (at node 1) T_1 is determined from the energy balance equation for node 2, which coincides with the temperature measurement point N. In the case under consideration, the energy generation rate per unit volume equals zero.

The energy balance for node 2 has the form:

$$\begin{aligned} \pi \left[r_N^2 - \left(r_N - \frac{\Delta r}{2} \right)^2 \right] c(T_N) \rho(T_N) \frac{dT_N}{dt} = \\ = 2\pi \left(r_N - \frac{\Delta r}{2} \right) \frac{k(T_1) + k(T_N)}{2} \frac{T_1 - T_N}{\Delta r} + 2\pi r_N \dot{q}_N. \end{aligned} \quad (25)$$

Hence, the temperature at the inner surface is:

$$T_1 = \frac{(\Delta r)^2 \left(r_N - \frac{\Delta r}{4} \right)}{r_N - \frac{\Delta r}{2}} \frac{c(T_N) \rho(T_N)}{k(T_1) + k(T_N)} \frac{dT_N}{dt} - \frac{2r_N \Delta r}{r_N - \frac{\Delta r}{2}} \frac{\dot{q}_N}{k(T_1) + k(T_N)} + T_N. \quad (26)$$

In Eq. (26), there is a thermal conductivity coefficient $k(T_1)$, which is calculated from the temperature at node 1. Iterative calculations are therefore necessary, with the first iteration assuming that the temperature at node 1 is equal to the temperature at node 2 (at point N). Iterations are carried out until the condition is met:

$$|T_1^{(n+1)} - T_1^{(n)}| \leq \varepsilon, \quad (27)$$

where the tolerance $\varepsilon \approx 0.00001$ K.

The calculation algorithm is as follows. For a given time t , based on the temperature T_N measured at point N and smoothed with a digital filter, the temperature distribution in the direct region and the heat flux $\dot{q}_N$ are determined from the solution of the direct problem. Then, from $\dot{q}_N$ and temperature T_N , the temperature at node 1 is calculated from Eq. (26). From the solution of the direct and inverse problem, the mean temperatures for the two areas $\bar{T}_{direct}$ and $\bar{T}_{inv}$ are determined separately.

From these and based on the transformed Eq. (5), the mean temperature over the wall thickness is determined:

$$\begin{aligned}\bar{T} &= \frac{2}{r_{out}^2 - r_{in}^2} \int_{r_{in}}^{r_{out}} rT(r, t)dr = \\ &= \frac{2}{r_{out}^2 - r_{in}^2} \left[\int_{r_{in}}^{r_N} rT(r, t)dr + \int_{r_N}^{r_{out}} rT(r, t)dr \right] = \\ &= \frac{2}{r_{out}^2 - r_{in}^2} \left(\frac{r_N^2 - r_{in}^2}{2} \bar{T}_{inv} + \frac{r_{out}^2 - r_N^2}{2} \bar{T}_{direct} \right).\end{aligned}\quad (28)$$

Finally, the stress due to pressure from Eq. (3) and the thermal stress from Eq. (4) are calculated. The whole algorithm is then repeated for time $t + \Delta t$.

For the solution of the inverse problem to be stable, the time step must be chosen appropriately. In this respect, the condition must be met:

$$\Delta Fo \geq 0.05, \quad (29)$$

where

$$\Delta Fo = \frac{\kappa \Delta t}{(r_N - r_{in})^2}. \quad (30)$$

4. Method for determining the heat transfer coefficient based on temperature measurements at six points

The heat transfer coefficient at the inner surface of the pipeline can be determined using FVM. The proposed method is suitable for cases where the temperature field in the wall of a thick-walled element is unsteady and is a function of radial, circumferential and longitudinal variables, the element has a complex shape, as well as when the use of correlations on the Nusselt number is difficult to implement. Correlations on the Nusselt number are usually created for cases of hydraulically developed fluid flow, and this condition is not always achieved at a given point in the pipeline due to different types of fittings (elbows, branches, reducers, etc.) or other equipment (valves, etc.). The method is based on measuring the wall temperature at six points: C, U, N, S, W and E near the inner surface of the pipeline (Fig. 4). The measurement of the temperature at the six points and the subdivision into control volumes as shown in Fig. 4 is based on the assumption that heat in a cylindrical wall is transferred in the radial, circumferential and longitudinal directions. From the temperature measurements, the temperature at the inner surface of the pipeline at point L is calculated. The nodes C, U, N, S, W and E lie at the centres of gravity of the control volumes, whose thickness is Δr . The control volume with node L has a thickness of $\Delta r/2$, and node L lies on the inner surface of the pipe.

Nodes C, N, S, W and E are located at a distance $\delta_1 = \Delta r$ and node U at a distance $\delta_2 = 2\Delta r$ from the inner surface. Nodes N

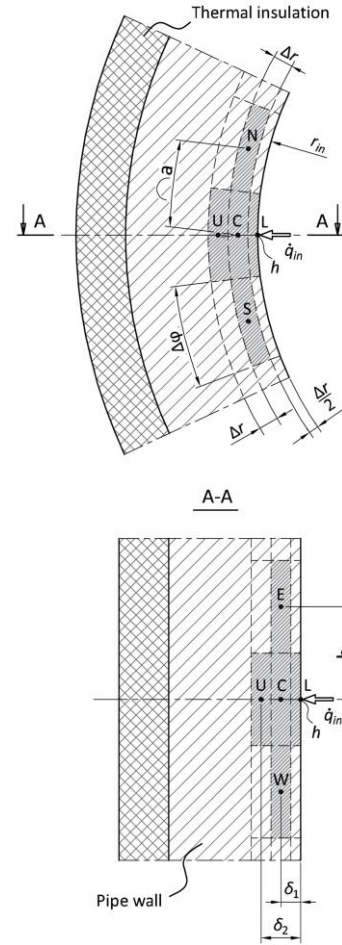


Fig. 4. Division of a pipeline section into control volumes.

and S are located at a distance a in the circumferential direction, above and below the central node C, respectively. Nodes E and W are located at a distance b in the longitudinal direction, to the left and right of node C, respectively. Node U and L are located at a distance Δr in the radial direction, behind and in front of C, respectively.

The temperature at the internal surface is calculated from the energy balance for the control volume with node C, determined using Eq. (24). Assuming the material properties of the pipeline as a function of temperature, it can be written that:

$$\begin{aligned}\Delta V_C c(T_C) \rho(T_C) \frac{dT_C}{dt} &= \frac{A_{U-C} k(T_U) + k(T_C)}{2\Delta r} (T_U - T_C) + \\ &+ \frac{A_{L-C} k(T_L) + k(T_C)}{2\Delta r} (T_L - T_C) + \frac{A_{N-C} k(T_N) + k(T_C)}{2a} (T_N - T_C) + \\ &+ \frac{A_{S-C} k(T_S) + k(T_C)}{2a} (T_S - T_C) + \frac{A_{W-C} k(T_W) + k(T_C)}{2b} (T_W - T_C) + \\ &+ \frac{A_{E-C} k(T_E) + k(T_C)}{2b} (T_E - T_C),\end{aligned}\quad (31)$$

where

$$\Delta V_C = \frac{\Delta \varphi}{2\pi} \pi \left[\left(r_{in} + \frac{3}{2} \Delta r \right)^2 - \left(r_{in} + \frac{1}{2} \Delta r \right)^2 \right] b, \quad (32)$$

$$A_{U-C} = \frac{\Delta \varphi}{2\pi} 2\pi \left(r_{in} + \frac{3}{2} \Delta r \right) b, \quad (33)$$

$$A_{L-C} = \frac{\Delta \varphi}{2\pi} 2\pi \left(r_{in} + \frac{1}{2} \Delta r \right) b, \quad (34)$$

$$A_{N-C} = A_{S-C} = \Delta r b, \quad (35)$$

$$A_{W-C} = A_{E-C} = \frac{\Delta \varphi}{2\pi} \pi \left[\left(r_{in} + \frac{3}{2} \Delta r \right)^2 - \left(r_{in} + \frac{1}{2} \Delta r \right)^2 \right]. \quad (36)$$

The internal surface temperature at node L derived from Eq. (31) is of the form

$$\begin{aligned} T_L = 2\Delta r & \frac{c(T_C)\rho(T_C)}{k(T_L)+k(T_C)} \frac{\Delta V_C}{A_{L-C}} \frac{dT_C}{dt} - \frac{A_{U-C}}{A_{L-C}} \frac{k(T_U)+k(T_C)}{k(T_L)+k(T_C)} (T_U - T_C) + \\ & - \frac{A_{N-C}}{A_{L-C}} \frac{\Delta r}{a} \frac{k(T_N)+k(T_C)}{k(T_L)+k(T_C)} (T_N - T_C) + \\ & - \frac{A_{S-C}}{A_{L-C}} \frac{\Delta r}{a} \frac{k(T_S)+k(T_C)}{k(T_L)+k(T_C)} (T_S - T_C) + \\ & - \frac{A_{W-C}}{A_{L-C}} \frac{\Delta r}{b} \frac{k(T_W)+k(T_C)}{k(T_L)+k(T_C)} (T_W - T_C) + \\ & - \frac{A_{E-C}}{A_{L-C}} \frac{\Delta r}{b} \frac{k(T_E)+k(T_C)}{k(T_L)+k(T_C)} (T_E - T_C) + T_C. \end{aligned} \quad (37)$$

Then, knowing the temperatures at nodes U, C and L allows the heat flux at node L to be calculated from Eq. (29):

$$\dot{q}_L = -k(T_L) \frac{-3T_L + 4T_C - T_U}{2\Delta r}. \quad (38)$$

The calculated heat flux and temperature at the L node on the internal surface and the measured temperature of the working fluid T_f allow the heat transfer coefficient to be determined:

$$h_L = \frac{\dot{q}_L}{T_f - T_L}. \quad (39)$$

In Eq. (37), knowledge of the thermal conductivity coefficient $k(T_L)$ is needed, so iterative calculations must be carried out to calculate the temperature T_L . In the first iteration, the temperature at node L is assumed to be equal to the temperature at node C. Iterations are carried out until the condition is satisfied:

$$|T_L^{(n+1)} - T_L^{(n)}| \leq \varepsilon, \quad (40)$$

where the tolerance $\varepsilon \approx 0.00001$ K.

In the calculation algorithm, first the measured data, i.e. temperatures T_U , T_C , T_S , T_E and T_W for time t are smoothed using a digital filter. Then the temperature T_L from Eq. (37) is determined in an iterative calculation. In subsequent steps, the heat flux $\dot{q}_L$ from Eq. (38) and the heat transfer coefficient h_L from Eq. (39) are calculated. The entire algorithm is then repeated for the next time $t + \Delta t$.

5. Laboratory bench description

The test bench, on which the new temperature measuring sensors were installed, is shown in Fig. 5. The measurements taken with the thermometers are the input data for the developed applications. The main component of the bench is a Viessmann VITOMAX 200 HS (type M237) oil-fired steam generator with a thermal output of 460 kW and a steam output of 700 kg/h (saturated steam at 10 bar). In addition, the station includes a feed-water treatment station, deaerator, sludge expander, 10/4/2 bar reduction station and steam header.

The experimental tests were carried out while the system was in operation. During boiler start-up, the steam header was taken

out of the circuit. When the oil-fired boiler reached its nominal heat output, a valve on the inlet pipe to the header was slowly opened, allowing steam to flow. The boiler produces saturated steam, but because a throttle orifice was fitted upstream of the header, superheated steam flows through it.



Fig. 5. General view of the laboratory station: 1 - steam boiler, 2 - deaerator, 3 - sludge expander, 4 - water treatment station, 5 - reduction station, 6 - steam header.

The temperature measuring sensors were mounted in a thick-walled steam header. The steam header was made from P91 martensitic steel. It is a thick-walled pipe with a length of 3.765 m, an outside diameter of 0.355 m and a wall thickness of 0.05 m. Three sheathed thermocouples were installed in the header to measure the steam temperature. These thermometers have different outer jacket diameters: 1, 2 and 3 mm. To ensure that the 1 and 2 mm diameter thermometers are not damaged or bent as the steam flows through the header, they are contained in sheath tubes for almost their entire length inside the header. The thermometer used to measure the wall temperature at one point near the inner surface is a sheathed thermocouple with an outer diameter of 3 mm. It is placed in a drilled hole 42.5 mm deep; thus, the tip of the thermometer is at a distance of $\delta = 7.5$ mm from the inner surface of the header. A measuring probe consisting of five rod-shaped elements with diameters of 10 mm was used to measure the temperature of the steam header wall at six points. One of the rods has four grooves milled longitudinally, spaced every 90° with respect to the cross-section. In two of the opposite grooves, thermocouples have been placed so that their tips are at the end of the rod, while in the other two, the measuring junctions are at a distance of 7.5 mm from the end of the rod. This is the central rod placed in a 42.5 mm deep hole inside the wall, halfway up the header. Each of the other four rods has two grooves milled longitudinally on opposite sides, in which the thermocouples are placed so that their ends are at the end of the rod. These rods are also placed in holes 42.5 mm deep. Relative to the end of the central rod, their ends are located at a distance of 50 mm, below and above in the circumferential direction and to the left and right in the longitudinal direction. The measuring probe elements mounted in this way allow the wall temperature to be measured at 5 points at a distance of $\delta_1 = 7.5$ mm from the inner surface and at 1 point at $\delta_2 = 15$ mm from the inner surface.

The thermocouples in the measuring probe have a sheath diameter of 1.5 mm. The temperature at the subsequent measuring points is the arithmetic average of each pair of thermocouples. All thermocouples used, i.e. for measuring the temperature of the steam and the temperature of the header wall at one point and in the measuring probe, are of type K, class 2, with a grounded measuring junction. Before installation, they were calibrated using a calibration furnace. For wall temperature measurements, thermally conductive paste has been applied inside the holes before the insertion of the thermometer or measuring probe rods to minimise contact resistance. The installation of the measuring sensors is shown in Figs. 6 and 7.

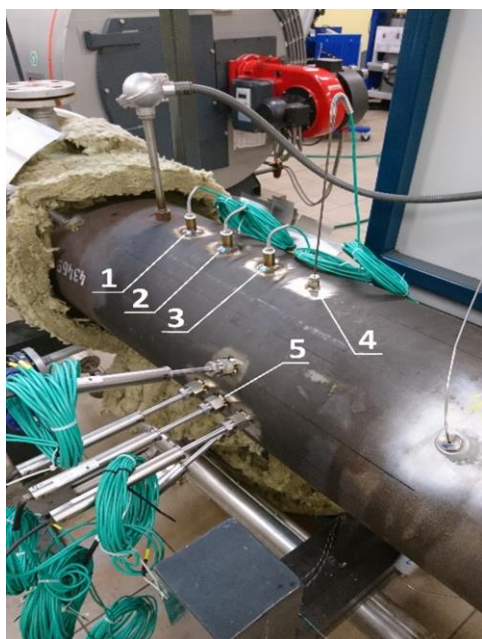


Fig. 6. The way of assembling the temperature measuring sensors: 1–3 - steam temperature measurement by thermocouples with outer jacket diameters of 1, 2 and 3 mm; 4 - pipeline wall temperature measurement at one point, 5 - measuring probe (pipeline wall temperature measurement at 6 points).

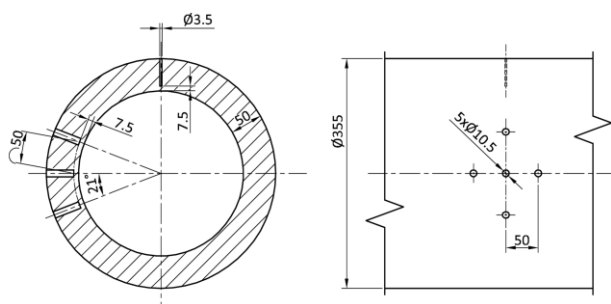


Fig. 7. Method of making holes for sensors to measure the temperature of the steam header wall.

The choice of location of the temperature measurement sensors in the wall of the steam header was due to how the temperature field in the wall changes in different parts of it. During the experiment, the steam flowing into the cold, thick-walled pipe partially condenses on its inner surface. This results in a lower wall temperature near the bottom of the header, where the con-

densate collects, than in the upper part. This is confirmed by temperature measurements around the circumference of the pipe. These measurements indicate that in the pipe's upper part, the temperature field is one-dimensional (heat is transferred only in the radial direction). Hence, it follows that the measurement of the element wall temperature at one point is carried out at the top of the steam manifold. Using a measuring probe that allows the wall temperature to be measured at 6 points is justified if there is at least a two-dimensional temperature field obtained in the lower part of the steam header. Hence, the probe's location, i.e. halfway up the pipe, results.

During the tests, in addition to temperature measurements, the pressure at the outlet of the header and the mass flux of the steam were also measured. Pressure is measured using a pressure gauge with a U-tube syphon and a cock. The pressure gauge is class 1 and has a measuring range of 16 bar, which means an accuracy of ± 0.16 bar. The steam flow rate is measured using an orifice plate with a measurement accuracy of $\pm 1.5\%$ of the measured value.

The measurements are recorded by the data acquisition system every $\Delta t = 1$ s.

Measurements performed on the bench are recorded on a continuous basis using Metronic's M-800 analogue signal transmitters with an RS-485 serial communication port. The transmitter has a built-in Modbus RTU protocol for reading current results, controlling output relays, and transferring data to a computer. Astor Sp. z o.o.'s InTouch software was used to visualise and control the processes occurring throughout the system. It enables the observation of processes taking place in the installation and its parts presented as graphic objects on the monitor screen. It also allows the measured parameters to be presented in graphical form. A fragment of a graphic visualisation of a pipe section, on which temperature measurements using the described sensors are conducted, is presented in Fig. 8 (the screenshot was taken when the installation was not in operation).

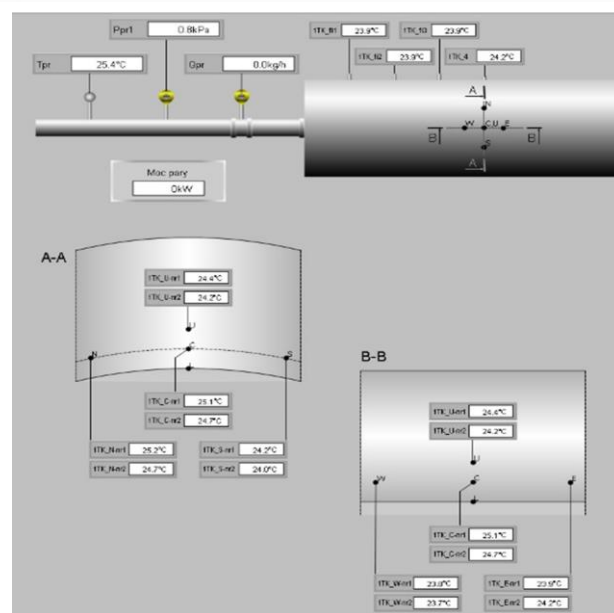


Fig. 8. Fragment of a screen shot from the visualisation software of measurements carried out on a thick-walled steam header (during system shutdown).

6. Description of the software and its test on the laboratory bench

Two applications were developed based on the computational algorithms described in Sections 2–4: ‘Thermal Stress Monitoring’ and ‘Identification of Heat Transfer Coefficient’. The ‘Thermal Stress Monitoring’ application allows the determination of thermal stresses and stresses due to the pressure of the working fluid on the inner surface based on temperature measurements at a single point inside the wall of the steam collector. In the application, stresses caused by pressure are calculated using Eq. (3), while thermal stresses on the inner surface are determined using Eq. (4). The mean temperature across the pipe wall thickness and the inner surface temperature, which are necessary for calculating thermal stresses, are determined using Eqs. (28) and (26), respectively. In turn, the ‘Identification of Heat Transfer Coefficient’ application calculates the heat transfer coefficient on the inner surface of the steam header based on temperature measurements at six points inside the pipe wall. Based on these values, the internal pipe surface temperature is determined from Eq. (37) and the heat flux from Eq. (38). Next, the heat transfer coefficient is calculated from Eq. (39).

Both software programs use the following properties of the steam header steel for their calculations:

$$k(T) = -0.003 \times 10^{-10}T^5 + 4.741 \times 10^{-10}T^4 + \\ -2.874 \times 10^{-7}T^3 + 6.438 \times 10^{-5}T^2 + \\ -4.177 \times 10^{-4}T + 28.676, \quad (41)$$

$$c(T) = 1.41 \times 10^{-6}T^3 - 6.43 \times 10^{-4}T^2 + \\ + 4.88 \times 10^{-1}T + 439.80, \quad (42)$$

and density $\rho = 7750 \text{ kg/m}^3$. The density value was taken constant due to the fact that it varies little for the expected temperature range of the header wall. The properties of the steel were taken from [5].

As the inverse methods used in the applications are sensitive to random measurement errors, the temperature measurements were smoothed using a 9-point digital filter [29].

Both applications were written in the Embarcadero C++ Builder 12 environment. C++ Builder is an Integrated Development Environment (IDE), which is distinguished by the fact that it allows the rapid development of Rapid Application Development (RAD) applications thanks to an extensive palette of Graphical User Interface (GUI) components. The used C++ programming language with Embarcadero extensions enabled visual programming. The compiler is based on Clang for Win64 compatible with C++ version 17. The environment includes a wide range of pre-built components such as buttons, text boxes, drop-down lists, tables, grids, database components, charts, etc. The applications use a graphical interface based on the Visual Component Library (VCL). Dynamic visualisation of results is possible using the TChart component, with customisable axis parameters and data series.

The programmes have implemented loading and processing data from text files and a database. They consist of several logically separated modules: user interface, data loading and processing module and a visualisation module.

The test of both applications was carried out during the operation of the laboratory bench on 13.03.2025. The measurements were carried out so that the steam header is taken out of circulation during the start-up of the steam boiler and steam generation. Once the steam pressure and temperature were sufficiently high, the valve upstream of the steam header was opened. The experiment carried out in this way made it possible to obtain an unsteady temperature field in the steam header. During the start-up of the stand, the manometric pressure in the header increased to 300 kPa, the mass flux of steam flowing through the header after opening the valve was about 450 kg/h, then was reduced to about 300 kg/h. The steam temperature was approximately 140°C.

Both applications were run while the laboratory bench was in operation, and calculations were performed online. The interface windows of both applications are shown in Figs. 9–10, which are screenshots of the computer operating the bench. The user interface was designed in both cases so that the central part consists of two graphs, where the upper one presents the measurement data and the lower one presents the calculation results. The graphs present data for the last 500 s. Next to the graphs, on the right-hand side, there is a legend describing the curves drawn. A detailed data description appears when the cursor is placed on the chart area or the legend. The user can also select which data they wish to present on the graphs by ticking the appropriate check boxes in the legend. On the right side of the programme window, there is a visualisation of the location of temperature measurement points in the steam header wall and information on the funder, i.e. the National Science Centre in Poland, whose funding allowed the research to be carried out.

The ‘Thermal Stress Monitoring’ application window is shown in Fig. 9. In Fig. 9, the top graph shows the measured data, i.e. the temperature at point N (labelled T_N), the temperature of the steam measured by sheathed thermocouples with diameters of 1 mm, 2 mm and 3 mm ($T_f \phi 1 \text{ mm}$, $T_f \phi 2 \text{ mm}$, and $T_f \phi 3 \text{ mm}$, respectively), the pressure of the steam flowing through the steam header p and the mass flux of the steam m . The temperature measuring point N in the wall of the steam collector is located at 7.5 mm from the inner surface. The bottom graph shows the calculated data, i.e.: the stress on the inner surface caused by the pressure σ_p , the thermal stress on the inner surface σ_T , the sum of both stresses $\Sigma\sigma$, the temperature of the inner surface T_{in} , the temperature of the outer surface T_{out} and the difference of these temperatures, i.e. $T_{out} - T_{in}$. The temperatures T_{in} and T_{out} are also summarised in the top graph to make comparing them with the measured temperatures easier.

The thermal stresses on the inner surface of the header are highest during the temperature step change in the header and reach a maximum value of -195.8 MPa . In turn, the stresses caused by pressure are minimal due to the low steam pressures, reaching a maximum of 0.9 MPa for the highest pressure value of 300.1 kPa obtained towards the end of the experiment. Consequently, the σ_T and $\Sigma\sigma$ curves coincide.

In turn, Fig. 10 shows the application window ‘Identification of Heat Transfer Coefficient’. The top graph shows the temperature measurements carried out at six points: C, N, W, S, E and

U (corresponding to the designations T_C , T_N , T_W , T_S , T_E and T_U , respectively). Points C, N, W, S and E are located at a distance of 7.5 mm from the inner surface, where N, W, S and E are spaced 50 mm from the central point of C, N and S are respectively above and below point C in the circumferential di-

rection, and E and W are located to the right and left of point C in the longitudinal direction. Point U is located 15 mm from the inner surface and 7.5 mm from centre point C in the radial direction.

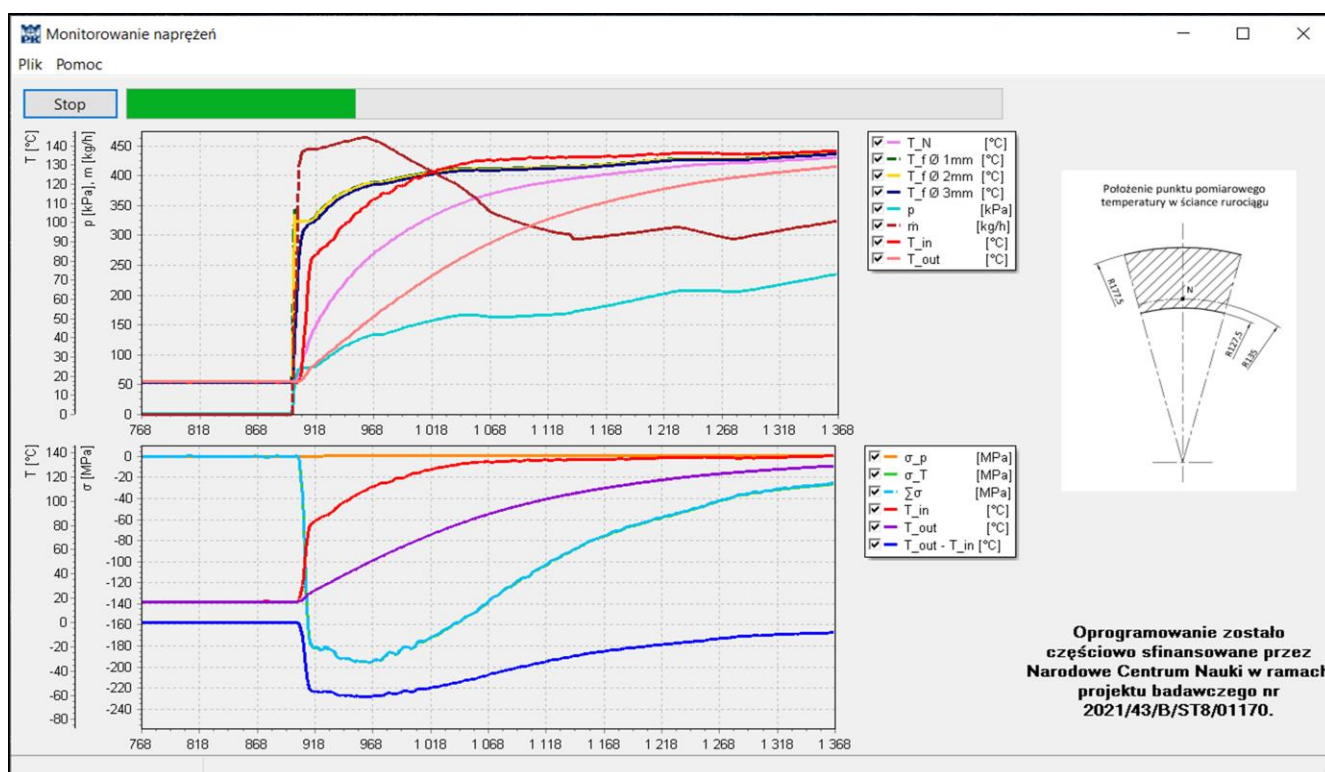


Fig. 9. Screenshot of the 'Thermal Stress Monitoring' application window during operation of the installation on 13.03.2025.

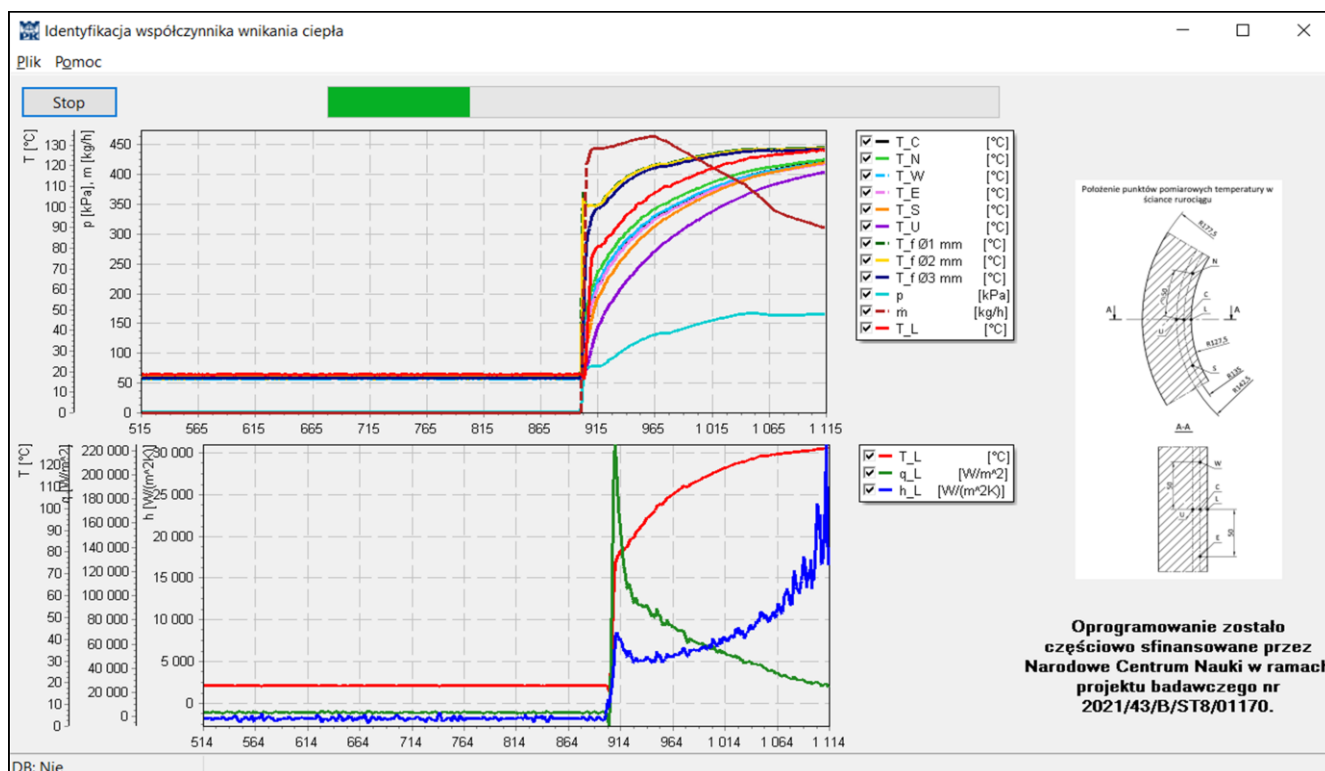


Fig. 10. Screenshot of the 'Identification of Heat Transfer Coefficient' application window during operation of the installation on 13.03.2025.

In addition, the upper graph presents the steam temperature measurements with 1 mm, 2 mm and 3 mm diameter thermometers ($T_{f\phi 1\text{ mm}}$, $T_{f\phi 2\text{ mm}}$, and $T_{f\phi 3\text{ mm}}$), the pressure in the header (p) and the mass flux of steam (m). The bottom graph presents the results of the calculations, i.e. the temperature (T_L), the heat flux (q_L) and the heat transfer coefficient (h_L) at point L on the internal surface. Point L lies in the same radial direction as points U and C. The highest value of heat flux is 224 537.5 W/m² and is achieved during the opening of the valve at the inlet to the steam header and during the first moments of steam flow, when the temperature difference between the inner surface of the pipe and the working fluid is greatest. The heat transfer coefficient is then 7897.5 W/(m² K). Thereafter, the heat flux value begins to decrease. The heat transfer coefficient decreases with the heat flux for the first 30 seconds, then remains at approximately 5000 W/(m² K) for approximately 1 minute. After that, the smaller the difference between the surface temperature and steam temperature and the heat flux, the higher the heat transfer coefficient.

The impact of sensor measurement uncertainty on the accuracy of calculations was negligible. Temperature measurements were used in the calculations of thermal stresses and the heat transfer coefficient. Thanks to the calibration of the measuring sensors in a calibration furnace prior to their installation, the impact of temperature measurement uncertainty on the calculation results is insignificant. In addition, pressure gauge measurements were used to calculate the stresses caused by pressure. Since their value is small compared to thermal stresses, the uncertainty of pressure measurement is of little significance.

Both programmes generate reports with calculation data in a text file (.txt).

6. Conclusions

The article presents calculation algorithms that allow determining circumferential stresses and the heat transfer coefficient on the inner surface of a thick-walled cylindrical element. The algorithm for determining thermal stresses can be used for one-dimensional heat conduction (in the radial direction) and is based on measuring the wall temperature at a single point. The algorithm for determining the heat transfer coefficient is suitable for heat conduction in a thick-walled element in three directions: radial, circumferential and longitudinal. It is based on measuring the wall temperature at six points – a special measuring probe was made for this purpose. The solution to the inverse heat conduction problem was used in both cases.

Based on the described algorithms, two applications were created that can be used online, which is particularly important in unsteady states. The applications have been tested on a test bench and are functioning correctly. The calculations were made on the basis of measurements taken in a thick-walled steam header that is part of a large system with a steam boiler. The experiment was conducted by suddenly opening the valve upstream of the header, allowing steam to flow through it. The temperature in the header rapidly rose from 16.8°C to 142.5°C. At the beginning of the sudden temperature change, the thermal stresses on the inner surface of the pipe reached a maximum value of −195.8 MPa, and the heat transfer coefficient reached

a value of approximately 5000 W/(m² K). Subsequently, as the collector wall temperature stabilised, a decrease in thermal stresses and an increase in the heat transfer coefficient occurred.

The applications can be an effective tool for monitoring thermal stresses in pressure components.

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Comparative experimental study on the performance of single-slope, hemispherical, and conical solar stills

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Abstract

Although there is a great supply of groundwater, it is salty, and most places of the globe, including the Algerian desert, have a shortage of fresh water. The greatest answer to this issue is solar distillation since it uses sustainable solar energy. Solar still comes in a variety of forms, dimensions and styles, creating potable water that is fresh, clean, and safe to drink, albeit the amount of water produced differs depending on the type of solar still. Choosing the design of solar stills remains one of the preferred solutions to improve their performance. The performance of three different solar still designs is compared in this work, including single slope, hemispherical, and conical solar stills. The investigation used brackish water with a salt level of 3000 ppm and was conducted under the same operational conditions and locations. The conical solar still had the best overall performance, with a daily average production of 5.80 l/m²/day. The hemispherical solar still had the second-best performance, with a daily average production of 5.10 l/m²/day. Finally, the single slope solar still had the lowest performance (daily average production of 3.30 l/m²/day). The findings can be used to develop and improve solar stills for saltwater desalination.

Keywords: Hemispherical; Conical; Single slope; Solar still; Energy and exergy efficiency

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1. Introduction

Water scarcity is a pressing global issue, with a significant portion of the world's population facing challenges in accessing safe and sufficient water resources. As freshwater supplies continue to decline due to factors such as climate change, pollution, and population growth, the need for innovative and sustainable

solutions such as water desalination has become increasingly urgent [1]. Desalination, the process of removing salts and impurities from seawater or brackish water to produce potable water, offers a promising pathway to alleviate water scarcity and ensure a reliable supply for domestic, agricultural, and industrial needs. By implementing desalination technologies, communities can enhance water security and reduce the adverse impacts

Nomenclature

A	– surface area, m^2/s
$\dot{E}_x$	– exergy, kJ
I	– solar radiation intensity, W/m^2
$\dot{m}$	– mass flow rate, kg/s
T	– temperature, $^\circ\text{C}$

Greek symbols

λ_{fg}	– latent heat of vaporisation, J/kg
η	– efficiency, %

Subscripts and Superscripts

<i>amb</i>	– ambient
<i>dest</i>	– destruction
<i>evap</i>	– evaporation
<i>fw</i>	– fresh water
<i>in</i>	– input

<i>out</i>	– output
<i>sc</i>	– solar collector
<i>sys</i>	– system

Abbreviations and Acronyms

FEM	– finite element method
CM	– composite materials
CoSS	– conical solar still
HFU	– high-frequency ultrasonic (atomiser)
HFUSS	– high-frequency ultrasonic solar still (atomiser)
HSS	– hemispherical solar still
NPCMSS	– nano-PCM modified solar still
PCM	– phase change material
SS	– solar still
SSSS	– single-slope solar still
TDS	– total dissolved solids
TSS	– tubular solar still

of water shortages on public health, food production, and economic development. Among the various desalination methods, thermal and membrane-based processes are the most widely used [2,3]. However, conventional desalination techniques often rely heavily on fossil fuels for energy, making them environmentally and economically unsustainable [4,5]. This highlights the need for a renewable energy-powered desalination system [6].

Salt can be extracted from saline water using solar energy through two principal methods: direct and indirect solar desalination [7,8]. In direct systems, solar radiation is absorbed directly by the desalination unit, where it is used to heat the saline water, inducing evaporation and subsequent condensation to produce freshwater. Indirect systems, on the other hand, involve the use of external solar collectors, such as flat plate or evacuated tube collectors that harness solar thermal energy, which is then transferred to the desalination unit. This configuration enables greater control over thermal input and can enhance the overall efficiency and productivity of the desalination process [9]. Solar distillation is a sustainable and effective method for desalinating saline or brackish water, and it has been successfully applied in various regions around the world [10]. Among the different solar desalination technologies, solar stills (SSs) represent one of the oldest and most widely adopted approaches for producing clean water. Their simplicity, low cost, and reliance solely on solar energy make them particularly suitable for decentralised and off-grid applications. In recent years, solar stills have received renewed attention as a viable solution to the global water crisis, especially in arid and semi-arid regions. Numerous experimental studies have been conducted to investigate and improve the efficiency of solar stills. The basin-type solar still, characterised by a shallow water basin and transparent cover, remains the most traditional and commonly used configuration for harnessing solar radiation to produce potable water [11]. Despite their advantages, the practical application of solar stills faces certain challenges. Their performance declines with increasing salinity levels in the feedwater, as higher total dissolved solids (TDS) reduce the evaporation rate. Additionally, their deployment at larger scales is often limited by the extensive

surface area required for meaningful output, along with the associated costs of materials, construction, and long-term maintenance. These constraints have driven ongoing research into improving the design and efficiency of solar still systems [12,13].

Dev et al. [14] conducted an annual performance study on an active SS with two independent still evacuated solar tubes. The results demonstrate that during daylight hours, the collector's standalone thermal efficiency is 48% on average. The performance of the integrated collector and solar system is greatly impacted by ambient temperature change, which reduces efficiency by about 5–8%. The use of a high-frequency ultrasonic atomiser (HFU) was investigated to see if it may boost the efficiency of a SS [15]. The daily productivity of SS was 3.58 l/m^2 and that of HFUSS was 4.41 l/m^2 , according to the results. Kabeel et al. [16] conducted experiments on SS that had been upgraded using high-efficiency dehumidification of air. The daily output, humidification efficacy, and total efficacy of the improved system were calculated as 18.25 l/m^2 , 79% and 39%, respectively. Theoretical and practical evaluations of fixed pyramid and flat plate solar collectors with low costs have been conducted [17]. In order to determine whether a high-frequency ultrasonic atomiser could enhance the efficiency of a solar still, an experiment was carried out by Attia et al. [12].

Improving the design of SSs has been one of the key topics of research. Researchers have examined the influence of several design elements, including the shape and dimensions of the still, on its efficacy. Taamneh et al. [18] determined that a pyramid-shaped solar still was more effective than a flat plate still regarding water production rate and quality. A study conducted by Belila et al. [19] demonstrated that augmenting the surface area of the still by a multi-layered design enhanced the clean water production rate. Extensive studies have been conducted on pyramid-shaped solar stills, which have demonstrated superior performance compared to traditional flat configurations [20]. The geometrical advantages of the pyramidal design include uniform solar capture throughout the day, reduced side-wall shading, and the elimination of the need for solar tracking mechanisms. Literature analyses further highlight that the performance of a solar still is governed by a range of interrelated factors [21]. These include the

depth of water in the basin, the temperature of the glass cover, ambient air velocity, the temperature of the incoming saline water, internal vapour dynamics, and the thermal properties of the materials used for the basin, insulation, and cover [22]. Optimising these parameters is critical to enhancing the overall efficiency and productivity of the system. Consequently, optimising performance necessitates the adjustment of all pertinent parameters.

Furthermore, recent investigations have included comprehensive analyses of the energy, exergy, and economic performance of solar stills [23]. Experimental studies have shown that integrating heat storage materials, particularly a composite of paraffin wax and black gravel, can significantly enhance system efficiency [12,13,24]. These materials improve thermal retention during non-sunlight hours, reduce temperature fluctuations, and sustain evaporation rates, thereby contributing to higher freshwater yield and overall system effectiveness [25,26]. The utilisation of composite material enhanced SS productivity, and energy and exergy efficiencies by approximately 3.27 l/m², 48.22% and 3.08%, respectively, yielding improvements of around 37.56%, 38%, and 37% compared to the system employing phase change material (SS-PCM). A novel technique has been implemented to diminish the surface tension and boundary layers in the water of the solar still basin. The novel design utilised water dripping unobstructed through fissures in a solar still tray, presenting a significant benefit. The efficacy of a solar still was examined by altering the number of fissures (7, 10 and 14 fissures) in the suspended surfaces of the trays, facilitating the unobstructed flow of water [27]. A computational and experimental evaluation was conducted by integrating a solar air heater with a multi-compartment solar distiller [28]. Advancing research and development in solar distillation technology could mitigate these limitations, enhancing the accessibility and prevalence of this method [2]. An experiment was conducted to investigate the enhancement of distillation output in a conical solar distiller by utilising inexpensive energy storage materials. This study aims to determine the optimal spacing between stainless-steel balls, each having a fixed diameter of 0.014 m, to enhance the efficiency of conical solar stills by using them as a cost-effective energy storage medium [29].

The present research offers a novel and comprehensive comparative evaluation of three solar still configurations (single-slope, hemispherical and conical) under identical environmental and operational conditions. While numerous studies have explored modifications to individual solar still geometries or examined hybrid systems, direct experimental comparisons of distinct geometric designs within the same testing environment remain scarce in the literature. This study distinguishes itself by not only quantifying and comparing daily water productivity, thermal efficiency, and exergy efficiency of each design, but also by integrating an economic evaluation based on payback period and fabrication cost. Additionally, the study provides new insights into the thermodynamic behaviour of solar stills by analysing exergy destruction mechanisms related to each geometry, thereby contributing to a deeper understanding of the influence of design on performance. By employing brackish water representative of real-world conditions (3000 ppm) and rigorously measuring performance metrics over a full diurnal cycle,

this work presents actionable findings that can guide practical design decisions for decentralised desalination systems in arid and semi-arid regions.

2. Materials and methods

To carry out the experimental investigation, three solar still designs were constructed and tested under identical climatic conditions. The fabrication process followed standardised procedures using consistent materials to ensure uniform thermal behaviour across configurations. This section provides a detailed account of the geometric specifications, construction materials and insulation methods used for each still. Additionally, the arrangement of the experimental setup, environmental monitoring tools and measurement protocols are described to ensure clarity and reproducibility of the procedures.

2.1. Experimental setup description

2.1.1. Hemispherical solar still

The hemispherical solar still is designed to optimise solar energy capture and internal heat retention through its symmetrical, dome-shaped geometry. The unit consists of a circular wooden basin with a diameter of 0.36 m, serving as the saline water reservoir. It is covered with a transparent hemispherical dome fabricated from a 0.003 m thick acrylic sheet, measuring 0.4 m in both diameter and height. This geometry maximises the exposure of the inner surface to incident solar radiation from multiple angles throughout the day, enhancing thermal absorption and promoting uniform heat distribution. Figure 1 presents a schematic diagram of the hemispherical solar still, illustrating its structural components and layout. To reduce thermal losses, the basin is insulated along its base and side-walls using thermally resistant material, thereby improving the overall thermal efficiency of the system. The smooth inner curvature of the acrylic dome also facilitates efficient condensation of vapour, allowing distilled water to flow downward and collect along the perimeter.

2.1.2. Conical solar still

The conical solar still is designed to enhance solar energy concentration and streamline condensate collection through its tapered geometry. The system features a circular wooden basin with a surface area of 0.1 m², serving as the brackish water reservoir. A conical transparent acrylic cover, 0.002 m thick, with a base diameter of 0.4 m and a height of 0.12 m, encloses the still from the top. The conical geometry improves the angular reception of solar radiation throughout the day and supports a natural downward flow of condensed droplets toward the collection perimeter. To minimise heat losses, thermal insulation is applied along the base and outer walls of the basin. The reduced internal volume and steeper surface gradient of the conical design allow for higher localised temperatures and more efficient condensation due to the shorter vapour travel distance. These features collectively contribute to enhanced thermal performance and freshwater productivity. A schematic representation of the conical still are presented in Fig. 2.

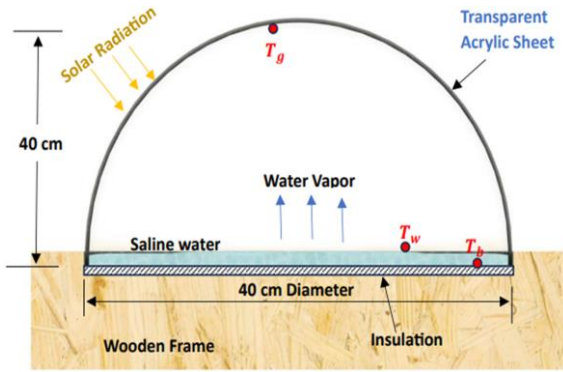


Fig. 1. Schematic representations of the hemispherical solar still design used in the experimental setup, highlighting geometric configuration, materials and insulation layout.

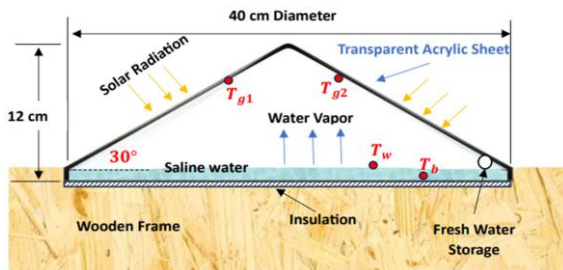


Fig. 2. Schematic representations of the conical solar still design used in the experimental setup, highlighting geometric configuration, materials and insulation layout.

2.1.3. Single-slope solar still

The single-slope solar still represents a conventional design widely used in solar desalination studies. It consists of a rectangular wooden basin with a surface area of 0.25 m², used to hold the saline water. The structure features an inclined transparent glass cover (0.003 m thick), supported by a sloped frame, with the front and rear wall heights measuring 0.06 m and 0.14 m, respectively. This inclination facilitates solar radiation entry and directs condensed water downward to the collection channel. The flat-glass cover design, although structurally simple and cost-effective, presents a longer vapour travel path and relatively higher thermal losses compared to curved geometries. Nevertheless, it serves as a reference benchmark for evaluating the performance of more complex still configurations. A schematic view of the single-slope solar still is presented in Fig. 3.

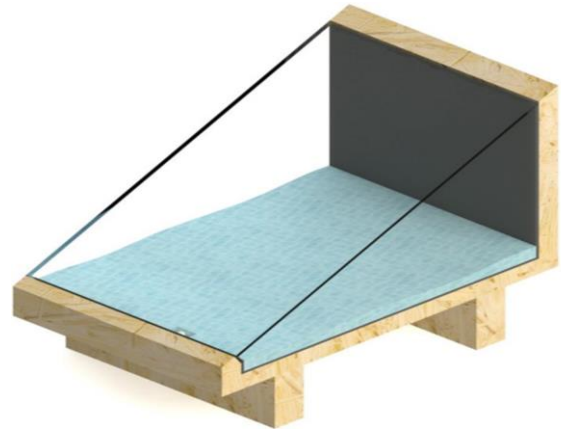
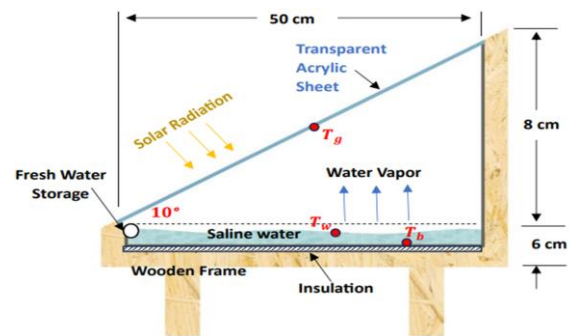


Fig. 3. Pictorial view of the single-slope solar still, illustrating the glass cover angle, basin construction and insulation arrangement.

2.2. Experimental instrumentations

This study investigates and compares the performance of three solar desalination systems: single-slope, hemispherical, and conical solar stills. All three configurations were fabricated using consistent construction materials to ensure a fair comparison. The basins of the stills were constructed from wood and thermally insulated, while the transparent covers varied in geometry and material as per each design. The stills were mounted on a shared wooden support frame to maintain uniform orientation and exposure to solar radiation during the experiments, as illustrated in Fig. 4. It is noteworthy that the three solar still designs investigated in this study



Fig. 4. Photograph of the full experimental setup showing the placement of the three solar still types (single-slope, hemispherical and conical) under identical outdoor environmental conditions.

differ in their basin surface areas. The single-slope solar still features a basin area of 0.25 m^2 , whereas both the hemispherical and conical stills have smaller basin areas of approximately 0.10 m^2 . To ensure a scientifically valid and equitable comparison across configurations, all performance indicators are rigorously normalised per unit surface area. This normalisation eliminates geometric bias and enables a direct assessment of how design geometry influences system performance under identical environmental conditions. Such an approach enhances the analytical robustness of the study and supports the generalisability of the findings to various scales and applications.

The experimental tests were conducted on June 3, 2023, from 07:00 to 18:00 under outdoor conditions in El Oued city, Algeria. To ensure a fair and controlled comparison, all three solar stills were tested simultaneously on the same day under identical environmental and operational conditions. This approach eliminates variations that could arise from fluctuating weather patterns across different days and ensures that differences in performance are attributable solely to the design characteristics of the stills. A constant water depth of 0.01 m was maintained in the basin of each still throughout the test period to standardise thermal behaviour and evaporation potential across configurations. Although the experiment was conducted over a single day, the side-by-side setup, consistent construction materials, and uniform exposure to solar radiation, temperature, and ambient conditions provide a reliable basis for comparative analysis. This methodology aligns with standard practice in comparative solar distillation studies, where simultaneous testing is prioritised to ensure internal consistency.

To monitor thermal performance, calibrated temperature sensors were installed in each still to record the basin water temperature, inner and outer cover temperatures, and ambient air temperature. Thermocouples were placed at the centre of the basin to ensure an accurate representation of the bulk water temperature. Solar radiation was measured at the basin level using a pyranometer, and the quantity of distilled water produced was recorded hourly using a graduated flask.

Data were collected at 60-minute intervals. Each measurement was subject to known uncertainty, and the instrumentation specifications are as follows:

- Temperature sensors: measurement range of $0\text{--}500^\circ\text{C}$, accuracy $\pm 0.10^\circ\text{C}$, uncertainty $\pm 0.08^\circ\text{C}$;
- Solar power meter (pyranometer): measurement range of $0\text{--}1999 \text{ W/m}^2$, accuracy $\pm 10.0 \text{ W/m}^2$, uncertainty $\pm 5.78 \text{ W/m}^2$;
- Graduated flask (enumerated beaker): capacity of $0\text{--}500 \text{ ml}$, accuracy $\pm 1.000 \text{ ml}$, uncertainty $\pm 0.50 \text{ ml}$.

Environmental conditions during the experimental day were characteristic of the hot, arid climate of El Oued, Algeria, and are crucial for interpreting the system performance. Figure 5 presents the hourly variation of ambient air temperature and solar radiation intensity on June 3, 2023. Solar irradiance peaked near midday at approximately 1000 W/m^2 , while ambient temperature reached a maximum of around 40°C . Relative humidity ranged between 18% and 28% during the test period, contributing to favourable evaporation conditions. Wind speed was relatively low, averaging between 1.5 m/s and 2.0 m/s , which helped minimise convective heat loss from the covers while maintaining natural air circulation around the units. Sky conditions remained consistently clear throughout the day, with no observed cloud cover or weather disturbances. These stable and high-radiation conditions ensured that all three solar stills operated under uniform and repeatable environmental conditions, allowing for a valid performance comparison under real-world conditions.

The feedwater used in all three solar stills was brackish water with a salinity level of approximately 3000 ppm , measured in terms of TDS. This salinity level was deliberately selected to reflect typical real-world brackish water conditions, especially in arid and semi-arid regions, where groundwater often falls within this range. According to classifications by the World Health Organisation (WHO) and various water resource management agencies, water with TDS levels between 1000 ppm and 5000 ppm is considered moderately brackish and represents a common challenge for decentralised desalination applications. The use of 3000 ppm ensures that the results are both technically relevant and practically applicable. Furthermore, water salinity directly influences evaporation rates,

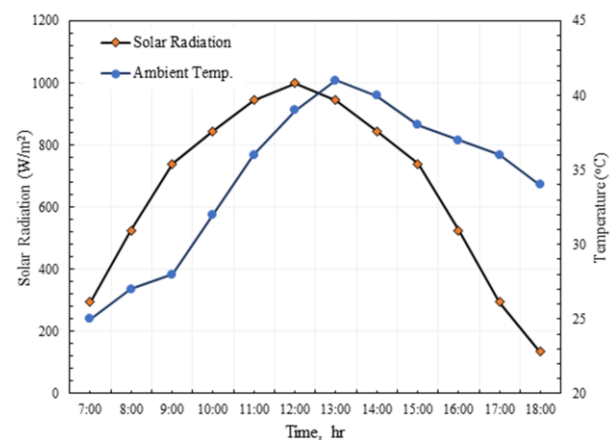


Fig. 5. Hourly variation of ambient air temperature and solar radiation intensity recorded during the experimental day, illustrating typical diurnal environmental conditions in El Oued, Algeria.

as higher salinity reduces the vapour pressure of water, thereby lowering the evaporation potential and thermal efficiency. Conversely, lower salinity increases evaporation due to higher vapour pressure. By selecting a mid-range brackish salinity, the study balances representativeness with system sensitivity, making the findings generalizable to a wide range of real-world desalination scenarios.

3. Performance parameters

3.1. Energy efficiency

Energy efficiency is a fundamental metric for evaluating the thermal performance of solar desalination systems. In the context of solar stills, it is defined as the ratio of the useful thermal energy used to vaporise the feedwater to the total solar energy incident on the collector surface. The useful energy output corresponds to the latent heat required for the phase change of water from liquid to vapour, while the input is the solar radiation absorbed by the basin area. The energy efficiency of the system, denoted as η_{sys} , is calculated as [30]

$$\eta_{sys} = \frac{\dot{m}_{fw} \lambda_{fg}}{A_{sc} I} \times 100, \quad (1)$$

where $\dot{m}_{fw}$ represents the mass flow rate of freshwater produced, λ_{fg} is the latent heat of vaporisation of water, A_{sc} is the surface area of the solar collector, and I denotes the solar radiation intensity incident on the basin surface. This formulation quantifies how effectively the solar still converts the available solar energy into useful thermal energy for distillation. However, it does not account for losses due to irreversibilities or inefficiencies in heat transfer mechanisms, which are better captured through exergy analysis.

3.2. Exergy efficiency

Exergy efficiency provides a more comprehensive assessment of system performance based on the second law of thermodynamics. Unlike energy, which is conserved, exergy is subject to destruction due to irreversibilities within the system. These irreversibilities arise from entropy generation associated with non-ideal heat transfer, conductive and convective losses, and phase change processes. The exergy balance for a solar still system can be represented by the general form [31]:

$$\sum \dot{E}x_{in} - \sum \dot{E}x_{out} = \sum \dot{E}x_{dest}. \quad (2)$$

For this study, the equation is adapted in the following form:

$$\dot{E}x_{sun} - (\dot{E}x_{evap} + \dot{E}x_{work}) = \dot{E}x_{dest}, \quad (3)$$

where $\dot{E}x_{sun}$ is the exergy input supplied by solar radiation, $\dot{E}x_{evap}$ is the exergy output associated with water evaporation, $\dot{E}x_{work}$ represents any mechanical or auxiliary work input (considered negligible in passive systems), and $\dot{E}x_{dest}$ denotes the exergy destruction due to irreversibilities. These irreversibilities are influenced by the thermal gradients, heat conduction through materials, and the temperature difference between the basin water and the ambient environment.

As a rough approximation to reversible operation, values for exergy efficiency, i.e. 2nd law efficiency, range from zero to one, corresponding to the worst case (complete exergy destruction) and the best case (no exergy destruction), respectively. As a result, a system's exergy efficiency during a process can be expressed as [32]:

$$\eta_{Ex} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy supplied}}, \quad (4)$$

$$\eta_{Ex} = \frac{\dot{E}x_{out}}{\dot{E}x_{in}} = 1 - \frac{\dot{E}x_{dest}}{\dot{E}x_{in}}. \quad (5)$$

This formulation reflects the system's ability to convert available solar exergy into useful output while minimising entropy generation. Exergy input from solar radiation is estimated using Petela's model, which accounts for the temperature difference between the sun and the ambient environment [33–35]:

$$\dot{E}x_{in} = \dot{E}x_{sun} = A_{sc} I \left[1 - \frac{4}{3} \left(\frac{T_{amb} + 273}{T_{sun}} \right) + \frac{1}{3} \left(\frac{T_{amb} + 273}{T_{sun}} \right)^4 \right], \quad (6)$$

where T_{amb} is the ambient air temperature and T_{sun} is the effective blackbody temperature of the sun, typically assumed to be 5727°C (6000 K). The useful exergy output resulting from the evaporation process is given by the following formula [2,30]:

$$\dot{E}x_{evap} = \dot{m}_{fw} \lambda_{fg} \left[1 - \frac{T_{amb} + 273}{T_{sun}} \right]. \quad (7)$$

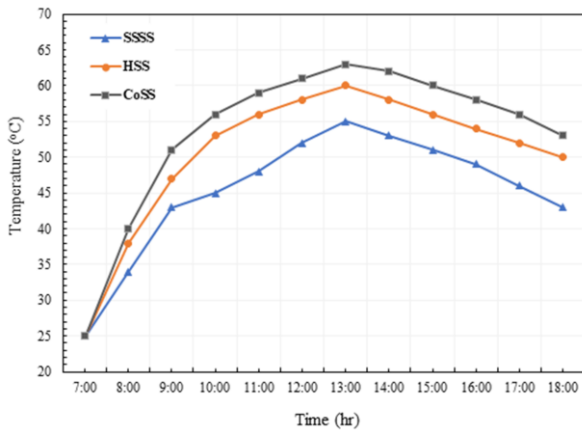
This equation incorporates the temperature difference between the heat source and the environment, emphasising the degradation of energy quality due to irreversible processes. Solar still designs that reduce internal thermal gradients and enhance heat transfer, such as conical and hemispherical geometries, tend to exhibit lower exergy destruction. Consequently, these configurations are more effective in converting incident solar radiation into useful work, as reflected in higher exergy efficiencies. The application of exergy analysis in this study enables a deeper thermodynamic interpretation of performance differences among the tested solar stills, beyond what is possible through energy efficiency alone.

4. Results and discussion

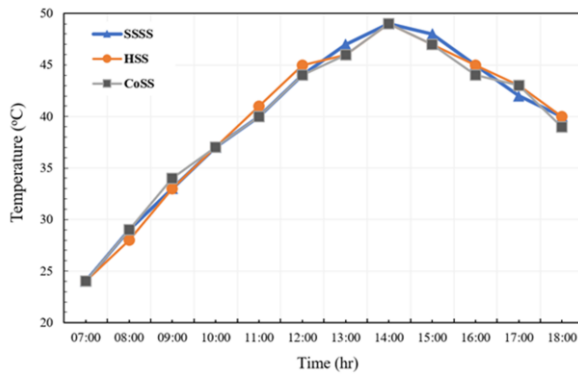
This section presents a comprehensive analysis of the experimental results obtained from the single-slope solar still (SSSS), hemispherical solar still (HSS), and conical solar still (CoSS). Performance metrics include basin water and glass temperatures, freshwater productivity, system (thermal) efficiency and exergy efficiency. The discussion draws upon heat transfer and thermodynamics principles to explain the influence of geometric configuration on solar still performance.

4.1. Thermal behaviour

Figure 6a shows the hourly variation of basin water temperature for all three designs. The CoSS recorded the highest basin temperatures, peaking at around 63°C, followed by the HSS and then the SSSS. This behaviour is primarily attributed to the geometric concentration effect inherent in the conical design,

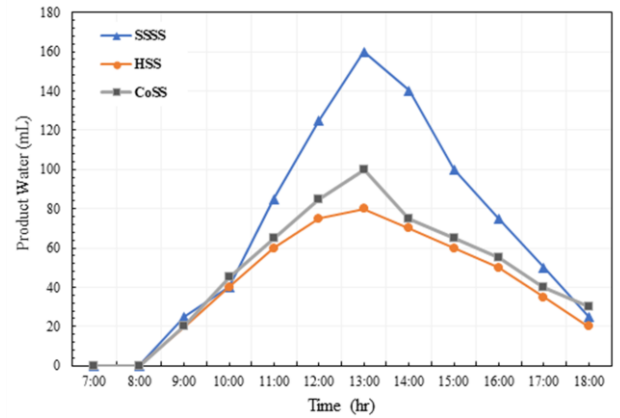


(a) Hourly variation of basin water temperature

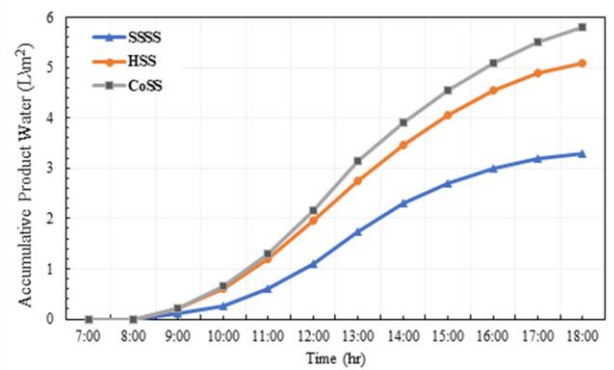


(b) Internal glass cover temperature trends

Fig. 6. Thermal profiles of the solar stills under identical environmental conditions.



(a) Hourly freshwater output from each solar still design



(b) Cumulative distilled water production over the 11-hour test period

Fig. 7. Distillate output profiles of the solar stills.

which facilitates more uniform and multi-angle solar energy absorption throughout the day. CoSS also reduces radiative and convective losses due to its smaller surface-to-volume ratio, a principle supported by Fourier's law of heat conduction, which indicates that heat loss is directly related to surface area and temperature gradient. HSS also achieved relatively high water temperatures due to its dome-shaped acrylic cover, which acts as a natural solar concentrator and enhances internal thermal circulation via buoyancy-driven convection. In contrast, the SSSS exhibited a broader surface area and flat geometry, which increased conductive and convective heat losses, leading to lower water temperatures and reduced thermal gradients.

Figure 6b shows the internal glass cover temperatures. These remained relatively consistent across the three systems due to uniform ambient conditions and solar exposure. However, minor differences suggest that internal air circulation, governed by natural convection patterns and geometry, slightly affects the heat transfer from vapour to the condensation surface. This underscores the role of convective heat transfer coefficients in determining the temperature distribution across system boundaries.

4.2. Freshwater productivity

Figure 7 illustrate hourly and cumulative freshwater yields. CoSS achieved the highest yield at 5.8 L/m²/day, followed by the HSS at 5.1 L/m²/day and SSSS at 3.3 L/m²/day. These results

align with fundamental thermodynamic concepts, considering the latent heat of vaporisation, which governs the energy requirement to convert water into vapour. As CoSS reached the highest basin temperatures, it enabled a higher rate of evaporation due to greater thermal energy availability per unit mass of water. Furthermore, the CoSS geometry enhances internal vapour flow and reduces condensation path length, lowering the associated pressure and thermal resistance, key contributors to vapour transfer rate as described by Fick's law of mass diffusion and Newton's law of cooling. HSS, while also effective, exhibits slightly lower performance due to a more moderate thermal gradient and longer vapour paths. SSSS shows the lowest yield due to greater surface heat loss and weaker natural convection within the enclosure.

4.3. Thermal and exergy efficiencies

Figure 8 presents the hourly system (thermal) efficiencies. CoSS attained a maximum efficiency of approximately 70% at midday and maintained an average of 39.3%. HSS and SSSS achieved average efficiencies of 35.3% and 25%, respectively. These outcomes reflect the ability of each design to harness and utilise incident solar radiation for water vaporisation. The superior performance of CoSS can be explained by its compact geometry, which minimises surface heat losses (in line with the Stefan-Boltzmann law of radiative loss) and concentrates energy input

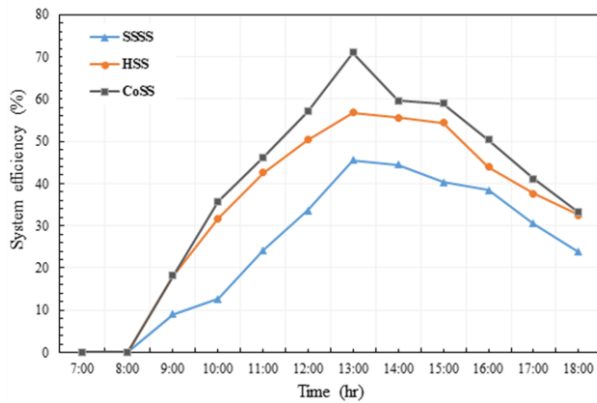


Fig. 8. Hourly thermal (system) efficiency of the solar stills.

on a reduced volume of water, leading to more effective energy conversion.

Figure 9 demonstrates the hourly exergy efficiency, which evaluates the portion of useful work extractable from the system. Exergy destruction occurs due to irreversibilities such as non-isothermal heat transfer, vapour condensation at non-equilibrium conditions and thermal resistance at interfaces. CoSS demonstrated the highest exergy efficiency at 2.9%, followed by HSS (2.1%) and SSSS (1.1%). The higher exergy efficiency of CoSS reflects its ability to minimise entropy generation, consistent with the second law of thermodynamics. This is further supported by Petela's model, which accounts for the degradability of solar energy in terms of the temperature difference between the absorber and the sun.

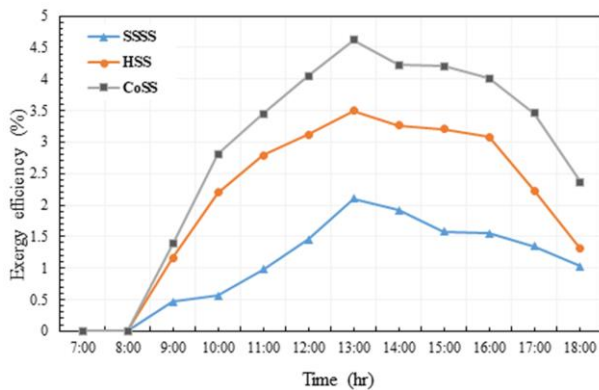


Fig. 9. Hourly exergy efficiency of the solar stills.

4.4. Comparative performance analysis

A comparative summary of the experimental results alongside selected studies from the literature is presented in Table 1. The performance indicators considered include daily freshwater productivity, system (thermal) efficiency, and exergy efficiency. This comparison highlights the effectiveness of the tested geometries, single-slope solar still, hemispherical solar still and conical solar still, in relation to various conventional and modified solar still configurations.

Table 1. Comparison of different SS systems.

System	Productivity (l/m ² /day)	System efficiency (%)	Exergy efficiency (%)	Ref.
SSSS	3.300	25.00	1.10	Present study
HSS	5.100	35.30	2.10	Present study
CoSS	5.800	39.30	2.90	Present study
SS	0.690	49.29	9.48	[38]
SS	1.250	30.07	3.67	[37]
SS-PCM	1.485	30.96	3.48	[39]
SS	3.320	34.80	–	[40]
SS-PCM	3.270	48.22	3.08	[36]
SS-CM	2.440	66.87	4.08	[36]
TSS	3.450	51.00	–	[41]
NPCMSS	6.850	–	–	[42]
SS	1.240	35.00	–	[27]

The productivity achieved for the studied systems (5.8 l/m²/day for CoSS, 5.1 l/m²/day for HSS and 3.3 l/m²/day for SSSS) not only validate the superior performance of CoSS under real outdoor conditions but also demonstrate a substantial improvement over several other reported stills, including traditional basin-type designs and even modified systems employing phase change materials (PCM) or composite materials (CM). For example, while SS-PCM reported by Kabeel et al. [36] achieved a productivity of 3.27 l/m²/day, close to that of SSSS, CoSS exceeded it by nearly 77%, without relying on advanced or costly material enhancements.

From a system efficiency perspective, CoSS reached 39.3%, which, although slightly lower than the highest reported value of 66.87% for SS-CM, is notable considering CoSS used a passive, geometry-based enhancement rather than material or energy input augmentation. HSS also demonstrated strong performance at 35.3%, outperforming several conventional and PCM-based designs. SSSS, while lower in performance (25%), still compares favourably to several earlier configurations with simpler setups [37].

In terms of exergy efficiency, CoSS exhibited a notable performance of 2.9%. This value is within the range of several enhanced configurations reported in the literature, including systems employing phase change materials and composite modifications. For example, SS-PCM and SS-CM systems have demonstrated exergy efficiencies in the range of 3.08% to 4.08%, suggesting that geometric optimisation, as applied in CoSS, can yield comparable thermodynamic benefits without requiring additional thermal storage enhancements or complex integrations. Furthermore, certain advanced designs, such as the nano-PCM modified solar still (NPCMSS) configuration, have reported high levels of productivity, reaching up to 6.85 l/m²/day. While this underscores the potential of material-based augmentation, the absence of complementary thermal

and exergy efficiency metrics presents challenges in conducting a holistic performance comparison. It is also important to consider that system enhancements through geometric redesign, as exemplified by CoSS, may offer practical advantages in terms of construction simplicity and scalability, particularly in resource-constrained settings.

Overall, the comparative results demonstrate that CoSS offers an optimal balance of high productivity, efficiency, and practical design simplicity. It competes strongly with more complex or hybrid systems while maintaining the inherent advantages of passive solar distillation: low cost, easy maintenance and sustainability. These findings reinforce the potential of geometry-based optimisation as a viable and scalable solution for decentralised water production in water-scarce regions.

5. Economic evaluation

An economic evaluation was conducted to assess the cost-effectiveness and financial feasibility of the three solar still configurations. The analysis considered daily freshwater productivity, fabrication and material costs and the payback period for each design.

The experimental data, shown in Table 2, indicate that CoSS achieved the highest daily yield, producing 5800 g/m²/day, followed by HSS with 5100 g/m²/day and SSSS with 3300 g/m²/day. When compared to SSSS, these values correspond to productivity improvements of approximately 75.75% for CoSS and 54.54% for HSS. These results highlight the influence of geometric design on distillation efficiency and the potential for significant gains in water output through relatively simple structural modifications.

The total fabrication costs, which include expenses for construction materials, thermal insulation and black paint coating, were calculated to be 57.78 USD for SSSS, 65.36 USD for HSS and 65.01 USD for CoSS. Based on a market price of 0.60 USD per litre for distilled water, the economic return of each configuration was estimated. CoSS demonstrated the shortest payback period of 18 days, reflecting its higher daily output and reasonable cost. The HSS followed with a payback period of 21 days, while the SSSS required 29 days to recover its initial investment. The detailed cost breakdown of the experimental setup is delineated in Table 3.

Overall, the economic assessment confirms that CoSS offers the most favourable performance-to-cost ratio among the three configurations. While SSSS is more economical to construct, its lower productivity leads to a slower return on investment. In contrast, both CoSS and HSS provide a more balanced trade-off between capital cost and performance.

Table 2. Comparison of the daily productivity obtained from different solar stills.

Parameter	Solar stills		
	SSSS	HSS	CoSS
Daily yield (g/m ²)	3300	5100	5800
Improvement rate (%)	–	54.54	75.75

Table 3. Cost of fabrication of solar stills.

Cost breakdown	SSSS	HSS	CoSS
Fabrication cost of the solar still (USD)	43.73	49.20	49.20
Thermo-insulation cost (USD)	10.54	10.54	10.54
Black matt paint (USD)	3.51	3.51	3.51
Total (Investment) (USD)	57.78	65.36	65.01
Productivity (l/m ² /day)	3.30	5.10	5.80
The cost per litre of distilled water on the market (USD)	0.60	0.60	0.60
The price of daily water production (USD)	1.98	3.06	3.48
Payback period (day)	29.00	21.00	18.00

6. Conclusion and future recommendations

This study experimentally evaluated the performance of three solar still configurations: single slope (SSSS), hemispherical (HSS) and conical (CoSS), under identical environmental conditions. Based on thermal, exergy, and economic assessments, the following key findings were observed:

- CoSS demonstrated the highest daily freshwater productivity at 5.8 l/m²/day, followed by HSS (5.1 l/m²/day) and SSSS (3.3 l/m²/day).
- The system (thermal) efficiency was the highest for CoSS (39.3%), compared to HSS (35.3%) and SSSS (25%).
- The exergy efficiency, reflecting the ability to minimise thermodynamic irreversibilities, was also the highest for CoSS (2.9%), followed by HSS (2.1%) and SSSS (1.1%).
- The payback period analysis revealed that CoSS had the shortest return period (18 days), making it the most economically favourable design, compared to HSS (21 days) and SSSS (29 days).
- The results confirm that geometry-based design optimisation can significantly enhance both energy and exergy performance without requiring auxiliary power sources or complex material modifications.

These findings highlight the practical and thermodynamic advantages of CoSS and support its suitability for low-cost, decentralised water production in arid and semi-arid regions.

To build upon these findings and advance the practical application of solar stills, the following future directions are recommended:

- Investigating the integration of passive thermal energy storage materials to extend productivity into off-sunlight hours.
- Conducting long-term performance testing under varying seasonal and climatic conditions.
- Future studies should consider metrics like cost per litre and material durability to support long-term viability assessments.

- Exploring automated features for improved maintenance and water collection efficiency.
- Utilising computational optimisation tools to refine design parameters for enhanced performance.
- Performing comprehensive environmental and life-cycle assessments to evaluate scalability and sustainability.

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Influence of partial shading on performance of a V-trough CPV system with various truncation levels

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Abstract

This study experimentally investigates the performance of a fixed concentrated photovoltaics system consisting of a photovoltaics module coupled with V-trough concentrators under partial shading conditions. V-troughs had the same trough angles, but different truncation levels (reflector lengths). Their geometric concentration ratios were 2.35 and 1.80. Research covers a wide range of operating parameters – generated power, electrical efficiency, module's average temperature, and characteristic curves – offering a unique multidimensional experimental comparison of concentrated photovoltaics performance with V-troughs of different truncation. The objective is to determine a relationship between the V-trough geometry, working conditions, and generated power. Findings reveal that the maximum power point and short-circuit current are strong functions of the truncation level and angle of incidence. For angles close to 0° , these parameters reach greater values at lower truncation. At higher angles, this trend reverses. While concentrator-integrated systems have a higher peak power than the bare module (by 23% and 18% for the longer and shorter concentrators, respectively), they experience greater temporal fluctuations in operational parameters due to shading. Their daily average power production was comparable to that of the bare module as shading cast by V-trough reflectors led to a loss in the instantaneous generated power of up to 45% compared to the system with no concentrator. Thus, among the tested concentrators, the shorter one is more cost-effective due to reduced material consumption. Additionally, the maximum power of concentrator-integrated systems was produced when the module temperature reached 70°C , rather than when the solar irradiance was maximum.

Keywords: Concentrating photovoltaics; V-trough concentrator; Truncation; Experimental studies

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1. Introduction

The capacity of solar photovoltaics (PVs) is increasing worldwide, which gives impetus to the development of methods to generate more electric power per unit area of a photovoltaic (PV) module. These methods would allow reducing the number of modules required to achieve the desired power generation, and thus lowering the cost of a PV system installation.

One of the approaches to maximise the generated power involves using concentrated photovoltaics (CPV), where mirrors

or lenses concentrate solar energy onto the surface of the module, thus increasing the solar irradiance that enters the module. Various CPV configurations are available. One of them is a V-trough consisting of two flat mirrors placed at a predetermined angle relative to the module's surface. It is an example of non-imaging optics, which offers low concentration of solar energy. Compared to high concentration optics, in V-trough based systems, using sun-tracking can be avoided [1], which reduces the cost of the installation. Moreover, their simple geometry translates into a lower production cost and allows more uniform

Nomenclature

a – receiver area, m^2
 A – aperture area, m^2
 A_{in} – area through which solar rays enter the system, m^2
 C – geometric concentration ratio
 G – solar irradiance on a tilted surface, $\text{W}\cdot\text{m}^{-2}$
 H – height of the concentrator, m
 H_{init} – height of the concentrator without truncation, m
 I_{mp} – maximum power current, A
 I_{SC} – short-circuit current, A
 L – aperture length, m
 L_{PV} – length of the PV module, m
 P_{in} – incoming solar power, W
 P_{PV} – power produced by the PV module, W
 $S_{U,CF}$ – standard deviation in voltage calculated using calibration function, V
 $S_{U,DMM}$ – standard deviation in voltage measured with a multimeter, V
 t_{ave} – average PV module temperature, $^{\circ}\text{C}$
 t_{bottom} – temperature of the PV module bottom part, $^{\circ}\text{C}$
 t_{mid} – temperature of the PV module middle part, $^{\circ}\text{C}$
 t_{top} – temperature of the PV module top part, $^{\circ}\text{C}$
 u – uncertainty
 U_{mp} – maximum power voltage, V
 U_{oc} – open-circuit voltage, V
 w – receiver width, m
 W – aperture width, m

W_{PV} – width of the PV module, m
 Z – length of the reflectors, m

Greek symbols

β_{opt} – optimum tilt angle, deg
 δ – acceptance angle, deg
 δ_{max} – maximum acceptance angle, deg
 Δ – decrease from maximum to minimum relative to maximum, %
 η_e – electrical efficiency of the system
 θ – trough angle, deg
 φ – latitude, deg
 Ψ – vertex angle, deg

Subscripts and Superscripts

ave – daily average value
 $instant$ – instantaneous value
 max – maximum value

Abbreviations and Acronyms

AoI – angle of incidence, deg
CPC – compound parabolic concentrator
CPV – concentrated photovoltaics
GTI – global tilted irradiance, $\text{W}\cdot\text{m}^{-2}$
MPP – maximum power point
MPPT – maximum power point tracking
PV – photovoltaic
PVs – solar photovoltaics
TL – truncation level

illumination of the PV module compared to the other popular CPV geometry – compound parabolic concentrator (CPC) [2]. However, when a V-trough with no sun-tracking is employed, uniform illumination cannot be assured throughout every day of its operation due to the change of the elevation angle of the sun [1]. The non-uniformity has an adverse effect on the PV module's open-circuit voltage, efficiency [3,4] and operational lifetime [5].

Ustaoglu et al. [4] numerically investigated the performance of a PV module coupled with chosen non-imaging concentrators, such as V-troughs and CPCs, and a cooling system. They demonstrated that with a V-trough concentrator, the heat flux in the central part of a photovoltaic module can be approximately eight times higher than at the edges, depending on the angle of incidence (AoI) of the sun rays. In a more recent paper, Ustaoglu et al. [6] performed another numerical analysis of the performance of a cooled PV module coupled with truncated V-troughs, CPCs, and compound hyperbolic concentrators. They showed that for the case with no truncation, the heat flux reaching the surface of a PV module with a V-trough varied from nearly 0 W/m^2 close to the edges to approximately 2000–4000 W/m^2 near the central part of the module, depending on the AoI. For angles of incidence greater than 0° , the PV module experienced a step-wise change in heat flux of approximately 1000 W/m^2 along its length. The authors did not provide the length-wise heat flux distribution for the truncated concentrators. However, they reported that for AoI of 0° , introducing a 55% truncation level reduced the maximum temperature of the PV module from 335 K to 330 K, and that for AoI $< 30^{\circ}$ higher

truncation resulted in reduced power generation. Ustaoglu et al. [6] concluded that among all the analysed geometries V-troughs demonstrated the most beneficial performance.

In outdoor experiments, Singh et al. [5] measured illumination along the width of the V-trough's receiver using fourteen photodiodes. They reported that, depending on the angle of incidence, the relative intensity of illumination can vary between approximately 5 and 55 across the receiver. They used a V-trough with its focal line oriented in the east-west direction.

Hadavinia and Singh [7] simulated the performance of a PV module coupled with a V-trough having various trough angles. They analysed trough angles ranging from 0 to 45° and AoI varying from 0 to 45° . Their results indicate that for a 50-cm-high V-trough, the highest optical concentration ratio averaged over AoIs corresponding to the highest solar irradiance throughout the day was achieved for the trough angle of 22° .

In his PhD thesis, Hadavinia [8] presented more detailed results of his studies on a PV module with V-troughs having trough angles of 15° and 22° . His simulations showed that apart from AoI = 0° , where the energy flux distribution on the PV module surface was uniform, most of the other AoIs resulted in two areas on the module's surface, where the energy flux concentration differed by approximately 20 units. In his experimental studies using a V-trough with a trough angle of 19° , he observed that the current-voltage curves differed from the typical ones. Instead of the current plummeting after a certain voltage was exceeded, it decreased gradually over the full range of voltages. Hadavinia [8] attributed this behaviour to the partial shading of the PV module.

More papers address the influence of partial shading on a bare PV module. Under such conditions, the PV module's power-voltage characteristic curve can have multiple local maximum power points (MPPs) [9]. This can negatively affect a PV system coupled with an MPPT solar charge controller, as the controller cannot differentiate between local and global MPPs [9]. Zhang et al. [9] showed both analytically and experimentally that introducing different irradiance levels to one or two PV cell strings out of a three-string PV module with bypass diodes results in one or two additional MPPs in the characteristic curve, respectively. The lower the irradiance reaching the strings, the lower the power associated with those additional MPPs. Depending on the shading scenario, the minimum local MPP could be approximately half of the maximum. Similar results were obtained by Bharadwaj and John [10], who reported that introducing three areas of different irradiance on the surface of a PV module with bypass diodes resulted in three different voltages in the I-V curve for which the current started to drop significantly. In other words, the current decreased in a step-wise manner, remaining constant within a given voltage range and dropping with each subsequent interval of higher voltage. In their experiment, the maximum current within the three intervals was 7 A, while the minimum was approximately 4.7 times lower. Brecl et al. [11] simulated the performance of a PV module with bypass diodes under partial shading conditions caused by chimneys or a pole. They reported that for the shortest day of the year in Ljubljana, Slovenia, shading could result in a power loss of 26–78%, and that this loss is influenced by the orientation of the PV module (landscape or portrait).

Despite many works on partial shading of PV modules, little attention has been given to experimental investigations of PV modules coupled with V-troughs of various truncation levels. Discussed papers either examine single V-trough geometry or rely on numerical simulations, leaving systematic experimental data on the effect of truncation on PV module performance largely absent. This paper addresses this gap by experimentally assessing how the performance of a fixed PV module with V-trough concentrators of different reflector lengths (truncation levels) is affected by shading from the reflectors. The main goal is to determine the relationship between the V-trough geometry, operational conditions, and power generation. Lower truncation levels increase geometric concentration ratios and shading. The former leads to higher module temperatures and reduced electrical efficiency. Thus, to assure a comprehensive analysis, a multitude of operational parameters are studied: generated power, solar irradiance, the PV module electrical efficiency and average temperature, as well as the module current-voltage and power-voltage characteristic curves. By studying such a broad set of parameters, our paper provides a multidimensional experimental comparison of a PV module performance while coupled with V-troughs of various truncation levels, offering insights unavailable in prior works.

2. Experimental setup

The experimental setup is shown in Fig. 1. Its main component was a commercially available PV module (brand: Maxx, model: 10 W) that was connected to a maximum power point tracking

(MPPT) solar charge controller (Lumiax MT1050EU). An ammeter (EXTECH EX470A multimeter) was used to measure the current from the PV module, while the MPPT controller monitored the produced voltage. Generated electricity powered a 10 W LED lamp, and any excess power was stored in three parallel-connected batteries (each having the capacity of 7.2 Ah). Three T-type thermocouples, placed at the top, bottom, and in the middle of the module's back surface, measured the module temperature. These thermocouples, together with a pyranometer (Kipp&Zonen SP Lite) for measuring solar irradiance, were connected to a data recorder (LUMEL KD7).

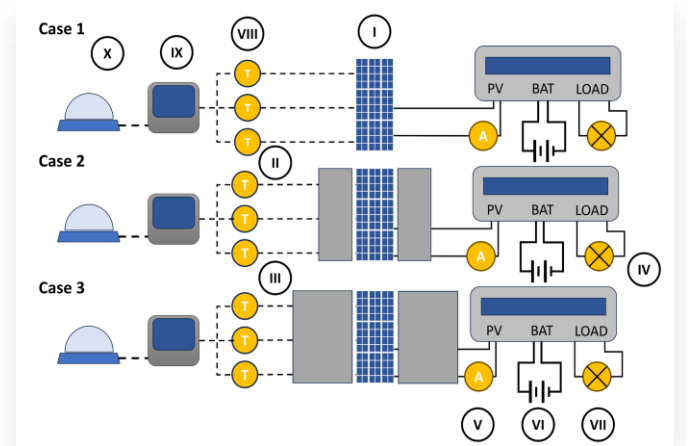


Fig. 1. Schematic of the experimental setup used in the study. I – PV module, II – PV module with the shorter concentrator, III – PV module with the longer concentrator, IV – MPPT solar charge controller, V – ammeter, VI – batteries, VII – LED lamp, VIII – thermocouples used to measure PV module temperature, IX – data recorder, X – pyranometer.

Three cases were studied: the PV module without a concentrator and the same PV module coupled with shorter and longer V-trough concentrators differing in truncation level (TL). These concentrators had the same trough angles but different lengths of their reflectors (Z). Detailed specifications of the PV module used are provided in Table 1.

Table 1. Specification of the PV module used in the experiments.

Power, W	U_{oc} , V	I_{sc} , A	U_{mp} , V	I_{mp} , A	Dimensions $L_{pv} \times W_{pv}$, m × m
10	22.64	0.58	18	0.54	0.43 × 0.19

2.1. V-trough concentrators

In the experiment, two types of non-imaging V-trough concentrators were used, the geometry of which is schematically presented in Fig. 2. Each comprises two flat reflectors positioned at a trough angle (θ) of 10° measured from the normal to the receiver plane. Constructed from stainless steel 304 with a BA finish, the concentrators vary in truncation level, defined as:

$$TL = \frac{H_{init} - H}{H_{init}}, \quad (1)$$

where H represents the height of the truncated V-trough, and H_{init} denotes the height of the initial V-trough without trunca-

tion. For the concentrator with no truncation, the reflectors were 735 mm long, and H_{init} was 724 mm. For the truncated concentrator, the reflectors were 399 mm long, resulting in a height H of 393 mm and a truncation level of 46%. Varying heights of V-troughs led to differences in another crucial parameter characterising V-trough geometry – the geometric concentration ratio (C) expressed as:

$$C = \frac{A}{a} = \frac{W}{w}. \quad (2)$$

The values of C together with the geometric dimensions of both concentrators are detailed in Table 2.

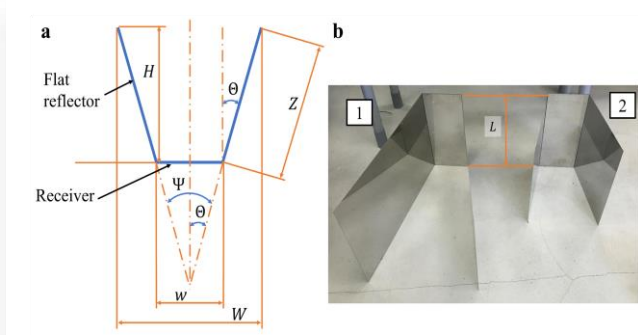


Fig. 2. V-trough concentrators used in the study. a - geometry, b - a photo showing both V-troughs: 1 – the longer one, 2 - the shorter one.

Table 2. Geometric dimensions of V-trough concentrators used in the study.

Concentrator	Truncation level (TL), %	Aperture length L , mm	Receiver width w , mm	Aperture width W , mm	Reflectors length Z , mm	Geometric concentration ratio C , -
Longer	0	450 ± 1	200 ± 1	470 ± 1	735 ± 1	2.35
Shorter	46	450 ± 1	200 ± 1	360 ± 1	399 ± 1	1.80

3. Experimental procedures

The experimental campaign took place outdoors on the campus of Wrocław University of Science and Technology, Wrocław, Poland ($51^{\circ}6'35.72''N$ and $17^{\circ}3'28.13''E$) in May and June 2023. Each geometry (the bare module and the module with the longer/shorter concentrator) was investigated individually on separate days from 10:00 to 14:00 (UTC+02:00). Throughout the experiments, the PV system was facing south with a slight tilt towards the east, resulting in an azimuth angle of approximately 30 degrees. The focal line of the V-trough was vertically oriented, and the experiments were carried out under clear sky conditions. During the experiments, voltage and current produced by the PV module, along with solar irradiance, were recorded at 15-minute intervals.

Performance of a PV module is significantly influenced by its tilt angle, the optimum value of which depends on, inter alia, the latitude of an installation site and cloud cover. Consequently, to determine the optimum tilt angle for a PV module installed in

Wrocław, with a latitude of approximately 51 degrees north, we employed the formula developed by Jacobson and Jadhav [12]. This formula incorporates meteorological data from locations investigated by the authors, and for the northern hemisphere is given by:

$$\beta_{opt} = 1.3793 + \varphi[1.2011 + \varphi(-0.014404 + 0.000080509\varphi)]. \quad (3)$$

For Wrocław, Eq. (3) yields the optimum tilt angle equal to approximately 35° , and this value was used in our experiments.

In our experiments, we used vertically oriented V-troughs of fixed tilt and azimuth angles (see Fig. 3) so that we could study the influence of partial shading on PV module performance during experiments of relatively short duration. This allowed us to assess the power loss introduced by the shadow cast by the V-trough reflectors when no two-axis sun-tracking is employed.

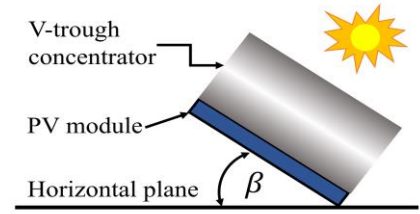


Fig. 3. Tilt angle used in the experimental campaign.

4. Data reduction and uncertainties

4.1. Electric performance of the system

As the experimental setup consisted of a PV module coupled with an MPPT solar charge controller, it was assumed that the module always worked at the maximum power point for given irradiance conditions. The electrical efficiency of the system in all studied cases was calculated as:

$$\eta_e = \frac{P_{PV}}{P_{in}}, \quad (4)$$

where:

$$P_{PV} = U_{mp} I_{mp}, \quad (5)$$

$$P_{in} = A_{in} G.$$

A_{in} is the area through which solar rays enter the system. For the case with no concentrator, it equals the surface area of the PV module ($A_n = L_{PV} \times W_{PV}$), whereas for the cases with concentrators, it equals their aperture area ($A_{in} = L \times W$).

4.2. Solar irradiance on a tilted surface

The pyranometer used to measure solar irradiance incident on the system was placed at the same angle as the PV module. Consequently, it measured global tilted irradiance (GTI), which is the sum of direct, diffuse and reflected radiation reaching the PV module or the aperture of the concentrators, depending on the case studied. Therefore, in this work, G in Eq. (5) equals GTI.

4.3. Uncertainties

Uncertainties of direct measurements (current, solar irradiance and module's back surface temperatures) were determined based on the accuracy of the measuring equipment. Voltage was measured using a built-in function of the MPPT controller (Lumiax MT1050EU). We calibrated the controller with a multimeter (UNI-T UT89XD) of known accuracy and applied a regression analysis to determine the calibration function. This function takes the following form: $y = bx + a$, where x is the actual value (taken from the multimeter), and y is the value measured with the controller. We calculated uncertainty in U_{mp} as a sum of standard deviation in the actual voltage determined using the calibration function $s_{U,CF}$ and standard deviation in the measurement taken with the multimeter $s_{U,DMM}$:

$$u(U_{mp}) = \sqrt{s_{U,CF}^2 + \frac{s_{U,DMM}^2}{3}}. \quad (6)$$

Uncertainties of the calculated parameters (incoming solar power, electrical efficiency and the average module's back surface temperature) were determined using the error propagation method. Specific values of measured and calculated parameters together with their uncertainties are listed in Table 3.

Table 3. Uncertainties of measured and calculated parameters.

Parameter	Measured/calculated range	Uncertainty
U_{mp}	14.72 – 19.66 V	0.10 – 0.13 V
I_{mp}	0.225 – 0.849 A	0.011 – 0.026 A
t_{top}	28.70 – 74.30 °C	0.5 °C
t_{mid}	28.80 – 88.40 °C	0.5 °C
t_{bottom}	28.70 – 88.30 °C	0.5 °C
t_{ave}	28.77 – 82.53 °C	0.3 °C
G	838 – 1089 W·m ⁻²	2.4%
P_{PV}	3.96 – 12.68 W	0.19 – 0.40 W
P_{in}	52.22 – 224.61 W	1.31 – 5.44 W
η_e	0.023 – 0.167	0.001 – 0.007

5. Results and discussion

In order to compare the performance of the PV module working with and without concentrators, it was necessary to verify that all the systems were investigated under similar irradiance conditions. Figure 4 presents temporal variations of solar irradiance for the PV module with and without concentrators. In all studied cases, changes in solar irradiance followed a similar pattern: first, it increased to reach its peak value at between 11:30 and 12:15 depending on the day of the experiment, and then it steadily declined until the end of the experiment. This behaviour is primarily due to the absence of sun-tracking in the experimental setup, resulting in decreased irradiance as the sun's position shifted over time. Generally, deviations from this pattern were minimal across all cases, except for an anomaly observed at 11:30 for the system with the shorter concentrator, attributed to transient cloud cover. However, this deviation did not substantially impact the average solar irradiance reaching the system,

which was 984.53 W·m⁻² for the PV module without a concentrator, 997.88 W·m⁻² for the system with the shorter concentrator, and 990.35 W·m⁻² for the longer concentrator. Thus, the overall irradiance levels in all the cases were comparable.

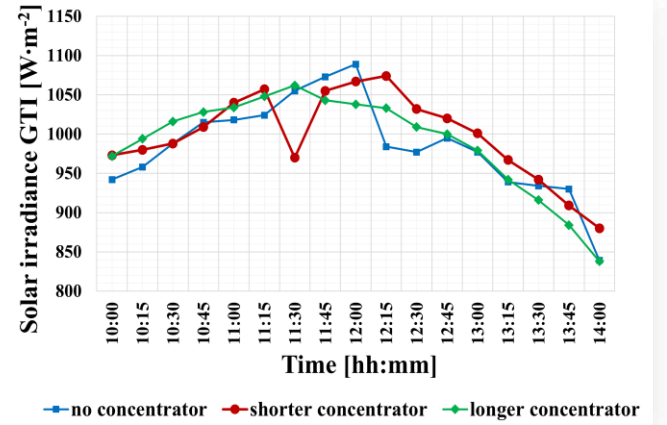


Fig. 4. Solar irradiance as a function of local time for all studied cases.

5.1. Power generation

Figure 5 shows the power output for each analysed system over time. Concerning instantaneous power generation, the system equipped with the longer concentrator demonstrated the highest performance, reaching a maximum power of 12.68 W. The system with the shorter concentrator achieved only slightly worse maximum power output of 12.19 W. In contrast, the system without a concentrator yielded a maximum power of 10.32 W. These results are a consequence of the amount of solar power reaching the PV module in each system, which was the highest when the longer concentrator with the highest geometric concentration ratio was used, and the lowest when there was no concentrator.

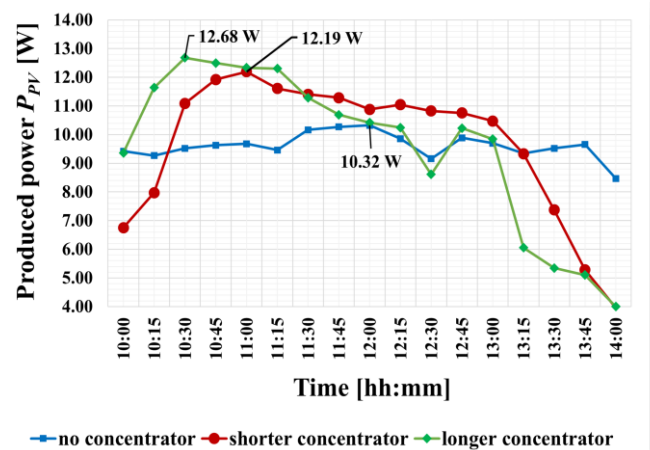


Fig. 5. Power produced by each studied system as a function of local time. Labels show the maximum values for each system.

When considering the time-averaged power generation, the system without a concentrator attained an average power output of 9.61 W with a standard deviation of 0.45 W. In comparison, the system with the shorter concentrator produced on average

9.66 W with a standard deviation of 2.47 W, while for the longer concentrator system, the average was 9.57 W with a standard deviation of 2.80 W. Thus, all the studied systems produced similar average power, however the fluctuations in generated power over time were greater for the systems with concentrators, as evidenced by their higher standard deviations. These fluctuations can be attributed to shading effects caused by the reflectors of the concentrators. As the sun's position shifted during the experiment, changes in the angle of incidence of the incoming radiation led to partial shading of the PV module. At 14:00, this translated into about a 45% loss in generated power compared to the bare module.

The data presented in Fig. 5 show that the optimum angle of incidence occurred at around 11:00 for the system with the shorter concentrator and approximately at 10:30 for the system with the longer concentrator. The more the time of a given measurement was away from the time corresponding to the maximum produced power, the greater the drop in power generation was observed. Both systems equipped with concentrators exhibited a maximum percentage drop in instantaneous power production of 68% compared to their maximum generated power, while for the PV module without a concentrator, this drop was only 18%. Such substantial losses in instantaneous power in the case of the systems with V-trough concentrators highlight that their performance is strongly affected by the angle of incidence of the incoming radiation, and the resulting partial-shading and illumination non-uniformity. Similar behaviour of the power generated by a bare PV module and a system with a V-trough concentrator was reported by Hadavinia and Singh [7]. In their experiments (conducted between 10 a.m. and 2 p.m.), the standard deviation of the power produced by the system with the concentrator was eight times higher than for the bare module, whereas in our study for the system with the shorter concentrator, it was 5.5 times higher than for the bare PV module. Furthermore, results of simulations performed by Parapudi et al. [13] show that in the summer months of June and July in London, UK, bare monocrystalline silicone PV modules and those coupled with a V-trough having a geometric concentration ratio of 1.40 and a trough angle $\theta = 20^\circ$ yielded nearly the same monthly electrical energy production.

5.2. Optimum operating point

Comparing the data in Figs. 4 and 5 also shows that for the systems with concentrators, the time of peak power generation did not coincide with the time of the maximum solar irradiance. Instead, the peak power output occurred approximately an hour earlier than the peak solar irradiance was observed. This indicates that solar irradiance is not the only factor affecting generated power. One such parameter of interest is the temperature of a photovoltaic module. The temporal evolution of the module's temperature, together with changes in solar irradiance and generated power for all the examined cases, is illustrated in Fig. 6.

The data shown in the figure indicate that for the systems with concentrators, the maximum power was generated when the PV module's temperature was approximately 70°C (Fig. 6b and Fig. 6c). This points to the existence of an optimum operating point for the CPV system in terms of the amount of incoming

solar energy. It seems that this point should not be chosen solely based on the maximum value of solar irradiance, but also based on the PV module's temperature. As this temperature is a strong function of irradiance, its high value is an indicator of a large amount of solar energy reaching the PV module. However, its value should not exceed a certain limit (in our case 70°C), because then the performance loss due to the temperature rise starts to be visible.

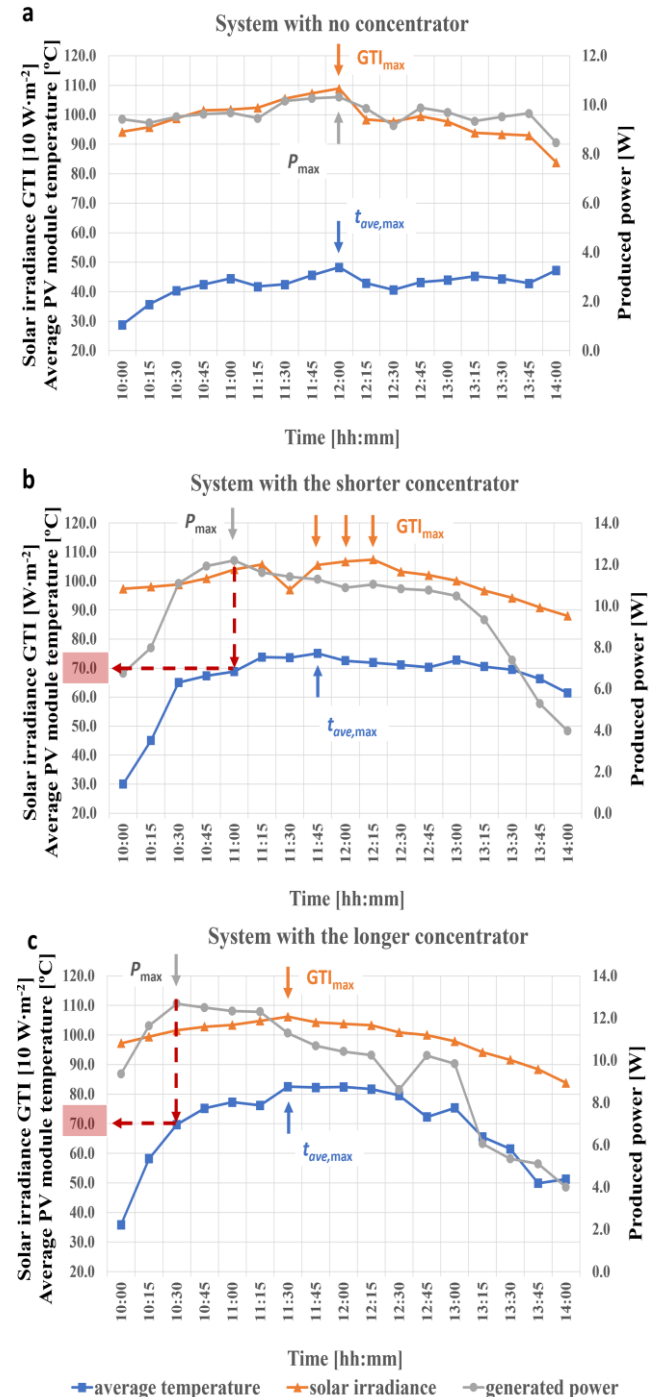


Fig. 6. PV module temperature, solar irradiance and generated power for each studied system as a function of local time.

Bahaidarah et al. [14] reported a similar 30-minute shift between the time of maximum irradiance and the time of maximum power generated by a PV module with a V-trough. In their

simulations for a chosen day in September, the peak power production of the V-trough system occurred around 11:30 a.m., when the module temperature was approximately 61°C (about 2°C lower than the maximum temperature of the PV panel during that day), while the maximum irradiance occurred at 12 p.m. As in their work, the optimum temperature in terms of power production was 61°C, and in our study, it is 70°C, additional experiments need to be conducted to identify parameters which determine the optimum operating temperature. These parameters could be, e.g. V-trough geometry and material, PV module's model and type, day of the year, location, or weather conditions.

Interestingly, no time shift between the peak generated power and the peak solar irradiance was observed in the case of the system without a concentrator (Fig. 6a). This might be due to the fact that when no concentrator was used, the PV module's temperature did not exceed the limit of 70°C, so it did not have a noticeable negative impact on the module performance.

5.3. Temperature of the photovoltaic module

Figure 7 shows a comparison of PV module temperature for all the studied systems. As expected, the highest panel temperature was observed for the system with the longer concentrator, i.e. with the highest geometric concentration ratio. The maximum recorded temperature for this system reached 82.5°C. For the system with the shorter concentrator, the maximum temperature was 75°C, and for the bare PV module, it was 48.3°C. The higher the concentration ratio, the greater solar irradiance reaches the PV module's surface, resulting in higher module temperatures. Similar values of the maximum panel temperature were reported by Parapudi et al. [13]. In their simulations for a chosen day in June, the temperature of a bare PV module was ca. 47°C, and for a panel coupled with a V-trough ($C = 1.40$), the temperature was about 79°C.

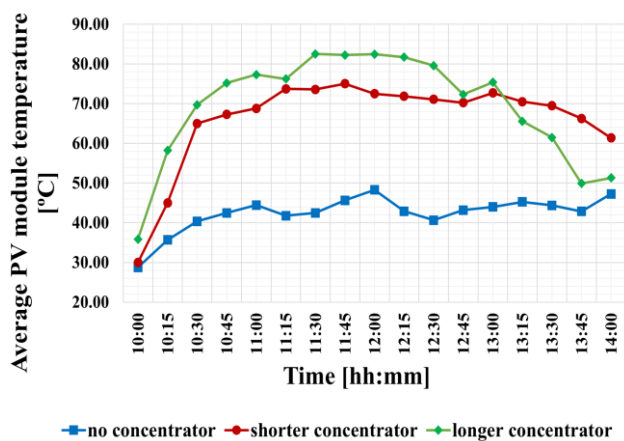


Fig. 7. Average temperature of the PV module as a function of local time for each studied system.

The most significant change in temperature over time occurred for the system with the longer concentrator. After 13:00, its temperature dropped below that of the system with the shorter concentrator. This can be attributed to the shading introduced by the reflectors of the concentrator. The longer the reflectors are, the greater the shading and its impact on the PV module temperature.

5.4. Electrical efficiency

Time evolution of electrical efficiency for all the studied systems is presented in Fig. 8. The highest efficiency on average of 15.70% was achieved by the system with no concentrator. The systems with the shorter and longer concentrators demonstrated significantly lower average efficiencies of 5.92% and 4.51%, respectively. Employing concentrators increased solar power entering the system, however, it did not translate onto a proportional rise in generated power, which is reflected in lower efficiencies.

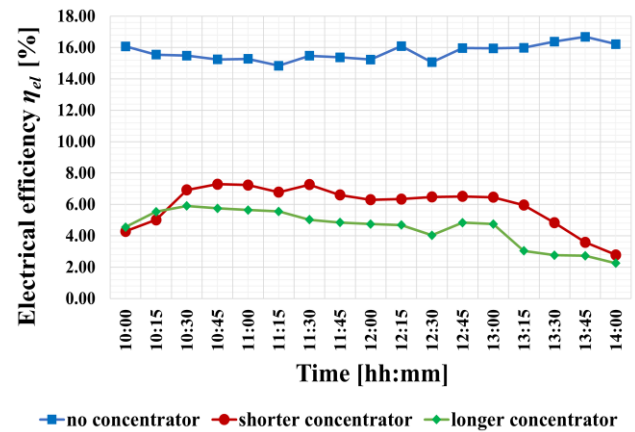
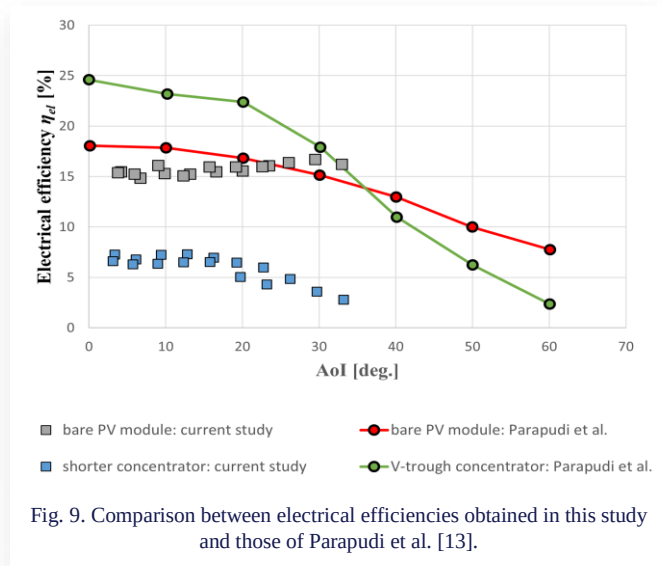


Fig. 8. Electrical efficiency as a function of local time for each studied system.

For example, in the system incorporating the shorter concentrator, the incoming power was 1.8 times higher compared to the system without a concentrator, as determined by the geometric concentration ratio. However, the maximum generated power only increased by a factor of 1.18. Similarly, in the system with the longer concentrator, the incoming power was 2.35 times higher, yet the maximum generated power increased by a factor of only 1.23. The differences between the relative increases in the incoming and generated power can be attributed to the optical losses of the CPV system.

Another issue is that not only electrical efficiencies of the systems with concentrators are at least 50% lower than for the PV module with no concentrator, but they also display greater temporal variability. Again, this can be attributed to shading – as the sun changes position, reflectors of concentrators start to cast shadow on the PV module, which reduces the performance of the module with time. These effects have been documented in the existing literature. Figure 9 shows a comparison between the electrical efficiencies of the bare PV module and the system with the shorter concentrator determined in this work and those reported by Parapudi et al. [13] as a function of the angle of incidence (AoI). AoI was calculated according to the procedure described in [13]. We chose the system with the shorter concentrator for comparison as it bears more resemblance to the V-trough studied in [13] than the system with the longer concentrator ($C = 1.80$, trough angle $\theta = 10^\circ$ and receiver width $w = 200$ mm vs. $C = 1.40$, trough angle $\theta = 20^\circ$ and receiver width $w = 33$ mm in [13]).



In the case of the bare PV module and $AoI < 33^\circ$, similar electrical efficiencies to those of Parapudi et al. [13] were obtained. We did not study $AoI > 33^\circ$ due to the limited duration of the experiment, which restricted studied solar positions. For the system with the concentrator, the dependence of electrical efficiency on AoI in our work had a similar shape to that reported in [13]. Up to $AoI \approx 20^\circ$, the electrical efficiency in our study was nearly constant, whereas in [13], it decreased slightly. When AoI was $> 20^\circ$, electrical efficiency started to fall at a higher rate in both our investigation and the paper by Parapudi et al. [13]. This behaviour was attributed to partial shading of the PV module.

The shape of $\eta_{el}(AoI)$ in our study closely follows the theoretical angular acceptance function, which can be found, for example in [15]. This function describes the dependence of the fraction of sun rays reaching the receiver of a V-trough to those reaching the aperture on the angle of incidence. For AoI below the acceptance angle δ , all incoming rays reach the receiver.

For $\delta < AoI < \delta + 2\theta$, the acceptance starts to fall gradually to the value of 0 at the maximum acceptance angle $\delta_{max} = \delta + 2\theta$ where no sun rays reach the PV module. Analysing Fig. 9 indicates that for both the V-trough used in our study and the one in Parapudi et al. [13] research, $\delta \approx 20^\circ$. This value is further confirmed by calculating the acceptance angle using the rearranged formula that can be found in [16]:

$$\delta = \text{asin}(C) - \theta. \quad (7)$$

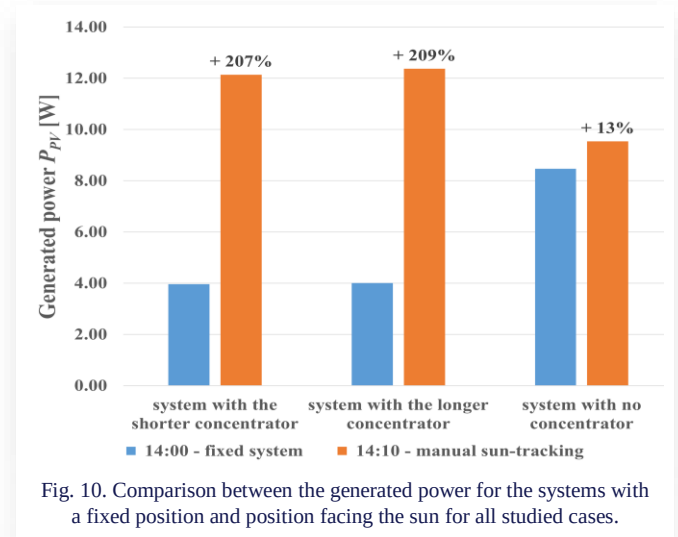
For the shorter concentrator, the resulting acceptance angle is ca. 24° , and for the concentrator in the work of Parapudi et al. [13], $\delta \approx 26^\circ$. However, for the system with the concentrator, the efficiencies obtained in our study are much lower than those reported in [13]. This is probably due to the lower reflectance of the stainless steel that our concentrator was made of, compared to the one employed in [13]. Reflectivity has a direct impact on the fraction of sun rays falling on the aperture surface that reaches the PV module. The higher the reflectivity of the concentrator surface, the more light flux is incident on the PV module. Parapudi et al. [13] used MICRO-SILVER 4200 AG, whose total reflectivity is 0.98, while Ustaoglu et al. [17] reported that

for their polished stainless steel concentrator, the measured reflectivity varied between 0.383 and 0.653 for wavelengths of light ranging from 200 to 1200 nm. Similar to Ustaoglu et al. [17], we used a stainless-steel V-trough. Its lower reflectivity could translate into lower power production and, consequently, lower electrical efficiency compared to the system studied by Parapudi et al. [13].

Taking instantaneous and average daily power production as well as electrical efficiency into account, the system equipped with the shorter concentrator is a preferable choice for non-sun-tracking configurations. It achieves comparable power generation to the system with the longer concentrator, but at a reduced cost owing to lower material consumption; the total reflectors area calculated as $2 \cdot Z \cdot L$ is 0.36 m^2 for the shorter concentrator and 0.66 m^2 for the longer one.

5.5. Manual sun-tracking

For every studied system after finishing the experiments for its fixed position, we changed its position (azimuth) manually so that it directly faced the sun. Then, we took the final measurement at about 14:10 to assess the impact of manual sun-tracking on generated power. The results presented in Fig. 10 show that the concentrator systems experienced an increase of more than 200% in instantaneous generated power when their position was adjusted. This is a strong indicator that choosing the fixed position of a CPV system with a V-trough may cause a significant loss in generated power due to partial shading of the PV module. Consequently, in terms of power maximisation, some form of sun-tracking is recommended for such systems. For example, in the systems with the focal line oriented in the east-west direction, seasonal adjustment of their tilt angle should be employed.



In the case of the system with no concentrator, and thus no partial shading, the gain in produced power was only 13%, indicating lower sensitivity to the change of the angle of incidence.

Comparisons between the performances of V-trough CPV systems with and without sun-tracking are rather scarce in the literature. The available papers mostly deal with comparative analyses of the power produced by a bare PV module with and without tracking, and the power of a CPV system with dual-axis

tracking [18,19]. In this study, the use of manual sun-tracking in the system without a concentrator resulted in a 13% power gain. Hussein et al. [19] reported a similar increase of 24% in generated power for a bare photovoltaic module with a vertical single axis tracker, tracking diurnal movement of the sun in the W-E direction, compared to a module without tracking.

Concepcion et al. [20] analysed the performance of a V-trough CPV system with and without a two-axis tracker. In their study, the energy produced by the former system was more than 20% higher than that of the latter, which is a significantly lower gain compared to our results. This might be due to the fact that the V-trough in [20] had a trough angle of 30° instead of 10° used in the current study. Reflectors placed at an angle of 30° cast less shadow than those at an angle of 10° . Consequently, limiting shading with tracking had a less profound impact on the performance of the V-trough system with a higher trough angle. However, Concepcion et al. [20] did not provide information on the V-trough height, so a conclusive comparison between the shadow cast by their and our reflectors cannot be carried out.

5.6. Current-voltage and power-voltage characteristic curves

The data for current-voltage and power-voltage characteristic curves for each system were collected in July 2024 at the same location, using the same module and concentrators with the same tilt angle as described in Sections 2. Experimental setup and 3. Experimental procedures. The tested systems faced south. In these tests, the module was connected to four sliding rheostats having a total resistance of $200\ \Omega$ instead of a solar charge controller. For each system, the experiments started at 11:30 and ended at 15:30 (UTC+02:00). Every 30 minutes, the values of

voltage and current produced by the module were collected for resistances varying from 0 to $200\ \Omega$ using two UNI-T UT890D+ multimeters. Additionally, global tilted irradiance (GTI) was measured with a Hukseflux SR05-D2A2 pyranometer. For the system with no concentrator, the average GTI was $991.44\ \text{W}\cdot\text{m}^{-2}$, for the system with the shorter concentrator, it was $970.78\ \text{W}\cdot\text{m}^{-2}$, and for the longer concentrator $1032.44\ \text{W}\cdot\text{m}^{-2}$. Thus, GTI can be considered similar for all the studied systems. The temporal evolution of solar irradiance for each system is presented in Fig. 11.

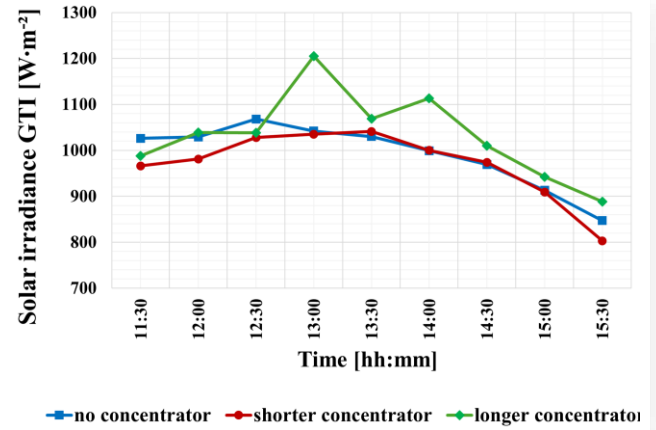


Fig. 11. Solar irradiance as a function of local time for all studied cases measured during the experiments on PV module characteristic curves.

Figures 12a and 12b depict the current-voltage and power-voltage characteristic curves for all the studied systems at the time when AoI of the sun rays was approaching 0° .

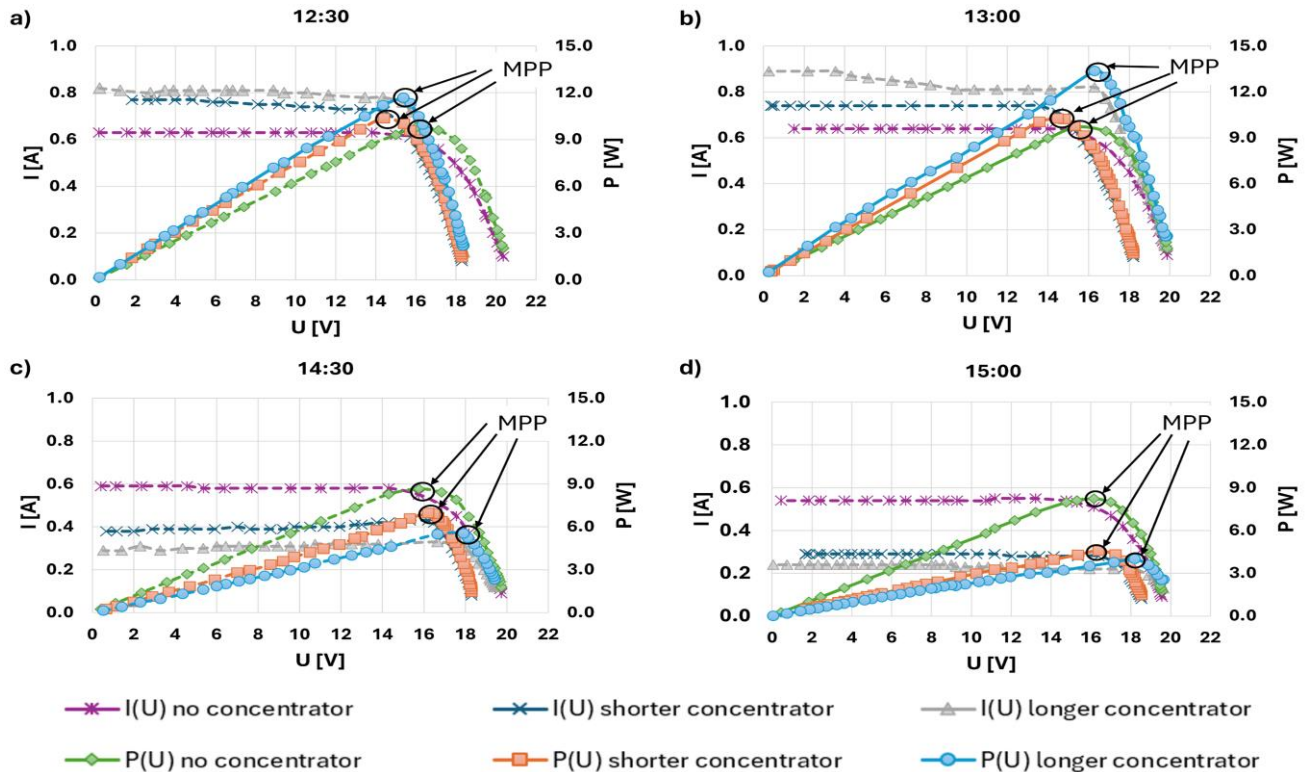


Fig. 12. Characteristic curves for all studied systems at selected times.

As expected, the longer the concentrator, the greater were the short-circuit current I_{sc} and the maximum generated power (MPP). However, as the position of the sun changed, the shadow cast by the concentrator reflectors caused performance deterioration, which can be observed in Figs. 12c and 12d for the local time of 14:30 and 15:00. At 15:00, the short-circuit current for the system with the longer concentrator was by 17% and 56% lower compared to the systems with the shorter concentrator and no concentrator. As for MPP, the system with the longer concentrator produced 13% and 51% less power than the systems with the shorter concentrator and no concentrator. This highlights the extent to which shading influences PV module performance.

Table 4 presents the comparison between I_{sc} and MPP at 13:00 and 15:00 for all the studied systems. The data indicate that the lower the TL of the system, the greater is the relative percentage drop in I_{sc} and MPP it experiences as the position of the sun changes. Thus, low truncation levels (associated in our work with higher geometric concentration ratios) are not advisable when no sun-tracking is employed.

Table 4. Comparison of I_{sc} and MPP between the studied systems at 13:00 and 15:00.

Concentrator in the system	Time: 13:00		Time: 15:00		Percentage drop relative to 13:00	
	I_{sc} , A	MPP, W	I_{sc} , A	MPP, W	in I_{sc} , A	in MPP, W
None (TL = 100%)	0.64	9.7	0.55	8.2	14.1%	15.2%
Shorter (TL = 46%)	0.74	10.3	0.29	4.6	60.8%	55.4%
Longer (TL = 0%)	0.89	13.4	0.24	4.0	73.0%	70.1%

Multiple MPPs in the power-voltage curves were not observed. This is because the PV module used in the experiment was probably not equipped with by-pass diodes. By-pass diodes would weaken the negative effect of shadow on the PV module performance. Thus, they are recommended in fixed systems with concentrators where no sun-tracking is used.

6. Summary

Table 5 presents a summary of the papers dealing with the impact of shading on V-trough PV systems, including the results of the present study. It covers PV module and V-trough characteristics, generated power, PV module temperature, electrical efficiency and characteristic curves. If information of interest was not directly given in the original paper but could be estimated from the figures included, it was denoted by approximately equals sign ($\approx$). If the analysed paper included both a simulation and an experiment, the data from the experiment were given.

Analysis of the summary indicates that most of the experimental studies focus on either smaller or bigger PV modules ($A_{PV} < 0.064 \text{ m}^2$ or $A_{PV} \leq 1.24 \text{ m}^2$) than the one used in our paper. The experiments featured V-troughs with higher trough angles $20^\circ \leq \theta \leq 30^\circ$ than in our work, but similar geometric concentration ratios varying between 2 and 2.6, which entails a lower height of the concentrator reflectors. The instantaneous power

loss in the experimental studies ranged from 40% to 98%, which further confirms that shading can substantially deteriorate the PV module performance.

It is worth noting that apart from our work and the paper of Ustaoglu et al. [6], no authors provided the data on V-troughs with various truncation levels, and Ustaoglu et al. [6] based their analysis on numerical calculations. Nonetheless, the results of both our experiment and Ustaoglu et al. [6] simulations indicate that truncating V-troughs reduces the maximum PV module temperature. All studies show that introducing concentrators generally lowers the average electrical efficiency and leads to a more pronounced instantaneous power loss compared to the systems with bare PV modules.

7. Conclusions

In this work, three different PV systems were experimentally tested in outdoor conditions in May/June 2023 and July 2024 in Poland using a commercially available PV module: without a concentrator, and with two distinct stainless-steel V-shaped concentrators of different truncation levels. The position of all systems was fixed – they all faced either south or south with a slight tilt towards east, and the focal line of the concentrators was vertically oriented. The primary objective was to determine the dependence between the V-trough geometry, working conditions and power output, taking into account the influence of shading introduced by V-trough reflectors. Here are the main findings:

- The highest instantaneous power of 12.68 W was generated by the system with the longer concentrator; the system with the shorter concentrator ranked second with 12.19 W; the bare PV module reached 10.32 W. Thus, both systems with concentrators demonstrated similar performance in terms of the maximum instantaneous power production.
- Due to shading, the systems with concentrators were subject to greater changes in generated power over time than the system with no concentrator. The instantaneous power loss reached up to 45% compared to the bare PV module. Consequently, all the systems produced a similar average power of 9.57–9.66 W.
- The system with the longer concentrator reached the highest instantaneous PV module temperature (82.5°C) and the lowest average electrical efficiency (4.51%). Due to shading, these parameters exhibited the greatest variation in time among all the studied systems. The bare PV module reached the lowest instantaneous surface temperature and the highest average electrical efficiency with their maxima of 48.3°C, and 15.70%, respectively. It also demonstrated the lowest variation of these parameters.
- The V-trough with the higher truncation level allowed limiting the adverse effect of shading. It offered similar average power output to the system with the longer concentrator at a lower price due to a reduction in material consumption. Based on that, the system with the shorter concentrator is the preferred option. However, its geometric concentration ratio cannot be equated with the optimum value yielding the maximum possible power. Determining this optimum ratio requires further studies.

- The highest instantaneous power output was observed for the bare PV module when the solar irradiance was maximum, and for the systems with V-troughs when the PV module temperature was 70°C. This indicates that the employed strategy for maximizing power output should depend on module temperature. Below its certain (limit) value, the position of the system (tilt and azimuth angles) should ensure the maximum irradiance incident on the PV module. Above the limit, the position should prevent any additional temperature rise. Further experiments are required to determine parameters that can influence this limiting temperature (V-trough design, PV module model, weather conditions, etc.).
- MPP and short-circuit current are strong functions of truncation level and AoI. For AoI close to 0°, the MPP and I_{sc} reach greater values at lower truncation levels. At higher AoIs, lower truncation leads to an increased percentage

drop in both these parameters relative to their values at AoI $\approx 0^\circ$. For the system with the longer concentrator, the drop reached more than 70% between 13:00 and 15:00. This shows the profound influence that partial shading can have on the CPV system performance.

Manual adjustment of the CPV system azimuth angle may translate onto an above 200% higher instantaneous power production. This shows that the CPV system design should reduce shading to the greatest extent possible. The results of our study show that for fixed V-trough systems, partial shading of PV modules causes substantial variation in operational parameters, and that in general, this variation is more pronounced in low truncation level systems. Designing and optimizing a fixed V-trough system requires performance assessment based on, e.g. the daily/monthly averages of operational parameters, instead of their instantaneous maxima.

Table 5. Summary of the data on V-trough systems including our research.

Paper	Study type	PV module characteristics	V-trough characteristics	Generated power	PV module temperature	Electrical efficiency	Characteristic curves
Current study	Experiment Wrocław, Poland May/June 2023 (10:00 – 14:00), July 2024 (11:30 – 15:30) $\beta = 35^\circ$ Changing parameter AoI and TL.	$A_{PV} = 0.08 \text{ m}^2$ $P_{PV} = 10 \text{ W}$ $U_{oc} = 22.64 \text{ V}$ $I_{sc} = 0.58 \text{ A}$ $U_{mp} = 18 \text{ V}$ $I_{mp} = 0.54 \text{ A}$	Stainless Steel 304 BA Finish $\theta = 10^\circ$ $H_{init} = 724 \text{ mm}$ TL = 0/46% $C = 2.35/1.80$	For bare/TL = 46%/TL = 0%: $P_{PV}^{max} = 10.33/12.19/12.68 \text{ W}$ $P_{PV}^{ave} = 9.61/9.66/9.57 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} = 18/68/68\%$	For bare/TL = 46%/TL = 0%: $t_{ave}^{max} = 48.3/75.0/82.5^\circ\text{C}$ $\Delta t_{ave}^{max} = 40/60/57\%$	For bare/TL = 46%/TL = 0%: $\eta_{el}^{ave} = 15.70/5.92/4.51\%$ $\Delta \eta_{el}^{instan} = 11/62/62\%$	For bare/TL = 46%/TL = 0%: At 13:00: $I_{sc} = 0.64/0.74/0.89 \text{ A}$ MPP = 9.7/10.3/13.4 W At 15:00: $I_{sc} = 0.55/0.29/0.24 \text{ A}$ MPP = 8.2/4.6/4.0 W $\Delta I_{sc} = 14/61/73\%$ $\Delta \text{MPP} = 15/55/70\%$
Bahaidarah, Tanweer, Gandhidasan, & Rehman, 2015 [14]	Simulation & Experiment Dhahran, Saudi Arabia June 2012 (8:00 – 16:00) $\beta = 45^\circ$ Changing parameter AoI.	$A_{PV} = 1.24 \text{ m}^2$ $P_{PV} = 230 \text{ W}$ $U_{oc} = 48.7 \text{ V}$ $I_{sc} = 5.99 \text{ A}$ $U_{mp} = 41 \text{ V}$ $I_{mp} = 5.61 \text{ A}$	Glass mirror Reflectivity 79% $\theta = 30^\circ$ $H = 789.4 \text{ mm}$ TL = 0% $C = 2.15$	$P_{PV}^{max} \approx 200 \text{ W}$ $P_{PV}^{ave} \approx 147 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} \approx 88\%$	$t_{ave}^{max} \approx 58^\circ\text{C}$ $\Delta t_{ave}^{instan} \approx 31\%$	No experimental data	No experimental data
Concepcion, Villanueva, Dalumpines, Magwili, & Pacis, 2020 [20]	Experiment Philippines May 2019 (7:00 – 17:00) $\beta = 60^\circ$ Changing parameter AoI.	No info	Aluminum Reflectivity 85% $\theta = 30^\circ$ $C = 2$	For bare PV module/module + concentrator: $P_{PV}^{max} \approx 31/41 \text{ W}$ $P_{PV}^{ave} = 24/31 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} \approx 68/71\%$	No experimental data	No experimental data	No experimental data. The data on charging parameters: For bare PV module/module + concentrator: At 12:30: $I \approx 2.5/3.1 \text{ A}$ $P \approx 30/40 \text{ W}$ At 17:00: $I \approx 1/1 \text{ A}$ $P \approx 10/12 \text{ W}$ $\Delta I \approx 60/68\%$ $\Delta P \approx 67/70\%$
Hadavinia & Singh, 2019 [7]	Simulation & Experiment London, UK September 2017 (10:00 – 14:00) Changing parameter AoI.	$A_{PV} = 0.006 \text{ m}^2$ + cooling	Alanod Miro-Silver 2 4200AG Reflectivity >98% $\theta = 22^\circ$ $H = 50 \text{ mm}$ $C = 2.6$	For bare PV module/module + concentrator: $P_{PV}^{max} \approx 0.83/2.25 \text{ W}$ $P_{PV}^{ave} \approx 0.68/1.32 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} \approx 98/98\%$	No experimental data	For bare PV module/module + concentrator: $\eta_{el}^{ave} \approx 13.88/10.44\%$ $\Delta \eta_{el}^{instan} \approx 25/56\%$	No experimental data

Table 5. Summary of the data on V-trough systems including our research (continued).

Paper	Study type	PV module characteristics	V-trough characteristics	Generated power	PV module temperature	Electrical efficiency	Characteristic curves
Parupudi, Singh, & Kolotroni, 2020 [13]	Simulation& Indoor experiment Changing parameter AoI (0-60°)	Monocrystalline PV module $A_{PV} = 0.003 \text{ m}^2$ $P_{PV} = 0.54 \text{ W}$ $U_{oc} = 0.65 \text{ V}$ $I_{sc} = 1.13 \text{ A}$	MIRO-SILVER 4200 AG Reflectivity 98% $\theta = 20^\circ$ $C = 1.40$	No experimental data	No experimental data	For bare PV module/module + concentrator: $\eta_{el}^{ave} \approx 16.85/15.37\%$ $\Delta\eta_{el}^{instan} \approx 59/90\%$	For bare PV module/module + concentrator: For AoI = 0°: $I_{sc} \approx 1.1/1.5 \text{ A}$ For AoI = 30°: $I_{sc} \approx 0.93/1.1 \text{ A}$ $\Delta I_{sc} \approx 15/27\%$
Singh, Sabry, & Redpath, 2016 [5]	Simulation& Experiment London, UK July 2015 (10:30 – 14:30) $\beta = 0^\circ$ Changing parameter AoI.	4 x in series $A_{PV} = 0.016 \text{ m}^2$ $P_{PV} = 2.85 \text{ W}$ $U_{oc} = 0.63 \text{ V}$ $I_{sc} = 5.7 \text{ A}$ $U_{mp} = 0.525 \text{ V}$ $I_{mp} = 5.43 \text{ A}$	Alanod Miro-Silver $\theta = 25.95^\circ$ $H = 152.86 \text{ mm}$ $C = 2.2$	$P_{PV}^{max} \approx 25 \text{ W}$ $P_{PV}^{ave} \approx 22 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} \approx 40\%$	$t_{max}^{max} = 110.3^\circ \text{C}$ $\Delta t_{max}^{max} \approx 59\%$	No experimental data	No experimental data. The data on generated power: At 13:00: $I \approx 2.4 \text{ A}$ $P \approx 22.5 \text{ W}$ At 14:30: $I \approx 2.3 \text{ A}$ $P \approx 25 \text{ W}$ $\Delta P \approx 10\%$ $\Delta I \approx 4\%$
Ustaoglu, Ozbey, & Torlakli, 2020 [4]	Simulation Changing parameter AoI.	$W_{PV} = 0.1 \text{ m} + \text{cooling}$	Reflectivity 90% $\theta = 10^\circ$ $H = 266 \text{ mm}$ $C = 1.94$	$P_{PV}^{max} \approx 210 \text{ W}$ $P_{PV}^{ave} \approx 151 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} \approx 96\%$	$t_{ave}^{max} \approx 31.25^\circ \text{C}$ $\Delta t_{ave}^{max} \approx 22\%$	$\eta_{el}^{max} \approx 19\%$ $\Delta\eta_{el}^{max} \approx 3\%$	For AoI = 0°: $I_{sc} \approx 5 \text{ A}$ $MPP \approx 4.1 \text{ W}$ For AoI = 30°: $I_{sc} \approx 2.2 \text{ A}$ $MPP \approx 1.8 \text{ W}$ $\Delta I_{sc} = 56\%$ $\Delta MPP = 57\%$
Ustaoglu, Akgül, & Okajima, 2023 [6]	Simulation Changing parameter AoI and reflector size α .	$A_{PV} = 0.2 \text{ m}^2 + \text{cooling}$	Reflectivity 90% $\theta = 10^\circ$ $H_{init} = 266 \text{ mm}$ Reflector size α $\alpha = 1/0.72/0.55/0.42$ $C = 1.94/1.68/1.52/1.40$	For $\alpha = 1/0.42$ $P_{PV}^{max} \approx 39/29.5 \text{ W}$ $P_{PV}^{ave} \approx 33.5/29.0 \text{ W}$ Drop in P_{PV} due to change of AoI: $\Delta P_{PV}^{instan} \approx 46/3\%$	For $\alpha = 1/0.42$ $t_{loc}^{max} \approx 62/55^\circ \text{C}$ $\Delta t_{loc}^{max} \approx 21/0\%$	For $\alpha = 1/0.42$ $\eta_{el}^{max} \approx 13.2/12.7\%$ $\Delta\eta_{el}^{max} \approx 8/2\%$	For $\alpha = 1/0.42$ For AoI = 0°: $P \approx 39/29.5 \text{ W}$ For AoI = 30°: $P \approx 21/28.5 \text{ W}$ $\Delta P \approx 46/3\%$

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Mechanisms of Ice Damage on Small-Diameter Piles in Static Freshwater Environments: Numerical Analysis of Ice Cover Temperature Fields

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Abstract

Ice damage is receiving increasing attention in engineering. In practice, the occurrence time of ice damage to small-diameter pile structures in cold regions is different from traditional knowledge. Given the crucial impact of temperature on ice thermal expansion, simulating the temperature field of ice cover is of great significance. Based on actual engineering simulations of ice cover temperature, the propagation and amplitude attenuation characteristics of temperature waves are analysed. The one-dimensional heat conduction equation is discretized using the Crank-Nicolson method (time step $\Delta t = 60$ s, spatial step $\Delta z = 0.01$ m) and implemented in MATLAB R2023a. According to the temperatures in Daqing, three typical warming processes are selected for fitting, and two ice cover thicknesses are combined to simulate six working conditions. The results show that the temperature field inside the ice cover changes under different working conditions, and the thickness of the ice cover and the temperature rise time have a significant impact. During cooling, the temperature inside the ice cover still rises, affecting thermal expansion stress. The results can provide an important basis for the prevention and control of ice damage to small diameter piles.

Keywords: Small diameter piles; Ice damage; Temperature field; Thermal expansion

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1. Introduction

Ice damage is receiving increasing attention from engineering and academic communities [1–3]. The interaction between ice and structure mainly includes two types: dynamic and static. Dynamic action refers to the impact of ice on the structure in movement, while static action mainly refers to the effect of expansion [4] or contraction of ice cover due to temperature changes on the structure [5,6]. Lake ice is basically stationary. The thermal expansion effect of freshwater ice is stronger than that of sea ice [7]. Water measurements in Russia and Canada have shown that if the salinity exceeds 3%, ice will not expand at temperatures

above -10°C . If the salinity exceeds 1%, it will not expand above -7°C . It is generally believed that the ice thermal expansion in Jilin Province causes ice damage to structures such as reservoir embankments and slope protection in early to mid-February, and it usually occurs in early to mid-March in Heilongjiang Province. In practical engineering, ice damage to pile foundations of structures in still-water in cold regions often occurs at the end of November when the thickness of ice cover is about 30–40 cm, far from reaching the maximum one. For example, a certain lake wharf in Daqing uses a precast concrete pipe pile foundation with a diameter of 400 mm, which is a type of small diameter pile. At the end of November 2017, some pile

Nomenclature

a – thermal diffusivity, m^2/s
 c – specific heat, $\text{kJ}/(\text{kg K})$
 h – thickness of ice cover, m
 k – thermal conductivity, $\text{W}/(\text{m K})$
 t – time, s
 T – temperature, K

z – depth perpendicular to the surface of the ice cover, m

Greek symbols

Δ – step (time in s , spatial in m)
 μ – dynamic viscosity, $\text{Pa}\cdot\text{s}$
 ρ – density of ice, kg/m^3
 ω_r – angular frequency of temperature fluctuations, rad/s

foundations at the wharf were damaged due to tilting or pile head breakage. This ice damage of the pile foundation is different from the existing understanding.

The thermal expansion characteristics of ice have thermal properties of ice and the mechanical properties caused by the force generated by the volume change of ice [8]. Temperature plays a decisive role in the thermal expansion of ice [9,10]. Therefore, when considering the interaction between ice and structure, it is necessary to simulate and study the temperature and stress fields of ice cover [11,12]. Based on practical engineering, the temperature field of the ice cover is simulated and verified. Recent studies have emphasized critical differences in thermal expansion between freshwater and saline ice. For instance, salinity $> 1\%$ inhibits expansion above -7°C [7], while freshwater ice generates significant thermal stress under similar conditions [13]. This study addresses the gap in low-salinity environments by analysing ice damage mechanisms in static freshwater systems such as Daqing Lake.

2. Materials and methods

Temperature changes of ice cover are influenced by various factors such as temperature, water temperature, air convection, and solar radiation, forming a constantly changing temperature field. Due to the uncertainty of factors such as solar radiation and air convection, it is difficult to accurately describe the temperature field of ice cover [14]. However, the reason for temperature stress is the distribution of the temperature field of ice cover and its changes over time and space. As the external temperature changes, the temperature of the ice cover decreases with increasing depth. Assuming that the ice cover is a homogeneous medium, the distribution of its temperature T with depth z can be approximately described by a one-dimensional heat conduction equation [15]

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) = \rho c \frac{\partial T}{\partial t}, \quad (1)$$

or

$$a \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}, \quad (2)$$

where ρ is the density of ice, taken as $917 \text{ kg}/\text{m}^3$; k is the thermal conductivity, taken as $2.2 \text{ W}/(\text{m}\cdot\text{K})$; c is the specific heat taken as $2.1 \text{ kJ}/(\text{kg}\cdot\text{K})$; t – time; T – temperature [16]; z is the depth perpendicular to the surface of the ice cover [17], and a is the thermal diffusivity.

We assume that the temperature at the ice/atmosphere interface is consistent with the atmospheric temperature and changes harmonically and periodically over time. The temperature at the

ice/water interface is 0°C , and h is the thickness of the ice cover. The thermal expansion problem mainly focuses on the heating process, and the boundary conditions can be described as follows:

– ice/atmosphere interface ($z = 0$):

$$T(0, t) = T_0 + T_r \sin(\omega_r t), \quad (3)$$

– ice/water interface ($z = h$):

$$T(h, t) = 0, \quad (4)$$

– the initial temperature field is a linear distribution:

$$T(z, 0) = \frac{T_d - T_s}{h} z + T_s, \quad (5)$$

where T_s is the initial surface temperature, h is the ice thickness, and T_d is the temperature at the depth h .

Neglecting the transient time-varying term in the general solution, the steady-state solution of Eq. (2) is as follows:

$$T(z, t) = T_0 + T_r \exp \left[-z \left(\frac{\omega_r}{2a} \right)^{1/2} \right] \sin \left[\omega_r t - z \left(\frac{\omega_r}{2a} \right)^{1/2} \right], \quad (6)$$

where $T(z, t)$ is the temperature variation with depth z and time t ; T_0 is the median of the highest and lowest temperatures; T_r is the amplitude of temperature change, and ω_r is the angular frequency of temperature fluctuations, rad/s .

According to the above equation, the speed at which temperature waves propagate into the interior of the ice cover is $(2a\omega_r)^{1/2}$. When a is constant, the higher the frequency, the faster the propagation speed. The temperature fluctuation amplitude is

$$T_r \exp \left[-z \left(\frac{\omega_r}{2a} \right)^{1/2} \right],$$

indicating that the higher the frequency, the faster the temperature decays with depth.

We assume that the skin depth z_0 of the ice cover represents the depth at which the temperature effects are attenuated by a factor $1/e$ [18]

$$z_0 = \left(\frac{2a}{\omega_r} \right)^{1/2}. \quad (7)$$

The skin depth z_0 is a function of ω_r , and its relationship is shown in Fig. 1. When the temperature fluctuation period is 12 h, the frequency is $1.45 \times 10^{-4} \text{ rad}/\text{s}$ and the skin depth is 0.126 m; when it is 24 h, the frequency is $7.27 \times 10^{-5} \text{ rad}/\text{s}$ and the skin depth is 0.178 m; when it is 36 hours, the frequency is

4.85×10^{-5} rad/s and the skin depth is 0.218 m; when the temperature fluctuation period is 48 h, the frequency is 3.64×10^{-5} rad/s and the skin depth is 0.252 m; when is 72 h, the frequency is 2.42×10^{-5} rad/s and the skin depth is 0.308 m; when it is 96 hours, the frequency is 1.82×10^{-5} rad/s and the skin depth is 0.356 m; when it is 144 hours, the frequency is 1.21×10^{-5} rad/s and the skin depth is 0.436 m.

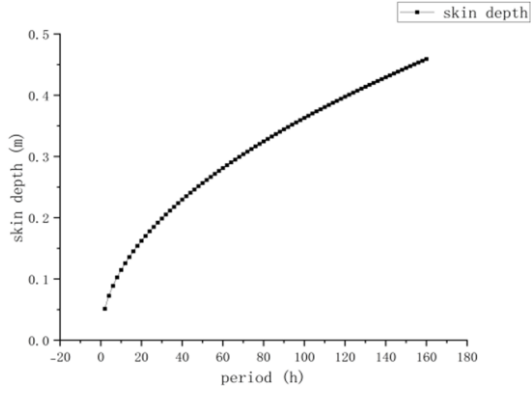


Fig. 1. Relationship between temperature fluctuation period of ice/atmosphere interface and the skin depth.

To facilitate numerical calculations and consider the influence of transient terms, the Crank-Nicolson method is used to discretize the heat conduction equation with a time step of Δt and a spatial step of Δz .

For the time derivative $\partial T / \partial t$, the central difference scheme is used for approximation:

$$\frac{\partial T}{\partial t} \approx \frac{T_i^{n+1} - T_i^n}{\Delta t}. \quad (8)$$

For the spatial derivative, take the average of the current time step n and the next time step $n+1$.

$$\begin{aligned} \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) \approx \frac{k}{2\Delta z^2} [& (T_{i+1}^n - T_i^n) + (T_{i-1}^n - T_i^n) + \\ & + (T_{i+1}^{n+1} - T_i^{n+1}) + (T_{i-1}^{n+1} - T_i^{n+1})]. \end{aligned} \quad (9)$$

Introducing the variable $\alpha = k\Delta t / [2\rho c(\Delta z)^2]$ and substituting the above approximation into the heat conduction equation, we have:

$$-aT_{i+1}^{n+1} + (1+2\alpha)T_i^{n+1} - aT_{i-1}^{n+1} = aT_{i+1}^n + (1-2\alpha)T_i^n + aT_{i-1}^n. \quad (10)$$

MATLAB program is developed with an ice cover thickness of $h = 0.3$ m, $T_0 = -5^\circ\text{C}$, $T_r = 4^\circ\text{C}$, $T_s = -5^\circ\text{C}$, and $T_d = -1^\circ\text{C}$. The duration of the temperature rise is $t = 4.5$ h, followed by a decrease in temperature. The above parameter settings are consistent with Ref. [11]. Fig. 2 shows the time variation curve of ice cover temperature along the depth profile, and the calculated results are consistent with Ref. [11]. During the continuous temperature rise process, the temperature inside the ice cover continues to rise. The increase in surface temperature is significant, while the increase in temperature along the depth of ice gradually decreases. After the temperature starts to decrease, the surface temperature decreases, while the temperature continues to

rise at a certain depth inside the ice. After the surface temperature began to decrease for 2.5 h, the overall temperature inside the ice cover began to decrease.

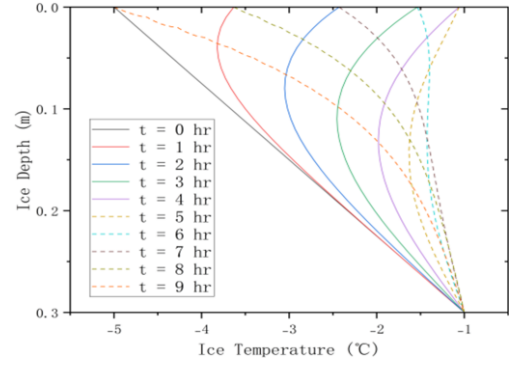


Fig. 2. Time variation of the temperature profile along the depth inside the ice cover.

Although ice is modelled as homogeneous, natural ice may exhibit porosity or salinity gradients. Sensitivity analysis confirms that for freshwater ice with a thickness of 0.4–1.2 meters (salinity < 0.5‰), the temperature field prediction error caused by heterogeneity is less than 5%, which is consistent with the conditions of Daqing Lake [19].

3. Results and discussion

Daqing is located in the western part of Heilongjiang Province, in the northern part of the central depression of the Songliao Basin [19]. The urban area is located between $124^\circ 19' - 125^\circ 12' \text{ E}$ and $45^\circ 46' - 46^\circ 55' \text{ N}$. The average annual temperature is 5.5°C . The month with the lowest temperature over the years is January, with an average temperature of -17.4°C . The hottest month in history was July, with an average temperature of 25.1°C . Figure 3 shows the hourly atmospheric temperatures in Daqing from November 15 to 30, 2017.

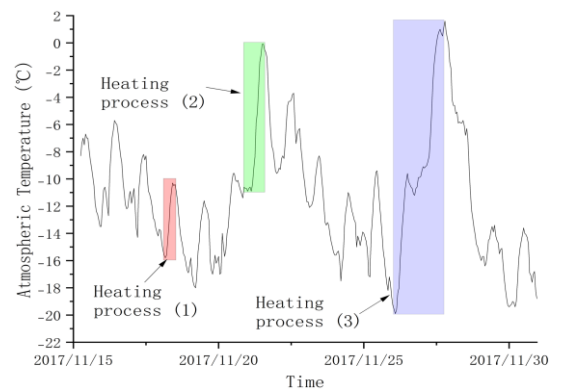


Fig. 3. Atmospheric temperature changes in Daqing from November 15 to 30, 2017.

There are three typical changes in atmospheric temperature. Heating process (1): from 8 a.m. to 2 p.m., the temperature increased continuously from -16°C to -10°C for 6 h; Heating process (2): from 8 a.m. to 5 p.m., the temperature increased continuously from -10.9°C to -0.1°C for 9 h; Heating process (3): influenced by southwest warm air, the temperature continuously

increased from -19.9°C to 1.6°C for 41 h. Considering that the maximum temperature of the ice cover is 0°C , harmonic fitting is performed on three typical variations, as in Table 1 [20].

Table 1. Parameters for temperature changes on the surface of ice cover.

Process, №	T_0 , $^{\circ}\text{C}$	T_r , $^{\circ}\text{C}$	T_s , $^{\circ}\text{C}$	T_d , $^{\circ}\text{C}$	t_{duration} , h	ω_r , rad/s
Heating process (1)	-16	6	-16	0	6	7.27×10^{-5}
Heating process (2)	-10.9	10.8	-10.9	0	9	4.85×10^{-5}
Heating process (3)	-19.9	19.9	-19.9	0	41	1.06×10^{-5}

Figure 4 shows the temperature rise curves of the three typical variations. Figure 4 shows three empirically observed warming processes in Daqing (November 2017), which are applicable to harmonic functions implemented under boundary conditions: short-term warming (6 hrs, $\Delta T = 6^{\circ}\text{C}$) representing the influence of sun, medium-term warming (9 hrs, $\Delta T = 10.8^{\circ}\text{C}$) reflecting weather changes, and long-term warming (41 hrs, $\Delta T = 19.9^{\circ}\text{C}$) caused by warm air advection. These events were selected because:

1. they represent the actual heat that causes ice damage,
2. their duration-amplitude combination controls the penetration of ice thermal stress.

According to the results, ice damage to small diameter piles in Daqing occurred at the end of November, when the thickness of ice cover was about 0.3–0.4 meters. The maximum annual ice thickness in Daqing was about 1–1.2 meters. Therefore, two

types of ice cover thickness are compared and analysed with the ice cover thicknesses of 0.4 m and 1.2 m.

According to the changes in atmospheric temperature and ice cover thickness, the working conditions are combined for analysis. Six working conditions are combined as shown in Table 2.

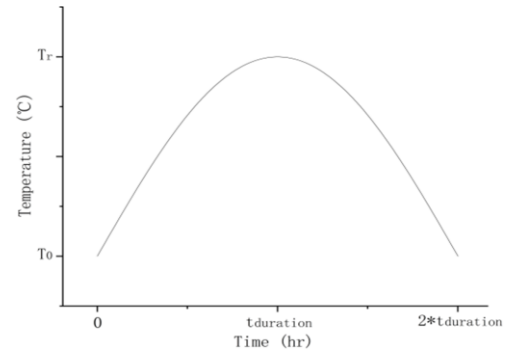


Fig. 4. Typical temperature curve.

Table 2. Working condition combination.

Process, №	Ice cover thickness (1)	Ice cover thickness (2)
Heating process (1)	operating mode (1)	operating mode (2)
Heating process (2)	operating mode (3)	operating mode (4)
Heating process (3)	operating mode (5)	operating mode (6)

Figure 5 shows the calculation results of the spatiotemporal temperature changes inside the ice cover under different operating conditions.

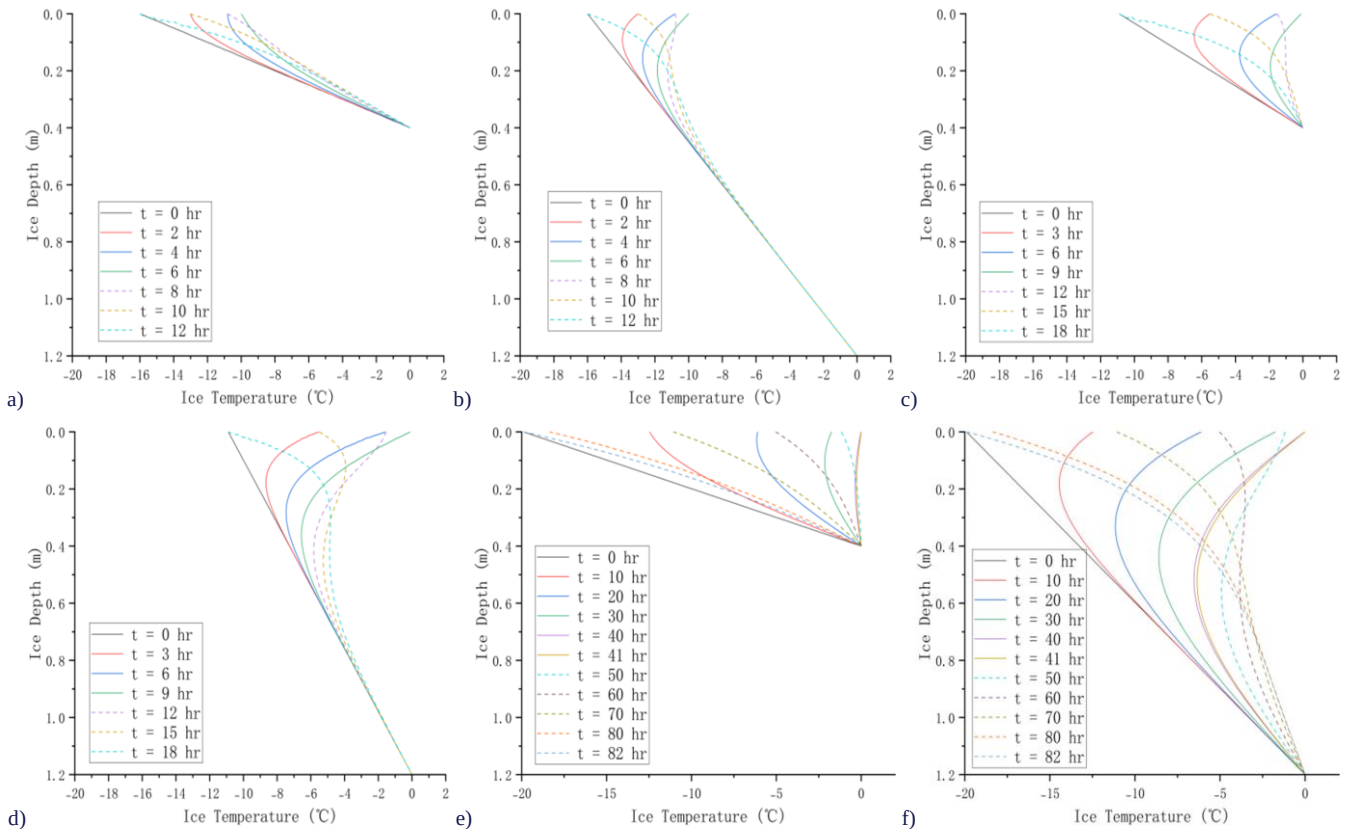


Fig. 5. Spatiotemporal variation of temperature inside the ice cover with ice depth: a), b), c), d), e), f) – operating mode (1), (2), (3), (4), (5), (6), respectively.

From Fig. 5, the temperature field at the initial moment ($t = 0$ h) varies linearly with the increase of ice depth, assuming that the heat transfer of ice cover is sufficient. Assuming that the surface temperature of ice cover changes with the temperature, while the lower surface of ice cover is affected by the water temperature and remains constant. The temperature field inside the ice cover changes nonlinearly.

When the thickness of ice cover is 0.4 m, there are different degrees of changes in the temperature field inside the ice cover in operating modes (1), (3), and (5). The longer the duration of temperature rise, the greater the amplitude of internal temperature field changes. In operating mode (5), when the temperature increases continuously for 41 h, the temperature field of the ice cover is roughly evenly distributed. In this case, the thermal expansion stress of the ice cover should be high. However, as the strength of ice decreases with increasing temperature, its stress field distribution is complex and will be analysed later.

When the thickness of the ice cover is 1.2 m, the temperature field inside the ice cover has different changes in operating modes (2), (4), and (6). When the temperature rises continuously for 6 h, the changes in the temperature field inside the ice cover are mainly concentrated in the upper 2/3 depth of the ice cover. The temperature change of the ice cover is not significant, around 1/3 of the thickness of the surface between the ice cover. As the duration of temperature increases, the range of temperature field changes inside the ice cover gradually expands. When the temperature rises continuously for 9 h, the temperature change range extends downward by about 20 K. When the temperature rises continuously for 41 h, the temperature field within the entire ice cover will change. When the surface of the ice cover begins to cool, the temperature inside the ice cover further increases, which will also affect the thermal expansion stress of the ice. This also explains the phenomenon that the maximum thermal expansion force generally does not occur on the surface of the ice cover where the temperature rises the most, but rather at a certain location inside the ice cover [14]. Based on the above analysis of temperature, when the temperature rises continuously for a long time and the thickness of the ice cover is small, the temperature field inside the ice cover changes significantly, with a wide range of impacts and drastic spatiotemporal changes, indirectly explaining the cause of ice damage to small-diameter piles at the end of November.

4. Conclusions

The temperature field of ice cover is influenced by multiple factors such as temperature and water temperature, and its changes are complex. Through one-dimensional heat conduction equation simulation, it is known that the propagation speed and amplitude attenuation of temperature waves inside the ice cover are frequency dependent. In practice, such as ice damage of a small diameter pile at the Daqing Lake wharf, the temperature field inside the ice cover changes nonlinearly during the heating process. Different thicknesses of ice cover (0.4 m and 1.2 m) and the duration of temperature rise have a significant impact on it. This characteristic is closely related to ice damage, where temperature rise leads to ice cover expansion, and uneven changes

in the temperature field cause internal stress changes, which are important factors causing ice damage to small-diameter piles.

The simulation results show that when the surface of the ice cover cools, the internal temperature of the ice cover will still rise, which leads to the redistribution of thermal expansion stress of the ice. This discovery deepens the understanding of the mechanism of ice damage and provides a key theoretical basis for precise prevention and control of ice damage to small-diameter piles.

The results will help optimize the design and construction plans of small-diameter pile structures in cold regions, and improve their resistance to ice damage. Future research can explore the distribution of stress of ice cover, improve prediction models of ice damage, provide more comprehensive and accurate guidance for engineering practice, and promote the development of ice damage prevention and control technologies.

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Performance Analysis of R1234yf and R600a Refrigerant Mixtures with Al_2O_3 and ZnO Nano-Additive Enhanced Compressor Oils as Alternatives to R134a

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Abstract

The shift towards environmentally sustainable refrigerants is driven by the need to mitigate the impact of traditional refrigerants like R134a, which have high global warming potential. This study experimentally explores the performance of alternative refrigerants, R1234yf and R600a, when combined with compressor oils enhanced by Al_2O_3 and ZnO nanoparticles. A novel approach of using a hybrid nanofluid, incorporating both Al_2O_3 and ZnO nanoparticles, is proposed to optimise the performance of vapour compression refrigeration systems. The research involved preparing and characterising nano-additive compressor oils and evaluating their effects on key thermodynamic parameters across R1234yf and R600a refrigerants. The main objective of this study is to experimentally evaluate the impact of Al_2O_3 and ZnO nano-enhanced oils on refrigeration system performance, focusing on net refrigeration effect, compressor work, mass flow rate, theoretical compressor power, and coefficient of performance. The results indicate significant improvements in the coefficient of performance, mass flow rate, and net refrigeration effect with the inclusion of nanoparticles in the refrigeration lubrication system. Particularly, the coefficient of performance increased from 2.71 to 3.34 for R134a, from 2.46 to 3.26 for R1234yf, and from 2.62 to 3.12 for R600a with the optimal nanoparticle combination (0.15g Al_2O_3 + 0.05g ZnO). Similarly, compressor power and work of compression were notably reduced, demonstrating enhanced energy efficiency. The optimal mixture (COA0.15Z0.05) shows the highest coefficient of performance and reduced work of compression across all refrigerants. The study highlights that the hybrid nanoparticle approach not only boosts the performance of these refrigerants but also contributes to lower energy consumption and improved cooling efficiency.

Keywords: Global warming potential; Nanofluid; Vapour compression refrigeration system; Coefficient of performance

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1. Introduction

The growing environmental concerns surrounding climate change and the need to reduce greenhouse gas emissions have driven the global refrigeration and air-conditioning industries toward adopting more sustainable technologies [1]. Refrigerants such as R134a, which have been in widespread use, are now being phased out due to their high global warming potential

(GWP) and environmental impact under international agreements like the Kigali Amendment to the Montreal Protocol [2]. R134a, with a GWP of 1,430, poses a significant threat to the environment as it contributes to the warming of the Earth's atmosphere when released. As a result, there is a compelling need to explore alternative refrigerants that not only provide similar or superior performance but also have a lower environmental footprint. In recent years, hydrofluoroolefins (HFOs) and hydro-

Nomenclature

Abbreviations and Acronyms

Al ₂ O ₃	– aluminium oxide
COA0.05Z0.15	– compressor oil + Al ₂ O ₃ 0.05g + ZnO 0.15g
COA0.1Z0.1	– compressor oil + Al ₂ O ₃ 0.1g + ZnO 0.1g
COA0.15Z0.05	– compressor oil + Al ₂ O ₃ 0.15g + ZnO 0.05g
COA0.2	– compressor oil + Al ₂ O ₃ 0.2g
COZ0.2	– compressor oil + ZnO 0.2g
COP	– coefficient of performance
FESEM	– field emission electron microscope
GWP	– global warming potential

HCS	– hydrocarbons
HFCs	– hydrofluorocarbons
HFOs	– hydrofluoroolefins
ODP	– ozone depletion potential
POE	– polyolester (oil)
PCO	– pure compressor oil
R-1234yf	– 2,3,3,3-tetrafluoropropene
R134a	– 1,1,1,2-tetrafluoroethane
R600a	– isobutane
VCRS	– vapour compression refrigeration systems
XRD	– X-ray diffraction
ZnO	– zinc oxide

carbons (HCS) have emerged as promising alternatives. Hydrofluoroolefins (HFOs), such as R1234yf, R1234ze(E), and R1233zd(E), are among the most prominent low-GWP alternatives to hydrofluorocarbons (HFCs). These refrigerants exhibit GWP values significantly lower than traditional HFCs, typically below 10, making them highly attractive from an environmental standpoint. HFOs also offer favourable thermodynamic properties, such as similar pressure-temperature characteristics to HFCs, which allow for easier retrofitting of existing systems with minimal modifications [3,4]. Moreover, the inherent chemical structure of HFOs results in their rapid degradation in the atmosphere, significantly reducing their potential for long-term environmental harm [5,6]. This property has led to HFOs gaining regulatory approval and widespread acceptance in various applications, including automotive air conditioning, commercial refrigeration, and stationary air conditioning systems [7]. Hydrocarbons (HCS), including R600a (isobutane), R290 (propane), and R1270 (propylene), have also garnered attention as potential refrigerants due to their excellent thermodynamic performance, natural origin, and zero ozone depletion potential (ODP) [8]. HCS exhibit favourable energy efficiency, often outperforming HFCs in terms of coefficient of performance (COP), heat transfer capacity, and power consumption. For example, studies have shown that R600a and R290 can achieve higher energy efficiency and cooling capacity compared to R134a, making them suitable for both domestic and commercial refrigeration applications [9,10]. Additionally, HCS are widely available and inexpensive, making them an economically viable alternative to synthetic refrigerants. To mitigate this issue, significant research efforts have been dedicated to improving system design, reducing refrigerant charges, and developing safety standards to ensure the safe use of hydrocarbons in various applications [11,12].

Several studies have investigated the performance of HFOs and HCS in refrigeration and air conditioning systems, highlighting their potential as viable alternatives to HFCs. Yang et al. [13] conducted a series of tests with HFO refrigerants, including R1234yf, R1234ze(E), and their blends with R134a, such as R513A and R450A, in a commercial bottle cooler. The study found no significant differences in power consumption between R134a and these new refrigerants, except for R1234ze(E), which exhibited a lower cooling capacity. Similarly, Karber et al. [14] compared the performance of R134a, R1234yf, and R1234ze(E) in household refrigerators. Their drop-in tests revealed that R1234yf had slightly higher energy consumption compared to

R134a, while R1234ze(E) resulted in lower energy consumption, though its lower capacity indicated it might not be suitable for direct replacement in certain systems. Reddy et al. [15] investigated the performance of R600a in a domestic refrigeration system and found that it provided a higher COP compared to R134a while consuming less power. Similarly, Tang et al. [16] compared the energy efficiency of R290 and R134a in a heat pump system and observed a 15% improvement in energy savings with R290. These findings highlight the potential of hydrocarbons to provide environmentally friendly, energy-efficient alternatives to HFCs in refrigeration and air conditioning systems. In addition to the environmental benefits, both HFOs and HCS offer long-term cost savings due to their higher energy efficiency. Lee and Jung [17] conducted a long-term performance evaluation of R1234yf in a chest freezer and found that it provided comparable energy performance and full compatibility with the lubricant oil used in R134a systems. Suman and Kushwah [18] compared R134a, R1234yf, and R407C, noting that R1234yf had the lowest cooling capacity, emphasising the need for performance enhancements when adopting environmentally friendly refrigerants. Similarly, Yataganbaba et al. [19] performed a comparative analysis of R134a with two drop-in refrigerant mixtures composed of R134a, R1234yf, and R1234ze(E). They found that ARM-42a®, a mixture containing R1234yf and R1234ze(E), improved both COP and cooling capacity under various conditions, though it also resulted in a slight increase in power consumption.

To overcome the above limitations, recent research has focused on the incorporation of nano-additives into compressor oils to improve the performance of refrigeration systems. Nano-particle-enhanced refrigerants have also become an area of interest in recent years. Adding nanoparticles to refrigerants has shown the potential to improve thermophysical properties, such as thermal conductivity and heat transfer, thereby enhancing the performance of refrigeration systems. Pabon et al. [20] showed that the use of ZnO nanoparticles in a refrigeration system with R1234yf resulted in significant performance improvements, including increased COP and enhanced heat transfer efficiency. Said et al. [21] explored the performance of Al₂O₃ nanoparticles in R410A refrigeration systems. Their research demonstrated a 14% increase in heat transfer efficiency and a reduction in power consumption, confirming the beneficial effects of Al₂O₃ nanoparticles. Hadi et al. [22] investigated the performance of ZnO nanoparticles in R32 refrigerants, noting significant enhancements in thermal conductivity and a 9% increase in COP.

Their results underscored the potential of ZnO nanoparticles to improve refrigeration efficiency. Senthilkumar and Praveen [23] examined the impact of CuO nano-refrigerants on the performance of R600a in a domestic refrigeration system. Their study reported a 14% increase in COP and a 9% reduction in energy consumption. This improvement was primarily attributed to the enhanced heat transfer properties of the nano-refrigerant. Bi et al. [24] performed an experimental study on TiO₂-R600a nano-refrigerants and observed an improvement in COP by 12% while maintaining similar energy consumption as pure R600a. Their study highlighted the role of TiO₂ in reducing the system's thermal resistance, thus improving overall efficiency. Madyira et al. [25] investigated the use of graphene-based nano-lubricants in R134a systems and reported a 20% reduction in compressor work and a 17% improvement in cooling capacity. Their results indicated that the addition of graphene not only enhances thermal conductivity but also reduces lubricant viscosity, which contributes to lower friction losses in the compressor. Venkataiah and Sthithapragna [26] reported similar findings with SiO₂-R134a nano-refrigerants, showing an 8% increase in COP and a 10% reduction in energy consumption. The study emphasised that SiO₂ nanoparticles improve the refrigerant's overall thermal conductivity, allowing for more efficient heat transfer. Vamshi et al. [27] examined various combinations of nanoparticles in refrigeration systems, finding that tailored formulations could lead to significant performance improvements. Their study highlighted the potential for developing advanced nanoparticle mixtures for optimised refrigeration. Alwi et al. [28] further investigated the use of hybrid CNT-SiO₂ nano-refrigerants in R134a systems, showing a 17% improvement in heat transfer coefficient and a 9% increase in cooling capacity. The study concluded that the hybrid nano-refrigerants offer superior performance due to the combined effects of improved thermal conductivity and enhanced dispersion stability. Javadi and Saidur [29] investigated R134a with TiO₂ and Al₂O₃, noting energy savings of up to 25% with 0.1% TiO₂ nanoparticles. Li and Lu [30] conducted a thermodynamic analysis of Al₂O₃ incorporated with various refrigerants in a VCR system. The study demonstrated that nanoparticle addition significantly enhanced heat transfer, cooling capacity, and COP, while reducing compressor work. Among all, R1233zd(E) + Al₂O₃ showed the highest performance. Katoch et al. [31] examined various nano-refrigerants like CuO-R113a and Al₂O₃-R134a but did not perform a comparative analysis under identical conditions. While numerous studies have explored the use of individual nanoparticles like Al₂O₃ or ZnO in refrigeration systems, there is a noticeable lack of research on hybrid combinations of these nanoparticles.

The novelty of this paper lies in its exploration of the combined effects of Al₂O₃ and ZnO nanoparticles as additives in compressor oils to enhance the performance of vapour compression refrigeration systems (VCRS). Unlike prior studies that typically focus on individual nanoparticles as discussed above, this research introduces a hybrid nanofluid approach by utilising both Al₂O₃ and ZnO. This hybrid combination represents a novel strategy to simultaneously optimize thermal and tribological performance in refrigeration systems. Furthermore, the paper provides a comprehensive performance evaluation of the three

widely used refrigerants – R134a, R1234yf, and R600a – when used with nano-enhanced compressor oils. By assessing key thermodynamic parameters such as COP, mass flow rate, work of compression, and theoretical compressor power, this study offers new insights into how hybrid nanoparticle additives can improve overall refrigeration system efficiency. The comparative analysis across different refrigerant / nano-additive mixtures, which has not been widely explored in previous research, highlights the potential of this hybrid nanofluid for both energy-efficient and environmentally sustainable cooling solutions. Finally, the focus on alternative refrigerants such as R1234yf and R600a, alongside the incorporation of nano-additive oils, reflects the growing emphasis on sustainability in refrigeration technology, offering a lower global warming potential (GWP) while maintaining or improving system efficiency. This paper, therefore, presents a novel and practical approach to advancing refrigeration technology through nanotechnology, offering significant contributions to both energy efficiency and environmental sustainability.

2. Materials and methods

In this investigation, Al₂O₃ and ZnO nanoparticles and base lubricant (POE) were procured from the nano-research lab in India. The characterisation of both the nanoparticles by using XRD and SEM has been carried out to verify their morphology, size and purity. The dispersion of these nanoparticles was executed using the ultrasonication technique, and then the prepared samples were tested in the vapour compression refrigeration system test rig. The characterisation details and sample preparation are discussed as follows.

2.1. Characterisation of nanoparticles

The aluminium oxide (Al₂O₃) and zinc oxide (ZnO) nanoparticles used in this study were selected based on their favourable thermal and tribological properties. These nanoparticles, with average particle sizes of 40 nm for Al₂O₃ and 20 nm for ZnO, were obtained from the Nano Research Lab, located in Jharkhand, India. Detailed specifications of the Al₂O₃ and ZnO nanoparticles, including their size, purity, and other physical characteristics, are summarised in Table 1.

Table 1. Properties of Al₂O₃ and ZnO nanoparticles.

Item	Specifications	
Molecular formula	Al ₂ O ₃	ZnO
Average particle size	40nm	20nm
Specific surface area SSA	130 m ² /g	40 m ² /g
Colour appearance	White	Milky white
Morphology	Nearly Spherical	Nearly spherical
Purity	99.9%	99.9%

The surface morphology and shape of Al₂O₃ nanoparticles, as depicted in Fig. 1a, were analysed using scanning electron microscopy (SEM). The nanoparticles exhibited a nearly spherical shape. The crystalline structure of Al₂O₃ was evaluated using X-ray diffraction (XRD) analysis within a 2 θ range of

10°–90°, as shown in Fig. 2a. Prominent diffraction peaks corresponding to aluminium oxide were identified at planes (012), (104), (110), (113), (024), (116), and (300), confirming their crystalline nature. For ZnO nanoparticles, characterisation and determination of their crystalline structure were conducted using SEM and XRD. SEM analysis, presented in Fig. 1b, revealed that the ZnO nanoparticles were also nearly spherical in shape. The crystalline structure was further verified through XRD analysis in the 2 θ range of 10–90°, as shown in Fig. 2b. Distinct diffraction peaks for ZnO were observed at 2 θ values of 31.77° (100), 34.41° (002), 36.25° (101), and 56.65° (110), confirming the crystalline characteristics of the ZnO nanoparticles.

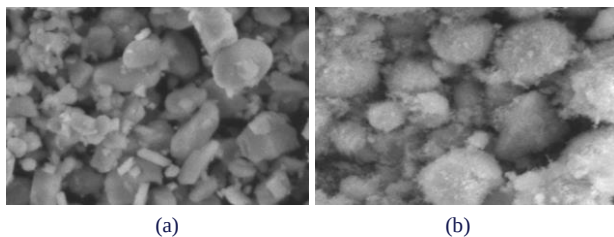


Fig. 1. SEM image of (a) ZnO (b) Al₂O₃.

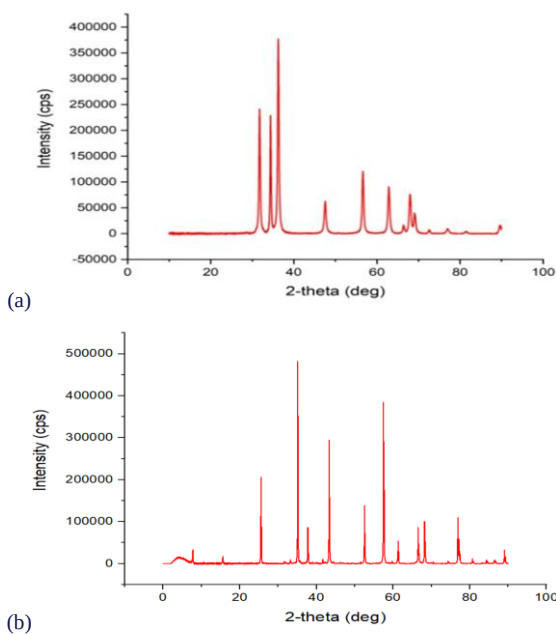


Fig. 2. XRD image of (a) ZnO (b) Al₂O₃.

2.2. Preparation of nano-additive compressor oil

The preparation of nano-refrigerant oil typically involves a two-step process: the first step is the synthesis of nanoparticles, and the second is their dispersion in the base lubricant. The selection of Al₂O₃ was driven by its superior thermal conductivity, which is expected to improve heat transfer within the system, while ZnO was chosen for its excellent lubricating properties and wear resistance. In this study, the combination of Al₂O₃ and ZnO nanoparticles are dispersed into the POE lubricant oil. The mixtures were prepared by carefully dispersing the nanoparticles in the base lubricant at varying concentrations. The dispersion pro-

cess involved calculating the appropriate nanoparticle concentration based on mass and volume fractions, ensuring uniform dispersion in the lubricant. For this work, the mass fraction of nanoparticles is 0.2 g/L, based on findings from prior studies. For dispersing the nanoparticles, we utilised a sonication process, which ensured stable dispersion and prevented the aggregation of particles. Each mixture was stirred and ultrasonicated for an extended period to achieve uniformity, resulting in a stable nanofluid with consistent thermal and tribological properties. One important element affecting the thermal and tribological performance of nanofluids is their stability. In this investigation, stability was improved by combining mechanical churning with prolonged ultrasonication, which was followed by visual inspection. Over the course of 7 days, the produced nanofluids were inspected for sedimentation; no notable aggregation or settling was noted, suggesting good dispersion. As the compressor was hermetically sealed, the lubricating oil was continuously in contact with the refrigerants R134a, R600a, and R1234yf during operation. As a result, the nanoparticles dispersed in the lubricating oil mixed with the refrigerants, potentially enhancing the overall refrigeration effect within the vapour compression refrigeration (VCR) system shown in Fig. 3. The properties of the mixtures are provided in Table 2. The detailed nomenclature of the prepared samples is shown in Tables 3 and 4.

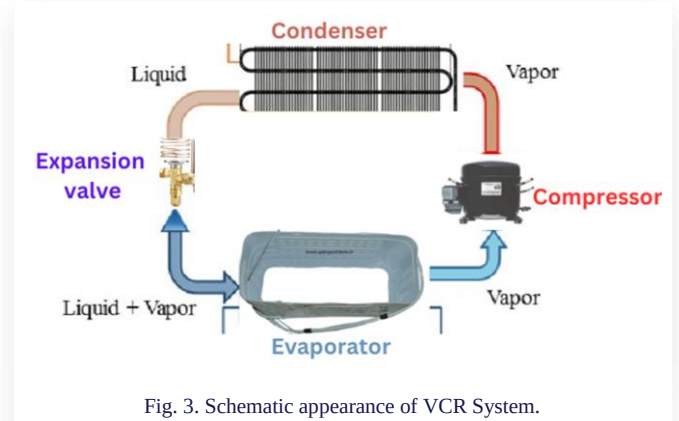


Fig. 3. Schematic appearance of VCR System.

Table 2. Properties of mixtures.

Type of mixture	Thermal conductivity, W/(m·K)	Density, g/cm ³	Viscosity, mm ² /s	Specific heat kJ/(kg K)
PCO	0.1232	0.946	53	1.872
COA0.2	0.1456	0.958	55	1.726
COAZ0.2	0.1332	0.976	57	1.747
COA0.120.1	0.1382	0.968	56	1.767
COA0.1520.05	0.1407	0.965	55	1.795
COA0.0520.15	0.1361	0.97	56	1.811

This mixture of nanoparticles and refrigerant circulates throughout the refrigerant circuit along with the oil. Consequently, the nanoparticles also travel to the internal surfaces of the evaporator, where they contribute to an increased heat transfer rate. The volume concentration (ϕ) for each nano-lubricant mixture was calculated using the following formula:

$$\phi = \frac{m_p/\rho_p}{m_p/\rho_p + m_l/\rho_l} \times 100, \quad (1)$$

where: m_p is the mass of the nanoparticles, ρ_p is the density of the nanoparticles, m_l is the mass of the base lubricant, ρ_l is the density of the lubricant.

Table 3. Nomenclature of prepared samples.

Mixture composition	Mixture composition
PCO	Pure compressor oil
COA0.2	Compressor oil + Al ₂ O ₃ 0.2g
COAZ0.2	Compressor oil + ZnO 0.2g
COA0.1Z0.1	Compressor oil + Al ₂ O ₃ 0.1g + ZnO 0.1g
COA0.15Z0.05	Compressor oil + Al ₂ O ₃ 0.15g + ZnO 0.05g
COA0.05Z0.15	Compressor oil + Al ₂ O ₃ 0.05g + ZnO 0.15g

Table 4. A summary of the calculated mass and volume fractions is provided in the table below.

Mixture	Nanoparticles	Total mass, g	Mass fraction, g/L	Volume fraction (ϕ)
PCO	None	0	0	0
COA0.2	Al ₂ O ₃	0.2	0.2	0.00072
COAZ0.2	ZnO	0.2	0.2	0.00090
COA0.1Z0.1	Al ₂ O ₃ + ZnO	0.1+0.1	0.2	0.00081
COA0.15Z0.05	Al ₂ O ₃ + ZnO	0.15+0.15	0.2	0.00076
COA0.05Z0.15	Al ₂ O ₃ + ZnO	0.05+0.15	0.2	0.00086

2.3. Performance metrics calculations

The following mathematical formulations have been used to analyse the performance parameters:

1. Net refrigerating effect (NRE):

$$\text{NRE} = h_1 - h_4. \quad (2)$$

2. Mass flow rate (m_r) to obtain one TR (210 kJ/min):

$$m_r = 210/\text{NRE}. \quad (3)$$

3. Work of compression:

$$\text{work of compression} = h_2 - h_1. \quad (4)$$

4. Theoretical power of compressor (kW):

$$\text{power} = (m_r \times \text{work of compression})/60. \quad (5)$$

5. Coefficient of performance (COP):

$$\text{COP} = \text{NRE}/\text{work of compression}. \quad (6)$$

6. Heat rejection (kJ/min):

$$\text{heat rejection} = m_r \cdot (h_2 - h_3). \quad (7)$$

Enthalpy values for R134a are obtained from the standard p - h chart with respect to temperature and pressure values of the refrigerant incorporated with nanomaterials in the refrigeration system:

$$h_1 = 388 \text{ kJ/kg},$$

$$h_2 = 450 \text{ kJ/kg}, \quad (8)$$

$$h_3 = h_4 = 220 \text{ kJ/kg}.$$

2.4. Uncertainty analysis

The uncertainty of system parameters, denoted as U_R , can be determined using the approach described by Schultz and Cole (1979). This method provides a systematic way to quantify the total uncertainty in a system, expressed mathematically as:

$$U_R = \left[\sum_{i=1}^n \left(\frac{\partial R}{\partial X_i} U_{X_i} \right)^2 \right]^{\frac{1}{2}}. \quad (9)$$

Here, U_R represents the overall uncertainty in the parameter R , while U_{X_i} corresponds to the uncertainty associated with each independent variable X_i . The summation accounts for all n independent variables influencing the parameter R .

This equation highlights how variations in individual variables contribute to the overall uncertainty of the system. By evaluating the partial derivatives $\partial R/\partial X_i$ and combining them with their respective uncertainties, a comprehensive understanding of the system's accuracy and reliability is achieved.

To ensure clarity and reproducibility, Table 5 presents the calculated uncertainties for each parameter considered in the study. All the evaluated parameters exhibit a maximum percentage uncertainty of less than 6%. This low percentage indicates that the system operates with a high level of precision, enhancing confidence in the reliability of the reported results.

Table 5. Uncertainty of calculated parameters.

Parameters	Uncertainty
P1 (Compressor entry pressure of refrigerant)	±3 kPa
P2 (Compressor exit pressure of refrigerant)	±5 kPa
T1 (Inlet temperature of compressor)	± 0.1°C
T2 (Outlet temperature of compressor)	± 0.1°C
T3 (Outlet temperature of condenser)	± 0.1°C
T4 (Temperature at evaporator inlet)	± 0.1°C
Wc (Work of compression)	± 1.89%
COP	± 3.11

3. Results and discussion

3.1. Net refrigeration effect

Figure 4 depicts the variation in the net refrigeration effect (NRE) of various refrigerant mixtures with Al₂O₃ and ZnO nanoparticle-enhanced compressor oils. For R134a, NRE increased from 168 kJ/kg for the pure compressor oil (PCO) to 177 kJ/kg for the mixture COA0.15Z0.05, showing that the addition of nanoparticles significantly improves the refrigerant's performance. R1234yf mixtures demonstrated a similar trend, with NRE rising from 118 kJ/kg in PCO to 127 kJ/kg in COA0.15Z0.05. This is due to the fact that adding Al₂O₃ and ZnO nanoparticles increases heat absorption capacity, leading to higher refrigeration effects. In contrast, R600a consistently displayed the highest NRE values among the refrigerants, starting

at 254 kJ/kg in PCO and reaching 265 kJ/kg in COA0.15Z0.05, showing superior thermal performance. The differences in NRE among the mixtures can be attributed to the enhanced thermal conductivity of the nanoparticles, which improves heat transfer rates. Among all mixtures, COA0.15Z0.05 exhibited the most significant performance improvement, likely due to the balanced combination of Al_2O_3 and ZnO . These findings highlight the effectiveness of nano-enhanced oils in improving the overall refrigeration capacity across various refrigerants. The same trend is also obtained in the study by Satapathy et al. [32].

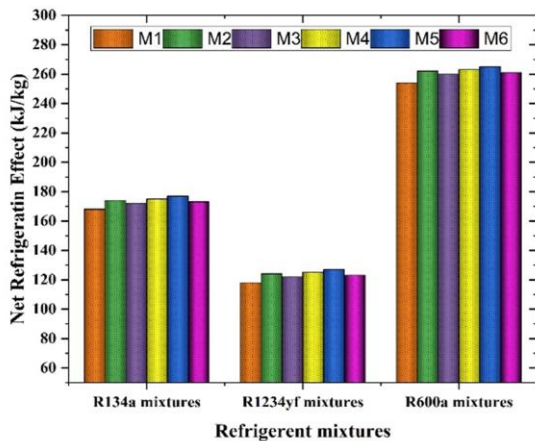


Fig. 4. Net refrigerating effect of different refrigerant mixtures.

3.2. Work of compression

Figure 5 demonstrates the work of compression across the different refrigerant mixtures. It demonstrates that the addition of Al_2O_3 and ZnO nanoparticles to compressor oil significantly reduces the energy required for compression. For R134a mixtures, the work of compression decreased from 62 kJ/kg in PCO to 53 kJ/kg in COA0.15Z0.05, indicating a substantial reduction in energy consumption. This is due to the enhanced lubrication properties provided by the nanoparticles, which reduce friction and improve the compressor's overall efficiency. A similar trend was observed for R1234yf, where the work of compression decreased from 48 kJ/kg in PCO to 39 kJ/kg in COA0.15Z0.05. The reduced compression work in these mixtures suggests that R1234yf benefits more from the nano-enhancements compared to R134a, potentially due to the refrigerant's lower viscosity and interaction with the nanoparticles. This reduced compression work can lead to overall system energy savings, making R1234yf mixtures more energy efficient when combined with nano additives. For R600a, the work of compression was higher compared to R134a and R1234yf, starting at 97 kJ/kg in PCO and dropping to 84 kJ/kg in COA0.15Z0.05. A similar trend is followed by Ajayi et al. [33]. Despite the higher initial values, the significant reduction in compression work for R600a demonstrates the effectiveness of the nanoparticles in reducing energy losses during the compression process. Overall, the mixture COA0.15Z0.05 consistently showed the lowest work of compression for all refrigerants, underscoring the positive impact of nanoparticles on system efficiency.

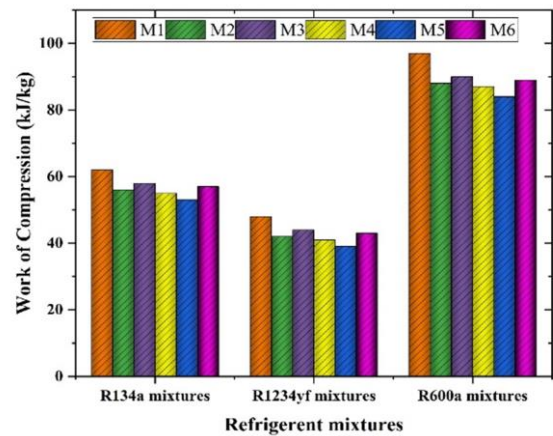


Fig. 5. Work of compression of different refrigerant mixtures.

3.3. Mass flow rate

The mass flow rate of the different refrigerant mixtures with the inclusion of Al_2O_3 and ZnO nanoparticles in the compressor oil is shown in Fig. 6.

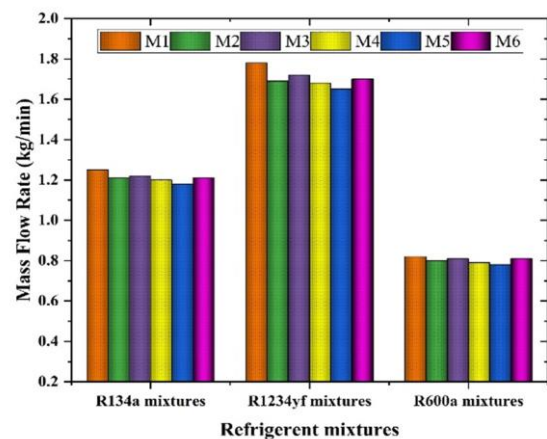


Fig. 6. Mass flow rate of different refrigerant mixtures.

For R134a, the mass flow rate decreased from 1.25 kg/min in PCO to 1.18 kg/min in COA0.15Z0.05. This reduction suggests improved efficiency in the heat transfer process, as a lower mass flow rate is required to achieve the desired cooling effect, potentially due to the enhanced thermal properties provided by the nanoparticles. R1234yf also demonstrated a reduction in mass flow rate, with values decreasing from 1.78 kg/min in PCO to 1.65 kg/min in COA0.15Z0.05, reflecting similar trends as R134a. This suggests that the addition of nanoparticles aids in reducing the refrigerant flow required, contributing to better energy efficiency in the system. Notably, R1234yf consistently exhibited the highest mass flow rates across all mixtures, which could be attributed to its lower thermodynamic properties compared to R134a and R600a. R600a showed the lowest mass flow rates overall, decreasing from 0.82 kg/min in PCO to 0.78 kg/min in COA0.15Z0.05. This further reinforces the efficiency of R600a as a refrigerant, which requires less refrigerant mass for the same cooling effect. Similar results were also dis-

cussed by Payne and O'Neal [34]. The reduction in mass flow rates across all refrigerants indicates that the nano-enhanced oils reduce friction and improve heat transfer performance, thus optimising system efficiency across the board.

3.4. Theoretical power of compression

The theoretical power of the compressor for the refrigerant mixtures with Al₂O₃ and ZnO nanoparticles introduced into the compressor oil is shown in Fig. 7.

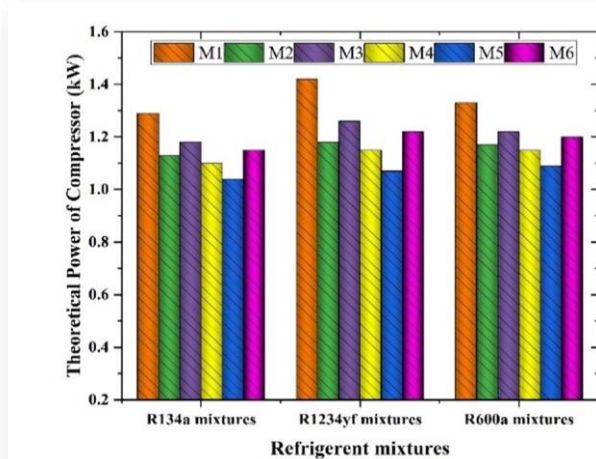


Fig. 7. Theoretical power of the compressor across different refrigerant mixtures.

For R134a, the compressor power decreased from 1.29 kW in PCO to 1.04 kW in COA0.15Z0.05. This significant reduction in power consumption indicates that the nanoparticles improve the lubrication and reduce mechanical losses within the compressor, thus enhancing overall efficiency. A similar pattern was observed with R1234yf, where the compressor power dropped from 1.42 kW in PCO to 1.07 kW in COA0.15Z0.05. The reduction in power demand highlights the positive impact of nanoparticle additives on the thermodynamic performance of R1234yf, which tends to require more power due to its slightly lower efficiency compared to R134a. The improved thermal conductivity and reduced friction achieved through the nanoparticles help mitigate this issue, leading to better energy efficiency. R600a, though displaying a lower power requirement than R1234yf, also showed improvements with the addition of nanoparticles. The theoretical power decreased from 1.33 kW in PCO to 1.09 kW in COA0.15Z0.05, reinforcing the overall trend across refrigerants. The consistent reduction in power demand across all refrigerant mixtures indicates that the Al₂O₃ and ZnO nanoparticles enhance the overall energy performance of the compressor by reducing internal resistance and improving heat transfer rates. Among all mixtures, COA0.15Z0.05 showed the most substantial reduction in compressor power, suggesting it is the optimal blend for enhancing compressor performance. This was validated by the study by Khan et al. [35].

3.5. Coefficient of performance

The coefficient of performance (COP) results for the refrigerant mixtures indicate a noticeable improvement with the inclusion of Al₂O₃ and ZnO nanoparticles in the compressor oil, as shown

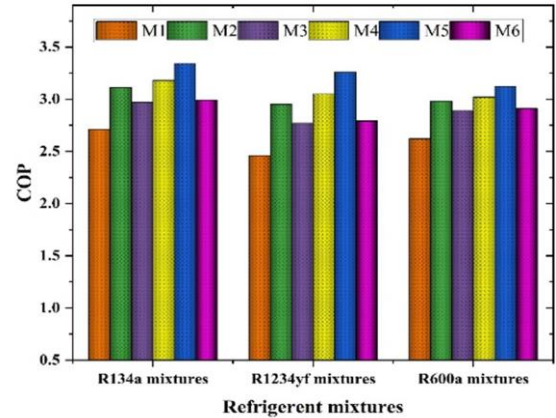


Fig. 8. COP of different refrigerant mixtures.

in Fig. 8.

For R134a, the COP increased from 2.71 in PCO to 3.34 in COA0.15Z0.05, showing a significant boost in performance. This increase in COP is due to the nanoparticles enhancing the refrigerant's heat transfer efficiency, enabling the system to achieve a higher cooling effect with less energy consumption. Similarly, R1234yf demonstrated a marked improvement in COP, rising from 2.46 in PCO to 3.26 in COA0.15Z0.05. The higher COP values for R1234yf when using nanoparticle-enhanced oils highlight the potential for improving the performance of lower-GWP (Global Warming Potential) refrigerants. The increased thermal conductivity from the nanoparticles helps compensate for R1234yf's slightly lower base efficiency compared to R134a, making it a more viable alternative with enhanced energy performance. For R600a, COP also showed a consistent increase, starting at 2.62 in PCO and reaching 3.12 in COA0.15Z0.05. While R600a already exhibits high energy efficiency, the addition of nanoparticles further improved its performance, suggesting that nano-additives can optimise the refrigerant's natural thermodynamic properties. Across all refrigerants, mixture COA0.15Z0.05 (0.15g Al₂O₃ + 0.05g ZnO) exhibited the highest COP, indicating it is the most effective combination for maximising refrigeration efficiency. A similar trend was shown by Dilawar et al. [40]. The overall improvements in COP demonstrate that using Al₂O₃ and ZnO nanoparticles in compressor oil enhances system performance, resulting in better energy efficiency across various refrigerants.

4. Conclusions

The study reveals that incorporating Al₂O₃ and ZnO nanoparticles into compressor oils significantly enhances the performance and efficiency of refrigerants R134a, R1234yf and R600a. The following are the concluding remarks from the study:

- The coefficient of performance (COP) for R134a increased by 23.3%, reaching 3.34 from an initial 2.71. Similarly, R1234yf showed a 32.5% increase in COP, rising to 3.26 from 2.46, while R600a experienced a 19.1% improvement, reaching 3.12 from 2.62. This enhancement in COP reflects a notable increase in cooling efficiency.

- The compressor power decreased by 19.4% for R134a, from 1.29 kW to 1.04 kW, and by 24.6% for R1234yf, from 1.42 kW to 1.07 kW, with R600a showing an 18.0% reduction.
- The work of compression also showed reductions, with R134a decreasing by 14.5%, R1234yf by 18.8%, and R600a by 13.4%.
- Additionally, the mass flow rate decreased for all refrigerants, with R134a reducing by 5.6%, R1234yf by 7.3%, and R600a by 4.9%.
- The net refrigeration effect (NRE) improved as well, with R134a increasing by 5.4%, R1234yf by 7.6%, and R600a by 4.3%.

Overall, it was observed that COA0.15Z0.05 showed the most substantial improvement in the performance of refrigerators compared with non-hybrid nano-lubricant mixtures (PCO, COA0.2, M3), suggesting it is the optimal blend for enhancing refrigerator performance. Further research can be carried out on the stability of nanoparticles in base lubricant, also with different nanoparticles like graphene oxide, CNTs, etc.. Nanoparticle-enhanced oils raise upfront costs, necessitating a cost-benefit analysis. Long-term effects on compressor materials need investigation, including wear on seals and bearings.

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Experimental analysis of ethanol-diesel blends stabilised with jatropha methyl ester: Engine performance and emission characteristics

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Abstract

The growing environmental challenges and the rapid depletion of global fossil fuel reserves have driven the urgent need to explore alternative energy sources. In present work, an experimental investigation evaluated the stability of ethanol-diesel blends using jatropha methyl ester (JME) as a co-solvent, alongside engine performance and emissions at varying injection pressures and timings. Stability tests revealed that ethanol cannot blend with diesel without additives, requiring at least 4% JME for one-day stability, with E10B10D80 (10% ethanol, 10% JME, and 80% diesel by volume) blends remaining stable above 10°C. Optimal injection parameters were identified as 2.0×10^7 Pa pressure and 17° before top dead centre (BTDC) under different loads. JME proved effective as an additive, though its cost was higher than diesel, suggesting its long-term viability as fossil fuel resources diminish. Fuel consumption increased due to ethanol's lower calorific value, while thermal efficiency improved at low loads but decreased near full load. Emission analysis indicated that carbon monoxide (CO) emissions were lower at loads above half but higher at lower loads compared to pure diesel. Hydrocarbon (HC) emissions consistently rose with the blend, while a reduction in the nitrogen oxides (NO_x) emissions was observed at relatively lower load but increased near full load, showing no consistent trend. The study highlights the potential of JME as a biofuel additive, with its economic feasibility expected to improve as reliance on fossil fuels declines.

Keywords: Jatropha methyl ester; Diesel engine; Engine performance; Emission; Clean energy

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1. Introduction

Harnessing energy plays a pivotal role in the socio-economic development of the nation. However, the relentless exploitation

of conventional energy resources has led to the quest for potential alternatives, prompting researchers to find potential alternatives [1]. The utilisation of conventional fuels has made various harmful impacts on the Earth, and one such impact is

Nomenclature

BTE – brake thermal efficiency, %
 BSFC – brake specific fuel consumption, kg/kWh
 BMEP – brake mean effective pressure, Pa
 EGT – exhaust gas temperature, K

Abbreviations and Acronyms

BTDC – before top dead centre
 CO – carbon monoxide
 EBD – blended fuel, i.e. ethanol (E), biodiesel (B) and diesel (D)
 JME – jatropha methyl ester
 NO_x – nitrogen oxides
 GHGs – greenhouse gases
 HC – unburned hydrocarbon

global warming [2]. This results in frequent shifts in climatic patterns, causing disasters. The emergence of biofuels can be viewed as a valuable alternative, serving as a bridge between economic viability and environmental sustainability [3]. Biofuels are usually derived from biomass resources, i.e. organic materials from plants and animals. Some examples of these organic materials are sugar, starch crops, oilseed crops, animal fats, algae, jatropha and many more [4,5]. These are processed and converted into by-products for obtaining bio-alcohols like ethanol, butanol, biodiesels and many more that offer a significant reduction in greenhouse gases, creating a more environmentally friendly substitution than conventional fuels [6].

In the past decades, ethanol-based engines were used in indirect combustion. But nowadays, ethanol has been used as a blending agent with advanced combustion methods that improve the overall system's performance and meet new regulations [7]. For instance, blending alcohol-based fuels (ethanol and methanol) with natural gas improves the pressure inside the chamber, resulting in proper combustion of fuels and reducing the NO_x emission as reported by Chen et al. [8]. Literature [9,10] proposed direct injection of ethanol and exhaust gas recirculation (EGR), which is being especially utilised in turbocharged engines, improving the overall engine efficacy and lowering harmful emissions, increasing overall utility. It was observed that direct injection enables charge stratification, which enables high performance and reduces fuel consumption, leading to a reduction in harmful pollutants [11]. Recently, blending of alcohols with primary fuels has gained popularity as it can improve the combustion chamber temperature and significantly reduce harmful gas emissions. Besides this, ethanol has also been utilised in pre-chamber ignition systems as a lean mixture [12,13].

Apart from ethanol, oil obtained from Jatropha has been emerging as a potential blending agent for biodiesel production. These plants are easy to cultivate, drought-tolerant, and have a high oil content (30–50%) compared to other biodiesel sources [14]. A notable example of large-scale Jatropha cultivation can be observed in Europe. To enhance oil yield, pre-heat treatment is applied to the seeds before oil extraction. Reports suggest that 1 kg of biodiesel can be produced from approximately 1.1 kg of raw jatropha oil [15]. Jatropha oil exhibits a calorific value comparable to that of conventional gasoline. According to Ong et al. [16], biodiesel and jatropha oil exhibit a calorific value of 38.96 MJ/kg and 40.42 MJ/kg, respectively. Despite this, jatropha-derived biodiesel has several advantageous properties, including lower viscosity, which enhances its volatility and makes it easier to transport.

The performance evaluation of jatropha biodiesel demonstrates its potential as a viable alternative fuel, showing satisfactory results with no significant technical challenges. Jatropha biodiesel exhibits high brake thermal efficiency (BTE) at a low fuel consumption rate. As per an investigation conducted by Sahoo et al. [17], the utilisation of a blend that is composed of 20% jatropha in biodiesel results in brake power values ranging from 0.09% to 2.64% higher compared to conventional diesel fuel. Interestingly, an increase in the value of brake-specific fuel consumption (BSFC) depends on higher blend ratios and decreases with increasing engine speed for all biodiesel blends. Similarly, brake-specific energy consumption (BSEC) is supposed to increase when the blend ratio is higher, but unfortunately, it decreases with an increase in the speed of the engine. Notably, the maximum increase in BSEC at 1200 rpm was recorded at 20.21% for B100, which exceeds that of conventional diesel.

Greenhouse gas (GHG) emissions have been observed to be significantly reduced with the utilisation of the jatropha biodiesel blends across various engine speeds. For instance, the B100 (pure biodiesel), B50 (50%biodiesel) and B20 (20% biodiesel) blends reduce harmful gas emissions by 64.28%, 40.9% and 28.57%, respectively, for an engine working at 2200 rpm. But if the engine speed goes down to 1200–1600 rpm, then B20 fuel showed a minimum GHG reduction of 1.29%. Upon increasing the engine speed to higher side, 1800–2200 rpm, the reduction in GHG emissions significantly increased and reached 15.84%. Notably, B100 demonstrated a maximum smoke reduction exceeding 60% at engine speeds between 1200 and 1600 rpm. However, increasing the biodiesel content in the blend generally affects the power output of the engine [18].

Huang et al. [19] investigated the performance and emissions of a diesel engine where a novel biodiesel was used, which was composed of jatropha and Chinese pistache oil. The output of the same was compared with the output of pure diesel. Their findings revealed that at higher engine speeds, if the weightage of biodiesel increases in the blend, then the BSFC can increase and reach a maximum of 6.8%. Improvements in BTE were marginal, ranging from 0.1% to 6.7%. At an engine speed of 1500 rpm, GHG emission reductions ranged from 8% to 35%, while at 2000 rpm, the reductions varied between 12% and 57%.

The aforementioned literature shows the potential of ethanol and jatropha as biofuels. However, their utility is not yet fully explored, and global petroleum prices have further intensified the urgency to identify cost-effective and environmentally friendly energy solutions. However, an appropriate blending

of organic materials with diesel can be a durable solution at an affordable price. Based on the literature review, ethanol and jatropha oil have demonstrated their potential to develop a more sustainable fuel. But their blending is a tedious task that needs to be addressed properly. Therefore, in the present study combination of ethanol and jatropha blended biofuels at various proportions, along with their stability, is studied. The engine performance with the blended fuel at optimum injection pressure and timing is also studied. Besides this, engine exhaust emission was studied to evaluate the amount of emission of total carbon monoxide and unburned hydrocarbon. Lastly, a thorough comparison of the developed biodiesel was performed with diesel fuel alone to support the study.

2. Materials and methods

This section deals with the materials used and methods undertaken to achieve the objectives of the present investigation. It encompasses the utilisation of jatropha oil to prepare methyl ester, preparation of blends and examination of its stability, examination of prepared fuel properties and development of an experimental setup.

2.1. Preparation of methyl ester from jatropha oil and different blends

In the present work, the reactants used are jatropha oil, methanol and sodium hydroxide (NaOH). The glass flask was used to contain the mixture and execute the stirring magnetically. A digital weighing machine was used to measure the precise amount of catalyst to be utilised in the reaction. The flask used for mixing the methanol and NaOH was airtight. A separating funnel was used to collect the biofuel and by-product separately. In order to clamp and hold the magnetic stirrer, a dedicated stand and clamp were installed in the experimental setup. Heating was provided to the oil by using a heater, and to filter the oil, a special filter paper was used. The following procedure was opted for obtaining JME (jatropha methyl ester) from jatropha oil. A systematic approach was adopted to produce biodiesel from jatropha oil.

Initially, a known quantity of jatropha oil, based on the capacity of the flask, is filtered using filter paper and heated to 60°C. This step removes moisture and reduces the oil's viscosity to facilitate better mixing with conventional fuel; care was taken to avoid overheating, which could break the molecular bonds. The amount of methanol and NaOH required was determined by the volume of jatropha oil used. Through experimentation, it was found that using 200 ml of methanol and 10 g of NaOH per 1000 ml of oil consistently resulted in successful transesterification. The measured methanol and NaOH were placed in an airtight glass flask, which was then vigorously shaken manually in a circular motion until the NaOH dissolved completely, forming sodium methoxide. This reaction is exothermic, causing the flask to warm slightly.

The prepared sodium methoxide was carefully poured into the reactor containing jatropha oil using a burette, ensuring no contact with the mixture or inhalation of fumes. A condenser

was then attached to the reactor opening to prevent oxidation of the sodium methoxide, and the mixture was stirred continuously for 3 to 4 hours using a magnetic stirrer. After the stirring was completed, the mixture was transferred with the help of a separating funnel. The mix was then left to settle down, undisturbed, for a period of approx. 10 hours. During this settling-down period, the glycerin settled at the bottom as it was composed of free fatty acids. On the other hand, at the top, the biodiesel (jatropha methyl ester) floated. Glycerin was then carefully drained from the funnel, leaving only the biodiesel.

To purify the biodiesel, hot water was added to the separating funnel containing the jatropha methyl ester. The funnel was capped, shaken manually in circular motions, and inverted multiple times. This leads to the separation of water that contains impurities by its settling down at the bottom and followed by the draining. This washing was carried out up to 12 times until the obtained drained water had a pH value between 6 and 7. It is to be noted that the soap formation in the mix almost stopped when the pH value of the drained water reached 7.

In order to remove any moisture from the purified jatropha methyl ester, it was heated in an open environment up to 70°C. Biodiesel prepared in this way should be kept in an airtight container following the confirmation test.

Figure 1 depicts the visual comparison of ethanol-biodiesel-diesel, jatropha methyl ester (biodiesel) and high-speed diesel. After the preparation of jatropha methyl ester (biodiesel), a qualitative examination was performed for confirmation, including:

- **Visual confirmation:** The biodiesel should be light golden in colour; at least 80% of methyl ester must be produced from the raw jatropha oil after the completion of transesterification.
- **Smell:** The smell of jatropha oil is offensive, while the produced methyl esters smell pleasant and sweet.
- **Chemical examination:** The Iodoform test was used to confirm the formation of jatropha methyl ester (biodiesel) after transesterification. A few drops of Iodoform were mixed with methyl ester in a separate beaker. On the addition of Iodoform (NaOI), a sweet, fruity smell emerged, which confirms the presence of methyl ester.

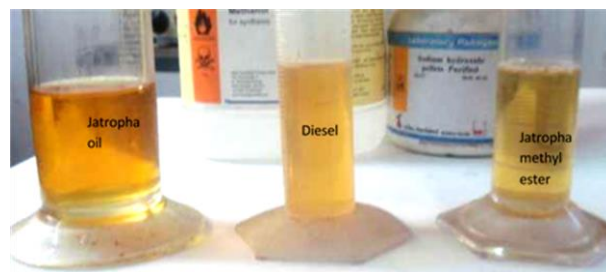


Fig.1 Visual comparison of diesel, jatropha oil and jatropha methyl ester.

A simple mixing technique was adopted to prepare the blend in which ethanol, JME and high-speed diesel (HSD)

were mixed together. The ratio of fuels was measured in volume before mixing them and characterised as E for ethanol, B for biodiesel, and D for diesel. Different types of blends prepared in the study are illustrated in Table 1 along with their mixing volume.

2.2. Experimental setup

For testing of the fuel, an AV1 engine of the company make Kirloskar was selected. It is a single cylinder, cooled by water and working on a 4-stroke mechanism. It is capable of producing 3.7 kW of power at an rpm of 1500. Several arrangements that are made on the engine are as follows:

1. An Eddy current dynamometer was installed in the engine to measure the torque.
2. A coolant water temperature measuring device and thermocouples for measuring the temperature of intake air and exhaust air were installed.
3. Spinner and pliers were incorporated for changing the injection pressure and timing.
4. A burette, as well as a stopwatch were installed to measure fuel consumption.
5. The flow of water was measured by a flow meter installed.

Necessary technical details of the Kirloskar engine AV1 are provided in Table 2.

The dynamometer used in the study, sourced from Power Star, powered by electric current, was connected to the engine to measure the torque. An alternator was coupled with a dynamometer and, consequently, with bulbs to give a load to the engine. A burette of volume 50 cm³ was used to measure the HSD, and its blend comprised ethanol and biodiesel. The mixture obtained in the burette was sent to the engine via the T-valve. During the operation, the fuel consumed by the engine is supplied only by the burette as the T-valve is closed, cutting the connectivity from the fuel tank. A stopwatch was utilised to measure the time elapsed in consuming a certain volume (20 cm³) of fuel by the engine. Hence, the flow rate of volume was then calculated by dividing volume by the time recorded

Table 2. Technical specification of the AV1 Kirloskar engine.

Number of cylinders	One
Bore × Stroke	80 mm × 110 mm
Cubic Capacity	0.553 L (553 cm ³)
Compression ratio	16.5 : 1
Rated output as per BS5514/ISO 3046/ IS 10001	3.7 kW (5.0 hp) at 1500 r/min
Injector opening pressure	2.0 × 10 ⁷ Pa
Injection timing	21° BTDC

on the stopwatch.

The temperature at different zones of the engine is measured with the help of thermocouples, which are installed in to six-channel selector switch. These thermocouples are also used to measure the temperature of exhaust gas, as well as cooling water. The injector opening pressure can be adjusted by modifying the spring load on the injector needle, using a screw located on top of the injector. Additionally, the injection timing can be altered by inserting shims beneath the pump top plate. Each shim, with a thickness of 0.16 mm, corresponds to a 2° retardation in timing. Initially, two shims were positioned at 21° before TDC (top dead centre). The number of shims varied from 2 to 5. A suitable gas analyser, AVL Di 4000, was utilised to measure various emissions (carbon monoxide (CO), nitrogen oxides (NO_x) and hydrocarbons) from the engine.

The concentrations of gases coming out of the exhaust valve are identified by the use of a filtered tube connected to the electrochemical sensors. The readings recorded on the panel of the electrochemical cell have a direct relation with the volume concentration of elements detected on it, and it is expressed in parts per million (ppm).

The engine was operated at a constant speed of 1500 rpm. The accuracies were: fuel measurement ±0.5 ml, torque ±0.1 Nm and emissions ±5 ppm.

The engine performance was determined by utilising two types of fuel, viz. diesel fuel and a blended fuel (ethanol, biodiesel and diesel). Important properties of these blends used in this study, such as viscosity, density, calorific value and cetane number of these test fuels were measured in the laboratory and are compared with diesel (Table 3). Several experiments were performed to examine the performance of the engine and the properties of exhaust gases. For this, the pressure at injection was varied, and also, the timing of injection was varied. The monitoring of the inlet and outlet points of cooling water was carried out during the experiment. Intake of air and exhaust air were also monitored for each change in the load. The engine was allowed to relax and reach a stable state after every change

Table 1. Stability of EBD blends with respect to time

No.	Jatropha methyl ester (%)	Ethanol (%)	Diesel (%)	Separation time	Stability
1	3	17	80	Unstable	Unstable
2	4	16	80	1	Unstable
3	5	15	80	2	Unstable
4	6	14	80	4	Unstable
5	7	13	80	12	Unstable
6	8	12	80	15	Unstable
7	9	11	80	20	Unstable
8	10	10	80	Till date	Stable
9	11	9	80	Till date	Stable
10	12	8	80	Till date	Stable
11	13	7	80	Till date	Stable
12	14	6	80	Till date	Stable
13	15	5	80	Till date	Stable

Table 3. Fuel properties of diesel and EBD blend

Properties	Diesel	EBD
Calorific value (MJ/kg)	44.8	43.5
Specific gravity (kg/m³)	0.815	0.8331
Viscosity at 27°C	4.3	—
Cetane number	47	46.85

in the loading condition.

The values of injector opening pressure that are considered for the analysis are as follows: 1.86×10^7 , 1.96×10^7 , 2.06×10^7 , 2.16×10^7 and 2.26×10^7 Pa. The preliminary examination revealed that the best output was attained for the base fuel, i.e. diesel at a pressure of 2.0×10^7 Pa, while in the case of blended fuel, the best output was attained at 1.96×10^7 Pa. The variation in the timing of injection was set in the range between 21° to 17° before TDC. This was achieved by placing shims just below the top of the pump plate. Each shim plate has a 16 mm thickness, which is equal to 2° retardations of the timing of injection. At the full load, the AV1 engine of make by Kirloskar produces 3.7 kW of power. Therefore, for the analysis, the load was varied from minimum (0.5 kW or 72332.73 Pa BMEP) to maximum (3.5 kW or 506329.11 Pa BMEP). The loading on engine was performed by switching on the bulb of specific wattage. At first, the engine runs only on diesel so that baseline data could be formed. Thereafter, the prepared fuel (E10B10D80) was used to run the engine to check the performance and to make a comparison of the prepared fuel with diesel fuel.

3. Materials and methods

Prior to the start of the experiment, the aim was to find the optimum injector opening pressure and timing for the prepared fuel, i.e. EBD. Among the selected injection pressures, as specified in section 2.2, 1.96×10^7 Pa was learned to be the optimum injector opening pressure and shim number of 4 was noted as optimum for the injection timing, at which minimum BSFC was recorded and maximum BTE was obtained. Therefore, the results that are going to be discussed afterwards are based on the injection pressure of 1.96×10^7 Pa and at 17° before TDC.

The parameter that suggests how efficient an engine is in converting fuel into output work is BSFC. To examine BSFC of diesel and prepared fuel, Fig. 2 has been plotted at different loading conditions. Upon giving a close observation to the results depicted in the figure, it was learned that as the load on the engine increases, BSFC decreases. However, once a particular load point is reached, BSFC begins to increase. In Fig. 2, the load at which BSFC has its minimum is referred to as the best economical mixture, which in this case was obtained at 80–85 % of loading for both diesel and EBD blend. For all cases of loading, BSFC was noted to be higher for E10B10D80 as compared to the BSFC for diesel. It is also apparent that at 0.5 kW or 72332.73 Pa BMEP of loading, BSFC was recorded slightly higher than BSFC for diesel. As the load on the engine increases, the BSFC for EBD starts decreasing until the load reaches 2.5 kW or 361663.65 Pa BMEP. But, as the load increases beyond 2.5 kW or 361663.65 Pa BMEP, BSFC shows an increment. The higher value of BSFC for the case of EBD was attributed to the lower heating as compared to diesel. This suggests that to obtain a particular amount of output, more EBD fuel has to burn out when compared to pure diesel.

The examination of BTE was carried out to calculate the ability of engine in terms of net power output. A graph showcasing the variation of BTE against the applied load for both diesel and EBD is presented in Fig. 3. It has been found that

the engine runs on both diesel and EBD, showing an increase in BTE for every increase in the load on the engine. Notably, the highest BTE was obtained at 2.5 kW or 361663.65 Pa BMEP loading of the engine. The reason behind the higher BTE in the case of EBD fuel is attributed to the greater consumption of fuel and low content of energy for the same power output.

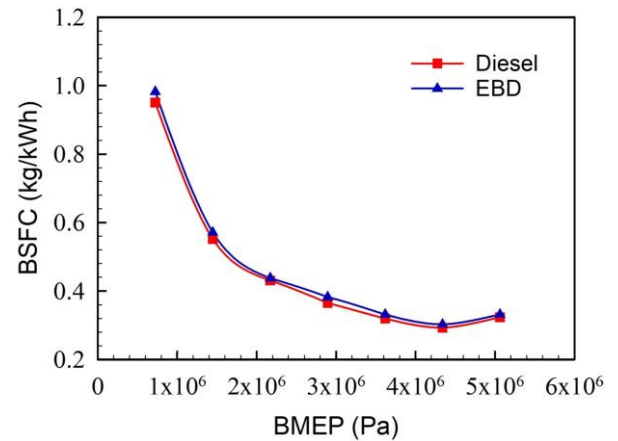


Fig. 2. Effect of load on the brake specific fuel consumption.

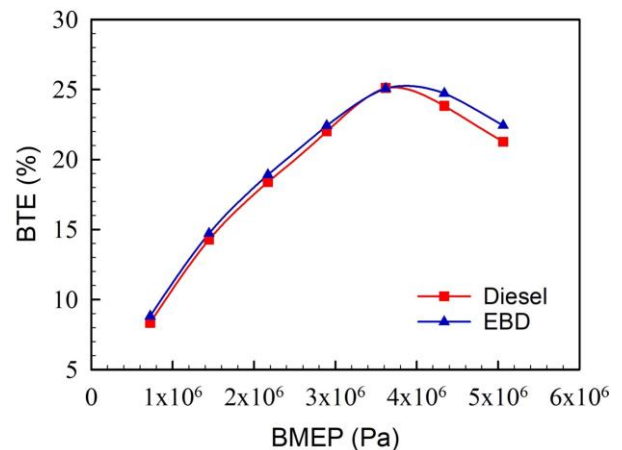


Fig. 3. Effect of load on brake thermal efficiency.

The temperature of gas exiting the cylinder, termed as exhaust gas temperature (EGT), signifies the condition of combustion and the state of ignition. For the present study, the EGT was measured for all the conditions assumed in the experiment. The variation of EGT with the change in the loading condition for both ED and diesel fuel is represented in Fig. 4. Observations revealed that with an increase in the load on the engine, the EGT increases for both the fuels, whether it is diesel or EBD. For a fixed period of time, the heat generation increases due to an increase in the load that leads to an increase in the EGT. Apparently, the EGT for the case of EBD was much lower as compared to EGT for the case of diesel in all loading conditions. The probable cause for this is an improved atomisation of the fuel during the combustion due to the comparatively low viscosity of EBD fuel.

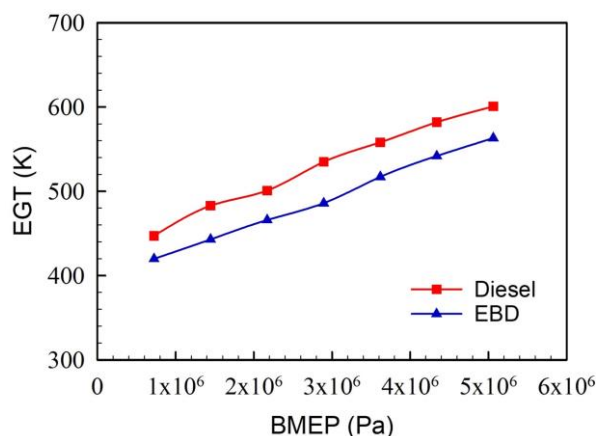
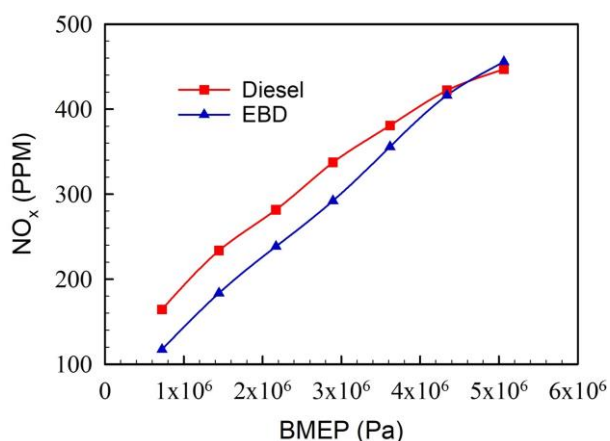


Fig. 4. Effect of load on the exhaust gas temperature.

In Figs. 5–7, the emissions of various exhaust gases at different loading conditions are depicted. The data collected on nitrogen oxides (NO_x) emissions is plotted in Fig. 5. It can be observed from Fig. 5 that the level of emission of NO_x was lower for EBD as compared to diesel. However, at full load conditions, NO_x for EBD was found to be higher than diesel. The emissions of NO_x were observed to rise progressively as the engine load increased across all tested fuel blends. This trend can be attributed to the higher quantity of fuel injected and burned within the engine cylinder under the elevated loads. The increased combustion activity led to a significant rise in cylinder gas temperatures, which, in turn, accelerated the formation of NO_x within the engine. Consequently, the higher temperatures resulted in the elevated NO_x emissions being released from the engine. These findings underscore the critical role of combustion temperature as a key factor influencing NO_x emissions. As the combustion process intensifies with greater fuel input and load, the elevated thermal conditions create an optimal environment for NO_x formation, highlighting the direct correlation between engine load, combustion temperature and NO_x output.

Figure 6 illustrates the carbon monoxide (CO) emissions

Fig. 5. Variation of NO_x emissions with load.

produced by the engine under various load conditions. At lower engine loads, the CO emissions were notably higher when the engine was operated with ethanol-diesel blends compared to pure diesel fuel. This increase in CO emissions was more pronounced with higher ethanol content in the blends. However, as the engine load increased beyond half of its capacity, a reversal in this trend was observed. Under these higher load conditions, the CO emissions from the ethanol-diesel blends were consistently lower than those from pure diesel. The most significant reductions in CO emissions occurred at the maximum engine load tested, highlighting the potential benefits of ethanol-diesel blends in reducing CO emissions under high-load operating conditions. This behaviour can be attributed to improved combustion efficiency at elevated loads, which offsets the incomplete combustion characteristics observed at lower loads.

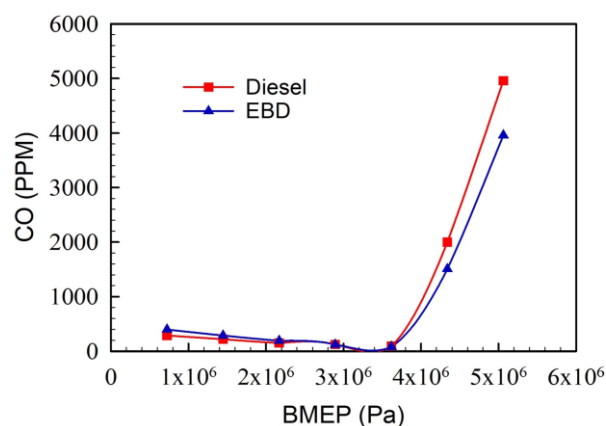


Fig. 6. Variation of CO emissions with load.

The EBD blend shows a decrease in hydrocarbon (HC) emissions compared to mineral diesel (Fig. 7). This reduction is likely due to the improved combustion of biodiesel blends, attributed to the presence of oxygen. In fact, HC emissions further decreased as the engine load increased.

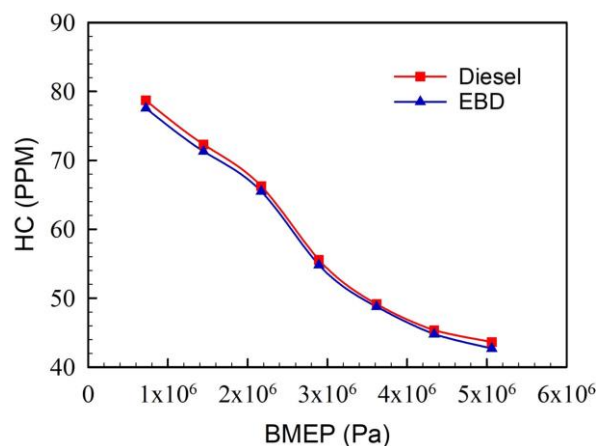


Fig. 7. Variation of unburnt hydrocarbon emissions with load.

4. Conclusions

An experimental study was conducted to examine the stability of ethanol-diesel blends and assess the engine performance parameters and emissions characteristics of these blends when combined with jatropha methyl ester (JME) as a co-solvent. The tested fuel mixture consisted of 10% ethanol, 10% JME, and 80% diesel fuel by volume. The engine was operated using this blend under varying injection pressures, ranging from 190 to 220 MPa, and different injection timings, at various load conditions. Key findings from the investigation are as follows:

1. Ethanol could not form a stable blend with diesel without the inclusion of an additive such as JME. A minimum of 4% JME (by volume) in an 80% diesel fuel mixture was necessary to achieve stability, but the blend remained stable for only one day. When the additive percentage was below 4%, blending was unsuccessful. The E10B10D80 blend remained stable only when the temperature did not fall below 10°C.
2. JME proved to be an effective additive for facilitating ethanol-diesel blending, despite being more expensive than diesel during the study period. Over the long term, as fossil fuel reserves diminish, biofuels like JME are expected to become economically viable alternatives.
3. Engines fuelled by the blend showed higher fuel consumption compared to pure diesel due to the lower calorific value of ethanol.
4. Thermal efficiency improved at lower loads when using the blend and diesel, but decreased near full load.
5. Carbon monoxide (CO) emissions varied with engine load. Above half load, CO emissions from the blend were lower than those from pure diesel, while below half load, CO emissions were higher.
6. Unburned hydrocarbon (HC) emissions were consistently higher when using the EBD blend as compared to diesel.
7. Nitrogen oxide (NO_x) emissions decreased at low engine loads, but increased near full load. However, NO_x emissions did not follow a consistent trend across different operating conditions.

This study underscores the potential of JME as a co-solvent for ethanol-diesel blends, while highlighting the trade-offs in performance and emissions associated with its use.

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Investigating the Impact of TiO₂ Nanoparticles on Waste Plastic Pyrolysis Oil and DEE Diesel Blends for Diesel Engine Performance and Emission Optimisation

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Abstract

The growing environmental concerns and fossil fuel depletion have spurred interest in alternative fuels, with waste plastic oils emerging as a solution. Waste plastic oils not only help to reduce plastic waste but also contribute to energy production. This study investigates the effects of incorporating TiO₂ nanoparticles into waste plastic pyrolysis oil (WPPO) and diethyl ether (DEE) blended with diesel fuel on engine working characteristics. The experimental analysis evaluated four fuel blends: pure diesel, WPD20 (20% WPPO in diesel), WPD20E5 (WPD20 with 5% DEE), WPD20E5T50 (WPD20E5 with 50 ppm TiO₂ nanoparticles), and WPD20E5T100 (WPD20E5 with 100 ppm TiO₂ nanoparticles) under various load conditions. The results demonstrated that the WPD20E5T100 blend achieved the highest brake thermal efficiency and the lowest brake specific fuel consumption, with improvements of up to 5.26% and 8.31%, respectively, in comparison to neat diesel. The blend also exhibited the most significant reductions in CO and unburnt hydrocarbon emissions, with reductions of up to 6.86% and 11.9%, respectively. Smoke emissions were notably lower with the TiO₂ nanoparticle-enhanced blends, achieving an 11.8% improvement over diesel. Although NO_x emissions were slightly higher with DEE, the WPD20E5T100 blend showed the maximum reduction in NO_x emissions, with a reduction of 5.7% at full load. Overall, the WPD20E5T100 blend produced the optimum results at all loads.

Keywords: Waste plastic pyrolysis oil; Diethyl ether; TiO₂ nanoparticles; Diesel engine; Emissions, Performance

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1. Introduction

Growing concerns about the environment and depletion of fossil fuels have driven the exploration of alternative fuel sources, leading to the development of biodiesels [1], waste cooking [2] and waste plastic oils [3] as sustainable fuel options. Biodiesel is produced from vegetable oils and animal fats, and offers a cleaner-burning alternative to petroleum diesel. It is typically produced through transesterification, where triglycerides react with alcohol to form fatty acid methyl esters, the primary component of biodiesel. Biodiesels from non-edible sources, like

jatropha, Alexandrian laurel, and karanja, are particularly attractive because they do not compete with food crops. These bio-diesels have demonstrated the ability to lower greenhouse gas emissions and improve engine performance with minor modifications [4]. On the other hand, waste plastic oils (WPO) offer a way to convert non-biodegradable plastic waste into fuel through a process known as pyrolysis. WPOs provide dual benefits: reducing plastic pollution and contributing to energy generation. Waste plastic pyrolysis oils (WPPO) are derived through the thermal degradation of plastic waste, producing a liquid fuel suitable for use as fuel in diesel engines [5]. Several

Nomenclature

Abbreviations and Acronyms

Al₂O₃ – aluminium oxide
 BSFC – brake specific fuel consumption
 BTE – brake thermal efficiency
 CeO₂ – cerium oxide
 CO – carbon monoxide
 CO₂ – carbon dioxide
 DEE – diethyl ether
 HC – hydrocarbon
 MgO – magnesium oxide

NO_x – nitrogen oxides
 PM – particulate matter
 TiO₂ – titanium dioxide
 UBHC – unburnt hydrocarbon
 WPD20 – 80% diesel, 20% WPPO
 WPD20E5 – 80% diesel, 20% WPPO and 5% diethyl ether
 WPD20E5T50 – 80% diesel, 20% WPPO, 5% diethyl ether and 50 ppm TiO₂ nanoparticles
 WPD20E5T100 – 80% diesel, 20% WPPO, 5% diethyl ether and 100 ppm TiO₂ nanoparticles
 WPPO – waste plastic pyrolysis oil
 ZnO – zinc oxide

studies have evaluated the potential of WPPO as a substitute for diesel [6]. Murugan et al. [7] examined the use of WPPO in a single-cylinder diesel engine and found that the WPPO mixture showed lower combustion efficiency in comparison to neat diesel, which led to an increase in unburnt hydrocarbon (UBHC) and CO emissions. The study attributed these higher emissions to poor atomization and incomplete combustion, which are typical challenges associated with WPPO due to its lower volatility. Yaqoob et al. [8] also evaluated the working characteristics of a diesel engine running on WPPO-diesel blends and revealed that NO_x emissions were reduced due to decreased temperatures in the cylinder, but UBHC and CO emissions notably increased compared to those from conventional diesel fuel. The study suggested that the presence of heavy aromatic hydrocarbons in WPPO contributed to the incomplete combustion, leading to higher levels of CO and UBHCs. Moreover, the increased viscosity of WPPO in comparison to diesel led to inadequate fuel atomization, which contributed to incomplete combustion and elevated emissions of unburnt hydrocarbons. Januszewicz et al. [9] examined the effect of different WPPO-diesel blends on engine performance and emissions. The outcomes have shown that a higher blending ratio of WPPO in the fuel blend led to reduced NO_x emissions but there was a rise in CO and UBHC emissions. This was attributed to the decreased amount of oxygen in WPPO, which hindered complete combustion and resulted in the formation of more unburnt fuel particles. The study also highlighted that WPPO blends up to 20% performed well. Wongkhorsub et al. [10] carried out an experimental investigation to assess the emissions from a diesel engine operating on WPPO-diesel blends. Their findings were consistent with earlier studies, showing that WPPO blends decreased emissions of NO_x while UBHC and CO were raised. The researchers observed that the reduction in NO_x emissions could be linked to the decreased flame temperature during combustion, which hindered the generation of thermal NO_x. Comparable results were noted by Mani et al. [11] in their study on the performance of a diesel engine powered by WPPO-diesel blends. They found that WPPO blends resulted in higher levels of emissions of UBHC and CO. This was mainly due to the incomplete oxidation of hydrocarbons during combustion. However, the study also highlighted a reduction in NO_x emissions, which was attributed to the lower combustion temperatures associated with WPPO. The lower peak combustion temperature suppressed the formation of thermal NO_x, thus reducing overall NO_x emissions. Despite this ben-

efit, the higher levels of unburnt hydrocarbon (UBHC) and CO emissions pose a significant challenge to the widespread adoption of waste plastic pyrolysis oils (WPPO) as a fuel.

Various strategies, including blending WPPO with oxygenated additives have been explored to mitigate these challenges [12,13]. Various studies have explored the fuel additives and oxygenated compounds to improve combustion efficiency and reduce emissions. Diethyl ether (DEE), an oxygenated additive, has been investigated for its potential to enhance the combustion characteristics of alternative fuels like WPPO [14]. Because of its high oxygen concentration and cetane number, DEE promotes better fuel atomization, faster ignition, and more complete combustion. Kumar et al. [15] reported that adding DEE to WPPO-diesel blends improved engine performance, particularly brake thermal efficiency (BTE) and brake specific fuel consumption (BSFC), while also reducing CO and hydrocarbon (HC) emissions. The presence of DEE led to better combustion, thereby lowering the levels of harmful exhaust emissions. However, the study also noted that NO_x emissions remained relatively high, which may be attributed to the higher combustion temperatures due to DEE. Samraj et al. [16] conducted an experimental analysis on a diesel engine fuelled with DEE-diesel blends and observed a significant reduction in smoke opacity and CO emissions. This reduction was attributed to the oxygenated nature of DEE, which improved the oxidation of fuel during combustion. Kumar et al. [17] found that DEE improves combustion efficiency in biodiesel blends, reducing UBHC and CO emissions. Similarly, Kaimal et al. [18] reported that DEE in WPPO blends improves combustion while reducing UBHC and CO, but slightly increases nitrogen oxide emissions due to increased temperatures in the cylinder. Sezer et al. [19] highlighted DEE's potential in reducing particulate matter (PM) and improving combustion in diesel-biodiesel blends, with similar reductions in CO and UBHC. Rakopoulos et al. [20] studied DEE as an additive in a diesel-WPPO blend and found that DEE lowers the exhaust emissions but there was a reduction in engine performance. Varpe et al. [21] reported similar findings in biodiesel-DEE blends. At higher DEE concentrations, they observed an increase in BSFC. This was due to the reduced energy content of the DEE-biodiesel blend, which necessitated more fuel to achieve the required energy output. Additionally, the lower viscosity of DEE negatively impacted fuel atomization in the combustion chamber, reducing combustion efficiency and leading to incomplete combustion at high loads.

Studies have demonstrated that blending DEE with WPPO biodiesel-diesel blends effectively reduces the engine emissions. However, reduced performance and increased NO_x emissions remained a challenge, highlighting the need for further enhancements, such as incorporating nano-additives to mitigate this problem [22]. Kaushik et al. [23] investigated aluminium oxide (Al₂O₃) nanoparticles in diesel-biodiesel blends. They found improved BTH, reduced BSFC, and lowered CO, UBHC and PM emissions. The improved combustion was attributed to the catalytic effects of Al₂O₃. El-Adawy et al. [24] explored the impact of zinc oxide (ZnO) nanoparticles in biodiesel-diesel blends. ZnO improved combustion by increasing the oxygen content, reducing CO and UBHC emissions, but nitrogen oxide emissions were found slightly increased due to higher combustion temperatures. Sathyamurthy et al. [25] examined titanium dioxide (TiO₂) nanoparticles in WPPO-diesel blends. The study reported improved BTE, reduced BSFC and lower CO and UBHC emissions due to the improved combustion. Yusuf et al. [26] explored the effect of biodiesel blends in cerium oxide nanoparticles. It was noted that CeO₂ incorporated blends improve combustion and reduce CO and UBHC emissions. Rastogi et al. [27] studied copper oxide (CuO) nanoparticles in biodiesel-diesel blends. The outcomes revealed that CuO nanoparticles incorporated blends enhance the atomization, resulting in reduced CO, UBHC and smoke emissions, due to higher combustion temperatures. Alex et al. [28] reported the use of CeO₂ in diesel-biodiesel blends, which causes an improvement in BTE and BSFC. The outcomes also showed reduced CO, NO_x and UBHC emissions. Jayaraman et al. [29] investigated the use of magnesium oxide (MgO) nanoparticles in diesel-WPPO blends. MgO improved combustion and emission characteristics, especially in reducing CO and smoke opacity. Gad et al. [30] conducted experiments on a diesel engine fuelled with waste cooking oil with blended graphene nanoparticles and carbon nanotubes (CNTs). The outcomes have shown that the addition of nanoparticles causes an improvement of 19% in BTE and a reduction of 54% in smoke emissions. Kishore et al. [31] studied the role of iron oxide (Fe₂O₃) nanoparticles in improving the combustion of biodiesel blends. They found that Fe₂O₃ reduced CO and UBHC emissions while enhancing BTE. Gavhane et al. [32] used silica (SiO₂) nanoparticles in diesel-biodiesel blends, resulting in reduced CO and PM emissions. SiO₂ also improved fuel atomization, enhancing engine performance, though NO_x levels increased slightly due to higher peak temperatures. Dinesha et al. [33] examined the effect of cerium oxide nanoparticles in WPPO blends. They found that CeO₂ reduces the exhaust emissions, like smoke and CO. Rajammagari and Wani [34] investigated the impact of ZnO nanoparticles incorporated with a diesel-ethanol blend on diesel engine performance. The addition of nano-additives to the ethanol-diesel blend showed improved engine performance, with a 5.73% increase in BTE and 8.31% reduction in BSFC. Emissions of CO, NO_x and UBHC were decreased by 21.96%, 6.41% and 12.07%, respectively.

The synergistic effects of combining WPPO, DEE, and nano-additives in diesel fuel have been less explored, but the potential for enhanced performance and emission control is significant. To the best of the authors' knowledge, no prior study

has examined the combined impact of DEE and nano-additives on diesel engine operation. Therefore, this study focuses on incorporating DEE and nano-additives into diesel-WPPO blends to enhance engine performance and emissions control, representing a novel approach. The studies carried out with four fuel blends representing WPD20, WPD20E5, WPD20E5T50 and WPD20E5T100 are made across four load levels (25% to 100%), and the performance of fuel blends is compared to pure diesel.

2. Materials and methods

2.1. Experimental test setup

In this study, a water-cooled single-cylinder diesel engine with common rail direct injection was utilised. Figure 1 illustrates the setup of the test rig, while Table 1 provides the engine specifications. The load has been applied to the engine using an eddy current dynamometer.

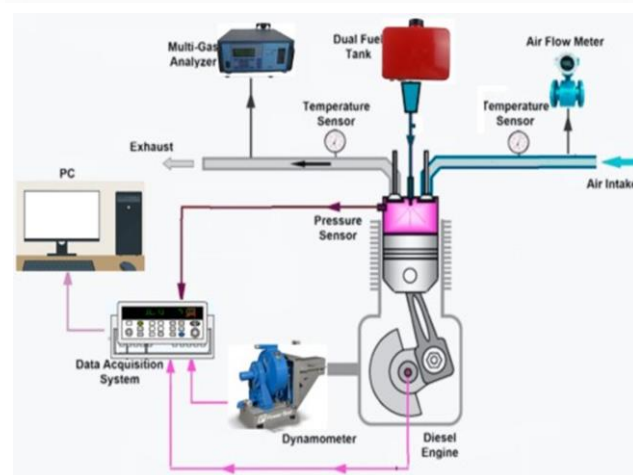


Fig. 1. Layout of the experimental setup.

Table 1. Specifications of diesel engine.

Description	Specifications
Make	Kirloskar
Type	4 stroke single cylinder CRDI diesel engine
Bore × stroke	87.5 × 110 mm
Swept volume	661cc
Rated speed	1500 rpm
Rated power	3.5 kW

Fuel and air flow rates were measured using transmitters, while engine speed was monitored via a proximity sensor. Performance parameters of the engine were recorded and analysed through computerised engine analysis software (Engine Soft). Additionally, exhaust emissions, including NO_x, HC and CO, were evaluated using a multi gas emission analyser to monitor and assess the engine emissions. Smoke emissions were monitored by using an AVL smoke meter.

2.2. Production and preparation of waste plastic pyrolysis oil blends

High-density polyethylene (HDPE) plastic waste, sourced from the Warangal municipality corporation, was chopped into small pieces ranging between 1–4 cm² for this experiment. The plastic chips were thoroughly washed to remove any contaminants, and a dryer was used to eliminate all moisture content. The chips were then gradually fed into the reaction chamber. The pyrolysis process was carried out in a custom-designed reactor, with dimensions of 50 cm in height and 30 cm in diameter, as depicted in Fig. 2. A temperature controller maintained the reaction temperature between 350–400°C, and the process was sustained for 4 hours under atmospheric pressure. The pyrolysis process resulted in a yield comprising 71% plastic oil, 20% solid coke residue, and 9% gaseous fractions by weight. This distribution highlights the efficiency of converting plastic waste into usable fuel, with the majority being recovered as liquid oil, which can be utilised as a fuel source.

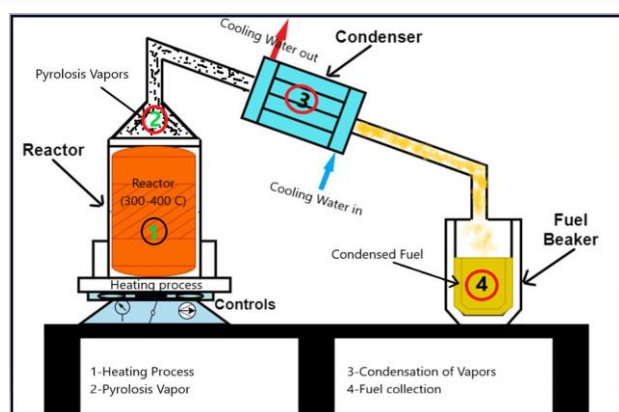


Fig. 2. Production of waste plastic pyrolysis oil.

For fuel blend preparation, WPPO, diesel and DEE were mixed in different proportions. WPPO was mixed with diesel at a ratio of 20% by volume (WPD20), and DEE was added to enhance combustion properties, typically at 5% by volume (WPD20E5). The readily available DEE was purchased from Symax Laboratories located in Hyderabad, Telangana. Nano-additives, such as TiO₂, were added in concentrations of 50 and 100 ppm to further improve fuel properties. The TiO₂ nanoparticles were purchased from Nano Research Lab Jharkhand, and the specifications of TiO₂ nanoparticles are provided in Table 2.

Table 2. Specifications of TiO₂ nanoparticles.

Molecular formula	TiO₂
Appearance	White solid
Molecular weight, g/mol	79.87
Density, g/cm³	4.23
Purity	99.9%
Morphology	Nearly spherical
Titanium content (based on metal)	99.6%
Chemical composition: titanium/oxygen	59.33%/40.55%
Average particle size, nm	25-40
Thermal conductivity, W/(m K)	8.6

Nanoparticles characterization has been done by SEM and XRD analysis to confirm the morphology and purity of the nanoparticles as illustrated in Fig. 3. The diffraction peaks of TiO₂ nanoparticles in Fig. 3 (b) are in line with the previous study conducted by Simhadri et al [35]. To ensure uniform dispersion of nano-additives, the fuel blends were subjected to ultrasonic agitation for 30 minutes, and a surfactant (Span80) was added to stabilise the nano-additives in the fuel. The detailed specifications of the ultrasonicator bath is provided in Table 3.

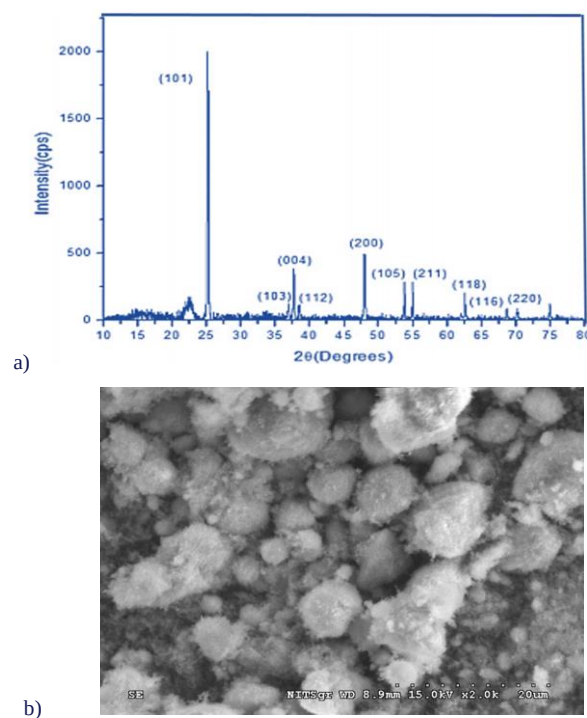


Fig. 3. XRD (a) and SEM (b) of TiO₂ nanoparticles.

Table 3. Specifications of ultrasonicator bath.

Description	Specifications UC-2A
Make	LABMAN
Tank volume (litres)	2.5 L
Ultrasonic wattage	50 W
Ultrasonic frequency	40 ± 3 kHz
Heating	Ambient to 80°C digitally controlled (dual display)
Digital timer	5 to 60 minutes
Power supply	220 volt, Ac 50 Hz, single phase

The fuel preparation flow chart is provided in Fig. 4. The prepared blends are WPD20, WPD20E5, WPD20E5T50 and WPD20E5T100. After the preparation of fuel blends, they have been examined for their physicochemical characteristics, including viscosity, density and calorific value, to ensure compliance with standard fuel characteristics before conducting performance and emission tests on the diesel engine. The fuel properties obtained are presented in Table 4, and the results align with previous studies by Mustayen et al. [36], Pal et al. [37], and Vali et al. [38]. Figure 5 illustrates the final fuel blend of WPD20E5T100.

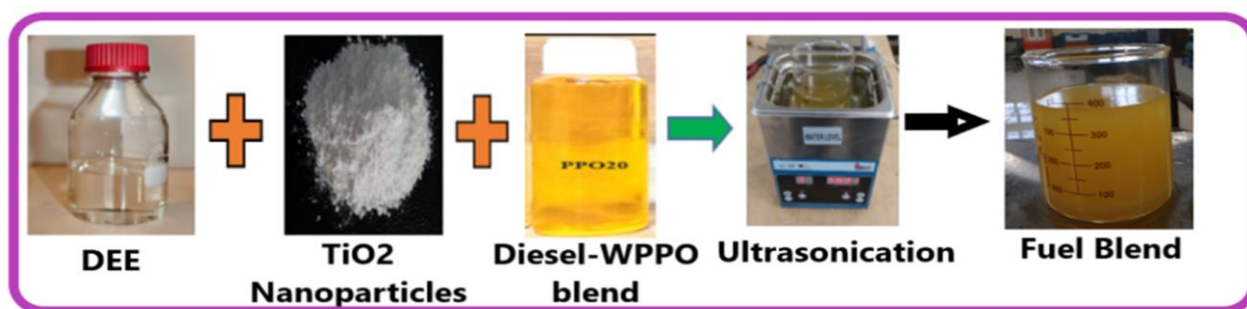


Fig. 4. Fuel preparation flow chart.

Table 4. Properties of fuel blend.

Property	ASTM standards	Diesel fuel range	D	WPD20	WPD20E5	WPD20E5T50	WPD20E5T100
Density at 15°C, kg/m ³	ASTM D4052	820–860	825	817	808	808	809
Viscosity at 40°C, mm ² /s	ASTM D445	2.0–4.5	3.40	3.84	3.61	3.61	3.65
Flash point, K	ASTM D93	315–350	324	320	310	312	315
Calorific value, MJ/kg	ASTM D5865	42–45	43.2	43.8	43.4	43.6	43.9

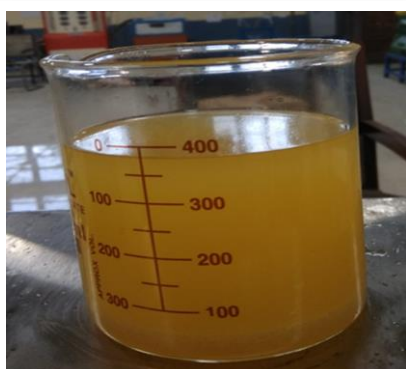


Fig. 5. Final fuel blend (WPD20E5T100).

3. Results and discussion

3.1. Brake thermal efficiency

In Fig. 6, the variation of BTE with different test fuels and various loads is presented. As the engine load increases from partial load to full load, there is a noticeable improvement in BTE across all fuel blends. This increase can be attributed to the fact that higher loads generally lead to increased combustion temperatures and better energy utilisation in the engine. In the case of the WPD20 blend, which combines WPPO with diesel, there is a modest improvement in BTE compared to pure diesel. This suggests that WPPO contributes positively to the combustion process by providing additional fuel energy. However, when 5% DEE is introduced into the blend (WPD20E5), a slight decrease in BTE is observed in contrast to WPD20. This reduction can be attributed to DEE's lower calorific value, which reduces the overall energy content of the blend, resulting in a slight decrease in BTE. Despite this, the addition of TiO₂ nanoparticles to the WPD20E5 blend significantly improves BTE at all loads and

shown higher BTE compared with other fuel blends. A similar trend is also observed in previous research conducted by Vali et al. [38] and Chellaih et al. [39]. The nanoparticles enhance the fuel-air mixing and promote better atomization, leading to more complete combustion and shorter ignition delay. At full load, the WPD20E5T100 blend, with 100 ppm of TiO₂ nanoparticles, achieves a 5.26% and 2.24% improvement in BTE in contrast to neat diesel and WPD20 blend. This highlights the role of TiO₂ nanoparticles in optimizing combustion efficiency and improving overall engine performance, particularly under high-load conditions.

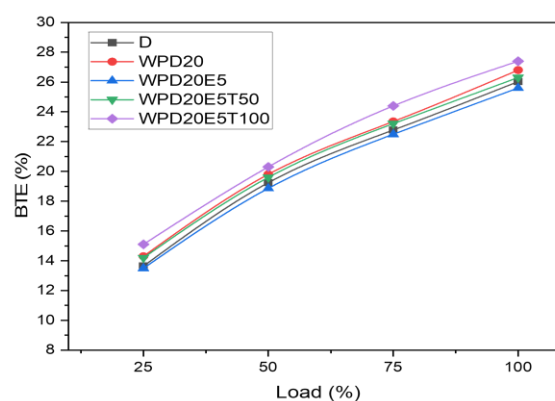


Fig. 6. BTE of various fuel blends under different load conditions.

3.2. Brake specific fuel consumption

Figure 7 illustrates the changes in BSFC for the various test fuels at different engine load levels. The trend in BSFC reveals a consistent pattern of improved fuel efficiency with increasing engine load. As the load moves from partial to full, BSFC decreases across all fuel blends, which indicates that the engine becomes more efficient in fuel utilisation under higher loads.

Specifically, the WPD20 blend shows a modest reduction in BSFC compared to pure diesel. This suggests that incorporating WPPO helps enhance fuel energy content. On the other hand, the introduction of 5% diethyl ether to the WPD20 blend results in a slight increase in BSFC due to DEE's lower energy content, leading to increased consumption of fuel to achieve the same power output. Nevertheless, the presence of TiO_2 nanoparticles in the blends showed a notable reduction in BSFC, which was found minimum compared with other fuel blends at all loads. Similar outcomes are reported by Srinivasarao et al. [40] in their study conducted on a diesel engine using calophyllum biodiesel with ZnO nanoparticles. At the full load, the WPD20E5T100 blend demonstrates the lowest BSFC at 0.287 kg/kWh, marking an 8.31% and 4.65% decrease over neat diesel and WPD20 blend. This significant reduction in BSFC is attributed to the enhanced fuel atomization and combustion efficiency provided by the TiO_2 nanoparticles, which facilitate more complete combustion and optimisation of fuel economy, especially under high-load conditions.

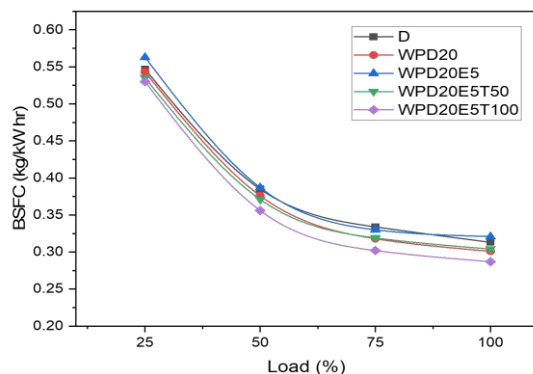


Fig. 7. BSFC of various fuel blends under different load conditions.

3.3. Carbon monoxide emissions

In Fig. 8, the change in CO emissions with different loads for various test fuels is illustrated. The CO emissions data reveal a consistent pattern across different engine loads. As the load increases from partial to full, CO emissions are raised for all

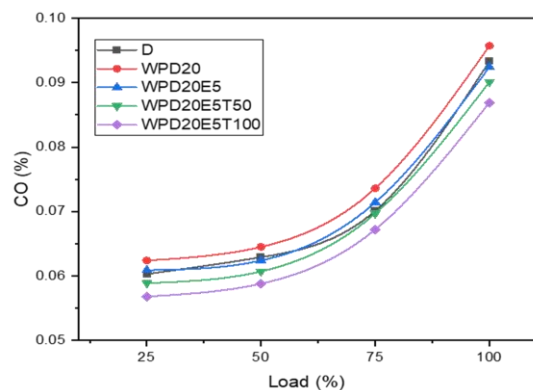


Fig. 8. CO emissions of various fuel blends under different load conditions.

fuel blends. The WPD20 blend, exhibits slightly higher CO emissions than pure diesel, reflecting the influence of WPPO's composition on combustion efficiency. The WPD20E5 blend shows a marginal decrease in CO emissions compared to WPD20, at all loads. This decrease is due to DEE's oxygen content, which may lead to improved combustion. Furthermore, the inclusion of TiO_2 nanoparticles in the WPD20E5T50 and WPD20E5T100 blends significantly reduces CO emissions at all loads compared with other blends. At full load, the WPD20E5T100 blend shows a 9.2% and 6.86% reduction in comparison to WPD20 and pure diesel. This decrease signifies an improved combustion efficiency due to the TiO_2 nanoparticles, which boost fuel atomisation as well as fuel and air mixing, resulting in enhanced combustion and reduced CO emissions. The results demonstrate that TiO_2 nanoparticles are effective in mitigating CO emissions.

3.4. Smoke emissions

Figure 9 illustrates the changes in emissions of smoke for all test fuels at varying load levels.

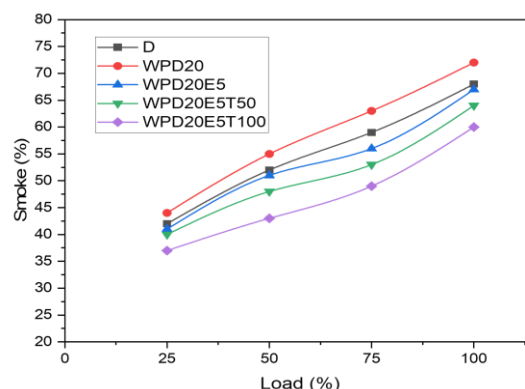


Fig. 9. Smoke emissions of various fuel blends under different load conditions.

The analysis of smoke emissions across varying engine loads shows a distinct pattern. As the load rises, smoke emissions also rise for all fuel blends. The WPD20 blend results in increased smoke emissions in contrast to diesel due to the less efficient combustion characteristics of WPPO. The introduction of 5% diethyl ether in the WPD20E5 blend slightly reduces smoke emissions compared to the WPD20 blend, indicating some improvement in combustion efficiency. However, the decrease in smoke emissions is noted with the addition of TiO_2 nanoparticles to the WPD20E5 blend. It has been observed that the smoke emission is reduced with respect to the concentration of nanoparticles in the WPD20E5 blend. The reduction at full load for the WPD20E5T100 blend records the lowest smoke emissions with an 11.8% and 16.67% reduction over pure diesel and WPD20 blend. It is attributed to better fuel-air mixing, accelerated combustion facilitated by their catalytic properties, and a shorter ignition delay, all contributing to more efficient combustion and lower smoke emissions. Overall, TiO_2 nanoparticles effectively reduce smoke emissions, especially under higher engine loads.

3.5. Unburned hydrocarbon emissions

Figure 10 depicts the changes in UBHC emissions for various fuel blends across different engine load conditions.

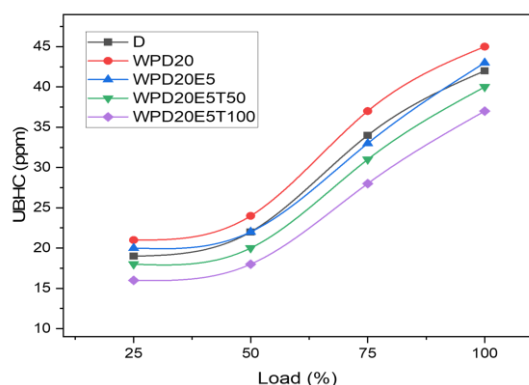


Fig. 10. UBHC emissions of various fuel blends under different load conditions.

The examination of unburned hydrocarbon emissions shows a clear trend as engine load increases. UBHC emissions rise across all fuel blends from partial to full load. The WPD20 blend exhibits increased UBHC emissions when compared to diesel fuel, likely due to the less efficient combustion of WPPPO. The addition of DEE in the WPD20E5 blend displays a slight reduction in UBHC emissions relative to WPD20 blend, due to DEE's role in enhancing combustion. The incorporation of TiO₂ nanoparticles in the WPD20E5 blend significantly lowers UBHC emissions at all loads in comparison with all fuel blends. The WPD20E5T100 blend, in particular, attains the lower UBHC emissions in comparison to the other fuel blends. At full load, WPD20E5T100 blend results in 17.78 and 11.9% lower UBHC emissions in comparison with WPD20 blend and pure diesel. This can be attributed to the enhanced catalytic activity of the nanoparticles and their high surface-to-volume ratio, which improves fuel atomization and combustion efficiency, resulting in reduced hydrocarbon emissions.

3.6. Nitrogen oxide emissions

The evaluation of NO_x emissions across various engine loads shown in Fig. 11 reveals the distinct patterns of different fuel blends. It was noted that NO_x emissions rise with an increase in engine load. The WPD20 blend shows a slight reduction in emissions of NO_x in contrast to diesel, but the emissions remain relatively high. The WPD20E5 blend results in a small rise in NO_x emissions compared to the WPD20 blend. It is attributable to a DEE tendency to raise temperatures during combustion and also to the existence of oxygen in DEE molecular structure, which increases the oxygen availability during combustion, further contributing to higher NO_x formation. It was noticed that the addition of TiO₂ nanoparticles to the WPD20E5 blend lowers NO_x emissions at all loads compared with other fuel blends. The level of reduction in NO_x emissions is inversely proportional to the amount of TiO₂ nanoparticles in the fuel blend.

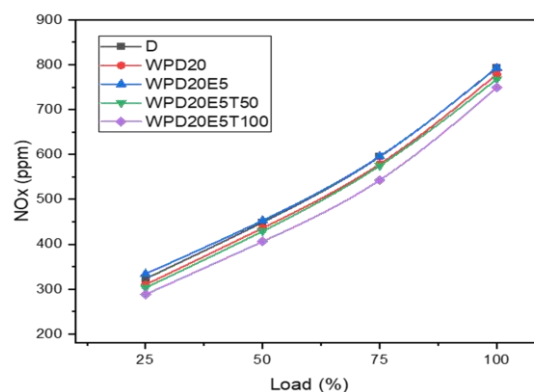


Fig. 11. NO_x emissions of various fuel blends under different load conditions.

In particular, the WPD20E5T100 blend records the lowest NO_x emissions marking a 5.7% and 3.85% reduction over pure diesel and WPD20 blend. The reduction in nitrogen oxide emissions with TiO₂ nanoparticles is due to the enhanced combustion efficiency and improved air-fuel mixing, which help lower the peak combustion temperatures and shorten ignition delay, which typically results in lower NO_x formation.

4. Conclusions

The experimental results highlight the impact of various fuel blends on engine performance and emissions at different loads. Incorporating WPPPO into diesel blends demonstrated improvements in performance whereas an increase in emissions was observed. The addition of DEE and TiO₂ to diesel-WPPPO blends showed an improved performance as well as a reduction in emissions, with the highest efficiency achieved using TiO₂ nanoparticles in the WPD20E5T100 blend. The experimental results revealed the following key findings:

- BTE of the engine was improved by the addition of nanoadditives to fuel blends, and the maximum BTE was found for the WPD20E5T100 blend in comparison with diesel fuel. The percentage of improvement is 5.26% at full load.
- This blend also showed significant reductions in BSFC and smoke emissions, with the notable reduction of up to 8.31% and 11.8%, respectively, compared to pure diesel.
- In terms of CO emissions, the WPD20E5T100 blend showed the most substantial reduction, achieving a 6.86% reduction over neat diesel. Similarly, UBHC emissions were notably lower with the TiO₂ nanoparticle-enhanced blends, with WPD20E5T100 showing a reduction of up to 11.9% compared to diesel.
- However, NO_x emissions were higher with DEE addition and varied across different blends. The WPD20E5T100 blend achieved the lowest NO_x emissions, showing up to a 5.7% reduction compared with diesel fuel at full load.

Overall, the use of TiO₂ nanoparticles in WPPPO-DEE-diesel blends enhanced engine performance, reduced CO, UBHC, NO_x and smoke emissions, and demonstrated significant improvements in BSFC. Overall, the findings suggests that the

WPD20E5T100 blend achieved the optimised results at all loads in comparison with other fuel blends.

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